HONG KONG INSTITUTE FOR MONETARY RESEARCH

A CURRENCY BOARD MODEL OF HONG KONG

by Yue Ma, Guy Meredith and Matthew S. Yiu

HKIMR Working Paper No.1/2002

January 2002

A Currency Board Model of Hong Kong*

Yue Ma
Economics Department, Lingnan University

Guy Meredith
International Monetary Fund

and

Matthew S. Yiu
Hong Kong Institute for Monetary Research

January 2002

Abstract

The need for a deeper understanding of the operation of Hong Kong's currency board arrangements was highlighted during the Asian financial crisis in 1998. A model-based approach built on hypothetical stochastic simulations would be useful for this purpose. This paper develops a new procedure of implementing stochastic simulations in a currency board model for Hong Kong. Our new procedure is useful in the context of a nonlinear model with forward-looking expectations under conditions of non-certainty-equivalence, such as the model of Hong Kong's currency board. A simple target-zone model of the exchange rate is used as an example to illustrate the difference between our new simulation procedure and existing procedures in the literature. Finally, the new procedure is applied to the currency board model to investigate the stochastic properties of endogenous variables under a wide range of shocks.

Keywords: *currency board, stochastic simulation, certainty equivalence, Hong Kong*
JEL codes:
F31 - Foreign Exchange
C15 - Statistical Simulation Methods: Monte Carlo Methods
E47 - Simulation (of a Monetary Model)

* Part of this paper was written when Meredith was Executive Director (Research) of the Hong Kong Monetary Authority (HKMA) and Director of the HKIMR, and Ma was Research Fellow at the HKIMR during July-August 2000. Early versions of the paper were presented at two workshops of the Research Department of the HKMA. The authors are grateful for the comments from two anonymous referees of the HKIMR Working Paper Series, Priscilla Chiu, Stefan Gerlach, Jimmy Ha, Daryl Ho, Chang Shu, Shu-ki Tsang and the workshop participants. We also thank Yu Qiao for research assistance provided. Ma thanks the HKIMR for their hospitality. The views presented in this paper are those of the authors and do not necessarily reflect those of the International Monetary Fund, the Hong Kong Institute for Monetary Research, its Council of Advisers, or Board of Directors.

1. Introduction

Hong Kong has built up experience on the design and operation of currency board arrangements over the years. Various authors have also articulated the design and operation of Hong Kong's arrangements. Yam (1998) depicted monetary developments of Hong Kong, including technical issues regarding the currency board arrangements. Balino and Enoch (1997) studied various currency board arrangements in a global context, including those of Hong Kong.

The need for a deeper understanding of currency board arrangements was highlighted during the Asian financial crisis in 1997-98, when Hong Kong was hit by an unprecedented sequence of financial shocks. This resulted in the introduction of technical measures in September 1998 to improve the functioning of Hong Kong's system (Yam, 1998). However, past experience provides only a limited perspective on the operations of Hong Kong's currency board arrangements. Such constraints are inevitable when the only guidepost for assessing the performance of the system is historical experience. In order to understand better the operation and design of the arrangements in Hong Kong, Meredith (1999) adopted a model-based approach to study their operation under a wide range of shocks, and to experiment with the design of the arrangements.[1]

This paper extends Meredith's (1999) model by investigating its rational expectations solutions in a stochastic environment. This involves performing hypothetical experiments on an artificial model of the currency board system. Such an approach has two advantages: (i) it allows an assessment of how the arrangements would work under conditions more volatile than those experienced since the technical measures were introduced in September 1998, and (ii) it permits a better understanding of the mechanisms at work under Hong Kong's currency board arrangements.

However, it is generally recognised that it is difficult to conduct stochastic simulations on a nonlinear model with forward-looking expectations.[2] Most of the existing literature implements such simulations under the 'certainty equivalence' assumption. 'Certainty equivalence' means that the expected values of the endogenous variables depend only upon the deterministic means of other variables. In practice, simulations are conducted by introducing shocks to the current period only, whilst imposing zero shocks (i.e. the mean of the shocks) to all future periods. Fair and Taylor (1983 and 1990) and Meredith (1999), for example, show how stochastic simulations are implemented under the certainty equivalence assumption. This procedure treats the agents as if they believed that no shocks would occur in the future. Their expectations are then formed on the basis of the deterministic levels of the model's predictions, as opposed to the stochastic means. This approach is generally acceptable in a linear model, as the deterministic and stochastic means of the variables will be the same.

[1] For the macroeconomic modelling of the Hong Kong economy, see, for example, Ma, et al. (1998) and Siregar and Walker (2000). However, no structural features of the currency board were incorporated in these papers.

[2] For a discussion of some theories of nonlinear exchange rate models, see Ma and Kanas (2000).

The situation becomes more complicated, however, when the forward-looking variables also depend upon the variance, or the higher moments, of the future distributions, representing a violation of the 'certainty equivalence' assumption. This will generally be the case in nonlinear models, making certainty equivalence an unsatisfactory assumption. This is likely to be particularly true when the nonlinearities involve the abrupt truncation of the distribution of a variable, as in the case of a target zone for an exchange rate or a zero lower bound for a nominal interest rate.[3]

This paper develops a new procedure of implementing stochastic simulations in the absence of certainty equivalence, such as the currency board model of Hong Kong initiated by Meredith (1999). Our procedure can be regarded as a generalisation of the solution methodology for linear rational expectations model to nonlinear model, where we are approximating the analytical solution with the regressions, rather than solving for it exactly. A simple target-zone model of exchange rates is used to illustrate the difference between our new procedure and existing techniques. Finally, the new procedure is applied to the currency board model to investigate the stochastic properties of endogenous variables under a wide range of shocks. A comparison of the simulation results of the new procedure with that of the existing procedure shows significant differences in the stochastic properties of the simulated variables, indicating the importance of making the correct assumption about the expectations formation.

Our paper serves three purposes. The first is to formally present the currency board model of Hong Kong initiated by Meredith (1999). Secondly, it develops a new stochastic simulation procedure for a nonlinear model with forward-looking expectations under conditions of non-certainty-equivalence. Thirdly, it applies the new simulation procedure in the context of stochastic simulations of the currency board model.

The current currency board model is in an experimental stage, with relationships that have been informally calibrated to yield results that are typical of the experience to date. A more formal calibration and/or estimation exercise would be needed to yield greater confidence in the specification of the key behavioural equations. The stochastic shocks that have been applied to the model have also been rather casually chosen to generate "interesting" results to illustrate the properties of the system. As such, they do not necessarily reflect the actual properties of historical shocks.

The remainder of the paper is organised as follows. Section 2 presents the structure of the currency board model; a flow chart of the model is presented in Figure 1, while the variable definitions are summarised in Appendix A. Section 3 discusses a new stochastic simulation procedure for a nonlinear model with forward-looking expectations under conditions of non-certainty-equivalence. Section 4 reports stochastic simulation results for the currency board model. Section 5 concludes.

[3] Whist this paper focuses on the non-certainty-equivalence aspect of the nonlinear models, other aspects of the nonlinearity such as the empirical distributions of the shocks and the implications of starting values of the model on its dynamic process are left to future research.

2. The Structure of the Currency Board Model of Hong Kong

This section describes the model used to perform the simulations of money market activity in Hong Kong under a currency board system. It is designed as a short-run model of the interaction between interest rates, the exchange rate, and interbank liquidity.[4] The Hong Kong Monetary Authority's (HKMA) discount window is one determinant of the supply of HK$ liquidity, as is the HKMA's intervention activity in the foreign exchange market under the currency board system. The relationships in the model are calibrated informally to be broadly consistent with the stylised facts of the operation of the arrangements since September 1998 (Yam, 1998). A more formal estimation strategy is constrained at this point by the lack of long time series for the relevant variables, but would be an interesting avenue for future research.

As discussed below, the model embodies model-consistent (or "rational") expectations. This means that expectations are consistent with the model's future predictions of the relevant variables based on information available when the expectations are formed.[5] Such expectations apply to two variables: (i) the future exchange rate, which determines the expected holding-period yield on HK$ versus US$ assets adjusted for expected exchange rate movements; and (ii) future overnight interest rates, which determine the current level of the one-month interest rate.

We first describe the main relationships in the model, followed by a brief discussion of calibration issues. A flow chart of the model is presented in Figure 1, which exhibits the structure of the model. The variable definitions are summarised in Appendix A.

2.1 Banks' Liquidity Preference Schedule

Behaviour in the interbank market is determined, in part, by a liquidity preference schedule that relates the level of aggregate clearing balances of the banks to overnight HIBOR (Hong Kong Interbank Offer Rate) (cf. Figure 1). The lower is overnight HIBOR, the lower is the opportunity cost to banks of holding liquidity in (non-interest-bearing) clearing balances, and the higher will be the demand for such balances. At the limit, as interbank market interest rates approach zero, the banks' demand for clearing balances would become infinite, as there would be no incentive to lend funds out.[6] In contrast, as interest rates rise, banks' demand for clearing balances are assumed to approach some minimum frictional level determined by uncertainties about end-of-day clearing.[7] We model this liquidity preference schedule as a rectangular hyperbola. The inverse liquidity preference schedule is given as follows:[8]

---

[4] Other sectors of the financial market such as the stock market have not yet been incorporated into the current version of the model.

[5] Rational expectations are distinct from "perfect foresight", as the latter refers to a situation where expectations equal future realisations of the variables. The model's predictions will not in general equal these realisations because the latter will depend on future shocks that were unanticipated when the expectations were formed.

[6] This would correspond to a liquidity trap for banks' clearing balances.

[7] Under the Real Time Gross Settlement System (RTGS) of Hong Kong, banks with negative overnight clearing balances would be in technical default. As all banks have nonnegative balances, and some banks receive funds late in the day that they are unable to lend out overnight, the end-of-day aggregate balance is always positive.

[8] The risk premium of the interest rate is not explicitly modelled yet and remains to future research.

$$r\_on = \alpha / (ab - ab^{min})\, e^{u\_ab}, \qquad (1)$$

where: ab = aggregate balance
ab^{min} = minimum frictional level of aggregate balance (set to zero for simplicity)
r_on = overnight HIBOR
u_ab = shocks to the aggregate balance.

α is a parameter that calibrates the equilibrium level of the aggregate balance at a reference interest rate. In the model, it is set to HK$1 billion when overnight HIBOR equals the US Fed Funds target rate.

2.2 Capital Flows: The Private Sector Demand for HK$ Assets

To complete the description of the money market, we need an equation describing capital inflows and outflows — i.e. the private sector's portfolio allocation between short-term HK$ and foreign currency assets. The interaction between this portfolio allocation decision and the banks' demand for clearing balances simultaneously determines HK$ interest rates (i.e. the overnight HIBOR, r_on) and the level of nonborrowed clearing balances (ab_nb), conditional on a given level of the exchange rate (cf. Figure 1). As discussed below, adding an equation to tie down the exchange rate, er, then gives a model that solves for all three variables of r_on, ab_nb and er simultaneously.

We assume that the share of private sector financial assets held in HK$s depends on the expected return on HK$ versus foreign currency assets. For the purposes of this model, the only return we consider is that on short-run liquid assets — changes in other returns that affect portfolio allocation are reflected in a disturbance term in the equation. Specifically, we assume that short-term portfolio allocation depends on an average of overnight and one-month HIBORs (adjusted for expected exchange rate movements) relative to equivalent US interest rates. The higher are HK$ rates, the greater will be the proportion of short-term assets held in HK$. Expected depreciation of the HK$, in contrast, reduces the return on domestic assets and results in capital outflows.[9]

The capital flow equation is modelled as a partial adjustment mechanism where the response of capital flows to a given interest differential increases over time according to a Koyck-type process.[10] Specifically:

$$k = \delta k^* + (1-\delta)\, k_{-1} + u\_k \qquad (2)$$

$$k^* = \alpha + (\beta/2)\, (UIP_{daily} + UIP_{monthly} + u\_r)$$

[9] There is no restriction imposed on the expected future spot exchange rate. This implies the credibility of the currency board arrangement is endogenously determined by the model. This issue will be investigated in future research.

[10] This implies that the capital markets are imperfect (see, for example, Barro, et al. 1995).

where:

UIP_{daily}	=	$r\_on - \Delta_1 er^e_{+1} - r\_ff$, (daily uncovered interest parity condition)
$UIP_{monthly}$	=	$r\_1m - \Delta_{30} er^e_{+30} - r\_ff - risk$ (monthly uncovered interest parity condition)
k	=	net stock of short-term HK$ assets held by the private sector
$k*$	=	long-run desired level of k
$\Delta_j er^e_{+j}$	=	j-period ahead expected depreciation of the HK$ (annualised changes)
r_1m	=	one-month HIBOR, calculated as the 30-day forward moving average of expected overnight HIBOR
r_ff	=	Fed Funds target rate
u_r	=	exogenous risk premium on HK$ assets
u_k	=	exogenous transitory shocks on capital flows.
$risk$	=	exogenous term premium on US$ assets.

2.3 Discount Window Borrowing

The aggregate balance shown in Figure 1 represents the sum of two components: the balance prior to discount window (DW) borrowing, and the amount due to such borrowing. To the extent that overnight HIBOR exceeds the base rate at the discount window, banks have an incentive to fund through DW borrowing as opposed to either in the interbank market or in the foreign exchange market. In the absence of nonpecuniary costs to using the discount window, banks would avoid borrowing in the market at rates exceeding the base rate, and instead would obtain funds through the discount window. Of course, the scope for DW borrowing is limited by available collateral in the form of banks' holdings of Exchange Fund paper. Until this constraint becomes binding, though, the discount window would put a short-run "cap" on overnight HIBOR at the level of the base rate if banks have no disincentive for such borrowing. After the banks have exhausted the first 50% of their eligible collateral, a premium of 500 basis points is applied on further borrowing.

In reality, the banks avoid extensive borrowing at the discount window, as indicated by instances when HIBOR has risen above the base rate without triggering very high levels of borrowing (see Figure 2). Rather than making the borrowing schedule perfectly elastic at the base rate, then, we assume that an increasing premium over the base rate is needed to induce additional borrowing from the discount window. A functional form based on a combination of a logistic and exponential function was chosen to reflect these properties and the observed data points in Figure 2:

$$ab\_dw = \theta_1 \left[\left(1 - e^{r\_bs - r\_on - 1}\right) / \left(1 + e^{r\_bs - r\_on - 1}\right)\right]^{\theta_2} + \theta_3 \, e^{u\_dw} \qquad (3)$$

where:

ab_dw	=	discount window borrowing
r_bs	=	base rate at discount window
r_on	=	overnight HIBOR
u_dw	=	shocks to discount window borrowing.

After eligible collateral for borrowing at the base rate is exhausted (the first 50% of Exchange Fund paper holdings), market interest rates would have to rise by another 500 basis points to induce additional borrowing in the second tranche of the discount window. In the model, we assume this second tranche is not operative. In part this is to simplify the model, and in part because these holdings of Exchange Fund paper are roughly equal to the amount the banks need to hold to satisfy intra-day collateral requirements under the RTGS clearing system. As such, they would not be continuously available as collateral for DW borrowing. In any event, the simulations suggest that it would be very rare that the first tranche of borrowing would be exhausted for the banking system as a whole.

2.4 Determination of the Base Rate in the Discount Window

Of course, the base rate, r_bs, is endogenous over the longer term in the face of a sustained rise in market interest rates. This is because the base rate equals the maximum of the Fed Funds target rate plus 1.5 percentage points, or a five-day moving average of overnight and one-month HIBORs. This is captured by the following relationships:

$$r\_bs^s_t = (1/2)\,(1/5)\,(r\_on_{t-1} + \ldots\ldots + r\_on_{t-5} + r\_1m_{t-1} + \ldots\ldots + r\_1m_{t-5}) \quad (4)$$

$$r\_bs_t = \max\,(\,r\_bs^s_t,\, r\_ff_t + 1.5) \quad (5)$$

$$r\_1m_t = (1/30)\,(r\_on + r\_on^e_{t+1} + \ldots\ldots + r\_on^e_{t+29}) + u\_r1m \quad (6)$$

where: r_bs^s = "shadow" base rate based on five-day moving average of overnight and one-month HIBORs

r_bs = actual base rate calculated as the higher of the shadow base rate or the Fed Funds target rate plus 1.5%

r_on^e = expected future level of overnight HIBOR

r_1m = one-month HIBOR, calculated as the 30-day forward moving average of expected overnight HIBOR

r_ff = Fed Funds target rate

u_r1m = the term premium shock.

2.5 Exchange Rate Determination

To close the model, we need a relationship that ties down the exchange rate. Since the implementation of the 'seven technical measures' for the currency board system on 7 September 1998, a convertibility undertaking (CU) rate on the weak side of the Hong Kong dollar against the US dollar has been introduced (Yam, 1998). While there is currently no formal strong-side intervention point, the Subcommittee on Currency Board Operations considered the options in this area in both meetings in October 1999 and July 2000 and "agreed that there would be scope to review this arrangement again, should the need arise" (HKMA, 2000). If a formal intervention point were to be established, it would be necessary to consider how wide the band should be, and the implied trade-off in terms of interest rate and exchange rate volatility.

To consider these important issues, in our model the HKMA is assumed, *counterfactually*, to have an intervention band with both "hard edges" under the currency board arrangements. In other words, there is no intervention as long as the exchange rate is within the band, but intervention takes place at *both* edges of the band in whatever amounts are necessary to prevent the band from being violated. It is noticed that this kind of intervention is passive intervention; it is triggered by market arbitrage mechanism (Tsang, 1999; Tsang and Ma, 2001).

To model this intervention strategy, a shadow spot exchange rate, er_s, is solved first in the absence of any HKMA intervention in the foreign exchange market, i.e. k_t is set to k_{t-1} in eq. (2). If the solved shadow rate er_s is within the intervention band, then er_s is also the actual market exchange rate er. That is illustrated in Figure 3a where the demand for Hong Kong dollars, k_t, is positively related to the actual spot exchange rate as given by (2), *ceteris paribus*. In the absence of intervention, the supply of Hong Kong dollar, represented by the vertical line, is set to k_{t-1}. Figure 3a shows the case where the equilibrium shadow exchange rate er_s is within the intervention band. As a result, the actual spot rate (er) equals the er_s.

However, if er_s is outside the intervention band, then intervention via manipulating the capital inflow variable, k_t, by the HKMA is triggered such that the market spot rate er stays within the band. That is illustrated in Figure 3b. The supply of Hong Kong dollars is cut from k_{t-1} to k_t to maintain the actual spot rate er at the edge of the band, i.e. 7.8 HK$/US$.

All HKMA's intervention is assumed to be *unsterilised*,[11] in that all purchases or sales of HK$s by the HKMA under the currency board arrangements affect the aggregate balance on a one-for-one basis (cf. Figure 1). As a result of this assumption, the nonborrowed component of the aggregate balance (ab_nb) and private-sector net demand for HK$ assets will be identical:

$$ab\_nb \equiv ab - ab\_dw \, , \tag{7}$$

$$ab\_nb = k \, . \tag{8}$$

2.6 Model Calibration

There are three key behavioural relationships in the model that must be calibrated prior to performing simulations – the banks' liquidity demand schedule; the discount window borrowing function; and the capital flow equation. At this point, the model has been informally calibrated to be broadly consistent with the stylised facts of money market operations since the technical measures were introduced in September 1998.[12] The main features are as follows:

- Banks' *liquidity preferences* in eq. (1) are specified such that the "normal" level of the aggregate balance is HK$1 billion, with a minimum frictional level of zero for simplicity.

[11] For sterilised intervention, one also has to be concerned about the timing of the sterilisation, which complicates the model behaviour considerably and is left for future research.

[12] Specifically, we tried to fit the first and the second moments of the observed time series in the model. However, a more formal calibration and/or estimation exercise would be needed to yield greater confidence in the specification of the key behavioural equations in the future research.

- The *capital flow equation* (2) is calibrated such that a one percentage point rise in the average interest rate differential causes a HK$10 billion shift in assets toward HK$s over time, with a daily adjustment speed of one tenth. In other words, the first-day inflow would amount to about HK$1 billion, rising to HK$10 billion over time according to the Koyck-lag adjustment process.

- The *demand for discount window borrowing* in eq. (3) is assumed to follow the function shown in Figure 2. The parameters were chosen to broadly fit the available data points. There are, however, few points with high interest rate gaps and consequently high DW borrowing, so it is difficult to estimate precisely the shape of the curve in this range.

It is also necessary to specify the stochastic processes that drive the shocks to the model. We assume there are five types of such shocks. The first two are "liquidity shocks" that shift the banks' liquidity preference schedule (u_ab) in eq. (1) and discount window borrowing schedule (u_dw) in eq. (3), respectively. Both of these shocks are assumed to follow a Gaussian distribution with a standard deviation of HK$400 million, resulting in volatility in the aggregate balance and DW borrowing that are typical of the actual data.

There are also two shocks to capital flows in eq. (2). We assume that capital flows are affected both by short-term shifts in portfolios due to transitory disturbances (u_k), as well as longer-term changes in the perceived risk premium on the HK$ (u_r). The transitory disturbances are handled by simply adding a white-noise error term to the equation for k, with a standard deviation of HK$800 million. The risk premium, in contrast, is modelled as a random walk, with the innovations generated by a Gaussian process with a standard deviation of eight basis points. This latter assumption leads to rather more volatility in interest rates than observed during normal times, and rather less than during past periods of speculative attacks. Parenthetically, an alternative would be to use a distribution with fatter tails than the normal, which would imply a more bimodal pattern of "small" versus "large" events. Indeed, recent research suggests that this type of distribution is more typical of financial markets than is the normal distribution. However, this is not implemented in the current version of the model and is left for future research.

Finally, the term premium shock, u_r1m, to the term structure of the one-month HIBOR in eq. (6) is also modelled as a random walk, similar to the risk premium shocks to the capital flows.

3. The Stochastic Simulation Procedure

The currency board model of Hong Kong can be expressed in a compact format as follows:

$$Y_t = \mathrm{F}(Y^e_{t+1}, Y_t, X_t, \varepsilon_t; \beta) \tag{9}$$

where $F(.)$ is a vector of linear and nonlinear functions defined in equations (1) to (8); Y_t is a vector of endogenous variables, including the serially correlated shocks; Y^e_{t+1} is a vector of expected values of Y_{t+1}; X_t is a vector of exogenous variables including lagged values of the shocks and Y_t; ε_t is a vector of independent stochastic shocks; and β is a vector of parameters. Both Y_t and X_t include the serially correlated component of the shocks. This technique treats the model solutions for both independent and serially correlated shocks in a unified framework.[13]

Similar to the solution of linear models with forward-looking expectations (for example, Blanchard and Kahn, 1980; McCallum, 1983 and 1998; and Ma, 1992), a reduced-form solution of the nonlinear model with forward-looking expectations (9) can be heuristically written as follows:

$$Y_t = G(X_t, \varepsilon_t) \tag{10}$$

$$X_{t+1} = H(X_t, \varepsilon_t) \tag{10a}$$

where $G(.)$ and $H(.)$ are vectors of nonlinear functions for (X_t, ε_t). This indicates that the expectations of Y_{t+1} depend upon the entire distribution of $(X_{t+1}, \varepsilon_{t+1})$ in a general solution.

3.1 A Deterministic Solution

A deterministic solution of model (9) may be obtained by, say, the Fair-Taylor algorithm (Fair and Taylor, 1983, 1990),[14] by setting $\varepsilon_t = 0$ for all t.

The Fair-Taylor algorithm involves both inner-loop and outer-loop iterations. The inner-loop iterations search for the model solution based on fixed expectations of Y^e_{t+1}:

$$Y_t = F(Y^e_{t+1}, Y_t, X_t, 0; \beta) \tag{11}$$

Since Y^e_{t+1} is fixed, this solution does not guarantee the solved Y_{t+i} (i>0) to be equal to the expected Y^e_{t+i}, i.e.:

$$Y_{t+i} \neq G(X_{t+i}, 0) \tag{12}$$

The equality is established by the outer-loop iterations. These search for the expected Y^e_{t+i}, until a model-consistent expectations solution is found, i.e., $Y_{t+i} = Y^e_{t+i}$, and

$$Y_t = F(Y_{t+1}, Y_t, X_t, 0; \beta) = G(X_t, 0) \tag{13}$$

---

13 Fair and Taylor (1983, 1990) deal with the serial correlated and uncorrelated shocks in two alternative model solution methods.

14 An alternative approach in the literature includes log-linearisation of the model around the equilibrium path. This approach is commonly adopted in the real business cycle models (cf. King, Plosser and Rebelo, 1988; Campbell, 1994; Uhlig, 1999).

3.2 A Stochastic Solution under Certainty Equivalence

It is generally recognised that it is difficult to conduct stochastic simulations in the context of a nonlinear model with forward-looking expectations. Most of the existing literature implements such simulations under the 'certainty equivalence' assumption. This means that the expected values of the endogenous variables depend only upon the deterministic means of other variables, as opposed to the stochastic means. Equation (10) can be used to illustrate this assumption. If certainty equivalence holds, then

$$Y^e_{t+i} = E[\mathrm{G}(X_{t+i}, \varepsilon_{t+i})] = \hat{\mathrm{G}}[E(X_{t+i}), E(\varepsilon_{t+i})] = \hat{\mathrm{G}}[E(X_{t+i}), 0], \text{ for any } i, \quad (14)$$

where $\hat{\mathrm{G}}[E(X_{t+i}), E(\varepsilon_{t+i})] = \hat{\mathrm{G}}[E(X_{t+i}), 0]$ is regarded as "deterministic mean", while $E[\mathrm{G}(.)]$ is the "stochastic mean".

Obviously if the model $\mathrm{F}(.)$ in (9) is a linear model, then its solutions, $\mathrm{G}(.)$ and $\mathrm{H}(.)$ in (10) and (10a), respectively, are also linear (see, for example, Blanchard and Kahn, 1980; McCallum, 1983 and 1998; and Ma, 1992). As a result, (14) always holds with $\hat{\mathrm{G}} \equiv \mathrm{G}$. That is, the deterministic and stochastic means of the variables are the same in a linear rational expectations model.

Substituting (14) into (9):

$$Y_t = \mathrm{F}(\hat{\mathrm{G}}[E(X_{t+1}), 0], Y_t, X_t, \varepsilon_t; \beta) = \mathrm{G}(X_t, 0) \quad (15)$$

Therefore, for each time period, the stochastic simulation of a nonlinear model is conducted by introducing shocks to the current period only, while imposing zero shocks in all future periods under certainty equivalence. By using the *solved* endogenous variables in the previous periods for the *lagged* endogenous variables, we can repeat this procedure for all simulation periods to get a stochastic solution path. Repeating this for numerous shocks will generate different solution path.[15] Effectively, the simulation treats agents as *if* they believed that no shocks would happen in the future.

The situation becomes more complicated when the forward-looking variables also depend upon the variance, or higher moments, of future distributions, violating the 'certainty equivalence' assumption. Equation (10) can again be used to illustrate this point. If certainty equivalence does not hold, then

$$Y^e_{t+i} = E[\mathrm{G}(X_{t+i}, \varepsilon_{t+i})] \neq \hat{\mathrm{G}}[E(X_{t+i}), 0], \text{ for any } i. \quad (16)$$

This case is analysed in the following sub-section.

[15] See Fair and Taylor (1983 and 1990) and Meredith (1999) for examples.

3.3 A Simulation-Regression Approach to Stochastic Simulations

In this sub-section, we present a new approach for conducting stochastic simulations on a nonlinear model with forward-looking expectations when certainty equivalence does not hold. In the next sub-section, we illustrate our approach in a simple target-zone exchange rate model.

To deal with the situation of non-certainty-equivalence, we start from certainty equivalence as a first approximation to the 'true' stochastic solution. However, the current values of the endogenous variables, Y_t, from this approximation solution do depend upon the entire distribution of other variables [cf. (15)]. To reveal the relationship between Y_t and X_t, we may run the following regression:

$$Y_t = \breve{G}(X_t) + \xi_t \tag{17}$$

where ξ_t is the regression residual.

Quite often, a nonlinear model cannot be solved for its analytical solution. In these circumstances, $\breve{G}(.)$ may be approximated by Taylor series expansion.

The functional form and parameters of $\breve{G}(.)$ now incorporate the entire distributions of $(X_{t+i}, \varepsilon_{t+i})$ $(i=0,1,2,....)$.[16] Therefore, $\breve{G}(.)$ is a better approximation than $\hat{G}$ for $G(.)$ in (15).[17]

Having estimated the expectations function $\breve{G}(.)$, we substitute it into (9) to obtain:

$$Y_t = F(E[\breve{G}(X_{t+1})], Y_t, X_t, \varepsilon_t; \beta) \tag{18}$$

which in fact is the reduced-form solution of the forward-looking expectations without the certainty-equivalence assumption. The difference between (15) and (18) is as follows. The expected values of Y^e_{t+1} in (18) depend upon the entire distribution (X_t, ε_t) via $\breve{G}(.)$. However, Y^e_{t+1} in (15) depend only upon the mean of (X_t, ε_t). Conducting stochastic simulations on model (18) will therefore get a closer solution to the 'true' stochastic solution than the solution in (15) under the certainty-equivalence assumption.

3.4 Example: A Simple Target-Zone Exchange Rate Model

To illustrate our new stochastic simulation procedure, we use a simple three-equation target-zone exchange rate model:

$$es_t = e_{t-1} + u_t \tag{19}$$

$$e_t = \begin{cases} -1 \ \ if \ \ es_t < -1 \\ es_t \ \ if \ \ -1 \le es_t \le 1 \\ 1 \ \ if \ \ es_t > 1 \end{cases} \tag{20}$$

[16] The serial correlation pattern, for example, of $(X_{t+i}, \varepsilon_{t+i})$ is well incorporated in $\breve{G}(.)$.

[17] This is true even if $\breve{G}(.)$ is approximated by a linear function. See Section 3.4 below for an explanation with a simple example.

$$e^e{}_t = e_{t+1} \tag{21}$$

This is a nonlinear model of an exchange rate target band. The first equation in (19) states that the "shadow" (i.e. unconstrained) exchange rate es_t is a random walk with innovation u_t.[18] Assume that u_t has the probability density function (pdf) $\psi(x)$ and the cumulative distribution function (cdf) $\Psi(x)$:

$$\Psi(x) = \int_{-\infty}^{x} \psi(z)dz$$

The second equation in (20) indicates that the actual exchange rate e_t is equal to the edges of the target band (-1 and 1 respectively) if es_t falls outside the band. Hence the band is modelled as the 'reflecting sticky barrier' in the statistical literature (cf. Cox and Miller, 1965). The third equation in (21) is the one-period ahead expectation of the exchange rate, $e^e{}_t$. In this illustrative model, the exchange rate expectation performs no other role, but its properties will depend importantly on whether or not certainty equivalence is assumed. These differences would affect the overall dynamics of a more general model.

The 'reflecting sticky barrier' implies that e_t has a doubly censored distribution. Conditional on the observation of e_{t-1}, denote the mean of e_t as $E(e_t|e_{t-1})$. Then by (20) we have:

$$E(e_t \mid e_{t-1}) = \int_{-\infty}^{-1} (-1)\psi(x)dx + \int_{-1}^{+1} x\psi(x)dx + \int_{-1}^{\infty} \psi(x)dx \tag{22}$$

For example, if e_t is standard Gaussian white noise, i.e., e_t ~ iid $N(0, 1)$, then we have (Rose, 1995, p.1395):

$$E(e_t|e_{t-1}) = [\Psi(1) - \Psi(-1)]\, e_{t-1} = a\, e_{t-1} \tag{23}$$

where $a = 0.68$.

This is the model-consistent, one-step ahead, expectations of e_{t-1}.

However, if this simple model is simulated with forward-looking expectations and certainty equivalence is assumed, then the future, unobserved, error terms (u) are all set to zero. This implies that $E(e_{t+1}|e_t)$ will always equal e_t – the expected future rate always equals the spot rate. But the expectation of the mean of $E(e_{t+1}|e_t)$ in a 'true' stochastic environment, i.e., under the non-certainty-equivalence assumption, will not equal the spot value [cf. (23)]. The intuition is as follows. Consider a case where e_t is already at the upper limit of the band. Then it can only go down, not up, and the mean of the expectation of e_t must be less than the spot value. Table 1 illustrates the differences between the two types of simulations.

In Table 1, columns (5) and (6) are always identical in the stochastic simulation with certainty equivalence. However, the stochastic simulation using forecasting rule (23) with non-certainty-equivalence generates

[18] This is a common assumption in the exchange rate target-zone literature (cf. Krugman, 1991), after the seminal paper of Meese and Rogoff (1983).

different results in column (8) from those in column (6). The simulation results illustrate that the squared-error of the forecast (SQEF) in column (9) is smaller than that in column (7) - a result that is consistent with our theoretical result.

In practice, one usually cannot observe the true value of the parameter of a in equation (23). However, under our stochastic simulation procedure outlined in sub-section 3.3, we may run a regression to estimate it.

If we add the expectations error back to the forecasting equation (23), then we have:

$$e_t = a\, e_{t-1} + \eta_t, \tag{24}$$

$$\text{with } E(\eta_t|\varepsilon) = 0, \tag{25}$$

where the information set $\varepsilon = \{e_{t-1}\}_{t=2,3,\ldots,T} = \{e_1, e_2, \ldots\ldots., e_{T-1}\}$.

We may apply ordinary least squares (OLS) estimation to (25):[19]

$$\hat{a} = \Sigma e_t e_{t-1}/\Sigma e_{t-1}^2 \tag{26}$$

Conditional on the observed information set ε, we have:

$$E(\hat{a}|\varepsilon) = a + \Sigma e_{t-1} E(\eta_t|\varepsilon)\,/\Sigma e_{t-1}^2 = a \tag{27}$$

By the law of iterated expectations (Greene, 2000, p.245):

$$E(\hat{a}) = E_\varepsilon[E(\hat{a}|\varepsilon)] = a + E_\varepsilon[\Sigma e_{t-1} E(\eta_t|\varepsilon)/\Sigma e_{t-1}^2\,] = a. \tag{28}$$

Thus, $\hat{a}$ is an unbiased estimator of a.

In this simple example, $\breve{G}(e_{t-1}) = \hat{a}\, e_{t-1}$, that is, $\breve{G}(.)$ is a linear function. However, it is still a better approximation than the estimator under the certainty-equivalence assumption.

In reality, analytical solutions in general cannot be obtained for most of the nonlinear models, such as the currency board model in this paper. In such circumstances, our simulation-regression procedure provides a practical approach to perform stochastic simulations. This advantage is illustrated in the next section, where the shadow spot rate, *er_s*, follows a complicated nonlinear process described in Section 2.5, instead of the simple random walk in (19). In other words, our procedure can be regarded as a generalisation of the solution methodology for linear rational expectations model to nonlinear model, where we are approximating the analytical solution with the regressions, rather than solving for it exactly.

[19] Estimating (17) is equivalent to estimating (25).

4. Stochastic Simulations of the Currency Board Model

In this section, we report the results of stochastic simulations of the currency board model of Hong Kong, which followed our new procedure outlined above. The currency model was calibrated around an initial baseline path as described in Section 2. A sequence of stochastic shocks, also discussed in Section 2, covering 360 days, or one calendar year,[20] was then fed into the model. The model was allowed to settle down to its baseline path for 30 days (i.e. one month) without any further shocks at the end of the simulation horizon. Finally, steady-state terminal conditions were imposed on the model solutions for 20 days beyond the model solution period.[21] The whole solution period is illustrated in Figure 4.[22]

Following our simulation-regression approach of stochastic simulation outlined in Section 3.3, the currency board model was first simulated under the assumption of certainty equivalence. Table 2 presents the summary statistics of the model solution under a bandwidth of ±20 pips, i.e. between 779.60 to 780.00 HK$ per 100 US$, as an example. Normality tests based on skewness and kurtosis (Davidson and MacKinnon, 1993)[23] have been conducted on the simulated endogenous variables. It shows that all endogenous variables have non-normal distributions, except the shadow and actual spot rates, er_s and er, respectively.

To obtain the stochastic characteristics of the model under different intervention bandwidths for exchange rate movements, the model was simulated repeatedly under four illustrative bandwidths: ±10, ±15, ±20, and ±25 pips, with a common weak-side boundary of 7.8 HK$/US$. The solutions for these four sets of stochastic simulations were saved and combined into a single joint dataset. Based on this dataset, the simulated values of those endogenous variables with forward-looking expectations were then regressed against a chosen set of exogenous variables, including the bandwidth as a control variable. The regression results are summarised in Table 3.[24]

Table 4 presents the summary statistics for the stochastic simulations performed on the currency board model augmented with the forecasting rules estimated in Table 3. It shows the results of the simulations with an intervention bandwidth of ±20 pips, i.e. between 779.60 to 780.00 HK$ per 100 US$, as an example. Table 4 reveals that, whilst this intervention band indeed does not allow the actual spot rate, er, to move outside the band, the shadow exchange rate, er_s, does move outside the band from time to time. This implies that HKMA intervention on both edges of the band is necessary under larger shocks.

One of the most interesting features of the simulated variables in Table 4 is that none exhibits a normal distribution according to tests based on skewness and kurtosis (Davidson and MacKinnon, 1993). Either

---

20 For the purposes of the simulations no distinction was made between trading days and non-trading days.

21 The long-run restrictions imposed on the model are endogenously determined constant levels or growth rates where appropriate, for the endogenous variables. It would be interesting to conduct sensitivity analysis for alternative long-run restrictions in the future research.

22 Several 'plain vanilla' deterministic shocks were simulated on the model and plausible results were generated. Due to space constraint, they are not reported here.

23 The skewness multiplied by $\sqrt{(N/6)}$ and the kurtosis multiplied by $\sqrt{(N/24)}$ both have a normal(0,1) distribution under the null that the random variable is normally distributed (see Davidson and MacKinnon 1993 for a derivation).

24 A linear functional form is chosen for regressions, with the bandwidth acting as a dummy variable. Various nonlinear functional forms have also been experimented but in general they generate numerical instability to the model.

the symmetric hypothesis (skewness test) or the normal tail thickness hypothesis (kurtosis test) is rejected at the 5% significance level for all the variables in Table 4. All variables indicate some degree of asymmetry except overnight HIBOR (r_on). However, overnight HIBOR displays a significantly thinner tail than the normal distribution. In fact all variables in Table 4 display thinner tails except discount window borrowing (ab_dw) and the expected 30-day ahead spot rate [$er^e(+30)$], with the latter two variables exhibiting fatter tails than the normal distribution. Given the fact that our currency board model is highly nonlinear, this is hardly a surprising result.

Finally, the simulation results in Table 2 are compared with that in Table 4. Both simulations are performed with the identical model under the same intervention bandwidth of ±20 pips, except assumptions about the expectations formation. Table 2 assumes certainty equivalence whilst Table 4 assumes non-certainty-equivalence. We find that none of the simulated variables has similar stochastic properties in the two simulations. For example, all the standard deviations in Table 4 are larger than those in Table 2. And none of the distributional shapes is similar for all variables except the expected 30-day ahead spot rate [$er^e(+30)$]. In both simulations, the distributions of $er^e(+30)$ exhibit significant asymmetry with the long tail in the strong side of the band, and display fatter tails than the normal distribution. However, for the remaining variables, no such similarities can be found. For example, the shadow and actual spot rates, er_s and er, respectively, exhibit normal distributions in Table 2, whilst none of them shows any normality in terms of their skewness and kurtosis in Table 4. The expected one-day ahead spot rate [$er^e(+1)$] displays significant asymmetry with the long tail on the weak side of the band in Table 2, whilst it exhibits significant long tail on the strong side of the band in Table 4. These results illustrate the importance of making the correct assumption about the expectations formation.

5. Conclusion

This paper develops and illustrates a new procedure for implementing stochastic simulations in the context of a currency board model for Hong Kong. Our new procedure is useful in the context of nonlinear models with forward-looking expectations under conditions of non-certainty-equivalence. A simple target-zone model of the exchange rate is used to illustrate this procedure. It is then applied to the actual model under illustrative exchange rate bandwidths, and the distributions of the endogenous variables are shown to significantly violate the normality assumption. A comparison of the simulation results of the new procedure with that of the existing procedure also shows significant differences in the stochastic properties of the simulated variables, indicating the importance of making the correct assumption about the expectations formation.

Our new procedure would also be useful in other applications of the currency board model. For example, it could be used to investigate the issue of alternative currency board arrangements, as discussed in Meredith (1999). Furthermore, it would be interesting to incorporate the recent literature on the 'simulation-based estimation' approach (e.g. Stern, 1997) into our modelling exercises. In such applications, the power of our new procedure could be more generally explored under the assumption of non-certainty-equivalence.

References

Balino, T. J. T. and C. Enoch (1997), "Currency Board Arrangements: Issues and Experience," IMF Occasional Paper No. 151.

Barro, R. J., N. G. Mankiw and X. Sala-i-Martin (1995), "Capital Mobility in Neoclassical Models of Growth," *American Economic Review*, 85: 103-15.

Blanchard, O. J. and C. M. Kahn (1980), "The Solution of Linear Difference Models Under Rational Expectations," *Econometrica*, 48: 1305-12.

Campbell, J. Y. (1994), "Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model," *Journal of Monetary Economics*, 33: 463-506.

Cox, D. R. and H. D. Miller (1965), The Theory of Stochastic Processes, London: Chapman and Hall.

Davidson, R. and J. G. MacKinnon (1993), Estimation and Inference in Econometrics, New York: Oxford University Press, Chapter 16.

Fair, R. C., and J. B. Taylor (1983), "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models," *Econometrica*, 51: 1169-86.

Fair, R. C., and J. B. Taylor (1990), "Full Information Estimation and Stochastic Simulation of Models with Rational-Expectations," *Journal of Applied Econometrics*, 5: 381-392.

Greene, W. H. (2000), Econometric Analysis, 4th ed, Upper Saddle River, NJ: Prentice Hall.

HKMA (2000), "Record of Discussion of the Meeting of the Exchange Fund Advisory Committee Sub-Committee on Currency Board Operations held on 5 July 2000," posted at the Hong Kong Monetary Authority website: http://www.info.gov.hk/hkma/eng/press/2000/20000802e3.htm.

King, R. G., C. I. Plosser, and S. T. Rebelo (1988), "Production, Growth and Business Cycles: I. The Basic Neoclassical Model," J*ournal of Monetary Economics*, 21: 195-232.

Krugman, P. (1991), "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 106: 669 - 82.

Ma, Yue (1992), "Policy Measurement for the Dynamic Linear Model with Expectations Variables: a Multiplier Approach," *Computational Economics*, 5: 303-12.

Ma, Yue, and A. Kanas (2000), "Testing Nonlinear Relationship among Fundamentals and Exchange Rates in the ERM," *Journal of International Money and Finance*, 19: 135-52

.

Ma, Yue, S. K. Tsang and S. H. Tang (1998), "The Impact of China Factor on the Hong Kong Economy pre-1997: A Macroeconomic Modelling Analysis," *International Review of Applied Economics*, 12: 89-106.

McCallum, B. T. (1983), "On Non-uniqueness in Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics*, 11: 139-68.

McCallum, B. T. (1998), "Solutions to Linear Rational Expectations Models: A Compact Exposition," *Economics Letters*, 61: 143-47.

Meese, R. A. and K. Rogoff (1983), "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *Journal of International Economics*, 14: 3-24.

Meredith, G. (1999), "Liquidity Management under Hong Kong's Currency Board Arrangements," Hong Kong Monetary Authority, paper presented at the International Workshop on *Currency Boards: Convertibility, Liquidity Management and Exit,* 9 October, Hong Kong Baptist University.

Rose, C. (1995), "A Statistical Identity Linking Folded and Censored Distributions," *Journal of Economic Dynamics and Control*, 19: 1391-403.

Siregar, R. Y., and W. C. Walker (2000), "Monetary Shocks and the Fundamental Determinants of the Real Exchange Rate under the Hong Kong Currency Board," *Asian Economic Journal*, 14: 1-21.

Stern, S. (1997), "Simulation-based Estimation," *Journal of Economic Literature*, 35: 2006-39.

Tsang, Shu-ki (1999), "Fixing the Exchange Rate through a Currency Board Arrangement: Efficiency Risk, Systemic Risk and Exit Cost," *Asian Economic Journal*, 13: 239-66.

Tsang, Shu-ki and Yue Ma (2001), "Currency Substitution and Speculative Attacks on a Currency Board System," *Journal of International Money and Finance*, forthcoming.

Uhlig, H. (1999), "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," in R. Marimon and A. Scott (eds) Computational Methods for the Study of Dynamic Economies, Oxford University Press, pp.30-61.

Yam, J. (1998), "A Modern Day Currency Board System," Hong Kong Monetary Authority pamphlet.

Table 1. Stochastic Simulations with and without Certainty Equivalence Assumption

(1) t	(2) e_{t-1}	(3) u_t	(4) es_t	(5) e_t	(6) $E(e_{t+1}\|e_t)$ with CE	(7) SQEF	(8) $E(e_{t+1}\|e_t)$ with NCE	(9) SQEF
0				0				
1	0	0.1	0.1	0.1	0.1		0.068	
2	0.1	-0.2	-0.1	-0.1	-0.1	0.04	-0.068	0.028
3	-0.1	1.5	1.4	1	1	1.21	0.68	1.14
4	1	-2.5	-1.5	-1		4		2.82
Sum						5.25		3.988

NB. i) SQEF: squared-error of forecast, i.e. $[e_{t+1} - E(e_{t+1}|e_t)]^2$.

ii) CE / NCE: certainty-equivalence / non-certainty-equivalence assumption.

iii) Column (4) is from eq. (19), column (5) from eq. (20), column (6) is from a stochastic simulation based on certainty equivalence, and column (8) is based on non-certainty-equivalence.

Table 2. Summary Statistics of the Stochastic Simulation of the Currency Board Model under the Certainty Equivalence Assumption

Sample size *N* = 391

	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis
r_on	5.9509	1.37962	1.5456	7.7220	-0.9221**	0.1133
r_1m	6.3195	0.63374	3.8971	7.8249	0.6701**	0.1141
ab	1.1000	0.41394	0.1771	2.7696	1.3282**	2.0690**
ab_dw	0.8945	1.02051	0.0000	4.1280	1.0244**	-0.3869
ab_nb	0.2054	1.06071	-1.6963	0.8000	-1.2362**	-0.4672*
k	0.2054	1.06071	-1.6963	0.8000	-1.2362**	-0.4672*
er	779.7969	0.06736	779.6566	780.0000	0.0132	0.0737
e_s	779.7971	0.06787	779.6566	780.0347	0.0777	0.2897
$er^e(+1)$	779.8041	0.04020	779.7405	779.9962	1.7614**	5.5184**
$er^e(+30)$	779.7902	0.02179	779.7405	779.8199	-1.4970**	0.5269**

NB: 1) Std Dev: standard deviation.

2) The skewness multiplied by $\sqrt{(N/6)}$ and the kurtosis multiplied by $\sqrt{(N/24)}$ both have a $N(0,1)$ distribution under the null that the random variable is normally distributed (see Davidson and MacKinnon, 1993 for a derivation).

3) */** indicates significant at the 10%/5% level.

4) The currency board model is simulated with an intervention bandwidth of ±20 pips for the Hong Kong dollar spot rate, i.e. between 779.60 to 780.00 HK$ per 100 US$, for the exchange rate movements as an example.

Table 3. Ordinary Least Squares Estimation of Forecasting Rules for the Currency Board Model of Hong Kong

(A) Dependent variable: *er(+30)*
Adjusted R^2 = .599

Variable	Estimated Coefficient	Standard Error	*t*-statistic	*p*-value
c	805.062	62.4402	12.8933	** [.000]
B	181.093	162.136	1.11692	[.264]
er	-.032207	.080048	-.402343	[.687]
r_on	-.130859E-02	.399054E-02	-.327924	[.743]
ab	.096456	.018368	5.25130	** [.000]
ab_dw	-.097466	.018437	-5.28653	** [.000]
$B^{*}er$	-.232681	.207867	-1.11937	[.263]
$B^{*}r\_on$	-.599512E-02	.011400	-.525909	[.599]
$B^{*}ab$	-.143880	.049043	-2.93378	** [.003]
$B^{*}ab\_dw$	.155643	.047294	3.29099	** [.001]

(B) Dependent variable: *er(+1)*
Adjusted R^2 = .654

Variable	Estimated Coefficient	Standard Error	*t*-statistic	*p*-value
c	500.974	62.2666	8.04563	** [.000]
B	-260.170	155.545	-1.67263	* [.095]
er	.357621	.079822	4.48023	** [.000]
r_on	.011947	.415586E-02	2.87480	** [.004]
ab	.661356E-02	.018560	.356331	[.722]
ab_dw	-.027901	.018042	-1.54649	[.122]
$B^{*}er$	.333278	.199414	1.67129	* [.095]
$B^{*}r\_on$	-.578602E-02	.011554	-.500785	[.617]
$B^{*}ab$	.051074	.048702	1.04869	[.295]
$B^{*}ab\_dw$	.010771	.046472	.231783	[.817]

(C) Dependent variable: *r_1mf = r_1m (+1) +......+ r_1m(+29).*
Adjusted R^2 = .450

Variable	Estimated Coefficient	Standard Error	*t*-statistic	*p*-value
c	13418.7	14403.5	.931627	[.352]
B	-227503.	38014.0	-5.98471	** [.000]
er	-16.9765	18.4647	-.919401	[.358]
r_on	-.176039	.931720	-.188940	[.850]
ab	-31.2212	5.28713	-5.90512	** [.000]
ab_dw	31.7400	5.39392	5.88440	** [.000]
$B^{*}er$	291.674	48.7370	5.98465	** [.000]
$B^{*}r\_on$	7.50482	2.47385	3.03367	** [.002]
$B^{*}ab$	38.3725	14.2139	2.69965	** [.007]
$B^{*}ab\_dw$	-41.6044	14.0916	-2.95242	** [.003]

NB. 1) */** indicates significant at the 10%/5% level.
2) Sample size: 1410.
3) All dependent variables are simulated variables.
4) Standard Errors are heteroskedastic-consistent.
5) *B*: intervention bandwidth of the exchange rate movements.

Table 4. Summary Statistics of the Stochastic Simulation of the Currency Board Model with Non-Certainty-Equivalence Assumption

Sample size N = 391

	Mean	Std Dev	Minimum	Maximum	Skewness	Kurtosis
r_on	4.0287	2.8313	0.0143	8.4152	-0.0692	-1.6022**
r_1m	6.2873	0.8931	4.2423	8.6842	0.3346**	-0.4240*
ab	4.0463	3.5349	0.0141	17.2310	0.7717**	-0.1935
ab_dw	0.7694	1.1950	0.0000	6.0473	1.8210**	2.7738**
ab_nb	3.2769	3.8037	-1.8247	12.3492	0.5698**	-0.6950**
k	3.2769	3.8037	-1.8247	12.3492	0.5698**	-0.6950**
er	779.8390	0.1232	779.6000	780.0000	-0.5387**	-0.8153**
e_s	779.8388	0.1296	779.4879	780.0847	-0.5716**	-0.5347**
$er^e(+1)$	779.7867	0.0657	779.6071	779.8980	-0.7986**	0.1205
$er^e(+30)$	779.7823	0.0426	779.6102	779.8413	-1.4000**	1.3329**

NB:
1) Std Dev: standard deviation.
2) The skewness multiplied by $\sqrt{(N/6)}$ and the kurtosis multiplied by $\sqrt{(N/24)}$ both have a $N(0,1)$ distribution under the null that the random variable is normally distributed (see Davidson and MacKinnon, 1993 for a derivation).
3) */** indicates significant at the 10%/5% level.
4) The currency board model is augmented with the forecasting rules estimated in Table 3. It shows a stochastic simulation with an intervention bandwidth of ±20 pips for the Hong Kong dollar spot rate, i.e. between 779.60 to 780.00 HK$ per 100 US$, for the exchange rate movements as an example.

Figure 1. The Flow Chart of the Currency Board Model of Hong Kong

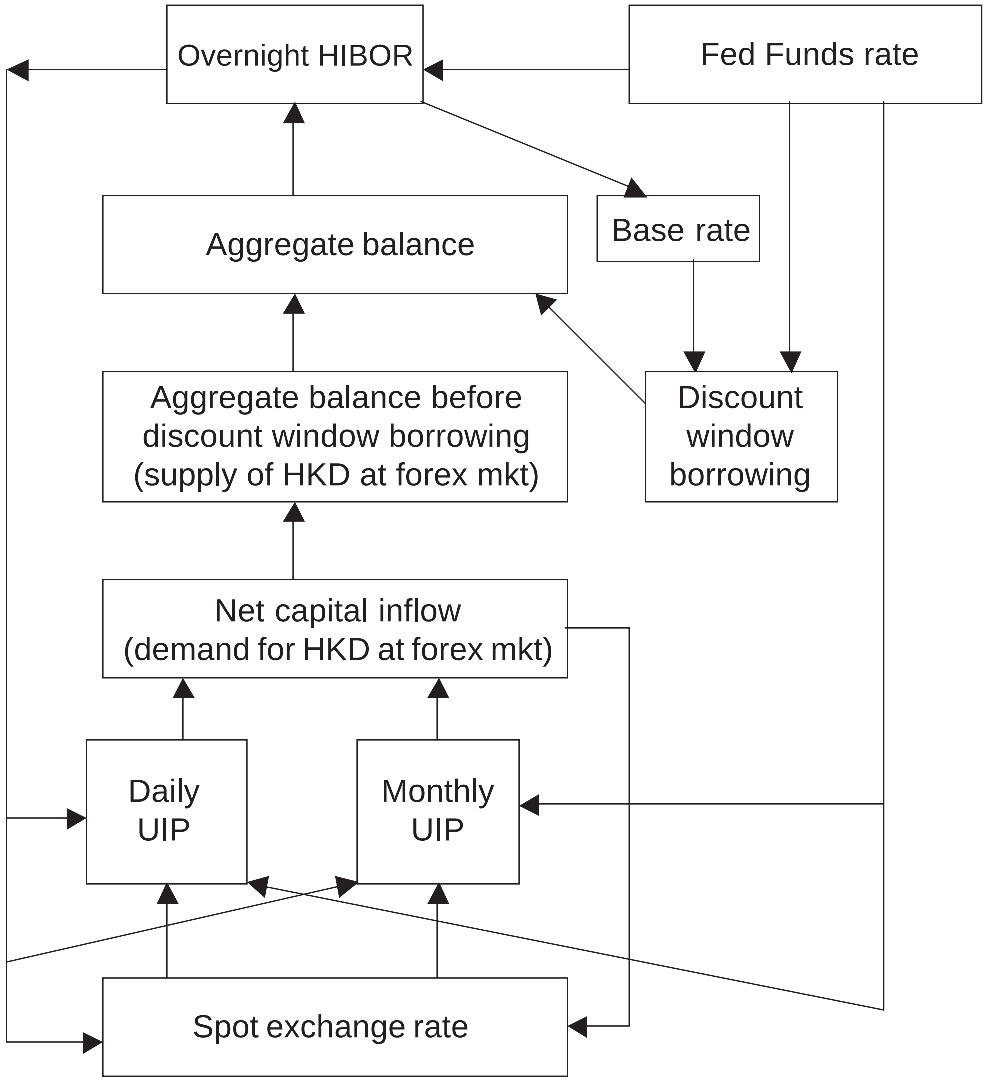


Figure 2: Discount Window Borrowing and Spread between Overnight HIBOR and Base Rate (7 Sept 1998 - 8 Sept 1999)

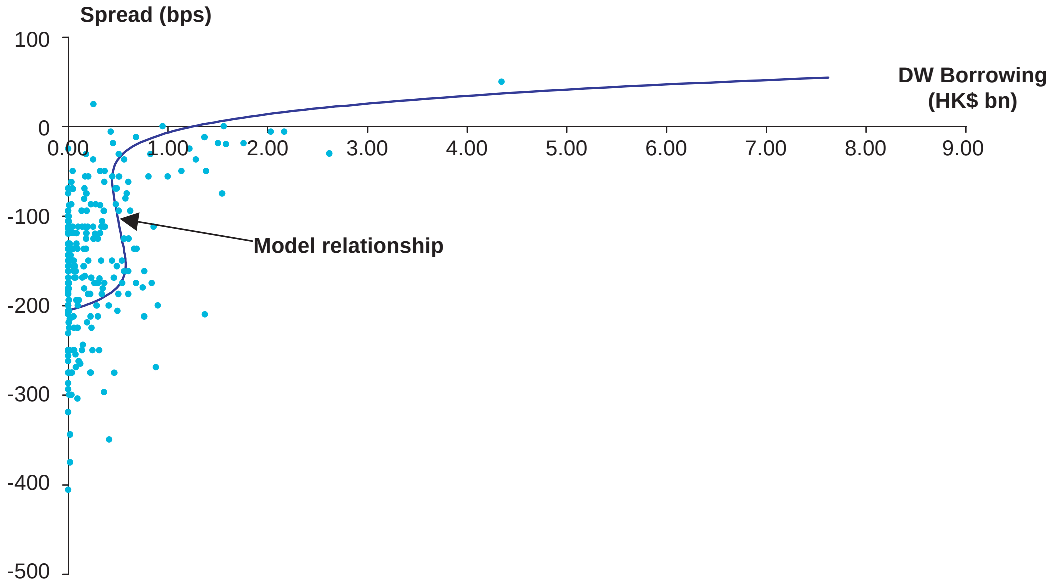


Figure 3a: No Intervention Is Required: Actual Spot Rate *(er)*=Shadow Spot Rate *(er_s)*

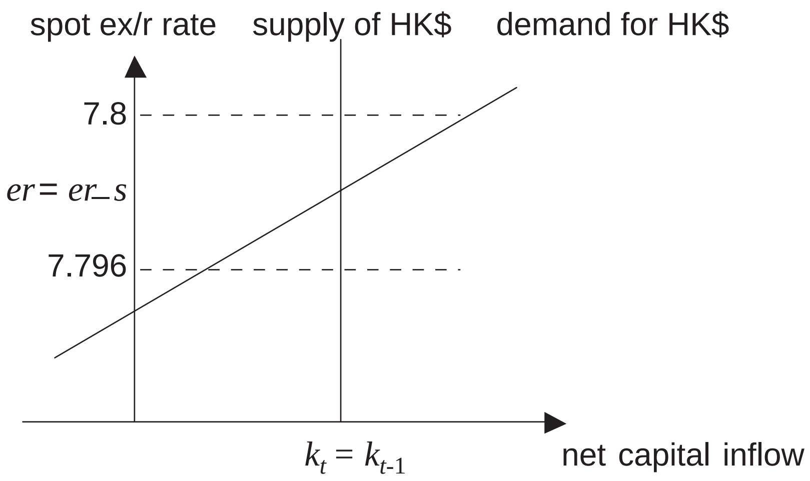
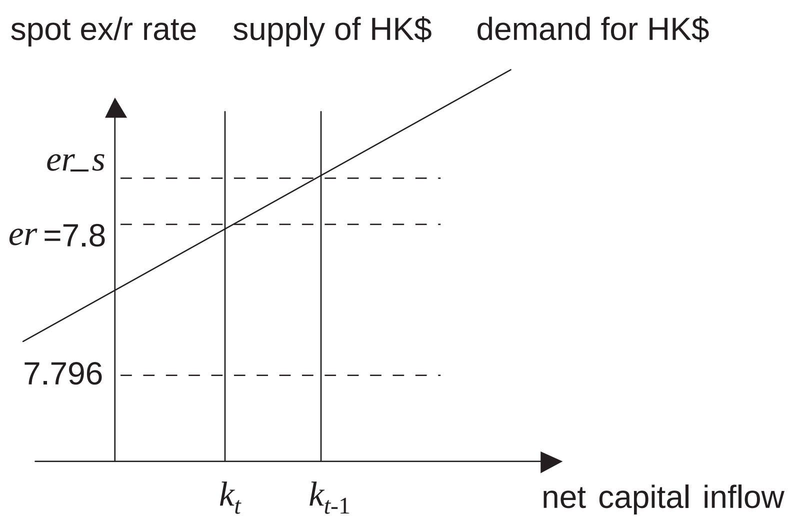
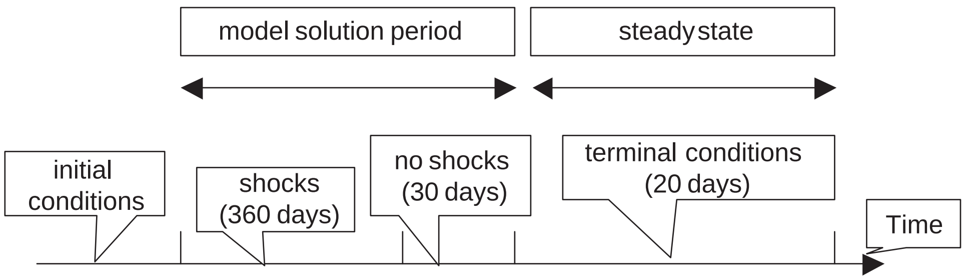


Figure 3b: HKMA Intervention Is Required as the Shadaw Spot Rate *(er_s)*>7.8: A Reduction of HK$ Supply from k_{t-1} to k_t



[7.796, 7.8] is the target zone.

Figure 4: Stochastic Simulation with Forward-Looking Expectations



Appendix A. Variable Definitions

$x^e(+i)$	*i*-period ahead expected variable *x*. Endogenous variables are indicated with '(E)'
ab	aggregate balance (E)
ab^{min}	minimum frictional level of aggregate balance (set to zero to simplicity)
ab_dw	discount window borrowing (E)
ab_nb	the nonborrowed component of the aggregate balance (E)
B	intervention bandwidth of the exchange rate movements
$\Delta_j er^e$	expected depreciation of the HK$ (annualised changes) (E)
er	actual spot exchange rate, HK$/US$ (E)
er_s	shadow spot exchange rate of *er*, HK$/US$ (E)
HIBOR	Hong Kong Interbank Offer Rate (E)
k	net stock of short-term HK$ assets held by the private sector (E)
k^*	long-run desired level of capital flows, *k* (E)
$risk$	exogenous term premium on US$ assets
r_bs	actual base lending rate at the discount window (E)
r_bs^s	"shadow" base rate based on five-day moving average of overnight and one-month HIBORs (E)
r_ff	Fed Funds target rate
r_on	overnight HIBOR (E)
r_on^e	expected future level of overnight HIBOR (E)
r_1m	one-month HIBOR, calculated as the 30-day forward moving average of expected overnight HIBOR (E)
u_ab	shocks to the aggregate balance
u_dw	shocks to discount window borrowing
u_k	exogenous transitory shocks on capital flows.
u_r	exogenous risk premium on HK$ assets
u_r1m	the term premium shock to the one-month HIBOR.