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BALANCED-BUDGET RULES AND AGGREGATE INSTABILITY: THE ROLE OF CONSUMPTION TAXES IN A MONETARY ECONOMY

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Balanced-Budget Rules and Aggregate Instability: The Role of Consumption Taxes in a Monetary Economy

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Abstract

This paper examines the stabilizing property of consumption taxation in a balanced-budget setting of a neoclassical one-sector cash-in-advance economy. We find that saddle-path stability is not a necessary outcome even though the utility function is additively separable between consumption and leisure. Both the existence of a Laffer curve and the indeterminacy outcome of consumption taxation depend on the elasticities of intertemporal substitution in consumption and of labor supply. Numerical examples show that consumption tax may lead to aggregate instability for the OECD countries under the current over-easy monetary policies.

Keywords: Balanced-Budget Rules, Consumption Tax, CIA Constraint, Indeterminacy
JEL Classification: E32, E63

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1. Introduction

In the context of recent economic downturns in Western developed economies, it is recognized that both the fiscal and monetary authorities are working together to keep their economies from falling into deeper recessions. On the fiscal side, balanced budget fiscal rules have been strongly proposed. For instance, in the EMU, member countries are urged to have fiscal deficits reduced to below 3% of their GDP as required by the Maastricht Treaty. The European debt crisis has pushed countries, such as Greece, Spain, and Italy, to undertake severe contractionary fiscal policies to reduce their budget deficits. In the US, balanced budget constitutional amendments have been proposed from time to time in the past couple decades (both in 1997 and 2011). The current "fiscal cliff" problem has raised the issue of balanced budgets once more. On the monetary side, both in the US and in Europe, central banks are pursuing extremely expansionary monetary policies to keep interest rates at record low levels. This is to alleviate the detrimental effects of severe contractionary fiscal policies on the real macroeconomy. Recent examples include the Quantitative Easing (QE) and Operation Twist (OT) of the Federal Reserve in US, and the "unlimited" purchase of euro-area government debts by the European Central Bank. In addition, the new Japanese prime minister, Shinzo Abe, plans to change the law to request the Japan Central Bank to set a 2% inflation target for the medium run. All these are evidence showing that the monetary side of the macroeconomic environment has been over-easy for the past few years. Figure 1 summarizes this fact in terms of the monetary base relative to GDP for these countries. Since 2007, both the US and the European Union have doubled their monetary base-GDP ratios.[1]

Existing macroeconomic literature has focused on analyzing the stability of balanced budget rules (BBR) on the fiscal side. In a one-sector neoclassical growth model, Schmitt-Grohé and Uribe (1997) "show numerically that indeterminacy arises within the range of capital and labor income tax rates observed for the United States and other industrialized countries" (p.977). The intuition of Schmitt-Grohé-Uribe self-fulfilling expectations goes as follows. When agents expect a higher future income tax, they reduce their labor supply by intertemporal substitution. This then lowers capital accumulation by reducing its expected return so that the marginal utility income goes up. As a result, current labor supply falls and so does output. With BBR and a given level of government spending, current income tax has to increase so that expectations are fulfilled. Recently, Giannitsarou (2007) replaces income taxation with consumption taxation in the Schmitt-Grohé-Uribe analysis and finds that the system becomes saddle-path stable. The economics of the Giannitsarou determinacy result is straightforward because consumption tax affects labor supply only through the intratemporal not the intertemporal substitution channel. But the opposing intratemporal income and substitution effects of consumption tax on labor supply completely offset each other in the Giannitsarou model. As a result, expectations are not fulfilled so that indeterminacy cannot occur. From the stability perspective, it seems that replacing, to some extent, income taxes with consumption taxes is the direction to go.[2] However,

[1] Figure 1 is borrowed from Ueda (2012).

[2] Another popular approach to comparing alternative taxation schemes is from the welfare perspective; for example, see Lu, Chen and Hsu (2011).

Nourry et al (2011) point out that in the absence of separable preferences, the intratemporal effects may not fully offset each other. Thus labor supply needs not be independent of the consumption tax so that indeterminacy cannot be ruled out. Their conclusion is that the indeterminacy of consumption taxation depends crucially on the specification of the preferences that dictates the magnitude of the intratemporal effects at work.

It is noted that all of the above analysis of BBR are cast within a real macroeconomic setting so that monetary factors are not present.[3] As we mention above, another important aspect of the current macroeconomic environment is that the monetary side is "over-expansionary." In this paper, we investigate whether a balanced-budget consumption-tax rule is dynamically stable in a monetary economy.[4] Specifically, we extend the Giannitsarou (2007) analysis to a monetary setting by imposing a standard cash-in-advance (CIA) constraint on consumption goods. We find that, in the presence of a monetary distortion, saddle-path stability is no longer a necessary outcome for the balanced-budget consumption-tax rule. The CIA constraint restores the intertemporal channel of consumption taxation on capital accumulation that is present in Schmitt-Grohé and Uribe (1997). Consequently, macroeconomic stability depends on the parameterization of the model, especially the intertemporal elasticity of substitution in consumption and the wage elasticity of labor supply. In the calibration exercise, we examine the effects of monetary growth on dynamic stability under non-linear taxation. If monetary policy is over expansionary (say the money growth rate goes up to 60%), then the majority of the OECD countries fall into the indeterminacy region.

To understand our findings, we note that the intuition for the Giannitsarou (2007) determinacy result is due to the absence of the intertemporal effect of consumption tax on labor supply. If an above average consumption tax is expected, then households reduce future consumption and increase current consumption due to intertemporal substitution. Since the consumption tax does not generate long-run distortion (the Euler equation), labor supply is unchanged. Given the pre-set level of government spending, the current consumption tax rate is lowered so that expectations of higher consumption tax are not fulfilled. However, in a monetary environment, when current consumption rises, the CIA constraint is tightened which creates an efficiency distortion that is absent in the real model. An intertemporal channel is generated with a resulting increase in the marginal utility of income, which in turn reduces the real wage. With constant labor supply, this then lowers capital accumulation and leads to a decline in income and consumption. As a result, the BBR requires an increase in the current consumption tax so that expectations are fulfilled. Intuitively, the strength of this intertemporal force on dynamic stability depends on the intertemporal elasticity of substitution in consumption, the elasticity of labor supply with respect to wages, and the money growth rate.

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[3] Since the work of Farmer (1997, 2000) and Sossounov (2000), it is recognized that money is a potential source of indeterminacy in dynamic macroeconomic models because its presence may potentially invalidate the Negishi (1960) theorem.

[4] There has been some research on the stability of the BBR in a finance constrained macroeconomy. See Gokan (2008) for an analysis on factor taxation, and Lloyd-Braga et al (2008) for an analysis on consumption taxation.

The organization of the paper is as follows. The general real model is reviewed in the next section. Section 3 develops the monetary model via a CIA constraint while Section 4 provides the equilibrium analysis. Section 5 discusses the roles played by money on macroeconomic stability in the presence of BBR. Section 6 extends the analysis to non-linear tax rules and presents the calibration results. Finally, Section 7 concludes.

2. The Benchmark Real Economy

Our analytical framework follows closely the standard one-sector neoclassical growth model with general preferences and technology. Our objective is to study the influence of consumption taxation on macroeconomic stability.

2.1 Households

The representative agent is endowed with one unit of time, and chooses the consumption paths c_t, working hours ℓ_t and capital k_t to maximize the discounted lifetime utility:

$$\int_0^{\infty} U(c_t, 1-\ell_t)\exp(-\rho t)dt, \tag{1}$$

where ρ denotes the discount rate. The utility function yields positive and diminishing marginal products, i.e., $U_i > 0 > U_{ii}$, for $i = 1,2$.[5] It also satisfies the normality condition for c and $(1-\ell)$ respectively as below, i.e.

$$\Delta_c = U_1U_{21} - U_2U_{11} > 0, \Delta_\ell = U_2U_{12} - U_1U_{22} > 0, \tag{2}$$

as well as the concavity condition:

$$U_{11}U_{22} - U_{12}^2 \geq 0.$$

The budget constraint is:

$$\dot{k}_t = (r_t - \delta)k_t + w_t\ell_t - (1+\tau_t)c_t, \tag{3}$$

[5] Throughout the paper, we use numerical subscripts to denote partial derivatives of a function.

where r_t and w_t are the rental rate of capital and wage rate of labor, respectively, $\delta \in (0,1)$ denotes the depreciation rate and τ is tax rate on the consumption spending.

The first-order conditions for the household problem are:

$$U_1 = (1+\tau_t)\Lambda_t, \tag{4}$$

$$U_2 = \Lambda_t w_t, \tag{5}$$

$$\dot{\Lambda}_t = \Lambda_t[\rho - (r_t - \delta)], \tag{6}$$

where Λ_t is the marginal utility of income and the transversality condition:

$$\lim_{t\to\infty} \Lambda_t k_t \exp(-\rho t) = 0,$$

is satisfied.

2.2 Firms

Assume that the constant-return technology adopted by the firm is $y = F(k, \ell)$ with $F_i > 0 > F_{ii}$, where $i = 1,2$. Since the factor market is perfectly competitive, we have:

$$r_t = F_1, \tag{7}$$

$$w_t = F_2. \tag{8}$$

2.3 Government

We consider an economy with a consumption tax only, where the government expenditure is financed by the balanced budget rule (BBR) as follows:

$$G_t = \tau_t c_t. \tag{9}$$

We focus on the case of exogenous government expenditure and endogenous consumption tax rule, i.e., $dG_t = 0$. From (9), we get:

$$\frac{d\tau_t}{dc_t} = -\frac{\tau_t}{c_t} < 0. \tag{10}$$

This is similar to the case studied by Schmitt-Grohé and Uribe (1997) and Giannitsarou (2007). Following Guo and Lansing (1998), we regard $d\tau_t/dc_t < 0$ as the regressive consumption tax rule.[6]

2.4 Equilibrium Analysis

Substituting (7), (8) and (9) into (3) and (6), we get:

$$\dot{\Lambda}_t = \Lambda_t\left(\rho + \delta - F_1\left(k_t, \ell_t\right)\right), \tag{11}$$

$$\dot{k}_t = F\left(k_t, \ell_t\right) - \delta k_t - \left(1 + \tau_t\right)c_t. \tag{12}$$

In addition, combining (4) and (5), we have:

$$\frac{U_2(c_t, 1-\ell_t)}{U_1(c_t, 1-\ell_t)} = \frac{F_2\left(k_t, \ell_t\right)}{1+\tau_t}. \tag{13}$$

We can then write ℓ_t as a function of c_t and k_t:

$$\ell_t = \ell\left(c_t, k_t\right), \tag{14}$$

With:

$$\ell_1 = \frac{\partial \ell_t}{\partial c_t} = -\frac{\widetilde{\Delta}_c + \tau'}{\Delta}\frac{U_2}{U_1},$$

$$\ell_2 = \frac{\partial \ell_t}{\partial k_t} = \frac{F_{21}}{\Delta} > 0,$$

where $\Delta = \frac{\Delta_\ell}{U_1^2}\left(1+\tau_t\right) - F_{22} > 0$ and $\widetilde{\Delta}_c = \frac{\Delta_c}{U_2 U_1}\left(1+\tau_t\right) > 0$. Next, using (4) and (14), we can rewrite the dynamic equation (32) in terms of c_t and k_t as follows:

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[6] When $d\tau_t/dc_t > 0$, it is known as the progressive tax rule.

$$\dot{c}_t = \frac{c_t}{\eta_t}\left\{\left[F_1(k_t,\ell_t)-(\rho+\delta)\right]-\frac{U_{12}\ell_2}{U_1}\left[F(k_t,\ell_t)-\delta k_t-(1+\tau_t)c_t\right]\right\}, \tag{15}$$

where $\eta_t = \sigma_t + \frac{c_t U_{12}}{U_1}\ell_1 + \frac{c_t}{1+\tau_t}\frac{d\tau_t}{dc_t}$ and $\sigma_t = -\frac{c_t U_{11}}{U_1} > 0$. Together with (12), this characterizes the dynamic system for studying the equilibrium dynamics of the model.

In steady state, we have $\dot{k}_t = \dot{\Lambda}_t = 0$. The steady state values of k_t and c_t, i.e., $\{k^*, c^*\}$ can be obtained from solving the following equation:

$$F_1\left(k^*, \ell\left(c^*, k^*\right)\right) = \rho + \delta, \tag{16}$$

$$F\left(c^*, k^*\right) = \left(1+\tau^*\right)c^* + \delta k^*. \tag{17}$$

Linearizing the $c-k$ system around the steady-state, we can compute the determinant and trace of the Jacobian coefficient matrix:[7]

$$Det = \frac{c^*}{\eta^*}\left[\underbrace{\left(F_{12}\rho - F_{11}F_2\right)}_{+}\ell_1 + \underbrace{\left(F_{11}+F_{12}\ell_2\right)}_{-}\left(1+\tau^* + c^*\frac{d\tau^*}{dc^*}\right)\right], \tag{18}$$

$$Trace = \rho - \frac{c^{*2}}{\eta^*\Omega}F_{12}U_{12}\frac{d\tau^*}{dc^*}, \tag{19}$$

With:

$$\Omega = U_1 F_{22} - \Delta_\ell\left(1+\tau^*\right)/U_1 < 0.$$

In order to have indeterminacy, we must have two roots with negative real parts. This in turn requires that $Det > 0$ and $Trace < 0$. We also note that as the consumption tax rule is more regressive (i.e., $d\tau^*/dc^*$ becomes more negative), the likelihood of indeterminacy increases (due to more likely to have $Det > 0$ and $Trace < 0$).

7 It is straightforward to show that $F_{11} + F_{12}\ell_2 = \frac{1}{\Delta}\left[\frac{F_{11}}{U_1^2}\left(1+\tau^*\right)\Delta_\ell\right] < 0$.

2.5 Macroeconomic Stability

Since our specification of preferences and technology are general, most of the results found in the existing literature can be included as special cases. In what follows, we provide our findings of the special cases that yield macroeconomic stability, i.e., no indeterminacy.

2.5.1 Separability of Consumption and Leisure

If the utility function is additively separable in consumption and leisure/hours worked, then $U_{12} = 0$. By (19), we then have:

$$Trace = \rho > 0.$$

Therefore, the system can never be indeterminate as the trace condition of indeterminacy is violated. Recall that Schmitt-Grohé and Uribe (1997) adopt the following instantaneous utility function in their taxation analysis:

$$U(c_t, \ell_t) = \log c_t - A\frac{\ell_t^{1+\chi}}{1+\chi},$$

which is additively separable in c_t and ℓ_t. This explains why when Giannitsarou (2007) applies the Schmitt-Grohé-Uribe analysis to consumption taxation, she cannot find any indeterminate steady-state equilibrium.

2.5.2 Non-Regressive Taxation

When consumption taxes are non-regressive, we have $d\tau/dc \geq 0$ so that $\ell_1 < 0$ and $\sigma > 0$. Then according to (18), we get:

$$Det = \underbrace{\frac{c^*}{\eta^*}}_{+}\left[\underbrace{\left(F_{12}\rho - F_{11}F_2\right)}_{+}\underbrace{\ell_1}_{-} + \underbrace{\left(F_{11} + F_{12}\ell_2\right)}_{-}\underbrace{\left(1+\tau^* + c^*\frac{d\tau^*}{dc^*}\right)}_{+}\right] < 0.$$

As a result, the determinant is always negative and the determinant condition of indeterminacy is violated. In other words, the system is always saddle-path stable. Our finding corroborates and complements the finding of Guo and Harrison (2004) for endogenous income taxation.

2.5.3 Intuition

Our general setting highlights the fact that the determinacy finding of Giannitsarou (2007) is an artifact of the additively separable utility function.[8] In her case, labor supply is independent of the consumption tax rate both intratemporally and intertemporally. So when households expect a higher consumption tax rate, they substitute current consumption for future consumption due to intertemporal substitution. Given constant government spending, the rise in current consumption suppresses the consumption tax rate because of the balanced-budget rule. This implies that expectations of a higher consumption tax rate can never be self-fulfilling.

On the other hand, under a general nonseparable concave utility function, then an expected higher future consumption tax rate reduces future labor supply due to intratemporal substitution. But this also implies a lower future income and hence lower future leisure (higher labor supply). Under separable logarithmic preferences, the two conflicting intratemporal effects offset each other which is the case studied by Giannitsarou (2007), and so indeterminacy cannot occur. The working of these two opposing intratemporal effects can be readily seen from the labor supply relation obtained from (8) and (13):

$$\hat{w}^s = \left(\frac{\Delta_c}{U_2 U_1} + \frac{1}{1+\tau^*} \frac{d\tau^*}{dc^*} \right) c^* \hat{c}_t + \frac{\Delta_\ell}{U_1 U_2} \ell^* \hat{\ell}_t ,$$

where $\hat{x} = d(\ln x)/dx$. The two terms inside the bracket of $\hat{c}_t$ on the RHS capture the above two opposing effects at work. Specifically, the first term is the intratemporal income effect while the second term identifies the intratemporal substitution effect. It is now clear that, if government spending is not at a pre-set level so that consumption taxes can be non-regressive (i.e., $d\tau/dc \geq 0$), then the substitution effect reinforces the stabilizing income effect. As a result, expectations again cannot be self-fulfilling.

3. The Basic Monetary Model

Our findings in the previous section highlight the fact that in order to have indeterminacy under consumption taxation, we must fulfill the following two necessary conditions:

1. Preferences must be non-separable in consumption and hours worked, i.e., $u_{12} \neq 0$ and

2. the consumption tax must be regressive, i.e., $d\tau/dc < 0$.

[8] See a similar analysis by Nourry et al (2011).

In this section, we study the stability nature of consumption taxes in a monetary economy. To highlight the stability role played by the monetary factor, we adopt a separable utility function in consumption and leisure so as to shut down the stabilizing income effect of consumption taxation.

3.1 Households

The representative agent is endowed with one unit of time, and chooses c_t, ℓ_t and k_t to maximize the discounted lifetime utility:

$$\int_0^{\infty}\left[u(c_t)+v(1-\ell_t)\right]\exp(-\rho t)dt. \tag{20}$$

The utility function yields positive and diminishing marginal products, i.e., $u^{'}>0>u^{''}$, and $v^{'}>0>v^{''}$. The budget constraint is:

$$\dot{k}_t = i_t - \delta k_t,$$

$$\dot{m}_t = r_t k_t + w_t \ell_t - i_t - (1+\tau_t)c_t - \pi_t m_t, \tag{21}$$

where r_t and w_t are the rental rate of capital and wage rate of labor, respectively, $\delta \in (0,1)$ denotes the depreciation rate, π_t is the inflation rate and τ_t is the endogenous tax rate on the consumption spending. Finally, the CIA constraint faced by the households is:

$$m_t \geq (1+\tau_t)c_t. \tag{22}$$

For the rest of the analysis, let's use λ_{jt} $(j=k,m)$ for the costate variables and ξ_t for the Lagrangian multiplier of the CIA constraint. The first-order conditions for this problem are as follows:

$$u^{'}(c_t) = (1+\tau_t)(\lambda_{mt}+\xi_t),$$

$$v^{'}(1-\ell_t) = \lambda_{mt} w_t,$$

$$\lambda_{kt} - \lambda_{mt} = 0,$$

$$\dot{\lambda}_{mt} = \lambda_{mt}(\rho+\pi_t) - \xi_t,$$

$$\dot{\lambda}_{kt} = (\rho + \delta)\lambda_{kt} - r_t \lambda_{mt},$$

and the transversality conditions are:

$$\lim_{t\to\infty} \lambda_{kt} k_t \exp(-\rho t) = \lim_{t\to\infty} \lambda_{mt} m_t \exp(-\rho t) = 0.$$

3.2 Firms

Assume that the constant-return technology adopted by the firm is $y = F(k, \ell)$ with $F_i > 0 > F_{ii}$, where $i = 1,2$. Since the factor market is perfectly competitive, we have:

$$r_t = F_1, \tag{23}$$

$$w_t = F_2. \tag{24}$$

3.3 Government

Government expenditure is financed by the following BBR:

$$G_t = \tau_t c_t + \mu_t m_t, \tag{25}$$

where μ is the nominal money growth rate and G is government expenditure. In the benchmark model, we simply assume that both μ and G are exogenous, and only τ_t is endogenous.[9]

3.4 Market Clearing Conditions

The goods market clearing condition is:

$$\dot{k}_t = F(k_t, \ell_t) - \delta k_t - c_t - G_t,$$

and the money growth rate satisfies:

[9] We consider the alternative formulation where μ is endogenous, with both τ and G exogenous. If G is endogenous, then the system is always determinate when the utility function is separable in c and ℓ, and the production function is neoclassical.

$$\frac{\dot{m}_t}{m_t} = \mu_t - \pi_t.$$

Finally, a binding CIA constraint yields:

$$m_t = (1+\tau_t)c_t.$$

4. Equilibrium Analysis

We have $\lambda_{kt} = \lambda_{mt}$ and hence the contemporaneous conditions become:

$$\xi_t = (r_t + \pi_t - \delta)\lambda_{kt}, \tag{26}$$

$$u'(c_t) = (1+\tau_t)(1+\pi_t + F_1 - \delta)\lambda_{kt}, \tag{27}$$

$$v'(1-\ell_t) = \lambda_{kt} F_2, \tag{28}$$

$$m_t = (1+\tau_t)c_t, \tag{29}$$

$$G = \tau_t c_t + \mu m_t. \tag{30}$$

Combining (27) and (28), we get the following characterization of intratemporal optimality:

$$\frac{v'(1-\ell_t)}{u'(c_t)} = \frac{F_2}{(1+\tau_t)(1+\pi_t + F_1 - \delta)}. \tag{31}$$

In addition, the above FOC equations allow us to write $(c_t, \ell_t, \tau_t, \pi_t, \xi_t)$ as a function of (λ_{kt}, k_t, m_t). Then we can solve (λ_{kt}, k_t, m_t) from the dynamic equations below:

$$\frac{\dot{\lambda}_{kt}}{\lambda_{kt}} = \rho + \delta - F_1, \tag{32}$$

$$\dot{k}_t = F(k_t, \ell_t) - \delta k_t - c_t - G, \tag{33}$$

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t. \tag{34}$$

4.1 Steady State

Since F exhibits constant returns to scale, we know that $F/k \equiv f$, F_1 and F_2 are functions of $x \equiv k/\ell$ only. From $\dot{\lambda}_k = 0$, we have:

$$F_1\left(x^*\right) = \rho + \delta,$$

which yields a unique x^* that is independent of τ^* and μ. By setting $\dot{m} = 0$, we get:

$$\pi^* = \mu. \tag{35}$$

The CIA constraint leads to:

$$m^* = \left(1 + \tau^*\right)c^*, \tag{36}$$

and then the BBR is:

$$G = \left[\tau^* + \mu\left(1 + \tau^*\right)\right]c^*. \tag{37}$$

From $\dot{k} = 0$, we have

$$\frac{c^*}{x^*\ell^*} = \frac{f\left(x^*\right) - \delta}{\left(1 + \mu\right)\left(1 + \tau^*\right)}. \tag{38}$$

Next, from (28), we get:

$$\frac{v'\left(1 - \ell^*\right)}{u'\left(c^*\right)} = \frac{F_2\left(x^*\right)}{\left(1 + \tau^*\right)\left(1 + \mu + \rho\right)}. \tag{39}$$

So (37)-(39) allow us to solve for $\left(c^*, \ell^*, \tau^*\right)$. Then (36) yields $m^*,$ and $k^* = x^*\ell^*$.

4.2 The Laffer Curve

According to (38) and (39), we know that:

$$\frac{dc^*}{d\tau^*} = -\frac{1+\chi^*}{\sigma^* + \chi^*}\frac{c^*}{1+\tau^*} < 0, \tag{40}$$

where $\sigma^* = -c^* u''(c^*)/u'(c^*) > 0$ and $\chi^* = -\ell^* v''(1-\ell^*)/v'(1-\ell^*) > 0$. Totally differentiating (37), we have:

$$\frac{dG}{d\tau^*} = \frac{1+\chi^* - (1+\tau^*)(1+\mu)(1-\sigma^*)}{(\sigma^* + \chi^*)(1+\tau^*)} c^*. \tag{41}$$

If $\sigma^* \geq 1$, $dG/d\tau^*$ is always positive. Only if $\sigma^* < 1$, there might exist an optimal $\tau^o \in (0,1)$ such that $dG/d\tau^* = 0$.[10] If τ^o exists, $dG/d\tau^* \gtreqless 0$ for $\tau^* \lesseqgtr \tau^o$. Then there exists a Laffer curve if $\sigma^* < 1$.

4.3 Stability

From (28), we can write ℓ_t as a function of λ_{kt} and k_t:

$$\ell_t = \ell(\lambda_{kt}, k_t), \ell_1 = -\frac{F_2}{v'' + \lambda_{kt}F_{22}} > 0, \ell_2 = -\frac{\lambda_{kt}F_{21}}{v'' + \lambda_{kt}F_{22}} > 0.$$

Using (29) and (30), we have:

$$c_t = c(m_t), c'(m_t) = 1+\mu > 0,$$

$$\tau_t = \tau(m_t), \tau'(m_t) = -G/c_t^2 < 0.$$

Finally, by (27), we can derive π as:

$$\pi_t = \pi(\lambda_{kt}, k_t, m_t),$$

10 The restrictions on the parameterization for $\tau^o \in (0,1)$ are

$$1+2[(1-\sigma^*)\mu - \sigma^*] > \chi^* > (1-\sigma^*)\mu - \sigma^*.$$

Where:

$$\pi_1 = -\frac{F_2}{\lambda_{kt}}\left(\ell_2 + \frac{1+\rho+\mu}{F_2}\right),$$

$$\pi_2 = -F_{12}\ell_2 - F_{11},$$

$$\pi_3 = \left[(1-\sigma_t)(1+\tau_t)(1+\mu)-1\right]\frac{1+\mu+\rho}{(1+\tau_t)c_t}.$$

Linearizing the system (32) - (34) around the steady state, we have:

$$\begin{pmatrix} \dot{\lambda}_{kt} \\ \dot{k}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} -\lambda_k^* F_{12}\ell_1 & -\lambda_k^*\left(F_{11}+F_{12}\ell_2\right) & 0 \\ F_2\ell_1 & \rho + F_2\ell_2 & -(1+\mu) \\ -\pi_1 m^* & -\pi_2 m^* & -\pi_3 m^* \end{pmatrix}\begin{pmatrix} \lambda_{kt}-\lambda_k^* \\ k_t - k^* \\ m_t - m^* \end{pmatrix}.$$

By computing the eigenvalues of the Jacobian matrix, we then derive the indeterminacy condition as follows.

Proposition 1 *In a monetary economy with balanced-budget consumption tax rules and additive preference, then indeterminacy can occur if and only if:*

$$(1-\sigma^*)(1+\tau^*)(1+\mu) > 1+\chi^*.$$

From (41), the necessary and sufficient condition for indeterminacy means that $\frac{dG}{d\tau^*} < 0$. That is to say, if and only if τ^o exists and $\tau^* > \tau^o$, could we have indeterminacy. For consumption tax rates on the upward-sloping side of the Laffer curve, the equilibrium is unique. This result works in an opposite way of Schmitt-Grohé and Uribe (1997), where the unique equilibrium exists for the labor income tax rate on the downward-sloping side of the Laffer curve. This is mainly due to the fact that consumption taxation and income taxation work in an opposite direction in the periodic budget constraint. (See Figure 2)

The intuition underlying indeterminacy can be understood as follows. Given the separability of preferences specification, as in Giannitsarou (2007), the intratemporal effects offset each other so that labor supply is independent of the consumption tax rate. Given expectations of a higher future consumption tax and the capital stock and rate of money growth, the only effect that remains is the intertemporal substitution effect which leads to an increase in current consumption. This is the end of the story in the real model so that expectations cannot be fulfilled. But in a monetary environment, the rise in consumption tightens the CIA constraint so that the marginal utility of income increases. Since labor supply does not change, the real wage falls so that capital is lowered according to (28). As a consequence, both income and consumption decline so that the initial stabilizing income effect is weakened. Thus the expectation of a higher consumption tax rate can be fulfilled under the BBR with money.

5. Money and Macroeconomic Instability

From the above analysis, it is clear that local indeterminacy cannot occur in the model when money is absent. Thus, it is important to understand the role played by money in leading to macroeconomic instability. In this section, by comparing with the real model, we point out that the instability role of money works through the following two channels:

1) money introduces back the non-separability of the utility function in c and m so that the conflicting intratemporal effects of consumption are regenerated;[11]

2) money increases the magnitude of the regressiveness of consumption taxation.

5.1 Non-Separability in Preferences

To simplify the algebra, we adopt the indivisible labor assumption and the Cobb-Douglas production function of the basic monetary model.[12] Specifically, the discounted lifetime utility function and the production technology are given by:

$$\int_0^\infty \left(u(c_t) - A\ell_t\right)\exp(-\rho t)dt, \tag{42}$$

And:

$$y_t = Bk_t^\alpha \ell_t^{1-\alpha}. \tag{43}$$

Therefore, it is straightforward to show that the dynamical system is:

[11] The idea that the CIA model and the money-in-the-utility-function model are somehow equivalent can be traced back to Feenstra (1986) and Wang and Yip (1992).

[12] It can be shown that all our results are valid for the general constant-return production technology.

$$\dot{x}_t = -\frac{x_t}{\alpha}\left(\rho + \delta - \alpha B x_t^{\alpha-1}\right),$$

$$\dot{k}_t = B k_t x_t^{\alpha-1} - \delta k_t - c(x_t, m_t) - G,$$

$$\dot{m}_t = [\mu - \pi(x_t, m_t)]m_t.$$

Linearizing the system around the steady state and solving for the eigenvalues (e_i) of the Jacobian matrix, we get:

$$e_1 = (\alpha - 1)Bx^{*\alpha-1} < 0,$$

$$e_2 = \rho + \frac{1-\alpha}{\alpha}(\rho + \delta) > 0,$$

$$e_3 = -\frac{\partial \pi}{\partial m} m^*.$$

Where:

$$\frac{\partial \pi}{\partial m} = \frac{1 + \pi^* + \rho}{c^*}\left[(1 + \pi^*)(1 - \sigma) - \frac{1}{1 + \tau^*}\right].$$

The necessary and sufficient condition for indeterminacy is $e_3 < 0$; otherwise, if $e_3 > 0$, the system is a saddle and we have determinacy.

Next, let us adopt the money-in-the-utility-function (MIUF) approach and specify the discounted lifetime utility as:

$$\int_0^\infty [\tilde{u}(c_t, m_t) - A\ell_t]\exp(-\rho t)dt. \qquad (44)$$

The dynamic system is observationally equivalent:

$$\dot{x}_t = -\frac{x_t}{\alpha}\left(\rho + \delta - \alpha B x_t^{\alpha-1}\right),$$

$$\dot{k}_t = Bk_t x_t^{\alpha-1} - \delta k_t - c(x_t, m_t) - G,$$

$$\dot{m}_t = m_t[\mu - \pi(x_t, m_t)].$$

The eigenvalues ($\tilde{e}_i$) of the Jacobian matrix of the linearized system are:

$$\tilde{e}_1 = \alpha(\alpha-1)Bx^{*\alpha-2} < 0,$$

$$\tilde{e}_2 = Bx^{*\alpha-1} - \delta = \rho + \frac{1-\alpha}{\alpha}(\rho+\delta) > 0,$$

$$\tilde{e}_3 = -\frac{\partial \pi}{\partial m} m^*,$$

Where:

$$\frac{\partial \pi}{\partial m} = -\frac{\tilde{u}_{12}\left(\frac{\tilde{u}_{12}}{\tilde{u}_1}c^* + \frac{\pi^*}{1+\tau^*}\right)}{\frac{\tilde{u}_{11}}{\tilde{u}_1}c^* + \frac{\tau^*}{1+\tau^*}} + \tilde{u}_{22}.$$

Since we only have one predetermined variable, the necessary and sufficient condition for indeterminacy is that $\tilde{e}_3 < 0$, i.e. $\partial\pi/\partial m > 0$. It is obvious that if $\tilde{u}_{12} = 0$, then $\tilde{e}_3 > 0$ and hence the system is always determinate.

Recall that the non-separability of preferences specification is a necessary condition for indeterminacy. So the reason that indeterminacy is possible under the CIA model is due to the fact that preferences are not separable in all its arguments.

5.2 Regressiveness of Consumption Taxation

In order to compare the regressiveness of consumption taxation, we recall (10) from the real model and calculate the elasticity of taxation to consumption:

$$\varepsilon_{\tau c} = \frac{d\tau_t}{dc_t}\frac{c_t}{\tau_t} = -1.$$

Then for the monetary model, by (29) and (30), we have:

$$\tilde{\varepsilon}_{\tau c} = \frac{d\tau_t}{dc_t}\frac{c_t}{\tau_t} = -\left(1 + \frac{\mu}{1+\mu}\frac{1}{\tau_t}\right) \leq -1 = \varepsilon_{\tau c},$$

and the magnitude is increasing in μ. Thus, as monetary policy becomes more expansionary so that μ increases, the regressiveness of consumption taxation rises (ie, $\tilde{\varepsilon}_{\tau c}$ becomes more negative). Since regressiveness of taxation is a necessary condition for indeterminacy, again the presence of money makes indeterminacy more likely in the model.

6. Non-Linear Taxation

Following Guo and Lansing (1998), we assume that the tax rate is a function of consumption relative to its steady-state level, i.e., $\tau = \tau\left(c_t/c^*\right)$.[13] With both μ and G exogenous in the system, the government budget constraint becomes:

$$G = \tau\left(c_t/c^*\right)c_t + \mu m_t. \tag{45}$$

Substituting the CIA constraint (29) into (45), we can write the government expenditure to be a function of c_t only. Denoting the elasticity of government expenditures around the steady-state as ζ^*, we have:

$$\zeta^* \equiv \frac{\partial G}{\partial c}\frac{c^*}{G} = 1 + \frac{\tau'}{\tau^* + \frac{\mu}{1+\mu}}.$$

Note that the taxation is progressive (regressive) if and only if $\zeta^* > 1$ (<1). Also recall that the Schmitt-Grohé-Uribe-Giannitsarou balanced budget rule implies that $\zeta^* = 0$.

6.1 Equilibrium Analysis

The dynamical system is given by the following differential equations:

$$\frac{\dot{\lambda}_{kt}}{\lambda_{kt}} = \rho + \delta - F_1,$$

[13] See Gokan (2012) for a study of non-linear factor taxation a finance constrained macroeconomy.

$$\dot{k}_t = F(k_t, \ell_t) - \delta k_t - [1 + \tau(c_t/c^*)]c_t - \mu m_t,$$

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t.$$

With the Guo-Lansing specification, the non-linear consumption tax is independent of c^* in the steady state, i.e., $\tau^* = \tau^*(1)$. Linearizing the system around the steady state, we have:

$$\begin{pmatrix} \dot{\lambda}_{kt} \\ \dot{k}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} -\lambda_k^* F_{12}\ell_1 & -\lambda_k^*(F_{11} + F_{12}\ell_2) & 0 \\ F_2\ell_1 & \rho + F_2\ell_2 & -(1+\mu) \\ -\pi_1 m^* & -\pi_2 m^* & -\pi_3 m^* \end{pmatrix} \begin{pmatrix} \lambda_{kt} - \lambda_k^* \\ k_t - k^* \\ m_t - m^* \end{pmatrix},$$

Where:

$$\ell_1 = -\frac{F_2}{v'' + \lambda_k^* F_{22}} > 0, \ell_2 = -\frac{\lambda_k^* F_{21}}{v'' + \lambda_k^* F_{22}} > 0,$$

$$\pi_1 = -\frac{F_2}{\lambda_k^*}\left(\ell_2 + \frac{1+\rho+\mu}{F_2}\right),$$

$$\pi_2 = -F_{12}\ell_2 - F_{11},$$

$$\pi_3 = -\frac{(1+\rho+\mu)(1+\mu)}{\{1+\zeta^*[(1+\mu)\tau^* + \mu]\}c^*}\left[\sigma^* + (\zeta^* - 1)\frac{\tau^* + \frac{\mu}{1+\mu}}{1+\tau^*}\right].$$

Then from the solution of the characteristic equation of the dynamical system, we have proposition 2.

Proposition 2 *In a monetary economy with non-linear balanced-budget consumption tax rule and separable utility function, indeterminacy can occur if and only if:*

$$\pi_3 > \frac{1+\rho+\mu}{m^*}\chi^*,$$

i.e.

$$\frac{(1+\mu)(1+\tau^*)(\sigma^*-1)}{1+\zeta^*\left[(1+\mu)\tau^*+\mu\right]}+\left(1+\chi^*\right)<0,$$

where $\sigma^*=-\dfrac{c^*u_{11}}{u_1}>0$ and $\chi^*=-\dfrac{v_{11}\ell^*}{v_1}>0$.

We first note that if consumption tax is progressive so that $\zeta^*>1$, then macroeconomic stability is guaranteed as in the real model. By applying (29) and (45), Proposition 2 implies that the necessary condition for indeterminacy under the preset government expenditure BBR rule (Schmitt-Grohé and Uribe, 1997) is $\sigma^*<1$:

Corollary 1 *From the Schmitt-Grohé-Uribe-Giannitsarou balanced budget rule and the CIA constraint, we have $\zeta^*=0$ and hence the necessary and sufficient condition for indeterminacy can be written as:*

$$(1+\mu)(1+\tau^*)(1-\sigma^*)>1+\chi^*.$$

So $\sigma^*<1$ is necessary for indeterminacy.

Again, as we explain in the previous sections, macroeconomic instability requires the intertemporal elasticity of substitution in consumption to be larger than unity in the Schmitt-Grohé-Uribe-Giannitsarou setting where $\zeta^*=0$. However, for the general case where we have non-zero elasticity of government expenditure, then $\sigma^*<1$ is no longer necessary for indeterminacy. For the general case, we rewrite the necessary and sufficient condition for indeterminacy of Proposition 2 as follows:

Corollary 2 *If the consumption tax is endogenous and non-linear, and the utility function is separable in consumption and leisure, the necessary and sufficient conditions for indeterminacy are as follows: 1) for $\sigma^*<1$,*

$$\zeta^*<1 \;\; and \;\; 1-\frac{1}{1-\zeta^*}<\frac{1}{(1+\mu)(1+\tau^*)}<1-\frac{1}{1-\zeta^*}\frac{\sigma^*+\chi^*}{1+\chi^*};$$

and 2) for $\sigma^*>1$, it changes to:

$$\zeta^*<0 \;\; and \;\; 1-\frac{\sigma^*}{1-\zeta^*}<\frac{1}{(1+\mu)(1+\tau^*)}<1-\frac{1}{1-\zeta^*}.$$

If the taxation rule is nonlinear to consumption and the consumption tax is regressive ($\zeta^* < 0$), then indeterminacy will occur even when the intertemporal elasticity of substitution is smaller than 1. Moreover, there exists a trade-off between μ and τ^*. If μ increases, the minimal level of τ^* required for indeterminacy will become smaller for any given σ^*, χ^* and ζ^*.

6.2 Parameterization

For the benchmark parameterization, we set $\rho = 0.04$, $\delta = 0.1$ in accordance with yearly US data. Also, we take the infinite Frisch labor supply elasticity $\chi^* = 0$ as in Hansen (1985), Schmitt-Grohé and Uribe (1997) and Giannitsarou (2007). For the intertemporal elasticity of substitution, the general consensus was, up until recently, that it is smaller than one. However, recent estimation provides robust results in the range between 2 and 3 (see Gruber 2006). Consequently, we choose $\sigma^* = 1/2.5$.

We first examine the stability effects of τ^* and ζ^* under a regime where the money growth rate is set at $\mu = 0.1$. The range of these two policy parameters are $\tau^* \in (0,1)$ and $\zeta^* \in (-1,1)$. Figure 3 reports the stability effect by depicting the indeterminacy region. For instance, when $\tau^* > 0.12$ or $\zeta^* \leq 0.25$, indeterminacy is possible under our benchmark parameterization. We then follow the presentation of Schmitt-Grohé and Uribe (1997) by highlighting the average values of τ^* and ζ^* of OECD countries in the figure. For τ^*, we adopt the consumption tax rates analyzed by Mendoza et al (1997). For ζ^*, we follow Nourry et al (2011) to use the output elasticity of government expenditure provided by Lane (2003). As the money growth rate is low in this case, none of the OECD countries locate in the indeterminacy region.

Figure 4 reports the stability effect by changing $\mu = 0.3$, which is a moderately high value. We find that more than half of the OECD countries fall into the indeterminacy region, including Italy, Canada, France, UK, Finland, Belgium, Germany Australia, Netherlands, Sweden, etc. It implies that the stabilization effect of consumption tax rule will be dominated if the money growth rate increases.

By further raising the money growth rate to a higher level, say $\mu = 0.6$, we find only three countries, Switzerland, Denmark and Norway, remain in the determinacy region. This is illustrated in Figure 5.

Finally, we provide a sensitivity analysis of the above findings with respect to two critical parameters: σ^* and χ^*. We present the analysis in Figures 6-8 under different rates of money growth.

As we can see, the effects of monetary growth on indeterminacy are robust with respect to alternative values of $\sigma^* = (1/4, 2/3)$ and $\chi^* = (0.5, 1.5, 5)$. On the other hand, for σ^* and χ^*, we find that the lower bound of the indeterminacy region in the (ζ^*, τ^*) space decreases when these two parameters decrease. In order words, indeterminacy is more likely when either σ^* or χ^* decreases. The intuition is straightforward. When σ^* decreases or the intertemporal elasticity of substitution in consumption increases, then for any given initial expected increase in consumption tax, the distortionary effect on the CIA constraint will be stronger due to a larger increase in current consumption from intertemporal substitution. When χ^* decreases or the Frisch elasticity of labor supply with respect to wage increases, then a given initial expected increase in consumption tax will lead to a larger reduction in the real wage for a constant labor supply. Both of these changes will contribute to the likelihood of indeterminacy of consumption taxation.[14]

7. Concluding Remarks

This paper extends the Giannitsarou (2007) analysis to a standard one-sector cash-in-advance economy to examine whether a balanced-budget consumption-tax rule is dynamically stable. In the absence of money, it is established that macroeconomic stability is guaranteed when, either the utility function of the representative household is separable in consumption and leisure or, when the distortionary taxes are non-regressive. We have found that, in the presence of a monetary distortion, the CIA constraint means that the intertemporal channel of consumption taxation affects capital accumulation, as occurs in the Schmitt-Grohé and Uribe model (1997). As a result, indeterminacy of the balanced-budget consumption-tax rule is possible and depends on the parameterization of the model, especially the intertemporal elasticity of substitution in consumption and the wage elasticity of labor supply. Nevertheless, with progressive consumption taxes, determinacy is the equilibrium outcome.

Quantitatively, following Schmitt-Grohé and Uribe (1997), we explore the indeterminacy outcome of OECD countries by highlighting their average values of consumption tax rates and output elasticities of government expenditure. We have shown that, as the money growth rate increases, the boundaries of the indeterminacy region shift to the right in the space of these two parameters. Consequently, when monetary policy is expanding, more and more countries fall into the indeterminacy region given their consumption tax rates and government spending. From a policy standpoint, our findings suggest that under the current over-easy monetary policies such as QE and OT, macroeconomic instability can be a major concern when implementing a BBR. It is our hope that this paper can make a positive contribution to the existing literature " suggesting that some frequently proposed policy feedback rules

14 See also Schmitt-Grohé and Uribe (1997) for a similar sensitivity finding of χ^* on indeterminacy in a real setting of factor taxation.

linking monetary and fiscal variables to the state of the economy can induce endogenous fluctuations and hence be destabilizing" (Schmitt-Grohé and Uribe, 1997, p.978).

An interesting extension to our analysis would be to explore the role of the price level on indeterminacy in the CIA economy. Specifically, it is known that the likelihood of indeterminacy is reduced when prices are sticky, as in Calvo (1983).[15] This type of analysis also opens the door for us to study other monetary policies such as interest rate rules. Nevertheless, we plan to leave these extensions to the BBR literature to future work.

[15] See Weder (2008) for such an analysis.

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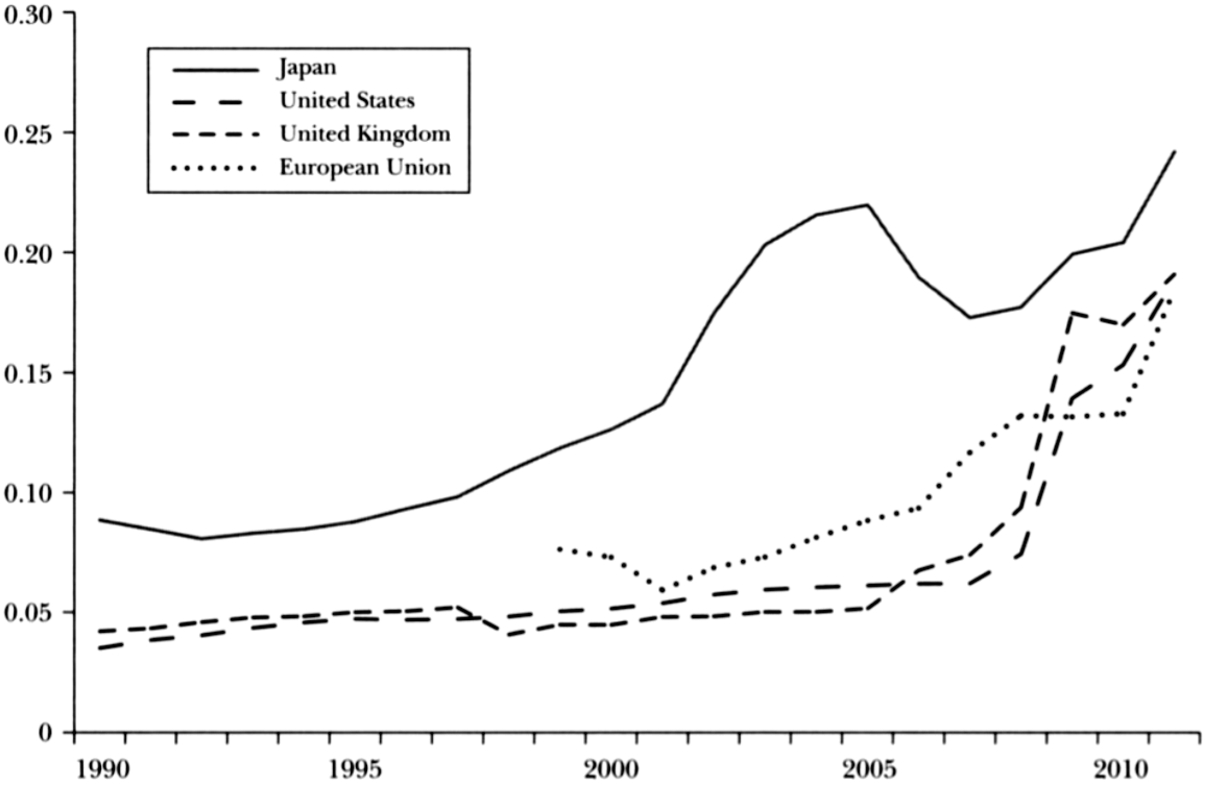
Figure 1. Monetary Base/GDP



Source: Datastream.

Note: Figure 1 shows the monetary base (that is, currency outstanding plus bank reserves) relative to GDP for four countries.

Figure 2. Laffer Curve and Regions of Determinacy/Indeterminacy

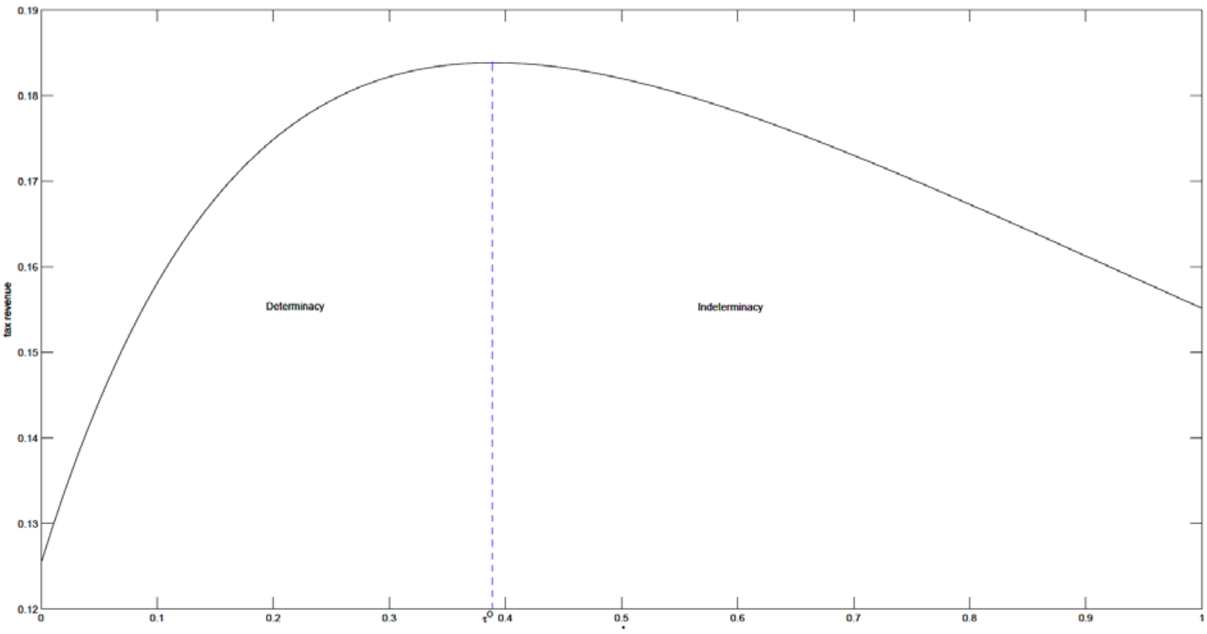


Figure 3. Indeterminacy Region with Non-Linear Consumption Tax Rule and μ = 0.1.

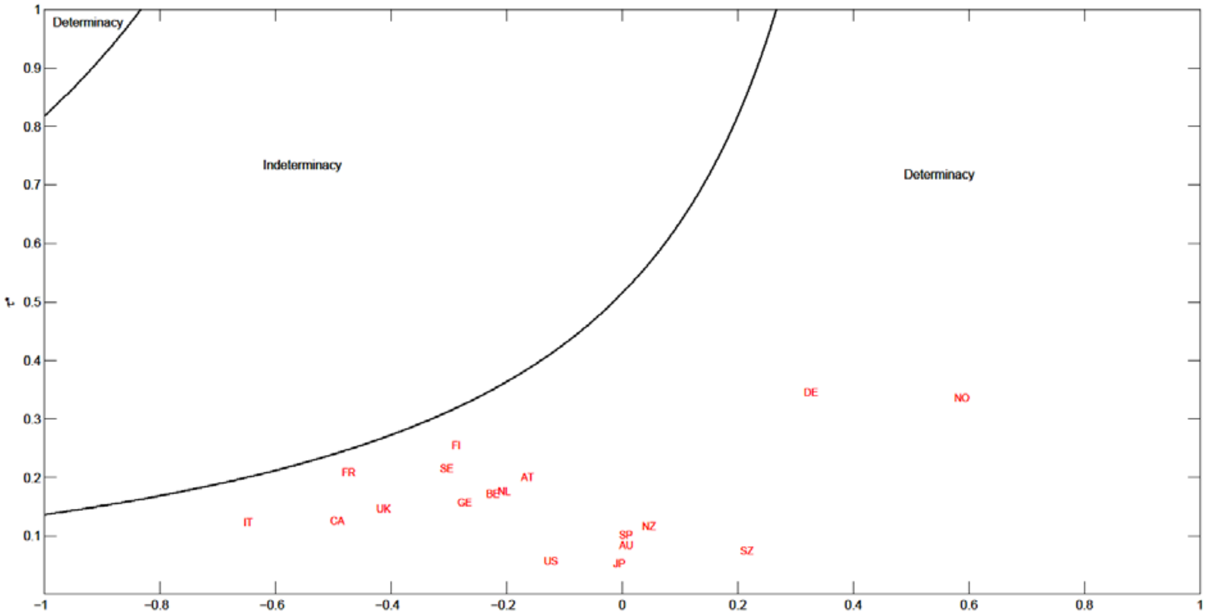


Figure 4. Indeterminacy Region with Non-Linear Consumption Tax Rule and μ = 0.3.

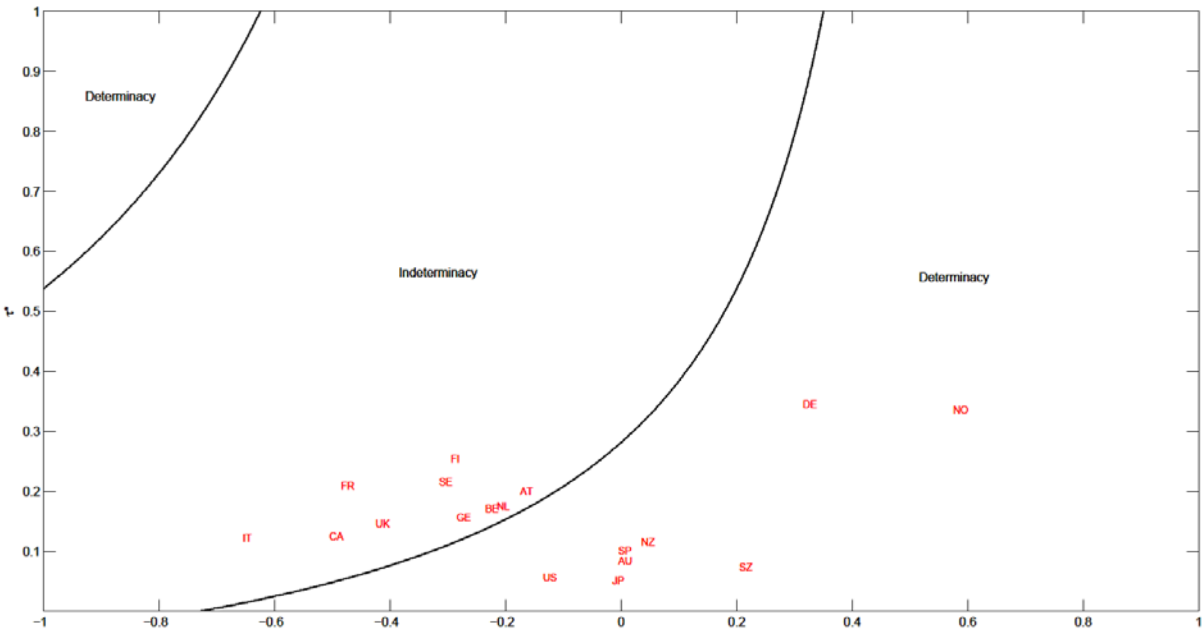


Figure 5. Indeterminacy Region with Non-Linear Consumption Tax Rule and μ = 0.6.

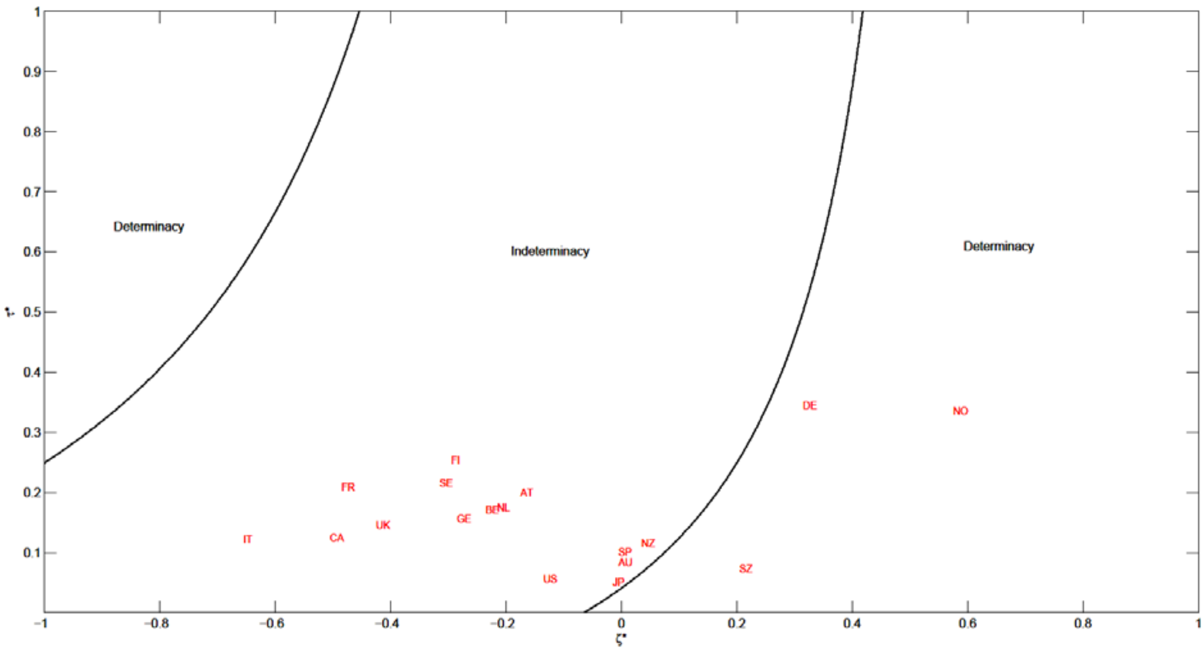


Figure 6. Indeterminacy Region with Non-Linear Consumption Tax Rule as χ^* and σ^* Change and μ = 0.1.

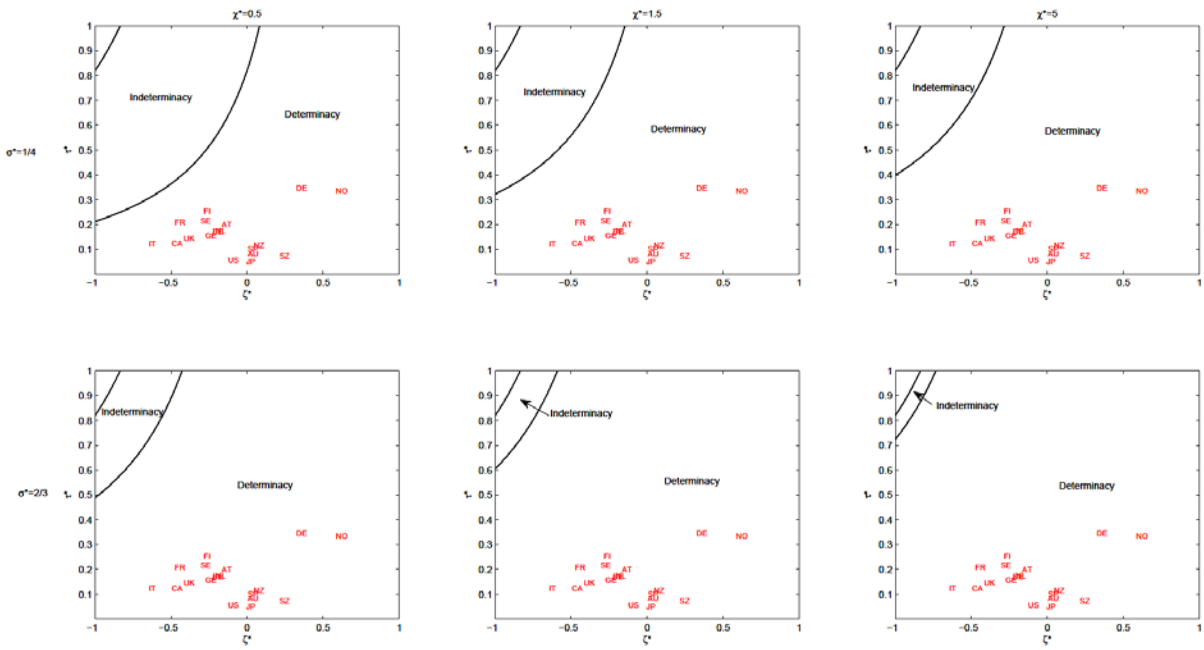


Figure 7. Indeterminacy Region with Non-Linear Consumption Tax Rule as χ^* and σ^* Change and μ = 0.3.

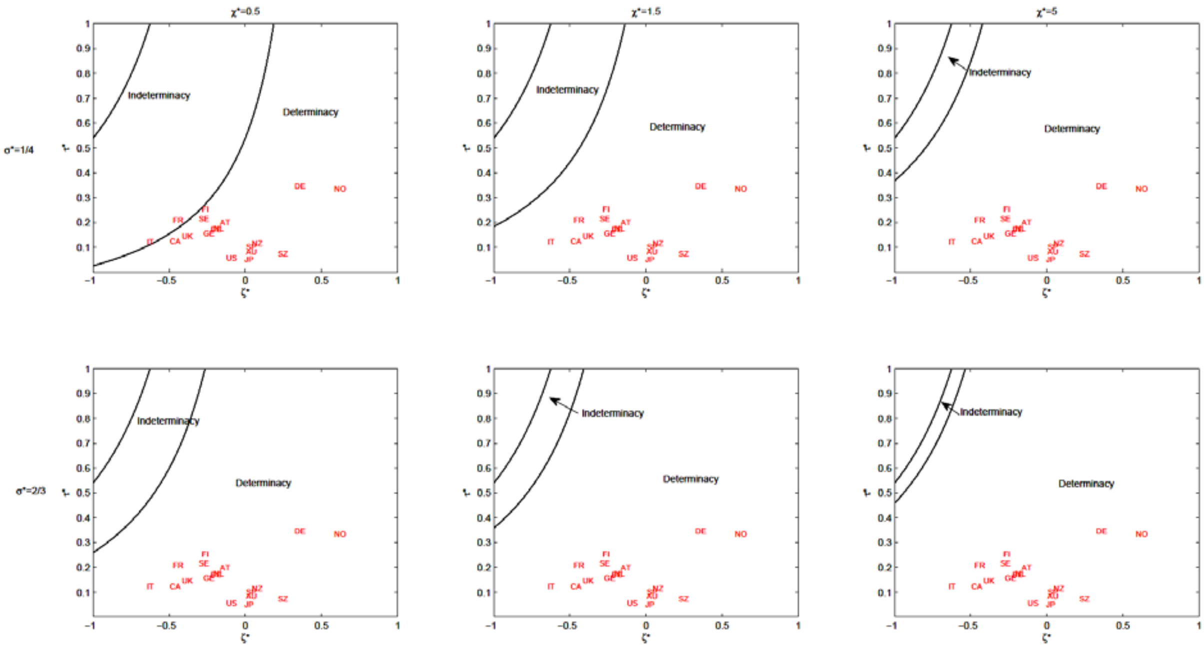


Figure 8. Indeterminacy Region with Non-Linear Consumption Tax Rule as χ^* and σ^* Change and μ = 0.6.