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MARKET LIQUIDITY AND FUNDING LIQUIDITY: AN EMPIRICAL INVESTIGATION

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Abstract

We provide empirical evidence that the relationship between market and funding liquidity display significant nonlinearities, consistent with theories of market trading with financially-constrained agents. Using data for the US equity market, we uncover nonlinearities that are consistent with a model with two extreme regimes: a lower regime characterized by the absence of correlation between market liquidity and funding liquidity, and an upper regime where the two variables are statistically positively correlated. Over the sample period the two variables are uncorrelated most of the time, since shocks to funding liquidity are economically small. This situation persists until agents are forced towards their capital constraints and shocks to funding liquidity becomes economically important.

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1. Introduction

A number of important theorems in finance rely on the ability of investors to trade any amount of securities without affecting their price. However, there are several frictions, such as trading costs and short sale restrictions, that impact price formation. The influence of market imperfections on security prices has long been recognized. Liquidity, a fundamental concept in finance and economics, by its very nature is difficult to define and even more difficult to estimate. In general it is defined as the ability to buy or sell large quantities of an asset quickly and at a low cost. Although its multidimensionality generates some confusion among market practitioners and policy makers, understanding and measuring liquidity is crucial to understand the workings of exchanges and the quality and efficiency of financial markets.

Kyle (1985) is among the first to note that liquidity relates to a number of transactional properties of the markets and liquidity measures used in empirical studies have spanned direct trading costs, e.g. quoted or effective bid-ask spreads, to indirect trading costs, e.g. price impact. Most of the empirical studies on liquidity have focused primarily on the *cross-sectional* determinants of market liquidity in mature financial markets (Benston and Hagerman, 1974; Stoll, 1978). As more data has recently become available, new studies have explored the *time-series* properties of market liquidity on a variety of financial markets. For example, Chordia *et al.* (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001) document empirical patterns affecting trading activity and market liquidity in equity markets. Chordia *et al.* (2001) study aggregate equity market spreads, depths and trading activity to document the influence of market returns, volatility and interest rates on liquidity. Fleming (1997, 2003), Fleming and Remolona (1999) and Brandt and Kavajecz (2004) among others, analyze market liquidity in the secondary US Treasury bond market.[1]

However, only very recently some studies have emphasized that the usual definitions of (market) liquidity can be restrictive and they do not capture the multidimensional aspects of financial assets trading. In fact, in all financial markets, trading requires capital. When a trader buys a security she can use the security as collateral and then borrow a fraction of its value against it. The difference between the security price and the collateral value must be financed by traders' own capital. When this additional realistic feature of financial markets trading is taken into account then liquidity is not a unique concept but assumes a dual perspective. In fact the ease with which an asset is traded (market liquidity) affects and is affected by the ease with which traders can obtain funds (funding liquidity).

From a theoretical point of view, Shleifer and Vishny (1997) show that investors' outflows from managed funds can amplify financial assets' negative ‘sentiment’ shocks. Geanakoplos (2003) proposes an asset

[1] Only in the past couple of years the literature has started to explore whether existing measures of market liquidity proposed in the extant literature are effectively able to capture current liquidity conditions (see Goyenko *et al.,* 2009 and the references therein).

pricing model where margin constraints are derived endogenously by the requirement that agents must repay their borrowings. Gromb and Vayanos (2002) explicitly incorporate in their model the fact that arbitrageurs face endogenous margin constraints in each market. Arbitrageurs provide liquidity because they absorb demand/supply pressure in each market. However, their ability to provide liquidity depends on their capital. Brunnermeir and Pedersen (2009) further elaborate on the relationship between market liquidity and funding liquidity and show that the two notions are mutually reinforcing and under certain circumstances margins can be destabilizing leading to cases of perverse liquidity spirals. More recently, Gârleanu and Pedersen (2009) explore how funding liquidity conditions affect asset prices. They show that securities with (nearly) identical cash flows but different margin requirements can be traded at different prices. Hence, funding liquidity conditions are of first order importance in explaining deviations from the Law-of-One-Price.

The implications of these recent important theoretical findings have not been fully investigated from an empirical point of view and, to the best of our knowledge, there has not been to date a thorough empirical analysis of the relationship between market and funding liquidity over a long period of time. The few notable exceptions are represented by Chordia *et al.* (2005) who show that the dynamics of market liquidity in the US equity and bond market is caused by exogenous primitive factors and Hendershott *et al*. (2007) who record that market makers' inventories affect the relative liquidity of stock buyers versus sellers in the US equity market. Furthermore, Coffey *et al.* (2009) and Mancini Griffoli and Ranaldo (2009) also investigate how funding liquidity conditions (and the measures put in place by the US Federal Reserve after the collapse of Lehman Brothers) have affected deviations from Covered Interest Parity (CIP) in the major currency markets during the 2008 financial crisis. However, a comprehensive empirical analysis of the dynamics of both market and funding liquidity is awaited.

This paper aims at filling this gap. Our study proposes a new empirical framework able to characterize the relationship between market liquidity and funding liquidity and which allows us to test some of the general predictions implied by recent theoretical models of market trading with financially-constrained agents. Our empirical results, obtained using daily data for the US equity market, are as follows: first, there is strong evidence that the relationship between market liquidity and funding liquidity is characterized by significant nonlinearities. While the existence of nonlinearities in this context is not novel *per se* from a theoretical point of view, our model documents this evidence from an empirical point of view. Second, consistent with the empirical implications of the models proposed by Geanakoplos (2003), Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), which we use to motivate our nonlinear regression, we find that, when shocks to funding liquidity are small, market liquidity conditions are not affected, hence market liquidity shocks are serially uncorrelated and they are not correlated with shocks to funding liquidity. When shocks to funding liquidity are large enough to force agents towards their capital constraints a positive relationship between market and funding liquidity conditions arise.

The rest of the paper is organized as follows. Section 2 provides an outline of the theoretical background and introduces the rationale for a nonlinear empirical framework. Section 3 describes the empirical framework used to analyze the relationship between market and funding liquidity. In Section 4 we report and discuss the empirical results, while Section 5 provides a discussion of the empirical findings. Section 6 concludes.

2. Market Liquidity and Funding Liquidity: The Rationale for a Nonlinear Empirical Framework

The idea that there may be nonlinearities in the market liquidity-funding liquidity relationship is not novel. From a theoretical point of view the studies by Shleifer and Vishny (1997), Geanakoplos (2003), Gromb and Vayanos (2002), Brunnermeir and Pedersen (2009) and Gârleanu and Pedersen (2009) suggest two key empirical implications: first, market liquidity and funding liquidity are mutually reinforcing and second, the effect of funding liquidity conditions on market liquidity is highly nonlinear. More specifically the rationale for a nonlinear relationship between market liquidity and funding liquidity can be appreciated if we consider financial markets with financially-constrained agents. In this context when agents' capital is large, a negative shock to funding liquidity has no effect on market liquidity. However, when capital becomes scarce, negative shocks to funding liquidity induce agents to liquidate their position, thus reducing market liquidity.

In its essence, this argument implies that, within a certain size of shocks to funding liquidity (and, consequently agents' capital), market liquidity and funding liquidity may move in an uncorrelated fashion and this situation can potentially persist for a long period of time. In some sense, this argument suggests the existence of an opportunity 'shadow cost of capital' which creates a band of inaction for the relationship between market liquidity and funding liquidity where the ability to borrow funds to carry out arbitrage strategies is not limited by the existence of increasingly tight margins.[2]

The crucial implication of the above analysis is that when limits to arbitrage to financially-constrained agents are taken into account, market liquidity and funding liquidity need not move together at all times and, indeed, they may even move in opposite directions within a bounded interval without giving rise to any significant statistical relationship. Arguments of this sort may be used to motivate the adoption of threshold-type models of the type originally proposed by Tong (1990) to empirically characterize the relationship between market and funding liquidity: these threshold models would allow for a band within which the relationship between market liquidity and funding liquidity may not be positive and statistically significant, while outside the band the process switches abruptly to become exactly consistent with the

[2] This is also in line with the limits to arbitrage theory of Shleifer and Vishny (1997) where agency frictions in professional money management lead to less aggressive trading than in a frictionless world, so that only limited arbitrage capital is allocated to the trading opportunities with the highest expected returns.

implications of the theoretical models and therefore characterize a strong positive and statistically significant relationship between the two variables.

Nevertheless, while threshold-type models are appealing in this context, various arguments can be made to rationalize multiple-threshold or smooth, rather than single-threshold or discrete, in the nonlinear relationship between market liquidity and funding liquidity at the aggregate or market level. First, the thresholds may be interpreted more broadly to reflect the individual shadow cost of arbitrage which is different for various market participants. Traders or financial institutions would be affected in different ways when a common shock to funding liquidity occurs in the broad market (Brunnermeier and Pedersen, 2009).

Second, one may argue that the assumption of instantaneous trade at the edges of the band of inaction should be replaced with the presumption that it takes some time to observe the realization of a shock to funding liquidity in the broad market and this may affect trade executions. This effect coupled with infrequent trading (Dumas, 1992) and limited participation to security markets imply that agents adjust their portfolios infrequently, with a different subset of agents adjusting in each period. Limited participation by non-financial corporations and unleveraged investors[3] implies that their portfolio shifts will be gradual, rather than abrupt (Lyons, 2001).

Overall, the arguments discussed above suggest funding liquidity constraints create a band within which market liquidity and funding liquidity may be unrelated or even move in opposite directions. Once funding liquidity shocks are large enough to make funding liquidity constraints bind, market liquidity becomes increasingly positively correlated with funding liquidity with the size of funding liquidity shocks. Hence, smooth rather than discrete adjustment may be appropriate in the present context, and time aggregation and nonsynchronous adjustment by heterogeneous agents are likely to result in smooth aggregate regime switching. This is indeed the kind of behavior we shall try to capture in our empirical framework, as discussed in the next section.

3. The Empirical Framework

A characterization of nonlinear relationship between market liquidity and funding liquidity is in terms of a smooth transition regression (STR) model (Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998). An STR model of market and funding liquidity may be written as follows:

$$\widehat{ML}_t = \alpha_1 + \left[\alpha_2 + \beta \widehat{FL}_t\right] \Phi\left(\widehat{FL}_t - \mu, \gamma\right) + \varepsilon_t \qquad (1)$$

3 For example, investors like mutual funds, pension funds or insurance companies, who do not have a comparative advantage over proprietary bank traders in implementing strategies heavily based on borrowed funds.

where $\widehat{ML}_t$ and $\widehat{FL}_t$ denote innovations to market and funding liquidity respectively, and ε_t is a disturbance term. The transition function $\Phi(\widehat{FL}_t - \mu, \gamma)$ determines the extent to which innovations in funding liquidity affect innovations in market liquidity is itself governed by the parameter γ, which effectively determines the magnitude of the impact of funding liquidity to market liquidity, and the transition variable. The transition variable $\widehat{FL}_t - \mu$ is assumed to be the difference between the innovation in funding liquidity at time t and a given threshold μ.

A simple transition function suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998) is the logistic function:

$$\Phi(\widehat{FL}_t - \mu, \gamma) = \frac{1}{1 + \exp\left[-\gamma\left(\widehat{FL}_t - \mu\right)\right]} \tag{2}$$

in which case (2) would be termed a logistic STR or LSTR model. The logistic transition function Φ is bounded between zero and unity, i.e. $\Phi: \Re \to [0,1]$, has the properties $\Phi[0] = 0$ and $\lim_{x \to \pm\infty} \Phi[x] = 1$, and is s-shaped. These properties of the LSTR model are particularly attractive in the present context because they allow a smooth transition between regimes and asymmetric impact of large shocks to funding liquidity on market liquidity, consistent with arguments put forward in the extant theoretical literature.

The transition parameter γ determines the speed of transition between the two extreme regimes, with lower absolute values of γ implying slower transition. The "lower" and "upper" regimes are defined as the regimes corresponding to the two extreme values of the transition function, where $\Phi(\cdot) = 0$ and $\Phi(\cdot) = 1$ respectively. Investigation of the properties of the model in these two extreme regimes sheds light on the stability and dynamic properties of the STR model. The arguments in the spirit of limits to arbitrage with financially-constrained agents suggest that the lower regime corresponds to $\widehat{FL}_t - \mu = 0$, where $\Phi(\cdot) = 0$ and Equation (1) becomes of the form:

$$\widehat{ML}_t = \alpha_1 + \varepsilon_{t+1}$$

The upper regime corresponds, for a given γ, to $\lim_{(\widehat{FL}_t - \mu) \to \pm\infty} \Phi(\widehat{FL}_t - \mu, \gamma)$, where equation (1) becomes linear regression where market liquidity is a function of funding liquidity via the parameter $\beta > 0$:

$$\widehat{ML}_t = (\alpha_1 + \alpha_2) + \beta\widehat{FL}_t + \varepsilon_{t+1}$$

This formulation has several virtues. First, the model nests the condition under which market liquidity innovations are serially uncorrelated, to which it would collapse in the absence of nonlinearity. Second, this specification captures the behavior of the relationship between market and funding liquidity which is implied by the theoretical considerations discussed in Section 2. Funding liquidity shocks may be small and persistent, however when capital is large, these have no effect on market liquidity. However, when capital becomes scarce, negative shocks to funding liquidity induce agents to liquidate their position, thus reducing market liquidity.

In short, the model allows for a significant positive relationship between market and funding liquidity shocks at all points where the transition function $\Phi(\cdot) = 1$. Hence, there is only one value of the transition function and one set of parameter restrictions that allow us to conclude that funding liquidity shocks significantly affect market liquidity shocks.

It is worth noting that Granger and Teräsvirta (1993) and Teräsvirta (1994) also suggest the exponential function as a plausible transition function for some applications, resulting in an exponential STR (ESTR) model, which implies symmetric behavior of the transition variable $\widehat{FL}_t - \mu$ no matter whether they are positive or negative. However, this case is not plausible in our context since only positive shocks to funding liquidity conditions are relevant and likely to affect market liquidity conditions. We do test for nonlinearities arising from the ESTR formulation as a test of specification of the estimated models in the section discussing the empirical analysis. Also, as a preliminary to our estimation of a nonlinear regression, we evaluate the adequateness of the linear regression by performing tests of linearity against nonlinearity of the smooth-transition type (for both ESTR and LSTR formulations) and by testing the hypothesis of symmetry directly.

It is important to emphasize that, while the empirical framework discussed above is inspired by the growing theoretical literature on the relationship between market and funding liquidity, we do not claim to provide a direct test of the models proposed by Gromb and Vayanos (2002), Geanakoplos (2003) and Brunnermeier and Pedersen (2009), but rather a test of their general predictions for the relationship between market and funding liquidity.

4. Empirical Results

4.1 Data, Summary Statistics

Our data set comprises daily observations of market liquidity and funding liquidity for the US equity market. The sample period spans from January 2, 1971 to December 31, 2005 because of data availability.

We measure market (il-)liquidity by the effective cost of trading proposed by Roll (1984) and estimated using the Gibbs sampler estimator proposed by Hasbrouck (2009). Roll (1984) suggests a model of security prices in a market with transaction costs. In this framework the efficient security price moves according to a random walk with serially uncorrelated public information shocks and the trade price at any time is a function of the efficient price plus (or minus) the effective cost. The model implies that an estimate of the effective cost is provided by $c = \sqrt{-Cov(\Delta p_t, \Delta p_{t-1})}$ where c is the estimated effective cost, Δp is the daily change in log-security prices and $Cov(\cdot)$ denotes the covariance operator. Hasbrouck (2009) introduces a Gibbs sampler estimator of the effective cost of trading assuming that the public information shock is normally distributed with zero mean and a given variance. The Gibbs sampler is used to jointly estimate the parameter models, the trade indicator and the efficient price. This measure is used since it has been found to be highly positively correlated with transaction-level data and it allows the analysis to be carried out over longer periods of time.[4]

The funding (il-)liquidity measure used in the empirical analysis is computed as short-term interest rate spread between collateralized and uncollateralized loans (Gârleanu and Pedersen, 2009). More specifically we use two proxies for the TED spread (Brunnermeier, 2009): the difference between the 3-month LIBOR and the 3-month T-bill rate and the difference between the 6-month LIBOR and the 6-month T-bill rate. All rates have been retrieved from *Reuters Datastream*. We have used this data since they exhibit the largest sample coverage. Alternative measures, such as LIBOR/GC-REPO spreads, would be only available from the mid-1990s.

From this data set, we construct the time series of interest, namely the level of market liquidity and funding liquidity, ML_t and FL_t respectively, both at the daily frequency.

In Table 1 Panel A), we report sample moments for the levels of the time series of interest. The summary statistics show that the selected market liquidity and funding liquidity measures have small and positive

4 Further technical details on the Gibbs sampler estimator can be found in Hasbrouck (2009). The time series of the effective cost of trading can be downloaded from Joel Hasbrouck's website on http://pages.stern.nyu.edu/126jhasbrou/Research/

mean with a large standard deviation. However, the first-order autocorrelation coefficient of all series is higher than 0.6 with values above 0.95 for the funding liquidity measures. These results are consistent with the stylized facts that the daily measures of market and funding liquidity are highly persistent processes.[5] These facts can be visualized in Figure 1 where the time series of both market and funding liquidity are plotted.

In order to remove the serial correlation of the market and funding liquidity time series we estimate an AR(5) model for all series and use the resulting residuals as zero-mean serially uncorrelated innovations (or shocks). The summary statistics of the innovation series are reported in Table 1, Panel B). In all cases the first-order autocorrelation coefficient is not different from zero at the conventional statistical level.

4.2 The Nonlinear Regression

In order to evaluate the validity of the assumption of linearity in the relationship between market and funding liquidity, we performed tests of linearity against the alternative of smooth transition nonlinearity, using the deviation of funding liquidity shocks from the 90th percentile of its distribution as the transition variable.[6] We followed the Teräsvirta (1994, 1998) decision rule to select the most adequate transition function for modelling nonlinearity in the present context. This is a testing procedure designed to test the hypothesis of linearity and to select the most adequate nonlinear function between a logistic and an exponential function. As shown by the results in Table 2, the general linearity test F_L strongly rejects the null hypothesis of linearity. Employing the Teräsvirta rule to discriminate between exponential and logistic formulations led us to conclude that a logistic function (LSTR model) is the most adequate parametric formulation (given that F_1, F_3 yield the lowest *p*-value). This finding is consistent with our priors discussed in Section 2.

Given the results from the linearity tests, we estimate the nonlinear regression (2) by nonlinear least squares under the restrictions $\alpha_1 = -\alpha_2$ (which we test formally below). In estimation, we followed the recommendation of Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998) of standardizing the transition variable $\widehat{FL}_t - \mu$ by dividing it by the sample standard deviation of the transition variable, $\hat{\sigma}_{\widehat{FL}_t}$, and using a starting value of 0.5 for the estimation algorithm.

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5 Asymptotic standard errors were calculated using an autocorrelation and heteroskedasticity consistent matrix of residuals throughout the paper (Newey and West, 1987). We also tested for unit root behavior of the time series examined by calculating several unit root test statistics. We were in each case unable to reject the stationarity at conventional nominal levels of significance confirming the fact that daily measures of market and funding liquidity are stationary but highly persistent processes.

6 We have also experimented with different threshold levels (75th, 95th and 99th percentiles) and the results are qualitatively and quantitatively similar to the ones reported in Table 2.

The results, reported in Table 3, Panel A) indicate that the relationship between market liquidity and funding liquidity is indeed highly nonlinear. The estimated transition parameter appears to be significantly different from zero, in each equation, both on the basis of the individual asymptotic standard errors as well as on the basis of Skalin's (1998) parametric bootstrap likelihood ratio test (see the *p*-values in square brackets in the last column of Panel A of Table 3).[7]

The estimates of the slope parameter β is correctly signed according to our priors based on the empirical implications of the theoretical studies, namely we find a positive and statistically significant estimated value of β when the transition function $\Phi(\cdot) = 1.$ In turn, these values imply that, for small shocks to funding liquidity (the transition variable), there is no relationship between market liquidity and funding liquidity, while for increasingly large shocks to funding liquidity (which are likely to make agents' capital constraints bind), the relationship between market liquidity and funding liquidity is positive and statistically significant.

These findings also imply that, since the relationship occurs rapidly for large shocks to funding liquidity, the bulk of the observations of market liquidity shocks is in the lower regime, generating unconditional absence of correlation between market liquidity and funding liquidity shocks. We shall return to the analysis of the distribution of market and funding liquidity shocks in the next section.

The estimated transition parameters also imply well-defined transition functions, as shown in Figure 2, which displays the plots of the estimated transition functions, $\Phi(\cdot)$, against the transition variable $\left(\widehat{FL}_t - \mu\right)/\sigma_{\widehat{FL}_t}$. The speed of the transition functions is made clear by the evidence that the limiting case of $\Phi(\cdot) = 1$ is attained for all proxies of funding liquidity, which is impressive given that we are dealing with daily data. The transition functions also confirm how most of the observations of market liquidity shocks are in the lower regime.

A battery of diagnostic tests is reported in Panel B) of Table 3. We report a likelihood ratio test ($LR1$) for the null hypothesis that $\alpha_1 = -\alpha_2$ and the results suggest that for both specifications $\widehat{FL}_t = \widehat{FL}_t^{3M}, \widehat{FL}_t^{6M}$ we could not reject the validity of these restrictions at the five percent significance level. As discussed in Section 3, these restrictions imply a condition in which market liquidity shocks are uncorrelated with funding liquidity and being equal to zero on average. For each of the estimated nonlinear regressions, we then tested the null hypothesis of no remaining nonlinearity (F_{NRN}), constructed as in Eithreim and Teräsvirta's (1996) and reported in the second column of Panel B. The null hypothesis of no remaining nonlinearity could not be rejected for any of the estimated models,

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7 Because the Skalin test of the null hypothesis that $\gamma = 0$ in the transition function may also be construed as a test of nonlinearity, these results confirm the presence of nonlinearity in the regression examined.

indicating that our parsimonious regression appears to capture satisfactorily the nonlinearity in the market liquidity-funding liquidity relationship. We also tested for the stability of the model by constructing the appropriate test proposed by Eithreim and Teräsvirta's (1996) for each nonlinear model. This is a test for the hypothesis of no structural break in the parameters (F_{NSB}) specifically designed for smooth transition models. The results, reported in the last column of Panel B, suggest no structural break in the parameters of the model, with *p*-values reasonably larger than the conventional five percent.

Overall, the nonlinear estimation results uncover strong evidence of nonlinearities in the relationship between market liquidity and funding liquidity, with market liquidity shocks increasingly positively correlated with funding liquidity shocks at a speed which depends upon the size of the funding liquidity shocks. The estimated models are in each case consistent with the priors established by existing theoretical studies on market trading with financially-constrained agents. It is worth emphasizing that this model does not imply that the relationship between market liquidity and funding liquidity holds all the time. On the contrary, given that the bulk of the observations lies in the lower regime, market liquidity shocks are *not* explained by funding liquidity shocks most of the time. The model implies that market liquidity and funding liquidity do not correlate when shocks to arbitrage capital are economically small enough to be ignored by agents who are not close to their funding constraints.

5. Discussion

Our empirical results provide clear evidence that the relationship between market liquidity and funding liquidity is characterized by important nonlinearities. This empirical result is novel and the proposed nonlinear model may be used to understand the properties of the dynamic relationship between market liquidity and funding liquidity. Our nonlinear regression was rationalized on the basis of the argument that the existence of limits to arbitrage due to funding constraints can allow market and funding liquidity to be uncorrelated within a certain range. According to existing theoretical studies, for small shocks to funding liquidity, market liquidity shocks are small and they are not affected by funding liquidity conditions. However, as funding liquidity shocks become larger, financially constrained arbitrageurs will face tighter borrowing conditions which will make funding constraints bind. This will induce market liquidity shocks to be highly positively correlated with funding liquidity shocks. Our nonlinear model parsimoniously captures this behavior and our estimation results uncover robust evidence that the US equity market has behaved in this fashion over the 1971-2005 sample period.

One aspect of the rationale behind this model is, therefore, that market participants face tighter trading conditions on the basis of the size of funding liquidity conditions and that this process induces the nonlinear dynamics we observe in the data. In Table 4, we report the average standardized funding liquidity shocks (first column), calculated as the average realized funding liquidity shocks divided by their standard deviation over the sample period. These shocks are by definition equal to zero on average and with a standard deviation of one.

Given that the transition function we estimate is bounded between zero and unity and may be viewed as the probability of being in one of the two extreme regimes (one regime with uncorrelated market and funding liquidity, and another regime where market and funding liquidity co-move positively), it is instructive to graph the estimated transition functions over time. In Figure 3, we plot the estimated transition function over the sample. The plots make clear how the model implies that the market and funding liquidity regressions are in the lower regime (defined, for simplicity, as the case where $\Phi(\cdot) \leq 0.5$) most of the time. The lower regime is the one characterized by serially uncorrelated market liquidity shocks which are not correlated with funding liquidity conditions, which, however, is associated with low and economically unimportant shocks to arbitrage capital. In some sense, therefore, these findings suggest that this regime does characterize the majority of the observations in the data, but only those observations where financial institutions' arbitrage capital is unlikely to be affected by funding constraints.

On the other hand, although fewer observations are in the upper regime (say when $\Phi(\cdot) > 0.5$), in this regime market liquidity shocks are highly positively correlated with funding liquidity shocks, suggesting that arbitrage capital affects trading conditions especially when arbitrageurs are close to their funding constraints.

It is instructive to calculate the size of the standardized funding liquidity shocks such that transition function $\Phi(\cdot) = 0.5$, which we term the minimum $\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$ ($min\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$) such that one may be in the upper regime. The calculations are given, for each of the two specifications, in the middle column of Table 4. The range of the minimum level required goes from about 2.25 to about 4.08. Indeed, this evidence seems consistent with the argument made in theoretical studies indicating that only large shocks to funding liquidity (larger than 2.25 times the standard deviation of funding liquidity shocks) affect arbitrage capital so that further funding liquidity shocks are likely to affect market liquidity conditions.

These shocks can hardly be achieved throughout the sample period, as illustrated in the last column of Table 4, which reports that more than 98 percent of the observations are below the minimum standardized funding liquidity shock we calculate.

An important caveat is, however, that while our empirical framework is inspired by the growing theoretical literature on the relationship between market and funding liquidity, we do not claim to provide a direct test of the models proposed by Gromb and Vayanos (2002), Geanakoplos (2003) and Brunnermeier and Pedersen (2009), but rather a test of their general predictions for the relationship between market and funding liquidity. We leave to future research the design of a framework where researchers can formally discriminate among different theories or frictions that predict the existence of nonlinearities in market liquidity/funding liquidity regressions.

6. Conclusions

Our empirical results provide confirmation that, for the US equity market over the past thirty years, market liquidity shocks are linked nonlinearly to funding liquidity shocks. The nonlinearities we uncover are consistent with a model with two extreme regimes: a lower regime characterized by the absence of correlation between market liquidity and funding liquidity, and an upper regime where the two variables are statistically positively correlated. This evidence is consistent with recent theoretical contributions on market trading with financially-constrained agents.

Although the results have been shown to be robust to a number of tests, several caveats are in order. While our empirical analysis is inspired by theoretical studies on market trading with financially-constrained agents, we do not claim that this paper provides a direct test of the specific hypotheses, but rather a test of their general predictions for the relationship between market liquidity and funding liquidity. Our approach is best interpreted as an empirical characterization of the dynamic relationship between market and funding liquidity motivated by theoretical contributions on market trading with financially-constrained agents or simply as an empirical investigation of a parsimonious model of market liquidity dynamics. In particular, although we have focused on a specific nonlinear formulation of the relationship between market liquidity and funding liquidity capable of capturing some of the key predictions of the theoretical models, experimentation with alternative nonlinear characterizations of the relationship is on the agenda for future research both to assess the robustness of our results and to further tighten the link between theory and empirical testing in a way that can allow us to discriminate among different arguments capable of rationalizing the existence of nonlinearities. Experimentation with multivariate versions of the nonlinear models used here that allow estimation of a general nonlinear model linking the joint dynamics of market liquidity and funding liquidity is also a promising area of research. Finally, our analysis has been confined to the US equity market, given our intention to shed light on the relationship between market and funding liquidity in a large equity market where the literature has established a large set of empirical stylized facts. However, future research may extend our framework to multiple markets in order to test the robustness of the current findings and record a new set of stylized facts for a variety of internationally integrated capital markets.

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Table 1. Summary Statistics

Panel A) ML_t is the effective cost of trading as in Hasbrouck (2009) and FL_t^{3M} and FL_t^{6M} are the 3-month and 6-month TED spreads computed as described in the main text. *Panel B)* $\widehat{ML}_t$ and $\widehat{FL}_t^{3M}$, $\widehat{FL}_t^{6M}$ are the residuals obtained from estimating a AR(5) model, with an intercept term, on the individual series ML_t, FL_t^{3M} and FL_t^{6M} respectively. Data to compute TED spreads are taken from *Reuters Datastream*. The sample period spans from January 2nd, 1971 to December 31st, 2005. Figures in parentheses are standard errors calculated by using an autocorrelation and heteroskedasticity consistent matrix of residuals, with three lags (Newey and West, 1987).

Panel A) Levels

	mean		standard deviation		AR(1)	
ML_t	0.425	(0.004)	0.329	(0.009)	0.645	(0.016)
FL_t^{3M}	1.045	(0.010)	2.012	(0.042)	0.980	(0.005)
FL_t^{6M}	1.057	(0.009)	1.951	(0.036)	0.984	(0.003)

Panel B) Innovations

	mean		standard deviation		AR(1)	
$\widehat{ML}_t$	–	–	0.077	(0.004)	>0.001	(0.002)
$\widehat{FL}_t^{3M}$	–	–	0.032	(0.003)	>0.001	(0.002)
$\widehat{FL}_t^{6M}$	–	–	0.023	(0.001)	>0.001	(0.002)

Table 2. Linearity Tests on the Linear Regression

The table reports the *p*-values from applying the linearity testing procedure suggested by Teräsvirta (1994, 1998). Assuming that a plausible, generic transition variable is q_t, the appropriate auxiliary regression for the linearity tests against a STR alternative is the following: $\hat{e}_t = \boldsymbol{\vartheta}_0'\mathbf{A}_t + \boldsymbol{\vartheta}_1'\mathbf{A}_t(q_t) + \boldsymbol{\vartheta}_2'\mathbf{A}_t(q_t)^2 + \boldsymbol{\vartheta}_3'\mathbf{A}_t(q_t)^3 + \xi_t$, where $\hat{e}_t$ is the estimated disturbance retrieved from the linear model being tested for linearity (in the present context it is the residual from the regression $\widehat{ML}_t = \delta_0 + \delta_1\widehat{FL}_t + e_t$), and $\mathbf{A}_t$ denotes the vector of explanatory variables in the model being tested, which in our case simply amounts to $\widehat{FL}_t^i$, $i = 3M, 6M$. The transition variable, q_t used in our tests is he deviation of funding liquidity shocks from from the 90th percentile of its distribution. The general test for linearity against STR is then the ordinary *F*-test of the null hypothesis: $H_{0L}: \boldsymbol{\vartheta}_1' = \boldsymbol{\vartheta}_2' = \boldsymbol{\vartheta}_3' = \mathbf{0}$. The choice between a LSTR and an ESTR model is based on a sequence of nested tests within H_{0L}. First, the null hypothesis H_{0L} must be rejected using an ordinary *F*-test (F_L). Then the following three hypotheses are tested sequentially: $H_{03}: \boldsymbol{\vartheta}_3' = \mathbf{0}$; $H_{02}: \boldsymbol{\vartheta}_2' | \boldsymbol{\vartheta}_3' = \mathbf{0}$; $H_{01}: \boldsymbol{\vartheta}_1' | \boldsymbol{\vartheta}_2' = \boldsymbol{\vartheta}_3' = \mathbf{0}$. Again, an *F*-test is used, with the corresponding test statistics denoted F_3, F_2, and F_1, respectively. The decision rule is as follows: if the test of H_{02} has the smallest *p*-value, an ESTR is chosen, otherwise an LSTR is selected. The *p*-values were calculated using the appropriate F distribution.

	F_L	F_3	F_2	F_1
$FL_t = FL_t^{3M}$	>0.01	>0.01	0.07	>0.01
$FL_t = FL_t^{6M}$	>0.01	>0.01	0.44	>0.01

Table 3. Nonlinear Regression: LSTR Estimation Results

Panel A) The table shows the results from the nonlinear regression $\widehat{ML}_t = \alpha_1 + [\alpha_2 + \beta\widehat{FL}_t]\Phi(\widehat{FL}_t - \mu,\gamma) + \varepsilon_t$ where $\alpha_1 = -\alpha_2$ and $\Phi(\widehat{FL}_t - \mu,\gamma) = \frac{1}{1+\exp\left[-\gamma\left(\frac{\widehat{FL}_t-\mu}{\sigma_{\widehat{FL}_t}}\right)\right]}$. Values in parentheses (SE_x) are asymptotic standard errors for the parameter x, calculated using an autocorrelation and heteroskedasticity consistent matrix of residuals (Newey and West, 1987). Values in brackets are *p*-values for the null hypothesis that $\gamma = 0$, calculated by the parametric bootstrap procedure suggested by Skalin (1998) using 5,000 replications. *Panel B).* $LR1$ is the likelihood ratio test for the null hypothesis that $\alpha_1 = -\alpha_2$. F_{NRN} is the test for the null hypothesis of no remaining nonlinearity, constructed as in Eitrheim and Teräsvirta (1996). F_{NSB} is a test for the null hypothesis of no structural break in the model's parameters, constructed as in Eithreim and Teräsvirta's (1996). For all test statistics, we report *p*-values, calculated using an autocorrelation and heteroskedasticity consistent matrix of residuals.

Panel A) Parameter estimates

	$\alpha_1 = -\alpha_2$	$\mathrm{SE}_{\alpha_1=-\alpha_2}$	β	SE_β	μ	SE_μ
$FL_t = FL_t^{3M}$	0.0043	(0.001)	0.2808	(0.06)	0.488	(0.09)
$FL_t = FL_t^{6M}$	0.0061	(0.002)	0.2341	(0.05)	0.203	(0.08)

	γ	SE_γ
$FL_t = FL_t^{3M}$	0.240	(0.044) [0.032]
$FL_t = FL_t^{6M}$	0.223	(0.029) [0.019]

Panel B) Diagnostic tests

	$LR1$	F_{NRN}	F_{NSB}
$FL_t = FL_t^{3M}$	0.42	0.25	0.37
$FL_t = FL_t^{6M}$	0.38	0.22	0.44

Table 4. Standardized Funding Liquidity Shocks

The first column of the table reports the mean of the standardized funding liquidity shocks calculated as liquidity shocks ($\widehat{FL}_t$) divided by the standard deviations of the same time series ($\sigma_{\widehat{FL}_t}$) over the sample period. $min\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$ denotes the minimum value of the standardized funding liquidity shock which leads to a shift from the lower regime to the upper regime, defined here as the value of the transition function $\Phi(\cdot) \equiv \frac{1}{1+\exp\left[-\gamma\left(\frac{\widehat{FL}_t-\mu}{\sigma_{\widehat{FL}_t}}\right)\right]} = 0.5.$ The last column reports the percentage of observations where the standardized funding liquidity shock is lower than or equal to $min\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$.The last row reports averages across specifications for each column.

	$\widehat{FL}_t/\sigma_{\widehat{FL}_t}$	$min\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$	%Obs $\widehat{FL}_t/\sigma_{\widehat{FL}_t} \leq min\left(\widehat{FL}_t/\sigma_{\widehat{FL}_t}\right)$
$\widehat{FL}_t = \widehat{FL}_t^{3M}$	>0.01	4.08	0.99
$\widehat{FL}_t = \widehat{FL}_t^{6M}$	>0.01	2.45	0.98
Average		*3.27*	*0.98*

Figure 1. Liquidity Measures

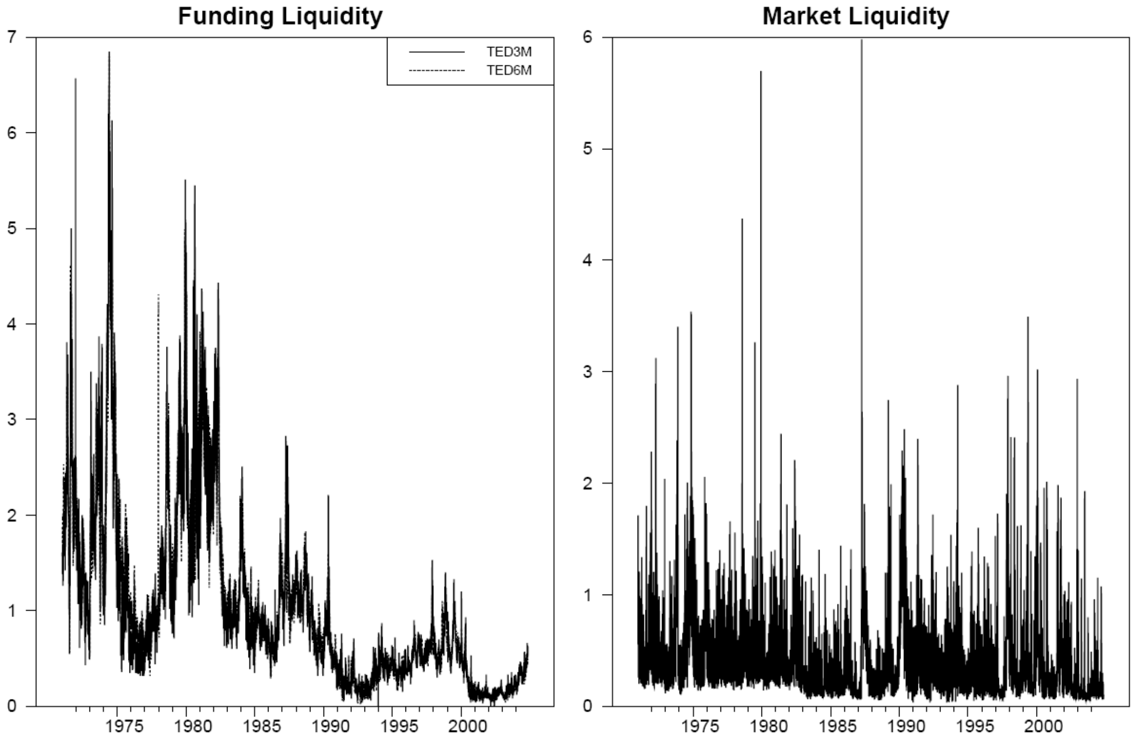

Figure 2. Estimated Transition Functions

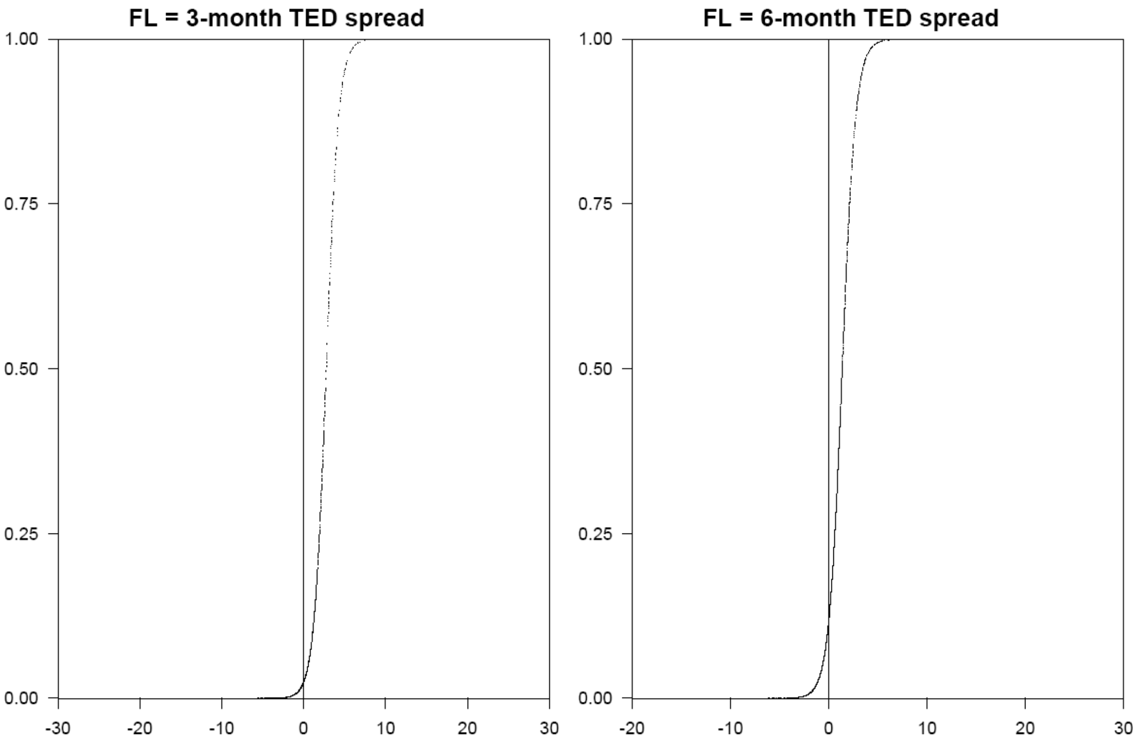

Figure 3. Funding Liquidity Shocks and Transition Functions