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Abstract

The New-Keynesian aggregate supply derives from micro-foundations an inflation-dynamics model very much like the tradition in the monetary literature. Inflation is primarily affected by: (i) Economic slack; (ii) Expectations; (iii) Supply shocks; and, (iv) Inflation persistence.

This paper extends the New Keynesian aggregate supply relationship to include fluctuations in potential output as an additional determinant of the relationship. Implications for monetary rules and for the estimation of the Phillips curve are pointed out.

Keywords: New-Keynesian Phillips Curve, Potential Output, Taylor rules.

JEL Classification: E1, E12, E60

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1. Introduction

The New Keynesian aggregate supply relationship typically links inflation surprises with fluctuations in the output gap, but there is no independent role for the fluctuations in potential output. Based on such construct, micro-based interest rules do not respond to fluctuations in potential output; they respond exclusively to fluctuations in the inflation rates and the output gaps.

Another branch in the inflation literature is concerned with the long-term level of potential output. The literature points out the long-run costs of inflation, based on some cross-country evidence. Findings point to some threshold effects in the relationship between inflation and growth. Consequently, above certain country-specific inflation thresholds, growth is negatively affected by mean inflation. (e.g., see Barro 1995 and Khan and Senhadji 2001). However, this long-run channel through which monetary policy affects potential output is not considered in this paper. Rather, the paper demonstrates how to bring the potential output into the aggregate supply relationship, by incorporating the effects of investment in capacity on aggregate supply.

The analysis in the paper is conducted in an optimization-based “New Keynesian” framework, à la Blanchard and Kiyotaki (1987), employing the analytical tools in the lucid exposition of Woodford (2003). Specifically, the model features imperfect competition in the product market, in which the producers mark up output prices over marginal costs, and also mark down wages below the marginal productivity of labor. We thus derive a version of the mark-ups of prices over wages in our model. Mark-ups turn out to be counter-cyclical – a very pronounced phenomenon in the European markets, as noted by Cohen and Farhi (2001). They note that “*European firms in bad times manage to keep the prices high, while their US counterparts are pressed into cuts and discounts of various forms.*” This is why the product market version of the Phillips curve (i.e., the relation between inflation and the output gap), in Europe, seems to be relatively more stable empirically than the labor market version (i.e., the relation between wage growth and unemployment).

Evidently, the equilibrium relation between inflation and excess capacity is significantly influenced by the degree of competition in the product market. A key feature of such equilibrium is the degree of strategic interactions between firms that set their prices *ex ante* and other domestic and foreign firms that set their prices so as to clear the markets *ex post*. This market-organization feature determines in turn the degree of price stickiness.

Understanding why nominal changes have real consequences (why a short-run aggregate supply relationship exists) has long been a central concern of macroeconomic research. Lucas (1973) proposes a model in which the effect arises because agents in the economy are unable to distinguish perfectly between aggregate and idiosyncratic shocks. He tests this model at the aggregate level by showing that the Phillips curve is steeper in countries with more variable aggregate maximal demand. Following Lucas (1973), Ball, Mankiw and Romer (1988) show that sticky-price Keynesian models predict that the Phillips curve should be steeper in countries with higher average rates of inflation and that this prediction also receives empirical support. Loungani, Razin and Yuen (2001), and Razin and Yuen (2001) show that both Lucas’s and Ball-Mankiw-Romer’s estimates of the Phillips curve slope depend on the degree of capital account restrictions.

The paper is organized as follows. Section 2 lays out the New-Keynesian analytical framework. Section 3 derives the aggregate supply relationship. Section 4 concludes with implications of the aggregate supply relationship to optimizing monetary rules, and implications of the empirical literature.

2. The Analytical Framework

Consider a closed economy with a representative household that is endowed with a continuum of goods-specific skills – uniformly distributed on the unit interval $[0, 1]$ – to be supplied to a differentiated product industry. Consumption goods are distributed on $[0, 1]$. The household seeks to maximize a discounted sum of expected utilities:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s; \xi_s) - \int_0^1 (v(h_s(j); \xi_s) dj + \phi(M_s / P)) \right],$$

where β is the subjective discount factor, C is the Dixit-Stiglitz (1977) index of household consumption, P the Dixit-Stiglitz price index, M/P the demand for real balances, ξ a preference shock, and $h(j)$ the supply of type- j labor to the production of good of variety j . As usual, we define the consumption index and its corresponding price index respectively as

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

and

$$P_t = \left\{ \int_0^1 p_t(j)^{1-\theta} dj \right\}^{\frac{1}{1-\theta}}, \quad (1)$$

where, $c(j)$ represents consumption of the j^{th} good, and $p(j)$ the price of $c(j)$. The elasticity of substitution among the different goods is $\theta > 1$ and the number of goods that are produced is equal to 1.

For our purpose, the relevant utility-maximizing conditions include an intra-temporal condition for the choice of labor supply of type j :

$$\frac{v_h(h_t(j); \xi_t)}{u_c(C_t; \xi_t)} = \frac{w_t(j)}{P_t} \quad (2)$$

and an inter-temporal condition for the consumption-saving choice:

$$\frac{u_c(C_t; \xi_t)}{u_c(C_{t+1}; \xi_{t+1})} = \beta(1 + r_t), \quad (3)$$

where r^t is the real rate of interest in period. As in the Dixit-Stiglitz (1977) model, demand for good j satisfies

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \quad (4)$$

The production function assumes the form

$$y_t(j) = k_t(j) f[A_t h_t(j) / k_t(j)],$$

where A_t is a random labor-augmenting productivity shock. We follow Woodford (2003, chapter 5) and assume (for tractability) that there is a separate capital stock, $k_t(j)$, for each good, rather than a single rental market for capital services that each producer has access to.

Investment spending is in the amount

$$I_t(j) = I[k_{t+1}(j) / k_t(j)] k_t(j),$$

The function $I[\cdot]$ is a convex function, as in the standard investment cost-of-adjustment textbooks.

$I_t(j)$ represents purchases by producer j of a Dixit-Stiglitz composite good:

$$I_t(j) = \left[\int_0^1 I_t^i(j)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1$$

For simplicity, the elasticity of substitution θ is the same as in the case of consumption purchases. The variable cost of supplying $y_t(j)$ is $w_t f^{-1}[y_t(j) / k_t(j)]$. This implies that the real marginal cost is:

$$s_t(j) = \frac{w_t(j)}{P_t A_t f^1[f^{-1}(y_t(j) / k_t(j))]}.$$

By using equation (2), we can replace the real wage above by the marginal rate of substitution. Imposing symmetry across firms (so that we can drop the index j), the above equation can be rewritten as

$$s(k, y, C; \xi, A) = \frac{v_h \left[f^{-1}(y_t / k_t) \frac{k_t}{A_t}; \xi \right]}{u_c(C_t; \xi) A_t f^1[f^{-1}(y_t / k_t)]} \quad (5)$$

2.1 Price Setting

Firms are monopolistically competitive in the goods markets, and each one of them behaves like a monopsony in the labor market (with producer j as the sole demander for labor of type- j). A fraction γ of the monopolistically competitive firms sets their prices flexibly at p_{1t} , supplying y_{1t} ; whereas the remaining fraction $1 - \gamma$ sets their prices one period in advance (in period $t - 1$) at p_{2t} , supplying y_{2t} . In the former case, the price is marked up above the marginal cost by a factor of $\mu = \left(\frac{\theta}{\theta - 1} > 1\right)$, so that

$$\frac{p_{1t}}{P_t} - \mu s(k_t, y_{1t}, Y - I_t; \xi_t, A_t) = 0 \quad (6a)$$

In the latter case, p_{2t} will be chosen to maximize expected discounted profit

$$E_t \left[\left(\frac{1}{1+i_{t-1}} \right) (p_{2t} y_{2t} - w_t h_t) \right] = E_{t-1} \left\{ \left(\frac{1}{1+i_{t-1}} \right) \left[Y_t P_t^\theta p_{2t}^{1-\theta} - w_t f^{-1}(Y_t P_t^\theta p_{2t}^\theta / k_t) \right] \right\},$$

where we have used the inverse demand function from equation (4) for y_{2t} and the inverse production function for h_t . One can show that p_{2t} satisfies

$$E_{t-1} \left\{ \left(\frac{1}{1+i_{t-1}} \right) Y_t P_t^{\theta-1} \left[\frac{p_{2t}}{P_t} - \mu s(k_t, y_{2t}, Y_t - I_t, \xi_t, A_t) \right] \right\} = 0 \quad (6b)$$

This condition has an intuitive interpretation. In the special case of perfect certainty, this is nothing but a standard equation describing price as a mark-up over marginal cost like equation (6a). With uncertainty, it can be interpreted as a weighted average of price mark-ups over marginal cost. This expected value is equal to zero. With price-pre-setting, the firm is committed to supply according to the realized demand. Hence, the realization of shocks will affect actual output, with negative shocks leading to excess capacity and positive shocks to over-capacity.¹

Our model predicts that the mark-ups of the producers who pre-set their prices will be counter-cyclical. Negative demand shocks will induce the flex-price firms to adjust their prices downward, attracting demand away from, and thus lowering the marginal costs and jacking up the price mark-ups of fix-price firms.

Given p_{1t} and p_{2t} , the aggregate price index in equation (1) can be rewritten as:

$$P_t = \left[\gamma p_{1t}^{1-\theta} + (1-\gamma) p_{2t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (1')$$

¹ Woodford (2003) assumes the Calvo price-setting framework. He derives an aggregate supply block, consisting of multiple dynamic equations. Our assumed price setting framework yields a single-equation aggregate supply relationship. This simple form focuses attention on the coefficient of the potential output variable in the aggregate supply relationship.

2.2 The Labor Market

The market for each type of goods-specific skill of labor service is characterized by workers as wage-takers and producers as wage-makers, as in the monopsony case. Figure 1 describes equilibrium in one such market. The downward-sloping marginal-productivity curve is the demand for labor. Supply of labor, S_h , is implicitly determined by the utility-maximizing condition for h , i.e.; see equation (2). The upward-sloping marginal factor cost curve is the marginal cost change from the producer point of view. It lies above the supply curve because, in order to elicit more hours of work, the producer has to offer a higher wage not only to that (marginal) hour but also to all the (intra-marginal) existing hours. Equilibrium employment occurs at a point where the marginal factor costs is equal to the marginal productivity. Equilibrium wage is given by B, with the worker's real wage marked down below her marginal product by a distance AB.²

Full employment obtains because workers are offered a wage according to their supply schedule. This is why our Phillips curve will be stated in terms of excess capacity (product market version) rather than unemployment (labor market version).

In fact, the model can also accommodate unemployment by introducing a labor union, which has monopoly power to bargain on behalf of the workers with the monopsonistic firms over the equilibrium wage. In such case, the equilibrium wage will lie somewhere between S_h and $M P_h$, and unemployment can arise — so that the labor market version of the Phillips curve can be derived as well. To simplify the analysis, we assume in this paper that the workers are wage-takers.

2.3 Investment

Profit maximization by producer j yields a first-order condition for investment

$$I'[k_{t+1}(j)/k_t(j)] = E_t \left(\frac{1}{1+i_t} \right) \frac{P_{t+1}}{P_t} \{ q_{t+1}(j) + (k_{t+2}(j)/k_{t+1}(j)) I'[k_{t+2}(j)/k_{t+1}(j)] - I[k_{t+2}(j)/k_{t+1}(j)] \},$$

where, q is the shadow value (because there is not any rental market) of an additional unit of capital.³ It is written in a Bellman-like equation as follows.

$$q_t(j) = w_t(j) \left(\frac{f(A_t h_t(j)/k_t(j)) - (A_t h_t(j)/(k_t(j))) - (A_t h_t(j)/k_t(j))}{A_t f'(A_t h_t(j)/k_t(j))} \right) \quad (7)$$

Note that if a rental market were to exist, $q_t(j)$ in equation (7) will be equal to the marginal productivity of capital, as expected.

² In the limiting case where the producers behave perfectly competitively in the labor market, the real wage becomes equal to the marginal productivity of labor and the marginal cost of labor curve is not sensitive to output changes. Thus, with a constant mark-up $\frac{\theta}{\theta-1}$, the Phillips curve becomes flat, i.e., no relation exists between inflation and excess capacity.

³ See Woodford (2003, chapter 5).

2.4 Potential Output

Potential output is defined as the price-flexible level of output. In the case where all prices are fully flexible (i.e., $\gamma = 1$), output will attain its natural level, Y_t^n , implicitly defined by

$$1 = \mu s(K_t, Y_t^n, Y_t^n - I_t; \xi_t, A_t)$$

Note that Y_t^n depends on the capital stock, K_t , and on current investment, I_t .

3. Aggregate Supply and Investment

This section derives the aggregate supply relationship. It has its roots in the expectations-augmented Phillips curve of the kind hypothesized by Friedman (1968) and Phelps (1970) for both closed and open economies. (See also Ball, Mankiw and Romer 1988 and Roberts 1995.)

In order to obtain a tractable solution, we log-linearize the equilibrium conditions around the steady state. In the steady state $\xi_t = 0$ and $A_t = \bar{A}$. We assume that $\beta(1+r) = 1$ and $i = r$ (i.e., inflation is set equal to zero in the shock-free steady state). Define $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right) \simeq \frac{x_t - \bar{x}}{\bar{x}}$ as the proportional deviation of any variable x_t from its deterministic steady state value \bar{x} . We can then log-linearize the model equations around the deterministic steady state equilibrium.

$$\hat{s}_t - \hat{s}_t^n = \omega(\hat{y}_t - \hat{y}_t^n) + \sigma^{-1}(\hat{y}_t - \hat{y}_t^n) - \sigma^{-1}\hat{I}_t - \omega_p(\hat{k}_t - \hat{k}_t^n) \quad (8)$$

where,

$$\omega = \omega_w + \omega_p,$$

$$\omega_w = \frac{v_{hh}}{v_h}, \omega_p = -\frac{f''}{f'^2}$$

and

$$\sigma = -\frac{cu_{cc}}{u_c}$$

Log-linearizing the two price-setting equations [(6a), (6b)], the investment rule, [equation (7)], and using equation (8), yields:

$$\log(P_{1t}) = \log(P_t) + (\omega + \sigma^{-1})(\hat{y}_{1t} - \hat{Y}_t^n) - \sigma^{-1}\hat{I}_t - \omega_p(\hat{k}_{1t} - \hat{k}_t^n) \quad (6a')$$

$$\log(P_{2t}) = E_{t-1}[\log(P_t) + (\omega + \sigma^{-1})(\hat{y}_{2t} - \hat{Y}_t^n) - \sigma^{-1}\hat{I}_t - \omega_p(\hat{k}_{2t} - \hat{k}_t^n)] \quad (6b')$$

Define, as standard, the inflation rate by, $\pi_t = \ln(P_t / P_{t-1})$, so that $\pi_t - E_{t-1}(\pi_t) = \log(P_t) - E_{t-1} \log(P_t)$.

Equations (6a') and (6b') can be combined to obtain the link between inflation surprise, and fluctuations in output gap, investment, and the stock of capital, as follows.

$$\pi_t - E_{t-1}(\pi_t) = \frac{\gamma}{(1-\gamma)} \left[\left(\frac{\omega + \sigma^{-1}}{1 + \theta\omega} \right) (\hat{Y}_t - \hat{Y}_t^n) - \frac{\sigma^{-1}}{1 + \theta\omega} \hat{I}_t - \frac{\omega_p}{1 + \theta\omega} \hat{K}_t \right] \quad (9)$$

Real marginal costs increase with a rise in the current level of economic activity; thus, they are affected by fluctuations in the aggregate capital stock and investment demand. Consequently, the fluctuations in investment and the capital stock are negatively correlated with surprise inflation, holding constant the output gap.

4. Aggregate Supply and Potential Output

Potential output is defined as the hypothetical output level that would result if prices and wages are completely flexible but other distortions, like taxes and imperfect competition, are left in place. That is potential output is normally lower than the efficient output level (the Pareto-optimal output level).

Log-linearizing the equation and rearranging yields:

$$\hat{Y}_{t+1}^n = \frac{1}{\omega + \sigma^{-1}} \left[\omega_p \hat{K}_{t+1}^n + \sigma^{-1} I_{t+1}^n + u_t \right] \quad (10)$$

where

$$u_t = \frac{\zeta_\xi}{\xi} d\xi_t + \frac{\zeta_A}{\xi} dA_t,$$

is a shock term.

$$\omega_p \hat{K}_{t+1}^n + \sigma^{-1} I_{t+1}^n = (\omega + \sigma^{-1}) \hat{Y}_{t+1}^n - u_t$$

Rearranging terms in equation (10), and substituting the resulting relationship into equation (9) yields the aggregate supply relationship we are seeking, as follows.

$$\pi_t - E_{t-1}(\pi_t) = \frac{\gamma}{(1-\gamma)} \left[\left(\frac{\omega + \sigma^{-1}}{1 + \theta\omega} \right) (\hat{Y}_t - \hat{Y}_t^n) - \frac{\omega + \sigma^{-1}}{1 + \theta\omega} \hat{Y}_t^n - u_t \right] \quad (11)$$

Equation (11) demonstrates that the fluctuations in potential output are negatively correlated with inflation surprises for a given realization of the shock, and a given level of the output gap. This means that the inflation-output trade-off is three-dimensional: among inflation output gaps and potential output.

In the absence of investment, however, the aggregate supply relationship reduces to the conventional relationship between surprise inflation and the output gap:

$$\pi_t - E_{t-1}(\pi_t) = \frac{\gamma}{(1-\gamma)} \left[\left(\frac{\omega + \sigma^{-1}}{1 + \theta\omega} \right) (\hat{Y}_t - \hat{Y}_t^n) \right] \quad (12)$$

Therefore the inflation-output trade-off becomes two-dimensional: between inflation rates and output gaps.

5. Conclusion

The New-Keynesian aggregate supply derives from micro-foundations an inflation-dynamics model very much like the tradition in the monetary literature. Inflation is primarily affected by: (i) economic slack; (ii) expectations; (iii) supply shocks; and (iv) inflation persistence.

In the open-economy literature, terms other than the output gap have already appeared in the aggregate supply function, once one substitutes the marginal costs by its determinants. Gali and Monaceli (2003) demonstrate that in the open economy case, while the marginal rate of substitution relates the real exchange rate with a consumption basket containing domestically produced and imported goods, the marginal product depends on domestic production. This drives a wedge among the CPI and the GDP deflators, leading to an additional term in the aggregate supply, the real exchange rate.

I conjecture that because potential output improves the inflation-output gap trade-off, the task of the monetary authority, which trades off inflation and output gaps is facilitated if they target potential output, as well as the inflation fluctuations and the output gaps. Therefore, a Taylor-like interest rule which targets not only the fluctuations in inflation rates and output gaps, but also the fluctuations in potential output is bound to raise the measure of consumer's welfare.

The conjecture is the subject of future research based on recent literature, which develops monetary policy rules from the measure of welfare of the representative consumer. Woodford (2003) reduces a dynamic optimization problem of the choice of the desired monetary rule, based on the representative consumer's welfare, into a linear-quadratic optimization problem. Extending this framework to an analysis of the interactions between monetary and fiscal policies, Benigno and Woodford (2003), the constrained-efficient allocation is achieved when output is equal to the would be price-flex no-wedge distortions level, with a zero price dispersion (or other costs of inflation), and real value of the initial government nominal debt equal to the deterministic steady state real debt.

The broader-scope aggregate supply that is derived in this paper also has implications for the empirical literature. The New Keynesian Phillips curve has also attracted renewed interest in much of the empirical research. Gali and Gertler (1999) and Gali, Gertler and Lopez-Salida (2001) present evidence that US and Europe inflation dynamics are consistent with a variant of the New Keynesian Phillips Curve. Potential output plays a role in the analysis only to the extent that it is needed to measure the output gap. Thus, the revised version of the aggregate supply relationship, which includes potential output as one of the inflation-output trade-off's determinants has an obvious implication for the empirical literature, as well. A new estimation strategy to come to grips with the broader scope of inflation-output trade off is warranted.

References

- Ball, Laurence (1994), "What Determines the Sacrifice Ratio?" in N. Gregory Mankiw, ed., *Monetary Policy*, University of Chicago Press.
- Ball, Laurence, N. Greg Mankiw and David Romer (1988), "The new Keynesian economics and the output-inflation trade-off," *Brookings Papers on Economic Activity*, 19: 1-65.
- Benigno, Pierpaolo and Michael Woodford (2003), "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," NBER Working Paper No.9905, Cambridge MA: National Bureau of Economic Research.
- Blanchard, Olivier and Nobu Kiyotaki (1987), "Monopolistic competition and the effects of aggregate demand," *American Economic Review*, 77: 647-66.
- Cohen, Daniel and Emmanuel Farhi (2001), "The Phillips curve across the Atlantic: It is the price curves that differ," CEPR Discussion Paper No.3100, December.
- Dixit, Avinash and Joseph E. Stiglitz (1977), "Monopolistic competition and optimum product diversity," *American Economic Review*, 67: 297-308.
- Friedman, Milton (1968), "The role of monetary policy," *American Economic Review*, 58: 1-17.
- Gali, Jordi and Mark Gertler (1999), "Inflation Dynamics: A Structural Econometric Approach," *Journal of Monetary Economics*, 44(2): 195-222.
- Gali, Jordi, Mark Gertler and David Lopez-salido (2001), "European Inflation Dynamics," *European Economic Review*, 45(7): 1237-70.
- Gali, Jordi and T. Monacelli (2003), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," NBER Working Paper No.8905, Cambridge MA: National Bureau of Economic Research.
- Gordon, Robert J. (1982), "Why Stopping Inflation may be Costly: Evidence from Fourteen Historical episodes," in Robert E. Hall, ed., *Inflation: Causes and Cures*, University of Chicago Press: 9-40.
- Khan, Mohsin S. and Senhadji, Abdelhak S. (2001), "Threshold Effects in the Relationship Between Inflation and Growth," *IMF Staff Papers*, 48:1-21.
- Lane, Philip R. (2001), "The new open economy macroeconomics: a survey," *Journal of International Economics*, 54: 235-66.
- Loungani, Prakash, Assaf Razin and Chi-wa Yuen (2001), "Capital mobility and the output-inflation tradeoff," *Journal of Development Economics*, 64: 255-74.
- Lucas, Robert E. (1973), "Some International Evidence on Output-Inflation Trade-offs," *American Economic Review*, 63: 326-34.
- Khan, Mohsin and Abdelhak Senhadji (2001), "Threshold Effects in the Relationship Between Inflation and Growth," *IMF Staff Papers*, 48(1): 1-21.
- Obstfeld, Maurice. and Kenneth Rogoff (1996), *Foundations of international macroeconomics*, Chapter 10, Cambridge: MIT Press.
- Okun, Arthur (1978), "Efficient Disinflationary Policies," *American Economic Review*, 68: 348-52.
- Phelps, Ned. S. (1970), *Microeconomic foundations of employment theory*, New York: Norton.
- Razin, Assaf and Chi-wa Yuen (2002), "The New Keynesian Phillips Curve: Closed Economy vs. Open Economy," *Economics Letters*, 75: 1-9.
- Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- Zhang, Lawrence Huiyan (2002), "Sacrifice Ratios with Long-Lived Effects," *Johns Hopkins University Working Paper*.

Figure 1. Labor Market Equilibrium

