HONG KONG INSTITUTE FOR MONETARY RESEARCH

A MODEL TO ANALYZE THE MACROECONOMIC INTERDEPENDENCE OF HONG KONG WITH CHINA AND THE UNITED STATES

Fuchun Jin

HKIMR Working Paper No.6/2002

March 2002

A Model to Analyze the Macroeconomic Interdependence of Hong Kong with China and the United States

Fuchun Jin*

Hong Kong Institute for Monetary Research
China Center for Economic Research, Peking University

March 2002

Abstract

Based on several stylized facts, this paper develops a macroeconomic model for Hong Kong that emphasizes its interdependence with China and the United States. Important macroeconomic variables such as the price level and nominal wage rate, as well as capital flow, are analyzed first in a static setting and then in a dynamic model. As an application, the model is used to analyze macroeconomic adjustment of Hong Kong in response to economic slowdown in the U.S. and a possible devaluation of the RMB. Theoretical results are obtained both under the assumption of adaptive and rational expectations.

* Corresponding address: China Center for Economic Research, Peking University, Beijing 100871, P.R. China (email: fjin@ccer.pku.edu.cn). This paper was written when the author was a visiting fellow at the Hong Kong Institute for Monetary Research (HKIMR). He wishes to thank the HKIMR and the research department of the Hong Kong Monetary Authority for their hospitality. Helpful comments and discussions from Stefan Gerlach, Priscilla Chiu, Matthew S. Yiu, David Parsely and Shang-Jin Wei as well as other participants at the HKIMR seminar are gratefully acknowledged. I wish to thank also the comments and suggestions of the two anonymous external reviewers of this manuscript. The views expressed in this paper are the author's only and do not necessarily reflect those of the HKIMR, its Council of Advisers, or Board of Directors.

1. Introduction

The spectacular economic development of Hong Kong during the last half a century demonstrates that it is undoubtedly one of the greatest success stories in the twentieth century. Free and dynamic, its outstanding performances in the economic arena are largely due to its governance by the rule of law, the neutrality and efficiency of its civil service, and the informational freedom that it enjoys. Equally important are the non-interventionist approach adopted by its government, and the transparency and consistency of its macroeconomic and fiscal policies, which provide a predictable business environment and a level platform for all players to utilize their competitive strengths as well as their comparative advantages. The collective results of those sound political and economic choices are the phenomenal growth experience of Hong Kong, and its remarkable strength and resilience in the face of political and economic shocks. These were most vividly shown during the recent events of the turnover of sovereignty from Britain back to China and the Asian financial crisis. In the latter case, Hong Kong has recovered from the setback much more rapidly than other economies in the region.

Although many observers are optimistic about future prospects and long-term trends in the Hong Kong economy, some are still uneasy about several recent developments in the global economy. As monetary conditions in Hong Kong are closely associated with those of the U.S., while real economic activities in Hong Kong are increasingly more and more dependent on those of Mainland China, a certain degree of prudence is undoubtedly warranted. Future challenges that Hong Kong will have to face are still many. The most obvious ones are, among others, as follows:

- There is a widespread belief of an economic slowdown in the U.S. In particular, Alan Greenspan, chairman of the U.S. Federal Reserve, had expressed this view on many occasions.

- The Japanese economy has been in recession for a number of years.

- China's imminent accession to the WTO.

- Speculations about a possible devaluation of the Chinese RMB.

- Periods of deflation in China.

- Greater integration of the Hong Kong economy with that of the Mainland.

- Introducing more of the highly educated persons of expertise from the Mainland to work in Hong Kong.

How might these events affect Hong Kong from a macroeconomic perspective, particularly with respect to the price level, wage rate and capital flow? Although there have been many articles written on these themes, they are mostly journalistic and therefore there is a general lack of analytical macroeconomic models for Hong Kong to answer those questions in a more rigorous manner. Consequently, in-depth studies at a theoretical level are few, and this has affected the quality of subsequent empirical research on the subject. The objective of this paper is to fill this gap.

In this paper, I develop a concise but analytically useful macroeconomic model for Hong Kong based on some stylized facts about the Hong Kong economy. The model emphasizes the interdependence of Hong Kong with China and the United States. Important macroeconomic variables for Hong Kong, such as the price level and nominal wage rate, as well as capital flow, are analyzed first in a static setting and then in a dynamic model. The model is then used to analyze macroeconomic adjustment of Hong Kong in response to a couple of contingent scenarios. These scenarios are economic slowdown in the U.S. and devaluation of the RMB. Results are obtained both under the assumption of adaptive and rational expectations.

For the moment, the purpose of the paper is rather modest, i.e., to think through the different issues in a conceptual way. Needless to say, it would be more interesting and highly desirable if one can obtain numerical results based on the model established in this paper and from past data or simulation. These will indeed be research topics for future work.

The paper is organized as follows. Some stylized facts about the Hong Kong economy are presented in Section 2. Section 3 describes a static macroeconomic model of Hong Kong. Static equilibrium values of the price level, wage rate and capital flow to Hong Kong are obtained in Section 4. Section 5 analyzes the stability of the steady state of Hong Kong. Section 6 extends the study to a dynamic model with rational expectations. Concluding remarks are presented in Section 7.

2. Stylized Facts about the Hong Kong Economy

We begin by describing several stylized facts about the Hong Kong economy, which constitute the basis upon which this paper will be built. These facts largely capture the unique characteristics of the Hong Kong economy, particularly at the macroeconomic level, and thus are essential in a model that addresses the Hong Kong economy.

- Free trade and free flow of capital into and out of Hong Kong. Most economists consider Hong Kong as one of the freest economies in the world.

- The interest rate of Hong Kong is determined by monetary policies of the U.S. The only recent exception was during the Asian financial crisis of 1997-1998 when there was an intervention in the financial market by the Hong Kong Monetary Authority.

- The Hong Kong dollar has been pegged to the U.S. dollar since October 1983 at an exchange rate of 7.8 Hong Kong dollars per U.S. dollar through a currency board.

- The tertiary sector, in particular the financial services sector, is a very important part of the Hong Kong economy.

- Prices respond rapidly to changing conditions of demand and supply. According to an IMF study based on data from 1973 to 1998, the price level in Hong Kong decreased 0.41% for every 1% increase in the unemployment rate. By contrast, during the same period, the figure was 0.19 for the U.S., 0.07 for the U.K., and 0.05 as an average for 14 countries in the OECD.

- Relatively flexible wage adjustment. This is because the labor market in Hong Kong is very competitive, and has relatively few institutional factors that can result in nominal wage rigidity. The government of Hong Kong has traditionally pursued a non-interventionist, laissez-faire approach in economic affairs. A high degree of flexibility in both the goods and services market and the labor market constitutes an important feature of the Hong Kong economy.

- Large amount of foreign exchange reserves in Hong Kong (over US$ 110 billion), which is more than eight times the amount of currency in circulation. Hong Kong is believed to be well capable of defending the exchange rate peg.

- Trade with the Mainland and the U.S. accounts for a very large fraction (over 50%) of the total trade of Hong Kong. Furthermore, "outward processing" accounts for a significant part of the trade between Hong Kong and the Mainland. Annual figures of four years from 1997 to 2000 indicate that on average imports from Mainland China and from the U.S. account for 41.24% and 7.27% of the total imports of Hong Kong respectively. On the export side, those to Mainland China and to the U.S. account for 34.51% and 23.04% of the total exports of Hong Kong. When re-exports are excluded from total exports, domestic exports from Hong Kong to Mainland China and to the U.S. account for 29.84% and 28.86%. The two-way trade with Mainland China accounts for 37.85% of the total two-way trade of Hong Kong, while that with the U.S. accounts for 14.90%.

- Property price, which is mainly affected by the land supply policy of the government, plays a very important role in the economic life of Hong Kong.

- There is strict foreign exchange control in China. RMB is not fully convertible and the capital account of China is closed.

- Exchange rate of the Chinese RMB is managed within a narrow band around 8.27 RMB against the U.S. dollar.

- Large foreign exchange reserves in China (over US$ 175 billion). The Chinese government had stated repeatedly during the Asian financial crisis that it would help Hong Kong in times of need. This further strengthens Hong Kong's capability of maintaining its linked exchange rate system.

This paper will incorporate most factors listed above, with those left out (outward processing and the importance of the property market) to be included in a future study about the general economic interdependence among the U.S., Hong Kong and China.

3. A Static Macroeconomic Model of Hong Kong

From a macroeconomic perspective, the structure of the Hong Kong economy is a relatively simple one. The nominal interest rate in Hong Kong is determined by monetary policies of the U.S., which is exogenous to the Hong Kong economy, while the nominal exchange rate is fixed by its currency board. The only variable that can change to balance the supply and demand for goods and services in Hong Kong is the price level, through its effects on the real interest rate and the real exchange rate. For a given expected

future price level, a positive aggregate demand shock to the Hong Kong economy that increases the current price level will raise the expected real interest rate and the real exchange rate of the Hong Kong dollar. Higher expected real interest rate discourages private investment, while higher real exchange rate reduces the competitiveness of Hong Kong's export products. The balance of the aggregate supply and aggregate demand can thus be restored.

In the labor market, the nominal wage rate in Hong Kong adjusts to balance the supply and demand for labor, with the latter being associated with the supply of goods and services through the production function relationship. Labor supply depends on the real wage rate and other factors. One of the important factors is the policy pursued by the government. More specifically to the case of Hong Kong, the inflow and outflow of highly skilled labor is one of the key factors that warrants particular attention.

The money supply in Hong Kong is not an autonomous variable. Rather, first and foremost, it changes to guarantee that the nominal exchange rate remains fixed at the pre-determined parity rate. It is important to realize that the quantity of money supply will also be at the level that is able to accommodate the transaction needs of the Hong Kong economy. In other words, the quantity equation of money should hold true for Hong Kong. In addition, in view of the characteristics of the currency board that is in place, a change of the monetary base should be equal to the change of the official settlement balance of Hong Kong, which is the sum of the balance on the current account and the capital account. Through this relationship, we can make inferences about the change of capital flow to Hong Kong in response to various changes that may happen in the world economy, particularly those in the U.S. and in Mainland China. More specifically, when the increase in the money stock exceeds the increase in the current account balance, we know that there must have been a capital inflow to Hong Kong. On the other hand, when the increase in the money stock is less than the increase in the current account balance, we know that there must have been a capital outflow.

Throughout this paper, I use subscript 1 to indicate variables of Hong Kong, 2 to indicate those of China, and 3 to indicate those of the United States.

Aggregate Demand for Goods and Services

The real aggregate demand for Hong Kong can be written as

$$y_1 = A_1 + NX_1 ,$$

where

$$A_1 = C_1 + I_1 + g_1$$

is the domestic absorption of Hong Kong, which consists of private consumption C_1, private investment I_1, and government purchases g_1, while

$$NX_1 = EX_1 - IM_1$$

is the net exports of Hong Kong to the rest of the world. For the sake of illustrative simplicity in this paper, I assume that the rest of the world consists of the Chinese Mainland and the U.S. only. One could easily make the paper more realistic by including other important economies in the world such as the EU and Japan.

$$A_1 = g_1 + by_1 - dr_1 ,$$

$$NX_1 = NX_{12} + NX_{13} ,$$

where NX_{12} and NX_{13} are net exports of Hong Kong to China and to the United States, respectively, which can further be expressed as,

$$NX_{12} = t_{12} + n_{12}y_2 - n_{21}y_1 + \alpha_1 q_{12} ,$$

$$NX_{13} = t_{13} + n_{13}y_3 - n_{31}y_1 + \alpha_2 q_{13} .$$

t_{12} and t_{13} are the autonomous net exports from Hong Kong to China and from Hong Kong to the United States, respectively. Both are mainly affected by the commercial policies pursued (or the lack thereof) by Hong Kong and the two countries involved. n_{12} is China's marginal propensity to import from Hong Kong, whereas n_{21} is Hong Kong's marginal propensity to import from China. Similarly, n_{13} is the U.S.'s marginal propensity to import from Hong Kong, whereas n_{31} is Hong Kong's marginal propensity to import from the U.S. q_{12} is the real exchange rate between Hong Kong and China, and q_{13} is the real exchange rate between Hong Kong and the U.S. The real exchange rates measure the competitiveness of Hong Kong products relative to those of China and the U.S.

$$q_{12} = e_{12} - p_1 + p_2 ,$$

$$q_{13} = e_{13} - p_1 + p_3 ,$$

where e_{12} is the nominal exchange rate between the currencies of Hong Kong and China, expressed as the number of Hong Kong dollars per unit of the Chinese RMB. Likewise, e_{13} is the nominal exchange rate between the currencies of Hong Kong and the U.S., expressed as the number of Hong Kong dollars per U.S. dollar. , p_1 , p_2 and p_3 are logarithms of the price levels in Hong Kong, China and the U.S., respectively. I assume that the Marshall-Lerner condition is satisfied for Hong Kong, i.e., the sum of the price elasticity of exports and the price elasticity of imports exceeds unity. We can thus bypass the possibility of the J-Curve and simply assume that α_1 and α_2 are positive. Without loss of generality, if we choose the price unit appropriately, we can normalize the price level in the U.S. so that P_3=1 and p_3=0. For notational simplicity, I further denote e_{13}=e_1 and e_{23}=e_2 . We can thus re-write the expressions for real exchange rates as

$$q_{13} = e_1 - p_1 ,$$

$$q_{12} = e_{12} - p_1 + p_2 = e_{13} - e_{23} + p_2 - p_1 = (e_1 - p_1) - (e_2 - p_2) .$$

Thus,

$$NX_1 = NX_{12} + NX_{13}$$

$$= (t_{12} + t_{13}) + n_{12}y_2 + n_{13}y_3 - (n_{21} + n_{31})y_1 + \alpha_1 q_{12} + \alpha_2 q_{13}$$

$$= t_1 + (\alpha_1 + \alpha_2)(e_1 - p_1) - \alpha_1(e_2 - p_2) + n_{12}y_2 + n_{13}y_3 - n_1 y_1 \ ,$$

where t_1=t_{12}+t_{13} represents the autonomous net exports of Hong Kong to the rest of the world, n_1=n_{21}+n_{31} is Hong Kong's marginal propensity to import from the rest of the world. Combining the expressions for A_1 and NX_1,

$$y_1 = A_1 + NX_1$$

$$= g_1 + by_1 - dr_1 + t_1 + (\alpha_1 + \alpha_2)(e_1 - p_1) - \alpha_1(e_2 - p_2) + n_{12}y_2 + n_{13}y_3 - n_1 y_1 .$$

Solving for y_1, we get

$$(1 - b + n_1)y_1 = g_1 + t_1 - dr_1 + (\alpha_1 + \alpha_2)(e_1 - p_1) - \alpha_1(e_2 - p_2) + n_{12}y_2 + n_{13}y_3$$

$$y_1 = \frac{1}{1 - b + n_1}\left[g_1 + t_1 - dr_1 + (\alpha_1 + \alpha_2)(e_1 - p_1) - \alpha_1(e_2 - p_2) + n_{12}y_2 + n_{13}y_3\right].$$

To simplify notations, let

$$f_1 = \frac{g_1 + t_1}{1 - b + n_1}, \quad \lambda = \frac{d}{1 - b + n_1}, \quad \gamma = \frac{\alpha_1 + \alpha_2}{1 - b + n_1}, \quad \delta = \frac{\alpha_1}{1 - b + n_1},$$

$$\eta_{12} = \frac{n_{12}}{1 - b + n_1}, \quad \eta_{13} = \frac{n_{13}}{1 - b + n_1}.$$

Aggregate demand is

$$y_1^d = f_1 - \lambda r_1 + \gamma(e_1 - p_1) - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 .$$

Real aggregate demand in Hong Kong will increase if Hong Kong pursues a trade policy that increases its autonomous net exports, if it pursues an expansionary fiscal policy, when its real interest rate is lower, or if there is a real exchange rate depreciation of its currency relative to the U.S. dollar. Real aggregate demand in Hong Kong will also increase if there is an increase in the real aggregate output in China or the U.S. It will decrease if there is a real exchange rate depreciation of the Chinese RMB relative to the U.S. dollar.

Real Interest Rate with Adaptive Expectations

$$r_1 = i_1 - ({}^{+1}_{+0}\pi^e_1) = i_1 - [({}^{+1}_{+0}p^e_1) - p_1] ,$$

where $({}^{+1}_{+0}p^e_1)$ is the logarithm of the expected price level in Hong Kong during the next period based on information available in the current period. $({}^{+1}_{+0}\pi^e_1)$ is therefore the expected rate of inflation. Assuming that expectations are formed adaptively, i.e.,

$$({}^{+1}_{+0}p^e_1) = ({}^{+0}_{-1}p^e_1) + \mu[p_1 - ({}^{+0}_{-1}p^e_1)] = \mu p_1 + (1-\mu)({}^{+0}_{-1}p^e_1) ,$$

where $0 < \mu < 1$, the expected rate of inflation is

$$({}^{+1}_{+0}p^e_1) - p_1 = (1-\mu)[({}^{+0}_{-1}p^e_1) - p_1] .$$

The *ex ante* real interest rate is therefore

$$r_1 = i_1 - (1-\mu)[({}^{+1}_{+0}p^e_1) - p_1] .$$

Nominal Interest Rate

Full capital mobility between Hong Kong and the U.S. implies the international asset market equilibrium. Namely, risk-neutral international investors through arbitrage will eliminate all expected profits. This entails the uncovered interest parity condition

$$i_1 = i_3 + E\left(\frac{de_1}{dt}\right).$$

Under Hong Kong's currency board, the Hong Kong dollar is pegged to the U.S. dollar at 7.8 Hong Kong dollars per U.S. dollar. Hong Kong, as well as Mainland China, has one of the largest foreign exchange reserves in the world. Based on this fact, I ignore the possibility of either the Hong Kong dollar being de-linked with the U.S. dollar, or it being devaluated or revaluated against the U.S. dollar. Under that assumption, the nominal exchange rate is expected to remain fixed relative to the U.S. dollar at the current parity rate. Thus, $E\left(\frac{de_1}{dt}\right) = 0$, and

$$i_1 = i_3 .$$

Aggregate Supply of Goods and Services

We assume a Cobb-Douglas aggregate production function for Hong Kong, with a constant level of capital stock (without loss of generality, it is normalized to 1.)

$$Y_1 = A_1 L_1^{1-\alpha} .$$

In logarithm, the aggregate production function is

$$y_1 = \hat{a}_1 + (1-\alpha)l_1 ,$$

where $\hat{a}_1$ and l_1 are logarithms of the total factor productivity and the level of employment, respectively. $\alpha > 0$ is the capital's share in the national income. The marginal product of labor is

$$\frac{\partial Y_1}{\partial L_1} = (1-\alpha)A_1 L_1^{-\alpha} .$$

The logarithm of the marginal product of labor is

$$\ln(1-\alpha) + \hat{a}_1 - \alpha l_1 .$$

In a competitive economy such as that of Hong Kong, profit maximization of firms entails that the real wage rate is equal to the marginal product of labor, i.e.,

$$w_1 - p_1 = \ln(1-\alpha) + \hat{a}_1 - \alpha l_1 .$$

The optimal amount of labor utilization can therefore be written as

$$l_1 = \frac{1}{\alpha}(p_1 - w_1) + \frac{1}{\alpha}[\ln(1-\alpha) + \hat{a}_1] .$$

Substituting the l_1 above to the logarithm of the Cobb-Douglas aggregate production function, we have the expression for the aggregate supply as

$$y_1 = \hat{a}_1 + (1-\alpha)l_1$$

$$= \frac{\hat{a}_1}{\alpha} + \frac{1-\alpha}{\alpha}(p_1 - w_1) + \frac{1-\alpha}{\alpha}\ln(1-\alpha) .$$

Let $\beta = \frac{1-\alpha}{\alpha}$ and $a_1 = \frac{\hat{a}_1}{\alpha} + \frac{1-\alpha}{\alpha}\ln(1-\alpha)$. The aggregate supply function can be written as

$$y_1^s = a_1 + \beta(p_1 - w_1) .$$

Aggregate Demand for Labor

From the discussion above, we also get the aggregate demand for labor as

$$l_1^d = \frac{1}{\alpha}(p_1 - w_1) + \frac{1}{\alpha}[\ln(1-\alpha) + \hat{a}_1] .$$

Aggregate Supply of Labor

The aggregate supply of labor should be positively related to the real wage rate. Let us assume it can be written as

$$l_1^s = \hat{l}_1 + \theta(w_1 - p_1).$$

An exogenous increase in the labor supply (shift of the labor supply curve) is represented by an increase in $\hat{l}_1$.

Balance of Payments

Current account balance is equal to the balance of trade if we ignore the difference between the investment income received and paid to the rest of the world, as well as the unilateral transfers. It can be written as

$$CA_1 = t_1 + (\alpha_1 + \alpha_2)(e_1 - p_1) - \alpha_1(e_2 - p_2) + n_{12}y_2 + n_{13}y_3 - n_1y_1.$$

Capital account balance can be decomposed into two parts. One part depends on the difference between the nominal interest rate of Hong Kong and that of the U.S., taking into consideration the expected appreciation of the U.S. dollar. All factors other than interest rates and the expectation of exchange rate changes that may affect net capital inflow to Hong Kong are represented by the term κ_1,

$$KA_1 = \kappa_1 + \kappa\left[i_1 - i_3 - E\left(\frac{de_1}{dt}\right)\right].$$

Change of the official settlement balance

$$\partial f_1 = \partial(CA_1) + \partial(KA_1).$$

Money Supply and the Balance of Payments

Under the current linked exchange rate arrangement in Hong Kong, an increase in the supply of U.S. dollars due to a surplus of the official settlement balance will put upward pressure on the Hong Kong dollar in the market. Whenever there is a deviation of the market exchange rate from 7.8 Hong Kong dollars per U.S. dollar, the three note-issuing banks (HSBC, BOC, Standard Chartered) will arbitrage away the profit opportunity. For example, if the market exchange rate becomes 7.7 HK$s per US$, by acquiring 1 US$ with 7.7 HK$s and exchange for 1 US$ worth of the Certificate of Indebtedness at HKMA, one of the three banks can issue 7.8 HK$s of banknotes and therefore obtain a profit of 0.1 HK$. This inevitably results in the sale of Hong Kong dollars (and simultaneous buy back of U.S. dollars) by the Hong Kong Monetary Authority and thus an increase in the monetary base. If we assume the money multiplier (the ratio of total nominal balance, say M1, to the base money) is equal to ψ, then

$$\partial m_1 - \psi - p_1 = \partial f_1.$$

4. Static Macroeconomic Equilibrium of Hong Kong

Goods and Services Market Equilibrium

Under the assumption of full capital mobility between Hong Kong and the U.S., as well as adaptive expectations, the aggregate demand for goods and services in Hong Kong can be expressed as

$$y_1^d = f_1 - \lambda\left[i_1 - (1-\mu)[({}^{+0}_{-1}p_1^e) - p_1]\right] + \gamma(e_1 - p_1) - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3$$

$$= \left[f_1 + \lambda(1-\mu)({}^{+0}_{-1}p_1^e)\right] - \lambda i_3 + \gamma e_1 - (\lambda - \lambda\mu + \gamma)p_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 ,$$

or,

$$y_1^d = f - \lambda i_3 + \gamma e_1 - \omega p_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 ,$$

where $f = f_1 + \lambda(1-\mu)({}^{+0}_{-1}p_1^e)$, and $\omega = \lambda - \lambda\mu + \gamma > 0$.

The aggregate supply of goods and services in Hong Kong is

$$y_1^s = a_1 + \beta(p_1 - w_1) .$$

Price level adjusts according to the following tâtonnement process

$$\frac{dp_1}{dt} = \tau(y_1^d - y_1^s)$$

$$= \tau\{f - \lambda i_3 + \gamma e_1 - \omega p_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 - a_1 - \beta(p_1 - w_1)\} ,$$

where τ indicates the speed of price adjustment. Equilibrium in the goods and services market is reached when the aggregate supply equals the aggregate demand

$$a_1 + \beta(p_1^* - w_1^*) = f - \lambda i_3 + \gamma e_1 - \omega p_1^* - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 .$$

$$p_1^* = \frac{\beta}{\omega + \beta}w_1^* + \frac{1}{\omega + \beta}[f - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 - a_1] .$$

Labor Market Equilibrium

The aggregate demand for labor is

$$l_1^d = \frac{1}{\alpha}(p_1 - w_1) + \frac{1}{\alpha}[\ln(1-\alpha) + \hat{a}_1].$$

The aggregate supply of labor is

$$l_1^s = \hat{l}_1 + \theta(w_1 - p_1).$$

Nominal wage rate adjusts according to the tâtonnement process

$$\frac{dw_1}{dt} = \sigma(l_1^d - l_1^s)$$

$$= \sigma\left[\frac{1}{\alpha}(p_1 - w_1) + \frac{1}{\alpha}[\ln(1-\alpha) + \hat{a}_1] - \hat{l}_1 - \theta(w_1 - p_1)\right],$$

where σ indicates the speed of nominal wage adjustment. Equilibrium in the labor market is reached when the supply of labor equals the demand for labor

$$\hat{l}_1 + \theta(w_1^* - p_1^*) = \frac{1}{\alpha}(p_1^* - w_1^*) + \frac{1}{\alpha}[\ln(1-\alpha) + \hat{a}_1],$$

or,

$$w_1^* - p_1^* = -\frac{\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\alpha}{1+\alpha\theta}a_1,$$

where $\bar{l}_1 = \hat{l}_1 - \ln(1-\alpha)$.

Proposition 1:

The steady state of the economy is reached when both $\frac{dp_1}{dt}=0$ *and* $\frac{dw_1}{dt}=0$. *In the steady state, the price level, nominal wage rate, real wage rate, employment level, and output are*

$$p_1^* = \frac{1}{\omega}[f-\lambda i_3+\gamma e_1-\delta(e_2-p_2)+\eta_{12}y_2+\eta_{13}y_3]-\frac{\beta}{\omega}\cdot\frac{\alpha}{1+\alpha\theta}\bar{l}_1-\frac{\alpha}{\omega}\cdot\frac{1+\theta}{1+\alpha\theta}a_1\,,$$

$$w_1^* = \frac{1}{\omega}[f-\lambda i_3+\gamma e_1-\delta(e_2-p_2)+\eta_{12}y_2+\eta_{13}y_3]-\frac{\alpha(\omega+\beta)}{\omega(1+\alpha\theta)}\bar{l}_1+\frac{\alpha(\omega-\theta-1)}{\omega(1+\alpha\theta)}a_1\,,$$

$$w_1^*-p_1^* = \frac{\alpha}{1+\alpha\theta}a_1-\frac{\alpha}{1+\alpha\theta}\bar{l}_1\,,$$

$$l_1^* = \frac{\alpha\theta}{1+\alpha\theta}a_1+\frac{1}{1+\alpha\theta}\bar{l}_1+\ln(1-\alpha)\,,$$

$$y_1^* = \frac{\alpha(1+\theta)}{1+\alpha\theta}a_1+\frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1\,,$$

while the steady state value of the nominal money balance is

$$m_1^* = p_1^*+y_1^*-hi_3$$

$$m_1^* = \frac{1}{\omega}[f-\lambda i_3+\gamma e_1-\delta(e_2-p_2)+\eta_{12}y_2+\eta_{13}y_3]+\frac{\alpha(\omega+\omega\theta-\beta)}{\omega(1+\alpha\theta)}\bar{l}_1+\frac{\alpha(\beta\omega-1-\theta)}{\omega(1+\alpha\theta)}a_1-hi_3\,.$$

In the steady state, while the price level, nominal wage rate and money supply in Hong Kong are affected by both demand side factors and supply side ones, real wage rate, aggregate output and labor utilization are affected only by productivity and labor supply.

Decreases in i_3, e_2, and increases in e_1, p_2, y_2, y_3 tend to increase the price level and the nominal wage rate, as well as the money supply. Increase in productivity raises the real wage rate, level of labor utilization and aggregate output, and decreases the price level, whereas its impact on nominal wage rate is ambiguous, depending on the sign of $\omega-\theta-1$.

Consider the case when an increase in $\bar{l}_1$ results in a greater increase in a_1, such as introducing highly educated persons of expertise from the Mainland to work in Hong Kong, which is likely to result in a general productivity increase. The result will be increases in the real wage rate, level of labor utilization and aggregate output.

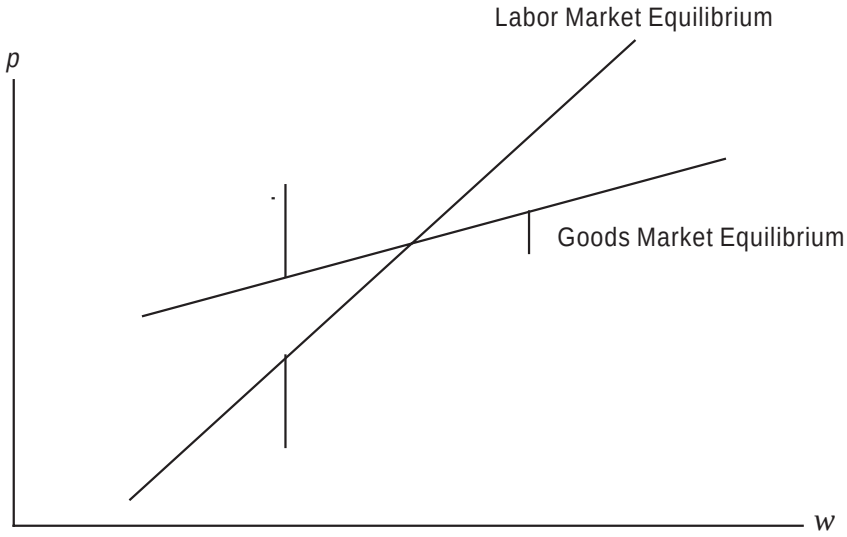
A Graph for Comparative Static Analysis

The locus for the goods and services market equilibrium are values of w_1 and p_1 such that

$$p_1 - \frac{\beta}{\omega+\beta} w_1 = \frac{1}{\omega+\beta}[f - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12} y_2 + \eta_{13} y_3 - a_1].$$

The locus for the labor market equilibrium are values of w_1 and p_1 such that

$$p_1 - w_1 = \frac{\alpha}{1+\alpha\theta}\bar{l}_1 - \frac{\alpha}{1+\alpha\theta} a_1 .$$



Capital Inflow to Hong Kong

One of the important implications of the linked exchange rate system in Hong Kong is that the money supply of Hong Kong is endogenous, changing to guarantee that the nominal exchange rate remains fixed at the pre-determined parity rate. Meanwhile, the quantity equation of money should also hold true for Hong Kong. In addition, change in the monetary base must be equal to the change in the sum of the balance on the current account and the balance on the capital account. Denote the money multiplier by ψ, then

$$\partial m_1 - \psi - p_1 = \partial f_1 ,$$

and thus,

$$\partial(KA_1) = \partial m_1 - \psi - p_1 - \partial(CA_1).$$

$\frac{\partial(KA_1)}{\partial y_3} > 0$ when $p_1 < \frac{\partial m_1}{\partial y_3} - \frac{\partial(CA_1)}{\partial y_3} - \psi$. As $\frac{\partial m_1}{\partial y_3} = \frac{\eta_{13}}{\omega}$ and $\frac{\partial(CA_1)}{\partial y_3} = n_{13}$, we get $\frac{\partial(KA_1)}{\partial y_3} > 0$ if and only if $p_1 < \frac{\eta_{13}}{\omega} - \psi - n_{13} = n_{13}\left[\frac{1-\omega(1-b+n_1)}{\omega(1-b+n_1)}\right] - \psi$.

Likewise, we can infer how capital flow to Hong Kong will respond to various changes in the world economy as stated in the following proposition:

Proposition 2:

Capital flow to Hong Kong critically depends on the price level in Hong Kong relative to those in the rest of the world. There will be capital inflow to Hong Kong, ceteris paribus, if,

a) y_3 *increases, and* $p_1 < n_{13}\Omega - \psi$;

b) y_2 *increases, and* $p_1 < n_{12}\Omega - \psi$;

c) p_2 *increases, and* $p_1 < \alpha_1\Omega - \psi$;

d) e_1 *increases, and* $p_1 < (\alpha_1 + \alpha_2)\Omega - \psi$;

e) e_2 *increases, and* $p_1 < -\alpha_1\Omega - \psi$;

f) a_1 *increases, and* $p_1 < \frac{\alpha}{\omega(1+\alpha\theta)}[\beta\omega + (1+\theta)(n_1\omega - 1)] - \psi$;

g) $\bar{l}_1$ *increases, and* $p_1 < \frac{\alpha}{\omega(1+\alpha\theta)}[\omega(1+\theta) + \beta(n_1\omega\alpha - 1)] - \psi$;

h) i_3 *increases, and* $p_1 < -\frac{\lambda}{\omega} - h - \psi$.

where $\Omega = \frac{1-\omega(1-b+n_1)}{\omega(1-b+n_1)} = \frac{1+d\mu-d-\alpha_1-\alpha_2}{\alpha_1+\alpha_2+d-d\mu}$.

---

5. Stability Analysis of the Steady State of Hong Kong

Rewrite the dynamic price level adjustment process

$$\frac{dp_1}{dt} = \tau(y_1^d - y_1^s) ,$$

and the dynamic nominal wage rate adjustment process

$$\frac{dw_1}{dt} = \sigma(l_1^d - l_1^s) ,$$

as a set of two variable first order homogeneous and autonomous differential equations in $(w_1 - w_1^*)$ and $(p_1 - p_1^*)$,

$$\begin{cases} \dfrac{dw_1}{dt} = -\dfrac{\sigma(1+\alpha\theta)}{\alpha}(w_1 - w_1^*) + \dfrac{\sigma(1+\alpha\theta)}{\alpha}(p_1 - p_1^*) , \\ \dfrac{dp_1}{dt} = \tau\beta(w_1 - w_1^*) - \tau(\omega+\beta)(p_1 - p_1^*) , \end{cases}$$

or,

$$\begin{cases} \dfrac{d(w_1 - w_1^*)}{dt} = a_{11}(w_1 - w_1^*) + a_{12}(p_1 - p_1^*) , \\ \dfrac{d(p_1 - p_1^*)}{dt} = a_{21}(w_1 - w_1^*) + a_{22}(p_1 - p_1^*) , \end{cases}$$

where

$$a_{11} = -\frac{\sigma(1+\alpha\theta)}{\alpha} < 0 ,$$

$$a_{12} = \frac{\sigma(1+\alpha\theta)}{\alpha} > 0 ,$$

$$a_{21} = \tau\beta > 0 ,$$

$$a_{22} = -\tau(\omega+\beta) < 0 .$$

The characteristic equation for the system of differential equations above is

$$\det(A - xI) = \det\begin{bmatrix} a_{11} - x & a_{12} \\ a_{21} & a_{22} - x \end{bmatrix} = x^2 - (a_{11} + a_{22})x + (a_{11}a_{22} - a_{12}a_{21})$$

$$= x^2 - tr(A)x + \det(A) = 0 \,,$$

where

$$tr(A) = a_{11} + a_{22} = -\frac{\sigma(1+\alpha\theta)}{\alpha} - \tau(\omega + \beta) < 0 \,,$$

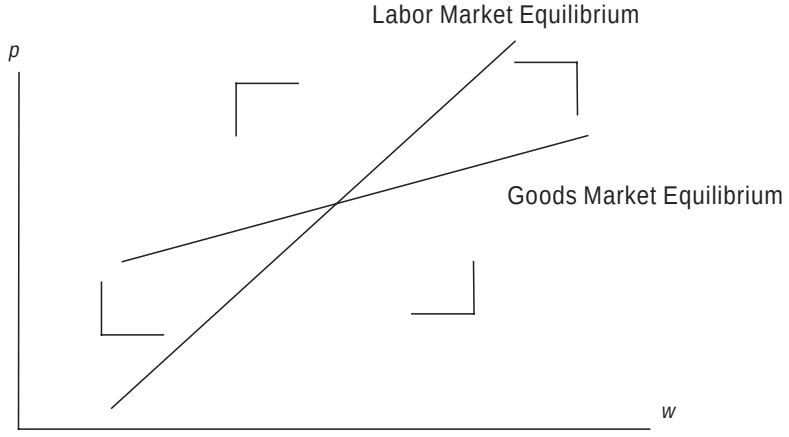
$$\det(A) = a_{11}a_{22} - a_{12}a_{21} = \tau\omega\frac{\sigma(1+\alpha\theta)}{\alpha} > 0 \,.$$

Stability of the solutions depends on

$$tr(A)^2 - 4\det(A) = \left(\frac{\sigma(1+\alpha\theta)}{\alpha} + \tau(\omega + \beta)\right)^2 - 4\tau\omega\frac{\sigma(1+\alpha\theta)}{\alpha}$$

$$= \left(\frac{\sigma(1+\alpha\theta)}{\alpha} - \tau(\omega + \beta)\right)^2 + 4\tau\beta\frac{\sigma(1+\alpha\theta)}{\alpha} > 0 \,.$$

The solutions are globally stable as $tr(A) < 0$, $\det(A) > 0$, and $tr(A)^2 - 4\det(A) > 0$.



Proposition 3:

The steady state, or the static macroeconomic equilibrium, of Hong Kong is globally stable. Suppose there is a shock to the economy and hence an initial deviation from the static macroeconomic equilibrium, the price level and the nominal wage rate will converge back to their steady state values after a period of adjustment.

6. A Dynamic Model with Rational Expectations

Up to this point, I have assumed that expectations about the price level determination are formed adaptively. In what follows, I will relax this assumption and use rational expectations instead.

Goods market equilibrium

$$a_1 + \beta(p_{1t} - w_{1t}) = f_1 - \lambda[i_3 - [E_t(p_{1,t+1}) - p_{1t}]] + \gamma(e_1 - p_{1t}) - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3,$$

$$(\beta + \lambda + \gamma)p_{1t} = \beta w_{1t} + \lambda E_t(p_{1,t+1}) + f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3.$$

Labor market equilibrium

$$w_{1t} - p_{1t} = -\frac{\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\alpha}{1+\alpha\theta}a_1.$$

Combining goods market equilibrium and labor market equilibrium, we get the price level

$$(\lambda + \gamma)p_{1t} = \lambda E_t(p_{1,t+1}) + f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta}a_1,$$

or,

$$p_{1t} = \frac{\lambda}{\lambda+\gamma}E_t(p_{1,t+1}) + \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12}y_2 + \eta_{13}y_3 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta}a_1\right]$$

$$p_{1t} = \frac{\lambda}{\lambda+\gamma}E_t(p_{1,t+1}) + \frac{1}{\lambda+\gamma}\eta_{13}y_{3t} + u_t,$$

where

$$u_t = \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12}y_2 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta}a_1\right]$$

is a random variable that represents supply and demand shocks other than change in the growth rate of US aggregate output.

We can thus obtain the behavior of the price level (and thus also the wage rate) in Hong Kong based on different assumptions about the behavior of the aggregate output of the U.S., and the possibility and the magnitude of a devaluation of the Chinese RMB. The results are described in the following proposition. Details of the proof are included in the Appendix.

Proposition 4:

(A) If the aggregate output of the U.S. follows an AR(1) process

$$y_{3t} = r + sy_{3,t-1} + v_t \,.$$

Then the price level in Hong Kong, when expectations are formed rationally, follows

$$p_{1t} = \frac{\lambda rs\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} + \frac{(rs-\lambda)s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-1} + \frac{\eta_{13}}{\gamma} y_t + u_t \,.$$

(B) If the growth rate of the aggregate output of the U.S. follows an AR(1) process

$$y_{3t} - y_{3,t-1} = r + s(y_{3,t-1} - y_{3,t-2}) + v_t \,.$$

Then the price level in Hong Kong, when expectations are formed rationally, follows

$$p_{1t} = \frac{(\lambda+\gamma)\lambda r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma^2} - \frac{\lambda s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-1} + \frac{(\lambda+\gamma)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3t} + u_t \,,$$

where, in both cases

$$u_t = \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12} y_2 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta} a_1 \right].$$

(C) If the probability of a devaluation of the RMB by the magnitude Δ is P, then the price level in Hong Kong, when expectations are formed rationally, follows

$$p_{1t} = -\frac{\delta\lambda P\Delta}{\gamma(\lambda+\gamma)} - \frac{\delta}{\gamma}e_2 + z_t - \frac{\delta}{\lambda+\gamma}\varepsilon_t ,$$

where

$$z_t = \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 + \delta p_2 + \eta_{12} y_2 + \eta_{13} y_3 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta}a_1 \right].$$

7. Concluding Remarks and Further Research

In this paper, I have developed a concise but analytically useful macroeconomic model for Hong Kong based on some stylized facts about the characteristics of the Hong Kong economy. The model particularly emphasizes the interdependence of Hong Kong with China and the United States. It is used to analyze macroeconomic adjustment of Hong Kong in response to several contingent scenarios under the assumption of adaptive and rational expectations.

This model can be considered as a "benchmark" model because of the simplifications made about the Hong Kong economy for analytical purposes. The model can be refined and extended in the following directions:

- I assumed a Cobb-Douglas aggregate production function for Hong Kong, with a constant level of capital stock. Capital inflow, which is an important variable analyzed in the model, will result in higher capital stock in the economy and thus raise the productivity of labor. Output can be affected through this important channel.

- The property and asset market plays an important role in the economy of Hong Kong. Higher property price will increase the cost of living and therefore affect the dynamics of labor demand and labor supply behavior. Property price will also affect the net worth of individuals and business. Through this balance sheet effect, it can affect investment and consumption in the economy.

- Trade with Mainland China and the U.S. accounts for a very large fraction of the total trade of Hong Kong. Furthermore, "outward processing" accounts for a significant part of the trade between Hong Kong and the Mainland. Refinement of the model by explicitly considering the tradable and non-tradable goods and the important role played by outward processing in the Hong Kong economy will likely generate new insights that are not provided in the current model.

- Empirical analysis and econometric tests can be performed based on the model, and numerical simulations can also be obtained to make predictions about the changes in the Hong Kong economy based on the model developed in this paper.

Inclusion of more detailed consideration of those factors listed above will surely generate much more realistic and informative results. They will be considered in the follow up of the current paper.

References

Beecham, B. Julian (1998), Monetary and Financial System in Hong Kong, 2nd ed., Hong Kong Institute of Bankers.

Buiter, W. H. and M. H. Miller (1981), "Monetary Policy and International Competitiveness: The Problems of Adjustment," *Oxford Economic Papers,* 33: 143-75.

Census and Statistics Department of the Hong Kong SAR Government, *Hong Kong Monthly Digest of Statistics,* various issues.

Cooper, Richard N. (1985), "Economic Interdependence and Co-ordination of Economic Policies," Chapter 23 in Ronald W. Jones and Peter B. Kenen (eds.) Handbook of International Economics, Vol. II, North-Holland.

Dodsworth, John and Dubravko Mihaljek (1997), "Hong Kong, China - Growth, Structural Change, and Economic Stability during the Transition," IMF Occasional Paper 152.

Dernburg, Thomas F. (1989), Global Macroeconomics, Harper & Row.

Dornbusch, Rudiger (1976), "Expectations and Exchange Rate Dynamics," *Journal of Political Economy, 84: 1161-76.*

Fleming, J. M. (1962), "Domestic Financial Policies under Fixed and Floating Exchange Rates," IMF Staff Papers, 9(3): 369-79.

Gandolfo, Giancarlo (1995), International Economics II : International Monetary Theory and Open-Economy Macroeconomics, 2nd Revised Edition, Springer Verlag.

Hong Kong Monetary Authority (1999), "Impact of RMB Devaluation on Hong Kong's Trade Sector," HKMA Research Department Internal Memorandum 14/1999.

Hong Kong Monetary Authority (2000), "Impact of China's WTO Accession on the Hong Kong Economy," HKMA Research Department Internal Memorandum 01/2000.

Hong Kong Monetary Authority (2000), Annual Report 2000.

Hong Kong Monetary Authority, *Monthly Statistical Bulletin,* various issues.

Hong Kong Monetary Authority, *Quarterly Bulletin,* various issues.

International Monetary Fund (2000), "People's Republic of China - Hong Kong SAR: Selected Issues and Statistical Appendix," Washington, D.C.

International Monetary Fund (2001), "People's Republic of China - Hong Kong SAR: 2000 Article IV Consultation - Staff Report and Public Information Notice on the Executive Board Discussion," Washington, D.C.

Jao, Y. C. (1997), Hong Kong as an International Financial Centre, City University of Hong Kong Press.

Kenen, Peter (1985), "Macroeconomic Theory and Policy: How the Closed Economy Was Opened," Chapter 13 in Ronald W. Jones and Peter B. Kenen (eds.) Handbook of International Economics, Vol. II, North-Holland.

Ma, Yue, Shu-Ki Tsang and Shu-Hung Tang (1998), "The Impact of the China Factor on the pre-1997 Hong Kong Economy: A Macroeconometric Analysis," *International Review of Applied Economics,* 12(1): 89-106.

McCallum, Bennett T. (1996), International Monetary Economics, Oxford University Press.

Mundell, Robert A. (1962), "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," *IMF Staff Papers,* 9(1): 70-79.

Mundell, Robert A. (1963), "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," *Canadian Journal of Economics and Political Science,* 29(4): 475-85.

Rivera-Batiz, Luis A. and Amadou N. R. Sy (2000), "Currency Boards, Credibility, and Macroeconomic Behavior," IMF Working Paper WP/00/97.

Stone, Ronald (1997), Economic Dynamics: Phase Diagrams and Their Economic Application, Cambridge University Press.

Sung, Yun-Wing (1998), Hong Kong and South China, the Economic Synergy, City University of Hong Kong Press.

Appendix

(A) We use the following AR(1) process

$$y_{3t} = r + sy_{3,t-1} + v_t$$

to represent the aggregate output of the U.S., then

$$\frac{1}{\eta_{13}}[(\lambda+\gamma)p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda+\gamma)u_t] = r + sy_{3,t-1} + v_t \text{ .}$$

Let the rational expectation solution for p_{1t} be in the form of

$$p_{1t} = \phi_0 + \phi_1 y_{3,t-1} + \phi_2 u_t + \phi_3 v_t \text{ .}$$

Then

$$p_{1,t+1} = \phi_0 + \phi_1 y_{3t} + \phi_2 u_{t+1} + \phi_3 v_{t+1},$$

$$E_t(p_{1,t+1}) = \phi_0 + \phi_1 y_{3t} = \phi_0 + \phi_1(r + sy_{3,t-1} + v_t),$$

$$(\lambda+\gamma)p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda+\gamma)u_t = \eta_{13}(r + sy_{3,t-1} + v_t),$$

$$(\lambda+\gamma)(\phi_0 + \phi_1 y_{3,t-1} + \phi_2 u_t + \phi_3 v_t) - \lambda[\phi_0 + \phi_1(r + sy_{3,t-1} + v_t)] - (\lambda+\gamma)u_t = \eta_{13}(r + sy_{3,t-1} + v_t) \text{ .}$$

Comparing coefficients,

$$(\lambda+\gamma)\phi_0 - \lambda(\phi_0 + \phi_1 r) = r\eta_{13},$$

$$(\lambda+\gamma)\phi_1 - \lambda s\phi_1 = s\eta_{13},$$

$$(\lambda+\gamma)\phi_2 - (\lambda+\gamma) = 0,$$

$$(\lambda+\gamma)\phi_3 - \lambda\phi_1 = \eta_{13}.$$

We get

$$\phi_0 = \frac{(\lambda+\gamma)r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma},$$

$$\phi_1 = \frac{s\eta_{13}}{\lambda+\gamma-\lambda s},$$

$$\phi_2 = 1,$$

$$\phi_3 = \frac{\eta_{13}}{\gamma}.$$

Therefore, the rational expectation price level will be

$$p_{1t} = \frac{(\lambda+\gamma)r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} + \frac{s\eta_{13}}{\lambda+\gamma-\lambda s} y_{3,t-1} + \frac{\eta_{13}}{\gamma} v_t + u_t$$

$$= \frac{\lambda rs\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} + \frac{(rs-\lambda)s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-1} + \frac{\eta_{13}}{\gamma} y_t + u_t.$$

(B) More realistically, we assume that the growth rate of the U.S. follows an AR(1) process

$$y_{3t} - y_{3,t-1} = r + s(y_{3,t-1} - y_{3,t-2}) + v_t,$$

$$p_{1t} = \frac{\lambda}{\lambda+\gamma} E_t(p_{1,t+1}) + \frac{1}{\lambda+\gamma}\eta_{13} y_{3t} + u_t,$$

$$\frac{1}{\eta_{13}}[(\lambda+\gamma)p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda+\gamma)u_t] = r + (1+s)y_{3,t-1} - sy_{3,t-2} + v_t.$$

Let the rational expectation solution for p_{1t} be in the form of

$$p_{1t} = \phi_0 + \phi_1 y_{3,t-1} + \phi_2 y_{3,t-2} + \phi_3 u_t + \phi_4 v_t.$$

Then

$$p_{1,t+1} = \phi_0 + \phi_1 y_{3t} + \phi_2 y_{3,t-1} + \phi_3 u_{t+1} + \phi_4 v_{t+1},$$

$$E_t(p_{1,t+1}) = \phi_0 + \phi_1 y_{3t} + \phi_2 y_{3,t-1} = \phi_0 + \phi_1[r + (1+s)y_{3,t-1} - sy_{3,t-2} + v_t] + \phi_2 y_{3,t-1},$$

$$(\lambda+\gamma)p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda+\gamma)u_t = \eta_{13}[r + (1+s)y_{3,t-1} - sy_{3,t-2} + v_t],$$

$$(\lambda+\gamma)(\phi_0 + \phi_1 y_{3,t-1} + \phi_2 y_{3,t-2} + \phi_3 u_t + \phi_4 v_t) - \lambda\{\phi_0 + \phi_1[r + (1+s)y_{3,t-1}$$

$$- sy_{3,t-2} + v_t] + \phi_2 y_{3,t-1}\} - (\lambda+\gamma)u_t = \eta_{13}[r + (1+s)y_{3,t-1} - sy_{3,t-2} + v_t].$$

Comparing coefficients,

$$(\lambda+\gamma)\phi_0 - \lambda(\phi_0 + r\phi_1) = \eta_{13} r,$$

$$(\lambda+\gamma)\phi_1 - \lambda(1+s)\phi_1 - \lambda\phi_2 = \eta_{13}(1+s),$$

$$(\lambda+\gamma)\phi_2 + \lambda s\phi_1 = -s,$$

$$(\lambda+\gamma)\phi_3 - (\lambda+\gamma) = 0,$$

$$(\lambda+\gamma)\phi_4 - \lambda\phi_1 = \eta_{13}.$$

We get

$$\phi_0 = \frac{(\lambda+\gamma)^2 r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma^2},$$

$$\phi_1 = \frac{(\lambda+\gamma+\gamma s)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma},$$

$$\phi_2 = -\frac{(\lambda+\gamma)s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma},$$

$$\phi_3 = 1,$$

$$\phi_4 = \frac{(\lambda+\gamma)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma}.$$

Therefore, the rational expectations price level will be

$$p_{1t} = \frac{(\lambda+\gamma)^2 r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma^2} + \frac{(\lambda+\gamma+\gamma s)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-1} - \frac{(\lambda+\gamma)s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-2} + u_t + \frac{(\lambda+\gamma)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} v_t$$

$$= \frac{(\lambda+\gamma)\lambda r\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma^2} + \frac{(\lambda+\gamma)\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3t} - \frac{\lambda s\eta_{13}}{(\lambda+\gamma-\lambda s)\gamma} y_{3,t-1} + u_t$$

$$= \frac{\lambda+\gamma}{\lambda+\gamma-\lambda s}\left(\frac{\lambda r\eta_{13}}{\gamma^2} + \frac{\eta_{13}}{\gamma} y_{3t} - s\frac{\lambda}{\lambda+\gamma}\cdot\frac{\eta_{13}}{\gamma} y_{3,t-1}\right) + u_t .$$

(C) Now let's investigate the impact of a possible devaluation.

$$p_{1t} = \frac{\lambda}{\lambda+\gamma} E_t(p_{1,t+1}) + \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 - \delta(e_2 - p_2) + \eta_{12} y_2 + \eta_{13} y_3 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta} a_1\right]$$

$$p_{1t} = \frac{\lambda}{\lambda+\gamma} E_t(p_{1,t+1}) - \frac{\delta}{\lambda+\gamma} e_{2t} + z_t ,$$

where

$$z_t = \frac{1}{\lambda+\gamma}\left[f_1 - \lambda i_3 + \gamma e_1 + \delta p_2 + \eta_{12} y_2 + \eta_{13} y_3 - \frac{\beta\alpha}{1+\alpha\theta}\bar{l}_1 + \frac{\beta\alpha}{1+\alpha\theta} a_1\right].$$

Let

$$e_{2t} = e_2 + \varepsilon_t ,$$

where

$$\varepsilon_t = \begin{cases} 0, & \text{Prob} = 1 - P \\ \Delta, & \text{Prob} = P \end{cases}$$

$$-\frac{1}{\delta}[(\lambda+\gamma) p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda+\gamma) z_t] = e_2 + \varepsilon_t$$

and δ_t is a small deviation of the exchange rate from the parity rate e_2.

Let the rational expectation solution for p_{1t} be in the form of

$$p_{1t} = \phi_0 + \phi_1 e_2 + \phi_2 z_t + \phi_3 \varepsilon_t .$$

Then

$$p_{1,t+1} = \phi_0 + \phi_1 e_2 + \phi_2 z_{t+1} + \phi_3 \varepsilon_{t+1},$$

$$E_t(p_{1,t+1}) = \phi_0 + \phi_1 e_2 + \phi_3 P\Delta,$$

$$(\lambda + \gamma) p_{1t} - \lambda E_t(p_{1,t+1}) - (\lambda + \gamma) z_t = -\delta(e_2 + \varepsilon_t),$$

$$(\lambda+\gamma)(\phi_0 + \phi_1 e_2 + \phi_2 z_t + \phi_3 \varepsilon_t) - \lambda(\phi_0 + \phi_1 e_2 + \phi_3 P\Delta) - (\lambda+\gamma) z_t = -\delta(e_2 + \varepsilon_t).$$

Comparing coefficients,

$$(\lambda+\gamma)\phi_0 - \lambda(\phi_0 + \phi_3 P\Delta) = 0,$$

$$(\lambda+\gamma)\phi_1 - \lambda\phi_1 = -\delta,$$

$$(\lambda+\gamma)\phi_2 - (\lambda+\gamma) = 0,$$

$$(\lambda+\gamma)\phi_3 = -\delta,$$

we get

$$\phi_0 = -\frac{\delta\lambda P\Delta}{\gamma(\lambda+\gamma)},$$

$$\phi_1 = -\frac{\delta}{\gamma},$$

$$\phi_2 = 1,$$

$$\phi_3 = -\frac{\delta}{\lambda+\gamma}.$$

Therefore, the rational expectation price level will be

$$p_{1t} = -\frac{\delta\lambda P\Delta}{\gamma(\lambda+\gamma)} - \frac{\delta}{\gamma} e_2 + z_t - \frac{\delta}{\lambda+\gamma}\varepsilon_t.$$