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Exchange Rates as Exchange Rate Common Factors*

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Abstract

Factor analysis performed on a panel of 23 nominal exchange rates from January 1999 to December 2010 yields three common factors. This paper identifies the euro/dollar, Swiss-franc/dollar and yen/dollar exchange rates as empirical counterparts to these common factors. These empirical factors explain a large proportion of exchange rate variation over time and have significant in-sample and out-of-sample predictive power.

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1. Introduction

The evolution of exchange rates through time is well described by a small number of common factors (Verdelhan, 2011) and these factors remain significant and quantitatively important after controlling for macroeconomic fundamental determinants of exchange rates (Engel, Mark and West, 2012). A further deepening of our understanding of exchange rates along these lines, however, is obstructed by a lack of identification of these common factors with variables that enter our economic models. This paper provides such an identification.

We identify the euro/dollar, Swiss-franc/dollar and yen/dollar exchange rates as empirical counterparts to three common factors extracted from a panel of 23 exchange rates against the U.S. dollar. Due to the euro's and yen's dominance in foreign exchange trading and the safe-haven role of the yen and the Swiss franc, our identification makes a certain amount of sense. Armed with this identification, we show that these empirical exchange rate factors can be usefully embedded in a prediction framework to produce forecasts that impressively beat the random walk with drift.[1] To partially preview our results, an out-of-sample forecasting exercise from June 2004 through December 2010 results in Theil's U-statistic values that lie below 1 for 70 percent of the currencies at the 3-month horizon and for 82 percent of the currencies at the 12-month horizon.

Ever since Meese and Rogoff (1983) initiated the research on out-of-sample fit/forecasting that has become standard procedure for exchange-rate model validation, work in this area has discovered (at least) three things. First, the particular time-period of the sample matters. Fundamentals-based models showed good ability to forecast exchange rates during the 1980s and early 1990s (Mark, 1995 and Chinn and Meese, 1995) but that predictive ability declined as observations from the 1990s and 2000s became available (Groen, 1999, Cheung et al., 2005). Second, the choice of fundamentals seems to matter. Earlier research focused on monetary and purchasing power parity (PPP) fundamentals and more recent work has incorporated monetary policy endogeneity via interest rate feedback rules (Molodtsova and Papell, 2009 and Molodtsova et al., 2008, 2011). Although there are institutional reasons to favor the Taylor-Rule approach, Engel, Mark and West (2007) conclude that while such models have some power to beat the random walk at long horizons, the results appear to be the strongest under PPP fundamentals. Third, sample size seems to matter. Rapach and Wohar (2001) and Lothian and Taylor (1996) report predictive power when working with relatively long time-series data sets by using observations extending back in time. To increase sample size while staying within the post Bretton Woods floating regime, a first-generation of papers (Mark and Sul, 2001, Rapach and Wohar, 2004, and Groen, 2005) found some predictive power using panel-data prediction methods.[2]

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1 In our data set, the random walk with drift is more difficult to beat than the driftless random walk in terms of mean squared prediction error.

2 The importance of cross-sectional information has been recognized since Bilson (1981) who used seemingly unrelated regression to estimate his exchange rate equation. Frankel and Rose (1996) initiated a literature on the panel data

We incorporate these lessons into the present paper first by sampling only exchange rates under the "euro" epoch. Forecasts of exchange rates since January 1999 have had more difficulty in beating the random walk than in some earlier periods so we are restricting our analysis to a relatively challenging time in terms of predictability. Second, we assess the value-added of the empirical factors approach by comparing it against the predictions of relatively successful PPP fundamentals. Third, we exploit panel data but in the fashion of recent work that has employed factor analysis. The importance and significance of the factors that we find after conditioning on the fundamentals suggests that there is a large body of "dark matter" that moves exchange rates and which is not accounted for in bi-lateral relations implied by two-country exchange rate models. But without an identification of the factors in terms of specific economic variables, it is not obvious how to address this dark matter. Hence, the identification provided by our paper can potentially help solve the exchange rate disconnect puzzle (Obstfeld and Rogoff, 2000) by informing future work on how to restructure exchange rate models.

The remainder of the paper is organized as follows. The next section develops the factor structure that underlies our analysis. In Section 3, we carry out the factor analysis on the exchange rate panel data. We find that the exchange rates in our sample are well described by three common factors. An in-sample analysis of the factors's explanatory power finds that they account for about two-thirds of the variance in exchange rate changes. The factor structure also implies that fundamentals-based predictive regressions employed in the literature suffer from omitted variables bias. The omitted variables are the common factors which are correlated with the fundamentals in a way that biases predictive tests of the null of no predictability towards an inability to reject the null of no predictability. We show that it is easier to reject the null with the in-sample test when one accounts for the factors. Section 4 carries out the identification of the empirical factors, develops a prediction framework that incorporates the empirical factors, and reports the results of an out-of-sample forecasting experiment. Section 5 concludes.

2. The Factor Structure

This section develops the factor structure that guides our empirical work. As in Engel, Mark and West (2012) but in contrast to other work with factors (e.g., Stock and Watson, 2002, 2006), our factors are extracted only from the exchange rate data and not from additional variables.

Let the log nominal exchange rates $\{s_{i,t}\}_{i=1}^{N}$ be driven by p common factors $\{f_{1,t}, f_{2,t}, \dots f_{p,t}\}$. Denote the $j-th$ factor loading for currency i by $\delta_{i,j}$ and let

$$F_{i,t} = \sum_{j=1}^{p} \delta_{i,j} f_{j,t}$$

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analysis of PPP, which is surveyed by Caporale and Cerrato (2006). Cerra and Saxena (2010) employed a panel data set with a large number (98) of countries in a study of the monetary model of exchange rates.

be the common exchange rate component for currency i. With this notation, nominal exchange rates have the factor structure

$$s_{i,t} = F_{i,t} + s_{i,t}^{o}. \tag{1}$$

We make the standard identifying restriction that the factors $\{f_{j,t}\}$ are mutually orthogonal and are orthogonal to the idiosyncratic component $s_{i,t}^{o}$. $s_{i,t}^{o}$ can either be a stationary process or, as is more likely the case, can be a unit-root process. We place further restrictions on $s_{i,t}^{o}$ as needed below.

Next, let the log real exchange rate between country i and the U.S. be

$$q_{i,t} = s_{i,t} + p_t^{*} - p_{i,t}, \tag{2}$$

where p^{*} is the log U.S. price level and $p_{i,t}$ is the log country i price level. Substituting (1) into (2) gives

$$q_{i,t} = F_{i,t} + q_{i,t}^{o}. \tag{3}$$

As an identifying restriction, we assume that the real exchange rate has the same factor structure as the nominal rate and that the idiosyncratic part of the real rate

$$q_{it}^{o} = s_{it}^{o} + p_t^{*} - p_{i,t} \sim I(0), \tag{4}$$

is a stationary process.

While it might appear that restricting $q_{i,t}$ and $s_{i,t}$ to have the identical factor structure is quite a strong assumption since it imposes orthogonality between price levels and the common factors driving nominal exchange rates, we will show below that it actually is not unreasonable. It is true that such an assumption would be indefensible if any of the countries experienced a hyper inflation during the sampling period, but that is not the case with our data. Price levels for the countries in our sample evolve relatively smoothly over time, unlike the exchange rate which behaves like an asset price. Secondly, a well known feature of real and nominal exchange rates is that their movements are highly correlated at both short and long horizons. Hence, imposing the identical factor structure on the real and nominal exchange rate, at least approximately, is not terribly unreasonable.

Although the factors are identical, the idiosyncratic components of the nominal and real exchange rates are allowed to differ. Looking at (4) and recalling that the idiosyncratic part of the real rate $q_{i,t}^o$ is covariance stationary, if relative price levels have a unit root, then $s_{i,t}^o$ also has a unit root and is cointegrated with $p_t^* - p_{i,t}$. Furthermore, if $s_{i,t}^o$ is not weakly exogenous, then its deviation from the relative price levels will have predictive power for future changes in $s_{i,t}^o$. We represent this idea using a $\beta_i > 0$ normalization with the restricted error-correction representation,

$$\Delta s_{i,t+1}^o = \alpha_i - \beta_i q_{i,t}^o + u_{i,t+1}. \quad (5)$$

Taking first differences of (1) and making use of (3), (4) and (5) gives

$$\Delta s_{i,t+1} = \alpha_i - \beta_i q_{i,t} + v_{i,t+1}, \quad (6)$$

where

$$v_{i,t+1} = \beta_i F_{i,t} + \Delta F_{i,t+1} + u_{i,t+1}. \quad (7)$$

Eq. (6) looks like an error-correction representation in which the deviation from PPP has predictive power for future changes in the nominal exchange rate. Restricting $\beta_i = \beta$ for all i is the PPP version of the panel short-horizon regression estimated by Mark and Sul (2001).

Under this factor structure, however, Mark and Sul's regression which treats $v_{i,t+1}$ as the regression error is subject to omitted variables bias because $E_t(q_{i,t} v_{i,t+1}) = \beta_i F_{i,t}^2 > 0$. The conditional correlation of the regression error with the real exchange rate causes the slope estimate to be biased towards zero. Hence, an econometrician who tests for predictive ability by regressing $\Delta s_{i,t+1}$ on $q_{i,t}$ and rejects the null hypothesis of no predictability if the t-ratio is sufficiently negative will confront a test that is biased towards an inability to reject the null. In our in-sample analysis, we will explicitly account for the factor structure in testing for predictive ability.

We briefly mention related work on exchange rates using factor analysis. Engel, Mark and West (2012) construct common factors from the exchange rates of 17 OECD countries. They assumed that $s_{i,t}^o \sim I(0)$ so that s_{it} is cointegrated with $F_{i,t}$ which they took to be a measure of the nominal exchange rate's central tendency.[3] Their analysis identified three common factors and employed them in the predictive regression

$$s_{i,t+k} - s_{i,t} = \alpha_i + \beta \underbrace{\left(F_{i,t} - s_{i,t}\right)}_{-s_{i,t}^0} + \varepsilon_{i,t+k}.$$

Using quarterly data from 1973 to 2007, they find that point predictions of the factor-based forecasts dominate random walk forecasts in mean-square error although they are not generally statistically significant. Lustig et al. (2011) are not interested in exchange rates per se but are interested in common factors driving excess currency returns (i.e., ex post deviations from uncovered interest parity) associated with the carry trade. In their analysis, their dominant factor is a global risk factor that is closely related to changes in volatility of equity markets around the world. Verdelhan (2011) extends those ideas to explaining exchange rate variation over time but he does not consider forecasting.

3. In-Sample Analysis

Our sample consists of 23 monthly exchange rates expressed as local currency prices of the U.S. dollar and consumer price indices of the associated countries.[4] We use the currencies of Australia, Brazil, Canada, Chile, Columbia, the Czech Republic, Denmark, the Euro, Hungary, Israel, Japan, Korea, Norway, New Zealand, the Philippines, Russia, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand, and the U.K. Because of the important role played by the euro in international finance, we begin the sample in January 1999 to draw observations only under the euro epoch. As seen in Table 1, the euro has consistently been the second most important currency (behind the U.S. dollar) in terms of foreign exchange market turnover. Although the time-span of our sample is relatively short, it does not extend across different regimes or institutional structures and is covers a period in which out-of-sample prediction has been a challenge. The sample ends in December 2010, which was the most recently available when the project began.

In the first subsection, we determine that there are 3 common factors in our exchange rate panel, we construct the factors and estimate the loadings. Then we decompose the variation in the exchange rate depreciation into components explained by each of the factors. In subsection 3.2, we estimate the *factor-augmented* PPP panel predictive regression (not subject to omitted variables bias) and show that an in-sample test of the null hypothesis of no predictability is more easily rejected than if one fails to account for the factors.

3 They also show that factor model forecasts will have lower mean-square prediction error than the random walk even when $\Delta s_{i,t}$ has almost no serial correlation.

4 We do not use monetary fundamentals simply due to data availability. For example, only 9 countries report their industrial production indexes. As a result, we use PPP fundamentals. In any event, Engel, Mark and West (2012) find that factor augmented PPP specification performance dominates Taylor rule and monetary fundamentals. Note that Australia and New Zealand report only quarterly CPIs which we interpolate in converting into monthly rates. The data source is Global Insight.

3.1 Factor Construction

Let the sample cover N countries and T time periods. To employ Bai and Ng's (2002) $IC_2(k)$ criterion to determine the number of factors, first use principal components to estimate k common factors from the nominal exchange rate *depreciations*, then construct the mean-squared deviation

$$V(k) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\Delta s_{i,t} - \sum_{j=1}^{k}\delta_{i,j}\Delta f_{j,t}\right)^2,$$

and choose k to minimize

$$IC_2(k) = ln(V(k)) + k\left(\frac{N+T}{NT}\right)ln(min(N,T)).$$

Doing so finds that log nominal exchange rates are driven by $k = 3$ common factors.

In Figure 1, we plot the integrated form of the factors ($\sum_{r=1}^{t}\Delta f_{i,r}$), which evolve smoothly and correspond to the log-level of the exchange rate. We see that there are periods, such as in the initial stages of the crisis (around 2009), when the factors all surge upwards. The estimated factors have the appearance of unit root processes and sometimes appear to trend together although their turning points do not coincide very tightly.

One of our identifying restrictions is that the same set of factors that drive nominal exchange rates also drives real exchange rates. To examine whether this restriction is reasonable or silly, we compare the nominal exchange rate factors to three factors estimated from log *real* exchange rate depreciations. Figures 2-4 plot the real and nominal factors together for comparison. Figure 2 shows the first common factor, Figure 3 the second factor and Figure 4, the third factor. The real and nominal factors are not exactly the same with somewhat more divergence between the real and nominal second factor. Overall the real and nominal factors are qualitatively very similar so we proceed with the empirical specification as described above.[5]

A quick assessment of the importance of the common factors in driving exchange rates is obtained by decomposing the variance of the depreciation into contributions from the factors and the idiosyncratic component. The orthogonality restrictions that we imposed for identification implies that the total depreciation variance is the sum of the component variances,

[5] We note that in log differences, the correlation between the real and nominal factors is 0.98 (1st factor), 0.99 (2nd factor) and 0.91 (3rd factor).

$$Var(\Delta s_{it}) = Var(\delta_{i,1}\Delta f_{1,t}) + Var(\delta_{i,2}\Delta f_{2,t}) + Var(\delta_{i,3}\Delta f_{3,t}) + Var(\Delta s^{o}_{i,t}). \quad (8)$$

Table 2 shows the results of this decomposition, from which the first factor is seen to account for nearly half of the variance of exchange rate changes. Taken together, common factor variation explains 66 percent of nominal depreciation variation and 64 percent of real depreciation variation. We note also that the proportion of variance in the nominal depreciation explained by each factor is very close to that explained in the real depreciation which again offers qualitative support for our identifying assumptions.

3.2 Testing for Predictability

In this subsection, we conduct an in-sample test of exchange rate predictability by estimating the factor-augmented PPP predictive regression (6) and testing the null hypothesis that the slope coefficient on the lagged real exchange rate $\beta,$ is zero. Inoue and Kilian (2004) argue that in-sample tests of predictability may be more credible than the results of out-of sample tests.

We make two points about the econometrics. First, we assume that the slope coefficients $\beta_i \sim iid(\beta, \sigma^2_\beta)$ are randomly distributed around β and estimate a common β by pooling across individuals in the panel. Second, we control for the omitted variables (the common factors) using the Greenaway-McGrevy et al. (2010) factor augmented fixed-effects panel regression estimator.[6]

Estimation proceeds as follows. From (6) and (7), we require estimates of $f_{j,t}$ and $\Delta f_{j,t}$. We estimate the $f_{j,t}$ using (3) and the $\Delta f_{j,t}$ from $\Delta s_{i,t}$ then include them in (6) and (7) to get the factor-augmented PPP regression

$$\Delta s_{i,t+1} = \alpha_i - \beta q_{i,t} + \sum_{j=1}^{3}\delta_{i,j}\hat{f}_{j,t} + \sum_{j=1}^{3}\phi_{i,j}\Delta\hat{f}_{j,t} + error_{i,t+1}. \quad (9)$$

Running least squares on (9) is Greenaway-McGrevy et al.'s first-stage estimator. Call the first-stage estimates of the constant and real exchange rate slope $b(1) = (\hat{\alpha}_i(1), \hat{\beta}(1))$. A second iteration proceeds by forming the residuals,

$$\hat{v}_{i,t+1}(1) = \Delta s_{i,t+1} - \hat{\alpha}_i(1) + \hat{\beta}(1)q_{i,t},$$

6 With stationary observations, the Greenaway-McGrevy et al. estimator is asymptotically equivalent to Bai's (2009) interactive fixed-effects estimator.

which from (7) is seen to be a function of six distinct factors $\{f_{j,t}\}_{j=1}^{3}$ and $\{\Delta f_{j,t}\}_{j=1}^{3}$. From this residual, estimate the three factors in levels and in differences, then employ them in (9). This results in updated coefficients, $b(2)=\left(\hat{\alpha}_i(2),\hat{\beta}(2)\right)$. If $|b(2)-b(1)|>c$ for some convergence criterion c, update $b(1)$ with $b(2)$ and repeat until convergence.

Table 3 reports estimation results on the full sample. Using the factor-augmented PPP regression, the null hypothesis of no predictability is easily rejected (t-ratio on slope of $q_{i,t}$ is 12). The table also shows the least-squares dummy variable (LSDV) estimate of (6) taking $v_{i,t+1}$ as the error. A full set of time dummies (common time effects) were included to obtain the LSDV results. Our argument that if the observations are generated by the factor structure, then ignoring the factors will bias the slope towards zero and make the test more difficult to reject is supported in the LSDV estimates. Note also that including the factors in the regression raises the $\overline{R}^2$ from approximately 0 (LSDV) to 0.8 (factor-augmented PPP) which is consistent with the results from the variance decompositions.[7] Even after controlling for PPP fundamentals, the common factors remain the most important component of exchange rate movements.

4. Out-of-Sample Prediction

We extend (6) and (7) to handle forecasts at different horizons by combining those equations and representing the prediction equation as

$$s_{i,t+k}-s_{i,t}=\alpha_i-\beta q_{i,t}+\left(\beta F_{i,t}+F_{i,t+k}-F_{i,t}\right)+u_{i,t+k}. \tag{10}$$

This factor-augmented PPP regression includes contemporaneous values of the factors and in its current form does not predict well out of sample (this was a problem confronting Engel, Mark and West, 2012). Moreover, forecasting the factors is not attractive because we don't know what the statistical factors are, exactly, nor what we should use as predictors.

One way to overcome this obstacle is to identify these statistical factors with the data. This is the task of the next subsection. In subsection 4.2 armed with this identification, we show that significant improvements in mean-square prediction error (MSPE) are attained by employing these empirical factors in place of the statistical factors in the factor-augmented PPP predictive regression.

7 One should not interpret the $\overline{R}^2$ to imply that 80 percent of the variation of exchange rates is predictable since out-of-sample predictive performance is never as good as what is implied by in-sample estimates.

4.1 Common Factor Identification

Since the factors are extracted from exchange rate data, we look to see if any particular exchange rate plays a dominant role in their evolution. We begin by regressing each of the 23 nominal depreciation rates $\Delta s_{i,t}$ on differences of the first factor $\Delta f_{1,t}$. The regression with maximal R^2 was for the euro/dollar rate. Hence, we identify the euro/dollar exchange rate as the first empirical factor. Next, we regress $\Delta f_{2,t}$ on each of the remaining 22 depreciations and $\Delta f_{1,t}$ and find that regressing on the Swiss-franc/dollar rate yields the maximal R^2. Hence, the Swiss-franc/dollar rate is identified as the second empirical factor. Similarly, looking for the highest R^2 when regressing $\Delta f_{3,t}$, on each of the remaining 21 depreciation rates and $\Delta f_{1,t}$ and $\Delta f_{2,t}$, identifies the dollar/yen rate as the third empirical factor. Figures 5-7 plot the integrated statistical factors next to the integrated empirical factors. The correspondence between the statistical and empirical factors is seen to be strikingly close. This identification also makes a certain amount of sense. The euro/dollar and yen/dollar exchanges account for the highest and second highest volume of foreign exchange transactions in the spot market (reported in Table 1) while both the yen and Swiss franc gain importance from the market perception of them as safe-haven currencies.[8]

4.2 Prediction with Empirical Factors

Armed with this identification, we label the empirical factors as $s_{21,t}$ (euro/dollar), $s_{22,t}$ (Swiss franc/dollar), and $s_{23,t}$ (yen/dollar). Our prediction model makes two modifications to (10). First, replace the statistical factors $\{f_{1,t}, f_{2,t}, f_{3,t}\}$ in $F_{i,t}$ with the empirical factors $\{s_{21,t}, s_{22,t}, s_{23,t}\}$. Second, omit the levels of the factors in the prediction equation and use only the changes. If (10) is the true data-generating process, then omission of the levels potentially leads to omitted variables bias and inconsistent tests of hypotheses about β, but it does not have serious consequences for evaluation of forecasts since we are focused on comparing MSPEs across different models. Hence, our $k-$period factor-augmented PPP prediction regression for currencies $i = 1,...,20$ is,

$$\hat{s}_{i,t+k} - s_{i,t} = \hat{\alpha}_{i,t} - \hat{\beta}_{i,t} q_{i,t} + \left(\sum_{j=21}^{23} \hat{\lambda}_{i,j,t} \left(\hat{s}^{(2)}_{j,t+k} - s_{j,t} \right) \right). \tag{11}$$

where $\hat{s}^{(2)}_{j,t+k}$ ($j = 21,22,23$) are (second stage) forecasted values of the empirical exchange-rate factors. The coefficient estimates in (11) are subscripted by t to make explicit that we do not use out-

8 Ranaaldo and Soderlind (2010) identify both the Swiss franc and yen as safe-haven currencies that appreciate against the U.S. dollar when U.S. stock prices and interest rates fall and when foreign exchange volatility increases.

of-sample information to generate the forecasts. Estimation of (11) is done by least-squares on a single-equation basis and proceeds in three stages.

Stage 1: For $j = 21,22,23,$ forecast the empirical factors with a pooled PPP predictive equation,

$$\hat{s}^{(1)}_{j,t+k} = s_{j,t} + \hat{a}_{j,t} - \hat{b}_t q_{j,t}.$$

The ‘1’ superscript in $\hat{s}^{(1)}_{j,t+k}$ indicates that this is the stage 1 prediction.[9]

Stage 2: Estimate (11) but omit the “own” exchange rate from the list of factors.[10] This gives,

$$\hat{s}^{(2)}_{21,t+k} = s_{21,t} + \hat{\alpha}_{21,t} - \hat{\beta}_{21,t} q_{21,t} + \left(\hat{\lambda}_{21,22,t}\left(\hat{s}^{(1)}_{22,t+k} - s_{22,t}\right) + \hat{\lambda}_{21,23,t}\left(\hat{s}^{(1)}_{23,t+k} - s_{23,t}\right)\right),$$

$$\hat{s}^{(2)}_{22,t+k} = s_{22,t} + \hat{\alpha}_{22,t} - \hat{\beta}_{22,t} q_{22,t} + \left(\hat{\lambda}_{22,21,t}\left(\hat{s}^{(1)}_{21,t+k} - s_{21,t}\right) + \hat{\lambda}_{22,23,t}\left(\hat{s}^{(1)}_{23,t+k} - s_{23,t}\right)\right),$$

$$\hat{s}^{(2)}_{23,t+k} = s_{23,t} + \hat{\alpha}_{23,t} - \hat{\beta}_{23,t} q_{23,t} + \left(\hat{\lambda}_{23,21,t}\left(\hat{s}^{(1)}_{21,t+k} - s_{21,t}\right) + \hat{\lambda}_{23,22,t}\left(\hat{s}^{(1)}_{22,t+k} - s_{22,t}\right)\right),$$

then iterate to convergence. That is, in step 2, replace $\hat{s}^{(1)}_{j,t+k}$ with $\hat{s}^{(2)}_{j,t+k}$ to obtain $\hat{s}^{(3)}_{j,t+k}$ and repeat until $|\,\hat{s}^{(\tau)}_{j,t+k} - \hat{s}^{(\tau-1)}_{j,t+k}| < 0.05k$ for each $t.$

Stage 3: Employ forecasts from stage 2 in (11) for final forecasts.

Before reporting the actual out-of-sample prediction results, it is instructive to examine the in-sample explanatory power of the factor-augmented PPP predictive regression. In Figure 8, we plot the actual and in-sample fitted depreciation rates for the pound/dollar rate at horizons of 1,4,8, and 12 months.[11] Fitted values from the PPP predictive regression are shown in circles. Especially at the longer horizons, augmenting the PPP regression by forecasted empirical factors improves in-sample predictive fit dramatically. In Figure 9, we plot the analogous fitted and actual values for the 12-month prediction horizon the New Zealand dollar/U.S. dollar rate, the Swedish kroner/U.S. dollar rate, the

9 Brazil and Thailand omitted from stage 1 estimation due to severe heteroskedasticity. See Mark and Sul (2011) for discussion of this problem.

10 Obviously this step would be fruitless if using the statistical factors $f_{j,t}$, since they are mutually orthogonal by construction. The empirical factors (exchange rates) can be, and are correlated with each other, however.

11 For $k = 1,$ estimate (11) from 2/99 to 12/10. For $k = 4,$ estimate from 5/99 to 12/10, and so forth. Hence, $\hat{\beta}_{i,t}$ in (11) becomes $\hat{\beta}_i$. Also, it is $\hat{s}^{(2)}_{j,t+k}$ that is included in the regression (not $s_{j,t+k}$).

Danish krone/U.S. dollar rate and Australian dollar/U.S. dollar rate. Similar improvements in fit are obtained by empirical factor augmentation.

For a quantitative assessment of the value-added gained by empirical factor augmentation, Table 4 shows $\bar{R}^2$ values from the PPP and the factor-augmented PPP predictive regressions at 1, 12, and 24 month horizons. The average $\bar{R}^2$ increases from -0.01 to 0.03 at the 1 month horizon, from 0.13 to 0.49 at the 12 month horizon and from 0.25 to 0.66 at the 24 month horizon. Denmark offers an example of striking improvement.

4.3 Out-of-Sample Forecast Evaluation

Out-of-sample forecasts are generated by nested versions of the factor-augmented PPP predictive regression (11). The models, in order of their restrictiveness are

- Random walk: $\hat{s}_{i,t+k} - s_{i,t} = \begin{cases} \hat{\alpha}_{i,t} & \textit{with drift} \\ 0 & \textit{without drift} \end{cases}$

- PPP : $\hat{s}_{i,t+k} - s_{i,t} = \hat{\alpha}_{i,t} - \hat{\beta}_{i,t} q_{i,t}$

- Factors-only: $\hat{s}_{i,t+k} - s_{i,t} = \hat{\alpha}_{i,t} + \sum_{j=21}^{23} \hat{\lambda}_{j,t} \left(\hat{s}^{(2)}_{j,t+k} - s_{j,t} \right)$

- Factor-augmented PPP: $\hat{s}_{i,t+k} - s_{i,t} = \hat{\alpha}_{i,t} - \hat{\beta}_{i,t} q_{i,t} + \left(\sum_{j=21}^{23} \hat{\lambda}_{i,j,t} \left(\hat{s}^{(2)}_{j,t+k} - s_{j,t} \right) \right)$

For each model and at each date we generate forecasts at horizons of 1 to 24 months. The first observation being forecasted is July 2004, so that 66 forecasts are generated regardless of horizon. That is, at horizon 1, initial estimation uses observations through June 2004. At horizon 2, initial estimation uses observations through May 2004, and so forth. Updating is done recursively. We employ Clark and West's (2006) test of equal MSPEs from nested models to assess relative forecast accuracy.[12]

Table 5 reports the Clark-West test results. To read the table, entries are the proportion of times (out of 24) that the null hypothesis of equal forecast accuracy is rejected at the *10 percent level*. Columns 1-3 report the proportion of rejections when the nested model is the random walk with drift and Columns 4-6 show the proportion of rejections when the nested model is the driftless random walk

[12] Clark and West show if the data are generated by a random walk, sampling error induces noise into estimates of the alternative model and hence into its forecast errors so we expect the mean-square prediction error of the model to exceed that of the random walk in this case. The Clark-West statistic corrects for this sampling variability induced upward bias in the mean-square prediction error. See also Rossi (2005) who studies forecast error bias induced by estimation error.

(the alternative hypothesis is the random walk is less accurate). For each of the predictive models (PPP, Factors-only, Factor-augmented PPP), an underscored entry indicates which version of the random walk (driftless or with drift) is more difficult to beat. Consider the Australian dollar results. PPP forecasts dominate the driftless random walk in 21 percent of the forecast horizons but never dominates the random walk with drift. Thus, the value 0.00 is underscored since for the PPP model, the random walk with drift poses a bigger challenge. Usually, the test result from Factors-Only and Factor-Augmented PPP yield the same conclusion about which version of the random walk poses the bigger challenge, but not always (case in point: Russia). Generally speaking, in our sample the random walk with drift is more difficult to beat than the driftless random walk.

A bold entry identifies the model with the best overall record against a particular benchmark--either the random walk with drift or without drift. For Australia, in forecasting against the random walk with drift, the Factor-Augmented model performs the best, hence the entry 0.71 is bolded.

Forecast accuracy relative to the random walk with drift. Looking at columns 1-3 Factors-Only or Factor-Augmented PPP dominate PPP except for South Africa, Taiwan, the U.K., and Japan. In several cases, PPP isn't significantly more accurate than the random walk with drift at any horizon. On average, Factors-Only is significantly more accurate than the random walk at 68 percent of the horizons and Factor-Augmented PPP in 66 percent.

Forecast accuracy relative to the driftless random walk. Looking at columns 4-6, Factors-Only significantly beats the driftless random walk in over 70 percent of the horizons for 11 of 23 exchange rates. This is true for PPP for 6 of 23 exchange rates. Factors-Only performs the best against the driftless random walk for 7 currencies, Factor-Augmented PPP performs best for 8 currencies. The two models are tied for Denmark, the euro, and Switzerland.

Relative forecast accuracy between Factors-Only and Factor-Augmented PPP. We cannot use the Clark-West tests against the random walk to assess relative forecast accuracy between Factors-Only and Factor-Augmented PPP. To make this comparison, Columns 7 and 8 report Clark-West test results between Factors-Only and Factor-Augmented PPP. In Column 7, we test the composite null that Factor-Augmented PPP is equal or less accurate than Factors-Only. Column 8 tests the null that Factors-Only is equally or less accurate than Factor-Augmented PPP. Looking at the results for Australia, 58 percent of the tests reject the null that Factor-Augmented PPP has equal or greater MSPE than Factors-only while none reject the null hypothesis that Factors-only has equal or greater MSPE than Factor-Augmented PPP. This speaks to Factor-Augmented as the better forecasting model for Australia. Looking at the bold entries in columns 7 and 8 finds that Factor-Augmented PPP dominates Factors-Only in 13 of 20 exchange rates.

Unadjusted Theil's U-Statistics. In Table 6, we provide additional information about forecasting performance across horizons by reporting unadjusted Theil's U-statistics for Factor-Augmented PPP against the random walk with drift. Theil's U is the MSPE of the candidate model divided by the MSPE of the random walk.[13] U-statistic values below 1 indicate superior forecast accuracy of the candidate model. Factor-Augmented PPP is seen to dominate the random walk in 8 of 23 cases at the 1-month forecast horizon and in 20 of 23 cases at the 12-, 18-, and 24-month horizons. The Theil's U values tend to decline as the forecast horizon lengthens.

Table 7 reports Theil's U-statistics as ratios of the MSPE from Factor-Augmented PPP relative to Factors-Only. For more than half of the exchange rates, Factor-Augmented PPP performs better than Factors-Only.

5. Conclusion

Common factors obtained by statistical factor analysis from exchange rates are known to “explain” currency price movements even after controlling for standard bi-lateral macroeconomic fundamentals. One implication is that conventional two-country exchange rate models cannot deliver satisfactory predictions about exchange rate determination. The development of a deeper structural understanding of exchange rate dynamics, however, is hindered by the lack of identification of these common factors.

In this paper, we provide an identification of the common factors and argue that the empirical factors are themselves exchange rates of the euro, the Swiss franc, and the yen against the U.S. dollar. This identification also makes economic sense. The euro and yen because the trading of those currencies dominate the foreign exchange market, and the Swiss franc and the yen because the market views them as safe-haven currencies. Beyond identification, we show that the explanatory and predictive power of the empirical factors are both quantitatively large and statistically significant during a sample period that has posed a challenge for exchange rate prediction.

13 Theil's U-statistics are biased along the argument of Clark and West (2006) in the sense that if the random walk is true, we expect Theil's U to be greater than 1. Hence, a U-statistic of 1 is actually evidence in favor of predictability. We have computed bias adjusted Theil's U-statistics which are available upon request.

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Table 1. Currency Distribution of Global Foreign Exchange Market Turnover: Percentage Shares of Average Daily Turnover in April (Total 200% due to Bilateral Rate). Source BIS Survey 2010

	2001	2004	2007	2010
U.S. dollar	89.9	88	85.6	84.9
Euro	37.9	37.4	37	39.1
Japanese yen	23.5	20.8	17.2	19
U.K. pound	13	16.5	14.9	12.9
Australian dollar	4.3	6	6.6	7.6
Swiss franc	6	6	6.8	6.4
Canadian dollar	4.5	4.2	4.3	5.3

Table 2. Variance Decomposition by Factor

	Nominal				Real			
Country	First	Second	Third	Total	First	Second	Third	Total
Australia	0.71	0.06	0.05	0.82	0.70	0.06	0.06	0.82
Brazil	0.19	0.39	0.00	0.57	0.16	0.40	0.00	0.56
Canada	0.45	0.08	0.07	0.60	0.41	0.06	0.14	0.62
Chile	0.29	0.23	0.01	0.52	0.26	0.21	0.00	0.47
Colombia	0.23	0.31	0.00	0.55	0.19	0.36	0.01	0.56
Czech	0.73	0.07	0.01	0.81	0.71	0.06	0.00	0.77
Denmark	0.81	0.11	0.00	0.93	0.79	0.11	0.01	0.91
Euro	0.81	0.11	0.00	0.93	0.81	0.11	0.00	0.93
Hungary	0.78	0.01	0.03	0.82	0.77	0.00	0.02	0.80
Israel	0.25	0.00	0.01	0.26	0.26	0.00	0.02	0.28
Japan	0.09	0.20	0.35	0.64	0.09	0.21	0.28	0.58
Korea	0.44	0.14	0.01	0.60	0.42	0.15	0.01	0.58
Norway	0.71	0.02	0.02	0.74	0.66	0.03	0.04	0.73
N. Zealand	0.61	0.02	0.04	0.68	0.61	0.03	0.03	0.67
Philippines	0.14	0.20	0.23	0.57	0.14	0.17	0.21	0.52
Russ. Fed.	0.30	0.00	0.02	0.32	0.27	0.00	0.00	0.27
Singapore	0.67	0.00	0.07	0.75	0.53	0.00	0.18	0.71
South Africa	0.33	0.03	0.02	0.38	0.30	0.03	0.03	0.37
Sweden	0.83	0.03	0.01	0.87	0.80	0.04	0.02	0.85
Switzerland	0.65	0.25	0.00	0.89	0.64	0.25	0.00	0.89
Taiwan	0.40	0.00	0.19	0.60	0.31	0.00	0.24	0.55
Thailand	0.32	0.01	0.32	0.65	0.36	0.01	0.21	0.58
U.K.	0.62	0.02	0.03	0.67	0.57	0.03	0.04	0.64
Average	0.49	0.10	0.06	0.66	0.47	0.10	0.07	0.64

Table 3. Predictive Regression

Method	$\hat{\beta}$	$t-$ratio	$\overline{R}^2$
Factor Augmented	0.128	12.055	0.801
LSDV	0.012	1.844	0.006

Table 4. In-Sample $\overline{R}^2$ from Factor-Augmented and PPP Predictive Regressions

	Factor Augmented	PPP	Factor Augmented	PPP	Factor Augmented	PPP
Horizon	1	1	12	12	24	24
Australia	0.02	-0.02	0.60	0.07	0.65	0.11
Brazil	0.04	-0.02	0.44	0.01	0.82	0.02
Canada	0.00	-0.01	0.42	0.06	0.45	0.04
Chile	0.04	0.00	0.55	0.25	0.80	0.40
Colombia	0.06	-0.01	0.61	0.07	0.80	0.08
Czech Rep.	0.05	-0.02	0.64	0.01	0.41	-0.01
Denmark	0.04	-0.02	0.68	0.05	0.92	0.16
Hungary	0.03	-0.02	0.51	0.03	0.42	0.06
Israel	0.01	0.00	0.35	0.32	0.66	0.56
Korea	-0.03	-0.01	0.25	0.18	0.52	0.30
Norway	0.02	-0.01	0.70	0.14	0.41	0.17
N.Zealand	0.02	-0.01	0.46	0.12	0.60	0.27
Philippines	0.07	-0.02	0.52	0.09	0.72	0.23
Russia	0.08	-0.01	0.21	-0.01	0.26	0.01
Singapore	0.01	-0.01	0.54	0.26	0.76	0.42
South Africa	0.00	-0.01	0.47	0.18	0.76	0.45
Sweden	0.03	-0.01	0.57	0.21	0.88	0.39
Taiwan	0.00	0.00	0.32	0.20	0.82	0.63
Thailand	0.05	-0.01	0.63	0.07	0.75	0.17
U.K.	0.00	0.00	0.38	0.23	0.83	0.49
Average	0.03	-0.01	0.49	0.13	0.66	0.25

Table 5. Clark-West Tests of Equal Forecast Accuracy

	RW with drift vs.			Driftless RW vs			Factors-only vs. Factor augmented PPP	
	PPP	Factors only	Factor augmented PPP	PPP	Factors only	Factor augmented PPP		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Australia	0.00	0.63	**0.71**	0.21	0.67	**0.92**	**0.58**	0.00
Brazil	0.00	**1.00**	0.38	0.00	**0.67**	0.17	0.13	**0.63**
Canada	0.00	0.63	**0.92**	0.54	0.71	**0.92**	0.00	0.00
Chile	0.54	0.71	**1.00**	0.21	0.54	**0.67**	**0.75**	0.00
Colombia	0.00	**1.00**	0.92	0.00	0.00	0.00	**0.33**	0.04
Czech	0.00	0.25	**0.29**	0.71	**0.92**	0.79	0.00	**0.21**
Denmark	0.00	**0.83**	**0.83**	0.08	**0.92**	**0.92**	**0.33**	0.00
Hungary	0.25	**0.71**	0.46	0.50	**0.88**	0.79	0.00	**0.17**
Israel	0.83	**1.00**	0.83	**0.83**	0.71	**0.83**	**0.71**	0.00
Korea	0.42	0.00	**0.83**	0.58	0.13	**0.83**	**0.83**	0.00
Norway	0.17	**0.58**	**0.58**	0.46	**0.96**	0.88	0.13	0.13
NZ	0.46	0.67	**0.79**	0.38	0.58	**0.92**	**0.67**	0.00
Philippines	0.33	**1.00**	0.92	0.00	**0.04**	0.00	**0.50**	0.00
Russia	0.46	**0.96**	0.00	0.08	0.63	**0.67**	0.00	0.00
Singapore	0.54	**0.88**	0.79	0.75	**0.88**	0.83	**0.46**	0.00
South Africa	**0.50**	0.00	0.00	**0.46**	0.00	0.08	**0.96**	0.00
Sweden	0.83	**0.96**	**0.96**	0.42	0.92	**0.96**	**0.92**	0.00
Taiwan	**0.96**	0.29	0.25	**0.92**	0.33	0.71	**0.38**	0.00
Thailand	0.00	**0.96**	0.92	0.00	**0.92**	0.67w	0.00	**0.21**
U.K.	**0.92**	0.88	**0.92**	0.88	0.63	**0.92**	**0.92**	0.00
Euro	0.00	**0.88**	**0.88**	0.08	**0.92**	**0.92**		
Switzerland	0.00	0.50	**0.50**	0.33	**0.88**	**0.88**		
Japan	**0.92**	0.42	0.42	**0.92**	0.58	0.58		
Average	0.35	**0.68**	0.66	0.41	0.63	**0.69**	**0.37**	0.06

Notes: Columns 1-6: Proportion of CW (Clark-West) rejections of null hypothesis that model's forecast accuracy is equal to that of random walk. The alternative is the model forecasts are more accurate than the random walk. Column 7: Proportion of CW rejections of composite null hypothesis that Factor-Augmented PPP forecasts are equally or less accurate than Factors-Only. Column 8: Proportion of CW rejections of null hypothesis that Factors-Only is equally or less accurate than Factor-Augmented PPP.

Table 6. Theil's U for Factor-Augmented PPP against Random Walk with Drift

	Forecast Horizon					
	1	3	6	12	18	24
Australia	1.013	**0.981**	**0.973**	**0.865**	**0.816**	**0.874**
Brazil	**0.978**	**0.962**	1.129	1.065	**0.674**	**0.431**
Canada	1.007	**0.983**	**0.963**	**0.978**	1.027	1.010
Chile	**0.970**	**0.909**	**0.884**	**0.700**	**0.458**	**0.334**
Colombia	1.033	1.052	1.082	1.045	**0.797**	**0.429**
Czech	1.031	1.038	**0.981**	**0.866**	**0.976**	1.207
Denmark	**0.992**	**0.964**	**0.911**	**0.695**	**0.587**	**0.655**
Hungary	1.023	1.014	**0.958**	**0.795**	**0.859**	**0.899**
Israel	1.007	**0.964**	**0.891**	**0.603**	**0.455**	**0.271**
Korea	1.014	**0.986**	**0.915**	**0.880**	**0.813**	**0.556**
Norway	1.004	**0.962**	**0.923**	**0.814**	**0.863**	**0.928**
NZ	1.005	**0.963**	**0.927**	**0.791**	**0.715**	**0.774**
Philippines	**0.994**	**0.948**	**0.876**	**0.798**	**0.669**	**0.463**
Russia	1.035	1.032	1.006	**0.959**	**0.954**	**0.887**
Singapore	1.031	1.013	**0.982**	**0.700**	**0.454**	**0.354**
South Africa	1.014	**0.982**	**0.970**	1.280	2.306	1.299
Sweden	**0.978**	**0.950**	**0.908**	**0.689**	**0.642**	**0.682**
Taiwan	1.009	**0.988**	**0.937**	**0.986**	1.177	**0.636**
Thailand	1.022	**0.973**	**0.983**	**0.907**	**0.760**	**0.779**
U.K.	**0.999**	**0.949**	**0.856**	**0.732**	**0.679**	**0.583**
Euro	**0.990**	**0.965**	**0.914**	**0.696**	**0.587**	**0.659**
Switzerland	**0.991**	1.017	**0.962**	**0.850**	**0.815**	**0.741**
Japan	1.034	1.038	1.044	**0.915**	**0.682**	**0.346**

Table 7. Ratio of Theil's U from Factors-Augmented PPP to Factors-Only

	Forecast Horizon					
	1	3	6	12	18	24
Australia	1.004	**0.991**	**0.969**	**0.886**	**0.897**	**0.964**
Brazil	1.054	1.098	1.348	1.537	1.561	1.334
Canada	1.015	**0.996**	**0.966**	**0.969**	**0.946**	**0.960**
Chile	1.002	**0.970**	**0.926**	**0.807**	**0.666**	**0.453**
Colombia	1.054	1.104	1.178	1.175	**0.955**	**0.732**
Czech	1.042	1.058	1.053	1.050	1.019	1.141
Denmark	1.000	**0.999**	**0.997**	**0.996**	**0.992**	**0.992**
Hungary	1.013	1.017	1.016	**0.983**	1.015	1.081
Israel	1.023	1.016	**0.954**	**0.744**	**0.786**	**0.728**
Korea	1.009	**0.977**	**0.903**	**0.824**	**0.720**	**0.590**
Norway	1.010	**0.995**	**0.965**	**0.942**	**0.990**	1.192
N. Zealand	**0.997**	**0.983**	**0.943**	**0.851**	**0.861**	**0.931**
Philippines	1.046	1.052	1.026	**0.965**	**0.814**	**0.827**
Russia	1.028	1.039	1.062	1.159	1.220	1.227
Singapore	1.018	1.022	1.013	**0.919**	**0.867**	**0.966**
South Africa	**0.998**	**0.961**	**0.895**	**0.835**	**0.837**	**0.657**
Sweden	**0.998**	**0.991**	**0.965**	**0.872**	**0.857**	**0.876**
Taiwan	1.012	0.993	0.943	1.030	1.076	**0.706**
Thailand	1.029	1.029	1.064	1.265	1.187	1.480
U.K.	1.006	**0.964**	**0.888**	**0.816**	**0.798**	**0.712**

Figure 1. Integrated Factors Estimated from Panel of Depreciation Rates

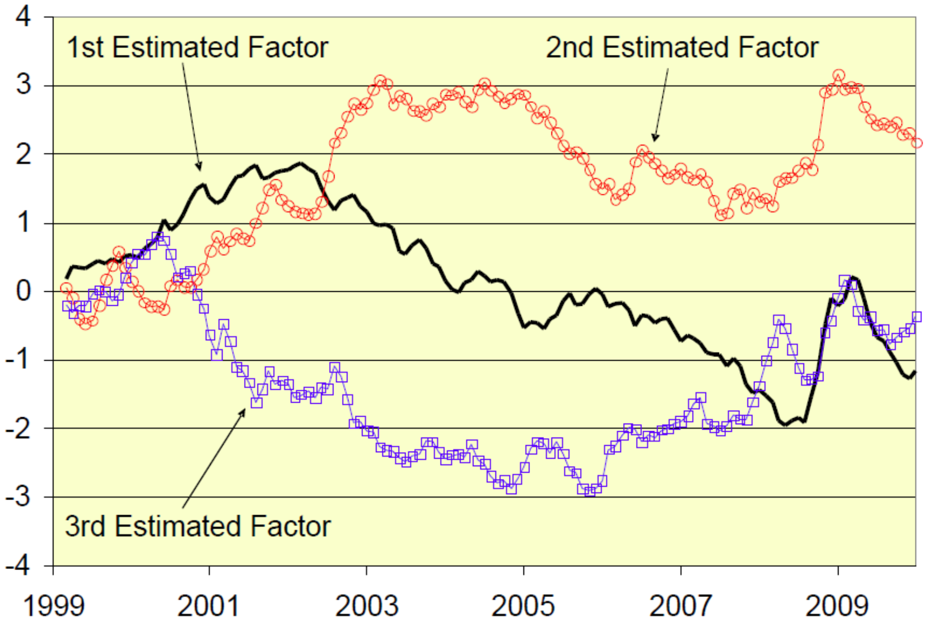

Figure 2. First Common Factor for Nominal Exchange Rate (Solid) and Real Exchange Rate (Circles)

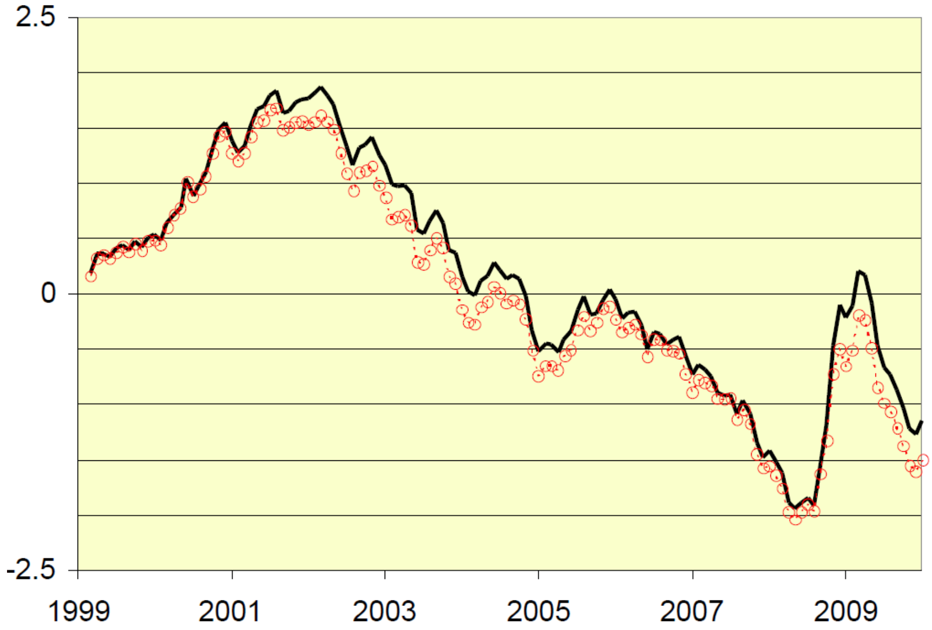

Figure 3. Second Common Factor for Nominal Exchange Rate (Solid) and Real Exchange Rate (Circles)

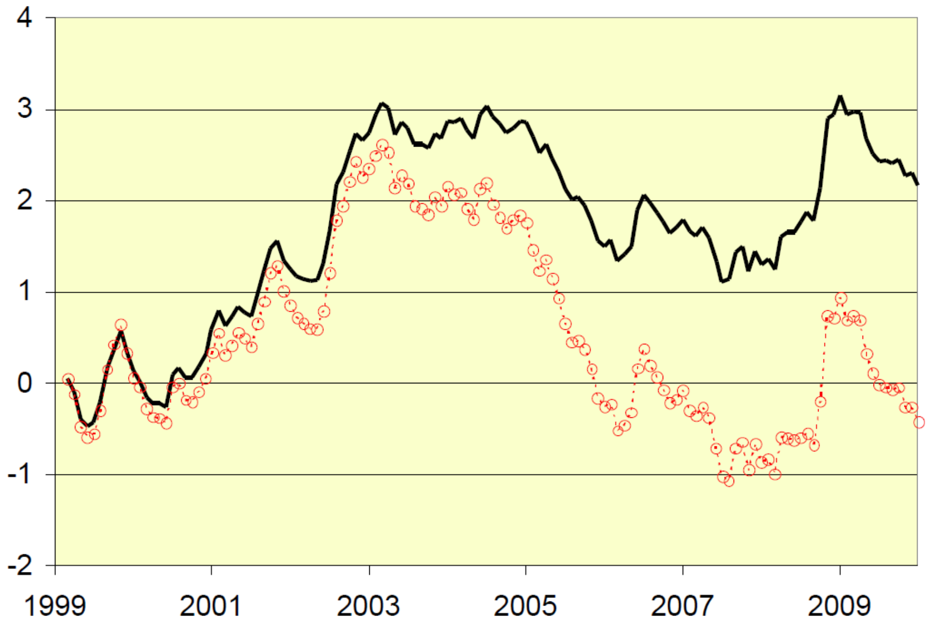

Figure 4. Third Common Factor for Nominal Exchange Rate (Solid) and Real Exchange Rate (Circles)

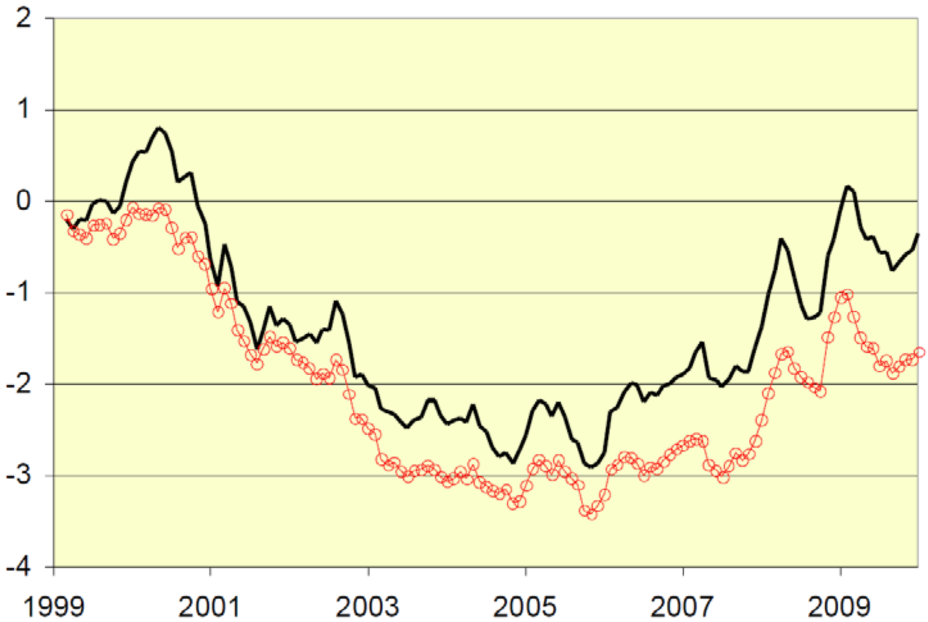

Figure 5. First Empirical and Statistical Nominal Exchange Rate Factor

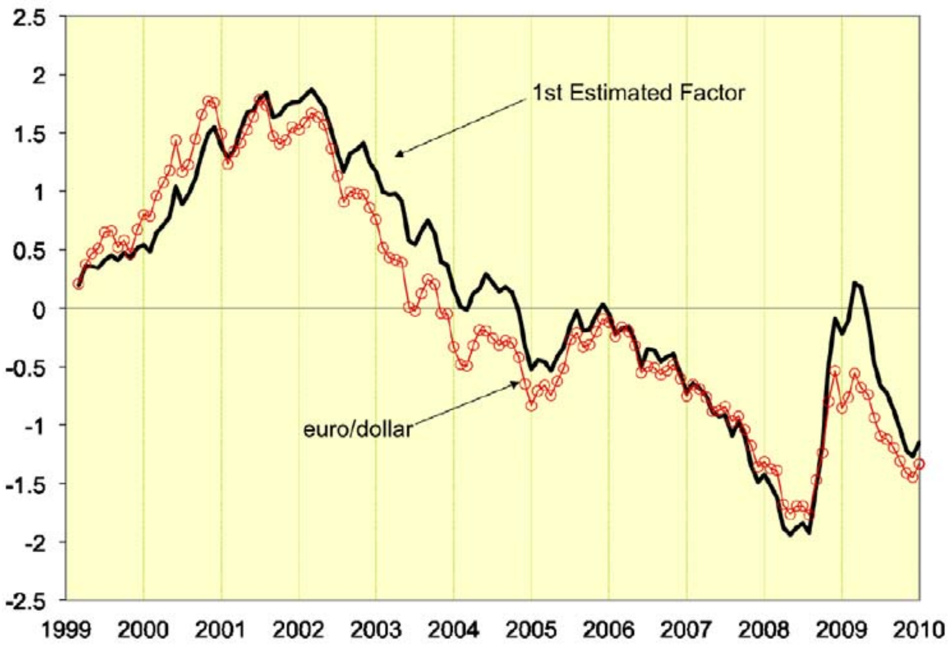

Figure 6. Second Empirical and Statistical Nominal Exchange Rate Factor

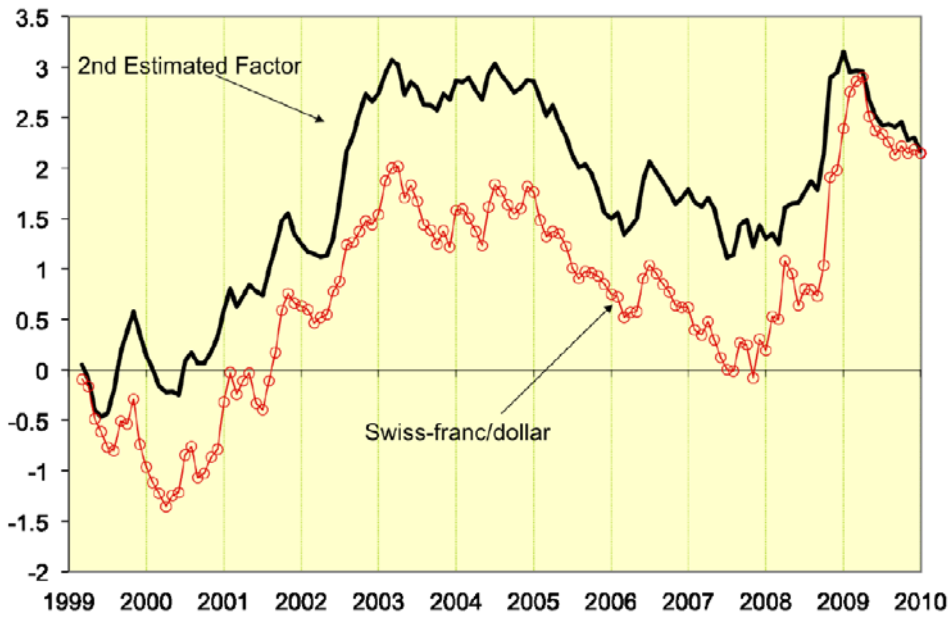

Figure 7. Third Empirical and Statistical Nominal Exchange Rate Factor

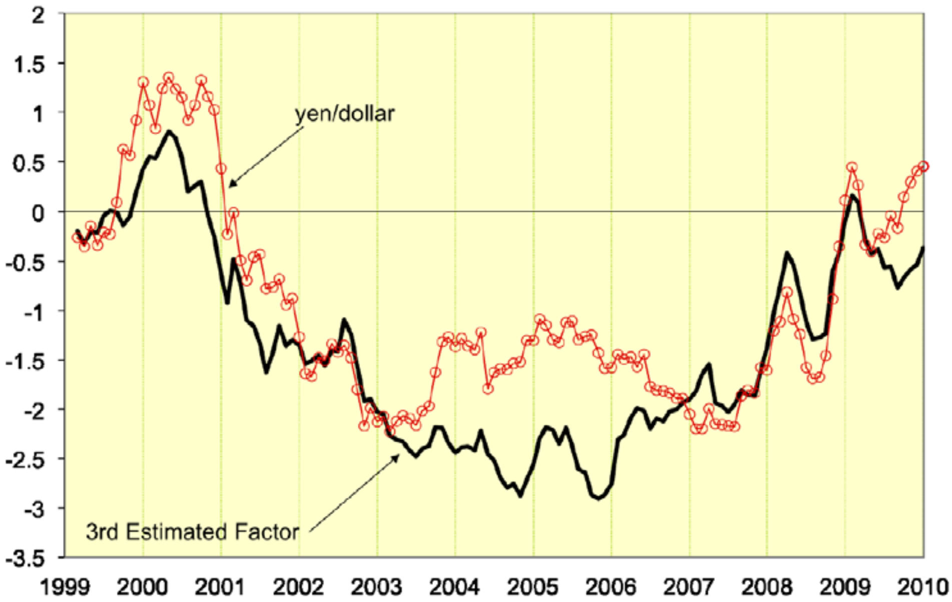

Figure 8. Actual and In-Sample Fitted Values for Pound-Dollar Rate

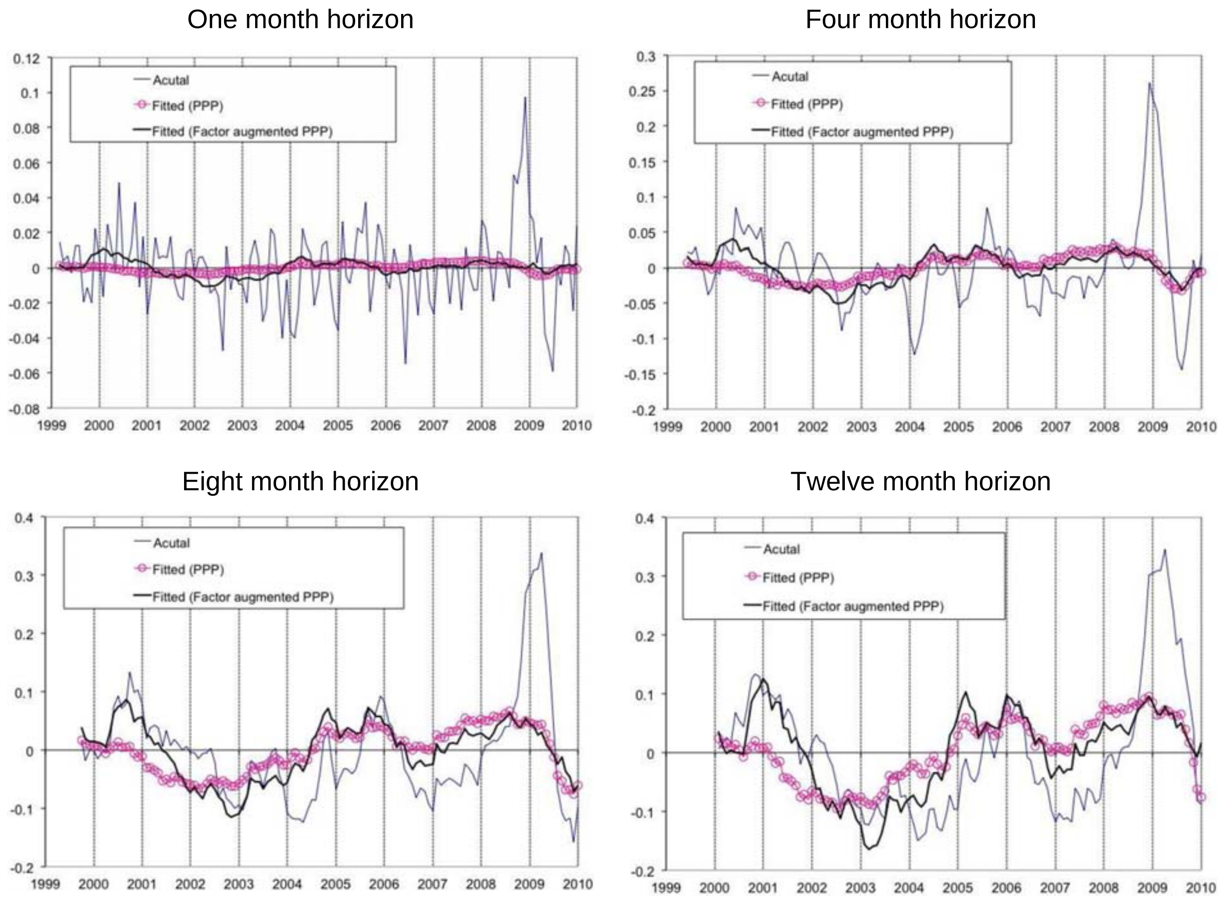

Figure 9. Actual and In-Sample Fitted Values at Twelve-Month Horizon