What Moves the Yield Curve? The Role of Inflation and Money at Different Horizons

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Abstract

We propose a dynamic three-factor model of the term structure in which the factors partition the yields into short-, medium- and long-maturity and model their dynamics in maturity- and in calendar-time. We investigate the nature of the yield changes by modeling their calendar-time dynamics as the joint response to 1) long-run forces producing enduring effects, 2) medium-run forces generating effects waning within business-cycle horizons and 3) short-run forces giving rise to very short-lived effects. These effects are tracked by the low-, medium- and high-frequency component, respectively, of the factor time functions, which we extract with a dynamic filter working in real time, in the time domain, and correcting for model uncertainty. Similarly, we decompose the evolution of the monetary policy rate, inflation and a global monetary liquidity index, we contrast the frequency components of the variables and test for (predictive) causality. Investigating U.S. data for the last three decades, we find that inflation is interrelated with all of the yields at business-cycle frequencies, while at low-frequencies since the end of the disinflation of the 1980s it is interrelated only with the short-term rates. Moreover, we find that while monetary policy exerts a limited effect on the long-rates at business-cycle horizons, it seems to generate enduring effects on their underlying pattern in the long-run.

JEL classification: G1, E4, C5

Keywords: Yield curve; Monetary Policy; Inflation; Global Liquidity; Frequency Decomposition; Model Uncertainty

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1 Introduction

Over the last few years, the unexpected response of the term structure of interest rates, in particular of the long-term rates, to the developments in the economic fundamentals of the most industrialized economies has drawn a lot of attention on the identification, and possibly the prediction, of the factors driving the forward (and then the spot) rates as a function of their maturity. Knowing the determinants of yield curve movements is especially relevant for central banks given that the short-term interest rate – and the transmission of its changes to longer maturities – is the main tool that monetary policy employs to maintain price and macroeconomic stability. From the viewpoint of the policymaker it is also important to know how much persistent the action exerted by such determinants is and if they affect in the same measure the entire maturity spectrum of the yield curve.

In this paper, we examine the behavior of short-, medium- and long-term rates and we assess how they respond to three types of determinants, namely short-, medium- and long-term forces. Moreover, we evaluate how the same three types of determinants shape the evolution of some macroeconomic variables which the economic theory suggests to be interrelated with the yield curve, and we investigate their links with the yields across the maturity spectrum and the time horizons testing, when possible, for predictive causality.

We focus on the last three decade developments using U.S. data. We model the yield curve starting from end-month U.S. nominal zero-coupon bond yields and we investigate their interrelation with U.S. headline consumer price inflation and its core component excluding the prices of food and energy. Moreover, we consider the target for the U.S. federal funds rate and, to account for the long-run effects of the monetary policy stances prevailing in the most industrialized and financially integrated economies, we consider a measure of global monetary liquidity.

To explain the movements of the yield curve we adopt the dynamic three-factor model proposed by Donati and Donati (2007), whose main characteristics are, first, that the factors partition the yield maturity spectrum into short-, medium- and long-term rates and model their dynamics in maturity- and in calendar-time; second, that the calendar-time dynamics of the factors is jointly determined by 1) long-run forces producing enduring effects, 2) medium-run forces generating effects waning within business-cycle horizons and 3) short-run forces giving rise to very short-lived effects. These effects are tracked by three frequency components partitioning the calendar-time functions of the factors. Specifically, the action of the long-run forces is tracked by the low-frequency component of these time-functions, the effect of the business-cycle forces is tracked by their medium-frequency component, and the effect of the short-lived forces is reproduced by their high-frequency component. We explicitly model the evolution of each frequency component of each variable of interest as the output of a linear, time-invariant, discrete-time, dynamic system identified starting from the data and controlled by exogenous inputs lying within three pre-specified frequency bandwidths. Such inputs are computed by a dynamic filter, namely an input-output state observer, which acts as a band-pass filter and then permits to perform the required spectral decompositions. In particular, the dynamic filter through a feedback control ensures that each frequency component evolves within its pre-specified frequency bandwidth, and that the sum of the three frequency components accurately reconstructs the actual history of the variable, thereby minimizing any information loss. We opted for this filtering approach because it has a number of advantages: 1) it works in the time domain and it allows to filter the data in real time, which means that the decomposition of a time function at time t_0 is performed without requiring the knowledge of the values that the time function will take on at time $t > t_0$ and without altering the outcomes of the decomposition already performed at time $t' < t_0$; 2) it permits to decompose also nonstationary time series; 3) it permits to extract all the selected frequency components jointly, and, finally, 4) it corrects for model uncertainty, that is for the deterioration in the empirical results stemming fromunavoidable model approximations and, possibly, misspecifications as well as for equally unavoidable data measurement errors. As a result, the proposed yield curve model is especially suitable to investigate the nature of the dynamic components of the yields. In addition, it fits the data with errors of mostly zero-mean and minimal standard deviation, and its parameter and variable estimates are robust to various forms of uncertainty.

The calendar-time dynamic models and the methodology to carry out the spectral decompositions have been designed to be applied to the time series of any economic variable. In particular, starting from the same frequency bandwidths, the decomposition of the time functions of the macroeconomic variables of interest allows the comparison of the respective low-, medium-, and high-frequency components thereby leading to a deeper understanding of the relationship between the developments in the economic fundamentals and the movements of the forward rates and yields.

This paper is organized as follows. In Section 2 we review the links with the literature. In Section 3 we introduce the data, the notation and the maturity-time dynamic yield curve model. In Section 4 we provide an overview of the methodological approach used to perform the spectral decomposition of the yield curve and the other macroeconomic variables and to model their calendar-time dynamics, while in Section 5 we provide an overview of the statistical properties of the extracted frequency components. In Section 6, we investigate the relationship between the yield curve and inflation. We show that over the last two decades U.S. inflation has fluctuated around an essentially constant long-run anchor with progressively smaller variability, as shown, using other approaches, also by Coley and Sargent (2005), Cecchetti et al. (2007) and Stock and Watson (2007). We find that, displaying a close interrelation with inflation, the target for the federal funds rate and the short-term rates have fluctuated around an essentially constant long-run level, but their variability has not decreased with time. In contrast, the long-term rates have steadily trended downwards and have exhibited progressively smaller fluctuations which have rendered their reactions to business-cycle fluctuations less noticeable than in the 1980s. Yet, they have kept on reacting to actual and expected consumer price changes. In Section 7 we examine the relationship between the yield curve and monetary policy. We find a close link between the policy rate and the short-term rates at all frequencies. However, the low-frequency components (LF) of the rates at longer maturities appear to respond to long-run forces other than those driving the LF of the policy rate and the short-end of the yield curve. In addition, both at business-cycle frequencies and at high-frequencies we cannot reject the hypothesis that the medium-frequency component (MF) of the policy rate does not Granger-cause the MF of the medium-term rates and of the long-term rates. In Section 8, we show that the LF of the inflation-adjusted policy rate and of the inflation-adjusted short-term rates have steadily trended downwards since early 1983 and that, when contrasted with the LF of real GDP annual growth, the latter has remained above the former two LF since early 1984. We speculate that this has stimulated the growing of U.S. external indebtedness and that the concomitant steady U.S. capital outflows coupled with the emergence of highly saving exporting countries have contributed to the build up of an "excessive" global monetary liquidity stock. We find evidence of a clear interrelation between the steady decline displayed by the LF of the medium- and long-term rates and the steady upturn displayed by global monetary liquidity thereby corroborating the argument put forth by several observers, e.g. Baks and Kramer (1999), Borio and Lowe (2001), King (2006), Warnock and Warnock (2006), Bollard (2007), Geithner (2007) and Bini Smaghi (2007) among others, that higher global liquidity has propped up – and reflected – rises in asset prices including those of U.S. Treasury securities. We argue that such effect has been evident on the bonds and notes not significantly affected by the monetary policy operations carried out by the Federal Reserve. We conclude by briefly discussing the implications of our results.

2 The link with the literature

This paper is related to the recent macro-finance literature (e.g. Ang and Piazzesi (2003), Rudebusch and Wu (2004), Diebold, Rudebusch, and Aruoba (2006), among others) in its quest for a clearer understanding of the macroeconomic determinants of the yield curve, but it builds on different assumptions. First, while the three non directly measurable, or latent, variables used to model the yield curve in large part of the recent literature typically reproduce the long-term rate and combinations of long-, intermediate- and short-term rates, and for such reason are labeled "level" slope" and "curvature" following Litterman and Scheinkman (1991), our model is designed in such a way that its three latent variables partition the maturity spectrum, thereby simplifying the exam of the interrelation between the macroeconomy and the yields as a function of their maturity. Second, we undertake that the three latent variables modeling the term structure reproduce the effects exerted on the forward and the spot rates not by a specific variable, as for example inflation or output, but by a large number of macroeconomic forces altogether. Given that such effects are virtually impossible to disentangle without a fully fledged dynamic model of the global economy, we treat them as an aggregate and divide the economic forces that move the yield curve according to how persistent their influence on the interest rate is.

This paper is also related to the literature which attempts to explain the puzzling behavior exhibited by U.S. long-term interest rates throughout 2004-05. When interpreted with the macro-finance empirical models of the literature this episode continues to appear a conundrum, as shown by Rudebusch, Swanson, and Wu (2006). Here we argue that investigating the relationship of the yield curve with the open, and not only with the domestic, macroeconomy and over different time horizons provides insight on the nature of the issue. To start with, the spectral decomposition reveals that the MF of the long-term rates was turning up from mid-2001 to mid-2004 − thereby anticipating the increase in the MF of headline inflation which went on from mid-2002 to mid-2006 − although due to the prevailing effect of their downward-trended LF, the 10-year-maturity zero-coupon bonds actually decreased amid fluctuations from early 2000 to mid-2003. Second, expanding on the analysis developed by Warnock and Warnock (2006), and considering international capital flows in conjunction with the concomitant slowly evolving, rising pattern, displayed by global monetary liquidity appears to help understand the slowly declining pattern exhibited by the LF of the long-term rates over the last two decades.

Finally, this paper is related to the literature studying the behavior of interest rates in the frequency domain. Part of this literature, e.g. Assenwacher-Wesche and Gerlach (2007), Sarno, Thortnon and Wen (2007), Granger and Rees (1968), examines the expectation theory of the term structure, which posits that the slope of of the yield curve reflects the market expectations of future changes in interest rates. This paper is more closely related to the studies examining the effectiveness of the "Bill Only" theory. Such literature started in the early 1950s after the decision by the Federal Open Market Committee to confine open market operations to short-term securities, preferably Treasury bills, against the assumption that changes in the availability of funds would first be reflected in the bill sector, and then would spread to other financial sectors (thereby affecting also longer-maturity rates). Our findings are in line with with the results obtained by Fand (1966), Sargent (1968), Dobell and Sargent (1969), Cargill and Meyer (1972), Pippenger (1974) and Brick and Thompson (1978) who, resting on considerations related to the actual implementation of monetary policy, provide evidence contradicting the hypothesis that long-term rates lag behind short-term rates, and argue that in efficient financial markets, long-term rates cannot depend on a distributed lag of short-term rates.

3 Data, notation and the yield curve model

We consider monthly data from 31 January 1980 to 30 September 2007 (333 monthly observations) and the following macroeconomic variables: 1) the annual growth rate of the U.S. consumer price index, henceforth denoted inflation or CPI, and the annual growth rate of the U.S. consumer price index excluding food and energy, henceforth denoted core inflation or $CCPI$, both from the U.S. Bureau of Labor Statistics; 2) the U.S. federal funds target rate from the Board of Governors of the Federal Reserve System, henceforth denoted policy rate or FFR; 3) a measure of global monetary liquidity obtained by dividing the sum of the nominal broad money stocks of the United States, the United Kingdom, the euro area, Japan and Canada with the sum of the nominal GDP of the same countries after having converted all these variables into U.S. dollars at current market exchange rates;[1] 4) month-end U.S. zero coupon bond yields of maturities $m = 6, 12, \ldots, 120$ months, regularly spaced at 6-month maturity intervals (20 spot rates for each point in time t) collected by the Bank for International Settlements.

3.1 The yield curve model

To model the yield curve we consider two time dimensions: calendar-time $t \in \mathbb{N}$ and maturity-time $m \in \mathbb{R}$. For any given point t in calendar-time, the spot rate $y(m,t)$, or continuously compounded yield to maturity m, of a zero coupon bond providing a unit redemption payment at time $t+m$, is defined by its relation with the price $P(m,t)$ of the bond as follows: $P(m,t) = e^{-y(m,t)m}$. We refer to spot rates as a function of their maturity-time m as the (spot) yield curve, or the term structure of interest rates. The spot rates are an average of the instantaneous forward rates. Specifically, the instantaneous forward rate $fw(m,t)$ is the marginal rate of return from the reinvestment of an m-period zero-coupon bond in an $(m+1)$-period zero-coupon bonds, so that the spot rates and the instantaneous forward rates are linked by the relation

$$y(m,t) = \frac{1}{m} \int_0^m fw(u,t)du \tag{1}$$

To model the term structure, we adopt the methodology proposed by Donati and Donati (2007), which has the characteristic of not imposing any requirement on the statistical distribution of the yields[2] and of correcting for model uncertainty. In particular, its advantage is that it permits to reduce the degradation in the quality of the empirical results which typically arises when the restrictions imposed by a theoretical model are not entirely met by the data, for example because the actual yields are not normally distributed and the data are affected by various measurement errors (see, e.g. Diebold, Piazzesi and Rudebusch, (2005)).

This yield curve model builds on two different dynamic processes acting simultaneously: one

[1] The nominal gross national product data and the money stock data are collected by the Bank for International Settlements (BIS). The broad money aggregates we consider are: the money stock M3 of the euro area and Canada, the money stck M2 plus the certificates of deposit of Japan, the money stock M4 of the United Kingdom and the money stock M2 of the United States. The bilateral exchange rates are collected by the Board of Governors of the Federal Reserve System.

[2] Being slightly asymmetric and increasingly flat with the lengthening of the maturity, the spot rates we consider reject the hypothesis of having a Gaussian distribution.

explains the evolution of the interest rates in maturity-time, while the other explains their behavior in calendar-time. For any given point t in calendar-time, first we model the dynamics of the forward rates in maturity-time, and then by applying eq. (1) we obtain the estimate of all the maturity spectrum of the spot rates. By reiterating the same process for all $t = 1, \ldots, 333$ we reconstruct the calendar-time series of the yields.

We begin by introducing the maturity-time model of the forward rates. The evolution of the forward rates in maturity-time is modeled in the state space with the following canonical, unforced, linear, time-invariant, continuous-time, dynamic systems of the 3^{rd}- order:

$$\begin{aligned} \begin{vmatrix} \dot{x_0}(m,t) \\ \dot{x_1}(m,t) \\ \dot{x_2}(m,t) \end{vmatrix} &= \begin{vmatrix} 0 & 0 & 0 \\ a & -a & 0 \\ 0 & b & -b \end{vmatrix} \begin{vmatrix} x_0(m,t) \\ x_1(m,t) \\ x_2(m,t) \end{vmatrix} && (2) \\ fw(m,t) &= x_2(m,t) + e(m,t) && (3) \end{aligned}$$

where state equation (2) consists of 3 first-order differential equations, 3 state variables $x_i(m,t)$, $i = 0, 1, 2$, a three−dimensional vector $\dot{\mathbf{x}}(m,t) \equiv d\mathbf{x}(m,t)/dm$ of state derivatives with respect to maturity-time, and a lower-triangular state transition matrix carrying in its principal diagonal the eigenvalues of the system. Specifically, the first eigenvalue is equal to zero, therefore the forward rates tend asymptotically to a constant value as $m \to \infty$. The other two eigenvalues, a and b, are time-invariant and coincide with the inverse of the time constants characterizing the impulse response of the system: $\tau_1 = 1/a$ and $\tau_2 = 1/b$. According to output equation (3), at point m in maturity-time, the level of the forward rate is determined by the state variable $x_2(m,t)$ plus an unknown reconstruction error $e(m,t)$ on whose distribution we make no assumptions.

The dynamic system of eqs. (2) and (3) has a unique solution,[3] which, for any point in time t, corresponds to the following forward rate curve:

$$\begin{aligned} fw(m,t) &= x_0(0,t) + [x_1(0,t) - x_0(0,t)]\, \tfrac{b}{b-a}\, \mathrm{e}^{-a\,m} + \\ &+ [(b-a)\; x_2(0,t) - b\, x_1(0,t) + a\, x_0(0,t)]\, \tfrac{1}{b-a}\, \mathrm{e}^{-b\,m} + e(m,t) \end{aligned} \qquad (4)$$

Given the relationship between forward and spot rates defined in eq. (1), from the the forward rate curve of eq. (4), for any point in time t, we obtain the following yield curve:

$$\begin{aligned} y(m,t) &= x_0(0,t) + \tfrac{b}{b-a}\, [x_1(0,t) - x_0(0,t)]\, \tfrac{1-\mathrm{e}^{-a\,m}}{a\,m} + \\ &+ \tfrac{1}{b-a}\, [(b-a)\; x_2(0,t) - b\, x_1(0,t) + a\, x_0(0,t)]\, \tfrac{1-\mathrm{e}^{-b\,m}}{b\,m} + \varepsilon(m,t) \end{aligned} \qquad (5)$$

where $x_i(0,t)$, $i = 0, 1, 2$ are the values of the state variables at the shortest maturity $m = 0$,

[3] See, e.g., C.-T. Chen, (1999).

i.e. the initial state values, and $\varepsilon(m,t)$ are the unknown spot rate reconstruction errors on whose distribution we make no assumptions.

3.2 Interpretation of the latent yield curve variables

The time-invariant parameter pair $(a,\,b)$ and, for any point in time t, the initial state values fully characterize the forward and the spot rate curves, because starting from $x_0(0,t)$, $x_1(0,t)$, and $x_2(0,t)$, the forward rates (and thus the spot rates) for the entire maturity spectrum, are obtained by solving eqs. (2) and (3) recursively. The model state variables are *latent* because even though they compress and summarize all the information contained in yield data, they are not directly accessible to measurement. Each initial state has its own economic meaning. Specifically, $x_0(0,t)$, which is maturity time m - invariant, thus $x_0(0,t) = x_0(\infty,t)$, corresponds to the *asymptotic final value* that both the forward and the spot interest rate curves take on at the longest maturity $m = \infty$. Then, the initial states $x_0(0,t)$ for $t = 1,\ldots,333$, summarize the information contained in the *long-end of the yield curve* (and thus in the long-term rates).

The initial state $x_2(0,t)$ defines the *starting value* of both the forward and the spot interest rate curves because it sets the value which the forward and the spot rates take on at the shortest maturity $m = 0$. Then, $x_2(0,t)$ for $t = 1,\ldots,333$, summarizes the information contained in the *short-end of the yield curve* (and thus in the short-term rates). Note that eqs. (2) and (3) imply that in maturity-time the state variable $x_2(m,t)$ tends asymptotically towards $x_0(m,t)$, hence $x_2(\infty,t) = x_0(\infty,t)$, according to a dynamic law characterized by the time constant $\tau_2 = 1/b$.

The initial state $x_1(0,t)$ links the starting value $x_2(0,t)$ to the asymptotic final value $x_0(0,t)$ therefore it *defines the shape* of the forward and spot rate curves. For example, if $x_1(0,t)$ is smaller than both $x_0(0,t)$ and $x_2(0,t)$ the yield curve exhibits an inverted hump, while if its level is included between the starting and the asymptotic final values, $x_2(0,t) < x_1(0,t) < x_0(0,t)$, the yield curve is positively and monotonically sloped. Then, $x_1(0,t)$ for $t = 1,\ldots,333$, summarizes the information contained in the *middle of the yield curve* (and thus in medium-maturity yields). Note that eqs. (2) and (3) imply that also the state variable $x_1(m,t)$ tends asymptotically towards $x_0(\infty,t)$, hence, $x_1(\infty,t) = x_2(\infty,t) = x_0(\infty,t)$, according to an exponential law specified by the function $\mathrm{e}^{-a\,m}$.

3.3 Estimate of the yield curve model

The dynamic system of eqs. (2) and (3) modeling the forward rates in maturity-time is designed in such a way that the knowledge of $fw(m,t)$ suffices to uniquely estimate the latent initial state values $x_i(0,t)$, $i = 1,2,3$, which are not directly accessible to measurement.[4] Given that the forward rate curve and the yield curve are both fully characterized by the same pair of parameters and, for any time t, by the same initial state values, we estimate $x_0(0,t)$, $x_1(0,t)$, and $x_2(0,t)$, for

[4] In other words, the dynamic system modeling the forward rates is "observable," see, e.g., C.-T. Chen, (1999).

$t = 1, \ldots, 333$, and we identify the parameters a and b by fitting the yield curve of eq. (5) to the data. Specifically, we start with an initial guess for the pair (a, b) − which we use to obtain the ordinary least squares estimates of the initial state values $x_i(0, t)$, $i = 1, 2, 3$, by employing the Householder transformations −and then we recursively apply the numerical algorithm based on the conjugate-gradient method contained in the software suite EicasLab to minimize the quadratic loss functional $Fspe = \sum_{m=0}^{20} \sum_{t=1}^{333} [\varepsilon(m,t)]^2$, where $\varepsilon(m,t)$ are the spot rate reconstruction errors. With the data we consider, the loss functional $Fspe$ is minimized by the unequal eigenvalues: $a = -0.037$ and $b = -0.073$, which correspond to the time constants $\tau_1 = 26.79$ months and $\tau_2 = 13.68$ months. This means that the yield curve of eq. (5) reconstructs the 6,660 yields characterizing the movements of the U.S. yield curve over the last three decades with a root mean square error of 5.301 basis points. The overall yield fitting results are summarized in Table 1 where we report the mean, standard deviation and the root mean squared error, expressed in basis points, of the reconstruction errors $\varepsilon(m, t)$.

Table 1: Summary statistics: the yield fitting errors

Maturity (*months*)	Mean (*basis points*)	Std. Dev. (*basis points*)	RMSE (*basis points*)	Autocorrelations Lag (1)	Lag (12)	Lag (30)
6	-0.644	3.945	3.997	0.749	0.102	0.056
12	0.120	2.513	2.516	0.731	0.117	-0.131
18	0.547	3.555	3.597	0.704	-0.015	0.013
24	0.404	3.111	3.137	0.722	0.041	-0.011
30	0.236	2.483	2.495	0.730	0.275	0.046
36	0.332	2.028	2.055	0.681	0.125	-0.108
42	0.565	1.960	2.040	0.550	0.023	-0.173
48	0.447	2.155	2.201	0.648	0.079	0.015
54	-0.056	2.274	2.275	0.736	-0.161	0.206
60	-0.675	2.502	2.592	0.601	-0.189	0.179
66	-0.979	2.867	3.030	0.535	0.212	0.007
72	-1.031	2.808	2.991	0.788	0.232	0.115
78	-0.791	2.602	2.719	0.817	0.094	-0.039
84	-0.409	2.553	2.586	0.758	-0.189	-0.127
90	-0.170	2.298	2.304	0.793	-0.235	-0.027
96	-0.207	2.014	2.025	0.852	0.014	-0.144
102	-0.387	2.051	2.087	0.887	0.014	0.011
108	-0.239	2.251	2.263	0.832	-0.131	0.136
114	0.674	2.826	2.906	0.755	0.084	0.068
120	2.261	4.168	4.742	0.855	0.336	0.147

This table reports the mean and the standard deviation of the fitting errors $|\varepsilon(m,t)| = |\tilde{y}(m,t) - y(m,t)|$ where $\tilde{y}(m,t)$ are the actual zero-coupon bond yields of maturity $m = 6, 12, \ldots, 120$ and t runs from 31 January 1980 to 30 September 2007, and $y(m,t)$ are the model-based yields of eq. 5. The fourth column reports the root mean squared fitting errors $RMSE = \sqrt{\frac{1}{333} \sum_{t=1}^{333} [\varepsilon(m,t)]^2}$ and the last three columns show the autocorrelation of the fitting errors at displacements of 1 month, 12 months and 30 months.

The model performs well in fitting all the maturities: the mean values of the fitting errors are negligible at all maturities, their standard deviations average 2.65 basis points and their root mean squared errors range between 2.02 and 4.74 basis points. All maturities exhibit a high, positive, first-order autocorrelation of the fitting errors which decreases, but not vanishes, with the lengthening of the displacement. Despite such persistent, yet contained, autocorrelation, the fitting performance of the model leaves virtually no room for any further reconstruction improvement[5] so that attempts to whiten the fitting errors completely would translate in an overfit of the model and, thus, into parameter instability. This becomes evident when we measure the explanatory power of the model with the norm ℓ^2. Specifically, the 6,660 yields in our sample measure $\|y\| = \sqrt{\sum_{i=0}^{20} \sum_{t=1}^{333} [y(m_i, t)]^2} = 625.29$, their reconstructed counterparts measure $\|\hat{y}\| = 625.27$ and the fitting errors measure $\|\hat{\varepsilon}\| = 4.33$. This means, first, that the model reconstructs the yields optimally and the fitting errors $\hat{\varepsilon}(m, t)$ are orthogonal to the reconstructed yields $\hat{y}(m, t)$, so that by the Pythagorean theorem $\|y\| = \sqrt{\|\hat{\varepsilon}\|^2 + \|\hat{y}\|^2}$, because $\frac{\|\hat{\varepsilon}\|^2 + \|\hat{y}\|^2}{\|y\|^2} = 100\%$; and, second, that the model explains as much as $\|\hat{y}\| / \|y\| = 99.995\%$ of the actual yields. Hence, what remains unexplained is far too small to justify the employment of another state variable without incurring in overstimation problems producing a too large sensitivity of the model parameters to small computation numerical errors.

4 The yield curve and the macro variables in the frequency domain

In this section, we recall the approach proposed by Donati and Donati (2007) to model in the frequency domain the yield curve and the other examined macroeconomic variables. We begin by providing a brief overview of the methodology, then we go through some of its technical aspects.

With the objective of putting in evidence that the movements of the yields, as of any other macroeconomic variable, are driven by a number of macroeconomic forces and that from the viewpoint of the policymaker it is important to know how much persistent the action exerted by such forces is, we distinguish between: 1) *long-run forces* producing enduring effects that may persist up to infinity, which in our model take at least 5.5 years to fully materialize; 2) *medium-run forces* producing transitory effects waning within business-cycle horizons, which in our model take at least 2.5 months to become fully evident and less than 5.5 years to abate; 3) *short-run forces* producing transitory and very short-lived effects which in our model get fully disclosed after minimum 1.2 months and die away within 2.5 months.

We identify the effects produced by these three types of forces by decomposing, at each point in time t, the time function of each variable into three components lying within three pre-specified frequency bands. Specifically, we partition the time function of the variable of interest into a *low-frequency component*, which tracks the action of the long-run forces, a *medium-frequency component*,

[5] On this issue see also Diebold, Rudebusch and Aruoba [2006], Bijörk and Christensen [1999], Bliss [1997] and Dahlquist and Svensson [1996].

which tracks the effect of the business-cycle forces, and a *high-frequency component*, which reproduces the effect of the short-lived forces. The frequency decomposition is carried out in such a way that the arithmetic sum of the low-, medium- and high-frequency components reconstructs the actual evolution of the variable.

In order to capture the cause-and-effect relationship between the economic forces and the evolution of the variable of interest in the frequency domain, each frequency components is modeled as the output of a linear, time-invariant, strictly causal, dynamic system subject to the action of inputs, or external causes, lying within the pre-specified frequency bandwidth. Of course, such inputs are unknown and need to be estimated starting from the data. We make no assumption on their statistical properties, and we reconstruct them by employing an input-output state observer with an embedded closed-loop control, which has been designed to act as a band-pass filter. In particular, the input-output state observer performs three tasks simultaneously: 1) it ensures that the frequency components of the variable of interest evolve within their specific bandwidth; 2) it ensures that the reconstructed variable stemming from the sum of the three frequency components tracks the patterns of actual variable; and finally 3) it corrects for model uncertainty, that is for the degradation in the model performance caused by unavoidable model simplifications and possible misspecifications and by the equally unavoidable data measurement errors.

This filtering approach, which works in the time domain, has been preferred to the techniques working in the frequency domain primarily because it does not require to perform the Fourier transformations of the signals. Moreover it has the advantage of working in real time. This means that the decomposition of the time function $z(\cdot)$ of the variable of interest carried out at the current time t_0, is performed without requiring the knowledge of the future values of $z(t)$, with $t > t_0$, and without altering the outcomes of the decomposition already performed at time $t' < t_0$. We opted for this approach also because it permits to extract from the data more than one frequency component at the same time, and in way that minimizes the information loss when switching from a frequency bandwidth to the other. In fact, given that the sum of the three frequency components reconstructs the actual pattern of the variable of interest, the oscillations whose period is neither significantly lower nor significantly higher than the selected frequency cuts end up with being captured by one of two neighboring frequency domains. In addition, we selected this approach because it can be applied also to nonstationary signals, and finally because it also corrects for model uncertainty.

Now we summarize the main technical aspects of the approach. Following Donati (1971), we partition the frequency range $[0 \div f_{max}]$ of the time function $z(t)$ of the variable of interest in the *low-frequency domain* $[0 \div f_{lf}]$, in the *medium-frequency domain* $[f_{lf} \div f_{mf}]$ and in the *high-frequency domain* $[f_{mf} \div f_{hf}]$ associating to each frequency domain the time intervals T_{lf}, T_{mf} and T_{hf}, respectively, which have been selected in such a way that $T_{lf} \cdot f_{lf} \gg 1$, $T_{mf} \cdot (f_{mf} - f_{lf}) \gg 1$ and $T_{hf} \cdot (f_{hf} - f_{mf}) \gg 1$ (see the Appendix for the details). Accordingly, we decompose the function $z(t)$ into a low-frequency component $z_{lf}(t)$, a medium-frequency component $z_{mf}(t)$ and a high-frequency component $z_{hf}(t)$, which, with a reasonable approximation, correspond to the partitions of the power spectrum $\Phi(f)$ of the signal $z(t)$ in the three ranges above. As a result, we obtain that

$$z(t) = z_{lf}(t) + z_{mf}(t) + z_{hf}(t) + we(t) \tag{6}$$

where $we(t)$, which is the residual of the frequency decomposition, is the component of the function $z(t)$ lying within the highest, residual, frequency domain $[f_{hf} \div f_{fmax}]$. We are not going to investigate $we(t)$ because from the point of view of this study, which works with monthly data, it is an unpredictable noise.

As mentioned above, the frequency components $z_{lf}(t)$, $z_{mf}(t)$ and $z_{hf}(t)$ are the outputs of three linear, time-invariant, strictly causal, dynamic systems, denoted M_{lf}, M_{mf} and M_{hf}, which are of the 2^{nd} order and have the following canonical representation:

$$\begin{aligned}
\begin{vmatrix} q_{j,1}(t+1) \\ q_{j,2}(t+1) \end{vmatrix} &= \begin{vmatrix} 1-a_j & -b_j \\ 1 & 1 \end{vmatrix} \begin{vmatrix} q_{j,1}(t) \\ q_{j,2}(t) \end{vmatrix} + \begin{vmatrix} u_j(t) \\ 0 \end{vmatrix} \\
z_j(t) &= \begin{vmatrix} 0 & 1 \end{vmatrix} \begin{vmatrix} q_{j,1}(t) \\ q_{j,2}(t) \end{vmatrix} \qquad j = lf, mf, hf
\end{aligned} \tag{7}$$

where $q_{j,1}(t)$, $q_{j,2}(t)$ are the two state variables of the system, u_j is its single input, and a_j, b_j are the system parameters which are in one-to-one correspondence with the system eigenvalues. In order to reconstruct the inputs u_j, for *j=lf, mf, hf,* steering the dynamics of the systems M_j, and thereby governing the evolution of the frequency components of the variable of interest, and to estimate the values taken by the state variables $q_{j,1}(t)$, $q_{j,2}(t)$ we use an input-output state observer. This has been designed to ensure that each modeled frequency component evolves within its frequency bandwidth, and that the sum of three frequency components tracks the actual time function $z(t)$ minimizing the residual *we(t)* of the spectral decomposition. The input-output state observer forms a linear, time-invariant, strictly causal, discrete-time, dynamic system of a dimension as big as the double of the order of the systems whose variables it estimates: since each M_j is of the 2^{nd} order and we use three frequency components to reconstruct the time function $z(t)$ of the variable of interest, the state observer is of the 12^{th} order, that is it works with 12 state variables as follows:

$$\mathbf{q}(t+1) = \mathbf{H}\mathbf{q}(t) + \mathbf{B}\mathbf{z}(t) \tag{8}$$

$$y(t+1) = \mathbf{G}\mathbf{q}(t+1) \tag{9}$$

As shown by eq. (8), at time t the state observer system receives as input the time function $z(t)$ of the variable of interest and through the 12- dimensional vector of state variables $\mathbf{q}(t)$ and the real, time-invariant, (12×12) - dimensional matrices $\mathbf{H}$ and $\mathbf{B}$ it computes the value of the state vector $\mathbf{q}(t+1)$. As shown by eq. (9) through the real, time-invariant, (12×12) - dimensional matrix $\mathbf{G}$ the state vector $\mathbf{q}(t+1)$ is turned into the output vector $y(t+1)$, which includes the one-step ahead estimates carried out at time t of the three pairs $(u_j,\ z_j)$ for j=*lf, mf, hf,* and thus of the predicted value $\hat{z}(t+1)$ taken on by the time function at time $t+1$. By solving recursively eq. (8), and iteratively imposing that $z(t+1)$ be strictly equal to the predicted value $\hat{z}(t+1)$, we obtain the out-of-sample forecasts of the frequency components z_j up to the desired forecast horizon. We are not going to elaborate further on this aspect here because the focus of this paper is on the spectral decomposition of $z(t)$ and not on its out-of-sample forecasting.

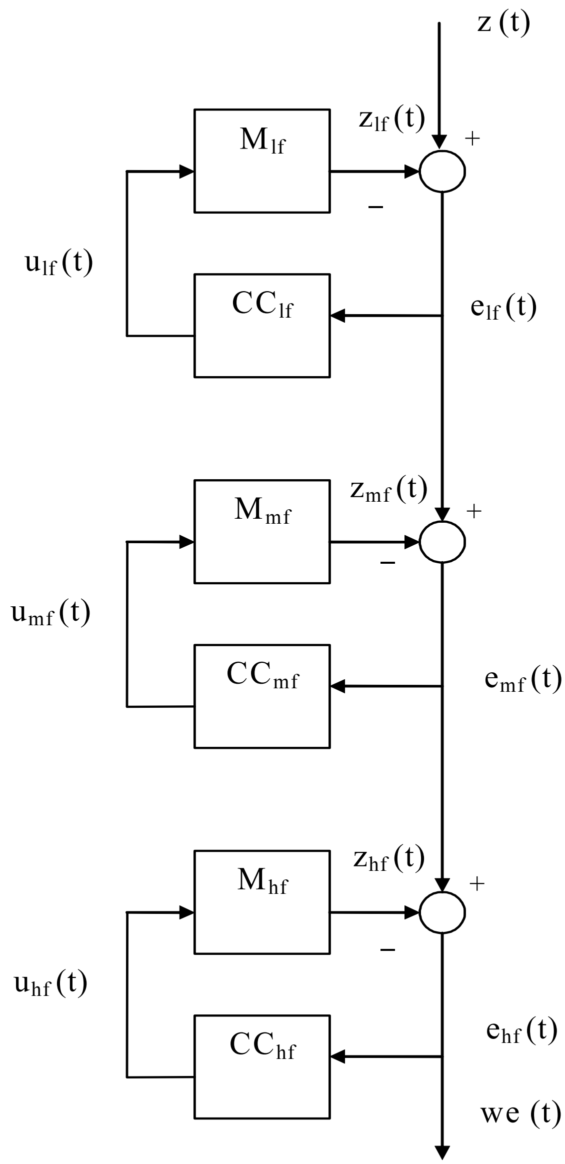
The 12 eigenvalues of the matrix $\mathbf{H}$ govern the behavior of the input-output state observer. We use 4 eigenvalues to extract[6] each of the three frequency components of $z(t)$. The frequency components are extracted by means of three successive loops as shown in Figure 1. To start with, the estimate of the low-frequency component $z_{lf}(t)$ produced by the dynamic system M_{lf} as in eq. (7), is contrasted with the actual value of $z(t)$. The state observer, by means of the feedback control system CC_{lf} embedded in it, reacts to the difference $z(t) - z_{lf}(t) = e_{lf}(t)$ and computes the input $u_{lf}(t)$ which then forces the dynamics of system M_{lf}. We exogenously impose the value 0.985 to the four eigenvalues of the input-output state observer that are used to extract the low-frequency component. This is equivalent to impose that $u_{lf}(t)$ lies within a bandwidth of angular frequency of 0.015 rad/month.[7] In this way we guarantee that $z_{lf}(t)$ evolves within the low-frequency domain $[0 \div f_{lf}]$. Next, the residual $e_{lf}(t)$ left after the low-frequency component has been extracted from $z(t)$, is contrasted with the medium-frequency component $z_{mf}(t)$. Through the input $u_{mf}(t)$, the feedback control system CC_{mf} guarantees that $z_{mf}(t)$ tracks $e_{lf}(t)$ within the selected medium-frequency domain. This is achieved by setting the four corresponding eigenvalues of the input-output state observer equal to 0.6, which is equivalent to impose that $u_{mf}(t)$ lies within a bandwidth of angular frequency $[0.015 \div 0.5]$ rad/month. The residual $e_{lf}(t) - z_{mf}(t) = e_{mf}(t)$ belongs to a higher frequency domain, which we contrast with the high-frequency component $z_{hf}(t)$. To compute the input $u_{hf}(t)$ that guarantees that the system output $z_{hf}(t)$ tracks $e_{mf}(t)$ within the pre-specified high-frequency bandwidth we use the feedback control system CC_{mhf} and we assign the value 0.2 to the four related eigenvalues of the input-output state observer. As a result, the power spectrum $u_{hf}(f)$ belongs to the angular frequency range $[0.5 \div 1.6]$ rad/month. The residual $e_{mf}(t) - z_{hf}(t) = we(t)$, which we do not investigate, receives the power of the time function $z(t)$ that lies within the angular frequency domain $[1.6 \div 3.14]$ rad/month.

The parameters (a_j, b_j) for $j = lf, mf, hf$, characterizing the dynamics of the systems M_j are

[6] See Donati and Donati (2007) for further details.

[7] Given the eigenvalue λ, we have that $e^{-\omega T} = 1 - \lambda$, where T is the sampling period, which in our case corresponds to 1 month, and ω is the angular frequency. Moreover, $\omega = 1/\tau = 2\pi f$ where τ is the time constant characterizing the observer impulse response and f is the frequency at which the frequency bandwidth of the observer is cut off as an effect of the eigenvalue λ.

Figure 1: Frequency decomposition of the time function $z(t)$



identified by minimizing a weighted quadratic loss functional defined on the out-of-sample forecast errors. Such errors are obtained when the systems M_j produce the estimates of $z_{lf}(t)$, $z_{mf}(t)$, $z_{hf}(t)$ without any feedback from the input-output state observer, that is when we impose that $e_{lf}(t) = e_{mf}(t) = e_{hf}(t) = 0$. We denote $fe(t,\tau) = z(t) - \hat{z}(t,\tau)$ the out-of-sample forecast error, where $\hat{z}(t,\tau)$ is the value of $z(t)$ that was forecasted $\tau = 1, \ldots, 24$ months before. Then we consider the weighted quadratic cost functional $Fer(t) = \sum_{\tau=1}^{24}[fe(t,\tau)]^2 w(\tau)$ where $w(\tau)$ is a negative exponential function that assigns to the forecast errors a weight decreasing with the lengthening of the prediction horizon τ. The values of the parameters (a_j, b_j), $j = lf, mf, hf$, are then identified by minimizing the quadratic cost functional $Ft = \sum_{t=48}^{333}[Fer(t)]^2$ through an iterative numerical minimization.

5 Estimate of the frequency components

In this section, we report some statistics on the frequency components extracted from the time functions of the variables of interest. In Table 2 we report the mean and the standard deviation, expressed in percentage points, of the residuals $we(t)$ from the frequency decompositions of the long-term rates (i.e. of $x_0(0,t)$), the medium-maturity rates (i.e. of $x_1(0,t)$), the short-term rates (i.e. of $x_2(0,t)$), the federal funds rate target (FFR), inflation (CPI), core inflation (CCPI) and the global monetary liquidity index (LIQ). The mean values of the decomposition residuals are very close to zero for all of the variables. This means that the data we consider do not contain any systematic information at very high frequencies, possibly because monthly observations obscure a substantial amount of very short-lived variations. The standard deviations of the decomposition residuals are small, thereby confirming that the information contained in such high frequencies is negligible. Only the residuals for the medium-maturity rates exhibit a standard deviation (of 0.25 percentage points) suggesting that the very high-frequency component of these especially volatile rates contains some, although modest, unpredictable information other than noise.

In Table 2 we report also the correlation coefficients between the frequency components partitioning the time functions of the same variables of interest. Ideally, the frequency components should be orthogonal to each other, and thus uncorrelated. Most of the presented correlation coefficients are very close to zero. However, it is worth recalling that, in order to show how the frequency components change over time, we perform the spectral decomposition over finite time intervals (see the Appendix) and that, by the Heisenberg uncertainty principle, this implies a necessarily limited resolution of the boundaries of each frequency bandwidth. In particular, this means that the frequency bandwidths overlap and that the oscillations of the variable of interest, whose periodicity is neither significantly lower nor significantly higher than two neighboring pre-specified frequency cuts, is divided between such two frequency components, which then turn out typically positively correlated. For this reason, we find that the correlation between the medium- and the high-frequency components of all variables but the medium-maturity rates, is small, but non-zero. Comin and Gertler (2006) to emphasize the interrelation between high and medium-frequency components combine them into a single "medium-term business cycle" frequency component. The high correlation (of 68%) between the medium- and the high-frequency component of core inflation indicates that this variable is little volatile, so that it is difficult to clearly disentangle between the power it displays within its non-low frequency bandwidths.

Table 2: Summary statistics: the frequency decomposition

Variable	Residuals		Correlations of FC		
	Mean	Std. Dev.	LF & MF	LF & HF	MF & HF
Long-rates	0.023	0.053	0.06	0.00	0.26
Mid-rates	0.009	0.254	0.10	0.11	0.08
Short-rates	0.013	0.126	-0.05	-0.02	0.22
FFR	0.014	0.122	-0.03	-0.05	0.38
CPI	0.010	0.111	-0.01	-0.04	0.28
CCPI	0.024	0.081	-0.09	0.22	0.68
LIQ	-0.002	0.004	-0.29	-0.05	0.20

This table reports the mean and the standard deviation of the residuals from the frequency decompositions of $x_0(0,t)$ which models the long-end of the yield curve, $x_1(0,t)$ which models the middle of the yield curve, $x_2(0,t)$ which models the short-end of the yield curve, the policy rate (FFR), inflation (CPI), core inflation (CCPI) and the global monetary liquidity index (LIQ). The last three columns of the table report the correlation between the low-frequency (LF) and the medium-frequency (MF) components, the low-frequency and the high-frequency (HF) components, and between the MF and the HF of the same variables of interest.

Table 3 presents a set of statistics on the volatility exhibited by each extracted frequency component, and thereby on the amount of power displayed by each considered variable within the three frequency bandwidths. While the reported statistics will be commented in detail in the following sections, we already highlight how, all in all, the considered variables have displayed a decline in the variability of their frequency components since the early 1990s. In particular, over the last fifteen years all the considered variables have exhibited a decline in the mean values and the standard deviations of their LF. Since we do not examine the behavior of global liquidity in the early 1980s, the mean and the standard deviation of its LF are computed only over the period 31 January 1992−30 September 2007. Similarly, the standard deviations of the MF and the HF of the long-term rates, the policy rate, inflation, and core inflation have declined over the period 31 January 1992−30 September 2007 compared with the period January 1985−30 September 2007. Only the MF and HF of the medium-maturity rates have, instead, increased. The combination of MF and HF denoted MFetHF, confirms that the response of the long-term rates to business-cycle and short-lived forces has progressively become more muted, and so did the response of core inflation. In contrast, the policy rate, the short-term rates and inflation have displayed an upturn in the volatility of their MFetHF over the last few years, starting from 2001.

Table 3: Summary statistics: standard deviations of the FC

Variable	LF				MF		HF		MFetHF		
	1985-2007		1992-2007		1985-2007	1992-2007	1985-2007	1992-2007	1985-2007	1992-2007	2001-2007
Long-rates	7.09	1.68	6.13	0.90	0.39	0.30	0.49	0.40	0.69	0.54	0.43
Mid-rates	6.65	2.06	5.46	1.06	1.02	1.08	1.84	1.93	2.17	2.29	2.26
Short-rates	4.95	1.23	4.25	0.29	1.44	1.44	0.63	0.56	1.69	1.69	1.74
FFR	5.08	1.40	4.28	0.38	1.34	1.33	0.57	0.48	1.65	1.64	1.67
CPI	2.99	0.28	2.85	0.09	0.62	0.41	0.64	0.51	1.01	0.67	0.81
CCPI	2.90	0.54	2.62	0.03	0.53	0.37	0.37	0.25	0.83	0.54	0.45
LIQ			1.50	0.17		0.06		0.02		0.06	0.06

This table in the first and in the third columns reports the mean and in the second and in the fourth columns reports the standard deviation of the LF of the variables considered over the periods 31 January 1985:30 September 2007 and 31 January 1992:30 September 2007, respectively. The remaining columns of the table report the standard deviations of the MF, the HF and a combination of the MF and the HF (MFetHF) of the considered variables over the same two time periods. The last column show also the standard deviation of the MFetHF for the period 31 January 2001:30 September 2007. All the statistics are expressed in percentage points.

Table 4: Summary statistics: persistence of the FC

Variable	LF				MFetHF			HF		
	Lag (1)	Lag (12)	Lag (24)	Lag (36)	Lag (1)	Lag (12)	Lag (24)	Lag (1)	Lag (12)	Lag (24)
Long-rates	0.98	0.79	0.59	0.42	0.91	-0.03	0.00	0.82	-0.18	0.07
Mid-rates	0.98	0.78	0.60	0.43	0.81	-0.14	0.12	0.71	-0.02	0.19
Short-rates	0.98	0.76	0.56	0.38	0.98	0.53	0.03	0.81	-0.07	-0.04
FFR	0.98	0.77	0.57	0.39	0.98	0.55	0.02	0.85	0.10	0.01
CPI	0.97	0.69	0.49	0.33	0.93	0.30	0.10	0.80	-0.14	0.06
CCPI	0.97	0.69	0.45	0.26	0.97	0.75	0.50	0.86	0.42	0.38
LIQ	0.97	0.72	0.52	0.37	0.97	0.74	0.47	0.81	0.14	0.07

This table reports the autocorrelation of the low-frequency component (LF), the combined medium- and high-frequency components (MFetHF) and the high-frquency components (HF) at displacements of 1 month, 12 months, 24 months, and for the LF of 36 months, extracted from the variables of interest.

Finally, Table 4 reports the autocorrelation of the extracted frequency components at displacements of 1 month, 12 months, 24 months, and for the LF also of 36 months. As expected, for each series the LF shows the highest persistence. For example, an increase of one percentage point in the LF of the medium-maturity rates at time t is likely to have declined only to 0.43 percentage points at time $t + 36$, i.e. after three years. Core inflation and the global liquidity index display the highest persistence of the effects produced by business-cycle and short-lived forces, combined. For example, if they increase by one percentage point above their LF level at time t, they are likely to be still around 0.50 percentage points above their LF level at time $t + 24$, i.e. after two years. In contrast, the MF and HF of the long-term rates exhibit significantly smaller persistence and so do the MF and HF of the medium-maturity rates, thereby suggesting that these rates are essentially driven by long-run, slowly evolving forces.

6 The yield curve and inflation in the frequency domain

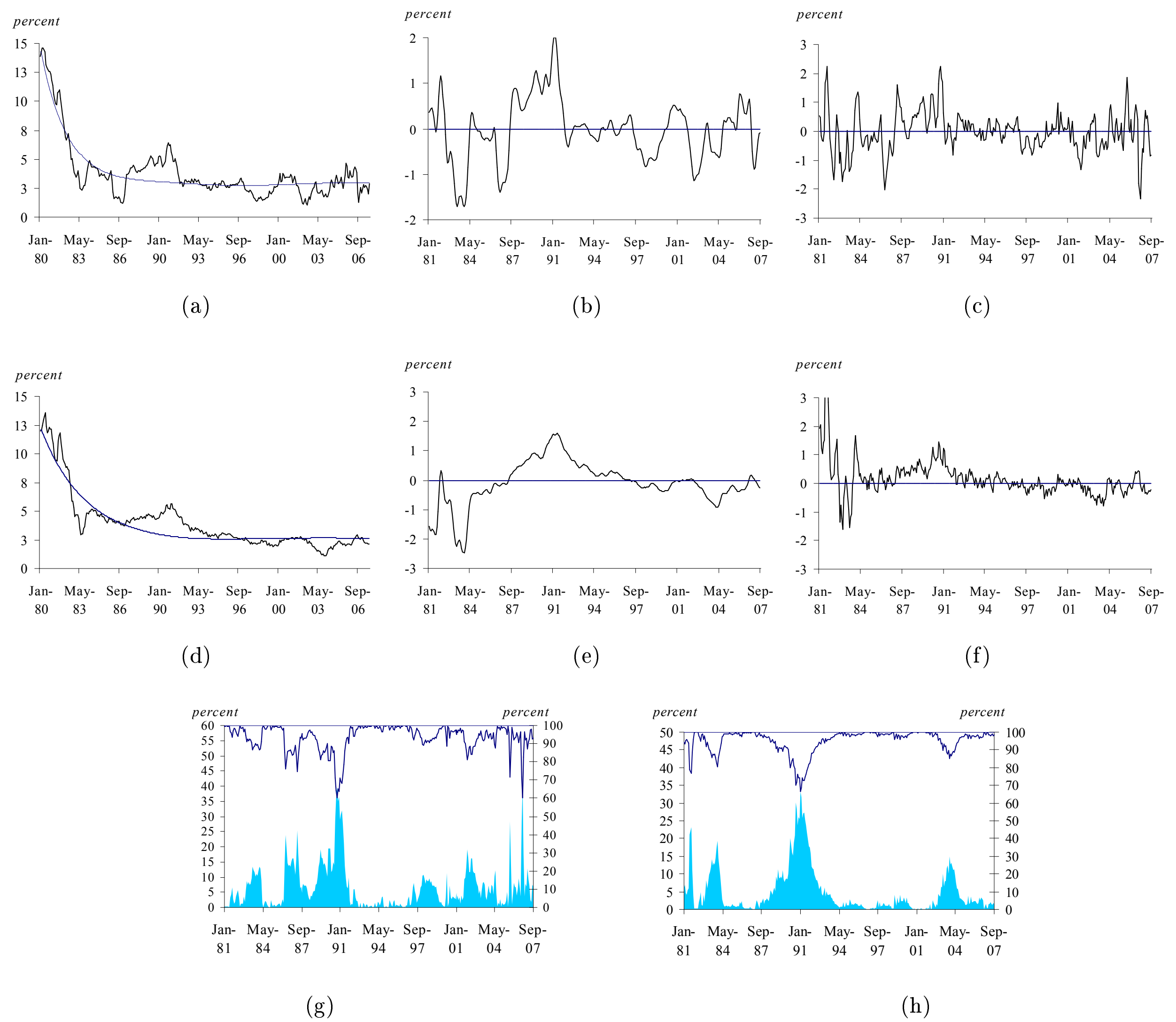
In this section we examine the relationship between the yield curve and inflation by investigating the interrelation among their frequency components. We show that over the last two decades U.S. inflation has fluctuated around a long-run anchor of about 3.0% and that the short-end of the U.S. yield curve has followed a similar pattern, fluctuating around a long-run level of about 4.0%. The long-end of the U.S. yield curve, although reacting timely to actual and expected consumer price changes, has instead displayed a steadily downward-trended underlying pattern and much less volatility.

6.1 The frequency decomposition of inflation

The frequency decomposition shows that during the disinflation of the 1980s the LF of annual inflation, which captures its enduring changes, underwent a marked decline (of about 11 percentage points) to stabilize, starting from 1992, at the level of 2.9% (with a standard deviation of 0.10 percentage points), as shown in panel (a) of Figure 2. Meanwhile, the LF of core inflation stabilized at the slightly lower level of 2.6% (with a standard deviation of 0.03 percentage points), as shown in panel (d) of Figure 2. These findings suggest that from the early 1990s onwards inflation has fluctuated around its almost flat low-frequency component. Similar results have been obtained using different methodologies by Coley and Sargent (2005), Cecchetti et al. (2007), who find that gross domestic product price inflation also followed an underlying trend anchored at about 2.2% over the past few years, and by Stock and Watson (2007), who investigate inflation by decomposing it into a permanent stochastic trend and a component modeling the transitory fluctuations around the trend.

At the same time, the MF and HF of inflation and core inflation have displayed progressively smaller variability as reported in Table 3 and shown in panels (b), (c), (e) and (f) of Figure 2. All in all, the exam of the amount of power displayed by inflation and core inflation within their frequency bandwidths across time reveals that: 1) over the three decades that we consider both inflation measures have exhibited most of their power within their low-frequency bandwidths, as shown on top (right-hand scale) of panels (g) and (h) of Figure 2, which partition the time functions of inflation and core inflation into their frequency components; 2) during the 1980s and early 1990s, when inflation was more volatile, the importance of their medium- and high-frequency bandwidths was higher, as shown on bottom (left-had scale) of panels (g) and (h) of Figure 2; finally, 3) some upturns in variability occurred also in recent years, during the deflation scare period of 2002-03 for what concerns core inflation, and since mid-2005 for what concerns inflation.

Figure 2: The frequency decompositions of inflation



This figure presents the spectral decomposition of U.S. headline inflation and core inflation. Panel (a) shows headline inflation and its LF; panel (b) shows its MF and panel (c) presents its HF. Panel (d) shows core inflation and its LF; panel (e) shows its MF and panel (f) presents its HF. Panel (g) shows the partition of inflation into its LF (top of the chart, right-hand scale) and the combination of its MF and HF (bottom of the chart, left-hand scale). Finally, Panel (h) shows the partition of core inflation into its LF (top of the chart, right-hand scale) and the combination of its MF and HF (bottomo of the chart, left-hand scale)and panel (g) shows the same tye of partition for core inflation.

Altogether, these facts suggest that over the last two decades U.S. inflation has generally remained well-anchored, possibly reflecting, as for example recently argued by Mishkin (2007), the pattern of U.S. survey-based long-run inflation expectations, which have stabilized at about 2.5% over the last few years.

6.2 The frequency decomposition of the yield curve

The exam of the frequency decompositions of the monetary policy rate and the short-term rates reveal a picture similar to the one displayed by inflation thereby suggesting that these variables are highly interrelated. Specifically, the LF of the federal funds rate target and of the short-end of the yield curve also underwent a marked decline in the 1980s to stabilize at about 4.3% (with standard deviations of 0.40 percentage points) and at 4.25% (with a standard deviation of 0.3 percentage points), respectively, since 1992, as reported in Table 3 and shown in panels (a) and (d) of Figure 3. In addition, the exam of the MF show that the monetary policy rate and the short-term rates have been fairly correlated with inflation (by about 60%) and core inflation (by about 40%), as shown in panels (b) and (e) of Figure 3 (since the medium-frequency components of the short-end of the yield curve and the federal funds rate are highly correlated, here we show each of them with a different inflation measure). Finally, the variability of the HF of the policy rate, the short-term rates and inflation have moved within the same ranges throughout all the years we investigate, as reported in Table 3, while core inflation has displayed much more contained volatility.

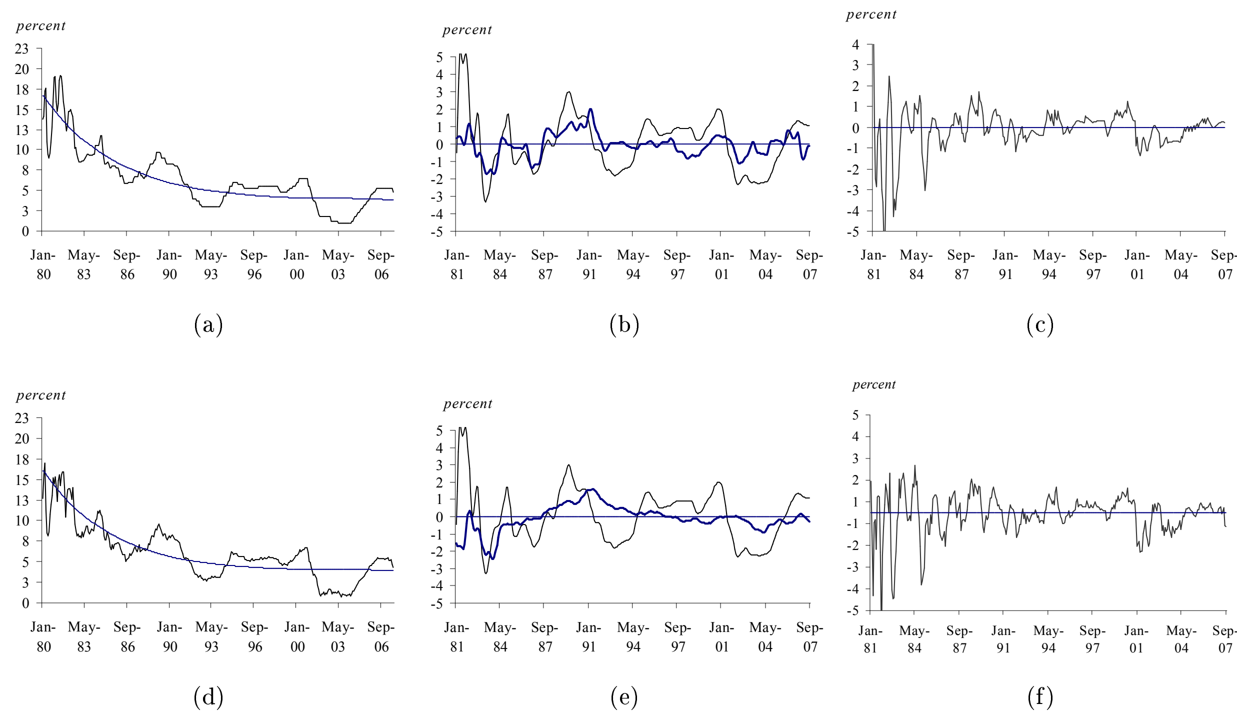
Granger causality tests run to investigate whether inflation and the federal funds rate target help predict each other at business-cycle frequencies, show that the hypothesis that the MF of inflation does not cause the MF of the policy rate cannot be rejected over the entire sample, and at increasingly longer lags since the 1990s, while we do reject the hypothesis that the MF of the policy rate does not Granger-cause the MF of inflation (see Table 5). Therefore, it appears that Granger causality runs one -way from the policy rate to inflation thereby supporting the view that through the years monetary policy has progressively more pre-emptively reacted to price pressures upturns and downturns.

Table 5: Summary statistics: Granger causality I

Period	1981:01-2007:09 Lags: 1		1981:01-2007:09 Lags: 18		1981:01-1991:12 Lags: 1		1992:01-2007:09 Lags: 7	
Null Hypothesis:	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob
MF inflation does not Granger cause MF policy rate	0.44	0.51	1.37	0.15	0.41	0.52	0.8	0.59
MF policy rate does not Granger cause MF inflation	7.68	0.00	2.70	0.00	7.33	0.00	1.33	0.24

This table reports pairwise Granger causality tests. Not to consider the first noisy estimates of the medium frequency (MF) components, the sample period begins on 31 August 1980.

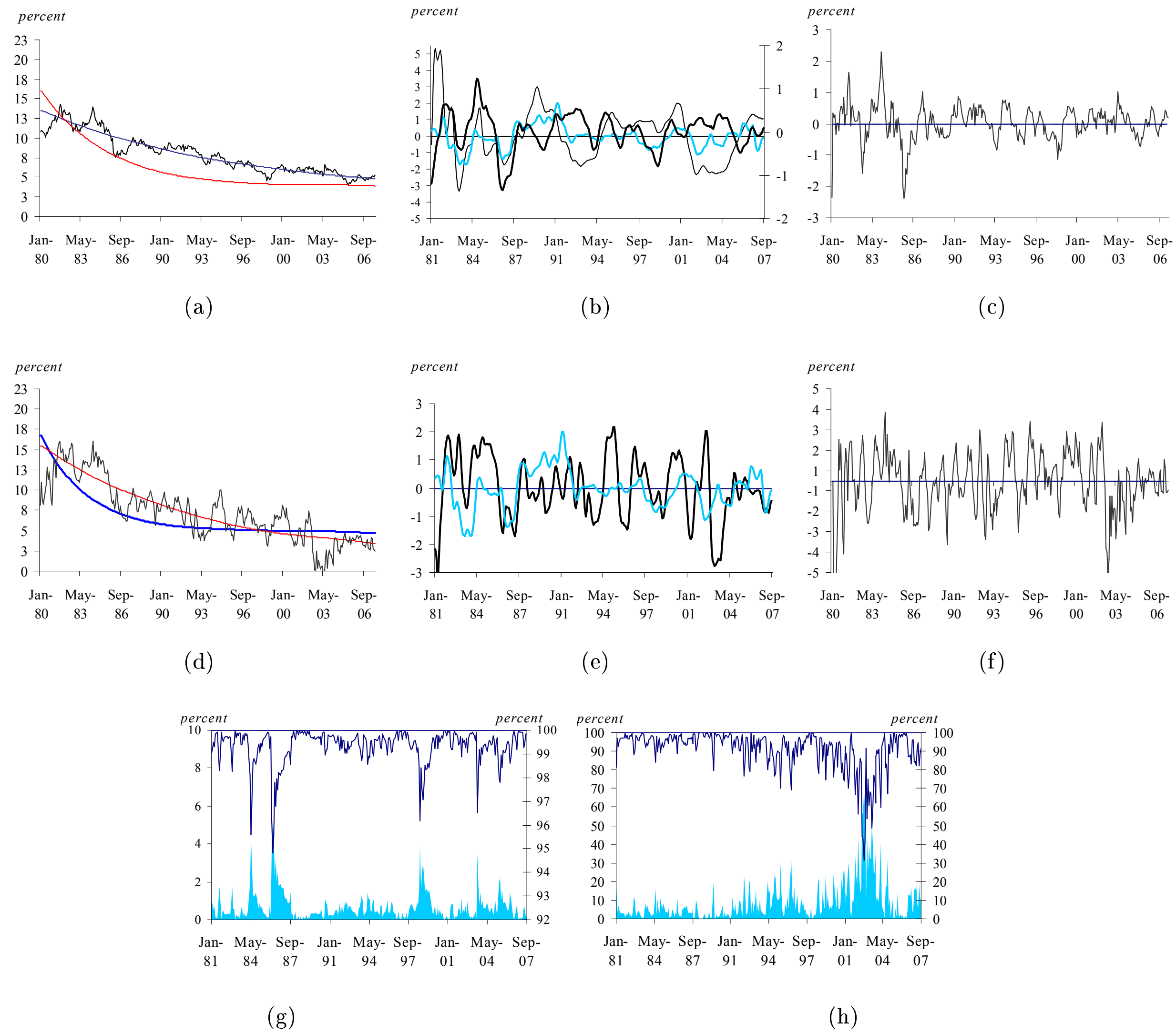
Figure 3: The frequency decompositions of the policy rate and the short-term rates



This figure presents the spectral decomposition of U.S. headline inflation and core inflation. Panel (a) shows headline inflation and its LF; panel (b) shows its MF and panel (c) presents its HF. Panel (d) shows core inflation and its LF; panel (e) shows its MF and panel (f) presents its HF. Panel (g) shows the partition of inflation into its LF (top of the chart, right-hand scale) and the combination of its MF and HF (bottom of the chart, left-hand scale). Finally, Panel (h) shows the partition of core inflation into its LF (top of the chart, right-hand scale) and the combination of its MF and HF (bottomo of the chart, left-hand scale).

The exam of the MF also shows how different the response can be when monetary policy reacts to core inflation as opposed to when it responds to a headline measure of inflation including all price items. Specifically, at the turn of the millennium the MF of the federal funds rate target and inflation trended downwards on the backdrop of the stock market correction of 2000 and the ensuing short recession of 2001. Yet, while the MF of headline inflation bottomed out in the middle of 2002, the MF of core inflation began bouncing back only in March 2004, as shown in panels (b) and (e) of Figure 3. The Federal Reserve, by focusing on the deflationary risks associated to the steadily downward-trended core inflation began tightening in June 2004, when headline inflation had been on the rise for almost two years and real output had been accelerating for almost three years. It is worth also noticing that business-cycle and short-lived forces appear to exert especially persistent effects on core inflation thereby rendering its dynamics more sluggish than that of headline inflation, as shown in Table 3.

Figure 4: The frequency decompositions of the medium- and long-term rates



This figure presents the spectral decomposition of medium- and long-term rates. Panel (a) shows the long-term rates, their LF and, on the bottom, the LF of the short-term rates; panel (b) shows the MF of the long-term rates (dark line, right-hand scale), the MF of inflation (light solid line, left-hand scale) and the MF of the policy rate (thin line, left-hand side); panel (c) presents the HF of the long-term rates. Panel (c) shows the medium-term rates, their LF and, on the bottom, the LF of the short-term rates; panel (e) shows the MF of the medium-term rates (dark line, right-hand scale), the MF of inflation (light solid line, left-hand scale); panel (f) presents the HF of the medium-term rates. Finally, Panel (h) shows the partition of the long-term rates into their LF (top of the chart, right-hand scale) and the combination of their MF and HF (bottomo of the chart, left-hand scale) and panel (g) shows the same tye of partition for the medium-term rates.

The medium- and the long-term rates appear interrelated with inflation developments less noticeably than the short-term yields (and the federal funds rate). To start with, their LF have continued to trend downwards also after the disinflation-period of the 1980s as opposed to stabilize at a constant level as the LF of the short-term rates, as shown in panels (a) and (d) of Figure 4.

Moreover, while the amount of power displayed by the medium-term rates within their medium-frequency has progressively increased, peaking during the deflation scare period of 2002 and 2003, the long-term rates appear on average less affected by transitory, medium- and short-run forces, than the rates at lower maturities, as shown in Table 3 and presented in panels (g) and (h) of Figure 4.

Nonetheless,throughout the entire period we investigate we cannot reject the hypothesis that the MF of inflation does not Granger-cause the MF of the long-term rates, while we do reject the hypothesis that the MF of the long-term rates does not Granger-cause the MF of inflation (see Table 6). Therefore, it appears that long-term rates typically react to changes in consumer prices in advance, and that they do so earlier than the short-term rates, as indicated by the comparison with the statistics reported in Table 5. This is possibly due to the fact that long-term incorporate market expectations, to which monetary policy itself reacts (see e.g. Goodfriend 1993), and because together with inflation they also anticipate the related monetary policy moves. Finally, the exam

Table 6: Summary statistics: Granger causality II

Period	1981:01-2007:09 Lags: 3		1981:01-2007:09 Lags: 18		1981:01-1991:12 Lags: 3		1992:01-2007:09 Lags: 15	
Null Hypothesis:	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob
MF inflation does not Granger cause MF long-rates	0.48	**0.70**	0.71	**0.81**	0.26	**0.85**	0.65	**0.83**
MF long-rates does not Granger cause MF inflation	2.55	0.06	1.45	0.11	2.89	0.04	0.92	0.54

This table reports pairwise Granger causality tests. Not to consider the first noisy estimates of the medium frequency (MF) components, the sample period begins on 31 August 1980.

of the MF suggests that during the "conundrum" period of 2004-05, labeled so by the then-Federal Reserve Board Alan Greenspan because the long-term rates did not appear to increase in response to a rising policy rate and sound economic conditions, the long-term rates reacted to increasing inflationary pressures. As shown in panel (e) of Figure 4, in in September 2000 the MF of the policy rate embarked on a decline that lasted until February 2002, and remained at a low level until May 2004. Yet, the MF of the long-term rates after an initial decrease, turned up in July 2001 and fluctuated, trending even slightly upwards, until mid-2004. When it finally began decreasing, the policy rate had already reverted upwards. Therefore, it seems that from mid-2001 to mid-2004, when the MF of the policy rate and the long-term rates moved in opposite directions, upwards the first and downwards the second, despite their exceptionally contained volatility the long-term rates were reacting to rising inflation.

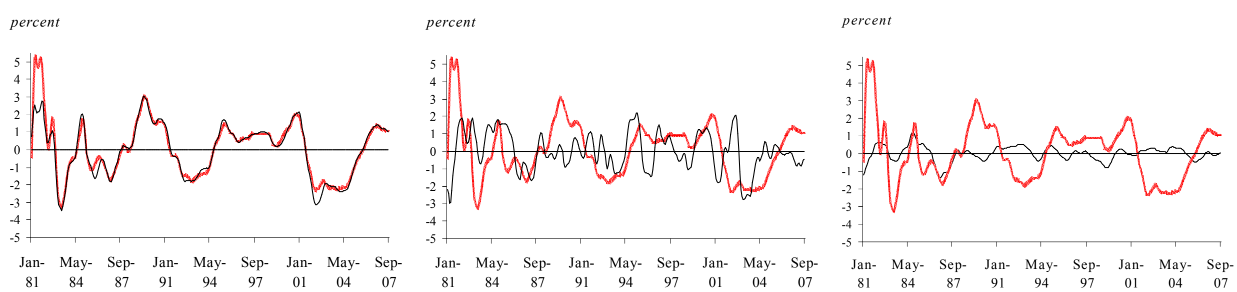
To conclude, actual and expected inflation appear to influence the movements of yields at all maturities. The target for the federal funds rate and the short-end of the yield curve seem noticeably interrelated with inflation both at low- and higher- frequencies, in line with the mandate of the Federal Reserve to maintain price stability. For what concerns the long-term rates, although but a decline in their variability has rendered their response to business-cycle forces more muted over the last few years, they appear to keep on anticipating by few months changes in consumer prices with their MF. Such decline in volatility suggests that the movements of the long-end of the yield curve have become more dependent on long-run forces, which however appear little related to inflation developments. In fact, the steadily declining LF of the long-term rates appears difficult to reconcile with the almost flat LF of inflation.

7 The yield curve and monetary policy in the frequency domain

In this section we examine the relationship between the yield curve and the federal funds target rate by investigating the interrelation of their frequency components. Against this background, we argue that while monetary policy influences the short-term rates, its direct effect on yields of longer maturities is limited.

Through the implementation of open market operations, the Federal Reserve aims to meet the target it sets for the federal funds rate by altering the conditions prevailing on the reserve market thereby encouraging its depositary institutions to trade federal funds. The most common form of open market operations are repurchase and reverse repurchase agreements. These operations are typically settled overnight or within few days and involve mostly, although not exclusively, the purchase and resale of short-term and medium-maturity U.S. Treasury securities. Trade in longer-term bonds and notes is less frequent.

Figure 5: The frequency decompositions of the policy rate and the yields



(a) (b) (c)

This figure presents the MF of the federal funds target rate and the short-term rates (panel (a)), the medium-mturity rates (panel (b)), and the long-term rates (panel (c)).

Given these characteristics, and in line with the findings of the literature (e.g. Hamilton (1997) and, more recently, Sarno, Thorton, Wen (2007)) we obtain that the MF of the federal funds rate target is highly correlated (by 96%) and virtually synchronous with the MF of the short-end of the yield curve, as shown in panel (a) of Figure 5, and that it is modestly correlated (by 21%) with the MF of the middle of the yield curve, as shown in panel (b) of Figure 5, although the medium-maturity yields seem to anticipate the changes in the policy rate at business-cycle frequencies, as reported in Table 7. This result needs not be in contrast with the expectation hypothesis of the term structure and is consistent with the findings of Sarno and Thorton (2003), who argue that if the market anticipates the changes in the policy rate, market rates may move in advance of the funds rate. However, we also obtain that the MF of the federal funds rate target is virtually uncorrelated (by 0.02%) with the MF of the long-term rates, as shown in panel (c) of Figure 5), but the long-term rates seem to anticipate the changes in the policy rate at business-cycle frequencies, as reported in Table 7.

Table 7: Summary statistics: Granger causality III

Period	1983:01-2007:09		1983:01-2007:09		1995:01-2007:09		1995:01-2007:09	
	Lags: 2		Lags: 10		Lags: 3		Lags: 10	
Null Hypothesis:	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob
MF mid-term rates do not Granger cause MF policy rate	3.59	0.03	1.29	0.23	10.31	3.E-06	3.56	0.00
MF policy rate does not Granger cause MF mid-term rates	0.46	**0.63**	0.47	**0.91**	0.74	**0.53**	0.77	**0.66**
	Lags: 3		Lags: 12		Lags: 3		Lags: 12	
Null Hypothesis:	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob	F-Stat	Prob
MF long-term rates do not Granger cause MF policy rate	2.45	0.06	1.61	0.09	3.81	0.01	2.31	0.01
MF policy rate does not Granger cause MF long-term rates	1.49	**0.22**	1.43	**0.15**	2.66	**0.05**	1.70	**0.07**

This table reports pairwise Granger causality tests. Not to consider the first noisy estimates of the medium frequency (MF) components, the sample period begins on 31 January 1983.

These findings seem to support the arguments originally made by Fand (1966) and confirmed for example by Dobell and Sargent (1969), according to which by operating essentially at the short- and medium-range of the maturity spectrum, central banks exert a necessarily limited influence on long-term rates. In particular, Dobell and Sargent (1969) suggest that, given the collinearity between interest rates at all maturities, "what can be done [...] is to investigate to what extent these rates appear to move independently." Therefore, we check whether monetary policy and the long-term rates have always moved in the same direction in response to changes in the state of the economy. We find that there are a number of episodes in which this has not occurred. Notably, this happened at the time of the 1957-1958 recession, when a substantial decline in the short-rates had very little repercussions on the long-rates (see Fand (1996)). Again, this happened at the time of the 1990 recession, when the policy rate was progressively decreased from 9.75% to 3.0% following economic growth and not inflation considerations and the long-term rates, after an initial decline, turned upwards, and held at high levels also when the policy rate had bottomed out, reflecting high fiscal indebtedness and the concerns of the market about the inflation outlook. Finally, this happened in 2004-05 giving rise to a conundrum as discussed in the preceding Section. Even in the most recent past, the MF of the policy rate and the long-end of the yield curve have moved in different directions, trending slightly downwards the first and rising the second, possibly reflecting economic growth and inflation developments, respectively (see panel (e) of Figure 4).

To conclude, we argue that the fact that the policy rate and the long-term rates move in the same direction when they react to the same economic developments, as for example inflation, does not mean that monetary policy directly controls the long-term rates. In fact, monetary policy and the long-term rates may react to different economic forces and in such case they appear to move in different directions. Finally, as already remarked in the preceding Section, beyond business-cycle horizons, the policy rate and the medium- and the long-term rates appear to have been driven by different long-run economic forces since the 1990s, as exemplified by the different patterns taken by their LF.

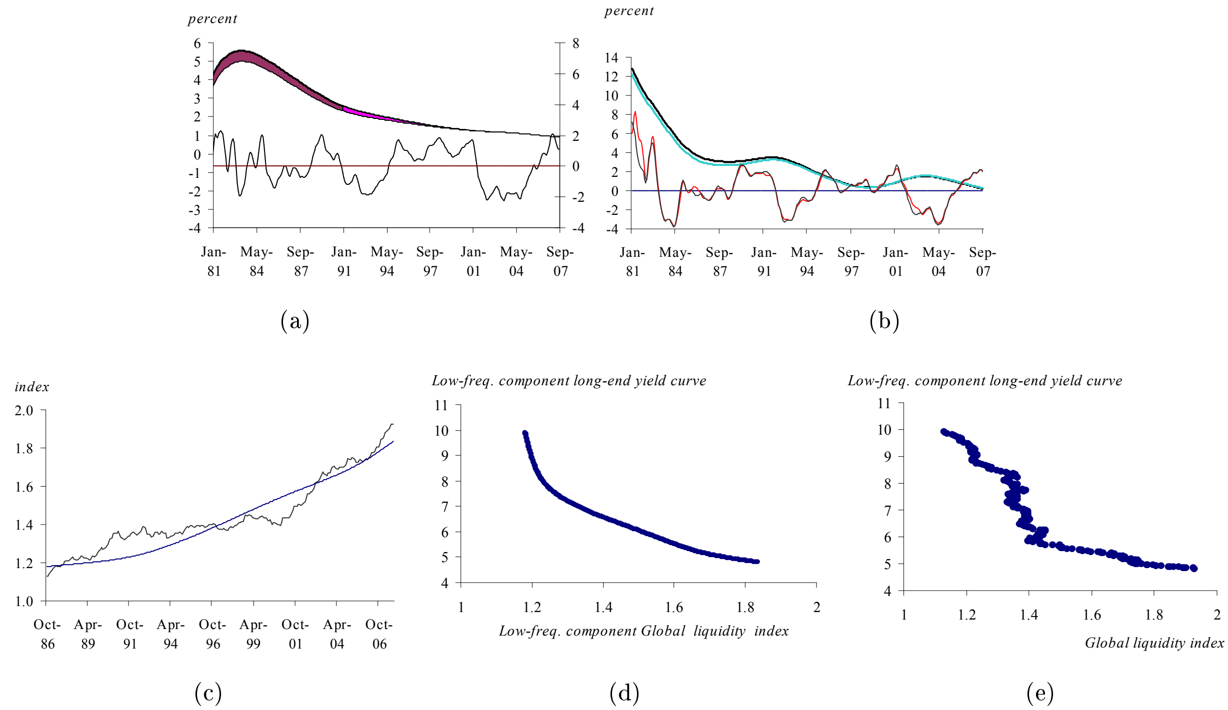
8 The yield curve, monetary policy and global liquidity

In this section we examine the relationship between the LF of the long-end of the yield curve and the secular evolution of global monetary liquidity.

The contrast of the frequency components of inflation and the short-term rates carried out in Section 6 suggests that the LF of the U.S. inflation-adjusted policy rate and short-term rates have steadily trended downwards for most of the period we investigate. In fact, since 1980 the LF of the inflation-adjusted policy and short-term rates, obtained by subtracting the LF of inflation from the LF of the policy rate and the short-end of the yield curve, have hovered around an average level included between 2.4% and 2.6%, and have moved along a downward trend after peaking in early 1983. In the meantime, since the end of the disinflation period of the 1980s, the MF of the real policy rate, obtained by subtracting the MF of inflation from the MF of the policy rate (which overlaps with the MF of the short-term rates) has remained above its LF level twice: from mid-1994 to mid-2001 and from early 2006 to today (see panel (a) of Figure 6). When we contrast the frequency components of the real short-term rates with the frequency components of real GDP annual growth, which we extract starting from the same frequency bandwidths and a similar dynamic model as for the other macro variables, we obtain that the difference between the LF of the short-term rate and real GDP growth has followed a steadily decreasing pattern (see panel (b) of Figure 6). Of course such estimates, while indicative, are inevitably imprecise. Yet, they point in the same direction of other findings, as for example the evaluation of the U.S. monetary policy stance made with the Taylor rule by Taylor (2007).

We speculate that this has stimulated the growing of U.S. external indebtedness – the U.S. current account, which was in balance or close to balance in the early 1990s, steadily deteriorated thereafter bottoming out at a 6.2% of GDP deficit in 2006 – and thereby to the outflows of financial capital. Meanwhile U.S. inflation remained moderate – thereby easily permitting the Federal Reserve to support the economy – thanks to a combination of factors including technological innovation, greater openness of emerging economies to international trade and the resulting upturn in competitive pressures and greater central bank transparency and credible anti-inflation commitments (see e.g. Borio (2006) and El-Erian (2007).

Figure 6: The long-term rates and global liquidity



This figure in panel (a) presents the LF (left-hand scale) and the MF (right-hand scale) of the real short-term rate; in panel (b) it shows the difference between the LF and MF of the real short-term rate and the those of real annual GDP growth; in panel (c) it shows the "excess" global liqudity index with its LF; in panel (d) it plots a scatter diagram with the LF of the "excess" global liqudity index on the x-axis and the LF of the long-term rates on the y-axis; finally, in panel (e) it shows the the "excess" global liqudity index on the x-axis and the LF of the long-term rates on the y-axis.

Under the assumption that inflation has increasingly been determined at the global level as shown by Borio and Filardo (2007), the high saving ratios of the rapidly developing economies running current account surpluses may have also contributed to offset the rise in inflationary pressures in the United States, notably because these countries spent the bulk of their trade profits not in purchasing products and services, but in acquiring financial assets to manage exchange rate regimes (see e.g. Geithner (2007)). Yet, while consumer price inflation in the most industrialized countries remained contained, the prices of financial assets and the real estate surged around the globe.

To investigate the relationship between the liquidity generated and exchanged on international markets (see, e.g. Bollard (2007)), globalization, and the upturn in asset prices, we examine the evolution of broad monetary aggregates, because they are characterized bya slow and persistent dynamics which appears especially suitable for the assessment of enduring, underlying trends. In particular, we use an “excess” global liquidity index obtained by dividing the sum of the nominal broad money stocks of the United States, the United Kingdom, the euro area, Japan, Canada and China with the sum of the nominal GDP of the same countries, being all variables converted into US dollars at current market exchange rates.[8] Such index exhibits a general upward-trended long-run dynamics (see panel c) of Figure 6) which underwent a noticeable acceleration in early 2001, when fiscal and external indebtedness of the United States were expanding, the long-term rates and the policy rates decoupled (see panel c) of Figure 5), and the MF of the real short-term rate as well as its difference with the MF of real GDP both moved into negative territory (see panel (a) and (b) of Figure 6).

The relationship between the yield curve and monetary liquidity is inherently non-linear and possibly circular. It is non-linear because higher liquidity directly props up the prices of the U.S. Treasury securities and only through them, which are inversely related to their yields, it pushes the interest rates down. It is possibly circular because lower interest rates may in turn support global liquidity rises: when the yields on U.S. Treasury securities become too low to compensate for their risk, investors may decide to reallocate their portfolios towards safer and more liquid asset as broad money. Moreover, low interest rate spur the demand for credit further. To check whether the build up of “excess” global liquidity of the last two decades may be related to the concomitant decline observed in long-term rates, we plot the global liquidity index and its LF and the LF of the long-term rates are plotted on two scatter diagrams (see panel (d) and (e) of Figure 6).

The resulting negatively sloped curve appears to corroborate the hypothesis that larger liquidity has been associated to decreases in long-term rates. In particular, the relationships displayed in panels (d) and (e) of Figure 6 suggest that the influence of (excess) global liquidity on long-term rates has progressively sated, given that the upturn in the global liquidity index since 2001 has been associated to smaller decreases in the LF of the long-term rates. It may also indicate that the declining trend underlying long-term rates has approached its lower bound. Finally, if the correlation between the two variables keeps on holding, it may also suggest that an upturn in the LF of the U.S. long-term nominal rates has to be associated to a sizable reduction in the (excess) global liquidity index.

9 Conclusion

In this paper we propose a frequency decomposition analysis to test theoretical assumptions and enhance our understanding on the relationship between the yield curve, monetary policy, inflation and global liquidity. We use a dynamic three-factor model to explain the term structure, designed in such a way that its three latent variables partition the maturity spectrum, and reproduce the

[8]See, e.g. "Excess global liquidity, asset prices and inflation" Inflation Report, Bank of England, February 2006.

effects exerted on the forward and the spot rates by a large number of macroeconomic forces divided according to how persistent their influence on the interest rates is. Similarly, we explicitly model in the state space the dynamics of the other macroeconomic variables of interest. To extract the frequency components from each variable, we explicitly model their evolutions under the assumption that they are driven by three types of forces: 1) long-run forces whose enduring effects drive the low-frequency component, 2) medium-run forces whose effects wane within business-cycle horizons and govern the medium-frequency component, and 3) short-run forces whose short-lived effects drive the high-frequency component. To explain the relationship between the economics forces and the frequency components, we model the latter as the outputs of linear, time-invariant, discrete-time, dynamic system identified starting from the data and controlled by exogenous inputs lying within three pre-specified frequency bandwidths. Such inputs are computed by a dynamic filter, which acts as a band-pass filter, works in real time and in the time domain, corrects for model uncertainty and ensures that the arithmetic sum of the frequency components reconstructs the actual history of the variable of interest.

We find evidence of a direct relationship holding at all frequencies between inflation, monetary policy and the short-end of the yield curve. In particular, this suggests that over the last three decades, through its direct control on short-term interest rates, the Federal Reserve has been able to efficiently maintain price stability. We obtain that also the long-term rates react to actual and expected changes in the consumer price level at business-cycle frequencies, although their long-run pattern appears to be driven by forces other than those governing inflation and monetary policy. Furthermore, we cannot reject the hypothesis that the policy rate does not (Granger) cause the long-term rates also at business-cycle frequencies.Yet, we argue that this does not mean that monetary policy does not influence the long-term rates. We suggest that the relationship between monetary policy and the long-term rates can be assessed at long-run horizons through the effect that monetary policy exerts of the monetary liquidity conditions. In the current increasingly integrated global economy the variable to examine is a indicator of global monetary liquidity. However, we argue that taking a global focus needs not diminish the effect of domestic policies. In fact, we find evidence that the pattern underlying the U.S. inflation-adjusted real short-term rate has been declining since early 1983, thereby holding at levels lower than the underlying pattern displayed by U.S. real GDP growth. We speculate that this has fostered the build up of U.S. external indebtedness and thereby the accumulation of "excess" global monetary liquidity. Not being spent to purchase consumer goods and services, such liquidity appears to have fed into the price of those U.S. Treasury securities not directly affected by open market operations. In fact, we find evidence of a long-run relationship between the progressive upturn in "excess" global monetary liquidity and the progressive decline observed in the underlying pattern of medium- and long-maturity yields. We conclude by quoting A. Greenspan (2007) "monetary policy should make even a fiat money economy behave "as though anchored by gold.""

Appendix I

Consider the discrete-time function $z(t)$ that we want to investigate in the frequency domain. The function $z(t)$ is a finite power sampled time function, satisfying the condition, $\lim_{N\to\infty} \frac{1}{2N} \sum_{t=-N}^{N} [z(t)]^2 = Pz < \infty$ where Pz is the mean power of the signal $z(t)$. Being the sampling unit considered in this paper equal to one month, the upper limit of the frequency domain is $f_{max} = 0.5\, cycles/month = \pi rad/month$. Given that the mean power Pz is finite, the function $z(t)$ is not Fourier transformable. In this case, if $z(t)$ follows an ergodic stationary process, Pz is distributed in the frequency domain with a power spectrum $\Phi(f)$, such that: $Pz = \int_0^{fmax} \Phi(f)\, df$. If $z(t)$ follows a nonstationary processes, as the macrovariables that we consider, the signal power is likely to be time-varying. In this case, as shown by Donati (1971), it is possible to define a *class* of time-varying power spectral functions $\varphi(f_i, t)$, where f_i, with $i \in (1, Nf)$, denotes the frequency values belonging to a finite set of Nf elements, with the following properties:

- $pz(t) = \sum_{i=1}^{Nf} \varphi(f_i, t)$ is a "*locally time averaged*" instantaneous power obtained by a suitable smoothing of the signal instantaneous power $[z(t)]^2$, such that: $Pz = \lim_{N\to\infty} \frac{1}{2N} \sum_{t=-N}^{N} pz(t)$;
- $\bar{\Phi}(f_i) = \lim_{N\to\infty} \frac{1}{2N} \sum_{t=-N}^{N} \varphi(f_i, t)$, is a "*locally frequency averaged*" power spectral value such that $Pz = \sum_{i=1}^{Nf} \bar{\Phi}(f_i)$. If the signal $z(t)$ is the realization of an ergodic stationary stochastic process, the power spectrum $\bar{\Phi}(f_i)$ corresponds to a "*locally frequency averaging*" of the stochastic process power spectrum $\Phi(f)$.

The elements $\varphi(f_i, t)$ of the time-varying power spectrum class are related to the criteria selected to perform the local averaging in the time and frequency domains. While different averaging criteria may be adopted, they should meet the following general rules:

1. A weighted averaging approach must be applied, with the weighting function defined in such a way that the averaged value may be attributed (even roughly) to a *finite interval,* whose amplitude is denoted T when referring to the *time interval,* and Δf when referring to the *frequency interval.* The intervals of amplitudes T and Δf define the *finite resolution* of the performed averages. As a result, in the time domain, two values $\varphi(f_i, t_1)$ and $\varphi(f_i, t_2)$ cannot differ significantly if the time instant difference $\|t_2 - t_1\|$ is not significantly larger than T. Similarly, in the frequency domain, two values $\varphi(f_1, t)$ and $\varphi(f_2, t)$ cannot be significantly different if $\|f_2 - f_1\|$ is not significantly larger than Δf.

2. The time-varying power spectral decomposition is possible only by adopting finite resolutions T and Δf such that $T \cdot \Delta f \gg 1$.

3. If we adopt the greatest time resolution $T = 1\, month$, which is equal to the sampling unit, then the required frequency resolution is $\Delta f = fmax$ and then no spectral decomposition is possible.

4. If we adopt the greatest frequency resolution, which in our case is $\Delta f = 1/333$ *cycles/month* since our data extend to 333 monthly observations, then the required time resolution coincides with all the time interval (running from 1980:01 to 2007:09) and no time-varying power spectrum may be considered, but only the power spectrum of the power averaged over all the available time series data.

Having clarified the above conditions, given the aim of this paper, we decide to opt for a good resolution in the time domain and then to accept a low resolution in the frequency domain. As a result, we decompose the time series $z(t)$ in only four spectral components: a low-, a medium-, a high-frequency component and a residual decomposition error which belongs to a residual very frequency domain, which we do not investigate.

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