

# Excessive Dollar Borrowing in Emerging Markets

## Balance Sheet Effects and Macroeconomic Externalities

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### Abstract

This paper shows that private borrowers in emerging markets take on excessive dollar debt as opposed to local currency debt because they fail to internalize that dollar debt increases macroeconomic volatility and exposes their economies to greater risk of financial crises.

In emerging markets, adverse shocks typically depreciate the exchange rate, thereby inflating the local currency value of dollar debts and reducing the working capital and net worth of dollar borrowing firms. In the presence of credit constraints, this leads to a decline in output. However, lower economic activity depreciates the exchange rate further, triggering a cycle of rising debt burden, falling output, and depreciating exchange rates, which is also known as ‘debt deflation process.’

Individual borrowers take the distribution of exchange rates as given, i.e. they do not internalize that lower economic activity depreciates the exchange rate, which in turn tightens credit constraints for all other borrowers. Therefore they undervalue the social costs of dollar debt and take on too much of it. We present a number of policy measures and ultimately advocate an unremunerated reserve requirement on dollar debt to correct the distortion.

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# 1 Introduction

In the aftermath of the East Asian crisis, researchers have investigated the causes underlying the observed financial turmoil. The literature on third generation currency crises (see e.g. Krugman, 1998, 1999) as well as a growing empirical literature (see e.g. Levy Yeyati, 2006) maintain that external dollar debt has played a major role in making emerging markets vulnerable to financial crises. These studies conclude that it would be efficient for countries to reduce their exposure to foreign currency debt so as to lower this risk.

From the standpoint of individual borrowers, however, the choice between local currency debt and dollar debt is a private insurance decision: exchange rates are counter-cyclical so local currency debt insures against aggregate fluctuations, but it commands an interest rate premium since international lenders are averse to emerging market risk. When borrowers choose the currency composition of their debt portfolio, they weigh the expected costs of fluctuations in consumption against any savings from the interest rate spread between the local currency and the dollar.

If borrowers have rational expectations and choose to take on the risk of holding dollar debt, why is this decision not socially efficient? In other words, why is it not efficient for countries to experience financial crises in some (unfortunate) states of nature?

The existing literature has offered three main categories of explanations. First, Eichengreen and Hausmann (1999) posit that markets for emerging market local currency debt simply do not exist. However, over the past decade a vibrant market in such bonds has developed (Burger and Warnock, 2006). A second explanation has been moral hazard, e.g. that firms take on risky dollar debt so as to take advantage of bailout guarantees (see Krugman, 1998). However, this view is contradicted by the high levels of dollar debt that can be observed in firms unlikely to be bailed out (Eichengreen and Hausmann, 1999). A third explanation (Caballero and Krishnamurthy, 2003) is limited domestic financial development. This results in socially excessive dollar borrowing if one of two conditions is met: either (i) firms do not know *ex ante* whether they will be net borrowers or net lenders in domestic financial markets during a crisis, which is often counter-factual, or (ii) the benchmark for social efficiency is an omnipotent social planner who can engage in compensatory transfers, rather than a constrained social planner.

This paper presents an alternative and complementary view. We show that individual firms fail to internalize the effects of their borrowing decisions on the tightness of external borrowing constraints, which are binding in low output states. Note that such pro-cyclical fluctuations in firms' *external* borrowing capacity lie at the heart of most third-generation models of currency crises (see e.g. Krugman, 1998, 1999; Chang and Velasco, 2001), underlining the importance of the externality that we identify.<sup>1</sup>

We show that domestic borrowers fail to internalize that the counter-cyclical payoffs of dollar debt raise not only the volatility of each agent's disposable income, but also

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<sup>1</sup>By contrast, the externality in Caballero and Krishnamurthy (2003) arises from the mis-pricing of dollar collateral in *domestic* financial markets. Furthermore, in our model decentralized agents' choice of dollar debt is socially excessive even compared to the more stringent benchmark of a constrained social planner who can control only those instruments that are also available to decentralized agents, and in particular who does not have access to compensatory transfers.

make aggregate demand, and by extension exchange rates, more volatile. Each agent takes the volatility of exchange rates as a given, when in fact his borrowing behavior has an important effect on such volatility. Higher exchange rate volatility means stronger depreciations and therefore a lower dollar value of domestic collateral in low output states. This tightens borrowing constraints in such states and increases the frequency and severity of financial crises.

International lenders only care about their private payoffs, not about the opportunity costs that binding borrowing constraints impose on borrowers. Therefore the price that they charge does not reflect the potential costs of binding collateral constraints. Misguided by the erroneous price signal, decentralized borrowers fail to internalize this negative externality to dollar debt and engage in excessive dollar borrowing.

The setting in which we analyze the problem is a small open emerging market economy with three time periods, labeled 0, 1 and 2. There are two goods, tradables and non-tradables, the relative price of which represents the real exchange rate.

In period 0, small domestic agents borrow from international lenders to finance investment. They have to allocate their debts between dollars and local currency, which are represented by the price of tradable and non-tradable goods, respectively. In period 1, domestic agents produce both tradable and non-tradable goods. An aggregate shock affects productivity in the tradable sector and by extension domestic income and aggregate demand in the emerging market.<sup>2</sup> The exchange rate adjusts in order to equilibrate aggregate demand and supply. In case of a negative shock, for example, aggregate demand for both non-tradable and tradable goods falls; by implication the price of non-tradables has to decline (i.e. the real exchange rate depreciates) so as to restore demand for non-tradables to the given supply and clear the market. As a result, the price of non-tradables is pro-cyclical and moves in parallel with aggregate demand, i.e. the real exchange rate appreciates in high states and depreciates in low states.<sup>3</sup> Consequently, the repayments on dollar debt (debt denominated in tradable goods) are high in low states and low in high output states, exacerbating the impact of aggregate shocks. By contrast, the repayments on local currency debt (debt denominated in non-tradable goods) move in parallel with aggregate demand, mitigating the impact of aggregate shocks and therefore providing excellent insurance against consumption risk.

If the interest rate on local currency equalled that on dollar debt, this insurance would be free and risk averse borrowers would only borrow in local currency (Korinek, 2007a). However, in emerging markets, lenders typically charge a risk premium on domestic currency debt because they are averse to emerging market currency fluctuations

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<sup>2</sup>This can be thought of e.g. as a productivity slowdown in the tradable sector, as experienced by Thailand in 1996/97, or as a devaluation of a trading partner or a competitor in export markets, which affected other East Asian countries after Thailand's devaluation and Argentina after Brazil's devaluation.

<sup>3</sup>We chose the given model of the real exchange rate solely for analytical convenience – as we will show below, the price of non-tradables will be linear in aggregate demand in our setup. Alternative models of the exchange rate would entail the same qualitative results, as long as the exchange rate depreciates on average in response to strong negative shocks. This condition is very likely to be satisfied in emerging markets, even if the official exchange rate regime is a peg, as demonstrated e.g. by Argentina in 2001/02.

(Dodd and Spiegel, 2005). Borrowers need to trade off this risk premium against the insurance benefits of local currency debt.

The counter-cyclical payoffs of dollar debt introduce an accelerator effect into the economy that raises the volatility of agents' disposable income, of aggregate demand, and by extension of exchange rates. In particular, the more dollar debt a country as a whole has contracted, the steeper the decline in aggregate demand in response to a negative shock of a given magnitude and the more the exchange rate depreciates.

International lenders' valuation of domestic collateral depends on the exchange rate. The more dollar debt the economy as a whole is exposed to, the larger the exchange rate depreciation and by extension the stronger the fall in the dollar value of collateral for a given shock size. This reduces the borrowing capacity of firms that own collateral denominated in local currency, i.e. it tightens borrowing constraints and makes financial crises more frequent and more severe. The price of local currency debt reflects the social cost of its direct insurance effects on agents' consumption, but not the indirect effect of more stable exchange rates on collateral values in crisis states. Decentralized agents thus fail to internalize the full social benefits of local currency debt or, by the same token, the full social costs of dollar debt. As a result they engage in excessive dollar borrowing from a social point of view.

Another way of viewing the externality is that if everybody in the economy had borrowed the socially efficient (low) amount of dollar debt, then each individual agent would have an incentive to take advantage of the resulting macroeconomic stability and borrow in dollars so as to save on the risk premium on local currency debt.

The paper discusses a number of policy remedies to correct the externality to dollar debt. We show that a tax on cross-border dollar borrowing that is proportional to the risk premium on local currency is superior to a constant tax on dollar debt, since the latter can lead to multiple equilibria, of which only one is socially efficient. Such a proportional tax can be implemented e.g. by imposing unremunerated reserve requirements in local currency on dollar lenders, similar to the regulations that Chile implemented in the 1990s to discourage excessive capital inflows (Gallego et al., 2002).

We also compare the insurance properties of local currency denominated debt with GDP-linked dollar debt. While the two instruments offer similar insurance characteristics in normal times, local currency debt better protects against financial crises, because consumption and the exchange rate decline more strongly than GDP in crisis states, since the current account balance increases. The relative insurance benefits and associated externalities of various forms of cross-border capital flows are analyzed in more detail in Korinek (2007b).

In methodology, this paper is most closely related to the literature on the financial accelerator in macroeconomic models (see for example Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). Building on these ideas, Krugman (1999) described financial crises as involving a debt deflation mechanism of falling exchange rates, tightening collateral constraints and falling output. Mendoza (2005) demonstrates that such a mechanism can quantitatively account for the dynamics of emerging market business cycles, i.e. for long periods of relatively smooth cycles interspersed by infrequent financial crises when financial constraints become binding.

These two works take the use of dollar debt as given. Our contribution to this literature is to endogenize borrowers' choice of debt denomination and to show that individual agents' failure to internalize the effects of the financial accelerator during crises creates an externality that leads to excessive dollar borrowing.

The remainder of our paper is organized as follows. Section 2 introduces a benchmark model with perfect risk markets and no borrowing constraints. We show that higher dollar debt increases the volatility of individual consumption, aggregate demand, and of the real exchange rate for a given volatility of the underlying fundamentals, but that decentralized agents' choice of dollar debt is efficient. In section 3, we introduce borrowing constraints that depend on collateral and show the central externality result of the paper, i.e. that agents in the decentralized equilibrium do not internalize the effects of their choice of debt denomination on the value of their collateral in low states and therefore engage in excessive dollar borrowing. Section 4 presents some comparative statics, such as the effects of changes in lenders' risk aversion, and in the economy's riskiness. Section 5 discusses a number of policy implications. Finally, section 6 concludes.

## 2 Benchmark Model

### 2.1 Analytical Environment

In our analytical model, we analyze an emerging market represented by a small open economy. We assume that there are three time periods, labeled by  $t = 0, 1, 2$ , and two sets of agents: First, the emerging market economy is inhabited by a continuum of domestic agents of mass 1. Secondly, there is a continuum of international lenders, who are large in comparison to the emerging market. We discuss each in detail below.

There are two perishable goods in the economy, tradable goods  $T$  and non-tradable goods  $N$ . Tradable goods can be moved costlessly across borders and can be borrowed or lent abroad. Non-tradable goods have to be consumed in the domestic economy in the period of their production. The prices of the two goods are denoted by  $p_{T,t}$  and  $p_{N,t}$ . We choose tradables as the numeraire so that  $p_{T,t} \equiv 1$ . By implication,  $p_{N,t}$  represents the price of non-tradables relative to tradables, which is a measure of the real exchange rate. The economy is subject to a random productivity shock that depends on the state of nature  $\omega \in \Omega$ , where  $\Omega$  is the set of potential outcomes. The shock is realized at the beginning of period 1 and is observed by all agents.

### 2.2 Domestic Agents

Domestic agents are risk averse and obtain utility from consuming tradable goods  $C_{T,t}$  and non-tradable goods  $C_{N,t}$  according to the utility function

$$U = E \left\{ \sum_{t=1}^2 \beta^t \hat{u}(C_t) \right\} \quad \text{where} \quad C_t = C_{T,t}^\sigma C_{N,t}^{1-\sigma}$$

$E$  is the expectations operator,  $\beta$  represents agents' discount factor, and  $\hat{u}(\cdot)$  is their period utility function, which satisfies  $\hat{u}'' < 0 < \hat{u}'$ .  $C_t$  is an aggregator of tradable and non-tradable consumption, where the parameters  $\sigma$  and  $1 - \sigma$  are the expenditure shares of tradables and non-tradables in agents' optimal consumption bundle.<sup>4</sup>

We assume that agents start out with an initial amount of wealth (or debt, for negative values) of  $W_0$ . Agents need to invest a fixed amount of tradables  $\bar{I}$  in both periods 0 and 1.<sup>5</sup> As a return on their investment, they produce a bundle of tradable and non-tradable goods of  $(Y_{T,t}^\omega, \bar{Y}_N)$ . The fixed amounts of production capture that production factors cannot be re-allocated instantaneously. Instead, relative prices have to adjust, i.e. the real exchange rate appreciates or depreciates.

The output of tradables  $Y_{T,1}^\omega$  is subject to the aggregate productivity shock  $\omega$  in period 1. This shock can stem e.g. from a general slowdown in productivity growth in that sector or from an adverse terms-of-trade shock, such as a neighboring country's devaluation or a fall in the world market price of the country's main exports. The expected value of first period output equals  $E[Y_{T,1}^\omega] = \bar{Y}_T$ . For simplicity, suppose that tradable production in period 2 is constant and also equals to  $Y_{T,2}^\omega = \bar{Y}_T$ .

In the given representative agent framework, there is no scope for a domestic credit market, since agents are all identical and have a common, strictly concave utility function. They are either all lenders or all borrowers. However, agents can borrow from abroad: they have a choice of foreign currency debt  $F_t$  or local currency debt  $L_t$ . Both kinds of debt are normalized to yield one unit of tradables in period  $t - 1$  from abroad.<sup>6</sup> However, they differ in the amounts to be repaid in period  $t$ . Foreign currency debt  $F_t$  mandates the repayment of an unconditional return of  $R^*F_t$  tradables in period  $t$ , where  $R^*$  is determined on world markets. The return  $R_{L,t}^\omega$  on local currency debt is indexed to the real exchange rate, i.e. is proportional to the price of non-tradables:  $R_{L,t}^\omega \cong p_{N,t}^\omega$ . Taking on an amount of  $L_t$  of local currency debt in period  $t - 1$  thus entails an obligation to repay  $R_{L,t}^\omega L_t$  in period  $t$ , where  $R_{L,t}^\omega$  depends on the realization of the real exchange rate.

In our benchmark model, we assume that there are no capital market imperfections, and that agents can take on their optimal amounts of borrowing in foreign and local currency while maintaining strictly positive consumption in all states of the world, i.e. there is no bankruptcy. In the following section, we will introduce borrowing constraints that limit the amount of borrowing that agents can take on depending on the state of their balance sheets.

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<sup>4</sup>The Cobb-Douglas form in which tradable and non-tradable goods enter utility implies that the real exchange rate is a linear function of the output shock. This in turn guarantees that the two assets, dollar debt and local currency debt, span the entire state space and therefore that risk markets are complete for domestic agents. While our externality result holds as long as both tradables and non-tradables are normal goods, the chosen specification allows us to derive the result analytically.

<sup>5</sup>Endogenizing the amount of investment would not change the basic results – agents would equalize the expected marginal product of investment to the cost of capital on international markets. In the presence of binding borrowing constraints, agents would reduce not only current consumption, but also investment, and would thereby have negative effects on future output. This could potentially increase the size of the externality even further.

<sup>6</sup>Naturally, foreigners can only provide domestic agents with tradable goods, since non-tradables cannot be moved across borders.

We can summarize domestic agents' optimization program in the following form:

$$\max_{\{C_t^\omega, C_{T,t}^\omega, C_{N,t}^\omega, F_t^\omega, L_t^\omega\}_{t=1,2}} E \left\{ \sum_{t=1}^2 \beta^t \hat{u}(C_t^\omega) \right\} \quad (1)$$

$$\text{s.t. } C_t^\omega = [C_{T,t}^\omega]^\sigma [C_{N,t}^\omega]^{1-\sigma} \quad (2)$$

$$\bar{I} \leq W_0 + F_1 + L_1 \quad (3)$$

$$C_{T,1}^\omega + p_{N,1}^\omega C_{N,1}^\omega + \bar{I} = Y_{T,1}^\omega + p_{N,1}^\omega \bar{Y}_N - R^* F_1 - R_{L,1}^\omega L_1 + F_2^\omega + L_2^\omega \quad (4)$$

$$C_{T,2}^\omega + p_{N,2}^\omega C_{N,2}^\omega = \bar{Y}_T + p_{N,2}^\omega \bar{Y}_N - R^* F_2^\omega - R_{L,2}^\omega L_2^\omega$$

## 2.3 International Lenders

International capital markets are populated by a continuum of identical competitive lenders. We assume that international lenders are large in comparison to the small emerging market economy, so that the equilibrium in international capital markets is exogenous to events in the emerging market economy.

As a result, lenders' pricing kernel  $M_t^\omega$  is given exogenously.<sup>7</sup> In equilibrium, the returns  $R_{i,t}^\omega$  on any asset  $i$  in which international lenders can invest have to satisfy the standard pricing condition

$$E \{ R_{i,t}^\omega M_t^\omega \} = 1 \quad (5)$$

This pins down the risk-free return that international lenders demand on dollar debt as  $R^* = 1/E[M_t^\omega]$ , which we assume constant over time.

## 2.4 Definition of Equilibrium

We take the equilibrium in international capital markets as given, since we assumed that the emerging market economy is small compared to international markets. Given international lenders' pricing kernel  $M_t^\omega$  and risk free return  $R^*$ , an equilibrium in the emerging market economy is defined as

- an allocation  $(C_t^\omega, C_{T,t}^\omega, C_{N,t}^\omega, F_t^\omega, L_t^\omega)$  and
- prices and returns  $(p_{N,t}^\omega, R_{L,t}^\omega)$  for  $t = 1, 2$  and all  $\omega \in \Omega$
- which are consistent with international lenders' pricing condition (5)
- which satisfy domestic agents' optimization problem (1) given (2)

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<sup>7</sup>In appendix ?? we provide a simple example of a consumption-based asset pricing model that yields such a pricing kernel. However, for an appropriate definition of  $M_1^\omega$ , most asset pricing models can be captured by the following pricing condition. Other common terms used for  $M_t^\omega$  in the asset pricing literature are intertemporal marginal rate of substitution or stochastic discount factor.

- and which clear goods markets at all times  $t = 0, 1, 2$  and in all states  $\omega \in \Omega$ , i.e.

$$\begin{aligned}
&\text{for non-tradables:} && C_{N,t}^\omega = \bar{Y}_N \\
&\text{for tradables:} && \bar{I} = W_0 + F_1 + L_1 \\
&&& C_{T,1}^\omega + \bar{I} = Y_{T,1}^\omega - R^* F_1 - R_{L,1}^\omega L_1 + F_2^\omega + L_2^\omega \\
&&& C_{T,2}^\omega = \bar{Y}_T - R^* F_2^\omega - R_{L,2}^\omega L_2^\omega
\end{aligned}$$

## 2.5 Determination of Lenders' Equilibrium Returns

The emerging market economy under study is small, and so international lenders' supply of  $F_1$  and  $F_2^\omega$  is horizontal at  $R^*$  for domestic agents, up to the point where borrowing constraints start to bind.

Since there is no uncertainty in period 2, local and foreign currency debt  $L_2^\omega$  and  $F_2^\omega$  both yield the riskless return  $R^*$  and one of the two is redundant. We set w.l.o.g.  $L_2^\omega = 0$ .

The price at which international lenders are willing to supply local currency debt  $L_1$  in the first period can be determined from lenders' equilibrium pricing condition  $E[R_{L,1}^\omega M_1^\omega] = 1$ . Let us define  $\rho$  as a measure of the risk premium on local currency debt so that

$$(1 - \rho)E[R_{L,1}^\omega] = R^*$$

As discussed above the payoff  $R_{L,1}^\omega$  is linear in the real exchange rate, and our definition of  $\rho$  implies that it is in expectation  $\frac{R^*}{1-\rho}$ . According to simple no-arbitrage considerations we can denote the state contingent return on local currency debt as

$$R_{L,1}^\omega = \frac{p_{N,1}^\omega}{E[p_{N,1}^\omega]} \cdot \frac{R^*}{1 - \rho} \quad (6)$$

Substituting this expression into lenders' equilibrium pricing condition  $E[R_{L,1}^\omega M_1^\omega] = 1$ , we find that

$$\rho = -R^* \text{Cov} \left( \frac{p_{N,1}^\omega}{E[p_{N,1}^\omega]}, M_1^\omega \right) = -\text{Cov} \left( \frac{p_{N,1}^\omega}{E[p_{N,1}^\omega]}, \frac{M_1^\omega}{E[M_1^\omega]} \right) \quad (7)$$

This capture the standard result that international lenders do not mind holding risk that is uncorrelated to their pricing kernel  $M_1^\omega$ . However, they do require a higher return for holding risk that is negatively correlated with their pricing kernel  $M_1^\omega$ .

If lenders were risk-neutral, i.e. the pricing kernel  $M_1^\omega$  was constant, or if the real exchange rate was uncorrelated to  $M_1^\omega$ , then  $\rho = 0$  and lenders would supply any finite amount of local currency debt at actuarially fair prices. On the other hand, if lenders are risk-averse, i.e. the pricing kernel is a non-degenerate random variable, and if the real exchange rate  $p_{N,1}^\omega$  is negatively correlated with the pricing kernel, i.e. on average depreciated when lenders value consumption highly and appreciated when they value consumption less, then lenders demand a positive risk premium  $\rho > 0$ . Empirically, there is a positive risk premium on local currency debt from emerging markets (see e.g. Dodd and Spiegel, 2005). For the rest of our analysis we thus assume that lenders' pricing kernel is such that  $\rho > 0$ .

## 2.6 Domestic Agents in Competitive Equilibrium

In competitive equilibrium, representative agents in the emerging market economy solve maximization problem (1). We can substitute the consumption aggregator  $C_t^\omega$ , the constraint for tradable consumption  $C_{T,t}^\omega$  and for first period dollar borrowing  $F_1$  to arrive at the following maximization problem:

$$\max_{C_{N,t}^\omega, L_1, F_2^\omega} E \left\{ \hat{u} \left( [Y_{T,1}^\omega + p_{N,1}^\omega (\bar{Y}_N - C_{N,1}^\omega) - \bar{I} + R^* (W_0 - \bar{I}) + L_1 (R^* - R_{L,1}^\omega) + F_2^\omega]^\sigma [C_{N,1}^\omega]^{1-\sigma} \right) \right. \\ \left. + \beta \hat{u} \left( [\bar{Y}_T + p_{N,2}^\omega (\bar{Y}_N - C_{N,2}^\omega) - R^* F_2^\omega]^\sigma [C_{N,2}^\omega]^{1-\sigma} \right) \right\} \quad (8)$$

In the decentralized equilibrium, individual agents take the aggregate price level  $p_{N,t}^\omega$  of non-tradables as well as the returns  $R^*$  and  $R_{L,1}^\omega$  as given. We obtain the following first-order conditions:

$$\begin{aligned} \text{FOC}(C_{N,t}^\omega): \hat{u}'(C_t^\omega) \left( \sigma [C_{T,t}^\omega]^{\sigma-1} (-p_{N,1}^\omega) [C_{N,t}^\omega]^{1-\sigma} + [C_{T,t}^\omega]^\sigma (1-\sigma) [C_{N,t}^\omega]^{-\sigma} \right) &= 0 \\ \text{FOC}(L_1): E \left\{ \hat{u}'(C_1^\omega) \sigma [C_{T,1}^\omega]^{\sigma-1} [C_{N,1}^\omega]^{1-\sigma} [R^* - R_{L,1}^\omega] \right\} &= 0 \\ \text{FOC}(F_2^\omega): \hat{u}'(C_1^\omega) \sigma [C_{T,1}^\omega]^{\sigma-1} [C_{N,1}^\omega]^{1-\sigma} &= \beta R^* \hat{u}'(C_2^\omega) \sigma [C_{T,2}^\omega]^{\sigma-1} [C_{N,2}^\omega]^{1-\sigma} \end{aligned}$$

The first expression relates the consumption of non-tradables and tradables in each period. The second condition equalizes the expected marginal utility provided by domestic currency debt  $L_1$  to its marginal cost. The third condition describes the intertemporal tradeoff between first- and second-period consumption.

We employ a recursive solution strategy: First, we describe the determination of the exchange rate determined implied by  $\text{FOC}(C_{N,t}^\omega)$  and use  $\text{FOC}(F_2^\omega)$  to solve for the equilibrium in periods 1 and 2, given a choice of debt composition  $L_1$  in period 0. Then, we use this solution to analyze agents' optimal choice of local currency debt in period 0.

### First Order Condition on $C_{N,t}^\omega$ : Determination of the Real Exchange Rate

The first order condition on  $C_{N,t}^\omega$  determines equilibrium in the market for non-tradable goods. In our setup, the domestic supply of non-tradable goods is fixed at  $\bar{Y}_N$ , a modeling assumption that reflects that production takes time and output cannot be adjusted instantaneously. Market clearing for non-tradable goods thus always requires that demand equals the given supply, i.e.  $C_{N,t}^\omega = \bar{Y}_N$ . Changes in market conditions thus affect prices rather than quantities in the non-tradable sector. Tradable goods, on the other hand, can be imported or exported if agents borrow or lend abroad.

If we substitute the market clearing condition for non-tradable goods into the first order condition on  $C_{N,t}^\omega$ , we find

$$p_{N,t}^\omega = \frac{1-\sigma}{\sigma} \cdot \frac{C_{T,t}^\omega}{\bar{Y}_N} = \varsigma \cdot \frac{C_{T,t}^\omega}{\bar{Y}_N} \quad (9)$$

In equilibrium the real exchange rate  $p_{N,t}^\omega$  is proportional to the economy's absorption of tradables  $C_{T,t}^\omega$ .<sup>8</sup> (To save on notation we define  $\varsigma = \frac{1-\sigma}{\sigma}$ .)

If e.g. a negative shock to agents' production of tradables  $Y_{T,1}^\omega$  occurs, tradable goods become scarcer and non-tradable goods relatively more abundant. For equilibrium to be restored, the relative price of tradables has to rise, i.e.  $p_{N,1}^\omega$  falls – the real exchange rate depreciates. Similarly, if tightening external borrowing constraints force agents to reduce their borrowing, they have to cut back on their tradable consumption  $C_{T,1}^\omega$  and export more (or import fewer) tradables, whereas the supply of non-tradable goods remains constant (non-tradable production cannot be shipped overseas) and the same quantity has to be consumed. Equilibrium again requires that the relative price of non-tradables  $p_{N,1}^\omega$  falls.

We can apply condition (9) that the real exchange rate is linear in tradable consumption to expressions (6) and (7) for the return on local currency debt  $R_{L,1}^\omega$  and the risk premium  $\rho$  to obtain

$$R_{L,1}^\omega = \frac{R^*}{1-\rho} \cdot \frac{C_{T,1}^\omega}{E[C_{T,1}^\omega]} \quad (10)$$

$$\rho = -R^* \text{Cov} \left( \frac{C_{T,1}^\omega}{E[C_{T,1}^\omega]}, M_1^\omega \right) \quad (11)$$

The first expression captures that the payoff of local currency debt is a linear function of domestic agents' consumption of tradable goods. This illustrates that local currency debt provides excellent insurance against consumption risk: domestic agents have to pay back more in precisely the states that their consumption is high, and pay back less when their consumption is low.

The second equation expresses lenders' risk premium as a function of the covariance of domestic tradable consumption, which drives the exchange rate, with lenders' pricing kernel  $M_1^\omega$ . The more domestic tradable consumption is negatively correlated with lenders' risk factors, the higher the risk premium that lenders demand as a compensation for taking on local currency debt.

In our further analysis below, we will be able to greatly save on notation by employing two transformations: Firstly, we substitute the utility function  $\hat{u}(\cdot)$  by the following function:

$$u(C_T) = \hat{u}(C_T^\sigma C_N^{1-\sigma}) = \hat{u}(C_T^\sigma \bar{Y}_N^{1-\sigma}) \quad (12)$$

where we take advantage of the fixed supply and the market clearing condition for

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<sup>8</sup>The simple linear relationship stems from the Cobb-Douglas aggregator (2). For other utility functions, we would obtain a non-linear but still monotonic relationship between  $p_{N,t}^\omega$  and  $C_{T,t}^\omega$ , so long as both tradable and non-tradable goods are normal. The real depreciation mechanism and the externality result of this paper are thus robust to any standard utility function. However, if (9) is non-linear and there are more than two shocks, risk markets would no longer be complete. This creates an additional externality that would warrant exchange rate intervention by the social planner. We will touch upon this issue in the following subsections, but we leave a more detailed analysis to future research.

non-tradables  $C_{N,t}^\omega = \bar{Y}_N$ . Secondly, we employ the transformation

$$N = L_1 \cdot \frac{R^*}{(1 - \rho)E[C_{T,1}^\omega]}$$

$L_1$  measures the amount of local currency debt in units of tradable goods. Local currency debt provides insurance against consumption risk, so we can think of  $N$  roughly as the amount of iNsurance that agents take on relative to their expected level of consumption.

In conjunction with condition (10) for the return on local currency debt this simplifies expression (4) for agents' first-period consumption to

$$C_{T,1}^\omega = Y_{T,1}^\omega - (1 + R^*)\bar{I} + R^*W_0 - N [C_{T,1}^\omega - (1 - \rho)E[C_{T,1}^\omega]] + F_2^\omega \quad (13)$$

This formulation illustrates that, for a given amount of total borrowing, raising  $N$ , i.e. increasing the amount of local currency debt at the expense of dollar debt, is equivalent to swapping the risky consumption stream  $C_{T,1}^\omega$  against lenders' certainty equivalent  $(1 - \rho)E[C_{T,1}^\omega]$ .

### First Order Condition on $F_2^\omega$ : Euler Equation

The first order condition on  $F_2^\omega$  is the Euler equation that determines agents' optimal intertemporal allocation across periods 1 and 2. Using the transformed utility function from above, it can be represented as

$$\text{FOC}(F_2^\omega): u'(C_{T,1}^\omega) = \beta R^* u'(C_{T,2}^\omega)$$

Making the standard assumption that  $\beta R^* = 1$ , this condition implies that  $u'(C_{T,1}^\omega) = u'(C_{T,2}^\omega)$ , i.e. domestic agents set  $F_2^\omega$  in every state such as to smooth out the shock and perfectly equalize consumption over the two periods:

$$C_{T,1}^\omega = C_{T,2}^\omega = \bar{Y}_T - R^*F_2^\omega$$

This results in an expression for the optimal amount of borrowing  $F_2^\omega$  of

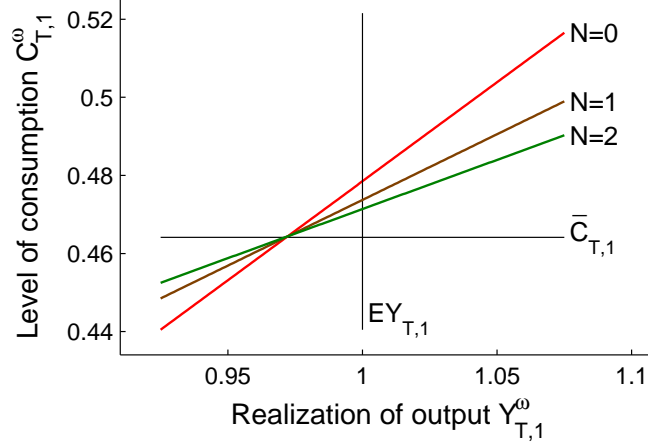
$$F_2^\omega = \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \quad (14)$$

Let us substitute this finding into agents' level of consumption (13) and solve for

$$C_{T,1}^\omega = \frac{\bar{Y}_T + R^*Y_{T,1}^\omega - R^*(1 + R^*)\bar{I} + (R^*)^2W_0 + (1 - \rho)NR^*E[C_{T,1}^\omega]}{1 + R^*(1 + N)} \quad (15)$$

This equation indicates how tradable consumption depends on the productivity shock  $Y_{T,1}^\omega$  for a given level of insurance  $N$ . It demonstrates that consumption is less sensitive to productivity shocks the more local currency debt domestic agents have taken on. This is also illustrated in figure 1.<sup>9</sup> The higher  $N$ , the flatter the lines in the figure,

<sup>9</sup> We used the following parameter values for this figure: We chose  $Y_{T,1}^\omega \sim N(\bar{Y}, s_Y^2)$  with  $\bar{Y}_T = \bar{Y}_N = 1$ ,  $s_Y = .05$ ,  $\sigma = .40$ ,  $R^* = 1.03$ ,  $\bar{I} = .25$  and  $W_0 = -.50$ .  $M_1^\omega$  was calibrated to be perfectly negatively correlated with  $Y_{T,1}^\omega$  and yield a risk premium of  $\rho = 3\%$  when all borrowing takes place in foreign currency.



**Figure 1:** The figure depicts the level of consumption  $C_{T,1}^\omega$  as a function of the output shock  $Y_{T,1}^\omega$  for several different levels of insurance  $N$ . As can be seen, more local currency debt makes consumption less sensitive to the shock: borrowers can trade in claims on their variable level of consumption  $C_{T,1}^\omega$  against lenders' certainty equivalent  $\bar{C}_{T,1} = (1 - \rho)E[C_{T,1}^\omega]$ , represented by the horizontal line. This 'rotates' consumption around  $\bar{C}_{T,1}$ . The vertical line represents the average level of output  $\bar{Y}_T$ .

i.e. the less consumption varies in response to the productivity shock  $Y_{T,1}^\omega$ . In addition, note that insurance is not free, and the payment of the risk premium reduces average consumption  $E[C_{T,1}^\omega]$ . Taking expectations of equation (15), we can solve for agents' average level of consumption

$$E[C_{T,1}^\omega] = \frac{(1 + R^*)\bar{Y}_T - R^*(1 + R^*)\bar{I} + (R^*)^2 W_0}{1 + R^*(1 + \rho N)} \quad (16)$$

The figure also illustrates another feature: from decentralized agents' perspective, taking on more local currency debt is equivalent to swapping the random payoff  $C_{T,1}^\omega$  against lenders' certainty equivalent  $(1 - \rho)E[C_{T,1}^\omega]$ . However, in general equilibrium the variables  $C_{T,1}^\omega$ ,  $\rho$ , and  $E[C_{T,1}^\omega]$  are all endogenous. Taking the derivative of expression (15) with respect to  $N$ , we find that

$$\frac{dC_{T,1}^\omega}{dN} = -\frac{R^*}{1 + R^*(1 + N)} [C_{T,1}^\omega - \bar{C}_{T,1}]$$

where

$$\bar{C}_{T,1} = \frac{d(1 - \rho)NE[C_{T,1}^\omega]}{dN} = (1 - \rho)E[C_{T,1}^\omega] - NE[C_{T,1}^\omega] \frac{d\rho}{dN} + (1 - \rho)N \frac{dE[C_{T,1}^\omega]}{dN} = (1 - \rho)E[C_{T,1}^\omega]$$

Increasing the amount of insurance  $N$  reduces borrowers' consumption in every state  $\omega$  by the random payoff  $C_{T,1}^\omega$  and replaces this by the constant payoff  $\bar{C}_{T,1}$ , which can be interpreted as the fixed increase in consumption that results in exchange for increasing the amount of insurance  $N$  and giving up some of the random  $C_{T,1}^\omega$ .

$\bar{C}_{T,1}$  consists of three parts: expected consumption discounted by the risk premium, the reduction in the risk premium for the pre-existing amount of insurance  $N$ , and the reduction in the payoff of the pre-existing  $N$  units of insurance that result from increasing  $N$ . Decentralized agents internalize the first term, i.e. they are aware that lenders charge a risk premium for taking on the risky component of  $C_{T,1}^\omega$ . However, individual agents do not internalize that their aggregate behavior affects the risk premium and the payoff of the existing units of insurance. In our benchmark model, it turns out that these two terms cancel out. Intuitively, the reason for this is that increasing  $N$  does not affect the relative costs and payoffs of the existing insurance arrangement: lenders do not care about whether they provide a defined contingent payoff through a small number of strongly fluctuating assets or a large number of mildly fluctuating assets. Mathematical details are provided in appendix B.

In the figure, the result is that borrowers' consumption seems to rotate around the point where  $C_{T,1}^\omega = \bar{C}_{T,1}$  when the amount of insurance  $N$  is raised: local currency debt increases income and consumption in low states with  $C_{T,1}^\omega < \bar{C}_{T,1}$  and reduces income and consumption in high states with  $C_{T,1}^\omega > \bar{C}_{T,1}$ .

The following proposition summarizes the implications of more insurance  $N$  for the variance of consumption and the real exchange rate as well as for the country's risk premium  $\rho$ :

**Proposition 1** *The more of its debt a country denominates in local currency, the lower the volatility of aggregate demand, consumption and the real exchange rate; as a result, the lower is the risk premium that international lenders' charge on local currency.*

**Proof.** Using expression (15) for  $C_{T,1}^\omega$ , we express the standard deviation of tradable consumption as

$$\text{Std}(C_{T,1}^\omega) = \frac{R^*}{1 + R^*(1 + N)} \cdot \text{Std}(Y_{T,1}^\omega) \quad (17)$$

This expression unambiguously decreases the higher the fraction of local currency-denominated debt, or insurance:

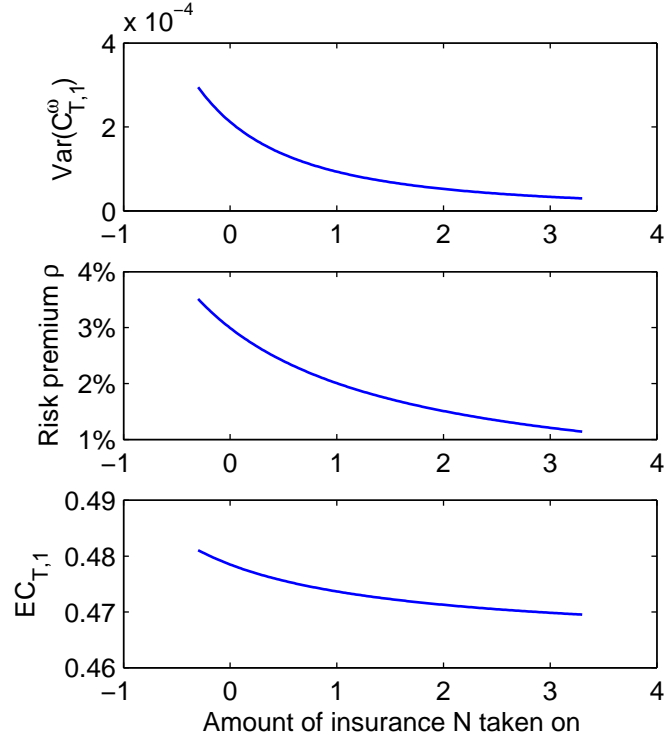
$$\frac{d\text{Std}(C_{T,1}^\omega)}{dN} = - \left( \frac{R^*}{1 + R^*(1 + N)} \right)^2 \cdot \text{Std}(Y_{T,1}^\omega)$$

Since the real exchange rate  $p_{N,1}^\omega = \frac{\$}{Y_N} \cdot C_{T,1}^\omega$  is linear in tradable consumption according to (9), the same results apply to the real exchange rate and the risk premium. ■

We have illustrated the functional relationship between the level of insurance  $N$  and (i) the variance of consumption  $\text{Var}(C_{T,1}^\omega)$ , (ii) the risk premium  $\rho$ , and (iii) average consumption  $E[C_{T,1}^\omega]$  in figure 2.

### First Order Condition on $L_1$ : Choice of Currency Denomination

Let us next focus on agents' decision regarding how much local currency debt to take on, i.e. on how much to pay for insurance. We substitute the return on local currency



**Figure 2:** The first pane of the figure depicts that the variance of consumption (and of the real exchange rate) is a decreasing function of the amount of insurance  $N$ . The second pane shows the dependence of the risk premium on the level of insurance  $N$ : as more insurance makes the country less risky, the risk premium falls. The third pane illustrates the fall in average consumption as more insurance is bought. This stems from the fact that insurance is costly – borrowers need to pay a risk premium on local currency debt.

$R_{L,1}^\omega = \frac{R^*}{1-\rho} \cdot \frac{C_{T,1}^\omega}{E[C_{T,1}^\omega]}$  from condition (10) into the first order condition on  $L_1$  and, using our transformed utility function, we obtain that

$$\text{FOC}(L_1): E \left\{ u'(C_{T,1}^\omega) [p_{N,1}^\omega - (1-\rho)E[p_{N,1}^\omega]] \right\} = 0$$

The product  $E[u'(C_{T,1}^\omega)X^\omega]$  represents borrowers' valuation of an asset with payoffs  $X^\omega$  in every state  $\omega$ . The condition above thus states that borrowers' valuation of one unit of local currency (payoff  $p_{N,1}^\omega$ ) has to equal the valuation of its certainty equivalent, i.e. of the expected value  $E[p_{N,1}^\omega]$  discounted by lenders' risk premium  $\rho$ . Using the fact that for any two random variables  $X^\omega$  and  $Y^\omega$ :  $E[X^\omega Y^\omega] = \text{Cov}(X^\omega, Y^\omega) + E[X^\omega]E[Y^\omega]$ , this yields that

$$\text{Cov} \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]}, \frac{p_{N,1}^\omega}{E[p_{N,1}^\omega]} \right\} = -\rho$$

Domestic agents hold local currency debt up to the point where the additional insurance effect per unit of local currency debt equals the cost of obtaining the insurance, which is the risk premium on local currency. The higher the risk premium, the less insurance agents thus take on. On the other hand, in the extreme case that  $\rho = 0$ , borrowers would perfectly insure against consumption risk by holding an infinite amount of local currency debt. As a result consumption would be constant and the covariance between the two variables would turn zero.

As we discussed above in equation (9), the real exchange in our model is linear in tradable consumption. This implies that

$$\frac{p_{N,1}^\omega}{E[p_{N,1}^\omega]} = \frac{C_{T,1}^\omega}{E[C_{T,1}^\omega]}$$

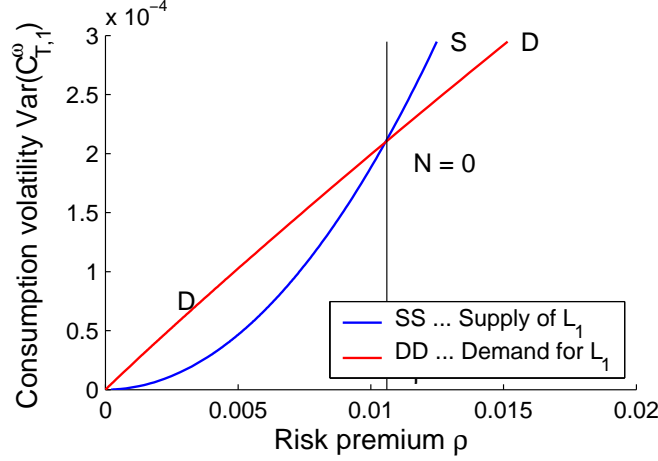
The covariance expression above then entails that agents insure to the point where the absolute value of the covariance between agents' marginal utility and their consumption equals the risk premium  $\rho$ . Approximating the utility function by a quadratic function<sup>10</sup> implies a marginal utility of  $u'(C_T) = \Gamma - C_T$ . This yields the condition

$$\text{Var}(C_{T,1}^\omega) = \rho E(C_{T,1}^\omega) E[u'(C_{T,1}^\omega)] \quad (18)$$

Domestic agents' optimal variance of tradable consumption is directly proportional to the risk premium on local currency debt, i.e. to the cost of insuring against volatility. In the case that  $\rho = 0$ , the optimal variance of tradable consumption would be zero, i.e. domestic agents would insure perfectly against consumption fluctuations.<sup>11</sup> In our analysis we assumed that lenders charge a positive risk premium  $\rho > 0$  on local currency debt. This implies that domestic agents' insure imperfectly. They opt for higher consumption volatility the higher the risk premium on local currency debt.

<sup>10</sup>We assume that  $u(C_T)$  is quadratic in tradable consumption, i.e. that  $u(C_T) = -\frac{1}{2}(\Gamma - C_T)^2$ , where  $\Gamma$  is a constant such that  $C_T < \Gamma$ . This implies that the original utility function  $\hat{u}(C)$  has to take the form  $\hat{u}(C) = -\frac{1}{2} \left[ \Gamma - \left( \frac{C}{\bar{Y}_N^{1-\sigma}} \right)^{\frac{1}{\sigma}} \right]^2$ , where  $C$  has to fulfill  $\Gamma^\sigma > \frac{C}{\bar{Y}_N^{1-\sigma}} > \left( \frac{1-\sigma}{2} \Gamma \right)^\sigma$ . The two inequalities guarantee that the original utility function  $\hat{u}(C)$  satisfies the standard neoclassical conditions  $\hat{u}'(C) > 0 > \hat{u}''(C)$ .

<sup>11</sup>In the given model, this would require an infinite amount of local currency debt and an infinite long position in dollars.



**Figure 3:** The figure depicts the supply locus  $SS$  and demand locus  $DD$  for local currency denominated debt. Equilibrium is determined by the intersection of the two, here indicated by the vertical line  $\rho^*$  and the resulting equilibrium value of  $N$ .

### Solution to the Decentralized Equilibrium

The solution to the decentralized equilibrium is given by the 4 equations (11), (13), (14) and (18) in the 4 variables  $(\rho, N, F_2^\omega, C_{T,1}^\omega)$ , where the last two equations and the last two variables are state-contingent. Conditions (3) and (9) then yield  $F_1$  and  $p_{N,1}^\omega$ , and from the market clearing condition for non-tradables we trivially obtain  $C_{N,t}^\omega = \bar{Y}_N$ .

We can collapse this system of equations into a system of two equations in two variables,  $\rho$  and  $\text{Var}(C_{T,1}^\omega)$ . The first equation is borrowers' optimality condition, or demand locus (18). In figure 3, this is the  $DD$  locus. It is slightly concave because of the term  $E[C_{T,1}^\omega]^\omega(\Gamma - E[C_{T,1}^\omega])$ , which is an inverted parabola.

The second equation can be derived from lenders' supply condition through the following steps. Using condition (15), we find that

$$\text{Cov}(C_{T,1}^\omega, M_1^\omega) = \frac{R^*}{1 + R^*(1 + N)} \cdot \text{Cov}(Y_{T,1}^\omega, M_1^\omega)$$

We have demonstrated above in (17) that a similar relationship holds for the standard deviation of consumption, i.e.  $\text{Std}(C_{T,1}^\omega) = \frac{R^*}{1 + R^*(1 + N)} \text{Std}(Y_{T,1}^\omega)$ . Putting the two together, it follows that

$$\rho E[C_{T,1}^\omega] = -R^* \text{Cov}(C_{T,1}^\omega, M_1^\omega) = -R^* \cdot \frac{\text{Cov}(Y_{T,1}^\omega, M_1^\omega)}{\text{Std}(Y_{T,1}^\omega)} \cdot \text{Std}(C_{T,1}^\omega) = -\xi \cdot \text{Std}(C_{T,1}^\omega) \quad (19)$$

for an appropriately defined constant  $\xi$ , where we use the observation that the covariance and standard deviation of  $Y_{T,1}^\omega$  are given. The risk premium that lenders require is thus linear in the standard deviation of consumption: the more consumption and the real exchange rate fluctuate, the higher the premium that international lenders require to hold local currency debt. We can consider this as international lenders' optimality condition or supply locus of local currency debt. In figure 3, this relationship is depicted as  $SS$ . The locus  $SS$  is convex since the axis in the figure depicts the variance  $\text{Var}(C_{T,1}^\omega)$ , i.e. the square of the standard deviation.

Note that the amount of local currency debt (as represented by  $N$ ) increases as we move from the top right of the figure to the bottom left along both the  $SS$  and  $DD$  loci, as local currency debt reduces volatility in the economy. In the limit, i.e. as  $N \rightarrow \infty$ , both loci end up in the origin. The concavity and convexity of the two schedules guarantee a unique non-degenerate equilibrium, as indicated in the figure.

## 2.7 Optimal Currency Composition of Debts

We analyze two different versions of the social planner's problem here. Firstly, we describe a constrained social planner who can determine the currency composition of borrowers' debts, but who leaves the decision regarding how much to consume and borrow to decentralized agents. This setup is of particular interest for policymakers who have to regulate foreign currency borrowing. In the next subsection, we will describe an all-powerful social planner who coordinates all borrowing and consumption allocations of agents.

The constrained social planner in the emerging market economy only has the capacity to coordinate the currency denomination of debts. Contrary to agents in the decentralized equilibrium, she internalizes the effects of her decisions on the volatility of the real exchange rate  $p_{N,t}^\omega$ . This has two equilibrium effects: First, a lower variance implies that the covariance of the real exchange rate with lenders' stochastic discount factor falls, which reduces the risk premium  $\rho$  that lenders charge, as described in condition (11). Secondly, as the volatility in the real exchange rate falls, each unit of local currency debt provides less insurance, since it co-varies less with the state of the economy. To obtain a given amount of insurance, it is thus necessary to hold more local currency debt.

The constrained social planner leaves the decision regarding how much to consume and how much to save to the decentralized agents. In the given setup, the amount of borrowing in period 0 is pre-determined by  $\bar{I}$ . Decentralized agents only have a choice over how much to consume and borrow/save in period 1, i.e.  $C_{T,1}^\omega$  versus  $F_2^\omega$ . Given the social planner's decision regarding  $L_1$  and the realization of the productivity shock  $Y_{T,1}^\omega$ , private agents' optimization problem in period 1 is

$$\begin{aligned} & \max_{C_{T,1}^\omega, F_2^\omega} E \{ u(C_{T,1}^\omega) + \beta u(\bar{Y}_T - R^* F_2^\omega) \} \\ \text{s.t. } & C_{T,1}^\omega = Y_{T,1}^\omega - (1 + R^*)\bar{I} + R^*W_0 + L_1(R^* - R_{L,1}^\omega) + F_2^\omega \end{aligned}$$

where we have substituted  $C_{N,t}^\omega = \bar{Y}_N$  and employed the transformed utility function  $u(\cdot)$  as in (12). We can substitute the constraint into the first period utility function and derive with respect to  $F_2^\omega$  to find agents' Euler equation

$$\text{FOC}(F_2^\omega) : u'(C_{T,1}^\omega) = \beta R^* u'(C_{T,2}^\omega)$$

They perfectly smooth their consumption over time. As in the decentralized equilibrium, this results in a level of second-period borrowing of

$$F_2^\omega = \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \quad (20)$$

Taking this into account, we can formulate the constrained social planner's maximization problem as

$$\begin{aligned}
& \max_{C_{T,1}^\omega, F_2^\omega, N, \rho, EC_{T,1}} E \{u(C_{T,1}^\omega) + \beta u(\bar{Y}_T - R^* F_2^\omega)\} \\
& \text{s.t. } C_{T,1}^\omega = Y_{T,1}^\omega - \bar{I} - R^*(\bar{I} - W_0) - N[C_{T,1}^\omega - (1 - \rho)EC_{T,1}] + F_2^\omega \\
& \quad F_2^\omega = \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \\
& \quad \rho EC_{T,1} = -R^* \text{Cov}(C_{T,1}^\omega, M_1^\omega) = -E[(C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)] \\
& \quad EC_{T,1} = E[C_{T,1}^\omega]
\end{aligned} \tag{21}$$

Here we have used the transformation  $N = L_1 \cdot \frac{R^*}{(1-\rho)E[C_{T,1}^\omega]}$  and substituted for the real exchange rate  $p_{N,1}^\omega = \frac{\bar{Y}_T}{Y_N} C_{T,1}^\omega$  according to equation (9). Since  $u(C_{T,1}^\omega) = u(C_{T,2}^\omega)$  and the constraint on  $F_2^\omega$  always has to hold with equality, we can simplify the constrained social planner's objective to  $(1 + \beta)u(C_{T,1}^\omega)$  or, dropping the constant,  $u(C_{T,1}^\omega)$ .

Note that we use the variable  $EC_{T,1}$  as a separate argument in the maximization problem, which is linked to expected consumption  $E[C_{T,1}^\omega]$  in the last constraint. This is necessary, since the expectations around the social planner's objective and for expected consumption in the covariance condition need to be calculated using two different integrating variables. This treatment leads to the following Lagrangian for the optimization problem:

$$\begin{aligned}
\mathcal{L}^{SP} = E \left\{ u(C_{T,1}^\omega) - \nu [\rho EC_{T,1} + (C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)] - \eta [EC_{T,1} - C_{T,1}^\omega] \right. \\
\left. - \mu^\omega \left[ C_{T,1}^\omega - Y_{T,1}^\omega + (1 + R^*)\bar{I} - R^* W_0 + N(C_{T,1}^\omega - (1 - \rho)EC_{T,1}) - \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*} \right] \right\}
\end{aligned}$$

where  $\mu^\omega$  is the shadow value of relaxing the constraint on first period consumption in state  $\omega$ ,  $\nu$  is the shadow value of the required risk compensation per unit  $N$  of insurance and  $\eta$  is the shadow value of increasing expected consumption  $EC_{T,1}$  by one unit. This results in the following first-order conditions:

$$\begin{aligned}
\text{FOC}(C_{T,1}^\omega) : \quad & u'(C_{T,1}^\omega) = \mu^\omega(1 + N + 1/R^*) + \nu(R^* M_1^\omega - 1) - \eta \quad \forall \omega \\
\text{FOC}(N) : \quad & E\{\mu^\omega C_{T,1}^\omega\} = (1 - \rho)E\{EC_{T,1}\} \\
\text{FOC}(\rho) : \quad & E[\mu^\omega]N EC_{T,1} = -\nu EC_{T,1} \\
\text{FOC}(EC_{T,1}) : \quad & E[\mu^\omega]N(1 - \rho) = \nu\rho + \eta
\end{aligned}$$

The first equation describes the consumption allocation in each state of the world. Agents' marginal utility of consumption has to equal  $1 + N + 1/R^*$  times the shadow price of relaxing the consumption constraint<sup>12</sup> plus the expected costs from increasing the risk premium on local currency debt, which depends on international lenders' pricing kernel

<sup>12</sup>This is because increases in  $C_{T,1}$  entail higher repayments on local currency debt and are smoothed over the ensuing periods. The exogenous components of the constraint on consumption (e.g.  $Y_{T,1}^\omega$ ) has to rise by  $1 + N + 1/R^*$  units for actual consumption to increase by one unit.

in that state, minus the costs from decreasing the average level of consumption. The second condition captures the optimal amount of insurance by equalizing the benefit and costs of the marginal unit of insurance. The last two first order conditions can be used to obtain the following expressions for the shadow prices

$$\nu = -N \cdot E[\mu^\omega] \quad \text{and} \quad \eta = N \cdot E[\mu^\omega] \quad (22)$$

We substitute these into the  $\text{FOC}(C_{T,1}^\omega)$  to obtain

$$(1 + \beta + N)\mu^\omega = u'(C_{T,1}^\omega) + NE\mu^\omega R^* M_1^\omega$$

Replacing this  $\mu^\omega$  in turn into the  $\text{FOC}(N)$  and dividing through the constant  $(1 + \beta + N)$  yields

$$E \{ [u'(C_{T,1}^\omega) + NE\mu^\omega R^* M_1^\omega] [C_{T,1}^\omega - (1 - \rho)EC_{T,1}] \} = 0$$

In order to simplify this equation, let us use the following lemma.

**Lemma 1** *The expectation  $E \{ bR^* M_1^\omega [C_{T,1}^\omega - (1 - \rho)EC_{T,1}] \}$  vanishes for any constant  $b$ .*

**Proof.** Opening up the square brackets and collecting terms, we find that

$$\begin{aligned} E \{ R^* M_1^\omega [C_{T,1}^\omega - (1 - \rho)EC_{T,1}] \} &= \rho R^* E[M_1^\omega] EC_{T,1} + R^* \text{Cov}(M_1^\omega, C_{T,1}^\omega) = \\ &= \rho EC_{T,1} - \rho EC_{T,1} = 0 \end{aligned}$$

where the last step follows from condition (11) determining the equilibrium risk premium on local currency debt. Multiplying the expression by any finite constant leaves the conclusion unaffected. The economic reason for this result is that international lenders are – by definition – indifferent between the random payoff  $C_{T,1}^\omega$  and its certainty equivalent  $(1 - \rho)E[C_{T,1}^\omega]$ . ■

Applying this lemma, we find that the social planner's optimal level of local currency debt is determined by the condition

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -\rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)] \quad (23)$$

This expression is already familiar from the competitive equilibrium case. The solution of the constrained social planner is given by the 4 equations (11), (13), (20) and (23) in the 4 variables  $(\rho, N, F_2^\omega, C_{T,1}^\omega)$ . These conditions are identical to the ones in the previous subsection determining the competitive equilibrium. We can summarize this result in the following proposition.

**Proposition 2** *The equilibrium allocations of the constrained social planner and of agents in the decentralized equilibrium coincide.*

The constrained social planner thus cannot improve on the market outcome. Let us next analyze the case of an unconstrained social planner who can determine all of agents' decisions.

## 2.8 Social Planner

In the following analysis, the social planner coordinates not only the currency composition of debts in the emerging market economy, but also agents' intertemporal pattern of consumption.

We will show that in general, the social optimum in this setup differs from the previous two cases. The reason is that, although the social planner faces the same two assets to insure consumption, i.e. local and foreign currency debt, she recognizes that by altering agents' intertemporal consumption allocation, she can affect the relative price of non-tradable goods and therefore the relative payoff of local currency debt. Deviating from agents' Euler equation in this way comes at a second-order cost, but provides a first-order insurance benefit to international lenders, which they will pass on to borrowers in the form of a lower risk premium. In other words, the social planner mitigates the incompleteness of risk markets through a second-best intervention in the intertemporal allocation of consumption, which affects the relative payoff of the risky asset, local currency debt. In doing so, she can improve risk sharing between borrowers and lenders. This is a general equilibrium effect that small agents do not internalize.<sup>13</sup>

Analytically, we drop the constraint on  $F_2^\omega$  from the constrained social planner's maximization problem (21) and include the utility of consumption in both periods as the objective. The resulting Lagrangian is

$$\begin{aligned} \mathcal{L}^{SP} = E \bigg\{ & u(C_{T,1}^\omega) + \beta u(\bar{Y}_T - R^* F_2^\omega) \\ & - \mu^\omega [C_{T,1}^\omega - Y_{T,1}^\omega + (1 + R^*)\bar{I} - R^* W_0 + N(C_{T,1}^\omega - (1 - \rho)EC_{T,1}) - F_2^\omega] \\ & - \nu [\rho EC_{T,1} + (C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)] - \eta [EC_{T,1} - C_{T,1}^\omega] \bigg\} \quad (24) \end{aligned}$$

Compared to the previous case of the constrained social planner, the first order condition  $\text{FOC}(C_{T,1}^\omega)$  changes, and the condition  $\text{FOC}(F_2^\omega)$  is added, since the social planner has to decide on the optimal amount of second period borrowing:

$$\begin{aligned} \text{FOC}(C_{T,1}^\omega) : \quad & u'(C_{T,1}^\omega) = \mu^\omega(1 + N) + \nu(R^* M_1^\omega - 1) - \eta \quad \forall \omega \\ \text{FOC}(F_2^\omega) : \quad & \beta R^* u'(C_{T,2}^\omega) = \mu^\omega \quad \forall \omega \end{aligned}$$

In the  $\text{FOC}(C_{T,1}^\omega)$ , the social planner internalizes the effect that making consumption more countercyclical to lenders' pricing kernel  $M_1^\omega$  reduces the risk premium on local currency debt because of its positive insurance effect on lenders. The condition  $\text{FOC}(F_2^\omega)$  implies that the social planner no longer equalizes  $u'(C_{T,1}^\omega) = u'(C_{T,2}^\omega)$ , but uses deviations from this rule as a second-best means of making up for missing insurance markets. The  $\text{FOC}(\rho)$  and  $\text{FOC}(EC_{T,1})$  are unchanged compared to the constrained social planner's case and imply that  $\nu = -NE[\mu^\omega]$  and  $\eta = NE[\mu^\omega]$ , as given in (22). Substituting the results into the  $\text{FOC}(C_{T,1}^\omega)$ , we obtain that

$$\mu^\omega = \frac{u'(C_{T,1}^\omega) + NE[\mu^\omega]R^* M_1^\omega}{1 + N} \quad (25)$$

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<sup>13</sup>For a more general discussion of this externality see e.g. Stiglitz (1982).

Economically,  $\mu^\omega$  is the sum of the benefit in terms of additional consumption from relaxing the constraint on  $C_{T,1}^\omega$  and the cost in terms of raising the risk premium. Using this  $\mu^\omega$  in the FOC(N), where the term with  $M_1^\omega$  vanishes because of lemma 1, and performing the same steps as above in the constrained social planner's problem, we obtain

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -\rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)] \quad (26)$$

In other words, the condition determining the optimum amount of local currency debt is the same as in both the competitive equilibrium and the constrained social planner's regime.

On the other hand, the social planner internalizes that the exchange rate is determined by agents' consumption allocations  $C_{T,1}^\omega$ , and that these in turn depend not only on the productivity shock and the amount of local currency debt, but also on her borrowing choices  $F_2^\omega$ , which are determined by the first order condition  $u'(C_{T,2}^\omega) = \mu^\omega$ . She can raise lenders' payoff in a given state  $\omega$  by borrowing more  $F_2^\omega$ , which raises agents' disposable income, their aggregate demand, and thus appreciates the real exchange rate. By the same token, reducing  $F_2^\omega$  depreciates the real exchange rate in that state.

If the emerging market economy and international lenders are only exposed to a common risk factor, then risk markets between borrowers and lenders are complete and the optimal amount of local currency debt  $L_1$  can deliver the optimum insurance outcome. In that case, the solution to the decentralized equilibrium and the social planner's solution coincide. Let us define this situation formally as

**Definition 1** *An emerging market's productivity shocks  $Y_{T,1}^\omega$  and international lenders' pricing kernel  $M_1^\omega$  are driven by a single common risk factor if fluctuations in the two variables are perfectly correlated, i.e. if*

$$|\text{Corr}(Y_{T,1}^\omega, M_1^\omega)| = 1 \quad (27)$$

Note that this condition is trivially satisfied if there are only two states of the world, i.e. if  $\Omega = \{\omega_1, \omega_2\}$ , and if  $Y_{T,1}^\omega, M_1^\omega$  are non-degenerate. This is because in such a probability space, we can find two suitable scalars  $a$  and  $b$  for any two random variables  $X^\omega, Y^\omega$  such that  $X^\omega = a + bY^\omega \forall \omega$ .

On the other hand, there are two ways in which the condition can be violated. First, if some component of the emerging market economy's productivity shock is uncorrelated with lenders' pricing kernel, lenders would provide free insurance against this risk factor. Knowing this, the social planner would decrease her borrowing in states with an uncorrelated negative productivity shock. This would depreciate the exchange rate and thereby lower the contingent repayment mandated by agents' holdings of local currency debt. The opposite considerations would apply during uncorrelated positive productivity shocks. Overall, this entails better insurance for domestic agents than the pure intertemporal consumption smoothing rule  $u'(C_{T,1}^\omega) = \beta R^* u'(C_{T,2}^\omega)$ .

Secondly, if some component of lenders' pricing kernel is uncorrelated with the productivity shock in the emerging market, then lenders would be willing to pay borrowers

to take on some of that risk. Borrowers would be willing to carry it against an appropriate compensation, which would take the form of a reduction in lenders' interest rate. The social planner thus borrows more  $F_2^\omega$  in states in which international lenders are affected by an uncorrelated negative shock; this appreciates the exchange rate in these states and increases the payoffs on local currency debt to lenders. In return for this insurance, lenders demand a lower risk premium  $\rho$  on local currency debt.

Before summarizing these findings in a proposition, let us note the following lemma. (We have relegated the proof to the mathematical appendix.)

**Lemma 2** *Let  $X^\omega, Y^\omega, Z^\omega$  be scalar random variables that are perfectly correlated, such that  $|\text{Corr}(X^\omega, Y^\omega)| = |\text{Corr}(Z^\omega, Y^\omega)| = 1$ . Then  $E[X^\omega] = E[Z^\omega]$  and  $E[X^\omega Y^\omega] = E[Z^\omega Y^\omega]$  if and only if  $X^\omega = Z^\omega$ .*

Note that under the given assumptions, the condition  $E[X^\omega Y^\omega] = E[Z^\omega Y^\omega]$  can also be written as  $E[(X^\omega - Z^\omega)Y^\omega] = \text{Cov}(X^\omega - Z^\omega, Y^\omega) = 0$ .

**Proposition 3** *If an emerging market's productivity shocks and lenders' pricing kernel are driven by a single common risk factor and borrowers' utility function is quadratic, then the decentralized equilibrium coincides with the social planner's solution.*

**Proof.**  $Y_{T,1}^\omega$  is the risk factor underlying the emerging market economy's productivity shocks. If there is only one risk factor driving both the emerging market economy and international lenders' pricing kernel, then  $|\text{Corr}(Y_{T,1}^\omega, M_1^\omega)| = 1$ . Given that borrowers' utility function is quadratic, all components of  $F_2^\omega$ ,  $C_{T,1}^\omega$  and  $\mu^\omega$  are linear combinations of the two random variables and hence also perfectly correlated with  $Y_{T,1}^\omega$ . Note that we can re-write equation (25) as

$$\mu^\omega = u'(C_{T,1}^\omega) - \frac{N}{1+N} E[u'(C_{T,1}^\omega)] \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega \right\}$$

If the term in curly brackets is zero, then the social planner's borrowing choices coincide with that of decentralized agents. We use lemma 2 to establish that this is true under the assumptions of the proposition. According to the lemma, it is sufficient to show that  $E \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} \right\} = E[R^* M_1^\omega]$  and  $\text{Cov} \left( \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega, C_{T,1}^\omega \right) = 0$  in order to prove that the term in square brackets vanishes and  $\mu^\omega = u'(C_{T,1}^\omega)$ . The first equality holds trivially. The second equality can be transformed to

$$\text{Cov} (u'(C_{T,1}^\omega), C_{T,1}^\omega) = \text{Cov} (C_{T,1}^\omega, R^* M_1^\omega) E[u'(C_{T,1}^\omega)]$$

The covariance term on the right hand side can be replaced by the risk premium according to equation (11), yielding the familiar condition

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -\rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)]$$

We have already demonstrated in equation (26) above that this condition holds in equilibrium. Therefore  $\mu^\omega = u'(C_{T,1}^\omega)$ , and all other equilibrium allocations coincide with the solutions to the decentralized equilibrium. ■

As discussed above, if productivity shocks and lenders' pricing kernel are at least partially driven by different risk factors, then this result breaks down. We can then substitute the shadow price  $\mu^\omega$  from (25) into the FOC( $F_2^\omega$ ) to obtain

$$u'(C_{T,2}^\omega) = u'(C_{T,1}^\omega) - \frac{N}{1+N} E[u'(C_{T,1}^\omega)] \cdot \left\{ \frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} - R^* M_1^\omega \right\}$$

If the marginal utilities of the two agents are affected by different risk factors, then in general  $\frac{u'(C_{T,1}^\omega)}{E[u'(C_{T,1}^\omega)]} \neq R^* M_1^\omega$ . In states in which domestic agents' marginal utility is relatively higher, the social planner borrows less than indicated by the intertemporal consumption smoothing condition  $u'(C_{T,1}^\omega) = u'(C_{T,2}^\omega)$ . As a result the real exchange rate depreciates relative to that case, and domestic agents' repayments on local currency debt are lower. If lenders' stochastic discount factor is relatively higher, the opposite results apply.

### 3 Borrowing Constraints and Financial Crises

In the previous section we emphasized the role of the exchange rate as a tool of macroeconomic adjustment: in the case of a negative productivity shock, aggregate demand is low and a depreciation of the local currency increases demand for local non-tradable goods so as to clear markets. We also noted that this created a pecuniary externality associated to borrowers' choice of debt denomination: the less insurance in the form of local currency debt borrowers take on, the more of the adjustment has to be carried out by exchange rates, i.e. the more volatile the local currency becomes.

High exchange rate volatility can impose real costs on the economy, and this has proven to be of particular importance for emerging markets. As we noted in the beginning of this paper, both the theoretical and empirical literature maintain that exchange rate depreciations in the presence of large amounts of dollar debt can have strongly contractionary effects (see e.g. Krugman, 1998, 1999; Levy Yeyati, 2006; Razin and Rubinstein, 2006). While the economic literature has identified a number of contractionary effects of depreciations (see e.g. Caves et al., 2002), this paper focuses on balance sheet effects, which have played a key role in the financial crises of the past decade.<sup>14</sup>

Contractionary balance sheet effects work through the following mechanism: When borrowers have contracted dollar liabilities, exchange rate depreciations simultaneously inflate the local currency value of their dollar debts and reduce the international value of local currency denominated income and collateral. Both factors deteriorate borrowers' balance sheets, and this in turn magnifies a variety of problems of asymmetric information that exist between borrowers and lenders, such as moral hazard and adverse selection, and can cause or exacerbate credit rationing (see Stiglitz and Weiss, 1981).

When borrowing constraints bind, dollar debt introduces an accelerator effect into the economy: Exchange rate depreciations deteriorate agents' balance sheets, which

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<sup>14</sup>For a detailed discussion of the importance of balance sheet effects versus other channels of contractionary depreciation see e.g. Frankel (2005). Note that the results of this paper do not depend on the exact channel through which depreciations have contractionary effects.

tightens their borrowing constraints and reduces aggregate demand. Reduced aggregate demand in turn depreciates the exchange rate further. As described in Mendoza (2005), the two effects then lead to a contractionary cycle of falling exchange rates and tightening borrowing constraints, similar to the debt deflation process in Fisher (1933).

Note that the effects of borrowing constraints are asymmetric. While the described debt deflation process entails that exchange rate depreciations have strongly negative welfare effects, there are no corresponding positive effects to appreciations in good states when borrowing constraints are not binding – loose constraints are loosened further. In such situations, higher exchange rate volatility imposes clear real costs on the economy.

In the given setup, agents' period 0 borrowing is fixed by the requirement that  $\bar{I} = F_1 + L_1 + W_0$ . For simplicity we assume that this amount always satisfies the period 0 borrowing constraint. However, agents can be subject to binding borrowing constraints in period 1. We assume that the amount of debt they can take on in period 1 is subject to the constraint

$$F_2^\omega + L_2^\omega \leq \kappa(Y_{T,1}^\omega + p_{N,1}^\omega \bar{Y}_N) \quad (28)$$

This specification implies that agents' maximum amount of borrowing is linear in their income. Such borrowing constraints are common in the macroeconomic literature.<sup>15</sup> Lenders often put an important weight on current income to determine the maximum amount of loans they are willing to grant, especially in mortgage and consumer credit markets (Arellano and Mendoza, 2003). In a more general setup, we could also include the amounts of production and/or consumption next period as parameters in the borrowing constraint. However, the exact source of the credit market imperfection is not essential to the results in this paper, and the given linear specification offers the benefit of analytical simplicity.

As in the benchmark model, we assume that borrowers' optimal debt composition is such that their consumption is positive in all states and there is no bankruptcy. Furthermore, we limit financial contracts to one period contracts so as to guarantee that the borrowing constraints apply to the whole firm, not only to marginal borrowing.

The term  $p_{N,1}^\omega$  in the collateral constraint is what can make exchange rate depreciations contractionary. A falling real exchange rate  $p_{N,1}^\omega$  reduces the dollar value of non-tradable collateral and makes lenders less willing to lend to domestic agents. If the constraint becomes binding, domestic agents' borrowing has to contract and their disposable income falls. As we noted earlier, this entails a further depreciation in  $p_{N,1}^\omega$ , which makes borrowing constraints even tighter, and so forth. In short, a Fisherian debt deflation process is set in motion. We impose the restriction on our parameters that  $\kappa\varsigma < 1$  so as to guarantee that the feedback effects between exchange rate and borrowing constraints become smaller from round to round and the economy converges to a non-degenerate equilibrium after a shock.

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<sup>15</sup>See for example Mendoza (2005); Mendoza and Smith (2006). Aghion et al. (1999) derive a linear credit multiplier from a moral hazard problem among borrowers, who face an incentive to strategically default on their loans. Higher wealth implies that borrowers have more to lose from lenders' efforts to collect, and they thus face a lower incentive to default. A related approach is presented in Bernanke et al. (1999), who show borrowing costs increase with falling net worth because of the higher associated probability of costly bankruptcy. As in our specification, this leads to a supply of debt that decreases in agents' income.

In the model presented here, the initial shock that triggers the described debt deflation process is a shock to tradable production  $Y_{T,1}^\omega$ , which lowers agents' income and therefore depreciates the exchange rate. In the spirit of the sudden stops described by Calvo (1998), borrowing constraints could also tighten because of a shock to lenders' willingness to supply credit, as captured e.g. by the credit multiplier  $\kappa$ . Both kinds of shocks can cause borrowing constraints to suddenly bind, and can set in motion the described Fisherian debt deflation mechanism.

### 3.1 Competitive Equilibrium

Agents in the competitive equilibrium solve the same maximization problem as in the unconstrained case (8) of the previous section, augmented by the borrowing constraint defined in (28). The resulting Lagrangian is<sup>16</sup>

$$\mathcal{L}^{CE} = E \left\{ u(Y_{T,1}^\omega - (1 + R^*)\bar{I} + R^*W_0 - R^*L_1[p_{N,1}^\omega - (1 - \rho)E(p_{N,1}^\omega)] + F_2^\omega) + \right. \\ \left. + \beta u(\bar{Y}_T - R^*F_2^\omega) - \lambda^\omega [F_2^\omega - \kappa(Y_{T,1}^\omega + p_{N,1}^\omega \bar{Y}_N)] \right\}$$

The corresponding first order conditions are

$$\begin{aligned} \text{FOC}(F_2^\omega) : u'(C_{T,1}^\omega) &= \beta R^* u'(C_{T,2}^\omega) + \lambda^\omega \\ \text{FOC}(L_1) : E \{ u'(C_{T,1}^\omega) [p_{N,1}^\omega - (1 - \rho)E(p_{N,1}^\omega)] \} &= 0 \end{aligned}$$

We note first that, despite of the additional constraint, the  $\text{FOC}(L_1)$  can be transformed to yield the same solution as condition (18) in the previous section

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = \rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)] \quad (29)$$

As long as the collateral constraint is not binding, agents' choice of second period borrowing equalizes the marginal utilities of period 1 and period 2 consumption. On the other hand, if the borrowing constraint is binding, agents borrow the maximum amount possible. In short, this implies that in equilibrium

$$F_2^\omega = \min \left\{ \frac{\bar{Y}_T - C_{T,1}^\omega}{R^*}, \kappa (Y_{T,1}^\omega + \varsigma C_{T,1}^\omega) \right\} \quad (30)$$

Applying the transformation  $N = L_1 \frac{R^*}{(1 - \rho)E[C_{T,1}^\omega]}$ , we can use these  $F_2^\omega$  to express agents' optimal first period consumption  $C_{T,1}^{\text{con}}$  in constrained states and  $C_{T,1}^{\text{unc}}$  in unconstrained states of the world as

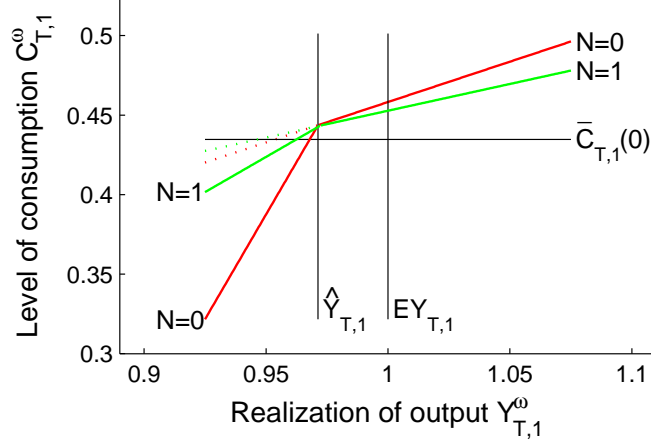
$$C_{T,1}^{\text{con}} = \frac{(1 + \kappa)Y_{T,1}^\omega - (1 + R^*)\bar{I} + R^*W_0 + (1 - \rho)NE[C_{T,1}^\omega]}{1 + N - \kappa\varsigma} \quad (31)$$

$$\text{and } C_{T,1}^{\text{unc}} = \frac{\bar{Y}_T + R^*Y_{T,1}^\omega - R^*(1 + R^*)\bar{I} + (R^*)^2W_0 + (1 - \rho)NR^*E[C_{T,1}^\omega]}{1 + R^*(1 + N)} \quad (32)$$

A direct implication is the following result:

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<sup>16</sup>We have made the usual simplifications, i.e. we set  $C_{N,1}^\omega = \bar{Y}_N$ , substituted the utility function from (12) and assumed w.l.o.g. that  $L_2^\omega = 0$ .



**Figure 4:** The figure depicts the level of consumption  $C_{T,1}^\omega$  as a function of the output shock  $Y_{T,1}^\omega$  for several different levels of insurance  $N$ . As can be seen, more local currency debt makes consumption less sensitive to the shock, and particularly so in those states where the credit constraint is binding. As a result, the variance of consumption is reduced.

**Proposition 4** *Under binding borrowing constraints, consumption is more sensitive to shocks. Local currency debt can reduce the sensitivity of consumption to shocks, and it can do so more strongly when borrowing constraints are binding. As a result, increasing local currency debt compared to foreign currency debt reduces the volatility of consumption.*

**Proof.** The sensitivity of consumption to shocks can be calculated as the derivative of the respective consumption variable to  $Y_{T,1}^\omega$ .

$$\begin{aligned} \frac{dC_{T,1}^{\text{con}}}{dY_{T,1}^\omega} &= \frac{1 + \kappa}{1 + N - \kappa\varsigma} & \frac{d^2C_{T,1}^{\text{con}}}{dY_{T,1}^\omega dN} &= -\frac{1 + \kappa}{(1 + N - \kappa\varsigma)^2} \\ \frac{dC_{T,1}^{\text{unc}}}{dY_{T,1}^\omega} &= \frac{R^*}{1 + R^*(1 + N)} & \frac{d^2C_{T,1}^{\text{unc}}}{dY_{T,1}^\omega dN} &= -\left(\frac{R^*}{1 + R^*(1 + N)}\right)^2 \end{aligned}$$

Clearly, constrained consumption is more sensitive to output shocks. This is because when borrowing constraints are binding, an increase in income is fully consumed and, in addition, increases agents' collateral and therefore the amount that they can borrow. When borrowing constraints are loose, on the other hand, only a fraction of the increase in income is consumed; the rest of it is saved so as to smooth consumption over the ensuing period.

Local currency debt mitigates the impact of shocks on consumption, and since this impact is stronger in borrowing constrained states, local currency debt has a stronger mitigating impact in those states. ■

Figure 4 illustrates the effects of local currency debt on consumption volatility graphically. In unconstrained states, local currency borrowing mitigates the sensitivity of consumption to shocks to roughly the same extent as in the benchmark case without credit constraints in figure 1 on page 12. On the other hand, in those states where

borrowing constraints are binding, i.e. to the left of the vertical  $\hat{Y}_{T,1}$ -line in the figure, the response of consumption to output shocks is magnified because of the debt deflation effect. Local currency debt mitigates these feedback effects and therefore smoothes the response.

Let us indicate the threshold values of output and consumption at which the borrowing constraint is marginally binding by hats, i.e. by  $\hat{Y}_{T,1}$  and  $\hat{C}_{T,1}$ . For realizations of the shock with  $Y_{T,1}^\omega < \hat{Y}_{T,1}$ , the constraint is binding, for realizations with  $Y_{T,1}^\omega \geq \hat{Y}_{T,1}$  it is loose. At the cutoff value  $\hat{Y}_{T,1}$ , the constrained and the unconstrained levels of consumption  $C_{T,1}^{\text{con}}$  and  $C_{T,1}^{\text{unc}}$  are equal and the constraint is marginally binding. We find  $\hat{Y}_{T,1}$  as the value of  $Y_{T,1}^\omega$  for which the two expressions (31) and (32) are equal.

$$\hat{Y}_{T,1} = \frac{(1 + \kappa\varsigma R^*) [(1 + R^*)\bar{I} - R^*W_0 - (1 - \rho)NE[C_{T,1}^\omega]] + \bar{Y}_T (1 + N - \kappa\varsigma)}{1 + \kappa [1 + R^* (1 + N - \varsigma)]}$$

Note that  $F(\hat{Y}_{T,1})$  represents the probability that borrowers will be constrained. Taking the derivative of  $\hat{Y}_{T,1}$  with respect to  $N$  (details are provided in appendix D), we find that

$$\frac{d\hat{Y}_{T,1}}{dN} = \frac{1 + \kappa\varsigma R^*}{1 + \kappa [1 + R^* (1 + N + \varsigma)]} \cdot [\hat{C}_{T,1} - \bar{C}_{T,1}]$$

where  $\bar{C}_{T,1} = \frac{d(1-\rho)NEC_{T,1}}{dN}$  is again the constant payoff that agents receive from increasing the amount of insurance  $N$  and forgoing the random realization of consumption  $C_{T,1}^\omega$ . The sign of the derivative  $d\hat{Y}_{T,1}/dN$  depends on the sign of  $\hat{C}_{T,1} - \bar{C}_{T,1}$ , i.e. more insurance reduces the threshold if the net payoff of insurance in the marginally constrained state is positive. There are two factors that could potentially turn the sign of this derivative negative: First, if lenders are strongly risk-averse, then the risk premium would be high and the certainty equivalent of consumption  $(1 - \rho)E[C_{T,1}^\omega]$  could fall below the threshold. Secondly, if the incidence of constraints is high, then the marginally constrained state  $\hat{C}_{T,1}$  can be pushed above the certainty equivalent.

### 3.2 Social Planner

Individual agents regard the exchange rate, and therefore the dollar value of their collateral and the tightness of borrowing constraints, as exogenously given. The social planner, by contrast, internalizes that exchange rates are driven by aggregate macroeconomic outcomes and that agents' behavior influences the tightness of borrowing constraints. In particular, the social planner recognizes that increased local currency borrowing makes the exchange rate less volatile and thereby mitigates the potential costs of borrowing constraint. Analytically, we add the borrowing constraint (28) to the social planner's optimization problem (24) from the benchmark case of the previous section.

The corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}^{SP} = E \bigg\{ & u(C_{T,1}^\omega) + \beta u(\bar{Y}_T - R^* F_2^\omega) - \lambda^\omega [F_2^\omega - \kappa(Y_{T,1}^\omega + \varsigma C_{T,1}^\omega)] \\ & - \mu^\omega [C_{T,1}^\omega - Y_{T,1}^\omega + (1 + R^*)\bar{I} - R^* W_0 + N(C_{T,1}^\omega - (1 - \rho)EC_{T,1}) - F_2^\omega] \\ & - \nu [\rho EC_{T,1} + (C_{T,1}^\omega - EC_{T,1})(R^* M_1^\omega - 1)] - \eta [EC_{T,1} - C_{T,1}^\omega] \bigg\} \end{aligned}$$

Here  $\lambda^\omega$  is the multiplier on the borrowing constraint. The resulting first-order conditions on  $C_{T,1}^\omega$  and  $F_2^\omega$  are:

$$\begin{aligned} \text{FOC}(C_{T,1}^\omega) : \quad & u'(C_{T,1}^\omega) + \lambda^\omega \kappa \varsigma = \mu^\omega (1 + N) + \nu (R^* M_1^\omega - 1) - \eta \quad \forall \omega \\ \text{FOC}(F_2^\omega) : \quad & \beta R^* u'(C_{T,2}^\omega) + \lambda^\omega = \mu^\omega \quad \forall \omega \end{aligned}$$

The first-order conditions  $\text{FOC}(N)$ ,  $\text{FOC}(\rho)$  and  $\text{FOC}(EC_{T,1})$  are unchanged from the benchmark case without borrowing constraints, and the latter two equations give rise to the same two expressions  $\nu = -NE[\mu^\omega]$  and  $\eta = NE[\mu^\omega]$  as in equation (22) of that section. Substituting these as well as the  $\text{FOC}(F_2^\omega)$  into the  $\text{FOC}(C_{T,1}^\omega)$  yields

$$(1 + N)\mu^\omega = u'(C_{T,1}^\omega) + \lambda^\omega \kappa \varsigma + NE[\mu^\omega] R^* M_1^\omega$$

In conjunction with the first order condition  $\text{FOC}(N)$ , where the term with  $M_1^\omega$  vanishes according to lemma 1, we then obtain

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -\rho E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)] - \kappa \varsigma E[\lambda^\omega (C_{T,1}^\omega - (1 - \rho)E[C_{T,1}^\omega])]$$

The first term on the right-hand side of this expression is identical to the equilibrium condition (29) for local currency debt in the decentralized equilibrium. The second term captures the effects of local currency debt on the incidence of collateral constraints, which decentralized agents do not internalize. Let us define the term  $\theta$  to capture this externality and re-write the condition for the social planner's optimum amount of local currency debt as

$$\text{Cov}(u'(C_{T,1}^\omega), C_{T,1}^\omega) = -(\rho - \theta) E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)] \quad (33)$$

$$\text{where } \theta = -\kappa \varsigma \cdot \frac{E[\lambda^\omega (C_{T,1}^\omega - (1 - \rho)E[C_{T,1}^\omega])]}{E[C_{T,1}^\omega] E[u'(C_{T,1}^\omega)]} \quad (34)$$

$\lambda^\omega$  is the shadow value of relaxing the borrowing constraint in period  $\omega$ , i.e. the wedge that the constraint introduces into the agent's Euler equation. It is positive in low productivity states  $Y_{T,1}^\omega < \hat{Y}_{T,1}$  in which borrowing constraints are binding, and zero otherwise. When constraints bind in low productivity states, consumption is generally also small, so that  $C_{T,1}^\omega < (1 - \rho)E[C_{T,1}^\omega]$ , as long as lenders' risk premium is not too large. The product inside of the expectations operator in the expression for  $\theta$  is thus negative in constrained states and zero in unconstrained states, in which the shadow value on the borrowing constraint is  $\lambda^\omega = 0$ . This implies that the externality term  $\theta$  is generally positive.

Using the quadratic utility approximation from earlier<sup>17</sup>, we can describe the social planner's choice of local currency debt by

$$\text{Var}(C_{T,1}^\omega) = (\rho - \theta)E[u'(C_{T,1}^\omega)]E[C_{T,1}^\omega]$$

For a given risk premium  $\rho$  and a positive externality term  $\theta$ , the social planner's solution entails less volatility in consumption than the decentralized equilibrium. The social planner reaches this outcome by contracting more local currency debt than decentralized agents. In equilibrium, this leads to a reduction in consumption volatility, and by extension to a decrease in exchange rate volatility and a reduction in international lenders' risk premium.

As in our discussion of the sensitivity of  $\hat{Y}_{T,1}$  to the amount of insurance  $N$ , there are exceptional circumstances in which the opposite results hold: when international lenders are strongly risk-averse and the probability of binding borrowing constraints is very high, the certainty equivalent  $(1 - \rho)E[C_{T,1}^\omega]$  can be less than consumption in those constrained states in which insurance is supposed to alleviate agents' borrowing constraints. In the extreme case that  $(1 - \rho)E[C_{T,1}^\omega] < \min \{C_{T,1}^\omega\}$ , more insurance would reduce consumption in all states – in such a situation, domestic agents unambiguously benefit from reducing their holdings of local currency debt. They would correspondingly increase their dollar debts and hold long positions of local currency, since lenders are willing to pay them a substantial premium for taking on this risk. However, decentralized agents would not internalize that the higher consumption obtained from these earnings would also mitigate collateral constraints through their effect on exchange rates and might therefore hold too little dollar debt.

More generally, we can distinguish between two effects of local currency debt: the insurance effect consists of the contingent payment  $-(C_{T,1}^\omega - E[C_{T,1}^\omega])$ ; it always reduces the overall costs of borrowing constraints by smoothing income, i.e. by providing positive payoffs in low states. The (negative) income effect of local currency debt is captured by the uncontingent payment of the risk premium  $\rho E[C_{T,1}^\omega]$ ; it lowers consumption in all states and increases the incidence and tightness of binding constraints.

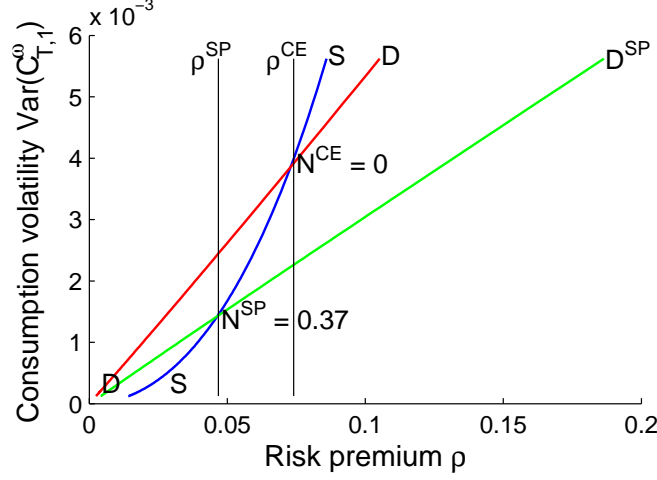
Decentralized agents only trade off the benefits and costs of insurance against consumption risk in choosing the composition of their debt portfolio. The social planner, on the other hand, also considers the insurance and income effects of local currency debt on the incidence of collateral constraints. Since the impact of these constraints is asymmetric (loosening binding constraints in low states is beneficial, while tightening loose constraints in high states is costless), the social planner's optimum entails more insurance, i.e. more local currency debt, than the decentralized equilibrium under regular circumstances. The following is the technical condition to guarantee that the externality term  $\theta$  is positive:

**Condition 1** *The externality term is positive  $\theta > 0$  if and only if*

$$\int_0^{(1-\rho)E[C_{T,1}^\omega]} \lambda^\omega [(1-\rho)E(C_{T,1}^\omega) - C_{T,1}^\omega] dC_{T,1}^\omega > \int_{(1-\rho)E[C_{T,1}^\omega]}^\infty \lambda^\omega [C_{T,1}^\omega - (1-\rho)E(C_{T,1}^\omega)] dC_{T,1}^\omega$$

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<sup>17</sup>The general result holds for all neoclassical utility functions, i.e. the relationship between the covariance term in (33) and the variance of consumption is always monotonic, but not necessarily linear.



**Figure 5:** The figure depicts lenders' and private borrowers' optimality loci  $SS$  and  $DD$ , as well as the social planner's optimality locus  $DD^{SP}$ . The competitive equilibrium and the social planner's solution are determined by the intersection of international lenders' locus with domestic agents' or the social planner's optimality locus, here marked by the vertical lines  $\rho^{SP}$  and  $\rho^{CE}$ . The resulting equilibrium values of  $N$  are indicated next to the intersection points.

As long as the left-hand side, i.e. the sum of all positive insurance payoffs, weighted by the shadow prices of relaxing the constraint in the respective state of the world, exceed the sum of weighted negative payoffs, more insurance reduces borrowing constraints and  $\theta$  is therefore positive.

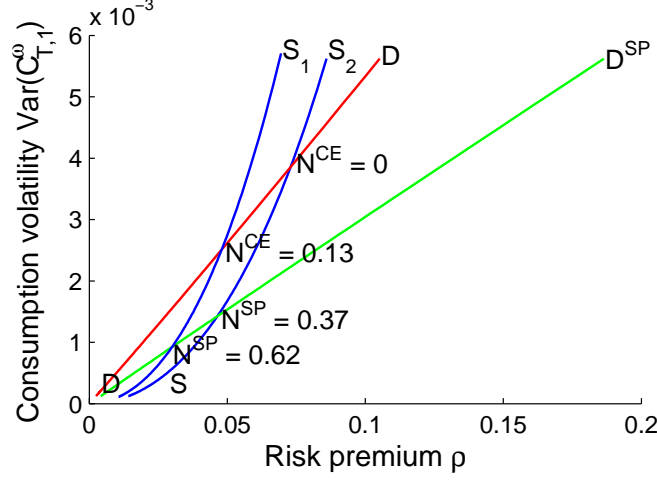
**Example 1** If  $\hat{C}_{T,1} < (1 - \rho)E[C_{T,1}^\omega]$ , then the payoffs to insurance in constrained states are always positive. In the inequality in condition 1, the left hand-side is positive and the right-hand side is zero. Hence the decentralized equilibrium in such an economy is always characterized by excessive dollar borrowing.

**Example 2** In an economy with only two states, any allocation in the decentralized equilibrium has to satisfy  $C_{T,1}^L < (1 - \rho)E[C_{T,1}^\omega]$ . Otherwise, lowering the amount of local currency debt would increase consumption in both states of the world because of a positive income effect, and make agents unambiguously better off. If agents are only constrained in the low state, then increasing the amount of local currency debt would unambiguously alleviate these borrowing constraints without impacting borrowing in the high state. Consequently, condition 1 is always satisfied in such an economy and decentralized agents would borrow excessively in dollars.

The implications for the optimum amount of local currency debt are the following:

**Proposition 5** *If the externality term  $\theta$  is positive, then the decentralized equilibrium is characterized by excessive dollar borrowing.*

Graphically, a positive externality term  $\theta$  implies that the social planner's optimality locus for local currency debt is to the right of decentralized agents' optimality locus in figure 5. As can be seen, the social planner's equilibrium then exhibits lower consumption volatility and a lower risk premium.



**Figure 6:** The figure depicts how changes in risk aversion shift lenders' optimality locus  $SS$ . An increase in lenders' risk aversion, for example, shifts this locus from  $SS_1$  to the right to  $SS_2$  and reduces the amount of local currency debt that borrowers contract. The resulting equilibrium values of  $N$  are indicated next to the intersection points.

## 4 Comparative Statics

In this section we analyze how changes to some of the parameters of the model affect the competitive equilibrium of the economy and the size of the associated externality.

### 4.1 Lenders and Changes in Risk-Aversion

We start out with an analysis of a shock to lenders' risk aversion, i.e. we transform lenders' pricing kernel by a multiplicative factor  $\alpha > 0$ , while keeping the mean constant:

$$M_\alpha^\omega = E[M_1^\omega] + (1 + \alpha) \cdot (M_1^\omega - E[M_1^\omega]) = \frac{1 + \alpha \cdot (R^* M_1^\omega - 1)}{R^*}$$

This operation can be regarded as a mean-preserving spread in  $M_1^\omega$ . While the interest rate  $R^* = \frac{1}{E[M_1^\omega]}$  for risk-free assets is unaffected, the transformation implies that lenders' risk aversion decreases for  $\alpha < 1$  or increases for  $\alpha > 1$ .

We have depicted the case of an increase in risk aversion by  $\alpha = .25$  in figure 6. The direct effect of this is to shift lenders' optimality locus  $SS_1$  by  $\alpha$  to the right to  $SS_2$ , i.e. for any given amount of exchange rate volatility, lenders demand a risk premium that is by a factor  $\alpha$  higher than before. Analytically, we can express the relationship as

$$\left. \frac{d\rho}{d\alpha} \right|_{d\text{Var}(C_{T,1}^\omega)=0} = \rho$$

Since the optimality loci of borrowers and of the social planner are also upward-sloping, the effect on the resulting equilibrium is magnified: because of the higher risk premium, agents are willing to hold less local currency debt, which makes their economy

more volatile, increases the risk premium further, and so forth. In the given example, the total effect on the risk premium is roughly twice as large as the direct effect. This holds for both the decentralized equilibrium and the social planner's solution. There are two additional indirect general equilibrium effects: First, since a change in the risk premium affects domestic agents' expected consumption  $E[C_{T,1}^\omega]$  and expected marginal utility of consumption  $E[u'(C_{T,1}^\omega)]$  for any given level of  $\text{Var}(C_{T,1}^\omega)$ . This entails a small shift to borrowers' optimality locus. Secondly, the threshold  $\hat{Y}_{T,1}$  below which agents are borrowing constrained will generally move as the equilibrium shifts. However, these two effects are of a second-order magnitude that is not visible in the figure. Let us summarize our results in the following proposition.

**Proposition 6** *An increase in lenders' risk aversion has the direct effect of raising the risk premium on local currency debt, which discourages the use of local currency debt. This has the equilibrium effect of raising volatility in the economy and increasing the risk premium further.*

## 4.2 Changes in the Economy's Riskiness

We model the effects of a change in the economy's riskiness as an increase in the standard deviation  $s_Y$  of the output shock  $Y_{T,1}^\omega$  by a factor  $\alpha$ , i.e.

$$Y_{T,1}^\omega \sim N(\bar{Y}_T, (\alpha s_Y)^2)$$

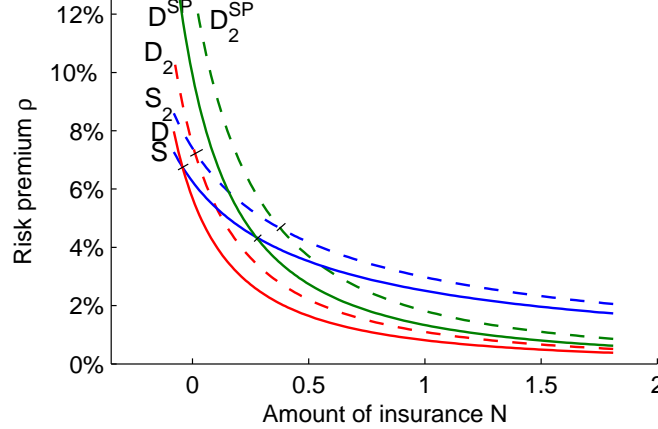
In the diagrams used so far, which depicted the variance of consumption against the risk premium  $\rho$ , the resulting effects are hard to discern: there are no direct effects on borrowers' or lenders' optimality loci. We can conclude from this that in equilibrium, both the variance of consumption and the risk premium  $\rho$  remain roughly constant.

However, the amount of local currency debt that has to be held to reach this equilibrium is higher. For a given amount of local currency debt, both the variance of consumption and the risk premium rise. We thus depict the effects of changes in the economy's riskiness in a diagram of the risk premium  $\rho$  as a function of the amount of insurance  $N$  in figure 7.

Let us first investigate the effects of an increase in volatility in the economy's output shock on borrowers' demand for local currency. Following the equation of their demand locus (18) that  $\text{Var}(C_{T,1}^\omega) = \rho E u'(C_{T,1}^\omega) E C_{T,1}$ , we find that

$$\left. \frac{d\rho}{d\alpha^2} \right|_{dN=0} = \frac{\left. \frac{d\text{Var}(C_{T,1}^\omega)}{d\alpha^2} \right|_{dN=0} - \rho \left. \frac{dEC_{T,1}Eu'(C_{T,1}^\omega)}{d\alpha^2} \right|_{dN=0}}{E(C_{T,1}^\omega)Eu'(C_{T,1}^\omega)} \approx \rho$$

In other words, the demand curve for local currency debt shifts approximately by the square of the increase in the standard deviation of the output shock. The reason is that the variance of consumption increases by approximately  $\alpha^2$  and that this term dominates any second order effects on average consumption and the average marginal utility of consumption. Thus, the risk premium that borrowers are willing to pay after an increase by the factor  $\alpha$  in the standard deviation of output is by a factor  $\alpha^2$  higher than before, for a given level of local currency debt. This shifts the demand curve



**Figure 7:** The figure illustrates the effects of changes in the economy's riskiness on the resulting equilibrium. An increase in the standard deviation of the economy's output shock (in the figure by a factor  $\alpha = 1.5$ ) shifts both demand and supply of local currency debt to the right. The result is an unambiguous increase in the amount of insurance.

for local currency debt up by the factor  $\alpha^2$ . In addition, as long as  $F(\hat{Y}_{T,1}) < 0.5$ , the incidence of borrowing constraints rises, which raises demand for insurance even further.

Similarly, lenders require a higher risk premium on local currency debt for a given level of insurance  $N$ , since the economy's riskiness rises and thus the covariance of the returns on local currency with their pricing kernel increases. Holding the amount of insurance  $N$  constant, we find that

$$\left. \frac{d\rho}{d\alpha} \right|_{dN=0} = -\frac{R^*}{EC_{T,1}} \cdot \text{Cov} \left( \left. \frac{dC_{T,1}^\omega}{d\alpha} \right|_{dN=0}, M_1^\omega \right) + \rho \left. \frac{dEC_{T,1}}{d\alpha} \right|_{dN=0} \approx \rho$$

The covariance of consumption with lenders' pricing kernel is increased by approximately the same factor as the increase in the standard deviation of the output shock. The fall in expected consumption is only a second-order effect. In total, the supply curve thus shifts upwards by approximately the factor  $\alpha$ .

As the figure demonstrates, the effect on the economy's equilibrium is that the amount of local currency debt unambiguously rises. The risk premium in the economy stays approximately constant.

**Proposition 7** *An increase in the volatility of the economy's output shock raises the amount of local currency debt that agents incur so as to offset the impact of the higher volatility on consumption.*

## 5 Policy Implications

The externality in this paper arises because of the interaction of foreign currency debts with borrowing constraints. Hence the first-best policy solution would be to address

the capital market imperfections that underlie these constraints, for example through better enforcement of creditor rights, streamlined bankruptcy proceedings etc. However, in the short to medium term, policymakers have to take many of the features of the institutional framework of emerging market economies as given.

Recognizing this constraint, a common policy prescription (see e.g. Goldstein and Turner, 2004) is that firms should match the currency denomination of their liabilities with that of their revenues. However, even if this rule is followed, the externality in my paper is still present: if firms in the tradable sector borrow in dollars, the volatility of the real exchange rate will be higher than if they borrow in local currency, and this will negatively impact the dollar value of nontradable firms' collateral in low states. Furthermore, some of the foreign currency denomination in firms' revenues might be fictitious, e.g. if the collateral that they receive from domestic clients in case of bankruptcy is non-tradable, such as real estate (Tirole, 2003).

Another key factor that gives rise to the externality is aversion to emerging market risk on the part of international lenders. Without the resulting risk premium on local currency debt, domestic borrowers would have no incentive to resort to dollar debt and cause externalities. In general, stronger institutions within emerging markets help to reduce international investors' risk aversion, but as discussed above this might not be achievable in the short run. A number of researchers have put forward proposals to deepen international markets in emerging market currencies by combining different currencies into one security with superior risk/return characteristics (see Dodd and Spiegel, 2005; Eichengreen and Hausmann, 2005). If this widens the pool of investors in emerging market currencies and therefore enhances risk diversification in international capital markets, the risk premium charged on individual domestic currencies could be reduced.

A policy prescription that has been widely followed during emerging market financial crises is to attempt to stabilize exchange rates (Calvo and Reinhart, 2002). The model in this paper discusses real exchange rates and assumes that the local currency is tied to the price of non-tradables, i.e. implicitly that monetary authorities stabilize the money price of non-tradable goods. In a more general setup, let us assume that the central bank stabilizes an index of tradable and non-tradable goods<sup>18</sup>

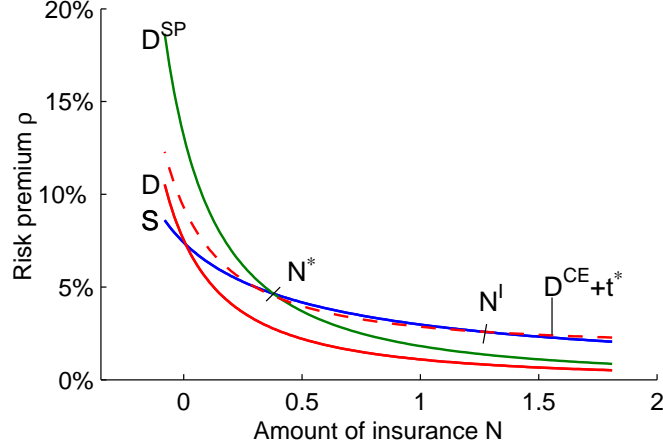
$$P = p_T^\phi p_N^{1-\phi}$$

where  $\phi$  is the relative weight on tradables. In this formulation, the analysis of sections 2 and 3 of this paper captures the case  $\phi = 0$ .

The other polar case that  $\phi = 1$  describes a central bank that stabilizes the money value of tradable goods, which, given a constant world market price of tradable goods, can be interpreted as a policy of pegging the exchange rate. Note that if a central bank credibly follows this policy, local currency and foreign currency debt in our model are equivalent. As a consequence, the risk premium on local currency debt would disappear. At the same time, local currency could no longer be used for insurance: the economy forgoes the exchange rate as a tool of macroeconomic adjustment. Still, the real exchange rate would fluctuate in this model, implying inflation in the money

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<sup>18</sup>For a detailed discussion of the mechanism involved, see e.g. Woodford (2003).



**Figure 8:** A linear tax  $t$  on dollar debt shifts the demand curve for local currency debt upwards in parallel, which can implement the social optimum  $N^*$ , but can also lead to the inefficient equilibrium  $N^I$ .

price of non-tradable goods in high productivity states and deflation in low productivity states. This situation corresponds to imposing the constraint  $L_1 = 0$  in our benchmark model and would clearly reduce welfare in the economy.

On the other hand, if the peg is not perfectly credible, an interest rate differential between local currency and dollar will capture depreciation expectations and investors' risk aversion. Our externality result still holds in this case: domestic borrowers will want to save on lenders' risk premium and engage in dollar borrowing, not recognizing that this will adversely affect the dollar value of their collateral in low productivity states, when central banks are most likely to abandon the peg.

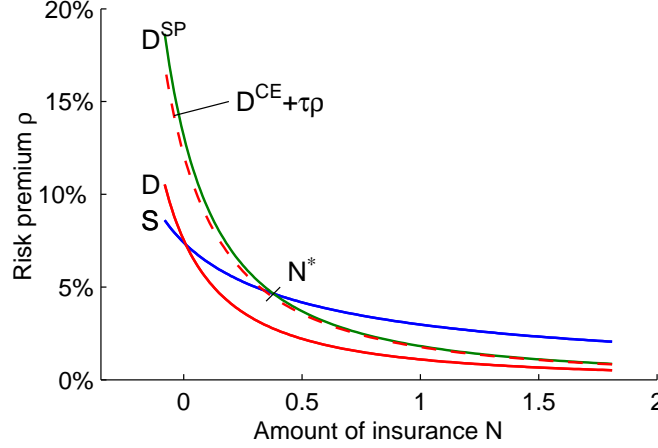
More generally, the monetary authorities could stabilize any price index with  $\phi \in [0, 1]$ . The case  $\phi = \sigma$ , for example, corresponds to a policy of targeting constant consumer prices. As long as  $\phi < 1$ , exchange rates are counter-cyclical, local currency debt insures borrowers against aggregate productivity shocks, and the externality is still present. However, for  $\phi \neq 0$  the exchange rate is a non-linear function of the shock and risk markets are no longer complete. We are currently working on a separate paper to analyze the implications.

## 5.1 Optimal Tax on Dollar Borrowing

We showed above in section 3.2 that the socially optimal level of dollar debt is in general less than the amount of debt that individual borrowers contract in the decentralized equilibrium. It follows naturally that the social planner could make agents internalize the externality associated with their level of dollar borrowing by imposing a tax on dollar debt.

The optimal fixed tax  $t^*$  on dollar borrowing is such that in equilibrium, borrowers pay a spread of  $\rho - \theta$  instead of  $\rho$  on local currency, so as to internalize the externality. For a tax  $t^*$  per unit of foreign currency, this implies that we require

$$[1 - (\rho - \theta)]E[R_{L,1}^\omega] = R^* + t^* \quad \text{or} \quad t^* = \theta E[R_{L,1}^\omega] \quad (35)$$



**Figure 9:** A tax  $\tau$  that is proportional to the spread on local currency always implements the social optimum. It shifts the demand curve for local currency debt upwards towards the social planner's demand and raises the equilibrium amount of local currency borrowing.

where the values of  $\theta$  and  $E[R_{L,1}^\omega]$  are the ones in the social planner's equilibrium. An example for such an optimal tax is shown in figure 8, for which we used the parameter values discussed in footnote 9 on page 11 and calibrated lenders' risk aversion such that they demand a risk premium of 3% when  $N = 1$ . Borrowers' risk aversion is calibrated to yield a zero level of local currency debt in the competitive equilibrium. The optimal per dollar tax rate in this example turns out to be  $t^* = 1.75\%$ .

Note that a constant per dollar tax  $t^*$  has the disadvantage that it leads to a situation of multiple equilibria, including an inefficient equilibrium with excessive local currency debt (in the figure denoted as  $N^I$ ). In that equilibrium, the volatility of the economy is low because of the high amount of domestic currency debt; the tax  $t$  on dollar debt is much higher than the externality and induces borrowers to take on an inefficiently large amount of local currency debt. However, this equilibrium is unstable. The reason for its existence is that the size of the externality  $\theta$  (in the figure the vertical difference between decentralized agents' and the social planner's demand curves) decreases approximately proportionally to the spread  $\rho$ , whereas the tax rate  $t^*$  is fixed.

This suggests that it would be more efficient to levy a tax that is proportional to the spread on local currency debt, i.e. a tax in the amount of  $\tau\rho$ . The optimal  $\tau$  for such a tax could be determined using the condition

$$[1 - (\rho - \theta)]E[R_{L,1}^\omega] = R^* + \tau\rho \quad \text{or} \quad \tau = \frac{\theta}{\rho}E[R_{L,1}^\omega] = \frac{\theta}{\rho(1 - \rho)}R^* \quad (36)$$

where the values of  $\theta$  and  $\rho$  are again the ones at the social planner's equilibrium. In the given example, the optimal  $\tau = 38\%$ . When we depict the shift in decentralized agents' demand that results from such a tax in a diagram, it almost coincides with the social planner's demand in figure 9.

In terms of implementation, there is a number of policy measures that have tax-like effects on dollar debt. These include the following:

- *Reserve requirements:* A country can impose reserve requirements on foreign currency lenders. In the simplest version, it would require lenders to hold an amount of  $\frac{t^*}{R^*-1}$  per dollar lent in an account without remuneration, so that the lost amount of interest would equal the optimal tax rate. A similar approach was used by Chile to regulate short-term capital flows (see e.g. Gallego et al., 2002).

A more refined version of this policy instrument could require lenders to hold an amount of  $\tau$  per dollar lent in an unremunerated account denominated in local currency (which could be inflation-indexed). In that case, the lost amount of interest would approximately equal the relative tax rate  $\tau$ . Furthermore, lenders would be obliged to carry some local currency risk even though their loan is in dollars.

- *Tax Deductability:* In most tax legislations around the world, interest rate payments are tax deductible for corporations. Selectively limiting this deductability on dollar, but not domestic currency debt would be equivalent to an increase in the effective interest rate for foreign currency borrowers. A full elimination of the tax deductability of interest payments on dollar debt, for example, would be equivalent to a rise in the dollar interest rate by  $\tau^C(R^* - 1)$ , where  $\tau^C$  represents the corporate tax rate applicable to that borrower.
- *Banking regulation:* Some countries, such as Malaysia, have used wide-ranging regulations on domestic banks that forbid them not only to lend in foreign currency, but also to lend to companies that have loans in foreign currency from other sources. Similarly, domestic regulators could subject lending to such companies to an additional tax or higher capital adequacy requirements. Such regulations would put strong restrictions on dollar debtors' access to domestic financial markets, which effectively makes it more costly for them to borrow in dollars.

In addition, international regulations such as e.g. the Basle capital adequacy standards could be used to make dollar debt more expensive to lenders by increasing their capital adequacy requirements on dollar debt relative to local currency debt lent to emerging market borrowers.

- *Bankruptcy regulations:* Foreign currency debt can be penalized through differential treatment debts in bankruptcy regulations. Although we have not modeled the effects of bankruptcy in our framework, the model could easily be extended to allow for an unfavorable treatment of dollar debt in the case of bankruptcy, which would equal the tax  $t^*$  in expected value. An additional benefit of such a setup might be that the probability of bankruptcy of borrowers in an economy varies over time, and that these variations would generally go in parallel with the externality discussed here.

Note that Broda and Levy Yeyati (2006) suggest a differential treatment for local and foreign currency debts in bankruptcy proceedings in order to mitigate another externality: they find that lenders to bankruptcy-prone borrowers have an incentive to denominate their loans in dollars, which on average appreciate in precisely those states of the world when bankruptcy is most likely. Lenders can thereby maximize the claim on borrowers' residual value in bankruptcy proceedings.

Most of these policy instruments could be circumvented via derivatives markets. Naturally, any regulations in debt markets would thus have to be accompanied by corresponding regulations in derivatives markets.

## 5.2 GDP-Indexed Debt Versus Local Currency Debt

An important policy question in the recent literature on emerging markets' debt structure was whether countries should rather issue local currency debt or GDP-linked debt.<sup>19</sup> Both types of debt are ways of reducing volatility in emerging markets by making debt repayments contingent on the state of the economy. While there are a number of aspects in which the two instruments differ, such as the ease of measurement of the variable upon which repayment is contingent or the ability of a country's government to affect this variable and engage in moral hazard, the discussion here focuses exclusively on the capacity of the two instruments to insure against risk.

In the model presented in this paper, we identify local currency debt as indexed to the real exchange rate  $p_{N,1}^\omega$ , i.e. it can be regarded as inflation-indexed debt. Similarly, we identify GDP-linked debt as indexed to the productivity shock  $Y_{T,1}^\omega$ . During normal times, the real exchange rate is a smoothed linear function of the productivity shock, as can be determined from equation (9) for the real exchange rate and expression (15) that gives consumption as a function of the output shock. The coefficient on the productivity shock in the equation for the exchange rate is

$$\frac{dp_{N,1}^\omega}{dY_{T,1}^\omega} = \frac{\varsigma}{\bar{Y}_N} \cdot \frac{R^*}{1 + R^*(1 + N)} = \text{const}$$

However, when borrowing constraints become binding during financial crises, the reaction of the exchange rate to an output shock is amplified by an accelerator effect. In such states, the sensitivity of the exchange rate to the productivity shock is significantly higher than in unconstrained states:

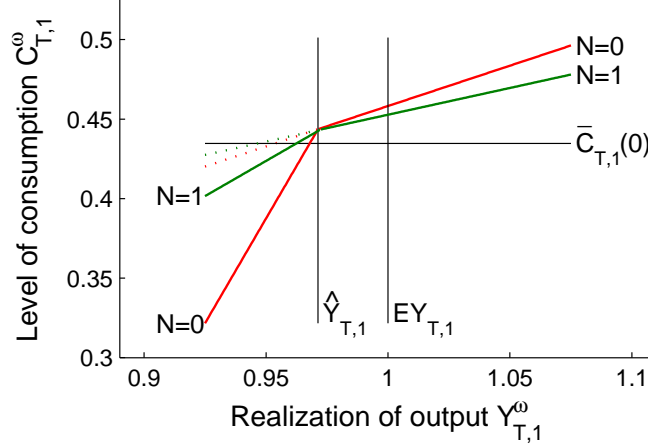
$$\frac{dp_{N,1}^\omega}{dY_{T,1}^\omega} = \frac{\varsigma}{\bar{Y}_N} \cdot \frac{1 + \kappa}{1 + N - \kappa\varsigma}$$

The reason for this multiplier effect is that, while agents would like to increase their borrowing to smooth the negative shock, binding borrowing constraints instead force them to cut back on borrowing. Consumption is reduced not only because of the lower output shock, but also because of the tightening borrowing constraints. Macroeconomically, the reduction in foreign borrowing requires a sharp reversal in the current account balance (see e.g. Calvo, 1998), which reduces aggregate consumption more than the fall in aggregate output. This fall in aggregate consumption is also reflected in the real exchange rate, which moves in parallel to consumption.

The countries affected by the East Asian crisis present a strong example for this: even though the variance of quarterly changes in their real exchange rates was smaller

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<sup>19</sup>See e.g. Borensztein et al. (2004) for the IMF's assessment of this issue or Griffith-Jones and Sharma (2006) for a study of GDP-linked bonds by the UN Department of Economic and Social Affairs.



**Figure 10:** The figure depicts the payoffs of local currency debt versus GDP-linked debt for an equal amount of expenditure on insurance premia. Local currency debt insures better against consumption risk in crisis states, since the exchange rate depreciation is amplified by binding borrowing constraints.

then the variance in growth over the past two decades, the depreciation in their real exchange rates during the Asian financial crisis was many times stronger than the fall in GDP.<sup>20</sup> Local currency debt would thus have provided much better insurance against the East Asian crisis than GDP-linked debt. We can summarize our observations in the following proposition.

**Proposition 8** *For a given expenditure on insurance premia, holding local currency debt provides higher payoffs in (constrained) crisis states than holding GDP-linked debt.*

Figure 10 provides an example for the payoffs of the two instruments, given an equal amount of expenditure on risk premia.

## 6 Conclusions

The goal of this paper was to analyze the borrowing decisions of private agents in emerging markets and to demonstrate that they borrow excessively in dollars because they fail to internalize the effects of their decision on the value of their collateral. We first discussed that the two key features of local currency denominated debt are that it provides insurance against aggregate shocks since exchange rates are counter-cyclical, and that borrowers have to pay a risk premium for this insurance. The insurance effect of local currency debt reduces the volatility of agents' disposable income. In aggregate, this makes demand, and by extension exchange rates, less volatile.

Our main result was that this pecuniary externality introduces a distortion into the economy when individual agents are subject to collateral-dependent borrowing constraints: stable exchange rates mitigate the negative effects of a low productivity shock

<sup>20</sup>Data from International Financial Statistics, 1985 – 2005, and author's calculations.

on the dollar value of each agent's collateral, making borrowing constraints less binding and financial crises less likely. Decentralized agents fail to internalize this positive externality to local currency debt (or, equivalently, negative externality to dollar debt) and engage in too much dollar borrowing from a social point of view.

Excessive dollar borrowing makes emerging market economies more volatile and increases both the incidence and the severity of financial crises. We have discussed a number of potential policy remedies to correct the distortion: While a linear tax on dollar debt can result in multiple equilibria, we showed that a tax on dollar debts that is proportional to the risk premium on local currency debt, implementable for example through unremunerated reserve requirements, provided a better solution. Similar effects can be obtained through banking and bankruptcy regulations, or changes in the tax deductability of dollar interest payments. Furthermore, we showed that local currency debt is a superior insurance instrument to GDP-linked dollar debt, since the current account reversals associated with financial crises entail that consumption and exchange rates typically fall more strongly than GDP during crisis periods.

There are a number of questions that are left for future research. Firstly, we modeled the risk premium on local currency as a function of the expected volatility in exchange rates only. If some component of the risk premium is due to other (e.g. institutional) factors, different considerations apply. For policy applications it would then be desirable to analyze how these other components of the risk premium can be reduced. Some examples for this are provided e.g. in Eichengreen and Hausmann (2005).

Secondly, while our basic model is real, a more detailed analysis of the potential role of monetary policy, especially under price stickiness, would be desirable for policy applications. Furthermore, currency depreciations in emerging markets sometimes occur because their governments resort to the printing press when they run into binding budget constraints (see e.g. Burnside et al., 2001). Our preliminary investigations indicate that this depreciation mechanism can complement the one presented in our model and strengthen our result, to the extent that the incidence of binding borrowing constraints on the government and on firms in the private sector is correlated.

Thirdly, we derived the main results of this paper in a model with complete risk markets. While this allowed us to focus on an in-depth analysis of the effects of collateral constraints, it is not a realistic assumption. Under incomplete risk markets, additional externalities to decentralized borrowers' choice of debt denomination arise and can lead to systematic biases in their debt portfolio. This research topic offers a number of promising insights that we are currently investigating.

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## A Proof of Lemma 1

**Proof of Lemma 1.** The proof from left to right works as follows. The assumptions on the correlations imply that we can write  $X^\omega = a + bY^\omega$  and  $Z^\omega = c + dY^\omega$  for some scalars  $a, b, c, d$ . Then the two equations in expectations read as

$$\begin{aligned} a + bE[Y^\omega] &= c + dE[Y^\omega] \quad \text{and} \quad aE[Y^\omega] + bE[(Y^\omega)^2] = cE[Y^\omega] + dE[(Y^\omega)^2] \\ \text{or} \quad (a - c) &= (d - b)E[Y^\omega] \quad \text{and} \quad (a - c)E[Y^\omega] = (d - b)E[(Y^\omega)^2] \end{aligned}$$

Eliminating alternatively  $(a - c)$  and  $(d - b)$  from the two equations yields

$$(a - c) = (a - c) \frac{(E[Y^\omega])^2}{E[(Y^\omega)^2]} \quad \text{and} \quad (d - b)(E[Y^\omega])^2 = (d - b)E[(Y^\omega)^2]$$

Note that, since  $Y^\omega$  has non-zero correlation with other variables, its variance must be positive, and therefore  $E[(Y^\omega)^2] > (E[Y^\omega])^2 > 0$ . The equations above can then only be satisfied if  $a = c$  and  $b = d$ , which implies that  $X^\omega = Z^\omega$ . The proof in the opposite direction is trivial. ■

## B Derivation of $\bar{C}_{T,1}$ in Benchmark Model

We defined earlier that  $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN}$  and re-wrote this as

$$\bar{C}_{T,1} = \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN} = (1-\rho)E[C_{T,1}^\omega] - NE[C_{T,1}^\omega] \frac{d\rho}{dN} + (1-\rho)N \frac{dE[C_{T,1}^\omega]}{dN}$$

$\bar{C}_{T,1}$  can be regarded as something like lenders' certainty equivalent of consumption, taking into account general equilibrium effects. (More precisely speaking,  $\bar{C}_{T,1}$  is the increase in borrowers' fixed payoff  $(1-\rho)NE[C_{T,1}^\omega]$  from contracting more local currency debt, taking into account all general equilibrium effects.

$\bar{C}_{T,1}$  consists of three different terms: The first one,  $(1-\rho)E[C_{T,1}^\omega]$ , captures that borrowers receive the expected value discounted by a risk premium in return for forgoing the random variable  $C_{T,1}^\omega$ . The second term,  $NE[C_{T,1}^\omega] \frac{d\rho}{dN}$ , internalizes that the risk premium actually falls the more insurance agents take on. The third term takes into account that, since agents have to pay for insuring, expected consumption itself falls the more insurance is taken on.

Let us analyze the two derivatives on the right hand side of this expression. We note first that we can write the two equations (11) and (16) defining the equilibrium values of  $\rho$  and  $E[C_{T,1}^\omega]$  in vector form as

$$\begin{pmatrix} \rho \\ E[C_{T,1}^\omega] \end{pmatrix} = \begin{pmatrix} -\frac{(R^*)^2 \text{Cov}(Y_{T,1}^\omega, M_1^\omega)}{E[C_{T,1}^\omega][1+R^*(1+N)]} \\ \frac{(1+R^*)\bar{Y}_{T,1} - (1+R^*)\bar{I} + R^*W_0}{1+R^*(1+\rho N)} \end{pmatrix}$$

Differentiating this system of equations with respect to  $N$  yields

$$\begin{pmatrix} 1 & \frac{\rho}{E[C_{T,1}^\omega]} \\ \frac{R^*NE[C_{T,1}^\omega]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \begin{pmatrix} \frac{d\rho}{dN} \\ \frac{dE[C_{T,1}^\omega]}{dN} \end{pmatrix} = - \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} \\ \frac{\rho R^*E[C_{T,1}^\omega]}{1+R^*(1+\rho N)} \end{pmatrix} \quad (37)$$

The determinant of the matrix that pre-multiplies the vector of differentials is

$$\left| \begin{pmatrix} 1 & \frac{\rho}{E[C_{T,1}^\omega]} \\ \frac{R^* N E[C_{T,1}^\omega]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \right| = 1 - \frac{\rho R^* N}{1+R^*(1+\rho N)} = \frac{1+R^*}{1+R^*(1+\rho N)}$$

We use this determinant to invert the matrix and left-multiply equation (37) by this inverse to obtain

$$\begin{aligned} \begin{pmatrix} \frac{d\rho}{dN} \\ \frac{dE[C_{T,1}^\omega]}{dN} \end{pmatrix} &= -\frac{1+R^*(1+\rho N)}{1+R^*} \begin{pmatrix} 1 & -\frac{\rho}{E[C_{T,1}^\omega]} \\ -\frac{R^* N E[C_{T,1}^\omega]}{1+R^*(1+\rho N)} & 1 \end{pmatrix} \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} \\ \frac{\rho R^* E[C_{T,1}^\omega]}{1+R^*(1+\rho N)} \end{pmatrix} = \\ &= -\frac{1+R^*(1+\rho N)}{1+R^*} \begin{pmatrix} \frac{\rho R^*}{1+R^*(1+N)} - \frac{\rho^2 R^*}{1+R^*(1+\rho N)} \\ -\frac{\rho(R^*)^2 N E[C_{T,1}^\omega]}{[1+R^*(1+\rho N)][1+R^*(1+N)]} + \frac{\rho R^* E[C_{T,1}^\omega]}{1+R^*(1+\rho N)} \end{pmatrix} = \\ &= -\begin{pmatrix} \rho R^* \frac{1+R^*(1+\rho N) - \rho - \rho R^*(1+N)}{[1+R^*(1+N)][1+R^*]} \\ \rho R^* E[C_{T,1}^\omega] \frac{1+R^*(1+N) - R^* N}{[1+R^*(1+N)][1+R^*]} \end{pmatrix} = \begin{pmatrix} -\frac{\rho(1-\rho)R^*}{1+R^*(1+N)} \\ \frac{\rho R^* E[C_{T,1}^\omega]}{1+R^*(1+N)} \end{pmatrix} \end{aligned}$$

It is easy to see that  $NE[C_{T,1}^\omega] \frac{d\rho}{dN} = (1-\rho)N \frac{dE[C_{T,1}^\omega]}{dN}$ . This implies that the second and third term in the expression above for  $\bar{C}_{T,1}$  cancel out, and we find that

$$\bar{C}_{T,1} = \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN} = (1-\rho)E[C_{T,1}^\omega]$$

Economically, the reason is the following.  $NE[C_{T,1}^\omega] \frac{d\rho}{dN}$  is the reduction in insurance costs for the pre-existing  $N$  units of insurance when the amount  $N$  is raised.  $(1-\rho)N \frac{dE[C_{T,1}^\omega]}{dN}$  is the reduction in the payoff of the pre-existing  $N$  units of insurance that stems from a fall in average consumption  $E[C_{T,1}^\omega]$  when the amount  $N$  is increased. The two terms cancel out because all in all, increasing  $N$  does not affect the relative costs and payoffs of the existing insurance arrangement. Lenders do not care about whether they provide a defined contingent payoff through a small number of strongly fluctuating assets or a large number of mildly fluctuating assets.

## C Derivation of $\bar{C}_{T,1}$ in Model with Borrowing Constraints

As in the benchmark model, we define  $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN}$  as the increase in borrowers' fixed payoff  $(1-\rho)NE[C_{T,1}^\omega]$  from contracting more local currency debt, taking into account all general equilibrium effects. This can again be decomposed into the three parts

$$\bar{C}_{T,1} = (1-\rho)E[C_{T,1}^\omega] - NE[C_{T,1}^\omega] \frac{d\rho}{dN} + (1-\rho)N \frac{dE[C_{T,1}^\omega]}{dN}$$

In the following, we first express the derivative  $\frac{dC_{T,1}^\omega}{dN}$  as an intermediate result. We then analyze the two derivatives  $\frac{dE[C_{T,1}^\omega]}{dN}$  and  $\frac{d\rho}{dN}$  and return to characterizing  $\bar{C}_{T,1}$ .

### C.0.1 Derivative $dC_{T,1}^\omega/dN$

We can derive consumption in constrained states  $C_{T,1}^{\text{con}}$  and in unconstrained states  $C_{T,1}^{\text{unc}}$  from equations (31) and (32) with respect to  $N$  as

$$\frac{dC_{T,1}^{\text{con}}}{dN} = -\frac{C_{T,1}^{\text{con}} - \bar{C}_{T,1}}{1 + N - \kappa\varsigma} \quad \text{and} \quad \frac{dC_{T,1}^{\text{unc}}}{dN} = -\frac{R^* [C_{T,1}^{\text{unc}} - \bar{C}_{T,1}]}{1 + R^*(1 + N)}$$

This can be summarized as

$$\frac{dC_{T,1}^\omega}{dN} = -x^\omega (C_{T,1}^\omega - \bar{C}_{T,1}) \quad \text{where} \quad x^\omega = \frac{dC_{T,1}^\omega}{dY_{T,1}^\omega} = \begin{cases} \frac{1}{1+N-\kappa\varsigma} & \text{if } Y_{T,1}^\omega < \hat{Y}_{T,1} \\ \frac{R^*}{1+R^*(1+N)} & \text{if } Y_{T,1}^\omega \geq \hat{Y}_{T,1} \end{cases} \quad (38)$$

Increasing the amount of insurance  $N$  raises consumption in states in which it is lower than  $\bar{C}_{T,1}$  and lowers it in those where consumption is higher than  $\bar{C}_{T,1}$ . We use  $x^\omega$  as the short notation for the multiplier of changes in income on consumption, as derived in proposition 4. As already discussed there,  $x^{\text{con}} > x^{\text{unc}}$ , i.e. the smoothing impact of more local currency debt  $N$  on consumption is stronger in constrained states than in unconstrained states.

### C.0.2 Derivative $dE[C_{T,1}^\omega]/dN$

The derivative of expected consumption can then be written as<sup>21</sup>

$$\begin{aligned} \frac{dE[C_{T,1}^\omega]}{dN} &= E \left[ \frac{dC_{T,1}^\omega}{dN} \right] = E [x^\omega (\bar{C}_{T,1} - C_{T,1}^\omega)] = E (x^\omega) \bar{C}_{T,1} - E [x^\omega C_{T,1}^\omega] = \\ &= E (x^\omega) (\bar{C}_{T,1} - E[C_{T,1}^\omega]) - \text{Cov} (x^\omega, C_{T,1}^\omega) \end{aligned}$$

Expected consumption generally falls as the amount of insurance  $N$  is increased, since insurance is costly. Algebraically, this is captured by the observation that  $\bar{C}_{T,1} < E[C_{T,1}^\omega]$ . However, as expressed by the covariance term, this direct effect is counteracted by the lower incidence of borrowing constraints, which tends to raise the level consumption: Since  $x^{\text{con}} > x^{\text{unc}}$ , the impact of insurance in constrained states, which is generally positive since constraints occur in low states, is stronger than the impact in unconstrained states, which is on average negative since unconstrained states are high states. This can reverse the sign of the derivative when the probability of borrowing constraints is sufficiently high.

### C.0.3 Derivative $d\rho/dN$

Taking the total differential of equation (11), which defines the risk premium that lenders charge for a given distribution of period 1 consumption, we obtain

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<sup>21</sup>Note that we apply the Leibniz rule in the first step, but the derivatives of the boundary between constrained and unconstrained states  $d\hat{Y}_{T,1}/dN$  cancel out, since at  $\hat{Y}_{T,1}$  the constrained and unconstrained values of consumption are identical, i.e. consumption is a continuous function of the output shock.

$$\frac{d\rho}{dN} E[C_{T,1}^\omega] = -\rho \frac{dE[C_{T,1}^\omega]}{dN} - R^* \text{Cov} \left( \frac{dC_{T,1}^\omega}{dN}, M_1^\omega \right)$$

Increasing the amount of insurance generally reduces the risk premium, as the (positive) covariance term in the expression outweighs the (generally negative) derivative term.

#### C.0.4 Solving for $\bar{C}_{T,1}$

Let us successively substitute both derivatives back into the expression for  $\bar{C}_{T,1}$ :

$$\begin{aligned} \bar{C}_{T,1} &= (1 - \rho)E[C_{T,1}^\omega] + N \left[ \rho \frac{dE[C_{T,1}^\omega]}{dN} + R^* \text{Cov} \left( \frac{dC_{T,1}^\omega}{dN}, M_1^\omega \right) \right] + \\ &\quad + (1 - \rho)N \frac{dE[C_{T,1}^\omega]}{dN} = \\ &= (1 - \rho)E[C_{T,1}^\omega] + NR^* \text{Cov} \left( \frac{dC_{T,1}^\omega}{dN}, M_1^\omega \right) + N \frac{dE[C_{T,1}^\omega]}{dN} = \\ &= (1 - \rho)E[C_{T,1}^\omega] + NR^* E \left[ \frac{dC_{T,1}^\omega}{dN} \cdot M_1^\omega \right] \end{aligned}$$

## D Derivative $d\hat{Y}_{T,1}/dN$

Taking the derivative of  $\hat{Y}_{T,1}$  with respect to  $N$  and using the definition that  $\bar{C}_{T,1} = \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN}$ , we find that

$$\begin{aligned} \frac{d\hat{Y}_{T,1}}{dN} &= \frac{\bar{Y}_T - (1 + \kappa\varsigma R^*) \frac{d(1-\rho)NE[C_{T,1}^\omega]}{dN} - \kappa R^* \hat{Y}_{T,1}}{1 + \kappa [1 + R^* (1 + N + \varsigma)]} = \\ &= \frac{1 + \kappa\varsigma R^*}{1 + \kappa [1 + R^* (1 + N + \varsigma)]} \cdot [\hat{C}_{T,1} - \bar{C}_{T,1}] \end{aligned}$$

For the last step, observe that at the cutoff value  $\hat{Y}_{T,1}$ , the constrained and unconstrained levels of borrowing  $F_2^{\text{con}}$  and  $F_2^{\text{unc}}$  have to equal, which implies

$$F_2^{\text{con}} = \kappa(\hat{Y}_{T,1} + \varsigma \hat{C}_{T,1}) = \frac{\bar{Y}_T - \hat{C}_{T,1}}{R^*} = F_2^{\text{unc}} \quad \text{or} \quad \bar{Y}_T - \kappa R^* \hat{Y}_{T,1} = (1 + \kappa\varsigma R^*) \hat{C}_{T,1}$$