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HOUSE PRICE, MORTGAGE PREMIUM, AND BUSINESS FLUCTUATIONS

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House Price, Mortgage Premium, and Business Fluctuations

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Abstract

This paper investigates the transmission mechanism of mortgage premium to characterize the relationship between the housing market and the business cycle for the U.S. economy. The model matches the main features of the U.S. housing market and business cycles well. The mortgage premium is crucial for the amplification and propagation of the model to match the data. If the Federal Reserve had exercised pre-emptive monetary policy in 2002Q1, the counterfactual analysis suggests that a higher interest rate would have stabilized house price and housing investment volatilities, but would have taken a big toll on real GDP: its volatility remains approximately the same, but the level of GDP contracts dramatically.

Keywords: Mortgage Premium, House Price, DSGE

JEL Classification: E3, E4, E5, G1

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1. Introduction

The 2007-2008 subprime crisis has raised concerns among academics and policy makers regarding the nature of house price boom in early the 2000s and the subsequent dramatic bust. The boom-bust cycles of asset prices have attracted considerable attention in recent years because large fluctuations in asset prices were found to exert substantial real effects on economic activity. For example, house price or housing wealth may affect consumption expenditures by way of wealth effect, liquidity effect, or expectations (Case *et al.* (2005), Iacoviello (2004), Mullbauer (2007), Aoki *et al.* (2004), Calomiris *et al.* (2009)); affect capital investment and housing investment via collateral constraints (Kiyotaki and Moore (1997), Davis and Heathcote (2005), Iacoviello (2006), Ortalo-Magne and Rady (1998, 1999, 2006)); or act as a transmission channel for monetary policy and other exogenous shocks (Mishkin (2001, 2007)). Moreover, Borio and Lowe (2002), Cecchetti *et al.* (2000), Detken and Smets (2003), and Mishkin (2008) argue that large swings in asset prices may cause financial instability, leading to systemic risk. Recent studies on the subprime crisis also find that change in house price is the single best indicator for predicting delinquency rate on mortgages and the subsequent decline of the housing market (Doms, *et al.* (2007), Mian and Sufi (2008), Dell'Ariccia *et al.* (2009)).

Given that the housing market is closely connected to economic activity, the purpose of this paper is twofold. First, using a dynamic stochastic general equilibrium (DSGE) model, we characterize the relationship between the housing market and business cycle regularities for the U.S. economy. To address this issue, we modify the models by Aoki *et al.* (2004) and Bernanke *et al.* (1999) to focus on financial frictions in the housing market. The main transmission mechanism of the model is the external finance premium (EFP) on housing, or specifically the mortgage premium, which is contingent on the ratio of borrower's net worth to housing investment and measures the severity of credit market friction. Calibrated to the U.S. economy, the model is used to investigate properties of the U.S. housing market and business cycles. Second, following the argument of Taylor (2007, 2009) and Ahrend *et al.* (2008), that the deviation of federal funds rates from the Taylor rule during 2002-2004 substantially boosted the U.S. house prices before the subprime crisis,[1] we conduct a counterfactual analysis to examine how the U.S. housing market and economic activity might have reacted had the Federal Reserve raised the interest rate beginning in 2002Q1.

The main findings are as follows. The model is able to capture the relationship between the U.S. housing market and business cycles: the relative standard deviation of residential investment is much larger than non-residential investment, the correlation between residential investment and GDP is smaller than that between non-residential investment and GDP, the volatility of house price is larger than that of GDP, and the mortgage premium is pro-cyclical. We also find that the mortgage premium is crucial for the

[1] The Federal Reserve cut the target federal funds rate from 6% in early 2001 to merely 1% in mid-2003 and maintained the target rate at 1% for another 4 quarters before reverting its downward trend.

amplification and propagation of the model to match the data of the U.S. housing market and business cycles. Counterfactual analysis shows that if the Federal Reserve had raised the interest rate in $2002Q1$, the pre-emptive monetary policy would have stabilized house price and housing investment volatilities, but at the cost of substantially suppressing housing investment. In particular, the tightening monetary policy aimed to contain the housing market boom takes a big toll on real GDP: its volatility remains approximately the same, but the level of GDP contracts dramatically.

A substantial literature has demonstrated that imperfect capital market plays an important role in the propagation of shocks over the business cycles (Bernanke and Gertler (1995), Bernanke *et al.* (1999), Hubbard (1998)). As demonstrated by Bernanke and Gertler (1989) and Bernanke *et al.* (1999), imperfection in credit markets that drives a wedge between the cost of external and internal funds -- the EFP -- amplifies the magnitude and persistence of business cycle fluctuations. Theoretical works along this line include Carlstrom and Fuerst (1997) and Gomes *et al.* (2003), who explore the implications of changes in EFP for stock returns (induced by movements in the price of capital goods) and how changes in the value of financial assets amplify business fluctuations. De Graeve (2008) compares the EFP implied by a DSGE model to various financial market interest rate spreads, such as corporate bond yield spreads (Baa-Aaa or BBB-AAA), prime loan spreads (prime loan rate minus federal funds rate), and high-yield bond spreads. However, few existing works have explored the implications of the EFP specifically for the housing market. The volatility of residential investment is typically larger than non-residential investment (Davis (2010)); therefore, the propagation effect of EFP may play a more important role in the housing market than in the capital market.

The proposed model is built on Bernanke *et al.* (1999) and Aoki *et al.* (2004). However, there are notable differences. To focus on measuring the mortgage premium, this paper abstracts from financial frictions in the corporate sector. We apply credit market frictions in the corporate sector to the housing sector where house prices, housing investment, and credit constraints interact in a general equilibrium framework. Aoki *et al.* (2004) modify Bernanke *et al.* (1999) by adding the housing production sector and allowing heterogeneous households. However, their purpose is to study the effect of the financial accelerator on consumption through development of mortgage credit markets, such as mortgage equity withdrawal (MEW). The current paper focusses on quantifying the propagation effects of mortgage premium and compare them with U.S. business cycle regularities.[2]

The remainder of this paper is organized as follows. Section 2 describes the environment of the model. Section 3 linearizes and calibrates the model to the U.S. economy. Section 4 simulates the model and

[2] Other related papers are as follows. Jin *et al.* (2010) construct a Bernanke *et al.* (1999) type model with both residential and commercial real estate sectors. However, financial friction works via the commercial real estate sector. Iacoviello (2005) and Iacoviello and Neri (2010) also construct DSGE models with both residential and commercial real estate sectors. However, the credit constraints in these models are based on imperfect contract enforceability as in Kiyotaki and Moore (1997).

discusses its properties in matching the relationship between the housing market and business cycles. Section 5 conducts counterfactual analysis. Finally, section 6 concludes.

2. The Model

The study considers a model with discrete time and an economy populated with infinitely-lived households, firms, intermediaries, and a central bank. The household sector is considered a composite homeowners and consumers. This greatly simplifies the analysis by separating consumption/savings and housing investment decisions, without losing the financial accelerator mechanism.[3] Homeowners are the only group of agents subject to financial frictions. The production sector includes three types of firms: house producers, intermediate goods producers, and retailers.

2.1 Household Sector

Similar to Aoki *et al.* (2004), we separate each household into two behavior types of agents: homeowners who buy housing goods, and consumers who rent housing goods. Homeowners borrow funds to purchase houses from housing producers and rent them to consumers. Rental payments by consumers are captured as imputed rent. Homeowners borrowing from financial intermediaries to finance house purchases face an external finance premium caused by information asymmetries. A transfer payment to consumers further links consumers and homeowners. Homeowners can alternatively be regarded as firms owned by households that rent houses to the household sector.

2.1.1 Consumers

Consumers rent housing services from homeowners and receive a transfer from homeowners each period. The transfer links consumers and homeowners in the household sector and, more importantly, prevents homeowners from accumulating large amounts of wealth, so that they will no longer be credit constrained.

Consumers maximize the following expected lifetime utility function,

$$\max E_0 \sum \beta^t (\ln C_t^h + j_t \ln H_t^h - \frac{\left(L_t^h\right)^\eta}{\eta} + \vartheta \ln \frac{M_t}{P_t}),$$

[3] Our focus is not to investigate the effect of house price on consumption. Separating consumers and homeowners simplifies house purchase decisions by abstracting the EFP determination from risk sharing. This disentangles the borrowing constraint's effect on housing investment from consumption.

where E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, C_t^h is the amount of consumption at t, H_t^h denotes the demand of housing services at t, L_t^h is labor hours and M_t/P_t is the real money balance held by consumers.[4]

At the beginning of each period, a consumer holds cash M_{t-1} carried from the last period, receives the amount $R_{t-1}^n D_{t-1}$ from intermediaries, earns labor income $W_t L_t^h$, and receives lump-sum transfers $P_t F_t^r$ from retailers, $P_t F_t^h$ from homeowners, and $P_t T_t$ from the government. The consumer allocates these funds to consumption $P_t C_t^h$, housing rental $X_t^h H_t^h$, deposits to the intermediary D_t, and money holding for the next period M_t, where R_t^n is the gross riskless rate of interest, P_t is the nominal price level, and X_t^h is the gross nominal rental rate on housing.

Denoting $x_t^h \equiv X_t^h/P_t$ to be the real housing rent rate, $d_t \equiv D_t/P_t$ the deposits in real terms, $w_t \equiv W_t/P_t$ the real wage, and $\pi_t \equiv P_t/P_{t-1}$ the gross inflation rate, the budget constraint can be expressed as follows:

$$C_t^h + x_t^h H_t^h + \frac{M_t - M_{t-1}}{P_t} = R_{t-1}^n d_{t-1}/\pi_t - d_t + w_t L_t^h + F_t^r + F_t^h + T_t.$$

Consumers do not face borrowing constraints, therefore, they can borrow and lend freely among themselves or directly from financial intermediaries. It follows that consumers's marginal rate of substitution can be used to define the (implicit) riskless rate for this economy:

$$R_t^n = \frac{1}{E_t\left[M_{t+1}\right]},$$

where $M_{t+1} = \beta U_c(t+1)/[U_c(t)\pi_{t+1}]$ is the intertemporal marginal rate of substitution. Given our specification of the utility function, the steady state riskless rate is given by $R^n = 1/\beta$.

[4] Note that we abstract from the liquidity services of real money balances to simplify the exposition. This model can be viewed as a money-in-utility function model, in which household utility function includes a money balance term, $\vartheta M_t/P_t$ and we take ϑ to be arbitrarily small. A justification is that an economy with cashless limit in principle provides a good approximation to the behavior of an economy with a very small fraction of monetary transactions (Woodford (1998)). For example, if financial innovation carries out transactions with sufficiently small cash balances, then fluctuations in money demand have only negligible effects upon the equilibrium price level under a Wicksellian policy regime.

2.1.2 Homeowners and the Financial Contracting with Intermediaries

Homeowners borrow funds to purchase houses from house producers and rent them to consumers. Homeowners are subject to credit constraints due to information asymmetries and have to pay a premium for external financing, i.e., a mortgage premium. Homeowners purchase houses H_{t+1}^{h} (carried over to time t+1) at a price Q_t from house producers at the end of period t and rent to consumers at price X_{t+1}^{h} at period $t+1$. At the end of period t (going into period $t+1$), a homeowner has available net worth in real terms N_{t+1}. To finance the difference between expenditure on housing and net worth, the homeowner borrows from intermediaries. In real terms, the house financing is given by

$$q_t H_{t+1}^{h} = N_{t+1} + b_{t+1}^{h},$$

where $q_t \equiv Q_t / P_t$ is the real house price.

We assume that the return on housing is subject to both idiosyncratic and aggregate risk. The *ex post* gross return per unit of housing is ωR_t^h, where R_t^h is the ex post aggregate real rate of return on housing and ω is an idiosyncratic disturbance across homeowners and over time with a continuous and continuously differentiable c.d.f., $F(\omega)$, over a non-negative support and $E(\omega)=1$. To model the agency problem in credit markets, we follow the "costly state verification" (CSV) literature by assuming that only homeowners observe the realization of ω. it is costly for intermediary to audit (ex post) a borrower's return on housing. The costs tend to be interpreted as bankruptcy costs.

Intermediaries allocate consumer savings to finance the housing purchases of homeowners and investment of intermediate goods producers. The intermediary sector is assumed to be competitive. By funding a large number of homeowners and intermediate goods producers, intermediaries are able to diversify idiosyncratic risk. Given that homeowners are risk neutral and consumers are risk averse, the loan contract between an intermediary and the homeowners will have homeowners bear all the aggregate risk.

Note that the realized return at period t+1 for a typical homeowner is given by $\omega R_{t+1}^{h} q_t H_{t+1}^{h}$, and pre-determined debt is $R_{t+1}^{L} b_{t+1}^{h}$, where R_{t+1}^{L} is the gross real loan interest rate. Following Bernanke *et al.* (1999), the optimal contracting problem at the end of period t between the homeowner and the intermediary is to specify a cutoff value $\overline{\omega}_t$ such that the intermediary receives $\overline{\omega}_t R_{t+1}^{h} q_t H_{t+1}^{h}$ if $\omega \geq \overline{\omega}_t$; and the intermediary receives $(1-m)\omega R_{t+1}^{h} q_t H_{t+1}^{h}$ in residual claim net of bankruptcy costs

$m\omega R^h_{t+1} q_t H^h_{t+1}$ if $\omega < \overline{\omega}_t$. In the latter case, the homeowner defaults and hands over all the housing investment returns. The cutoff value $\overline{\omega}_t$ is therefore implied by

$$\overline{\omega}_t R^h_{t+1} q_t H^h_{t+1} = R^L_{t+1}(q_t H^h_{t+1} - N_{t+1}). \tag{1}$$

Competition among risk-neutral financial intermediaries ensures that the expected return on each loan contract is equal to the opportunity cost of funds:

$$\left(1 - F(\overline{\omega}_t)\right) R^L_{t+1}(q_t H^h_{t+1} - N_{t+1}) + \left(1 - m\right)\int_0^{\overline{\omega}_t} \omega R^h_{t+1} q_t H^h_{t+1} dF(\omega) = R_{t+1}(q_t H^h_{t+1} - N_{t+1}), \tag{2}$$

where $R_t \equiv R^n_t / \pi_t$, is the gross real riskless interest rate. On the left hand side the first term is the expected repayment to the intermediary under the non-default state and the second term is the expected repayment net of bankruptcy costs in the default state. Together with (1), by eliminating R^L_{t+1}, we can express the expected return of an intermediary as a function of the cutoff value $\overline{\omega}_t$,[5]

$$[\Gamma(\overline{\omega}_t) - mG(\overline{\omega}_t)] R^h_{t+1} q_t H^h_{t+1} = R_{t+1}(q_t H^h_{t+1} - N_{t+1}), \tag{3}$$

where $\Gamma(\overline{\omega}_t) \equiv \left(1 - F(\overline{\omega}_t)\right)\overline{\omega}_t + \int_0^{\overline{\omega}_t} \omega dF(\omega)$ is the expected gross share of profits going to the lender, $mG(\overline{\omega}_t) = m\int_0^{\overline{\omega}_t} \omega dF(\omega)$ is the expected monitoring cost.

The equation (3) implies a set of restrictions, one for each realization of R^h_{t+1}. This says that, with aggregate risk, the cutoff value $\overline{\omega}_t$ depends on the ex post realization of R^h_{t+1}. Note that a rise in $\overline{\omega}_t$ increases the payment in non-default state, but simultaneously raises the probability of default. Thus, equation (3) suggests that if the realization of homeowner's return R^h_{t+1} is lower, the non-default payment R^L_{t+1} and the cutoff value $\overline{\omega}_t$ will be higher, resulting in a higher probability of default ($F(\overline{\omega}_t)$). This implies a counter-cyclical probability of default.

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5 We impose the assumption that the hazard rate $h(\omega) \equiv dF(\omega) / \left(1 - F(\omega)\right)$ increases in ω, which is a relatively weak restriction for most distributions. Following Bernanke *et al.* (1999), this assumption ensures that expected return to the intermediary, the left hand side of (3), reaches a maximum at a unique interior value of $\overline{\omega}$. For the values of $\overline{\omega}$ above the maximum, expected return to the intermediary decreases, due to a higher probability of default. For the values of $\overline{\omega}$ below the maximum, the expected return is an increasing function and is concave.

Given the state-contingent values of $\overline{\omega}_t$, the optimal contracting problem can be written as

$$\max[1-\Gamma(\overline{\omega}_t)]R^h_{t+1}q_t H^h_{t+1}, \tag{4}$$

subject to the intermediary's participation constraint (3).

Let $s_t \equiv E_t(R^h_{t+1}/R_{t+1})$ be the expected discounted return to housing investment. The first order necessary condition yields the following optimal purchase for housing,

$$q_t H^h_{t+1} = \varphi(s_t)N_{t+1}, \text{ with } \varphi(1) = 1, \varphi' > 0, \tag{5}$$

that the optimal purchase decision of housing depends on homeowner's net worth and the wedge between expected return on housing and riskless rate. Given the value of H^h_{t+1} that satisfies (5), the schedule for $\overline{\omega}_t$ is uniquely determined by the intermediary's expected return in (3).

We also express (5) as

$$E_t(R^h_{t+1}) = \Omega(\phi_t)R_{t+1}, \Omega'(\cdot) < 0, \tag{6}$$

where

$$\phi_t \equiv \frac{N_{t+1}}{q_t H^h_{t+1}},$$

the ratio of homeowner's net worth to housing expenditure, or the inverse of the leverage ratio.

Equation (6) relates the gross marginal expected return of the homeowner to the overall gross marginal cost of funds, where the latter is the product of the *mortgage premium* $\Omega(\phi_t)$ times the gross riskless rate R_{t+1} . The premium $\Omega(\phi_t)$ depends inversely on the ratio of homeowner's net worth to housing expenditure, in other words, an increasing function of the leverage ratio. This also demonstrates the importance of house price in driving the dynamics of mortgage premium through changes in the leverage ratio. The specific form of the function $\Omega(\cdot)$ depends on the primitive parameters of the costly state verification problem, such as the proportional bankruptcy cost m.[6]

Because homeowners are risk neutral, their demand for houses depends on the expected return on housing and expected marginal finance cost. One unit of housing purchased at time t and rented at time $t+1$ for a typical homeowner yields the expected gross rate of return on housing

$$E_t(R_{t+1}^h) \equiv E_t\left[\frac{x_{t+1}^h + (1-\delta_h)q_{t+1}}{q_t}\right], \tag{7}$$

where $0 < \delta_h < 1$ is the depreciation rate of houses. Together with (6), we have homeowner's optimal demand for housing.

Finally, we describe the evolution of homeowners' aggregate net worth over time. Due to risk neutrality and constant-returns-to-scale technology, the demand for housing is linear in homeowners' net worth, which facilitates aggregation. After the realization of return on housing, the aggregate value of homeowners in the middle of period t is given by

$$V_t = R_t^h q_{t-1} H_t^h - \Omega(\phi_{t-1}) R_t (q_{t-1} H_t^h - N_t).$$

We assume that the population of homeowners is stationary. To ensure that bankrupt homeowners are able to start over, we assume that homeowners are endowed with L_t^o unit of labor, supplied inelastically to intermediate goods firms, and that they receive a small wage income $w_t L_t^o$ at the end of each period. Thus, the aggregate net worth of homeowners at the end of period t, after paying the transfer F_t^h to consumers, is given by[7]

$$N_{t+1} = V_t + w_t L_t^o - F_t^h,$$

Note that the realized return on housing R_t^h depends on the price of houses at time t, q_t, thus the

6 This leads to a similar acceleration mechanism as in Kiyotaki and Moore (1997), where borrowing constraints are directly tied to borrowers' collateralizable assets due to imperfect enforceability. The difference is that the EFP on housing in our model is also linked to the default rate of borrowers.

7 In our quantitative analysis below, the wage income $w_t L_t^o$ is of negligible size, and has no effect on the dynamics of net worth. The transfer rule is $F_t^h = \chi(N_t / q_{t-1} H_t^h), \ \chi' > 0$.

mortgage premium and the price of houses both determine the level of homeowners' net worth, which in turn feedbacks to the premium and house price. Furthermore, since the debts are in nominal terms, an unexpected deflation, for example, will erode homeowners' net worth, which raises the mortgage premium and affects economic aggregates through the financial accelerator.

2.2 Production of Houses

The house production sector uses linear technology by transforming I_t^h units of consumption goods into the same units of new houses. New houses are sold to homeowners and intermediate goods producers at the end of period t at price q_t. The maximization problem of a house producer is

$$\max q_t I_t^h - \Phi(I_t^h/H_{t-1})H_{t-1},$$

where $\Phi(\cdot)$ is a convex adjustment cost of producing new houses. In equilibrium, the q-theory of investment on housing implies that the house price is given by

$$q_t = \Phi'(I_t^h/H_{t-1}). \tag{8}$$

Thus, the stock of houses in the economy evolves according to

$$H_t = I_t^h + (1-\delta_h)H_{t-1}. \tag{9}$$

2.3 Intermediate Goods Producers and Retailers

Suppose there are intermediate goods producers who employ capital goods, housing services, and labor to produce intermediate goods, and retailers who combine varieties of intermediate goods to produce final goods. These producers do not face borrowing constraints.

Let the intermediate-goods producers maximize the following lifetime utility function

$$E_0\left(\sum_{t=0}^{\infty}\gamma^t \ln C_t^k\right),$$

where $\gamma \in (0,1)$ is the discount factor, and C_t^k is the amount of consumption at t. We assume that intermediate-goods producers have access to constant returns to scale technology, hiring K_{t-1} units of capital goods, L_t units of labor from the household sector, and commercial real estate services H_{t-1}^k to produce Y_t units of intermediate goods:[8]

$$Y_t = A_t K_{t-1}^{\mu} (H_{t-1}^k)^{\upsilon} L_t^{(1-\mu-\upsilon)}, \tag{10}$$

where A_t is an aggregate productivity shock, and $0 < \mu, \upsilon < 1$. The intermediate goods producers consume, purchase houses for production, repay debts, pay labor wages, and invest in capital goods. Their sources of funds include sales of intermediate goods to retailers and borrowing from intermediaries. The flow of funds constraint is given by

$$C_t^k + q_t(H_t^k - (1-\delta_h)H_{t-1}^k) + R_{t-1}^n b_{t-1}^k / \pi_t + w_t L_t + I_t^k + \xi_t^k + \xi_t^h + F_t^r = \frac{Y_t}{X_t} + b_t^k.$$

Intermediate goods producers sell to retailers at the wholesale price P_t^w and then these intermediate goods are transformed into composite final goods, priced at P_t. We denote $X_t = P_t / P_t^w$ to be the markup of final goods over intermediate goods. $R_{t-1}^n b_{t-1}^k / \pi_t$ is the amount of debt in real terms, implying that debts are set in nominal terms. Finally, there is an adjustment cost for the stock of capital goods and housing,

$$\xi_t^k = \frac{\psi_k}{2\delta_k}\left(\frac{I_t^k}{K_{t-1}} - \delta_k\right)^2 K_{t-1}, \ \xi_t^h = \frac{\psi_h}{2\delta_h}\left(\frac{I_t^{kh}}{H_{t-1}^k} - \delta_h\right)^2 H_{t-1}^k,$$

where $I_t^{kh} = H_t^k - (1-\delta_h)H_{t-1}^k$, $\psi_k / 2\delta_k$ and $\psi_h / 2\delta_h$ respectively measure the marginal adjustment cost for capital and housing, and the depreciation rates for capital and housing satisfies $0 < \delta_k, \delta_h < 1$. The stock of capital goods evolves according to

$$K_t = I_t^k + (1-\delta_k)K_{t-1}.$$

We next turn to retailers. Let there be a continuum of retailers indexed by $i \in [0,1]$. They purchase intermediate goods Y_t from intermediate-goods producers at a price P_t^w in a competitive market,

8 The commercial real estate markets are by no means free from agency problems. For many cases, commercial real estates are much riskier than residential real estates. Here we abstract from agency problems in commercial real estate markets and focus on the implications of financial frictions in residential housing markets. By so doing, the house purchasing decisions of homeowners and their returns entirely come from the provision of home services.

transform them into differentiated goods $Y_t(i)$, and sell $Y_t(i)$ at the price $P_t(i)$. We assume that each period the price can be adjusted with a probability $1-\theta$. The optimal $P_t^*(i)$ solves:

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\Lambda_{t,k}\left(\frac{P_t^*(i)}{P_{t+k}}-\frac{X}{X_{t+k}}\right)Y_{t+k}^*(i)\right\}=0,$$

where $Y_{t+k}^*(i)=(P_t^*(i)/P_{t+k})^{-\varepsilon}Y_{t+k}^f$, $\Lambda_{t,k}=\beta(C_t^h/C_{t+k}^h)$ is the discount rate of consumers, and X is the steady state value of the markup X_t. Therefore, retailer profits are given by $F_t^r=(1-1/X_t)Y_t^f$ which are distributed to consumers.

Finally, the differentiated goods are costlessly transformed to final goods according to the following CES technology

$$Y_t^f=(\int Y_t(i)^{(\varepsilon-1)/\varepsilon}di)^{\varepsilon/(\varepsilon-1)},$$

where $\varepsilon>1$ is the elasticity of the demand of the differentiated goods. In symmetrical equilibrium, $Y_t(i)=Y_t$ implies that $Y_t^f=(\int Y_t(i)^{(\varepsilon-1)/\varepsilon}di)^{\varepsilon/(\varepsilon-1)}=Y_t$. Given the aggregate final goods production function, the aggregate price level evolves according to

$$P_t=(\theta P_{t-1}^{\varepsilon}+(1-\theta)\left(P_t^*\right)^{1-\varepsilon})^{1/(1-\varepsilon)}.$$

Log-linearizing these equations leads to the following New Keynesian Phillips curve

$$\hat{\pi}_t=\beta E_t\hat{\pi}_{t+1}+\frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{X}_t+\hat{u}_t, \tag{11}$$

where variables with hats are log deviations from steady-state values, and $\hat{u}_t$ is a cost push chock which follows the autoregressive process,

$$\ln u_t=\rho_u\ln u_{t-1}+\varepsilon^u.$$

2.4 The Central Bank

The central bank sets the interest rate according to the general Taylor rule:

$$\ln(\frac{R_t^n}{R^n}) = \alpha_\pi \ln(\frac{\pi_t}{\pi}) + \alpha_Y \ln(\frac{Y_t^f}{Y^f}) + \ln(v_t), \quad (12)$$

where α_π and α_Y are the corresponding coefficients for inflation gap and output gap; R^n, π, and Y^f are the steady states of R_t^n, π_t, and Y_t^f respectively; and v_t is a monetary shock which follows the autoregressive process,

$$\ln v_t = \rho_v \ln v_{t-1} + \varepsilon^v.$$

Finally, the government budget constraint is given by

$$\frac{M_t - M_{t-1}}{P_t} = T_t.$$

2.5 Equilibrium

There are four markets in this economy. The housing market clearing condition requires that $H_t^k + H_t^h = H_t$. The labor market clears when the demand from intermediate goods producers equals the supply from consumers, $L_t = L_t^h + L_t^o$. Final goods market equilibrium requires that the sum of the consumption of consumers and intermediate goods producers, investment for home production and capital goods, adjustment costs, and monitoring costs equal the aggregate output, $Y_t^f = I_t^k + I_t^h + C_t^k + C_t^h + \xi_t^k + \xi_t^h + m\int_0^{\overline{\omega}_t} \omega dF(\omega) R_t^h q_{t-1} H_t^h$. Finally, the credit market clears by the Walras Law.

The equilibrium consists of an allocation $\{Y_t^f, I_t^k, I_t^h, K_t, H_t^k, H_t^h, C_t^k, C_t^h, b_t^k, b_t^h, L_t^h, L_t^o, M_t, D_t, F_t^r, F_t^h, \overline{\omega}_t\}$ and a sequence of prices and co-state variables $\{R_t^h, R_t^n, P_t, X_t^h, Q_t, W_t\}$ that satisfy the optimality conditions of consumers, homeowners, house producers, intermediate goods producers, and retailers. Finally, all markets clear.

3. Linearization and Parameter Values

A system of difference equations derived from the first order conditions, market-clearing conditions, the laws of motion for four exogenous shocks, and the policy rule of the central bank, characterize equilibrium prices and quantities of the model economy. We log-linearize the Euler equations and market clearing conditions around the non-stochastic steady state and rewrite all variables as percentage deviations from the steady state. The appendix lists the linearized system of the model. Here we discuss some parameters in the linearized equations that are of particular interest to our purpose. The variables without time subscripts denote their steady state values.

We first consider the linearization of the mortgage premium. Log-linearizing (6), we have

$$E_t \hat{R}^h_{t+1} = \hat{R}_{t+1} - \Theta\left\{\hat{N}_{t+1} - (\hat{q}_t + \hat{H}^h_{t+1})\right\},$$

where $\Theta = -\phi\,\Omega^{'}(\phi)/\Omega(\phi) > 0,\ \ \phi = N/qH^h \leq 1$. By definition, the linearized mortgage premium is given by $\hat{s}_t = E_t\hat{R}^h_{t+1} - \hat{R}_{t+1}$, and thus we have the linearized mortgage premium,

$$\hat{s}_t = -\Theta\left\{\hat{N}_{t+1} - (\hat{q}_t + \hat{H}^h_{t+1})\right\}.$$

The parameter Θ measures the elasticity (in absolute value) of the mortgage premium to variations in the net worth of homeowners relative to housing expenditure. The higher the net worth of the homeowner in the investment project, the lower the associated agency cost is. Empirical or theoretical studies give no immediately available value for this parameter. We experiment with several measures and therefore set $\Theta = 0.25$. We set the steady state ratio of homeowner's net worth to housing expenditure to be $\phi = 0.7$, following Aoki *et al.* (2004). The steady state mortgage premium s is taken to be the average value of historical mortgage risk premium.

The historical average spread between the one-year adjustable-rate mortgage (ARM) rate and the one-year Treasury bond rate during 1984Q1-2008Q4 is 1.12%. Therefore, we set the steady state of mortgage premium $s = 0.0112$. The data of these mortgage rates and Treasury bond yields are respectively from the Federal Home Loan Mortgage Corporation (Freddie Mac) and the Board of Governors of the Federal Reserve.

The log-linearization of (8) yields

$$\hat{q}_t = \Psi(\hat{I}_t^h - \hat{H}_t),$$

where $\Psi = \left(\Phi(I^h/H)\right)' / \left(\Phi(I^h/H)\right)''$ is the elasticity of house price to the housing investment ratio. Bernanke *et al.* (1999) suggest that a reasonable range for the value of the corresponding parameter in their model (for physical capital) is 0 and 0.5. Here we set $\Psi = 0.17$.

As the appendix shows, we express the state-contingent cutoff $\hat{\varpi}_t$ as a function of the linearized mortgage premium

$$\hat{\varpi}_t = \left[\frac{\Omega(\overline{\varpi})'}{\Omega(\overline{\varpi})}\overline{\varpi}\right]^{-1}\hat{s}_t,$$

and thus the linearized default rate $F(\varpi_t)$ is given by

$$\hat{f}_t = \frac{F(\overline{\varpi})'}{F(\overline{\varpi})}\overline{\varpi}\hat{\varpi}_t = \frac{F(\overline{\varpi})'}{F(\overline{\varpi})}\left[\frac{\Omega(\overline{\varpi})'}{\Omega(\overline{\varpi})}\right]^{-1}\hat{s}_t,$$

where $F(\overline{\varpi})'/F(\overline{\varpi})\left[\Omega(\overline{\varpi})'/\Omega(\overline{\varpi})\right]^{-1}$ measures the percentage change in the default rate with respect to a percentage change in mortgage premium. For the value of steady state default rate, the historical foreclosure rates across all types of mortgages from the Mortgage Bankers Association (MBA) is 1.15%. Thus, we set the steady state default rate $f = 0.0115$.[9]

We set the remaining parameters values following previous works. The discount rate of households β equals 0.99, while the discount rate of intermediate-goods producers γ is set to be 0.98, satisfying $\beta > \gamma$, which guarantees that intermediate goods producers are borrowers. Labor supply elasticity η is set to 1.01, as in Iacoviello (2005).

We set $\theta = 0.75$ such that the average length of price adjustments for intermediate goods is four quarters. The depreciation rate for capital goods δ_k is 0.03, and that for housing δ_h is 0.005 as in Iacoviello (2004). For the share of capital and real estate in the production function of intermediate goods,

9 Empirically, there are various reasons for mortgage defaults, including imperfect information, (community) contagion effect, adverse selection, etc. (among others, see Foote *et al.* (2008), Harding *et al.* (2009), Haughwout *et al.* (2008)).

i.e., μ and υ, we follow Iacoviello (2004) by setting them to 0.3 and 0.03, respectively. The parameter relating to the marginal adjustment cost for capital, ψ_k, is set to 0.1 to match the relative deviation of capital investment to aggregate output, which is 3.29. As for parameters in the Taylor rule, we adopt the values, $\alpha_\pi = 1.5$ and $\alpha_Y = 0.2$, following Iacoviello (2005).

There are four shocks in our model: monetary shock, inflation shocks, preference shock, and productivity shock, denoted (v_t, u_t, j_t, A_t). The steady state values of u_t, v_t, and A_t are set to be unity, while that of j_t is set to be 0.5 to match the observation that the steady state value of entrepreneurial real estate holding H_t^k accounts for approximately 10% of the total entrepreneurial asset.

The autocorrelation of monetary and preference shocks are set to be $\rho_u = 0.59$ and $\rho_j = 0.85$, as in Iacoviello (2005). The autocorrelation of monetary shock, $\rho_v = 0.32$, is taken from the average value of estimates by Ireland (2003). The autocorrelation of productivity shock is set to be $\rho_A = 0.9$, as in most works along this line of research. Among the standard deviations of these four shocks, σ_u and σ_v are normalized to be unity, and then we adjust σ_A and σ_j to match the relative deviation of housing investment to aggregate output, which is 6.28.

4. Properties of the Model

This section investigates the goodness-of-fit of our model in matching the regularities of U.S. business cycles.

In Figure 1 the solid lines represent impulse responses of the model with mortgage premium, while the dashed lines represent those without. When the effect of mortgage premium is not present, homeowners' net worth and the house price are less sensitive to changes in exogenous shocks, thus weakening the transmission mechanism of the model.

When mortgage premium is at work, the last column of Figure 1 shows that the house price declines in response to increased interest rate and a positive shock to inflation, and rises in response to a higher preference to housing and a positive technology shock. To see transmission mechanism of the model, consider the effects of a tightening monetary policy (a rise in v_t). A higher interest rate lowers house prices, contributing to a substantial decline in homeowners' net worth, and leading to a higher mortgage premium and default rate. These effects then feedback to house price. A positive shock to housing demand raises house prices, which strengthens homeowners' net worth, resulting in a lower mortgage premium and default rate.

Table 1 lists volatilities and correlations of some key aggregate variables generated from our model, together with the de-trended data documented by Davis (2010). The behaviors of house prices and housing investments over business cycles generate several interesting observations. First, both residential investment (I_t^h) and house prices are more volatile than real incomes, with relative standard deviations (with respect to real GDP) 6.28 and 1.37, respectively; second, residential investment is much more volatile than that of non-residential investment (I_t^k): the relative standard deviation of the former is 6.28, almost twice as large as the latter, 3.29; third, residential investment and house price have approximately the same correlations with GDP (0.64 and 0.65), but are smaller than the correlation of non-residential investment with GDP (0.8).

The model captures these data properties well in terms of the relative volatilities of capital investment and housing investment, and house prices. The contemporaneous correlations between these variables and aggregate output are also in line with the data. In particular, housing investment and house price are less pro-cyclical than capital investment, and residential investment is much more volatile than non-residential investment, both in data and in the model. The relative standard deviation of house price is 1.37 in the data, while it is 1.24 in the model. Hence, the model explains approximately 90% of actual house price volatility.

We then examine how well the model-generated mortgage premium matches the data. We use the one-year ARM rate net of the one-year Treasury bond yield to represent the mortgage premium. As the last row of Table 1 shows, the relative standard deviation of the premium is 0.31 in the data and 0.29 in the model. Hence, the model explains approximately 94% of the mortgage premium volatility. Furthermore, the correlation of mortgage premium with GDP in the data is -0.46. The model-generated correlation captures the moderate counter-cyclical mortgage premium, which is -0.23.

By (6) and (7), house price can be expressed as

$$q_t = E_t\left[\left(\frac{x_{t+1}^h + (1-\delta_h)q_{t+1}}{R_{t+1}}\right)\frac{1}{\Omega\left(N_{t+1}/q_t H_{t+1}^h\right)}\right].$$

When $\Omega\left(N_{t+1}/q_t H_{t+1}^h\right) = 1$ for all t, there is no financial acceleration effect and the model collapses into a typical real business cycle model. The last two columns of Table 1 also show the properties of major aggregate variables when mortgage premium plays no role in the transmission mechanism in the model. Clearly, all variables fail to capture the key properties of relative standard deviations and correlations with output. The result shows that the financial accelerator mechanism is crucial to help amplify initial shocks and explaining housing and business cycles in the U.S.

4.1 The Boom and Bust of House Price during 2002-2008

Given that our model is able to capture aggregate fluctuations, particularly the U.S. housing market, we now examine whether the model is able to match the properties of the U.S. housing market for a specific period: 2002Q1-2008Q4, i.e., the boom-bust cycle of house price before and after the subprime crisis.

As documented in Table 2, U.S. house prices surged above the trend (de-trended by the HP filter) by 6.11% during 2002Q1-2005Q4 and fell by -12.99% during 2006Q1-2008Q4. The simulated changes in house price during these two periods are respectively 3.64% and -7.84%. This means that the model explains house price fluctuations by approximately 60% in both periods. Note that we have allowed all of the four shocks (monetary shock, cost push shock, preference shock, and technology shock) to take effect. Therefore, the result also suggests that alternative channels through which the boom-bust house price cycle during 2002Q1-2008Q4 were at work.[10]

Figure 2 and 3 illustrate of the data and simulated paths for house price and housing investment during the period 2002Q1-2008Q4. Compared with the simulation without mortgage premium, the model replicates the surge and plunge of house price and housing investment of the subprime crisis reasonably well, as shown in the data.

5. The Subprime Crisis and House Price Cycle - A Counterfactual Exercise

Many authors have argued that the extremely lax monetary policy in the early 2000s significantly influenced the housing market boom before its crash, and that the Federal Reserve should have raised the interest rate beginning in 2002 (Taylor (2007, 2009) and Ahrend *et al.* (2008)). This section conducts some counterfactual exercises to determine the possible dynamics of macroeconomic variables would have been like if the interest rate had been raised in 2002Q1. The appendix describes the counterfactual analysis procedure.

We first examine the counterfactual exercise for house price and housing investment if the federal funds rate had been raised by 0.5% beginning in 2002Q1. Table 3 shows that house price and housing

[10] Alternative explanations have been proposed, such as liquidity, leverage, risk premium, financial innovations, market-based financial institutions, etc. See, for example, Adrian and Shin (2008, 2010), Blanchard (2009), Brunnermeier (2008), Kiyotaki and Moore (2005), Krishnamurthy (2010), Shleifer and Vishny (2011)).

investment would not have increased much before the crisis: they are lower than the benchmark simulation by 15.7% and 10.2% during the period 2002Q1-2005Q4, respectively. In the aftermath of the crisis, a higher interest rate would have mitigated falling house prices and housing investment by 3% and 0.8%, respectively. Figure 2 and 3 plot the simulated counterfactual house prices together with the data. In sum, a pre-emptive strike of the Federal Reserve in the early 2000s would have stabilized the boom-bust cycle of the housing market, albeit with asymmetric effect: a tightening monetary policy exerts a much larger effect on the housing market before the crisis; however, it does not appear to alleviate the precipitous decline in housing market activity.

We further apply the counterfactual exercise to the moment properties of output, inflation, housing investment, capital investment, and house price had the interest rate risen by 0.5% beginning in 2002Q1. From Table 4, it is apparent that the effects of a pre-emptive monetary policy to the economy as a whole are mixed: it lessens changes in volatility and declines in means for some variables at a certain period, but amplifies them for other variables. For example, a higher interest rate would stabilize house prices during 2002Q1-2005Q4 (its standard deviation declines from 1.82 to 0.93), but worsen the magnitude of its decline during 2006Q1-2008Q4 (its mean declines from -0.12 further to -0.88). Similarly, monetary policy would stabilize housing investment through the period 2002Q1-2008Q4, but substantially suppress housing investment for the entire period. In particular, tightening monetary policy aimed to contain the housing market boom takes a big toll on real GDP: its volatility remains nearly the same, but the level of GDP contracts dramatically.

The result echoes the finding of Assenmacher-Wesche and Gerlach (2008a, 2008b), that using a single monetary policy tool, such as interest rate, cannot effectively stabilize all asset prices and economic activity at the same time. They argue that interest rate, a monetary policy tool that tends to affect all sectors of the economy, is not suitable for serving a specific purpose. As demonstrated in our results, had the Federal Reserve raised the interest rate to specifically deal with the housing market, monetary policy would inevitably causes considerable adverse effects on economic activity.

6. Conclusion Remarks

This paper investigates the role of mortgage premium in the propagation mechanism of a DSGE model with financing frictions in the housing market. We modify the models by Aoki *et al.* (2004) and Bernanke *et al.* (1999) to evaluate whether our model matches the properties of the U.S. housing market and business cycles. Similar to the argument of Taylor (2007, 2009), that the house price rally before the crash around 2007 can be largely attributable to the extremely lax monetary policy of the Federal Reserve, we also conduct counterfactual analysis to determine what would happen if the Federal Reserve had raised the interest rate in 2002Q1.

We find that the model captures the cyclical properties of the housing market well, including the relative standard deviation of residential investment is approximately twice that of non-residential investment; the correlation between residential investment and GDP is smaller than that between non-residential investment and GDP; the volatility of house price is larger than that of GDP; and mortgage premium is countercyclical and house price is pro-cyclical in the housing market. Finally, the pre-emptive tightening monetary policy aimed to contain the housing market boom does stabilize the volatilities of house price and housing investment, but it comes at a heavy cost of suppressing the level of GDP.

Therefore, the interest rate does not appear to be the right tool for stabilizing the housing market because tightening monetary policy has a far-reaching effect on the economy as a whole. As mentioned above, Assenmacher-Wesche and Gerlach (2008a, 2008b) expressed a similar view. This points out a possible direction for future research: what are suitable policy tools for stabilizing asset prices? Some candidates may include policy tools specific to the housing market, such as the loan-to-value ratio. Our model can be further expanded to study this issue.

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Table 1. Business Cycle Properties of Data and the Model

	Data		Model		Model without Financial Accererator	
	Std	Corr	Std	Corr	Std	Corr
I_t^k	3.29	0.80	3.27	0.88	6.88	0.27
I_t^h	6.28	0.64	6.22	0.38	4.52	0.21
C_t	0.54	0.83	0.47	0.91	0.79	0.24
q_t	1.37	0.65	1.28	0.31	0.80	0.22
Mortgage Premium	0.31	-0.46	0.29	-0.23	0.00	0.00

Note: Sources of *relative standard deviation* (Std) and *Correlation* ($Corr$) with GDP are taken from Davis (2010), and the standard deviation and correlation between mortgage premium and output are calculated by HP-filtered data with a smoothing parameter set to be 1600. The relative volatility of the variable *x* is defined as std(*x*)/std(*Y*), where *Y* is the real GDP.

Table 2. Changes of House Price and Housing Investment: Data and Model

	Data	Model
Period	House Price	House Price
2002Q1 – 2005Q4	6.11	3.64
2006Q1 – 2008Q4	-12.99	-7.84
	Housing Investment	Housing Investment
2002Q1 – 2005Q4	14.19	16.52
2006Q1 – 2008Q4	-12.49	-19.92

Note: The changes in house price and housing investment **over a certain period of time** is expressed in terms of deviation from the estimated trends.

Table 3. Counterfactual Analysis: Changes in House Price and Housing Investment When Federal Funds Rate Had Increased 0.5% during 2002Q1-2008Q4

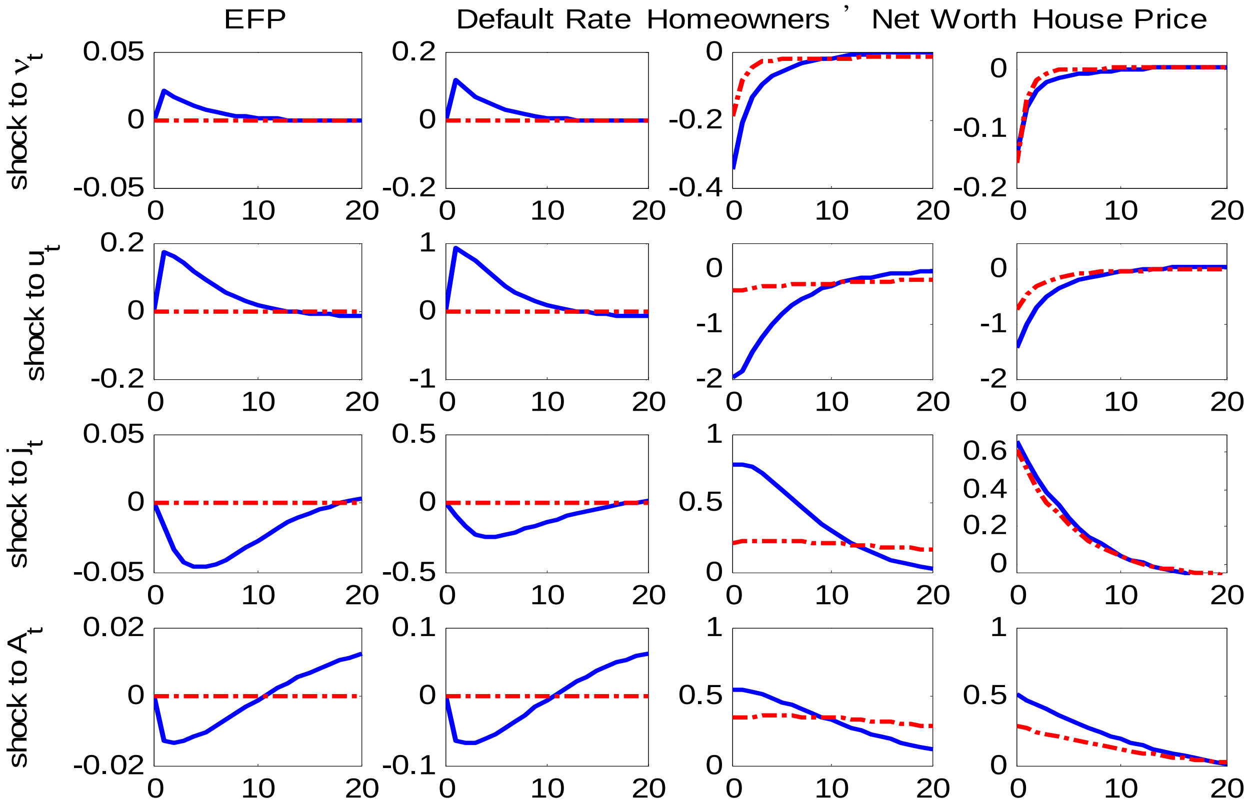
	Model	Counterfactual
Period	House Price	House Price
2002Q1 – 2005Q4	3.64	3.07 (-15.7%)
2006Q1 – 2008Q4	-7.84	-7.60 (+3%)
	Housing Investment	Housing Investment
2002Q1 – 2005Q4	16.52	14.83 (-10.2%)
2006Q1 – 2008Q4	-19.92	-19.76 (+0.8%)

Table 4. Counterfactual Analysis: Moment Properties When Federal Funds Rate Had Increased 0.5% during 2002Q1 – 2008Q4

Variables		y_t	π_t	I_t^h	I_t	q_t
	2002Q1 – 2005Q4					
Model	Mean	-0.50	-0.05	3.23	1.94	1.88
	Standard Deviation	0.72	0.03	6.44	2.55	1.82
Counterfactual	Mean	-1.20	-0.11	1.40	-1.58	0.89
	Standard Deviation	0.70	0.03	5.86	2.58	0.93
	2006Q1 – 2008Q4					
Model	Mean	0.88	-0.06	2.02	-0.48	-0.12
	Standard Deviation	0.63	0.04	9.76	5.04	2.12
Counterfactual	Mean	-0.04	-0.12	-3.79	-0.23	-0.88
	Standard Deviation	0.64	0.04	4.99	9.68	2.11

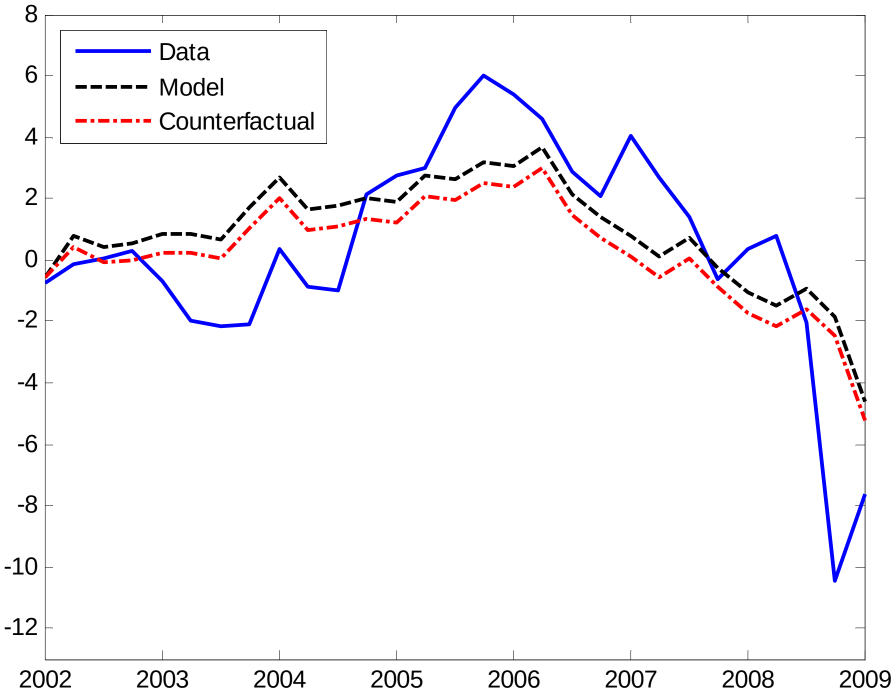
Note: The variables are expressed in deviation from the estimated trends.

Figure 1. Model-Generated Impulse Responses



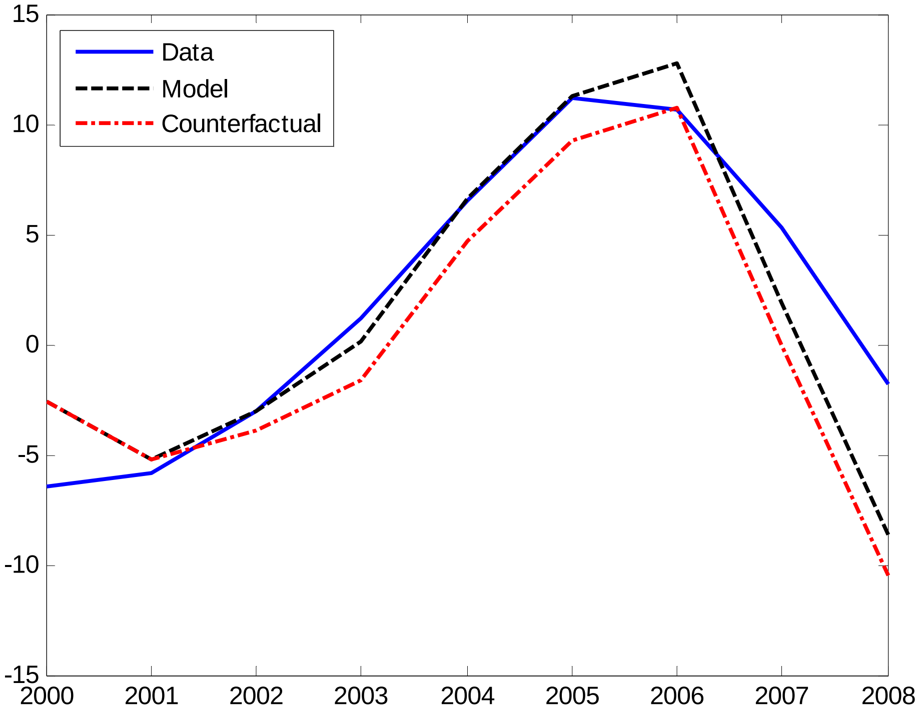
Note: The solid lines represent impulse responses of the model with financial accelerator, while the dashed lines represent those without.

Figure 2. Counterfactual Experiment - House Price



Note: The house price is expressed in terms of deviation from the estimated trend.

Figure 3. Counterfactual Experiment- Housing Investment



Note: The housing investment is expressed in terms of deviation from the estimated trend.

Appendix 1. First Order Conditions and the System of Equations for the Economy

Solving the maximization problem of consumers yields the following first order conditions respectively for consumption, housing demand, and labor supply:

$$\frac{1}{C_t^h} = E_t\left(\frac{\beta R_t^n}{\pi_{t+1}C_{t+1}^h}\right),$$

$$\frac{x_t^h}{C_t^h} = \frac{j_t}{H_t^h},$$

$$\frac{w_t}{C_t^h} = \left(L_t\right)^{\eta-1}.$$

Solving the maximization problem of intermediate goods producers yields the following first order conditions respectively for consumption, investment, housing demand for production, and labor demand:

$$\frac{1}{C_t^k} = E_t\left(\frac{\gamma R_t^n}{\pi_{t+1}C_{t+1}^k}\right),$$

$$\frac{1}{C_t^k}\left[1+\frac{\psi_k}{\delta_k}\left(\frac{I_t^k}{K_{t-1}}-\delta_k\right)\right] = \frac{\gamma}{C_{t+1}^k}E_t\left(\frac{\mu Y_{t+1}^f}{X_{t+1}K_t}+\left[1+\frac{\psi_k}{\delta_k}\left(\frac{I_{t+1}^k}{K_t}-\delta_k\right)\right](1-\delta_k)\right)+$$
$$\frac{\gamma}{C_{t+1}^k}\left[\frac{\psi_k}{\delta_k}\left(\frac{I_{t+1}^k}{K_t}-\delta_k\right)\frac{I_{t+1}^k}{K_t}-\frac{\psi_k}{2\delta_k}\left(\frac{I_{t+1}^k}{K_t}-\delta_k\right)^2\right],$$

$$\frac{q_t}{C_t^k} = E_t\left(\frac{\gamma}{C_{t+1}^k}\left[\frac{\upsilon Y_{t+1}^f}{X_{t+1}H_t^k}+\left[\frac{\psi_h}{\delta_h}\left(\frac{I_{t+1}^{kh}}{H_t^k}-\delta_h\right)\frac{I_{t+1}^{kh}}{H_t^k}-\frac{\psi_h}{2\delta_h}\left(\frac{I_{t+1}^{kh}}{H_t^k}-\delta_h\right)^2\right]+(1-\delta_h)q_{t+1}\right]\right),$$

$$w_t = \frac{(1-\upsilon-\mu)Y_t^f}{X_tL_t}.$$

There are 22 endogenous variables, R_t^n, R_t, π_t, H_t^h, H_t^k, H_t, C_t^h, C_t^k, I_t^k, I_t^h, K_t, R_t^h, x_t^h, q_t, Y_t^f, N_t, L_t, b_t^k, b_t^h, X_t, s_t, and $\overline{\omega}_t$ in our model. We solve the systems using 22 equations, including the first order conditions, budget constraints, and market clearing conditions.

$$\frac{1}{C_t^h} = E_t\left(\frac{\beta R_t^n}{\pi_{t+1} C_{t+1}^h}\right),$$

$$\frac{x_t^h}{C_t^h} = \frac{j_t}{H_t^h},$$

$$\frac{\frac{(1-\upsilon-\mu)Y_t^f}{X_t L_t}}{C_t^h} = (L_t)^{\eta-1},$$

$$q_t = \Phi'(I_t^h / H_{t-1}),$$

$$H_t = I_t^h + (1-\delta_h)H_{t-1},$$

$$H_t = H_t^k + H_t^h,$$

$$b_{t+1}^h = q_t H_{t+1}^h - N_{t+1},$$

$$N_{t+1} = V_t - F_t^h = R_t^h q_{t-1} H_t^h - \Omega\left(\frac{N_t}{q_{t-1}H_t^h}\right) R_t b_t^h - \chi\left(\frac{N_t}{q_{t-1}H_t^h}\right),$$

$$E_t(R_{t+1}^h) = E_t\left[\frac{x_{t+1}^h + (1-\delta_h)q_{t+1}}{q_t}\right],$$

$$E_t(R_{t+1}^h) = \Omega(N_{t+1}/q_t H_{t+1}^h) R_{t+1},$$

$$C_t^k + q_t(H_t^k - (1-\delta_h)H_{t-1}^k) + \frac{R_{t-1}^n b_{t-1}^k}{\pi_t} + w_t L_t + I_t^k + \xi_t^k + \xi_t^h = \frac{Y_t^f}{X_t} + b_t^k,$$

$$I_t^k = K_t - (1-\delta_k)K_{t-1},$$

$$\frac{1}{C_t^k} = E_t\left(\frac{\gamma R_t^n}{\pi_{t+1} C_{t+1}^k}\right),$$

$$\frac{1}{C_t^k}\left[1 + \frac{\psi_k}{\delta_k}\left(\frac{I_t^k}{K_{t-1}} - \delta_k\right)\right] = \frac{\gamma}{C_{t+1}^k}\left[\frac{\psi_k}{\delta_k}\left(\frac{I_{t+1}^k}{K_t} - \delta_k\right)\frac{I_{t+1}^k}{K_t} - \frac{\psi_k}{2\delta_k}\left(\frac{I_{t+1}^k}{K_t} - \delta_k\right)^2\right]$$
$$+ \frac{\gamma}{C_{t+1}^k} E_t\left(\frac{\mu Y_{t+1}^f}{X_{t+1} K_t} + \left[1 + \frac{\psi_k}{\delta_k}\left(\frac{I_{t+1}^k}{K_t} - \delta_k\right)\right](1-\delta_k)\right),$$

$$\frac{q_t}{C_t^k} = E_t \frac{\gamma}{C_{t+1}^k}\left[\frac{\upsilon Y_{t+1}^f}{X_{t+1} H_t^k} + \left(\frac{\psi_h}{\delta_h}\left(\frac{I_{t+1}^{kh}}{H_t^k} - \delta_h\right)\frac{I_{t+1}^{kh}}{H_t^k} - \frac{\psi_h}{2\delta_h}\left(\frac{I_{t+1}^{kh}}{H_t^k} - \delta_h\right)^2\right) + (1-\delta_h)q_{t+1}\right],$$

$$Y_t^f = A_t K_{t-1}^{\mu} (H_{t-1}^k)^{\upsilon} L_t^{(1-\mu-\upsilon)},$$

$$Y_t^f = I_t^k + I_t^h + C_t^k + C_t^h + \xi_t^k + \xi_t^h + m\int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^h q_{t-1} H_t^h,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa X_t + u_t,$$

$$\ln(\frac{R_t^n}{R^n}) = \alpha_\pi \ln(\frac{\pi_t}{\pi}) + \alpha_Y \ln(\frac{Y_t^f}{Y^f}) + \ln(\nu_t),$$

$$R_{t+1}^n = E_t\left(R_{t+1}\pi_{t+1}\right),$$

$$s_t = E_t\left(\frac{R_{t+1}^h}{R_{t+1}}\right),$$

$$\Omega(\overline{\omega}_t) = s_t.$$

Appendix 2. Steady State

$$R^n = \frac{1}{\beta},$$

$$K = \frac{\gamma\mu}{1-\gamma(1-\delta_k)}\frac{Y^f}{X} = \zeta_1 Y^f,$$

$$q = \Phi'(I^h/H),$$

$$q = \frac{\gamma\upsilon}{1-\gamma(1-\delta_h)}\frac{Y^f}{XH^k} = \zeta_2\frac{Y^f}{H^k},$$

$$x^h = \frac{jC^h}{H^h} = \zeta_3\frac{C^h}{H^h},$$

$$wL + F^r = \left[(1-\mu-\upsilon) + X - 1\right]\frac{Y^f}{X} = S'Y^f,$$

$$R^h = \frac{x^h + (1-\delta_h)q}{q} = \Omega(\phi)/\beta,$$

$$\chi(\phi) = x^h - \delta_h,$$

$$N = \frac{1}{1-\Omega(\phi)R^n}\left\{\left(R^h - \Omega(\phi)R^n\right)H^h - \chi(\phi)\right\} = \frac{\chi(\phi)}{\Omega(\phi)/\beta - 1},$$

$$b^h = qH^h - N > 0,$$

$$L = \left(\zeta_1^{-\mu}(H^k)^{-\upsilon}\right)^{\frac{1}{1-\mu-\upsilon}} Y^{f\,\frac{1-\mu}{1-\mu-\upsilon}},$$

$$C^h = \left[(1-\mu-\upsilon)\right]\frac{1}{X(L)^\eta}Y^f = \zeta_4 Y^f.$$

By

$$C^h = (wL + F^r) + \chi(\phi) - \delta_h x^h H^h + (1/\beta - 1)b^k + (1/\beta - 1)(qH^h - N)$$

$$= S'Y^f + \chi(\phi) - \delta_h\zeta_3 C^h - (1 - 1/\beta)b^k + (1/\beta - 1)\left[\frac{\zeta_3}{x^h}C^h - \frac{\chi(\phi)}{R^h - 1}\right],$$

then

$$b^k = S^{''}Y^f + \left[\frac{\chi(\phi)}{R^h - 1} - \frac{\chi(\phi)}{(1/\beta - 1)}\right].$$

Moreover,

$$\begin{aligned} C^k &= \frac{\mu + \upsilon}{X}Y^f - \delta_k K - \delta_h q H^k + (1 - 1/\beta)b^k \\ &= \left(\frac{\mu + \upsilon}{X} - \delta_k \zeta_1 - \delta_h \zeta_2 + S^{''}(1 - 1/\beta)\right)Y^f + \left[\frac{\chi(\phi)}{R^h - 1}(1 - 1/\beta) + \chi(\phi)\right], \end{aligned}$$

where $\phi = \dfrac{N}{qH^h}$, $\zeta_1 = \dfrac{\gamma\mu}{[1 - \gamma(1 - \delta_k)]X}$, $\zeta_2 = \dfrac{\gamma\upsilon}{[1 - \gamma(1 - \delta_h)]X}$, $\zeta_3 = j$, $S^{'} = [(1 - \mu - \upsilon) + X - 1]\dfrac{1}{X}$.

Normalizing $Y^f = 1$, we have

$$H^h = \frac{\zeta_3 \zeta_4}{x^h}, H^k = \zeta_2,$$

and all other variables can be solved. Let

$$H^h = \frac{\frac{\zeta_3 \zeta_4}{x^h}}{(\frac{\zeta_3 \zeta_4}{x^h} + \zeta_2)}, H^k = \frac{\zeta_2}{(\frac{\zeta_3 \zeta_4}{x^h} + \zeta_2)},$$

where $\zeta_4 = [(1 - \mu - \upsilon)]\dfrac{1}{X(L)^\eta}$, $S^{''} = \dfrac{\left[1 + \delta_h \zeta_3 - (1/\beta - 1)\frac{\zeta_3}{x^h}\right]\zeta_4 - S^{'}}{(1/\beta - 1)}$, so that

$$\frac{I^h}{Y^f} = \delta_h.$$

Appendix 3. Log-Linearization

$$0 = \frac{I^k}{Y^f}\hat{I}^k_t + \frac{I^h}{Y^f}\hat{I}^h_t + \frac{C^k}{Y^f}\hat{C}^k_t + \frac{C^h}{Y^f}\hat{C}^h_t - \hat{Y}^f_t,$$

$$\hat{C}^h_t = \hat{C}^h_{t+1} - \hat{R}^n_t + \hat{\pi}_{t+1},$$

$$\hat{x}^h_t = (\hat{j}_t - \hat{H}^h_t) + \hat{C}^h_t,$$

$$\hat{C}^k_t = \hat{C}^k_{t+1} - \frac{\gamma}{\beta}\hat{R}^n_t + \hat{\pi}_{t+1},$$

$$\hat{C}^k_t = \hat{C}^k_{t+1} - [1-\gamma(1-\delta_k)](\hat{Y}^f_{t+1} - \hat{X}_{t+1} - \hat{K}_t) + \psi_k(\hat{I}^k_t - \hat{K}_{t-1} - \gamma(\hat{I}^k_{t+1} - \hat{K}_t)),$$

$$\hat{q}_t = (1-\delta_h)\gamma\,\hat{q}_{t+1} + [1-\gamma(1-\delta_h)](\hat{Y}^f_{t+1} - \hat{X}_{t+1} - \hat{H}^k_t) + \psi_h\gamma(\hat{I}^{kh}_{t+1} - \hat{H}^k_t)) + \hat{C}^k_t - \hat{C}^k_{t+1},$$

$$\hat{Y}^f_t = \hat{A}_t + \upsilon\hat{H}^k_{t-1} + \mu\hat{K}_{t-1} + (1-\mu-\upsilon)\hat{L}_t,$$

$$\hat{Y}^f_t = \hat{X}_t + \eta\hat{L}_t + \hat{C}^h_t,$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{X}_t + \hat{u}_t,$$

$$\hat{K}_t = \delta_k\hat{I}^k_t + (1-\delta_k)\hat{K}_{t-1},$$

$$\frac{b^k}{Y^f}\hat{b}^k_t = \frac{C^k}{Y^f}\hat{C}^k_t + \frac{qH^k}{Y^f}(\delta_h\hat{q}_t + \hat{H}^k_t - (1-\delta_h)\hat{H}^k_{t-1}) + \frac{I^k}{Y^f}\hat{I}^k_t + \frac{R^n b^k}{Y^f}(\hat{R}^n_{t-1} + \hat{b}^k_{t-1} - \hat{\pi}_t) - \frac{(\mu+\upsilon)}{X}(\hat{Y}^f_t - \hat{X}_t),$$

$$E_t\hat{R}^h_{t+1} = \hat{R}_{t+1} - \Theta\left\{\hat{N}_{t+1} - (\hat{q}_t + \hat{H}^h_{t+1})\right\},$$

$$E_t\hat{R}^h_{t+1} = (1-\rho)\hat{x}^h_t + \rho E_t\hat{q}_{t+1} - \hat{q}_t,$$

$$\hat{q}_t = \Psi(\hat{I}^h_t - \hat{H}_{t-1}),$$

$$\hat{H}_t = \delta_h\hat{I}^h_t + (1-\delta_h)\hat{H}_{t-1},$$

$$\hat{N}_{t+1} = R^h[(1+\phi)\hat{R}^h_t - \phi\,\Theta(\hat{q}_{t-1} + \hat{H}^h_t) + (1+\phi\,\Theta)\hat{N}_t - \phi\hat{R}_t] - (R^h - 1)\hat{F}^h_t,$$

$$\hat{F}^h_{t+1} = \varkappa\left\{\hat{N}_{t+1} - (\hat{q}_t - \hat{H}^h_{t+1})\right\}$$

$$\hat{b}^h_{t+1} = \frac{H^h}{b^h}\hat{q}_t + \frac{H^h}{b^h}\hat{H}^h_{t+1} - \frac{N}{b^h}\hat{N}_{t+1},$$

$$\frac{H^k}{H}\hat{H}^k_t + \frac{H^h}{H}\hat{H}^h_t = \hat{H}_t,$$

$$\hat{R}^n_t = \alpha_\pi\hat{\pi}_t + \alpha_Y\hat{Y}^f_t + \hat{v}_t,$$

$$\hat{R}^n_{t+1} = \hat{R}_{t+1} + E_t\hat{\pi}_{t+1},$$

$$\hat{s}_t = E_t \hat{R}^h_{t+1} - \hat{R}_{t+1},$$

$$\hat{s}_t = \frac{\Omega(\overline{\omega})'}{\Omega(\overline{\omega})} \overline{\omega} \hat{\overline{\omega}}_t, \Rightarrow \hat{\overline{\omega}}_t = \left[\frac{\Omega(\overline{\omega})'}{\Omega(\overline{\omega})} \overline{\omega} \right]^{-1} \hat{s}_t = s_1 \hat{s}_t,$$

$$\hat{f} = \frac{F(\overline{\omega})'}{F(\overline{\omega})} \overline{\omega} \hat{\overline{\omega}}_t = s_2 \hat{\overline{\omega}}_t,$$

where $\Theta = -\frac{\Omega'(\phi)}{\Omega(\phi)}\phi$, $\rho = \frac{x^h}{x^h + (1-\delta_h)}$, $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$, $\varkappa = \frac{\chi'(\phi)}{\chi(\phi)}\phi$, $s_1 = \left[\frac{\Omega(\overline{\omega})'}{\Omega(\overline{\omega})} \overline{\omega} \right]^{-1}$, $s_2 = \frac{F(\overline{\omega})'}{F(\overline{\omega})} \overline{\omega}$, $\Psi = \frac{\left(\Phi(I^h/H)\right)'}{\left(\Phi(I^h/H)\right)''}$.

Appendix 4. Procedure for Counterfactual Analysis

We proceed with the counterfactual analysis according to the following procedure:

(1) Use the procedure proposed by Schmitt-Grohé and Uribe (2004) to solve the model. Let $X_t = [r_t, \pi_t, h_t, Y_t]'$ be the deviation of federal funds rate from the mean, the deviation of inflation rate from the mean, the de-trended housing starts, and the de-trended of GDP from $2002Q1-2008Q4$. The model solution corresponding to X_t can be expressed as $X_t^* = AS_t$ and $S_t = \rho_S S_{t-1} + e_t$, where $S_t = [v_t u_t \ j_t A_t]'$ is the vector of state variables. Substitute X_t into X_t^*, and then solve for $S_t = A^{-1} X_t$ and $e_t = S_t - \rho_S S_{t-1}$, where $e_t = [\varepsilon^v \varepsilon^u \varepsilon^j \varepsilon^A]'$ (called estimated shocks).

(2) Feed the estimated shocks e_t into the linearized decision rules of the model and check whether the model can replicate the behavior of real house price and housing investment from 2002Q1-2008Q4.