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# The Real Exchange Rate, Real Interest Rates, and the Risk Premium

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# Abstract

The well-known uncovered interest parity puzzle arises from the empirical regularity that, among developed country pairs, the high interest rate country tends to have high expected returns on its short term assets. At the same time, another strand of the literature has documented that high real interest rate countries tend to have currencies that are strong in real terms – indeed, stronger than can be accounted for by the path of expected real interest differentials under uncovered interest parity. These two strands – one concerning short-run expected changes and the other concerning the level of the real exchange rate – have apparently contradictory implications for the relationship of the foreign exchange risk premium and interest-rate differentials. This paper documents the puzzle, and shows that existing models appear unable to account for both empirical findings. The features of a model that might reconcile the findings are discussed.

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This study concerns two prominent empirical findings in international finance that have achieved almost folkloric status. The interest parity puzzle in foreign exchange markets finds that over short time horizons (from a week to a quarter) when the interest rate (one country relative to another) is higher than average, the securities of the high-interest rate currency tend to earn an excess return. That is, the high interest rate country tends to have the higher expected return in the short run. A risk-based explanation of this anomaly requires that the securities in the high-interest rate country are relatively riskier, and therefore incorporate an excess return as a reward for risk-bearing.

The second stylized fact concerns evidence that when a country's relative real interest rate rises above its average, its currency in levels tends to be stronger than average in real terms. Moreover, the strength of the currency tends to be greater than is warranted by rational expectations of future short-term real interest differentials. One way to rationalize this finding is to appeal to the influence of expected future risk premiums on the level of the exchange rate. That is, the country with the relatively high real interest rate has the lower risk premium and hence the stronger currency. When a country's real interest rate rises, its currency appreciates not only because its assets pay a higher interest rate but also because they are less risky.

This paper produces evidence that confirms these empirical regularities for the exchange rates of the G7 countries (Canada, France, Germany, Italy, Japan and the U.K.) relative to the U.S. However, these findings, taken together, constitute a previously unrecognized puzzle regarding how cumulative excess returns or foreign exchange risk premiums affect the level of the real exchange rate. Theoretically, a currency whose assets are perceived to be risky not only currently but looking forward to the distant future should be weaker, ceteris paribus. The evidence cited implies that when a country's relative real interest rate is high, the country's securities are expected to yield an excess return over foreign securities in the short run; but, because the high-interest rate currency tends to be stronger, over longer horizons it is the foreign asset that is expected to yield an excess return. This behavior of excess returns in the foreign exchange market poses a challenge for conventional theories of the foreign exchange risk premium.

In brief, when one country's interest rate is high, its currency tends to be stronger than average in real terms, it tends to keep appreciating for awhile, and then depreciates back toward its long-run value. But leading models of the forward-premium anomaly do not account for the behavior of the level of the real exchange rate: they predict that the high-interest rate currency will be weaker than average in real terms and appreciate over both the short- and long-run. A risk-based explanation for the empirical regularities requires a reversal of the risk premium – the securities of the high-interest rate country must be relatively riskier in the short-run, but expected to be less risky than the other country's securities in the more distant future.

Figure 1, which will be explained in detail later, illustrates the point dramatically. The chart plots estimates based on a vector autoregression for the U.S. relative to data constructed as a weighted average of the other G7 countries. The line labeled bQj shows the estimates of the slope coefficient of a regression of the real exchange rate in period t+j on the U.S.-Foreign real interest differential in period t. When the U.S. real interest rate is relatively high, the dollar tends to be strong in real terms (the real exchange rate is below its long run mean.) Over the ensuing months, in the short run, the dollar on average appreciates even more when the U.S. real interest rate is high, before depreciating back toward its long-run mean. The other two lines in the chart represent hypothetical behavior of the real exchange rate implied under two different models. The line labeled bRj shows how the real exchange rate would behave if uncovered interest parity held (based on the VAR forecasts of future real interest differentials.) Relative to the interest parity norm, the actual real exchange rate behavior is notably different: when the U.S. real interest rate is high, (1) the real value of the dollar (in levels) is stronger than implied by interest parity (the real exchange rate is lower); (2) the dollar continues to appreciate in the short run (while interest parity implies a depreciation.) The line labeled Model illustrates the implied behavior of the real exchange rate in a class of models based on risk averse behavior of representative agents in each economy. These models have been developed to account for the uncovered interest parity puzzle – the subsequent short-run depreciation of the currency that tends to accompany a relatively high Home interest rate. Referring to the line labeled bQj, the models are built to explain the initial negative slope of the line. However, the models miss the overall picture badly, because they predict the effect of interest rates on the level of real exchange rates with the wrong sign and therefore get the subsequent dynamics wrong as well.[1]

The literature on the forward premium anomaly is vast. Classic early references include Bilson (1981) and Fama (1984). Engel (1996) surveys the early work that establishes this puzzle, and discusses the problems faced by the literature that tries to account for the regularity. There have been many recent important contributions, including prominent papers by Backus, Foresi, and Telmer (2002), Lustig and Verdelhan (2007), Burnside et. al. (2010a, 2010b), Verdelhan (2010), Bansal and Shaliastovich (2010), Backus et. al. (2010). Below, we survey the implications of the recent theoretical work for real exchange rate behavior.

Dornbusch (1976) and Frankel (1979) are the original papers to draw the link between real interest rates and the level of the real exchange rate in the modern, asset-market approach to exchange rates. The connection has not gone unchallenged, principally because the persistence of real exchange rates and real interest differentials makes it difficult to establish their comovement with a high degree of uncertainty. For example, Meese and Rogoff (1988) and Edison and Pauls (1993) treat both series as non-stationary and conclude that evidence in favor of cointegration is weak. However, more recent work that examines the link between real interest rates and the real exchange rate, such as Engel and West (2006), Alquist and Chinn (2008), and Mark (2009), has tended to reestablish evidence of the empirical link. Another

[1] The models referred to here tend to treat the real exchange rate as nonstationary, in contrast to the evidence we present in section 2. As explained below, the line in Figure 2 refers to the model's prediction for the stationary component of the real exchange rate.

approach connects surprise changes in real interest rates to unexpected changes in the real exchange rate. There appears to be a strong link of the real exchange rate to news that alters the expected real interest differential – see, for example, Faust et. al. (2007), Andersen et. al. (2007) and Clarida and Waldman (2008).

The behavior of exchange rates and interest rates described here is closely associated with the notion of "delayed overshooting". The term was coined by Eichenbaum and Evans (1995), but is used to describe a hypothesis first put forward by Froot and Thaler (1990). Froot and Thaler's explanation of the forward premium anomaly was that when, for example, the Home interest rate rises, the currency appreciates as it would in a model of interest parity such as Dornbusch's (1976) classic paper. But they hypothesize that the full reaction of the market is delayed, perhaps because some investors are slow to react to changes in interest rates, so that the currency keeps on appreciating in the months immediately following the interest rate increase. Bacchetta and van Wincoop (2010) build a model based on this intuition. Much of the empirical literature that has documented the phenomenon of delayed overshooting has focused on the response of exchange rates to identified monetary policy shocks.[2] But in the original context, the story was meant to apply to any shock that leads to an increase in relative interest rates. Risk-based explanations of the interest parity puzzle have not confronted this literature's finding that high interest rate currencies are strong currencies. Our empirical findings are consistent with Froot and Thaler's hypothesis of delayed overshooting, but with one important modification. The empirical methods here allow us to uncover what the level of the real exchange rate would be if uncovered interest parity held, and to compare the actual real exchange rate with this notional level. We find the level of the real exchange rate is excessively sensitive to real interest differentials. That is, when a country's real interest rate increases, its currency appreciates more than it would under uncovered interest parity. Then it continues to appreciate for a number of months, before slowly depreciating back to its long run level.

Section 1 develops the approach of this paper. Section 2 presents empirical results. Section 3 develops some general conditions that have to be satisfied in order to account for our empirical findings. We discuss the difficulties encountered by representative agent models of the risk premium, and illustrate the problem by showing that some recent models based on non-standard preferences are unable to match the key facts we develop.[3] In section 4, we consider various caveats to our findings. Finally, in the concluding section, we discuss some alternative models that may be able to account for these empirical regularities.

[2] See, for example, Eichenbaum and Evans (1995), Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), and Bjornland (2009).

[3] "Representative agent models" may be an inadequate label for models of the risk premium that are developed off of the Euler equation of a representative agent under complete markets, generally taking the consumption stream as exogenous.

# 1. Excess Returns and Real Exchange Rates

We develop here a framework for examining behavior of excess returns and the level of the real exchange rate. We relate the concepts here to economic theories of risk and return in section 3.

Our set-up will consider a Home and Foreign country. In the empirical work of section 2, we always take the US as the Home country (as does the vast majority of the literature), and consider other major economies as the Foreign country. Let $i_t$ be the one period nominal interest in Home. We denote Foreign variables throughout with a superscript *, so $i_t^*$ is the Foreign interest rate. $s_t$ denotes the log of the foreign exchange rate, expressed as the Home currency price of Foreign currency. $E_t s_{t+1}$ refers to the expectation, conditional on time *t* information, of the log of the spot exchange rate at time $t+1$. We define the "excess return", $\lambda_t$, as:

$$\lambda_t \equiv i_t^* + E_t s_{t+1} - s_t - i_t \tag{1}$$

This definition of excess returns corresponds with the definition in the literature. We can interpret $i_t^* + E_t s_{t+1} - s_t$ as a first-order log approximation of the expected return in Home currency terms for a Foreign security. As Engel (1996) notes, the first-order log approximation may not really be adequate for appreciating the implications of economic theories of the excess return. For example, if the exchange rate is conditionally log normally distributed, then $\ln\left(E_t(S_{t+1}/S_t)\right) = E_t s_{t+1} - s_t + \frac{1}{2}\mathrm{var}_t(s_{t+1})$, where $\mathrm{var}_t(s_{t+1})$ refers to the conditional variance of the log of the exchange rate. Engel (1996) points out that this second-order term is approximately the same order of magnitude as the risk premiums implied by some economic models. However, we proceed with analysis of $\lambda_t$ defined according to equation (1) both because it is the object of almost all of the empirical analysis of excess returns in foreign exchange markets, and because the theoretical literature that we consider in section 3 seeks to explain $\lambda_t$ as defined above including possible movements in $\mathrm{var}_t(s_{t+1})$.

The well-known uncovered interest parity puzzle comes from the empirical finding that the change in the log of the exchange rate is negatively correlated with the Home less Foreign interest differential, $i_t - i_t^*$. That is, estimates of $\mathrm{cov}(s_{t+1} - s_t, i_t - i_t^*) = \mathrm{cov}(E_t s_{t+1} - s_t, i_t - i_t^*)$ tend to be negative. As Engel (1996) surveys, and subsequent empirical work confirms, this finding is consistent over time among pairs of high-income, low-inflation countries.[4] From equation (1), we note that the relationship

[4] Bansal and Dahlquist (2000) find that the relationship is not as consistent among emerging market countries, especially those with high inflation.

$\operatorname{cov}(E_t s_{t+1} - s_t, i_t - i_t^*) < 0$ is equivalent to $\operatorname{cov}(\lambda_t, i_t - i_t^*) < -\operatorname{var}(i_t - i_t^*) < 0$. That is, when the Home interest rate is relatively high, so $i_t - i_t^*$ is above average, the excess return on Home assets also tends to be above average: $\lambda_t$ is below average. This is considered a puzzle because it has been very difficult to find plausible economic models that can account for this relationship.

Let $p_t$ denote the log of the consumer price index at Home, and $\pi_{t+1} = p_{t+1} - p_t$ is the inflation rate. The log of the real exchange rate is defined as $q_t = s_t + p_t^* - p_t$. The ex ante real one-period interest rates, Home and Foreign, are given by $r_t = i_t - E_t \pi_{t+1}$ and $r_t^* = i_t^* - E_t \pi_{t+1}^*$. Note also $E_t q_{t+1} - q_t = E_t s_{t+1} - s_t + E_t \pi_{t+1}^* - E_t \pi_{t+1}$. We can rewrite (1) as:

$$\lambda_t = r_t^* - r_t + E_t q_{t+1} - q_t \tag{2}$$

We take as uncontroversial the proposition that the real interest differential, $r_t - r_t^*$, and excess returns, $\lambda_t$, are stationary random variables without time trends, and denote their means as $\bar{r}$ and $\bar{\lambda}$, respectively. We will also stipulate that there is no deterministic time trend or drift in the log of real exchange rates, so that the unconditional mean of $E_t q_{t+1} - q_t$ is zero. Rewriting (2):

$$q_t - E_t q_{t+1} = -(r_t - r_t^* - \bar{r}) - (\lambda_t - \bar{\lambda}) \tag{3}$$

Iterate equation (3) forward, applying the law of iterated expectations, to get:

$$q_t - \lim_{j \to \infty} \left( E_t q_{t+j} \right) = -R_t - \Lambda_t \tag{4}$$

where

$$R_t \equiv \sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \bar{r}) \tag{5}$$

and

$$\Lambda_t \equiv \sum_{j=0}^{\infty} E_t (\lambda_{t+j} - \bar{\lambda}) \tag{6}$$

We label $R_t$ as the "prospective real interest differential". It is the expected sum of the current and all future values of the Home less Foreign real interest differential (relative to its unconditional mean). It is important to note that $R_t$ is not the real interest differential on long-term bonds, even hypothetical infinite-horizon bonds. $R_t$ is the difference between the real return from holding an infinite sequence of short-term Home bonds and the real return from the infinite sequence of short-term Foreign bonds. An investment that involves rolling over short term assets has different risk characteristics than holding a long-term asset. Hence we coin the phrase "prospective" real interest differential to avoid the trap of calling $R_t$ the long-term real interest differential.

Similarly, $\Lambda_t$ is the expected infinite sum of excess returns on the Foreign security. We label this the "level excess return" or "level risk premium", to make reference to its influence on the level of the real exchange rate.

The left-hand side of (4), $q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right)$, can be interpreted as the transitory component of the real exchange rate. In fact, according to our empirical findings reported in section 2, we can treat the real exchange rate as a stationary variable, so $\lim_{j\to\infty}\left(E_t q_{t+j}\right) = \bar{q}$. As is well known, even if the real exchange rate is stationary, it is very persistent. Engel (2000), in fact, argues that it may be practically impossible to distinguish between the stationary case and the unit root case under plausible economic conditions. We proceed in examining $q_t - \bar{q}$, assuming stationarity, but note that our methods could be applied to the transitory component of the real exchange rate, taken as the difference between $q_t$ and a measure of the permanent component, $\lim_{j\to\infty}\left(E_t q_{t+j}\right)$. In section 4, we note how Engel's (2000) interpretation implies that in practice it may not be possible to distinguish a permanent and transitory component, but make the case that the economic analysis of that paper argues for treating the real exchange rate as stationary.

In section 3, we discuss the common assumption in theoretical models of excess returns that the real exchange rate is equal to the difference between the marginal utility of a Home consumer and Foreign consumer. Stationarity of the real exchange rate is completely compatible with a unit root in the log of consumption, or in the marginal utility of consumption. It requires simply that Home and Foreign marginal utilities of consumption be cointegrated, which is a natural condition among well-integrated economies such as the highly developed countries used in this study. It is analogous to the assumption made in almost all closed-economy models that we can treat the marginal utilities of different consumers within a country as cointegrated.

Under the stationarity assumption, we can write (4) as:

$$q_t - \bar{q} = -R_t - \Lambda_t \tag{7}$$

From this formulation, we see that the level excess return, $\Lambda_t$, captures the potential effect of risk premiums on the *level* of the real exchange rate, holding the prospective real interest differential constant.

In the next section, we present evidence that $\text{cov}(R_t, r_t - r_t^*) > 0$ and $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$. Taken together, these two findings imply from (7) that $\text{cov}(q_t, r_t - r_t^*) < 0$, which jibes with the concept familiar from Dornbusch (1976) and Frankel (1979) that when a country's real interest rate is high (relative to the foreign real interest rate, relative to average), its currency tends to be strong in real terms (relative to average.) But if $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$, the strength of the currency cannot be attributed entirely to the prospective real interest differential, as it would be in Dornbusch and Frankel (who both assume uncovered interest parity, or that $\lambda_t \equiv 0$.) The relationship between excess returns and real interest differential plays a role in determining the relation between the real exchange rate and real interest rates.

It is entirely unsurprising that we find $\text{cov}(R_t, r_t - r_t^*) > 0$. This simply implies that there is not a great deal of non-monotonicity in the adjustment of real interest rates toward the long run mean.

The central puzzle raised by this paper concerns the two findings, $\text{cov}(\lambda_t, r_t - r_t^*) < 0$ and $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$. The short-run excess return on the Foreign security, $\lambda_t$, is negatively correlated with the real interest differential, consistent with the many empirical papers on the uncovered interest parity puzzle. But the level excess return, $\Lambda_t$, is positively correlated. Given the definition of $\Lambda_t$ in equation (6), we must have that for at least $j = 0$ and possibly for some $j > 0$, $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) < 0$, but for other $j > 0$, $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$. The sum of the latter covariances must exceed the sum of the former to generate $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$. As we discuss in section 3, our risk premium models of excess return are not up to the task of explaining this finding. In fact, while they are constructed to account for $\text{cov}(\lambda_t, r_t - r_t^*) < 0$, they have the counterfactual implication that $\text{cov}(\Lambda_t, r_t - r_t^*) < 0$.

The empirical approach of this paper can be described simply. We estimate VARs in the variables $q_t$, $i_t - i_t^*$, and $i_{t-1} - i_{t-1}^* - (\pi_t - \pi_t^*)$. From the VAR estimates, we construct measures of $E_t\left(i_t - i_t^* - (\pi_{t+1} - \pi_{t+1}^*)\right) = r_t - r_t^*$. Using standard projection formulas, we can also construct estimates of $R_t$. To measure $\Lambda_t$, we take the difference of $q_t - \bar{q}$ and $R_t$. From these VAR estimates, we calculate our estimates of the covariances just discussed. As an alternative approach, we estimate VARs in $q_t$,

$i_t - i_t^*$, and $\pi_t - \pi_t^*$, and then construct the needed estimates of $r_t - r_t^*$, $R_t$, and $\Lambda_t$. The estimated covariances under this alternative approach are very similar to those from the original VAR. Our approach of estimating undiscounted expected present values of interest rates from VARs is presaged in Mark (2009) and Brunnermeier et. al. (2009).

# 2. Empirical Results

We investigate the behavior of real exchange rates and interest rates for the U.S. relative to the other six countries of the G7: Canada, France, Germany, Italy, Japan, and the U.K. We also consider the behavior of U.S. variables relative to an aggregate weighted average of the variables from these six countries.[5] Our study uses monthly data. Foreign exchange rates are noon buying rates in New York, on the last trading day of each month, culled from the daily data reported in the Federal Reserve historical database. The price levels are consumer price indexes from the Main Economic Indicators on the OECD database. Nominal interest rates are taken from the last trading day of the month, and are the midpoint of bid and offer rates for one-month Eurorates, as reported on Intercapital from Datastream. The interest rate data begin in June 1979. Most of our empirical work uses the time period June 1979 to October 2009. In some of the tests for a unit root in real exchange rates, reported in the Appendix, we use a longer time span from June 1973 to October 2009. It is important for our purposes to include these data well into 2009 because it has been noted in some recent papers that there was a crash in the "carry trade" in 2008, so it would perhaps bias our findings if our sample ended prior to this crash.[6] We treat the real exchange rates as stationary throughout our empirical analysis.

The Appendix details evidence that allows us to reject a unit root in real exchange rates. It is well known that real exchange rates among advanced countries are very persistent.[7] There is no consensus on whether these real exchange rates are stationary or have a unit root. Two recent studies of uncovered interest parity, Mark (2010) and Brunnermeier, et. al. (2009) estimate statistical models that assume the real exchange rate is stationary, but do not test for a unit root. Jordà and Taylor (2010) demonstrate that there is a profitable carry-trade strategy that exploits the uncovered interest parity puzzle when the trading rule is enhanced by including a forecast that the real exchange rate will return to its long-run level when its deviations from the mean are large. That paper assumes a stationary real exchange rate and includes statistical tests that cannot reject cointegration of $s_t$ with $p_t - p_t^*$.

5 The weights are determined by the value of each country's exports and imports as a fraction of the average value of trade over the six countries.

6 See, for example, Brunnermeier, et. al. (2009) and Jordà and Taylor (2009).

7 See Rogoff (1996) for example.

### 2.1 Fama Regressions

Table 1 reports results from the standard "Fama regression" that is the basis for the forward premium puzzle. The change in the log of the exchange rate between time t+1 and t is regressed on the time t interest differential:

$$s_{t+1} - s_t = \zeta_s + \beta_s (i_t - i_t^*) + u_{s,t+1} \tag{8}$$

Under uncovered interest parity, $\zeta_s = 0$ and $\beta_s = 1$.

We can rewrite this regression as:

$$i_t - (i_t^* + s_{t+1} - s_t) = -\zeta_s + (1-\beta_s)(i_t - i_t^*) + u_{s,t+1}$$

The left-hand side of the regression is the ex post excess return on the home security. If $\zeta_s = 0$ but $\beta_s < 1$, then the high-interest rate currency tends to have a higher excess return. There is a positive correlation between the excess return on the Home currency and the Home-Foreign interest differential.

The Table reports the 90% confidence interval for the regression coefficients, based on Newey-West standard errors. For five of the six currencies, the point estimate of $\beta_s$ is negative (Italy being the exception). Of those five, the 90% confidence interval for $\beta_s$ lies below one for four (France being the exception, where the confidence interval barely includes one.) For four of the six, zero is inside the 90% confidence interval for $\zeta_s$. (In the case of the U.K., the confidence interval barely excludes zero, while for Japan we find strong evidence that $\zeta_s$ is greater than zero.)

The G6 exchange rate (the weighted average exchange rate, defined in the data section) appears to be less noisy than the individual exchange rates. In all of our tests, the standard errors of the coefficient estimates are smaller for the G6 exchange rate than for the individual country exchange rates, suggesting that some idiosyncratic movements in country exchange rates get smoothed out when we look at averages. Table 1 reports that the 90% confidence interval for this exchange rate lies well below one, with a point estimate of -1.467.[8]

[8] The intercept coefficient, on the other hand is very near zero, and the 90% confidence interval easily contains zero.

### 2.2 Fama Regressions in Real Terms

The Fama regression in real terms can be written as:

$$q_{t+1} - q_t = \zeta_q + \beta_q\left(\hat{r}_t - \hat{r}_t^*\right) + u_{q,t+1} \tag{9}$$

In this regression, $\hat{r}_t - \hat{r}_t^*$ refers to estimates of the ex ante real interest rate differential, $r_t - r_t^* \equiv i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)$. We estimate the real interest rate from VARs. As noted above, we consider two different VAR models. Model 1 is a VAR with 3 lags in the variables $q_t$, $i_t - i_t^*$, and $i_{t-1} - i_{t-1}^* - (\pi_t - \pi_t^*)$. From the VAR estimates, we construct measures of $E_t\left(i_t - i_t^* - (\pi_{t+1} - \pi_{t+1}^*)\right) = r_t - r_t^*$. Model 2 is a 3-lag VAR in $q_t$, $i_t - i_t^*$, and $\pi_t - \pi_t^*$.

There are two senses in which our measures of $\hat{r}_t - \hat{r}_t^*$ are estimates. The first is that the parameters of the VAR are estimated. But even if the parameters were known with certainty, we would still only have estimates of $r_t - r_t^*$ because we are basing our measures of $r_t - r_t^*$ on linear projections. Agents certainly have more sophisticated methods of calculating expectations, and use more information than is contained in our VAR.

The findings for the Fama regression in real terms are similar to those when the regression is estimated on nominal variables. For four of the six currencies, the estimates of $\beta_q$, reported in Table 2A, are negative, and all are less than one. In addition, the estimated coefficient for the G6 aggregate is close to -1. This summary is true for both VAR models.

Table 2A reports three sets of confidence intervals. All of the subsequent tables also report three sets of confidence intervals for each parameter estimate. The first is based on Newey-West standard errors, ignoring the fact that $\hat{r}_t - \hat{r}_t^*$ is a generated regressor. The second two are based on bootstraps. The first bootstrap uses percentile intervals and the second percentile-t intervals.[9]

From Table 2A, all three sets of confidence intervals are similar. For the individual currencies, for both Model 1 and Model 2, the confidence interval for $\beta_q$ lies below one for Germany, Japan, and the U.K. It contains one for Canada and Italy, and contains one for France except using the second confidence interval.

[9] See Hansen (2010). The Appendix describes the bootstraps in more detail.

The findings are clear using the G6 average exchange rate: the coefficient estimate is $-0.93$ when the real interest estimate comes from Model 1, and $-0.91$ when Model 2 is used. All of the confidence intervals lie below one, though they all contain zero. For both models, the estimate of $\zeta_q$ is very close to zero, and all confidence intervals contain zero.

In summary, the evidence on the interest parity puzzle is similar in real terms as in nominal terms. The estimate of the coefficient $\beta_q$, tends to be negative and there is strong evidence that it is less than one. Even in real terms, the country with the higher interest rate tends to have short-run excess returns (i.e., excess returns and the interest rate differential are positively correlated.)

The Fama regression finds a strong negative correlation between $s_{t+1} - s_t$ and $i_t - i_t^*$. It is well known that for the currencies of low-inflation, high-income countries, $s_{t+1} - s_t$ is highly correlated with $q_{t+1} - q_t$, which suggests $q_{t+1} - q_t$ is negatively correlated with $i_t - i_t^*$. Since $i_t - i_t^* = r - r_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*)$, for exploratory reasons we consider a regression of $q_{t+1} - q_t$ on $\hat{r}_t - \hat{r}_t^*$ and $\hat{E}_t(\pi_{t+1} - \pi_{t+1}^*)$, where the latter is our measure of the expected inflation differential generated from the VARs. These regressions are reported in Table 2B. Specifically, Table 2B reports the estimation of:

$$q_{t+1} - q_t = \zeta_q + \beta_{q1}\left(\hat{r}_t - \hat{r}_t^*\right) + \beta_{q2}\hat{E}_t\left(\pi_{t+1} - \pi_{t+1}^*\right) + u_{q,t+1} \tag{10}$$

The estimates of $\beta_{q1}$ tend to be negative, and generally more negative than those reported in Table 2A for equation (9). The real surprise from Table 2B is that the estimates of $\beta_{q2}$ are negative for all currencies in both models. Though they are not always significantly negative individually, and we do not calculate a test of their joint significance, it is nonetheless telling that all of the coefficient estimates are negative. This implies that the currency of the country that is expected to have relatively high inflation is expected to appreciate in real terms. We return to this finding in section 5.

### 2.3 The Real Exchange Rate, Real Interest Rates, and the Level Risk Premium

Table 3 reports estimates from

$$q_t = \zeta_q + \beta_q\left(\hat{r}_t - \hat{r}_t^*\right) + u_{q,t} \tag{11}$$

In all cases (all currencies, for both Model 1 and Model 2), the coefficient estimate is negative. In virtually all cases, although the confidence intervals are wide, the coefficient is significantly negative.[10]

Recall from equation (7) that $q_t - \bar{q} = -R_t - \Lambda_t$, where $R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r})$ and $\Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$. If there were no excess returns, so that $q_t - \bar{q} = -R_t$, and $\hat{r}_t - \hat{r}_t^*$ were positively correlated with $R_t$, then there is a negative correlation between $q_t$ and $\hat{r}_t - \hat{r}_t^*$. That is, under uncovered interest parity, the high real interest rate currency tends to be stronger. For example, this is the implication of the Dornbusch-Frankel theory in which real interest differentials are determined in a sticky-price monetary model.

But we can make a stronger statement – there is a relationship between the real interest differential, $\hat{r}_t - \hat{r}_t^*$, and our measure of the level excess return, $\hat{\Lambda}_t$ (where $\hat{\Lambda}_t$ is our estimate of $\Lambda_t$ based on the VAR models.) Our central empirical finding is reported in Table 4. This table reports the regression:

$$\hat{\Lambda}_t = \zeta_\Lambda + \beta_\Lambda\left(\hat{r}_t - \hat{r}_t^*\right) + u_{\Lambda t} \quad (12)$$

[11]

In all cases, the estimated slope coefficient is positive. The 90 percent confidence intervals are wide, but with a few exceptions, lie above zero. The confidence interval for the G6 average strongly excludes zero. To get an idea of magnitudes, a one percentage point difference in annual rates between the home and foreign real interest rates equals a 1/12th percentage point difference in monthly rates. The coefficient of around 32 reported for the regression when we take the U.S. relative to the average of the other G7 countries translates into around a 2.7% effect on the level risk premium. That is, if the U.S. real rate increases one annualized percentage point above the real rate in the other countries, the dollar is predicted to be 2.7% stronger in real terms from the level risk premium effect.

This finding is surprising in light of the well-known uncovered interest parity puzzle. In the previous two subsections, we have documented that when $r_t - r_t^*$ is above average, the Home currency tends to have excess returns. That seems to imply that the high interest rate currency is the riskier currency. But the estimates from equation (12) deliver the opposite message – the high interest rate currency has the lower level risk premium. $\Lambda_t$ is the level risk premium for the Foreign currency – it is positively correlated with $r_t - r_t^*$, so it tends to be high when $r_t^* - r_t$ is low.

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10 The exceptions are that the third confidence interval contains zero for Model 1 for France, and Models 1 and 2 for the U.K.

11 To be precise, $\hat{\Lambda}_t$ is calculated as the difference between $q_t$ and our VAR estimate of $R_t$. To calculate our estimate of $R_t$, given by the infinite sum of equation (5), we demean $r_{t+j} - r_{t+j}^*$ by its sample mean. We use the sample mean rather than maximum likelihood estimate of the mean because it tends to be a more robust estimate.

We can write

$$\operatorname{cov}(\Lambda_t, r_t - r_t^*) \equiv \sum_{j=0}^{\infty} \operatorname{cov}[E_t(\lambda_{t+j}), r_t - r_t^*] \tag{13}$$

The short-run interest parity puzzle establishes that $\operatorname{cov}(\lambda_t, r_t - r_t^*) < 0$. Clearly if $\operatorname{cov}(\Lambda_t, r_t - r_t^*) > 0$, then we must have $\operatorname{cov}[E_t(\lambda_{t+j}), r_t - r_t^*] > 0$ for at least some $j > 0$. That is, in order for $\operatorname{cov}(\Lambda_t, r_t - r_t^*) > 0$, we must have a reversal in the correlation of the short-run risk premiums with $r_t - r_t^*$ as the horizon extends.

This is illustrated in Figure 2, which plots estimates of the slope coefficient in a regression of $\hat{E}_t(\lambda_{t+j-1})$ on $\hat{r}_t - \hat{r}_t^*$ for $j = 1, \ldots, 100$:

$$\hat{E}_t(\lambda_{t+j-1}) = \zeta_{\lambda j} + \beta_{\lambda j}(r_t - r_t^*) + u_{\lambda,t}^j$$

For the first few *j*, this coefficient is negative, but it eventually turns positive at longer horizons.

The Figure also plots the slope of regressions of $\hat{E}_t(r_{t+j-1} - r_{t+j-1}^*)$ on $\hat{r}_t - \hat{r}_t^*$ for $j = 1, \ldots, 100$:

$$\hat{E}_t(r_{t+j-1} - r_{t+j-1}^*) = \zeta_{rj} + \beta_{rj}(r_t - r_t^*) + u_{r,t}^j$$

These tend to be positive at all horizons.

The Figure also includes a plot of the slope coefficients from regressing $\hat{E}_t(q_{t+j} - q_{t+j-1})$ for $j = 1, \ldots, 100$:

$$\hat{E}_t(q_{t+j} - q_{t+j-1}) = \zeta_{qj} + \beta_{qj}(r_t - r_t^*) + u_{q,t}^j$$

Since $\hat{E}_t\left(q_{t+j} - q_{t+j-1}\right) = \hat{E}_t\left(r_{t+j-1} - r_{t+j-1}^*\right) + \hat{E}_t\left(\lambda_{t+j-1}\right)$, these regression coefficients are simply the sum of the other two regression coefficients that are plotted. In this case, the regression coefficients start out negative for the first few months, but then turn positive for longer horizons.

To summarize, when the Home real interest rate relative to the Foreign real interest rate is higher than average, the Home currency is stronger in real terms than average. Crucially, it is even stronger than would be predicted by a model of uncovered interest parity. Excess returns or the foreign exchange risk premium contribute to this strength. If Home's real interest rate is high – in the sense that the Home relative to Foreign real interest rate is higher than average – the level risk premium on the Foreign security is higher than average.

We can project the future path of the real exchange rate when Home real interest rates are high using the facts that the currency tends to be stronger than average (the finding of regression (12)), that it continues to appreciate in the short run (the famous puzzle, confirmed in the findings from regression (9)), and that the real exchange rate is stationary so it is expected to return to its unconditional mean. When the Home real interest rate is high, the Home currency is strong in real terms, and expected to get stronger in the short run. However, eventually it must be expected to depreciate back to its long run level.

One implication of these dynamics is similar to Jorda and Taylor's (2009) findings about forecasting nominal exchange rate changes. They find that the nominal interest differential can help to predict exchange rate changes in the short run: the high interest rate currency is expected to appreciate (contrary to the predictions of uncovered interest parity.) But the forecasts of the exchange rate can be enhanced by taking into account purchasing power parity considerations. The deviation from PPP helps predict movements of the nominal exchange rate as the real exchange rate adjusts toward its long-run level.

Figure 1 presents a slightly different perspective. This chart plots the slope coefficients from regressions of $-\hat{R}_{t+j}$ and $q_{t+j}$ on $r_t - r_t^*$ for the G6 average exchange rate.[12] That is, it plots the estimated slope coefficients from the regressions:

$$-\hat{R}_{t+j} = \zeta_{Rj} + \beta_{Rj}(r_t - r_t^*) + u_{R,t}^j$$

$$q_{t+j} = \zeta_{Qj} + \beta_{Qj}(r_t - r_t^*) + u_{Q,t}^j \, .$$

If interest parity held, the behavior of the real exchange rate should conform to the plot for $-\hat{R}_{t+j}$. That line indicates that the U.S. dollar tends to be strong in real terms when $r_t - r_t^*$ is high, and then is expected to depreciate back toward its long-run mean. The line for the regression of $q_{t+j}$ on $r_t - r_t^*$ shows three things: First, when $r_t - r_t^*$ is above average, the dollar tends to be strong in real terms, and much stronger than would be implied under uncovered interest parity. Second, when $r_t - r_t^*$ is above average, the dollar is expected to appreciate even more in the short run. This is the uncovered interest parity puzzle. Third, when $r_t - r_t^*$ is above average, the dollar is expected to reach its maximum

12 The plots for most of the other real exchange rates look qualitatively very similar.

appreciation after around 5 months, then to depreciate gradually. The line labeled "Model" is discussed in the next section.

We turn now to the implications of these empirical findings for models of the foreign exchange risk premium.

## 3. The Risk Premium

There is no reconciliation to the central puzzle of this paper - $\operatorname{cov}(\delta_t, r_t^d) < 0$ and $\operatorname{cov}(\Lambda_t, r_t^d) > 0$ - if the interest differential contains all relevant information for forecasting the exchange rate. That is, suppose the Fama regression equation (9) is treated as though it determines conditional expectations:

$$E_t q_{t+1} - q_t = \zeta_q + \beta_q (r_t - r_t^*) \tag{14}$$

If this interpretation were correct, then from equation (2), the risk premium is perfectly correlated with the real interest differential:

$$\lambda_t = r_t^* - r_t + E_t q_{t+1} - q_t = \zeta_q + (\beta_q - 1)(r_t - r_t^*) \tag{15}$$

The uncovered interest rate parity puzzle finds $\beta_q < 1$, so (15) implies $\lambda_t$ and $r_t - r_t^*$ are perfectly negatively correlated. It follows that, under this approach, $R_t \equiv \sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \bar{r})$ and $\Lambda_t \equiv \sum_{j=0}^{\infty} E_t (\lambda_{t+j} - \bar{\lambda})$ are perfectly negatively correlated. Since real interest rates are strongly positively serially correlated, so $\operatorname{cov}(R_t, r_t - r_t^*) > 0$, equation (15) must imply $\operatorname{cov}(\Lambda_t, r_t - r_t^*) < 0$ if $\beta_q < 1$. But the evidence of Table 4 shows the opposite, that $\operatorname{cov}(\Lambda_t, r_t - r_t^*) > 0$. The assumption embodied in equation (14) rules out our key empirical finding – that the correlation of the short-run risk premium and the level risk premium with $r_t - r_t^*$ are of opposite signs.

Taking as given the stronger condition that $\beta_q < 0$, there is a stronger implication. Iterating equation (14) forward,

$$q_t - \bar{q} = -\beta_q R_t \tag{16}$$

Then we must have that the real exchange rate, $q_t$, is perfectly positively correlated with $R_t$. Given the strong positive serial correlation of real interest rates, this implies we must have $\operatorname{cov}(r_t - r_t^*, q_t) > 0$. That is, we must have a positive correlation of the real interest differential with the real exchange rate, which contradicts the empirical evidence.

Two strands of the international finance literature are potentially in conflict. Models of the uncovered interest parity puzzle (interpreted as finding $\beta_q < 0$) that rely on the implicit assumption of equation (14) necessarily are at loggerheads with the literature that finds a currency tends to be stronger in real terms when its relative real interest rate is above the long-run average.

Figure 1 illustrates the problem. As already noted, this chart plots the slope coefficients from regressions of $-\hat{R}_{t+j}$ and $q_{t+j}$ on $r_t - r_t^*$ for the G6 average exchange rate – these are the lines labeled bRj and bQj, respectively. The third line, labeled Model, is an example of the theoretical regression coefficients of $q_{t+j} - \lim_{k\to\infty}\left(E_t q_{t+k}\right)$ on $r_t - r_t^*$ implied by the models that assume perfect correlation between the interest differential and the risk premium, which are discussed below in section 3.3.[13] The models are built to account for the empirical finding that the Home currency tends to appreciate in the short run when the home interest rate is high. But the models leave the correlation of the level of the real exchange rate and the real interest differential with the wrong sign, and imply monotonic adjustment rather than the hump-shaped dynamics apparent in the data.

Subsequently, we use the notation $\delta_t \equiv E_t q_{t+1} - q_t$ for the expected rate of real depreciation, and $r_t^d \equiv r_t - r_t^* - \bar{r}$ for the Home less Foreign short-term real interest differential. In essence, the assumption underlying equation (14) is that a single factor drives $\delta_t$ and $r_t^d$:

$$\delta_t = a_1 \phi_{1t} \tag{17}$$

$$\lambda_t = g_1 \phi_{1t} \tag{18}$$

$$r_t^d = c_1 \phi_{1t}, \quad c_1 \equiv a_1 - g_1 \tag{19}$$

so that

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[13] This line refers to the implied slope coefficient in the regression of $q_{t+i} - \lim_{j\to\infty}\left(E_t q_{t+j}\right)$ on $r_t^d$

$$\delta_t = \frac{a_1}{c_1} r_t^d = \beta_q r_t^d \tag{20}$$

as in equation (14).[14] Without loss of generality, we will assume $\mathrm{var}(\phi_{1t}) = 1$ and $a_1 > 0$.

A model that allows the short-run risk premium and level risk premium to covary with the real interest differential with opposite signs requires a model with at least two factors. However, while two factors are necessary, they are not sufficient.

As Fama noted, the finding of a negative coefficient in the "Fama regression" (8) implies that the variance of the risk premium is greater than the variance of expected depreciation: $\mathrm{var}(\lambda_t) > \mathrm{var}(\delta_t)$. The single-agent models of the risk premium have been built to embody this property. When there is a single factor driving the risk premium and expected depreciation, we must have $g_1 > a_1$. This condition is necessary for the slope coefficient in (20), $\frac{a_1}{c_1} = \frac{a_1}{a_1 - g_1}$, to be negative.

### 3.1 A Two-Factor Model

The problem arises from the fact that while $\mathrm{cov}(\lambda_t, r_t^d) < 0$, we have found $\mathrm{cov}(E_t \lambda_{t+j}, r_t^d) > 0$ for large enough *j*. In order to account for this, we need a model that allows at least two factors to drive real interest rates and risk premiums:

$$\delta_t = a_1 \phi_{1t} + a_2 \phi_{2t} \tag{21}$$

$$\lambda_t = g_1 \phi_{1t} + g_2 \phi_{2t} \tag{22}$$

$$r_t^d = c_1 \phi_{1t} + c_2 \phi_{2t} \qquad g_1 \equiv a_1 - c_1,\ g_2 \equiv a_2 - c_2 . \tag{23}$$

$\phi_{1t}$ and $\phi_{2t}$ are independent. Without loss of generality, we assume $a_1, a_2 > 0$, and $\mathrm{var}(\phi_{1t}) = \mathrm{var}(\phi_{2t}) = 1$.

Using these equations, we get:

$$\mathrm{cov}(\delta_t, r_t^d) = a_1 c_1 + a_2 c_2 = a_1(a_1 - g_1) + a_2(a_2 - g_2) \tag{24}$$

[14] We drop the constant terms hereinafter because they play no role in explaining the puzzles.

Models of the uncovered interest parity puzzle are designed to account for $\mathrm{cov}(\delta_t, r_t^d) < 0$. Given our normalizations, this requires either $g_1 > a_1$ or $g_2 > a_2$.

Define $\Phi_{1t} \equiv \sum_{j=0}^{\infty} E_t(\phi_{1t+j} - \bar{\phi}_1)$ and $\Phi_{1t} \equiv \sum_{j=0}^{\infty} E_t(\phi_{1t+j} - \bar{\phi}_1)$ Iterating (22) forward:

$$\Lambda_t = g_1\Phi_{1t} + g_2\Phi_{2t} \tag{25}$$

We find:

$$\mathrm{cov}(\Lambda_t, r_t^d) = (a_1 - g_1)g_1 \,\mathrm{cov}(\phi_{1t}, \Phi_{1t}) + (a_2 - g_2)g_2 \,\mathrm{cov}(\phi_{2t}, \Phi_{2t}) \tag{26}$$

Models typically assume positive serial correlation in the macroeconomic factors driving returns, so they have $\mathrm{cov}(\phi_{1t}, \Phi_{1t}) > 0$ and $\mathrm{cov}(\phi_{2t}, \Phi_{2t}) > 0$. These conditions are necessary for equation (23) to account for the real interest differential given that we see $\mathrm{cov}(R_t, r_t^d) > 0$. We have established that $\mathrm{cov}(\delta_t, r_t^d) < 0$ requires either $g_1 > a_1$ or $g_2 > a_2$. Since we have normalized $a_1, a_2 > 0$, then letting $g_1 > a_1$, our finding that $\mathrm{cov}(\Lambda_t, r_t^d) > 0$ requires $0 < g_2 < a_2$.

This condition is necessary for finding $\mathrm{cov}(\Lambda_t, r_t^d) > 0$, but we also need that $\phi_{2t}$ is more persistent than $\phi_{1t}$. Comparing (24) and (26), we also must have $\mathrm{cov}(\phi_{2t}, \Phi_{2t}) > \mathrm{cov}(\phi_{1t}, \Phi_{1t})$. The appendix establishes necessary and sufficient conditions in order to derive both $\mathrm{cov}(\delta_t, r_t^d) < 0$ and $\mathrm{cov}(\Lambda_t, r_t^d) > 0$.

The conclusion that $0 < g_2 < a_2$ is necessary is a challenge for the literature's models of risk premiums based on risk aversion of a representative agent. Those models formulate preferences in order to generate volatile risk premiums. Volatile risk premiums are important not only for understanding the uncovered interest parity puzzle, but also a number of other puzzles in asset pricing regarding returns on equities and the term structure.[15] In the single factor models, the condition that $g_1 > a_1$ delivers the necessary variation in the risk premium. The models are designed in such a way that the risk premium, $\lambda_t$, loads more heavily on the exogenous state variables that drive returns than does the expected rate of depreciation, $\delta_t$. The problem is that the condition $0 < g_2 < a_2$ requires that the risk premium respond less to $\phi_{2t}$ than does expected depreciation.

15 See for example Bansal and Yaron (2004).

### 3.2 Relation of Excess Returns to Stochastic Discount Factors

Here we briefly review the basic theory of foreign exchange risk premiums and relate the factors driving the risk premium to the state variables driving stochastic discount factors. See, for example, Backus et. al. (2001) or Brandt et. al. (2004).

Under the assumption of absence of arbitrage, there exists a stochastic discount factor, $M_{t+1}$ such that the returns on any asset *j* denominated in units of Home consumption satisfy $1 = E(_t M_{t+1} e^{r_{j,t+1}})$ for all *j*.[16] Applying this relationship to returns on Home and Foreign riskless real bonds, expressing returns in units of Home consumption, we have:

$$1 = e^{r_t} E_t M_{t+1} \tag{27}$$

and

$$1 = e^{r_t^*} E_t M_{t+1} D_{t+1} \tag{28}$$

where $D_{t+1} \equiv Q_{t+1} / Q_t$. Under log normality

$$r_t = -E_t m_{t+1} - \tfrac{1}{2} \operatorname{var}_t m_{t+1} \tag{29}$$

and, using $d_{t+1} = q_{t+1} - q_t$:

$$r_t^* = -E_t m_{t+1} - E_t d_{t+1} - \tfrac{1}{2} \operatorname{var}_t m_{t+1} - \tfrac{1}{2} \operatorname{var}_t d_{t+1} - \operatorname{cov}_t (m_{t+1}, d_{t+1}) \tag{30}$$

Taking differences, we get

$$\lambda_t = r_t^* - r_t + E_t d_{t+1} = -\tfrac{1}{2} \operatorname{var}_t d_{t+1} - \operatorname{cov}_t (m_{t+1}, d_{t+1}) \tag{31}$$

For returns expressed in units of the Foreign consumption basket, there exists a stochastic discount factor $M_{t+1}^*$ that satisfies

$$1 = e^{r_t^*} E_t M_{t+1}^* \tag{32}$$

[16] See Cochrane (2005), for example.

and

$$1 = e^{r_t} E_t M^*_{t+1} D^{-1}_{t+1} \tag{33}$$

Clearly for any $M_{t+1}$ that satisfies (28), there must me a $M^*_{t+1}$ that satisfies (32) defined by $M^*_{t+1} = M_{t+1} D_{t+1}$. Or,

$$d_{t+1} = m^*_{t+1} - m_{t+1} \tag{34}$$

In general, there is not a unique stochastic discount factor that satisfies equations (27) and (28) for returns in Home units, or (32) and (33) for returns in Foreign units. The discount factor is unique when markets are complete. The models we consider in the rest of section 3 assume complete markets.

From (31) and (34), we have:

$$\delta_t = E_t m^*_{t+1} - E_t m_{t+1} \tag{35}$$

$$\lambda_t = \tfrac{1}{2}(\operatorname{var}_t m_{t+1} - \operatorname{var}_t m^*_{t+1}) \tag{36}$$

We follow Backus et. al. (2001) and consider affine pricing models. The models we examine express the log of the discount factors, $m_{t+1}$ and $m^*_{t+1}$, as linear functions of state variables $z_{it}$:

$$-m_{t+1} = \delta + \sum_{i=1}^{k} \gamma_i z_{it} + \sum_{i=1}^{k} \eta_i z_{it}^{1/2} \varepsilon_{it+1} \tag{37}$$

$$-m^*_{t+1} = \delta^* + \sum_{i=1}^{k} \gamma^*_i z_{it} + \sum_{i=1}^{k} \eta^*_i z_{it}^{1/2} \varepsilon^*_{it+1} \tag{38}$$

We assume $\varepsilon_{it}$ and $\varepsilon^*_{it}$ are i.i.d. over time, with mean zero and variance equal to one. The $\varepsilon_{it}$ are mutually independent as are the $\varepsilon^*_{it}$, but $\varepsilon_{it}$ and $\varepsilon^*_{it}$ could be correlated for each *i*. We assume the factors follow the processes:

$$z_{it+1} = (1-\varphi_i)\theta_i + \varphi_i z_{it} + \upsilon_{it+1} \tag{39}$$

where $0 < \varphi_i < 1$, and $\upsilon_{it}$ are i.i.d. over time with mean zero and variance equal to one. Equations (37)-(39) are a special case of the general formulation in Backus et. al. (2001), but encompass all of the models we subsequently discuss. This formulation allows for independent factors to influence $m_{t+1}$ and $m^*_{t+1}$ because some of the $\gamma_i$, $\gamma^*_i$, $\eta_i$, and $\eta^*_i$ may be zero.

From these equations, we have:

$$\delta_t = E_t(m^*_{t+1} - m_{t+1}) = \delta - \delta^* + \sum_{i=1}^{k}(\gamma_i - \gamma^*_i) z_{it} \tag{40}$$

$$\lambda_t = \tfrac{1}{2}(\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^*_{t+1})) = \tfrac{1}{2}\sum_{i=1}^{k}(\eta_i^2 - \eta_i^{*2}) z_{it} \tag{41}$$

We can see that the risk premium and the expected depreciation in this setting are driven by the state variables that determine the stochastic discount factors. In section 3.1, we showed that to account for $\mathrm{cov}(\delta_t, r^d_t) < 0$ and $\mathrm{cov}(\Lambda_t, r^d_t) > 0$, we need *k* to equal at least two. If, for example, $k = 2$, and normalizing $\gamma_1 - \gamma^*_1 > 0$ and $\gamma_2 - \gamma^*_2 > 0$, the necessary conditions derived in the previous section can be expressed as $0 < \gamma_1 - \gamma^*_1 < \tfrac{1}{2}(\eta_1^2 - \eta_1^{*2})$ and $0 < \tfrac{1}{2}(\eta_2^2 - \eta_2^{*2}) < \gamma_2 - \gamma^*_2$.

The no-arbitrage conditions, equations (27)-(28) and (32)-(33) must hold under very general conditions, but they also do not by themselves give us much insight into the economic determinants of the risk premium. As Backus et. al. (2010) say, “It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency units: It is less a tautology that we can write down sensible stochastic processes for variables that are consistent with the carry trade evidence.” We next turn to models built on utility-maximizing representative agents with rational expectations in the Home and Foreign countries. In these models, $M_{t+1}$ and $M^*_{t+1}$ are the intertemporal marginal rate of substitutions for Home and Foreign agents, respectively.

In this case, we can interpret $\lambda_t$ as a risk premium that rewards investors for taking on undiversifiable risk. For Home agents, the excess return on Foreign bonds is given by $r^*_t - r_t + E_t d_{t+1} + \tfrac{1}{2}\mathrm{var}_t\, d_{t+1} = -\mathrm{cov}_t(m_{t+1}, d_{t+1})$. For Foreign investors, the excess return on Foreign bonds is $r^*_t - r_t + E_t d_{t+1} - \tfrac{1}{2}\mathrm{var}_t\, d_{t+1} = -\mathrm{cov}_t(m^*_{t+1}, d_{t+1})$. The excess return on Foreign bonds is not the same for Home and Foreign agents because the investors consider returns in different units. Intuition is aided by taking a simple average of these excess returns, which are equal to $\lambda_t$:

$$\lambda_t = -\mathrm{cov}_t\left(\frac{m_{t+1} + m^*_{t+1}}{2}, d_{t+1}\right) \tag{42}$$

As with any asset, the excess return is determined by the covariance of the return with the stochastic discount factor. If the foreign exchange return, $d_{t+1}$ is positively correlated with the Home discount factor, $m_{t+1}$, Home investors require a compensation for risk so the Foreign security has an excess return relative to the Home bond. For Foreign investors, the foreign exchange return on a Home bond is $-d_{t+1}$. If $-d_{t+1}$ is positively correlated with the Foreign discount factor, $m^*_{t+1}$, the Home asset is relatively risky, implying a lower excess return on the Foreign bond.

The challenge is to construct a model of the Home and Foreign representative agents' stochastic discount factors of the form of equations (37) and (38), with $k \geq 2$, where $0 < \gamma_i - \gamma_i^* < \frac{1}{2}(\eta_i^2 - \eta_i^{*2})$ and $0 < \frac{1}{2}(\eta_j^2 - \eta_j^{*2}) < \gamma_j - \gamma_j^*$ for some *i* and *j*.

### 3.3 Models of Foreign Exchange Risk Premium Based on the Stochastic Discount Factor

In this section, we examine models of the risk premium, $\lambda_t$, that are based on specifications of the stochastic discount factors derived from underlying models of preferences. Verdelhan (2010) builds a model based on the Campbell-Cochrane (1999) specification of external habit persistence to explain the familiar uncovered interest parity puzzle. The Appendix shows that this model cannot account for the main empirical findings of this paper because the risk premium and expected depreciation are driven by a single factor.

We focus attention on the "long-run risks" model of Bansal and Yaron (2004), based on Epstein-Zin (1989) preferences. Colacito and Croce (2011) have recently applied the model to understand several properties of equity returns, real exchange rates and consumption. Bansal and Shaliastovich (2010) and Backus, Gavazzoni, Telmer and Zin (2010) demonstrate how the "long-run risks" model based on can account for the interest-parity anomaly. We will see, however, that the formulations of the model considered in these papers are not able to account for the empirical findings of Section 2 of this paper, principally because they are designed to account for the finding that $\mathrm{var}(\lambda_t) > \mathrm{var}(\delta_t)$.

These papers directly extend equilibrium closed-economy models to a two-country open-economy setting. The closed economy models assume an exogenous stream of endowments, with consumption equal to the endowment. The open-economy versions assume an exogenous stream of consumption in each country. These could be interpreted either as partial equilibrium models, with consumption given but the relation between consumption and world output unmodeled. Or they could be interpreted as general equilibrium models in which each country consumes an exogenous stream of its own endowment and there is no trade between countries. Under the latter interpretation, the real exchange rate is a shadow price, since in the absence of any trade in goods, there can be no trade in assets that have any real payoff.

The consumption streams in each country are taken to follow unit root processes, as in the closed economy analogs, but with no assumption of cointegration. That implies that relative consumption levels and real exchange rates have unit roots, implications which do not have strong empirical support. However, relative real interest differentials and excess returns in these models are stationary. The moments that are of concern to us, $\mathrm{cov}(\lambda_t, r_t^d)$ and $\mathrm{cov}(\Lambda_t, r_t^d)$, are well defined, and the analysis of the necessary conditions for $\mathrm{cov}(\lambda_t, r_t^d) < 0$ and $\mathrm{cov}(\Lambda_t, r_t^d) > 0$ of section 3.1 applies.

Bansal and Shaliastovich (2010) apply the “long-run risks” model to the uncovered interest parity puzzle. In each country, households are assumed to have Epstein-Zin (1989) preferences. The Home agent’s utility is defined by the recursive relationship:

$$U_t = \left\{(1-\beta)C_t^\rho + \beta\left[E_t\left(U_{t+1}^\alpha\right)^{\rho/\alpha}\right]\right\}^{1/\rho} \tag{43}$$

In this relationship, $\beta$ measures the patience of the consumer, $1-\alpha$ is the degree of relative risk aversion, and $1/(1-\rho)$ is the intertemporal elasticity of substitution. Bansal and Shaliastovich focus on the case of $\alpha < \rho$, which corresponds to the case in which agents prefer an early resolution of risk, and in which the intertemporal elasticity of substitution is greater than one, $0 < \rho < 1$. A stronger assumption, $\alpha < 0$, is required to account for the uncovered interest parity puzzle, so we will adopt this assumption (as does the literature) in the subsequent discussion.

We will consider a somewhat more general version of the long-run risks model than is present in the literature, in order to explore channels through which it might be able to account for our empirical findings.

Assume an exogenous path for consumption in each country. In the Home country (with $c_t \equiv \ln(C_t)$):

$$c_{t+1} - c_t = \mu + l_t + \sqrt{u_t^h + u_t^c}\,\varepsilon_{t+1}^x \tag{44}$$

The conditional expectation of consumption growth is given by $\mu + l_t$. The component $l_t$ represents a persistent consumption growth modeled as a first-order autoregression:

$$l_{t+1} = \varphi_l l_t + \sqrt{w_t^h + w_t^c}\,\varepsilon_{t+1}^l \tag{45}$$

The innovations, $\varepsilon_{t+1}^{x}$ and $\varepsilon_{t+1}^{l}$ are assumed to be uncorrelated within each country, distributed $i.i.d.\ N(0,1)$, but each shock may be correlated with its Foreign counterpart ($\varepsilon_{t+1}^{*x}$ and $\varepsilon_{t+1}^{*l}$, which are mutually uncorrelated.)

In the Foreign country, we have:

$$c_{t+1}^{*} - c_{t}^{*} = \mu^{*} + l_{t}^{*} + \sqrt{u_{t}^{f} + u_{t}^{c}}\,\varepsilon_{t+1}^{*x} \tag{46}$$

$$l_{t+1}^{*} = \varphi_{l}^{*} l_{t+1}^{*} + \sqrt{w_{t}^{f} + w_{t}^{c}}\,\varepsilon_{t+1}^{*l} \tag{47}$$

The conditional variances are written as the sum of two independent components. The component with the *h* superscript is idiosyncratic to the Home country. An *f* superscript refers to the Foreign idiosyncratic component. The one with the *c* superscript is common to the Home and Foreign country. Conditional variances are stochastic and follow first-order autoregressive processes:

$$u_{t+1}^{i} = (1-\varphi_{u}^{i})\theta_{u}^{i} + \varphi_{u}^{i} u_{t}^{i} + \sigma_{u}^{i}\varepsilon_{t+1}^{iu},\ i = h, f, c \tag{48}$$

$$w_{t+1}^{i} = (1-\varphi_{w}^{i})\theta_{w}^{i} + \varphi_{w}^{i} w_{t}^{i} + \sigma_{w}^{i}\varepsilon_{t+1}^{iw}.\ i = h, f, c \tag{49}$$

The innovations, $\varepsilon_{t+1}^{iu}$ and $\varepsilon_{t+1}^{iw}$, $i = h, f, c$ are assumed to be uncorrelated, distributed *i.i.d.* with mean zero and unit variance.

We can log linearize the first-order conditions as in Hansen, Heaton, and Li (2005). We will ignore terms that are not time-varying or that do not affect both the conditional means and variances of the stochastic discount factors, lumping those variables into the catchall terms $\Xi_t$ and $\Xi_t^{*}$.

The Home discount factor is given by:

$$-m_{t+1} = \gamma_{u}^{r}(u_{t}^{h} + u_{t}^{c}) + \gamma_{w}^{r}(w_{t}^{h} + w_{t}^{c}) + \lambda_{x}^{r}\sqrt{u_{t}^{h} + u_{t}^{c}}\,\varepsilon_{t+1}^{x} + \lambda_{l}^{r}\sqrt{w_{t}^{h} + w_{t}^{c}}\,\varepsilon_{t+1}^{l} + \Xi_{t} \tag{50}$$

The Foreign discount factor is given by:

$$-m_{t+1}^{*} = \gamma_{u}^{*r}(u_{t}^{f} + u_{t}^{c}) + \gamma_{w}^{*r}(w_{t}^{f} + w_{t}^{c}) + \lambda_{x}^{*r}\sqrt{u_{t}^{f} + u_{t}^{c}}\,\varepsilon_{t+1}^{*x} + \lambda_{l}^{*r}\sqrt{w_{t}^{f} + w_{t}^{c}}\,\varepsilon_{t+1}^{*l} + \Xi_{t}^{*} \tag{51}$$

The parameters in these log-linearization are:

$$\gamma_u^r = \alpha(\alpha-\rho)/2 \qquad \gamma_u^{*r} = \alpha^*(\alpha^*-\rho^*)/2 \qquad \gamma_w^r = \alpha(\alpha-\rho)\omega_l^2/2 \qquad \gamma_w^{*r} = \alpha^*(\alpha^*-\rho^*)\omega_l^{*2}/2$$
$$\lambda_x^r = 1-\alpha \qquad \lambda_x^{*r} = 1-\alpha^* \qquad \lambda_l^r = -(\alpha-\rho)\omega_l \qquad \lambda_l^{*r} = -(\alpha^*-\rho^*)\omega_l^*$$
$$\omega_l = \beta/(1-\beta\varphi_l) \qquad \omega_l^* = \beta^*/(1-\beta^*\varphi_l^*)$$

Bansal and Shaliastovich assume that the long-run expected growth component of consumption, $l_t$, is the same in the Home and Foreign countries on the grounds that long-run growth prospects are nearly identical across countries.[17] They also assume identical parameters in the two countries. Under these assumptions, Bansal and Shaliastovich find:

$$\delta_t = \gamma_u^r(u_t^h - u_t^f) \tag{52}$$
$$\lambda_t = \tfrac{1}{2}(\lambda_x^r)^2(u_t^h - u_t^f) \tag{53}$$

Bansal and Shaliastovich (2010) assume agents have a preference for early resolution of risk, $\alpha < \rho$, and that the intertemporal elasticity of substitution is greater than one, which requires $0 < \rho < 1$. As Bansal and Shaliastovich (2010) explain, these parameter choices are needed in order for this model to account for variance asset pricing facts, such as the term structure of interest rates.[18] Further, Bansal and Shaliastovich assume $\alpha < 0$, in which case the model can generate $\mathrm{cov}(\delta_t, r_t^d) < 0$. But this restriction then implies $0 < \gamma_u^r < \tfrac{1}{2}(\lambda_x^r)^2$, so expected depreciation is less volatile than the risk premium. This in turn means that we must have $\mathrm{cov}(\Lambda_t, r_t^d) < 0$. Effectively, under this formulation, the risk premium and expected depreciation are driven by a single factor, $u_t^h - u_t^f$, but we have seen that a single factor model cannot explain both $\mathrm{cov}(\delta_t, r_t^d) < 0$ and $\mathrm{cov}(\Lambda_t, r_t^d) > 0$.

Backus et. al. (2010) do not impose the restriction that long-run expected growth in the Home country, $l_t$, is identically equal to the corresponding variable in the Foreign country, $l_t^*$. But they do assume identical parameters for Home and Foreign household preferences, and assume identical parameters in the stochastic processes for consumption growth. In this case, equations (52) and (53) generalize (ignoring terms involving $\Xi_t$ and $\Xi_t^*$) to:

$$\delta_t = \gamma_u^r(u_t^h - u_t^f) + \gamma_w^r(w_t^h - w_t^f) \tag{54}$$
$$\lambda_t = \tfrac{1}{2}(\lambda_x^r)^2(u_t^h - u_t^f) + \tfrac{1}{2}(\lambda_l^r)^2(w_t^h - w_t^f) \tag{55}$$

[17] As noted above, Bansal and Shaliastovich do not assume cointegration of the consumption processes, so shocks to the level of consumption in each country result in permanent level differences.

[18] See Bansal and Yaron (2004).

Assuming $\alpha < 0$ and $0 < \rho < 1$ as above, we find $\text{cov}(\delta_t, r_t^d) < 0$ is $\alpha < 0$ and $0 < \rho < 1$, but in this case the coefficients on $u_t^h - u_t^f$ and $w_t^h - w_t^f$ in the risk premium equation (55) are larger in absolute value than the coefficients in the equation for expected depreciation, (54). This can be seen because under these restrictions, $\gamma_u^r > 0$, $\gamma_w^r > 0$, $\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2 = -(1 - 2\alpha + \alpha\rho)/2 < 0$ and $\gamma_w^r - \frac{1}{2}(\lambda_l^r)^2 = -\rho(\rho - \alpha)\omega_l^2 / 2 < 0$. We obtain

$$r_t^d = \left(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2\right)(u_t^h - u_t^f) + \left(\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2\right)(w_t^h - w_t^f) \tag{56}$$

$$\Lambda_t = \tfrac{1}{2}(\lambda_x^r)^2 \left(\frac{u_t^h - u_t^f}{1 - \varphi_u}\right) + \tfrac{1}{2}(\lambda_l^r)^2 \left(\frac{w_t^h - w_t^f}{1 - \varphi_w}\right) \tag{57}$$

In equation (57), we have used the fact that Backus et. al. assume the persistence of the variance shocks is the same in the two countries, and the same for each component so that we can define $\varphi_u \equiv \varphi_u^h = \varphi_u^f = \varphi_u^c$ and $\varphi_w \equiv \varphi_w^h = \varphi_w^f = \varphi_w^c$. Therefore, the model also predicts $\text{cov}(\Lambda_t, r_t^d) < 0$ under these parameter assumptions, and contrary to the data.

The underlying difficulty with these models is that they have been designed to account for the empirical observation that $\text{var}(\lambda_t) > \text{var}(\delta_t)$. That relationship must hold, as Fama (1984) pointed out, for $\text{cov}(\delta_t, r_t^d) < 0$. In order to account for this relationship, the literature has engineered models in which $\lambda_t$ and $\delta_t$ react to the same exogenous factors, but $\lambda_t$ reacts more. In other words, preferences are specified so that if we have $\delta_t = a_1\phi_{1t} + a_2\phi_{2t}$ and $\lambda_t = g_1\phi_{1t} + g_2\phi_{2t}$, the models are designed so that the absolute values of $g_1$ and $g_2$ are greater than the absolute values of $a_1$ and $a_2$, respectively. We showed, however, that to account for our finding that $\text{cov}(\Lambda_t, r_t^d) > 0$, we need one of the $g_i$ to be smaller in absolute value than the corresponding $a_i$. The assumptions that are built in to account for a volatile risk premium preclude the possibility that $\text{cov}(\Lambda_t, r_t^d) > 0$.

We can consider more general assumptions than those in the literature, but the long-run risks model does not appear promising. The Appendix demonstrates the following propositions:

*Proposition A* Assume Home and Foreign households have identical preferences. Then under the model described in equations (46)-(51), with the parameter restrictions $\alpha < 0$ and $0 < \rho < 1$, we cannot have both $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$.

*Proposition B* Assume there are no common factors driving the variances of consumption growth: $u_t^c = w_t^c = 0$. Then under the model described in equations (46)-(51), with the parameter restrictions $\alpha < 0$ and $0 < \rho < 1$, we cannot have both $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$.

The assumptions of Proposition A are more general than even those in Backus et. al. (2010). That paper assumes that both preferences and the parameters of the stochastic processes driving consumption are identical in the Home and Foreign countries. Proposition A shows that even if we allow for different parameters in the processes for consumption (and the variances of consumption), the restrictions on parameters that allow the model to account for the interest parity puzzle preclude the model from delivering $\text{cov}(\Lambda_t, r_t^d) > 0$.

However, if we rely on taste differences across countries, the model might be able to account for the empirical findings of this paper. The Appendix derives two examples of parameter assumptions from the long-run risks model that will deliver $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$. With enough degrees of freedom to choose parameters in a model, that is not surprising, but the question remains open on whether the choice of preference parameters that make the model consistent with both $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$ are, first, empirically verifiable, and, second, consistent with other empirical regularities.

The consistency with other empirical regularities is perhaps called into question by Proposition B, which shows that the model with idiosyncratic factors only is not adequate. The preference parameters in the two countries must be chosen in a way that the loadings on common factors can deliver $\text{cov}(\Lambda_t, r_t^d) > 0$. The necessity of different loadings on common factors is strongly reminiscent of the findings of Lustig et. al. (2008). That paper finds that to account for the cross-sectional differences in risk premiums, in an affine pricing model, there must be differences in the responses of the stochastic discount factors across countries to common risk factors. However, as the Appendix shows, in order to explain $\text{cov}(\Lambda_t, r_t^d) > 0$, the long-run risks model must have the *opposite* relation as the one posited in Lustig et. al. (2008) to explain their cross-sectional findings. That is, in response to the common factor, we find low interest rate currencies must be less risky, which is the opposite of Lustig et. al.'s finding that low interest rate currencies need to load more on the common factor.

In short, existing models developed to account for the interest-parity puzzle are not adequate for understanding the empirical findings highlighted in this paper. In particular, there must first be some feature of the model that allows for the risk premium to be volatile in the short run, so that the risk premium reacts more to one of the state variables than does expected depreciation. Many models have been built to provide plausible accounts of this relationship, which implies that the high interest rate currency is riskier in the short run. Somehow, however, we also need models that can explain why in the long run, the high interest rate currency is expected to be less risky. The challenge is to provide a plausible mechanism that also is consistent with other empirical asset pricing regularities.

# 4. Other Issues

## 4.1 Whose Price Index?

The empirical approach taken in section 2 requires taking a stand on the appropriate price index used to deflate nominal returns for the Home and Foreign investor. In each country, we deflated nominal returns using the consumer price index measure of inflation. The theory of the risk premium discussed in section 3.3, however, applies to a representative agent, but the theory does not give us any guide as to which real world price index best represents the model's representative agent.

However, Engel (1993,1999) presents evidence that there is very little within-country variation in prices compared to the variation of the real exchange rate, at least for the U.S. relative to other advanced countries. The real exchange rate is given by $q_t = s_t + p_t^* - p_t$. In turn each log price index is a weighted average of individual consumer goods prices: $p_t = \sum_{i=1}^{N} w_i p_{it}$ , $p_t^* = \sum_{i=1}^{N} w_i^* p_{it}^*$ . The papers show, in essence, that there is very high correlation between $s_t + p_{it}^* - p_{it}$ for almost all goods, and these are very highly correlated with $q_t$. On the other hand, relative prices of goods within a country, $p_{it} - p_{jt}$, generally have much lower variance than $s_t + p_{it}^* - p_{it}$. The implication is that if we consider price indexes that use different weights than the CPI weights, the constructed real exchange rate will still be highly correlated with $q_t$.

This suggests that there probably is not much to be gained by ascribing some other price index to the representative investor. That is, changing the weights on the goods in the price index is unlikely to have much effect on the measurement of real returns on Home and Foreign assets for Home and Foreign investors.

## 4.2 The Method When Real Exchange Rates are Non-Stationary

If the real exchange rate is non-stationary, the empirical method used here can be adapted. The forward iteration that is the foundation of the empirical study, $q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t$, does not require that the real exchange rate be stationary. Instead, we could measure $\lim_{j\to\infty}\left(E_t q_{t+j}\right)$ as the permanent component of the real exchange rate. The level risk premium, $\Lambda_t$, could then be constructed as $-R_t - \left(q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right)\right)$.

The Appendix presented evidence that the real exchange rate is stationary, so there is no permanent component. Another approach, potentially, is to measure the permanent component using the Beveridge-Nelson (1981) decomposition, or some related method.[19] However, Engel (2000) discusses the problem of near observational equivalence of stationary and non-stationary representations of the real exchange rate. Suppose the real exchange rate is the sum of a pure random walk component, $\omega_t$, and a transitory component, $\rho_t$. Engel (2000) argues, based on an economic model and evidence from disaggregated prices, that it is plausible that U.S. real exchange rates contain a transitory component that itself is very persistent (though stationary) and very volatile (high innovation variance.) There may be a random walk component related to the relative price of nontraded goods, but this component has a low innovation variance. The transitory component, $\rho_t$, dominates the forecast variance of real exchange rates even for reasonably long horizons because it is so persistent and volatile.

In this case, there are two dangers in trying to separate a transitory component from the permanent component. On the one hand, even if the real exchange rate were stationary, so that $\omega_t = 0$, the econometrician may not reject a random walk because of the high persistence of $\rho_t$. The permanent-transitory decomposition might mistakenly determine that there is a permanent component that accounts for most of the variation of the real exchange rate, with little role for a transitory component.

The other danger is the opposite – that the econometrician uses powerful enough methods to detect the stationarity of $\rho_t$, but does not tease out the random-walk component, $\omega_t$. In this case, the econometrician might conclude that all movements in the real exchange rate are transitory (though quite persistent).

We have rejected a unit root in the real exchange rate, and so conclude that there is only a $\rho_t$ component. However, if the $\omega_t$ component has such a small innovation variance that it is undetectable, then for our purposes it is reasonable to measure the transitory component, $\rho_t$, by the actual real exchange rate. If Engel's (2000) characterization of U.S. real exchange rate dynamics is correct, then there is not much to be gained by trying to undertake a permanent-transitory decomposition, and analyzing the transitory component rather than the actual real exchange rate.

[19] See Morley, Nelson, and Zivot (2003) for a discussion of the relationship between the Beveridge-Nelson decomposition and more restrictive state-space decompositions.

### 4.3 The Term Structure

There are two possible ways to see connections between this study and studies of the term structure. First, is there a relationship between the findings here, and those of Alexius (2001) and Chinn and Meredith (2004) that interest-parity holds better at long horizons using long-term interest differentials? (Note that Bekaert, Wei, and Xing (2007) do not find evidence to support this claim.) The answer is no, not directly. Our study does not derive any relationship between long-term interest rates and exchange rate changes. It is critical to realize that the prospective real interest rate, $R_t$, is not a long-term rate but instead an infinite sum of expected short-term rates. The two differ because long-term interest rates incorporate a term premium. Our study has not offered any insights into the relationship between the term premium and exchange rates.[20]

Another connection is that there is an analogy to the uncovered interest parity puzzle in the term structure literature. The long-short yield differential can predict excess returns. However, the literature on the term structure does not have evidence such as that in Figure 2 – that the expected excess return reverses signs at some horizon. We can draw an analogy between the Fama regression for exchange rates and a version of the empirical work that establishes the term structure anomaly. Let $p_{t,n}$ be the log of the price of a bond with *n* periods to maturity at time *t,* that has a payoff of one (in levels) at maturity. If an investor holds that bond for one period, the return is $p_{t+1,n-1} - p_{t,n}$. The expected excess return is given by $\lambda_t^b = E_t p_{t+1,n-1} - p_{t,n} - r_t$, where $r_t$ is the return on a one-period bond. The yield to maturity of the bond with *n* periods to maturity is given by $y_{t,n} = -p_{t,n} / n$. Then consider the regression:

$$y_{t+1,n-1} - y_{t,n} - = \alpha + \beta \frac{1}{n-1}(y_{t,n} - r_t) + u_{t+1} \tag{58}$$

If the expectations hypothesis of the term structure held, we would find $\alpha = 0$ and $\beta = 1$. Instead, the empirical literature tends to find $\beta < 1$, and sometimes $\beta < 0$.[21] Equation (58) is equivalent to:

$$p_{t+1,n-1} - p_{t,n} - r_t = \alpha' + (1-\beta)(y_{t,n} - r_t) + u_{t+1}' \tag{59}$$

[20] To be sure, many papers, such as those of Verdelhan (2010) and Bansal and Shaliastovich (2010) build models that are meant to account for both the term premium and the uncovered interest parity puzzle.

[21] See for example Campbell and Shiller (1991) and Dai and Singleton (2002).

where $\alpha' = -\alpha(n-1)$, and $u_t' = -(n-1)u_t$. The expected value at time *t* of the left-hand side of this regression is $\lambda_t^b$, so $\beta < 1$ implies $\mathrm{cov}(\lambda_t^b, r_t - y_{t,nt}) < 0$. This is analogous to the finding in the foreign exchange literature that $\mathrm{cov}(\lambda_t, r_t^d) < 0$.

We can rewrite the equation for the risk premium and iterate forward to get:

$$p_{t,n} = -E_t \sum_{j=0}^{n-1} r_{t+j} - E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b \tag{60}$$

The equivalent to our finding in foreign exchange markets that $\mathrm{cov}(r_t^d, \Lambda_t) > 0$ would be evidence that $\mathrm{cov}(r_t - y_{t,nt}, E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b) > 0$. Just as $\mathrm{cov}(\Lambda_t, r_t^d) > 0$ requires $\mathrm{cov}(E_t \lambda_{t+j}, r_t^d) > 0$ for some *j*, a necessary condition for $\mathrm{cov}(E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b, r_t - y_{t,nt}) > 0$ is that $\mathrm{cov}(E_t \lambda_{t+j}^b, r_t - y_{n,t}) > 0$ for some *j*. However, so far such evidence has not been established in the term structure literature.

# 5. Conclusions

When a country's interest rate is relatively high, the country's short-term bonds tend to have a higher return than their counterpart in another country, at least among the U.S.-G7 country pairs. At the same time, when that country's interest rate is high, its currency tends to be very strong – stronger than would be implied under uncovered interest parity. The first empirical regularity suggests that the returns on the high-interest rate currency incorporate a risk premium, but the second empirical finding suggests the opposite – that the overall effect of currency risk is to strengthen the high-interest rate currency. These two facts are difficult to understand by looking only at the behavior of a particular risk-averse agent in each country. Section 3 demonstrates the difficulties to this approach.

There have been several recent studies that attempt to build theoretical models to account for the uncovered interest parity that do not rely on modeling the preferences of agents, but instead model the interaction of more than one group of agents. These papers build models that are designed to explain the first empirical fact – the uncovered interest rate parity puzzle.

Frankel and Froot (1990) introduce a model with two groups of agents, chartists and fundamentalists, each following ad hoc behavioral rules. The chartists base their asset choice on extrapolation of the exchange rate returns from the past, while the fundamentalists form their expectations based on a model of macroeconomic fundamentals. Neither group of agents have expectations that are rational – the fundamentalists expectations do not take into account the effect of the chartists on the equilibrium exchange rate.

Alvarez, Atkeson and Kehoe (2008) build a two-country model in which the fraction of agents in each country that participate in financial markets varies over time. In each country, agents are constrained to use their own local currency to purchase their consumption good which is also produced locally. In asset markets, agents can trade interest earning bonds denominated in their own currency for interest earning foreign currency bonds. They can also trade currencies in the asset market. The household is split into two parts – one that deals in the goods market and the other that deals in the asset market. For a fixed cost each period, the household can transfer money from the asset market to the goods market to be used for goods purchases. The fraction of households that transfer money each period is endogenous in equilibrium, and depends on the money growth rate. The risk premium in this model is linked to the variance of the stochastic discount factor of the “active” households – the ones that have paid the fixed cost to transfer money between asset and goods markets.

The model of Bacchetta and van Wincoop (2010) in which some agents are slow to adjust their portfolio produces dynamics that accord with some of the empirical findings of this paper. In the model, there is a group of agents that does not change its portfolio when the state of the economy changes. Portfolios adjust only when new agents enter the financial market in the overlapping generation model developed here. When the Home short-term interest rate is high, the Home currency appreciates, but only new entrants to the market choose portfolios based on the higher short-term interest rate. Over time, more agents enter the market, and the currency then is expected to appreciate further for several months before depreciating.

None of these papers explicitly delineates how the level of the real exchange rate is related to interest rates because they are primarily concerned with the short-run excess returns. However, the mechanism in these papers seems intuitively to be incompatible with our findings that the high-interest rate currency has a low level risk premium. These models essentially rely on underreaction to explain the exchange rate dynamics in response to an interest rate increase. When the interest rate rises, one group of agents does not react – the chartists in Frankel and Froot, and the inactive agents in Alvarez et. al. and in Bacchetta and van Wincoop. The second group of agents does not take full advantage of the profit opportunity that might arise, either because this group is itself not fully rational (the fundamentalists in Frankel and Froot) or because it is risk averse (the active agents in Alvarez et. al. and Bacchetta and van Wincoop.) These dynamics imply that in the short run, the currency appreciates less than it would if all agents were fully active. Apparently, the models cannot account for the fact that the currency initially appreciates more than it would under uncovered interest parity, which is a key finding of this paper. Bachetta and van Wincoop do compare the dynamics of the real exchange rate in their model compared to one in which all the portfolios are actively managed. The currency never appreciates in their model at any horizon as much as it would under active management, so the model does not deliver the key fact that the high-interest rate currency incorporates a level risk premium that strengthens the currency beyond its no-risk-premium level. Since none of the papers presents directly the implications of their models for the statistics derived in this paper, it is not clear how they perform in this regard, but at an intuitive level they do not seem well equipped to deal with the level risk premium puzzle.

Several recent papers have explored the implications for rare, large currency depreciations for the uncovered interest parity puzzle. Farhi and Gabaix (2009) present a full general equilibrium model of rare disasters and real exchange rates. Their model implies that when the Home real interest rate is high, the Home currency is weak in real terms, and so cannot account for the levels puzzle presented here.[22] This correlation occurs during "normal times" in their model – the anticipation of a future disaster leads to a simple positive correlation between the real interest differential and the real exchange rate. Nonetheless, there are two caveats that must be considered in light of Farhi and Gabaix and the related literature. The first is that if rare disasters are important, than the linear VAR technology used in this paper may not correctly capture the stochastic process for real exchange rates and real interest rates. Farhi et. al. (2009) and Burnside et. al. (2010a) extract information from options to infer expectations about rare large movements in exchange rates. Moreover, if these large rare events are important, then the lognormal approximations that lie behind our analysis of the risk premium in sections 3.3 and 3.4 are not correct. Higher order cumulants matter for the risk premium in that case.[23]

It may be that it is necessary to abandon the assumption that all agents have fully rational expectations. Some version of the model proposed by Hong and Stein (1999) may account for the empirical results uncovered here, which perhaps could be described as a combination of overreaction and momentum trading. That is, the short-term behavior of the real exchange rate under high interest rates incorporates overreaction in that the currency appreciates more than it would under interest parity. But perhaps momentum trading leads to expectations of further appreciation in the short run when the interest rate is high. Burnside et. al. (2010b), and Gourinchas and Tornell (2004), are recent approaches that have relaxed the assumption of full rationality in some way. Ilut (2010) adopts an optimizing approach in which ambiguity averse agents who are averse to uncertainty may underreact to good news and overreact to bad news.

Finally, Table 2b may contain a clue toward a model that helps to resolve the empirical puzzles. In that table, we find that an increase in Home relative to Foreign expected inflation tends to be associated with an expected real appreciation of the Home currency, holding the real interest differential constant. This regularity for the G7 country pairs is very different than what has been found to hold in relatively high inflation emerging economies.[24] In those countries, higher expected inflation in a country tends to be

[22] See also Guo (2009), Gourio et. al. (2010) and Gourinchas et. al. (2010).

[23] See Martin (2010).

[24] See Bansal and Dahquist (2000).

reflected in a higher expected nominal currency depreciation and a higher local nominal interest rate. The net effect is to make uncovered interest parity, in nominal terms, hold better empirically for emerging economies than developed countries. To contrast, here we find that higher expected Home inflation leads to an appreciation, not a depreciation, of the Home currency. Moreover, this expected appreciation is in real terms, and does not work through the effect on the current real interest differential (since the regression controls for the latter variable.) This suggests some sort of interaction between monetary policy and real exchange rates, but the connection to excess returns is not straightforward.

High real interest rates tend to strengthen a currency. That is common wisdom in foreign exchange markets. It fits the textbook description of exchange rate behavior, and is consistent with the empirical evidence in this paper and in other recent studies. This regularity cannot be ignored when we try to explain the uncovered interest parity puzzle. The high interest rate country may have short run excess returns (the uncovered interest parity puzzle), but it has a strong currency as well.

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**Table 1. Fama Regressions:** $s_{t+1} - s_t = \zeta_s + \beta_s (i_t - i_t^*) + u_{s,t+1}$

1979:6-2009:10

| *Country* | $\hat{\zeta}_s$ | 90% c.i. ($\hat{\zeta}_s$) | $\hat{\beta}_s$ | 90% c.i. ($\hat{\beta}_s$) |
|---|---|---|---|---|
| Canada | -0.045 | (-0.250,0.160) | -1.271 | (-2.355,-0.186) |
| France | -0.028 | (-0.346,0.290) | -0.216 | (-1.603,1.171) |
| Germany | 0.192 | (-0.136,0.520) | -1.091 | (-2.583,0.401) |
| Italy | 0.032 | (-0.325,0.389) | 0.661 | (-0.359,1.680) |
| Japan | 0.924 | (0.504,1.343) | -2.713 | (-4.036,-1.390) |
| United Kingdom | -0.410 | (-0.768,-0.051) | -2.198 | (-4.225,-0.170) |
| G6 | 0.054 | (-0.184,0.292) | -1.467 | (-3.164,0.231) |

Notes: 90 percent confidence intervals in parentheses.

**Table 2A. Fama Regression in Real Terms:** $q_{t+1} - q_t = \zeta_q + \beta_q\left(\hat{r}_t - \hat{r}_t^*\right) + u_{q,t+1}$

1979:6-2009:10

| | ***Model 1*** | | | | ***Model 2*** | | | |
|---|---|---|---|---|---|---|---|---|
| <u>*Country*</u> | $\hat{\zeta}_q$ | *90% c.i.(* $\hat{\zeta}_q$ *)* | $\hat{\beta}_q$ | *90% c.i.(* $\hat{\beta}_q$ *)* | $\hat{\zeta}_q$ | *90% c.i.(* $\hat{\zeta}_q$ *)* | $\hat{\beta}_q$ | *90% c.i.(* $\hat{\beta}_q$ *)* |
| Canada | 0.021 | (-0.170,0.212)<br>(-0.159,0.166)<br>(-0.135,0.157) | 0.138 | (-1.222,1.498)<br>(-1.908,1.632)<br>(-1.800,1.676) | 0.022 | (-0.170,0.214)<br>(-0.159,0.168)<br>(-0.136,0.165) | 0.149 | (-1.225,1.523)<br>(-1.827,1.683)<br>(-1.739,1.668) |
| France | -0.080 | (-0.399,0.239)<br>(-0.281,0.052)<br>(-0.294,0.054) | -0.576 | (-2.269,1.117)<br>(-2.240,0.719)<br>(-2.602,1.125) | -0.075 | (-0.397,0.247)<br>(-0.286,0.064)<br>(-0.290,0.064) | -0.526 | (-2.265,1.213)<br>(-1.964,0.733)<br>(-2.485,1.260) |
| Germany | -0.042 | (-0.341,0.257)<br>(-0.209,0.082)<br>(-0.207,0.081) | -0.837 | (-2.689,1.015)<br>(-3.458,0.313)<br>(-3.419,0.411) | -0.043 | (-0.341,0.255)<br>(-0.206,0.086)<br>(-0.205,0.079) | -0.912 | (-2.753,0.929)<br>(-3.622,0.252)<br>(-3.531,0.373) |
| Italy | 0.075 | (-0.234,0.384)<br>(-0.149,0.274)<br>(-0.124,0.261) | 0.640 | (-1.056,2.336)<br>(-1.136,2.087)<br>(-1.328,2.358) | 0.082 | (-0.229,0.393)<br>(-0.141,0.278)<br>(-0.116,0.276) | 0.733 | (-0.960,2.426)<br>(-1.038,2.115)<br>(-1.228,2.465) |
| Japan | 0.108 | (-0.201,0.417)<br>(0.008,0.336)<br>(0.015,0.326) | -1.314 | (-2.860,0.232)<br>(-3.300,0.254)<br>(-3.441,0.379) | 0.110 | (-0.197,0.418)<br>(0.006,0.325)<br>(0.012,0.326) | -1.358 | (-2.911,0.195)<br>(-3.167,0.111)<br>(-3.485,0.263) |
| United Kingdom | -0.241 | (-0.603,0.121)<br>(-0.574,-0.074)<br>(-0.611,-0.067) | -1.448 | (-3.042,0.146)<br>(-3.614,-0.127)<br>(-3.846,-0.039) | -0.228 | (-0.588,0.132)<br>(-0.555,-0.073)<br>(-0.588,-0.069) | -1.347 | (-2.915,0.221)<br>(-3.369,0.002)<br>(-3.725,0.027) |
| G6 | -0.048 | (-0.287,0.191)<br>(-0.210,0.080)<br>(-0.202,0.068) | -0.933 | (-2.548,0.682)<br>(-2.932,0.409)<br>(-3.005,0.527) | -0.047 | (-0.286,0.192)<br>(-0.189,0.082)<br>(-0.181,0.077) | -0.914 | (-2.511,0.683)<br>(-2.909,0.447)<br>(-2.861,0.588) |

Notes: 90 percent confidence interval in parentheses. The first confidence interval is based on Newey-West. The second two are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

**Table 2B. Fama Regression in Real Terms:** $q_{t+1} - q_t = \zeta_q + \beta_{q1}\left(\hat{r}_t - \hat{r}_t^*\right) + \beta_{q2}\hat{E}_t\left(\pi_{t+1} - \pi_{t+1}^*\right) + u_{q,t+1}$

1979:6-2009:10

| | | | ***Model 1*** | | | |
|---|---|---|---|---|---|---|
| *Country* | $\hat{\zeta}_q$ | *90% c.i.(* $\hat{\zeta}_q$ *)* | $\hat{\beta}_{q1}$ | *90% c.i.(* $\hat{\beta}_{q1}$ *)* | $\hat{\beta}_{q2}$ | *90% c.i.(* $\hat{\beta}_{q2}$ *)* |
| Canada | -0.001 | (-0.210,0.208) | -0.666 | (-2.005,0.673) | -3.362 | (-5.318,-1.406) |
| | | (-0.161,0.239) | | (-2.279,1.385) | | (-5.364,-1.214) |
| | | (-0.164,0.224) | | (-1.973,1.089) | | (-5.714,-1.330) |
| France | -0.078 | (-0.400,0.244) | -0.600 | (-2.316,1.116) | -0.560 | (-2.332,1.212) |
| | | (-0.320,0.076) | | (-2.183,0.779) | | (-2.462,1.816) |
| | | (-0.310,0.045) | | (-2.559,1.090) | | (-2.424,1.548) |
| Germany | 0.280 | (-0.110,0.670) | -0.598 | (-2.422,1.226) | -3.009 | (-5.179,-0.839) |
| | | (-0.092,0.631) | | (-2.523,1.314) | | (-5.715,-0.197) |
| | | (-0.046,0.675) | | (-2.415,1.257) | | (-5.702,-0.480) |
| Italy | 0.020 | (-0.335,0.375) | 0.581 | (-1.192,2.354) | -0.306 | (-1.332,0.720) |
| | | (-0.280,0.366) | | (-1.249,2.014) | | (-1.499,1.200) |
| | | (-0.246,0.315) | | (-1.544,2.411) | | (-1.390,0.996) |
| Japan | 0.975 | (0.465,1.485) | -2.612 | (-4.140,-1.084) | -3.956 | (-6.060,-1.852) |
| | | (0.510,1.527) | | (-4.131,-0.890) | | (-6.057,-1.889) |
| | | (0.574,1.505) | | (-4.029,-0.989) | | (-5.955,-1.668) |
| U.K. | -0.408 | (-0.763,-0.053) | -2.170 | (-3.943,-0.397) | -3.203 | (-7.334,0.928) |
| | | (-0.738,-0.096) | | (-3.791,-0.448) | | (-5.486,-0.438) |
| | | (-0.746,-0.149) | | (-3.918,-0.376) | | (-7.626,1.493) |
| G6 | 0.080 | (-0.190,0.350) | -1.299 | (-3.102,0.504) | -3.017 | (-5.567,-0.467) |
| | | (-0.143,0.271) | | (-3.008,0.343) | | (-5.270,-0.335) |
| | | (-0.132,0.268) | | (-3.221,0.655) | | (-5.844,-0.191) |

| | | | Model 2 | | | |
|---|---|---|---|---|---|---|
| Country | $\hat{\zeta}_q$ | 90% c.i.( $\hat{\zeta}_q$ ) | $\hat{\beta}_{q1}$ | 90% c.i.( $\hat{\beta}_{q1}$ ) | $\hat{\beta}_{q2}$ | 90% c.i.( $\hat{\beta}_{q2}$ ) |
| Canada | 0.000 | (-0.209,0.209) | -0.642 | (-1.983,0.699) | -3.405 | (-5.386,-1.424) |
| | | (-0.162,0.250) | | (-2.201,1.419) | | (-5.401,-0.874) |
| | | (-0.154,0.233) | | (-1.943,1.139) | | (-5.851,-0.934) |
| France | -0.074 | (-0.398,0.250) | -0.560 | (-2.322,1.202) | -0.631 | (-2.350,1.088) |
| | | (-0.313,0.084) | | (-2.034,0.794) | | (-2.406,1.402) |
| | | (-0.312,0.086) | | (-2.476,1.300) | | (-2.346,1.487) |
| Germany | 0.267 | (-0.121,0.655) | -0.674 | (-2.487,1.139) | -2.900 | (-5.058,-0.742) |
| | | (-0.057,0.598) | | (-2.711,1.102) | | (-5.586,-0.398) |
| | | (-0.021,0.630) | | (-2.577,1.146) | | (-5.611,-0.674) |
| Italy | 0.020 | (-0.337,0.368) | 0.668 | (-1.095,2.431) | -0.351 | (-1.387,0.685) |
| | | (-0.276,0.382) | | (-1.149,2.091) | | (-1.402,1.204) |
| | | (-0.254,0.343) | | (-1.409,2.424) | | (-1.386,0.840) |
| Japan | 0.961 | (0.451,1.471) | -2.635 | (-4.173,-1.096) | -3.881 | (-5.977,-1.785) |
| | | (0.510,1.493) | | (-4.119,-1.029) | | (-5.920,-1.820) |
| | | (0.551,1.484) | | (-4.126,-0.964) | | (-5.955,-1.668) |
| U.K. | -0.407 | (-0.761,-0.053) | -2.154 | (-3.922,-0.386) | -3.247 | (-7.325,0.831) |
| | | (-0.711,-0.105) | | (-3.750,-0.185) | | (-5.647,-0.586) |
| | | (-0.729,-0.148) | | (-3.866,-0.176) | | (-7.720,1.685) |
| **G6** | 0.081 | (-0.190,0.352) | -1.293 | (-3.086,0.500) | -3.034 | (-5.589,-0.479) |
| | | (-0.141,0.270) | | (-3.020,0.353) | | (-5.240,-0.317) |
| | | (-0.124,0.277) | | (-3.168,0.564) | | (-5.648,0.257) |

Notes: 90 percent confidence interval in parentheses. The first confidence interval is based on Newey-West. The second two are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

**Table 3. Regression of $q_t$ on $\hat{r}_t - \hat{r}_t^*$: $q_t = \zeta_q + \beta_q\left(\hat{r}_t - \hat{r}_t^*\right) + u_{q,t}$**

1979:6-2009:10

| | ***Model 1*** | | ***Model 2*** | |
|---|---|---|---|---|
| Country | $\hat{\beta}_q$ | *90% c.i.( $\hat{\beta}_q$ )* | $\hat{\beta}_q$ | *90% c.i.( $\hat{\beta}_q$ )* |
| Canada | -48.517 | (-62.15,-34.88) | -48.962 | (-62.73,-35.19) |
| | | (-94.06,-31.41) | | (-92.51,-33.11) |
| | | (-140.54,-27.34) | | (-139.93,-29.36) |
| France | -20.632 | (-32.65,-8.62) | -20.388 | (-32.42,-8.35) |
| | | (-44.34,-1.27) | | (-42.53,-3.73) |
| | | (-54.26,1.75) | | (-52.83,-0.46) |
| Germany | -52.600 | (-67.02.-38.18) | -52.738 | (-67.10,-38.37) |
| | | (-85.97,-25.35) | | (-85.87,-25.88) |
| | | (-105.29,-19.38) | | (-105.62,-19.06) |
| Italy | -39.101 | (-51.92,-26.28) | -39.550 | (-52.46,-26.64) |
| | | (-67.63,-16.36) | | (-67.29,-17.78) |
| | | (-90.01,-13.70) | | (-87.39,-15.45) |
| Japan | -19.708 | (-29.69,-9.72) | -19.669 | (-29.72,-9.61) |
| | | (-42.01,-1.05) | | (-42.79,-0.92) |
| | | (-46.53,-4.33) | | (-46.23,-3.94) |
| United Kingdom | -18.955 | (-31.93,-5.98) | -18.387 | (-31.01,-5.76) |
| | | (-40.19,-3.08) | | (-38.63,-3.61) |
| | | (-55.94,4.08) | | (-52.82,4.95) |
| G6 | -44.204 | (-55.60,-32.80) | -44.032 | (-55.34,-32.72) |
| | | (-73.17,-23.62) | | (-74.89,-22.06) |
| | | (-82.87,-21.74) | | (-82.93,-22.75) |

Notes: 90 percent confidence interval in parentheses. The first confidence interval is based on Newey-West. The second two are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

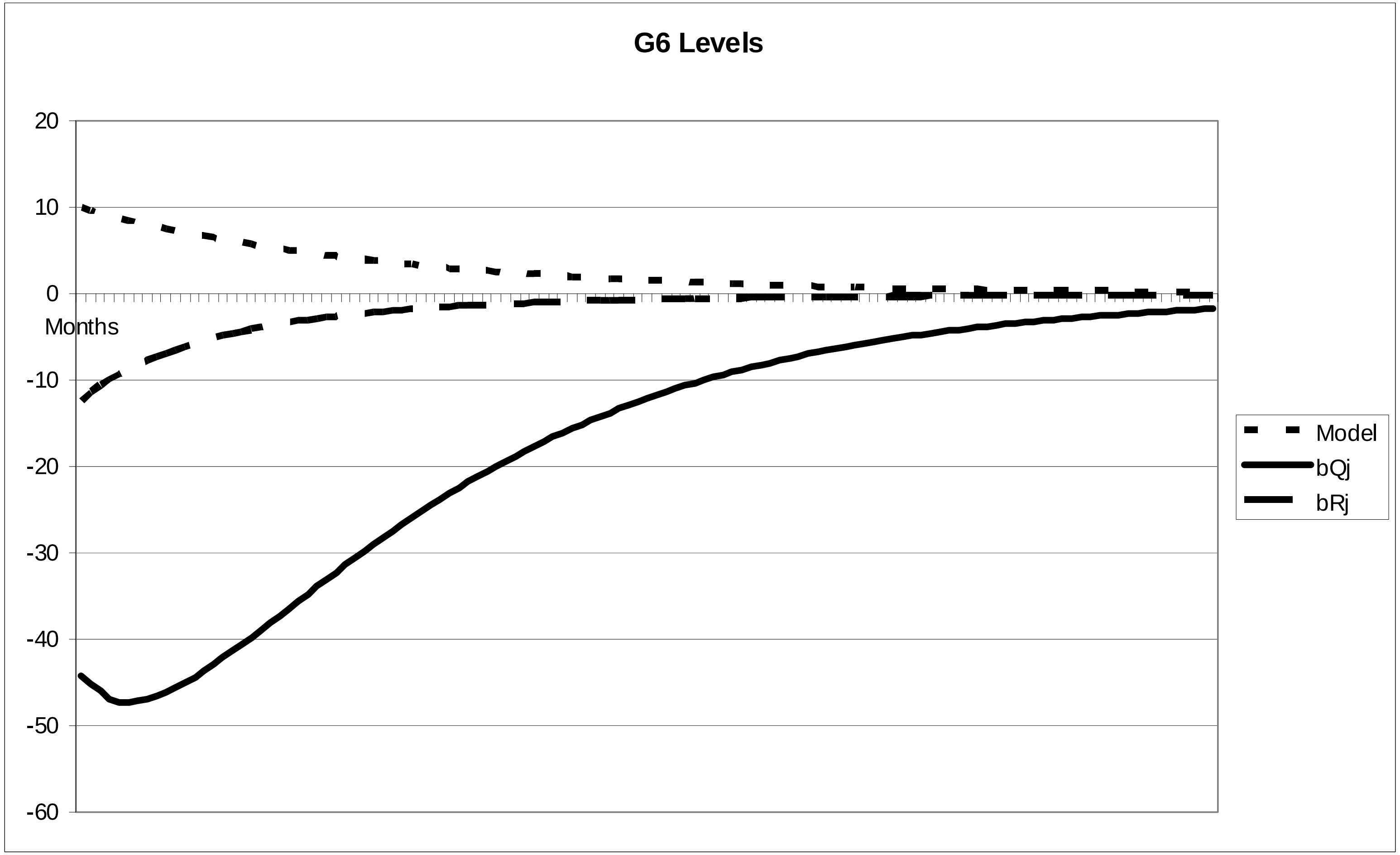
**Table 4. Regression of $\hat{\Lambda}_t$ on $\hat{r}_t - \hat{r}_t^*$: $\hat{\Lambda}_t = \zeta_\Lambda + \beta_\Lambda\left(\hat{r}_t - \hat{r}_t^*\right) + u_{\Lambda t}$**

1979:6-2009:10

| | *Model 1* | | *Model 2* | |
|---|---|---|---|---|
| Country | $\hat{\beta}_\Lambda$ | *90% c.i.(* $\hat{\beta}_\Lambda$*)* | $\hat{\beta}_\Lambda$ | *90% c.i.(* $\hat{\beta}_\Lambda$*)* |
| Canada | 23.610 | (15.12,32.10) | 24.192 | (15.64,32.75) |
| | | (12.62,51.96) | | (13.35,53.16) |
| | | (11.96,63.71) | | (12.99,71.13) |
| France | 13.387 | (1.06,25.72) | 14.045 | (1.84,26.25) |
| | | (-2.56,36.25) | | (0.80,35.39) |
| | | (-6.98,42.40) | | (-3.60,41.27) |
| Germany | 34.722 | (19.66,49.78) | 34.816 | (19.77,49.87) |
| | | (9.34,57.59) | | (10.30,59.11) |
| | | (3.68,69.36) | | (5.701,73.54) |
| Italy | 27.528 | (17.58,37.48) | 28.400 | (18.40,38.40) |
| | | (14.98,48.32) | | (16.00,48.83) |
| | | (12.51,58.54) | | (13.26,57.41) |
| Japan | 15.210 | (4.76,25.66) | 15.208 | (4.71,25.70) |
| | | (-0.45,37.08) | | (-0.99,37.77) |
| | | (0.91,38.87) | | (1.50,38.48) |
| United Kingdom | 14.093 | (0.33,27.86) | 13.575 | (0.17,26.98) |
| | | (0.39,34.46) | | (-0.11,33.13) |
| | | (-8.70,46.45) | | (-8.70,44.32) |
| G6 | 31.876 | (20.62,43.13) | 31.876 | (20.78,42.97) |
| | | (16.89,54.62) | | (17.39,55.49) |
| | | (16.78,60.89) | | (16.33,59.36) |

Notes: 90 percent confidence interval in parentheses. The first confidence interval is based on Newey-West. The second two are bootstrapped. The first reports a percentile interval bootstrap and the second a percentile-t interval bootstrap. See Appendix for details.

**Figure 1.**




The line labeled bRj plots estimates of $\beta_{Rj}$ from the regression $-\hat{R}_{t+j} = \zeta_{Rj} + \beta_{Rj}(r_t - r_t^*) + u_{R,t}^j$

The line labeled bQj plots estimates of $\beta_{Qj}$ from the regression $q_{t+j} = \zeta_{Qj} + \beta_{Qj}(r_t - r_t^*) + u_{Q,t}^j$

The line labeled Model plots the regression coefficient of $q_{t+j}$ on $r_t - r_t^*$ implied by a class of models discussed in Section 3

**Figure 2.**

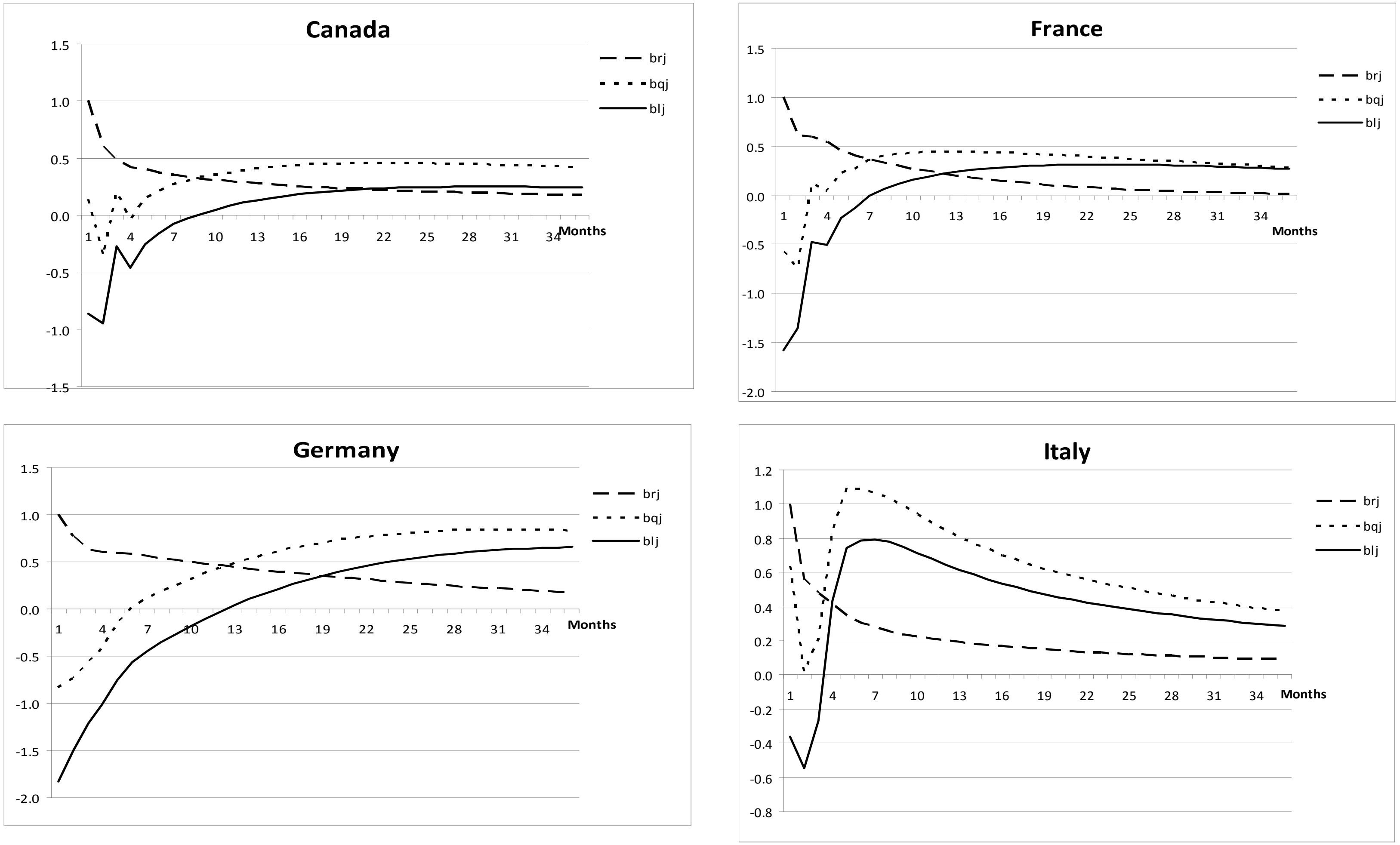
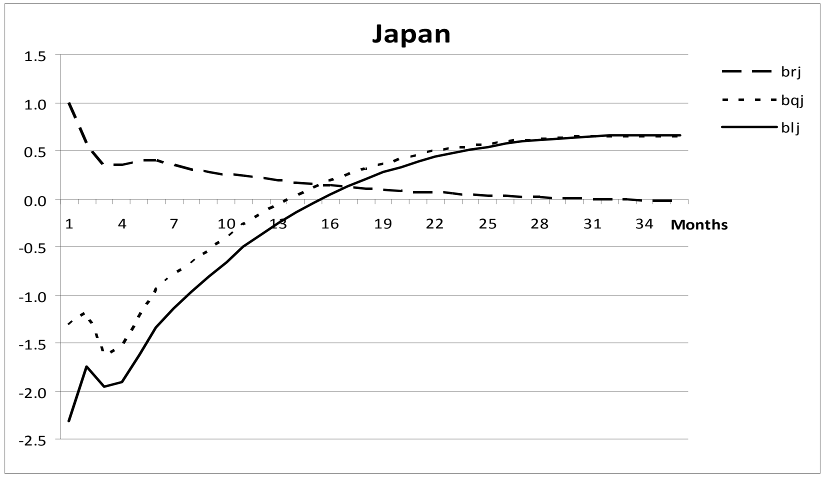
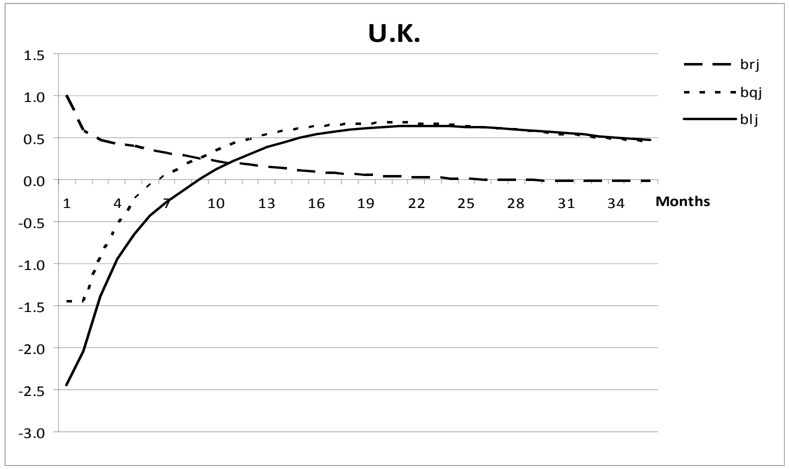
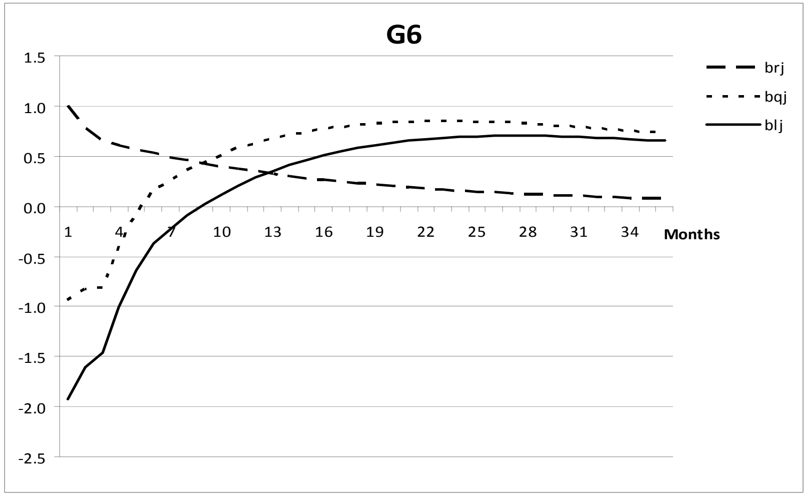


**Figure 2 (Continued)**




Figures plot the slope coefficients of these regressions:

brj refers to slope in $\hat{E}_t(r_{t+j-1} - r^*_{t+j-1}) = \zeta_{rj} + \beta_{rj}(r_t - r^*_t) + u^j_{r,t}$

bqj refers to slope in $\hat{E}_t(q_{t+j} - q_{t+j-1}) = \zeta_{qj} + \beta_{qj}(r_t - r^*_t) + u^j_{q,t}$

blj refers to slope in $\hat{E}_t(\lambda_{t+j-1}) = \zeta_{\lambda j} + \beta_{\lambda j}(r_t - r^*_t) + u^j_{\lambda,t}$

# Appendix

**Appendix to Section 2. Evidence on Stationarity of Real Exchange Rates**

Table A1 presents standard ADF tests for a unit root. The null is not rejected for any currency except the U.K. pound at the 10 percent level. The table also includes tests for a unit root based on the GLS test proposed by Elliott et. al. (1996). These tests show stronger evidence against a unit root – the null is rejected at the 5% level for three currencies, at the 10% level for two others, and not rejected for the Canadian dollar or Japanese yen. However, the test statistic is based on the assumption that there may be a trend in the real exchange rate under the alternative, which is not a realistic assumption for these real exchange rates.

We next follow much of the recent literature on testing for a unit root in real exchange rates by exploiting the power from panel estimation. The lower panel of Table A1 reports estimates from a panel model. The null model in this test is:

$$q_{it} - q_{it-1} = \mu_i + \sum_{j=1}^{k_i} c_i (q_{it-j} - q_{it-j-1}) + \varepsilon_{it} \tag{1}$$

Under the null, the change in the real exchange rate for country *i* follows an autoregressive process of order $k_i$. Note that the parameters and the lag lengths can be different across the currencies. Under the alternative:

$$q_{it} - q_{it-1} = \mu_i + \alpha q_{it-1} + \sum_{j=1}^{k_i} c_i (q_{it-j} - q_{it-j-1}) + \varepsilon_{it} \tag{2}$$

with a common $\alpha$ for the currencies.

We estimate $\alpha$ for the six currencies from (2).[25] We find the lag length for each currency by first estimating a univariate version of (2), and using the BIC criterion. The estimated value of $\alpha$ is reported in the lower panel of Table 1, in the row labeled “no covariates”.

This table also reports the bootstrapped distribution of $\alpha$. The bootstrap is constructed by estimating (1), then saving the residuals for the six real exchange rates for each time period. We then construct 5000 artificial time series (each of length 440, corresponding to our sample of 440 months) for the real exchange rate by resampling the residuals and using the estimates from (1) to parameterize the model.

[25] We do not include the average G6 real exchange rate as a separate real exchange rate in this test.

The lower panel of Table A1, in the row labeled “no covariates” reports certain points of the distribution of $\alpha$ from the bootstrap. We see that we can reject the null of a unit root at the 5 percent level.

We also consider a version of the panel test in which we include covariates. Specifically, we investigate the possibility that the inflation differential (with the U.S.) helps account for the dynamics of the real exchange rate. We follow the same procedure as above, but add lagged own relative inflation terms to equation (2). To generate the distribution of the estimate of $\alpha$, we estimate a VAR in the change in the real exchange rate (as in (1)) and the inflation rate. For each country, the real exchange rate and inflation rates depend only on own-country lags under the null. The bootstrap proceeds as in the model with no covariates.

The bottom panel of Table A1 reports the estimated $\alpha$ and its distribution for the model with covariates in the row labeled “with covariates”. Adding covariates does not alter the conclusion that we can reject a unit root at the 5 percent level.

Based on these tests, we will proceed to treat the real exchange rate as stationary, though we note that the evidence favoring stationarity is thin for the Canadian dollar and Japanese yen real exchange rates.

**Table A1. Tests for Unit Root in Real Exchange Rates**

Univariate Unit Root Tests, 1973:3-2009:10

| *Country* | *ADF* | *DF-GLS* |
|---|---|---|
| Canada | -1.771 | -1.077 |
| France | -2.033 | -2.036* |
| Germany | -2.038 | -2.049* |
| Italy | -1.888 | -1.914† |
| Japan | -2.071 | -0.710 |
| United Kingdom | -2.765† | -2.076* |
| G6 | -2.052 | -1.846† |

* significant at 5% level, † significant at 10% level

Panel Unit Root Test, 1973:3-2009:10

| *Model* | *Estimated Coefficient* | *1%* | *5%* | *10%* |
|---|---|---|---|---|
| No Covariates | -0.01705* | -0.02199 | -0.01697 | -0.01485 |
| With Covariates | -0.01703* | -0.02174 | -0.01697 | -0.01455 |

* significant at 5% level

**Appendix to Section 2.2: Bootstraps**

For both bootstraps in the results reported in Tables 2A, 2B, 3, and 4, we construct pseudo-samples using the VAR estimates.[26] For each pseudo-sample, we estimate the VAR. We estimate all of the regression coefficients reported in Tables 2A, 2B, 3 and 4, and calculate the Newey-West standard errors for each of those regressions. We repeat this exercise 1000 times.

The first confidence interval based on the bootstraps (the second confidence interval reported for each coefficient estimate) uses the coefficient estimates reported in the tables. Let $\hat{\beta}$ refer to any of the coefficient estimates reported in Tables 2A, 2B, 3 and 4. From the regressions on the pseudo-samples, we order the coefficient estimates from these 1000 replications from smallest to largest - $\hat{\beta}_1$ is the smallest and $\hat{\beta}_{1000}$ be the largest. The confidence interval reported in the tables is based on $[\hat{\beta} - \left(\hat{\beta}_{950} - \hat{\beta}\right), \hat{\beta} + \left(\hat{\beta} - \hat{\beta}_{50}\right)]$. That is, the reported confidence interval corrects for the asymmetry in the distribution of $\hat{\beta}_i$ from the regressions on the pseudo-samples.

Hansen (2010) argues that the first bootstrap method performs poorly when the $\hat{\beta}_i$ do not have a symmetric distribution. Instead, he recommends the following procedure. As above, let $\hat{\beta}$ refer to the estimated coefficient in the data, and $\hat{\sigma}$ to be the Newey-West standard error in the data. For each pseudo-sample *i*, we will record analogous estimates: $\hat{\beta}_i$ and $\hat{\sigma}_i$. $\theta_i$ is defined by: $\theta_i = \dfrac{\hat{\beta}_i - \hat{\beta}}{\hat{\sigma}_i}$. We arrange these $\theta_i$ from smallest to largest, so that $\theta_1$ is the smallest and $\theta_{1000}$ is the largest. The third confidence interval reported for each coefficient estimate is given by $[\hat{\beta} - \hat{\sigma}\theta_{950}, \hat{\beta} - \hat{\sigma}\theta_{50}]$. It turns out that our two bootstraps generally produce very similar confidence intervals.

[26] Initial values are set at the sample means. We generate samples of 865 observations, then use the last 365 observations, corresponding to the length of the time series we use in estimation.

**Appendix to Section 3.1: Necessary and Sufficient Conditions for $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$**

The model is given by equations (21)-(23) and (25). Assume the necessary conditions developed in the paper, $g_1 > a_1$ and $0 < g_2 < a_2$, are satisfied. We now derive necessary and sufficient conditions for $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$.

From equation (24), $\text{cov}(\delta_t, r_t^d) < 0$ requires $\dfrac{a_2 \,\text{var}(\phi_{2t})}{a_1 \,\text{var}(\phi_{1t})} < \dfrac{g_1 - a_1}{a_2 - g_2}$.

From equation (26), $\text{cov}(\Lambda_t, r_t^d) > 0$ requires $\dfrac{g_1 - a_1}{a_2 - g_2} < \dfrac{g_2 \,\text{cov}(\phi_{2t}, \Phi_{2t})}{g_1 \,\text{cov}(\phi_{1t}, \Phi_{1t})}$.

Combining these two conditions gives us the necessary and sufficient conditions for $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$:

$$\frac{a_2 \,\text{var}(\phi_{2t})}{a_1 \,\text{var}(\phi_{1t})} < \frac{g_1 - a_1}{a_2 - g_2} < \frac{g_2 \,\text{cov}(\phi_{2t}, \Phi_{2t})}{g_1 \,\text{cov}(\phi_{1t}, \Phi_{1t})} \tag{3}$$

must hold, as well as $g_1 > a_1$ and $0 < g_2 < a_2$.

In the models based on utility maximization that we consider, the factors $\phi_{1t}$ and $\phi_{2t}$ follow first-order autoregressions:

$$\phi_{it+1} = (1 - \varphi_i)\theta_i + \varphi_i \phi_{it} + \sigma_i \varepsilon_{it+1}$$

where $0 < \varphi_i < 1$, and $\varepsilon_{it} \sim NID(0, 1)$. In this case, condition (3) becomes

$$\frac{a_2}{a_1} < \frac{g_1 - a_1}{a_2 - g_2} \frac{\text{var}(\phi_{1t})}{\text{var}(\phi_{2t})} < \frac{g_2 (1 - \varphi_1)}{g_1 (1 - \varphi_2)} \tag{4}$$

**Appendix to Section 3.3: Verdelhan's (2010) Model Using Campbell-Cochrane Preferences**

In Verdelhan (2010) there are two symmetric countries. The objective of Home household *i* is to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j (C_{i,t+j} - H_{t+j})^{1-\gamma} / (1-\gamma) \tag{5}$$

where $\gamma$ is the coefficient of relative risk aversion, and $H_t$ represents an external habit. $H_t$ is defined implicitly by defining the "surplus", $s_t \equiv \ln\left((C_t - H_t)/C_t\right)$, where $C_t$ is aggregate consumption, and $s_t$ is assumed to follow the stochastic process:

$$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \mu(s_t)(c_{t+1} - c_t - g)\text{ , } 0 < \phi < 1 \tag{6}$$

Here, $\phi$ and $\overline{s}$ are parameters, and $c_t \equiv \ln(C_t)$ is assumed to follow a simple random walk:

$$c_{t+1} = g + c_t + u_{t+1}\text{, where } u_{t+1} \sim i.i.d.\ N\left(0, \sigma^2\right) \tag{7}$$

$\mu(s_t)$ represents the sensitivity of the surplus to consumption growth, and is given by:

$$\mu(s_t) \equiv \frac{1}{\overline{S}}\sqrt{1-2(s_t - \overline{s})} - 1\text{, when } s_t \le s_{\max}\text{ , 0 elsewhere.} \tag{8}$$

The log of the stochastic discount factor is given by:

$$m_{t+1} = \ln(\beta) - \gamma\left[g + (\phi - 1)(s_t - \overline{s}) + (1 + \mu(s_t))(c_{t+1} - c_t - g)\right] \tag{9}$$

When the parameters $\overline{S}$ and $s_{\max}$ are suitably normalized, Verdelhan shows we can write the expected rate of depreciation as:

$$\delta_t = -\gamma(1-\phi)(s_t - s_t^*) \tag{10}$$

where $s_t^*$ is the Foreign surplus. The excess return is given by:

$$\lambda_t = -(\gamma^2\sigma^2/\bar{S}^2)(s_t - s_t^*) \tag{11}$$

Under the assumption of Verdelhan (2010) that $\gamma(1-\phi) < \gamma^2\sigma^2/\bar{S}^2$, this model can account for the empirical finding of $\text{cov}(\lambda_t, r_t^d) < 0$. However, this assumption that the risk premium responds more to the current state, $s_t - s_t^*$, than the expected depreciation will not allow us to account for $\text{cov}(\Lambda_t, r_t^d) > 0$.

**Appendix to Section 3.3: Proofs of Propositions A and B**

*Proposition A*

Now consider the generalization of Backus et. al. (2010) in which all preference parameters are assumed to be identical across the two countries ($\alpha = \alpha^*$, $\rho = \rho^*$, $\beta = \beta^*$), but the stochastic processes for the random variables are not assumed identical, the model still cannot account for both $\text{cov}(\delta_t, r_t^d) < 0$ and $\text{cov}(\Lambda_t, r_t^d) > 0$.

We will prove this result for the more general assumptions that $\frac{\gamma_u^{*r}}{\gamma_u^r} = \left(\frac{\lambda_x^{*r}}{\lambda_x^r}\right)^2$ and $\frac{\gamma_w^{*r}}{\gamma_w^r} = \left(\frac{\lambda_l^{*r}}{\lambda_l^r}\right)^2$. The case of identical preferences is a special case. When preferences are identical, the relevant parameters become:

$$\begin{array}{lllll}
\gamma_u^r = \alpha(\alpha-\rho)/2 & \gamma_u^{*r} = \gamma_u^r & \gamma_w^r = \alpha(\alpha-\rho)\omega_l^2/2 & \gamma_w^{*r} = \gamma_w^r(\omega_l^{*2}/\omega_l^2) & \omega_l = \beta/(1-\beta\varphi_l) \\
\lambda_x^r = 1-\alpha & \lambda_x^{*r} = \lambda_x^r & \lambda_l^r = -(\alpha-\rho)\omega_l & \lambda_l^{*r} = \lambda_l^r(\omega_l^*/\omega_l) & \omega_l^* = \beta/(1-\beta\varphi_l^*)
\end{array}$$

It is clear in this case that $\frac{\gamma_u^{*r}}{\gamma_u^r} = \left(\frac{\lambda_x^{*r}}{\lambda_x^r}\right)^2$ since both ratios equal one. We can also see under identical preferences that $\frac{\gamma_w^{*r}}{\gamma_w^r} = \left(\frac{\lambda_l^{*r}}{\lambda_l^r}\right)^2 = \left(\frac{\omega_l^{*r}}{\omega_l^r}\right)^2 \frac{\omega_l^{*r}}{\omega_l^r}$.

For convenience, define $\gamma_{uR} \equiv \frac{\gamma_u^{*r}}{\gamma_u^r} = \left(\frac{\lambda_x^{*r}}{\lambda_x^r}\right)^2$ and $\gamma_{wR} \equiv \frac{\gamma_w^{*r}}{\gamma_w^r} = \left(\frac{\lambda_l^{*r}}{\lambda_l^r}\right)^2$. We can write the equation for the log of the Foreign pricing kernel as:

$$-m_{t+1}^* = \gamma_u^r\gamma_{uR}(u_t^f + u_t^c) + \gamma_w^r\gamma_{wR}(w_t^f + w_t^c) + \lambda_x^{*r}\sqrt{u_t^f + u_t^c}\,\varepsilon_{t+1}^{*x} + \lambda_l^{*r}\sqrt{w_t^f + w_t^c}\,\varepsilon_{t+1}^{*l} + \Xi_t^* \tag{12}$$

We find:

$$\delta_t = \gamma_u^r (u_t^h - \gamma_{uR} u_t^f + (1-\gamma_{uR}) u_t^c) + \gamma_w^r (w_t^h - \gamma_{wR} w_t^f + (1-\gamma_{wR}) w_t^c) \tag{13}$$

$$2\lambda_t = (\lambda_x^r)^2 (u_t^h - \gamma_{uR} u_t^f + (1-\gamma_{uR}) u_t^c) + (\lambda_l^r)^2 (w_t^h - \gamma_{wR} w_t^f + (1-\gamma_{wR}) w_t^c) \tag{14}$$

$$r_t^d = (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2)(u_t^h - \gamma_{uR} u_t^f + (1-\gamma_{uR}) u_t^c) + (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2)(w_t^h - \gamma_{wR} w_t^f + (1-\gamma_{wR}) w_t^c) \tag{15}$$

$$\begin{aligned} 2\Lambda_t = (\lambda_x^r)^2 &\left( \left( \frac{1}{1-\varphi_u^h} \right) u_t^h - \left( \frac{\gamma_{uR}}{1-\varphi_u^f} \right) u_t^f + \left( \frac{1-\gamma_{uR}}{1-\varphi_u^c} \right) u_t^c \right) \\ &+ (\lambda_l^r)^2 ( \left( \frac{1}{1-\varphi_w^h} \right) w_t^h - \left( \frac{\gamma_{wR}}{1-\varphi_w^f} \right) w_t^f + \left( \frac{1-\gamma_{wR}}{1-\varphi_w^c} \right) w_t^c ) \end{aligned} \tag{16}$$

From these equations, we derive:

$$\begin{aligned} \mathrm{cov}(\delta_t, r_t^d) = \gamma_u^r (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) \,\mathrm{var}(u_t^h - \gamma_{uR} u_t^f + (1-\gamma_{uR}) u_t^c) \\ + \gamma_w^r (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) \,\mathrm{var}(w_t^h - \gamma_{wR} w_t^f + (1-\gamma_{wR}) w_t^c) \end{aligned} \tag{17}$$

$$\begin{aligned} \mathrm{cov}(2\Lambda_t, r_t^d) &= (\lambda_x^r)^2 (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) \left( \left( \frac{1}{1-\varphi_u^h} \right) \mathrm{var}(u_t^h) + \left( \frac{(\gamma_{uR})^2}{1-\varphi_u^f} \right) \mathrm{var}(u_t^f) + \left( \frac{(1-\gamma_{uR})^2}{1-\varphi_u^f} \right) \mathrm{var}(u_t^f) \right) \\ &+ (\lambda_l^r)^2 (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) \left( \left( \frac{1}{1-\varphi_w^h} \right) \mathrm{var}(w_t^h) + \left( \frac{(\gamma_{wR})^2}{1-\varphi_w^f} \right) \mathrm{var}(w_t^f) + \left( \frac{(1-\gamma_{wR})^2}{1-\varphi_w^c} \right) \mathrm{var}(w_t^c) \right) \end{aligned} \tag{18}$$

Under the parameter assumptions of $\alpha < 0$ and $0 < \rho < 1$, we find $\gamma_u^r > 0$, $\gamma_w^r > 0$, $\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2 < 0$ and $\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2 < 0$. From (17) and (18), it is then clear that $\mathrm{cov}(\delta_t, r_t^d) < 0$ and $\mathrm{cov}(\Lambda_t, r_t^d) < 0$.

*Proposition B*

Next consider the case in which there are no common shocks, but allow Home and Foreign to have different preference parameters and different parameters in the equations describing the stochastic processes for consumption.

With no common shocks, the equations for the stochastic discount factors simplify to:

$$-m_{t+1} = \gamma_u^r u_t^h + \gamma_w^r w_t^h + \lambda_x^r \sqrt{u_t^h}\, \varepsilon_{t+1}^x + \lambda_l^r \sqrt{w_t^h}\, \varepsilon_{t+1}^l + \Xi_t \tag{19}$$

$$-m_{t+1}^* = \gamma_u^{*r} u_t^f + \gamma_w^{*r} w_t^f + \lambda_x^{*r} \sqrt{u_t^f}\, \varepsilon_{t+1}^{*x} + \lambda_l^{*r} \sqrt{w_t^f}\, \varepsilon_{t+1}^{*l} + \Xi_t^* \tag{20}$$

We then find:

$$\delta_t = \gamma_u^r u_t^h - \gamma_u^{*r} u_t^f + \gamma_w^r w_t^h - \gamma_w^{*r} w_t^f \tag{21}$$

$$2\lambda_t = (\lambda_x^r)^2 u_t^h - (\lambda_x^{*r})^2 u_t^f + (\lambda_l^r)^2 w_t^h - (\lambda_l^{*r})^2 w_t^f \tag{22}$$

$$r_t^d = (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) u_t^h - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2) u_t^f + (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) w_t^h - (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2) w_t^f \tag{23}$$

$$2\Lambda_t = \left(\frac{(\lambda_x^r)^2}{1-\varphi_u^h}\right) u_t^h - \left(\frac{(\lambda_x^{*r})^2}{1-\varphi_u^f}\right) u_t^f + \left(\frac{(\lambda_l^r)^2}{1-\varphi_w^h}\right) w_t^h - \left(\frac{(\lambda_l^{*r})^2}{1-\varphi_w^f}\right) w_t^f \tag{24}$$

From these equations, we conclude:

$$\begin{aligned} \operatorname{cov}(\delta_t, r_t^d) = {} & \gamma_u^r (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) \operatorname{var}(u_t^h) + \gamma_u^{*r} (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2) \operatorname{var}(u_t^f) \\ & + \gamma_w^r (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) \operatorname{var}(w_t^h) + \gamma_w^{*r} (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2) \operatorname{var}(w_t^f) \end{aligned} \tag{25}$$

$$\begin{aligned} \operatorname{cov}(2\Lambda_t, r_t^d) = {} & \left(\frac{(\lambda_x^r)^2}{1-\varphi_u^h}\right) (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) \operatorname{var}(u_t^h) + \left(\frac{(\lambda_x^{*r})^2}{1-\varphi_u^f}\right) (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2) \operatorname{var}(u_t^f) \\ & + \left(\frac{(\lambda_l^r)^2}{1-\varphi_w^h}\right) (\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) \operatorname{var}(w_t^h) + \left(\frac{(\lambda_l^{*r})^2}{1-\varphi_w^f}\right) (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2) \operatorname{var}(w_t^f) \end{aligned} \tag{26}$$

Under the parameter assumptions of $\alpha < 0$ and $0 < \rho < 1$, we find $\gamma_u^r > 0$, $\gamma_w^r > 0$, $\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2 < 0$ and $\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2 < 0$. Likewise, if $\alpha^* < 0$ and $0 < \rho^* < 1$, then $\gamma_u^{*r} > 0$, $\gamma_w^{*r} > 0$, $\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2 < 0$ and $\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2 < 0$. From (25) and (26), it is then clear that $\operatorname{cov}(\delta_t, r_t^d) < 0$ and $\operatorname{cov}(\Lambda_t, r_t^d) < 0$.

**Appendix to Section 3.3: Examples with Preference Differences in which $\operatorname{cov}(\delta_t, r_t^d) < 0$ and $\operatorname{cov}(\Lambda_t, r_t^d) > 0$**

It is clear from the previous proposition that we must introduce common shocks in order to account for $\operatorname{cov}(\Lambda_t, r_t^d) > 0$.

*Example 1*

One simple case is as in the model of Bansal and Shaliastovich (2010) in which $l_t = l_t^*$, but unlike this paper, we now allow differences in preferences and parameters in the stochastic processes driving consumption. In this case the equations for the stochastic discount factors simplify to

$$-m_{t+1} = \gamma_u^r (u_t^h + u_t^c) + \lambda_x^r \sqrt{u_t^h + u_t^c}\, \varepsilon_{t+1}^x + \Xi_t \tag{27}$$

$$-m_{t+1}^* = \gamma_u^{*r} (u_t^f + u_t^c) + \lambda_x^{*r} \sqrt{u_t^f + u_t^c}\, \varepsilon_{t+1}^{*x} + \Xi_t^* \tag{28}$$

Then we have:

$$\delta_t = \gamma_u^r u_t^h - \gamma_u^{*r} u_t^f + (\gamma_u^r - \gamma_u^{*r}) u_t^c \tag{29}$$

$$2\lambda_t = (\lambda_x^r)^2 u_t^h - (\lambda_x^{*r})^2 u_t^f + ((\lambda_x^r)^2 - (\lambda_x^{*r})^2) u_t^c \tag{30}$$

$$r_t^d = (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) u_t^h - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2) u_t^f + \left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right] u_t^c \tag{31}$$

$$2\Lambda_t = \left(\frac{(\lambda_x^r)^2}{1-\varphi_u^h}\right) u_t^h - \left(\frac{(\lambda_x^{*r})^2}{1-\varphi_u^f}\right) u_t^f + \left(\frac{(\lambda_x^r)^2 - (\lambda_x^{*r})^2}{1-\varphi_u^c}\right) u_t^c \tag{32}$$

We find:

$$\begin{aligned}\operatorname{cov}(\delta_t, r_t^d) &= \gamma_u^r (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2)\operatorname{var}(u_t^h) + \gamma_u^{*r} (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\operatorname{var}(u_t^f) \\ &\quad + \left(\gamma_u^r - \gamma_u^{*r}\right)\left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right]\operatorname{var}(u_t^c)\end{aligned} \tag{33}$$

$$\begin{aligned}\operatorname{cov}(2\Lambda_t, r_t^d) &= (\lambda_x^r)^2 (\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2)\frac{\operatorname{var}(u_t^h)}{1-\varphi_u^h} + (\lambda_x^{*r})^2 (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\frac{\operatorname{var}(u_t^f)}{1-\varphi_u^f} \\ &\quad + \left((\lambda_x^r)^2 - (\lambda_x^{*r})^2\right)\left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right]\frac{\operatorname{var}(u_t^c)}{1-\varphi_u^c}\end{aligned} \tag{34}$$

Under the parameter assumptions $\alpha < 0$ and $0 < \rho < 1$, and $\alpha^* < 0$ and $0 < \rho^* < 1$, we have found $\gamma_u^r > 0$, $\gamma_w^r > 0$, $\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2 < 0$ and $\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2 < 0$, and $\gamma_u^{*r} > 0$, $\gamma_w^{*r} > 0$, $\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2 < 0$ and $\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2 < 0$. Examining equation (33), it is clear that we can find $\operatorname{cov}(\delta_t, r_t^d) < 0$ if $\operatorname{var}(u_t^h)$ and/or $\operatorname{var}(u_t^f)$ are sufficiently large compared to $\operatorname{var}(u_t^c)$, irrespective of the sign of the expression that multiplies $\operatorname{var}(u_t^c)$.

Then, from equation (34), to find $\operatorname{cov}(\Lambda_t, r_t^d) > 0$, we need to have

$$\left((\lambda_x^r)^2 - (\lambda_x^{*r})^2\right)\left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right] > 0 \tag{35}$$

and $\varphi_u^c$ sufficiently large relative to $\varphi_u^h$ and $\varphi_u^f$. To demonstrate that we can find a set of coefficients that gives us (35), assume without loss of generality that $\alpha^* < \alpha$ (which are both less than zero.) Under these assumptions, $(\lambda_x^r)^2 - (\lambda_x^{*r})^2 < 0$. A necessary and sufficient condition for $(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2) < 0$ is $\alpha(2-\rho) < \alpha^*(2-\rho^*)$. These conditions then give us the inequality (35).

Note that the conditions $\alpha^* < \alpha$ and $\alpha(2-\rho) < \alpha^*(2-\rho^*)$ imply that $\rho$ must be small relative to $\rho^*$. We have assumed the home country is less risk averse ($1-\alpha < 1-\alpha^*$), so it must also have a lower intertemporal elasticity of substitution, $\frac{1}{1-\rho} < \frac{1}{1-\rho^*}$.

Under these assumptions, suppose there is a decline in the common shock $u_t^c$. Since $(\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \frac{1}{2}(\lambda_x^{*r})^2) < 0$, the Foreign interest rate rise relative to the Home interest rate ($r_t^d$ falls for positive values of $u_t^c$). Because $(\lambda_x^r)^2 - (\lambda_x^{*r})^2 < 0$, the risk premium on the Foreign bond (relative to the Home bond), $\lambda_t$, must fall. This is the opposite of the movements in relative interest rates and risk premium required to explain the cross-sectional portfolio results in Lustig, Roussanov, and Verdelhan (2008).

*Example 2*

Another example that can produce the result that $\text{cov}(\delta_t, r_t^d) < 0$ *and* $\text{cov}(\Lambda_t, r_t^d) > 0$ comes when we assume there are no idiosyncratic components. Then the equations for the stochastic discount factors simplify to

$$-m_{t+1} = \gamma_u^r u_t^c + \gamma_w^r w_t^c + \lambda_x^r \sqrt{u_t^c}\,\varepsilon_{t+1}^x + \lambda_l^r \sqrt{w_t^c}\,\varepsilon_{t+1}^l + \Xi_t \tag{36}$$

$$-m_{t+1}^* = \gamma_u^{*r} u_t^c + \gamma_w^{*r} w_t^c + \lambda_x^{*r} \sqrt{u_t^c}\,\varepsilon_{t+1}^{*x} + \lambda_l^{*r} \sqrt{w_t^c}\,\varepsilon_{t+1}^{*l} + \Xi_t^* \tag{37}$$

Then we have:

$$\delta_t = (\gamma_u^r - \gamma_u^{*r})u_t^c + (\gamma_w^r - \gamma_w^{*r})w_t^c \tag{38}$$

$$2\lambda_t = ((\lambda_x^r)^2 - (\lambda_x^{*r})^2)u_t^c + ((\lambda_l^r)^2 - (\lambda_l^{*r})^2)w_t^c \tag{39}$$

$$r_t^d = \left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right]u_t^c + \left[(\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) - (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2)\right]w_t^c \tag{40}$$

$$2\Lambda_t = \left(\frac{(\lambda_x^r)^2 - (\lambda_x^{*r})^2}{1-\varphi_u^c}\right)u_t^c + \left(\frac{(\lambda_l^r)^2 - (\lambda_l^{*r})^2}{1-\varphi_w^c}\right)w_t^c \tag{41}$$

We find:

$$\begin{aligned}\text{cov}(\delta_t, r_t^d) = &\left(\gamma_u^r - \gamma_u^{*r}\right)\left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right]\text{var}(u_t^c) \\ &+ \left(\gamma_w^r - \gamma_w^{*r}\right)\left[(\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) - (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2)\right]\text{var}(w_t^c)\end{aligned} \tag{42}$$

$$\begin{aligned}\operatorname{cov}(2\Lambda_t, r_t^d) &= \left((\lambda_x^r)^2 - (\lambda_x^{*r})^2\right)\left[(\gamma_u^r - \tfrac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \tfrac{1}{2}(\lambda_x^{*r})^2)\right]\frac{\operatorname{var}(u_t^c)}{1-\varphi_u^c} \\ &+ \left((\lambda_l^r)^2 - (\lambda_l^{*r})^2\right)\left[(\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) - (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2)\right]\frac{\operatorname{var}(w_t^c)}{1-\varphi_w^c}\end{aligned} \tag{43}$$

We will further assume that the persistence of consumption growth, $\varphi_l$, in the Home country is equal to the persistence of consumption growth, $\varphi_l^*$, in the Foreign country, and $\beta = \beta^*$, which imply $\omega_l = \omega_l^*$. The only parameters that are different between Home and Foreign are the ones determining relative risk aversion and the intertemporal elasticity of substitution, $\alpha \neq \alpha^*$ and $\rho \neq \rho^*$. Under these assumptions, we can show that if $\operatorname{cov}(\delta_t, r_t^d) < 0$, we may still be able to derive a set of parameter restrictions that give us $\operatorname{cov}(\Lambda_t, r_t^d) > 0$.

Assume as in the previous example that that $\alpha^* < \alpha$ (which are both less than zero.) Under these assumptions, $(\lambda_x^r)^2 - (\lambda_x^{*r})^2 < 0$. A necessary and sufficient condition for $(\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \frac{1}{2}(\lambda_x^{*r})^2) < 0$ is $\alpha(2-\rho) < \alpha^*(2-\rho^*)$. Under these assumptions, as in the previous example, we then have $\left((\lambda_x^r)^2 - (\lambda_x^{*r})^2\right)\left[(\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2) - (\gamma_u^{*r} - \frac{1}{2}(\lambda_x^{*r})^2)\right] > 0$. If $\frac{\operatorname{var}(u_t^c)}{1-\varphi_u^c}$ is sufficiently large relative to $\frac{\operatorname{var}(w_t^c)}{1-\varphi_w^c}$, we will have $\operatorname{cov}(\Lambda_t, r_t^d) > 0$ irrespective of the sign of the coefficient multiplying $\frac{\operatorname{var}(w_t^c)}{1-\varphi_w^c}$.

We need to show with these parameters that we can obtain $\operatorname{cov}(\delta_t, r_t^d) < 0$. We will do this by showing $\left(\gamma_w^r - \gamma_w^{*r}\right)\left[(\gamma_w^r - \frac{1}{2}(\lambda_l^r)^2) - (\gamma_w^{*r} - \frac{1}{2}(\lambda_l^{*r})^2)\right]$ can be negative. Then if $\operatorname{var}(w_t^c)$ is sufficiently large relative to $\operatorname{var}(u_t^c)$, we will have $\operatorname{cov}(\delta_t, r_t^d) < 0$. (This latter condition then requires $\varphi_u^c$ be greater than $\varphi_w^c$ for the condition in the previous paragraph, $\frac{\operatorname{var}(u_t^c)}{1-\varphi_u^c} > \frac{\operatorname{var}(w_t^c)}{1-\varphi_w^c}$, to hold.)

We can write:

$$\begin{aligned}&\left(\gamma_w^r - \gamma_w^{*r}\right)\left[(\gamma_w^r - \tfrac{1}{2}(\lambda_l^r)^2) - (\gamma_w^{*r} - \tfrac{1}{2}(\lambda_l^{*r})^2)\right] \\ &= \frac{\omega_l^2}{4}\left[\alpha(\alpha-\rho) - \alpha^*(\alpha^*-\rho^*)\right]\left[\rho(\alpha-\rho) - \rho^*(\alpha^*-\rho^*)\right]\end{aligned}$$

In order to get $\left[\alpha(\alpha-\rho)-\alpha^{*}(\alpha^{*}-\rho^{*})\right]\left[\rho(\alpha-\rho)-\rho^{*}(\alpha^{*}-\rho^{*})\right]<0$, we need only find parameters (for example) such that

$$\alpha(\alpha-\rho)-\alpha^{*}(\alpha^{*}-\rho^{*})<0$$
$$\rho(\alpha-\rho)-\rho^{*}(\alpha^{*}-\rho^{*})>0$$

and such that previous assumptions, $\alpha^{*}<\alpha<0$, $0<\rho<1$, $0<\rho^{*}<1$, and $\alpha(2-\rho)<\alpha^{*}(2-\rho^{*})$, are satisfied. For example, $\alpha^{*}=-1.5$, $\alpha=-1$, $\rho=0.1$, $\rho^{*}=0.9$ satisfy all of these conditions.