A Model to Analyse Financial Fragility[*][†]

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Abstract

Our purpose in this paper is to produce a tractable model which illuminates problems relating to individual bank behaviour and risk-taking, to possible contagious interrelationships between banks, and to the appropriate design of prudential requirements and incentives to limit 'excessive' risk-taking. Our model is rich enough to include heterogenous agents (commercial banks and investors), endogenous default, and multiple commodity and credit markets. Yet, it is simple enough to be effectively computable. Financial fragility emerges naturally as an equilibrium phenomenon.

In our model a version of the liquidity trap can occur. Moreover, the Modigliani-Miller proposition fails either through the nominal frictions or through incentives for differential investment behaviour because of capital requirements. In addition, a non-trivial quantity theory of money is derived, liquidity and default premia co-determine interest rates, and both regulatory and monetary policies have non-neutral effects.

In the second part, we present comparative static results of a parameterised version of the model. This may shed light on the effect of regulatory policies on commercial banks and financial fragility. They also indicate how monetary policy may affect financial fragility, thus highlighting the trade-off between financial fragility and economic efficiency.

[*]The views expressed are those of the authors and do not necessarily reflect those of the Bank of England, London School of Economics, or the University of Oxford.

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1 Introduction

It is a truism that the structure of a model needs to reflect the practical purposes which drive the research in the first place. In our case, we work for the Financial Stability Division of the Bank of England; our aim is to construct a model which illuminates problems relating to individual bank behaviour and risk-taking, to possible contagious inter-relationships between banks, and to the appropriate design of prudential requirements and incentives to limit 'excessive' risk-taking.

In order to reduce a model of the aggregate economy to manageable proportions, a common simplification is to assume that each sector has agents which behave identically, so that they can be presented in 'representative agent' format. So, in most models the banking system is represented by a single agent, which can either be viewed as a set of perfectly competitive identical banks, or, on occasions, as a single, monopolistic bank.

While the representative agent approach has many uses and advantages, applying it to the banking system inevitably obscures many of the economic and behavioural relationships, notably between banks, in which a regulatory authority is closely interested. For example, with a single 'representative' bank, there can be no interbank market. Again, either the whole banking system, as represented by the one agent, fails, or the whole banking system survives in face of some assumed shock. Typically in reality individual banks have differing portfolios, often reflecting differing risk/return preferences. So, typically, failures occur with the greatest probability amongst the riskiest banks. Such failures in turn generate interactions in the system more widely that may threaten the survival of other banks, a process of contagion. This may have several channels, both in interbank relationships more directly, and via changes in asset market flows and prices that may involve other sectors, e.g. persons and companies. Such interactions can hardly be studied in a model with a single representative bank, since many of these interactions, e.g. the interbank market, are ruled out by definition.

So the main innovation in our model is to incorporate a number of commercial banks. Each bank is distinguished by a unique risk/return preference. Since each bank is, and is roughly perceived as being, different, it follows that there is not a single market either for bank loans or bank deposits.[1] Instead, we assume that there is a separate market, with differing interest rates, in each case. We also allow individual non-bank agents to differ, with differing utility functions, and, hence differing attitudes towards potential bankruptcy; we also model in their case the incentives for avoiding bankruptcy, as we do in the case of the banks. Again we assume that the banks can observe the agents' differing riskiness with noise. This means that each borrower faces a different credit market. If each bank had its own individual, idiosyncratic information on each borrower, then if there were H borrowers and B banks, there would be $\frac{H!B!}{(H-1)!(B-1)!}$ bilateral markets for borrowing. If we assume, instead, that each bank has the same information on each borrower, an implausible assumption, then competition between banks would mean that there would be H separate markets. Alternatively, we can assume that borrowers have been pre-allocated at time t=0 to a particular bank, and that that allocation provides each respective bank with specialised information; such additional information allows them, for asymmetric information standard reasons, to lend cheaper than any other bank. Consequently there are B separate credit markets between each bank and a subset of borrowers that were initially randomly allocated.

This means that, instead of a single market for deposits/loans, we have multiple markets

[1] There would be such a single market if there was 100% complete deposit insurance in place, since then all deposits would be similarly riskless in any bank. We could adjust the market to take account of deposit insurance, with varying coverage, in future extensions.

for deposits (by separate bank) and for loans (by borrower and bank). Given the optimising conditions for the individual banks, after assuming an initial allocation of capital, the open market operations of the Central Bank, etc., etc., deposits may not be sufficient in each individual bank (plus capital), to finance that bank's asset portfolio (of cash, loans and Central Bank (public sector) debt), although within the banking sector as a whole outside liabilities must equal outside assets, and interest rates and/or cash flows adjust until that happens. So, deficit (surplus) banks borrow (lend) on the interbank market. In reality the interbank market is also segmented with banks of differing riskiness either borrowing at different rates, or facing limited 'caps' on such borrowing. At this stage in our exercise, however, we shall assume a single, undifferentiated interbank market with a common interest rate.

Since our focus is on financial fragility, the governance (public sector) institutions which we introduce are a financial regulator and a Central Bank; these two may, or may not, be the same institution, but will be assumed to cooperate where necessary. We abstract from fiscal policy. The financial regulator sets the penalties/incentives on bankruptcy in both the banking and the non-banking private sector, and also the required (minimum) capital adequacy ratios.

The Central Bank is established at time $t = 0$ with an allocation of (public sector, safe, fixed nominal value) debt as its assets. Against this it has as its liabilities cash, commercial banks reserve deposits and Central Bank debt. Deposits and Central Bank debt are held by the commercial banks only, in an initial allocation, against an equivalent initial allocation of capital. Central Bank open market operations exchange its own (interest-bearing) debt for (non-interest bearing) deposits. Moreover, the Central Bank can lend, or borrow, in the interbank market.

In principle the non-bank public can insist on converting its commercial bank deposits into currency or into Central Bank deposits. It is this convertibility commitment that forces commercial banks to hold Central Bank deposits. Again we assume an initial allocation of Central Bank cash to the public, and model the public's choice between (safe) cash, which is non-interest-bearing, and deposits, which are risky but interest-bearing, and can be used for expenditures, and other risky (non-liquid) aspects.

The present model is based on the model introduced by Tsomocos (2003a) and (2003b) which introduced a commercial banking sector and capital requirements in a general equilibrium model with incomplete markets, money and default. However, we depart by introducing the possibility of capital requirements' violation and consequent penalties and a secondary market for the banks' equity. Moreover, we introduce limited access to consumer credit markets, thus allowing for different interest rates across the commercial banking sector. Finally, we simplify the model by removing the intratemporal loan markets and allow only for intertemporal borrowing and lending.

The closest precursor to this approach is the work of Shapley and Shubik (1977), Shubik (1973) and Shubik (1999) who introduced a central bank in a strategic market game. Shubik (1973) also emphasised the virtues of explicitly modeling each transaction (see also Grandmont (1983), Grandmont and Laroque (1973), Grandmont and Younes (1972 and 1973) who introduced a banking sector into general equilibrium with overlapping generation). The commercial banking sector follows closely Shubik and Tsomocos (1992). The modeling of money in an incomplete markets framework is akin to a series of models developed by Dubey and Geanakoplos (1992, 2003a, and 2003b) and by Drèze and Polemarchakis (2000) and Drèze, Bloise, and Polemarchakis (2002). Finally, default is modelled as in Dubey, Geanakoplos and Shubik (2000), Shubik (1973), and Shubik and Wilson (1977), namely by subtracting a linear term from the objective function of the defaulter proportional to the debt outstanding.[2]

For the rest we have tried to keep this model as simple, standardised and parsimonious as we can. The structure of the paper is as follows. In section 2, we set out the basic form of the model. In sections 3 and 4, we formally define the budget sets of the non-bank public, commercial banks and Monetary Equilibrium with Commercial Banks and Default (MECBD). In section 5, we show under which conditions a MECBD is achieved. We provide conditions such that trade would be beneficial to traders even under the presence of positive interest rates, and asset markets will be active irrespective of the possibility of positive default in equilibrium. Thus in a MECBD positive default and financial fragility are compatible with the orderly functioning of markets. So, given default and financial vulnerability, there is room for economic policy to improve upon the ensuing inefficiencies.

A formal definition of financial fragility is proposed in section 6, borrowed from Tsomocos (2003a and 2003b). Also, the Keynesian liquidity trap holds in equilibrium in which commodity prices stay bounded whereas volume of trade in the asset markets tends to infinity whenever monetary policy is loosened. Also, the constrained inefficiency of equilibria is shown; thus justifying the role of economic policy whenever financial fragility is present in the economy. In section 7, we note that Hicksian elements of the demand for money are active in equilibrium. We establish the regulatory and monetary non-neutrality that characterise the lack of the classical dichotomy between the real and nominal sectors of the economy. We also show that a non-trivial quantity theory of money holds and the liquidity structure of interest rates depends on aggregate liquidity and default in the economy.

Using the principles derived, we proceed in section 8 to analyse concrete comparative statics changes in computable general equilibrium models. We rationalise the outcomes by tracing the new equilibrium. The results are based on the aforementioned principles that must hold simultaneously in equilibrium. For example, a change in monetary policy or capital requirements must satisfy contemporaneously the quantity theory of money, the liquidity structure of interest rates and the Fisher relation. Finally, we conclude in section 9 and all the proofs are relegated in the appendix.

This framework is more elaborate, but we believe that it also offers new insights into the analysis of financial fragility and systemic risk. We doubt whether contemporaneous models, without heterogenous agents, can adequately handle analysis relating to liquidity, default and contagion. After we gain experience with this model through parametric examples it is then logically conceivable to be able to derive our comparative statics results in a more general context.

2 The Model

2.1 The Economy

Consider the standard general equilibrium model with incomplete markets in which time extends over two time periods. The first period consists of one state and the second period consists of S possible states. Figure 1 describes the state space of the model. At $t = 0$, non-bank private sector (NBPS), commercial banks and the authorities take their decisions expecting the realisation of any one of the S possible future scenarios to occur. At t=1 one of the S state occurs and then again the economic actors take the appropriate decisions. A

[2] Dubey and Geanakoplos (1992) have studied more general default specification and also introduced the gains-from-trade hypothesis that guarantees the positive value of fiat money in finite horizon.

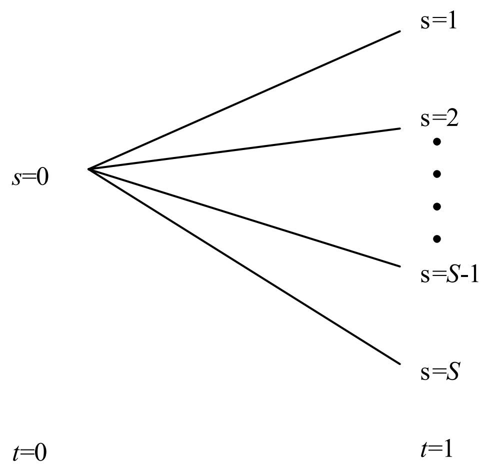

Figure 1: State Space

detailed explanation of the sequence of events is contained in section 2.3 and figure 2. NBPS and commercial banks transact, maximising their respective objective functions, whereas the central bank and the regulator are modelled as 'strategic dummies' (i.e. their choices are exogenously fixed and are common knowledge to economic agents). NBPS trade in commodities, financial assets, consumer loans, deposits and shares of commercial banks. Commercial banks lend to the consumers and take deposits, they borrow and lend in the interbank credit market, invest in the asset market and issue equity in the primary market. The central bank operates in the interbank market via OMOs. The regulator fixes the bankruptcy code for households and commercial banks exogenously and sets the capital-adequacy requirements for commercial banks.

Formally, the notation that will be used henceforth is as follows:
$t \in T = \{0, 1\}$ = time periods,
$s \in S = \{1, ..., S\}$= set of states at $t = 1$,
$S^* = \{0\} \cup \{S\}$= set of all states,
$h \in H = \{1, ..., H\}$= set of economic agents (households/investors),
$b \in B = \{1, ..., B\}$=set of commercial banks,
$l \in L = \{1, ..., L\}$=set of commodities,
$R_+^L \times R_+^{SL}$ = commodity space indexed by $\{1, ..., S\} \times \{1, ..., L\}$
$e^h \in R_+^L \times R_+^{SL}$= endowments of households
$m^h \in R_+^L \times R_+^S$= monetary endowments of households
$e^b \in R_+^{S^*}$=capital endowments of commercial banks
$u^h : R_+^L \times R_+^{SL} \to R$= utility function of agent $h \in H$,
χ_{sl}^h = consumption of commodity l in state s by $h \in H$.
The standard assumptions hold:

(A1) $\forall s \in S^*$ and $l \in L$, $\sum_{h \in H} e^h_{sl} > 0$,

(i.e. every commodity is present in the economy).

(A2) $\forall s \in S^*$ and h $(b) \in H$ (B), $e^h_{sl} > 0$ $(e^b_{sl} > 0)$ for some $l \in L$, $s \in S^*$,

(i.e. no household (commercial bank) has the null endowment of commodities (capital) in any state of the world).

(A3) Let A be the maximum amount of any commodity sl that exists and let $\mathbf{1}$ denote the unit vector in R^{SL}. Then $\exists Q > 0 \ni u^h(0, ..., Q, ..., 0) > u^h(A\mathbf{1})$ for Q in an arbitrary component

(i.e. strict monotonicity in every component).[3] Also, continuity and concavity are assumed.

Let $u^b(\pi^b_0, \pi^b_1, ..., \pi^b_s) : R^{S^*}_+ \rightarrow R=$ objective function of commercial banks.
$\pi^b_s =$ monetary holdings of b at $s \in S^*$.

A straightforward assumption is imposed.

(A4) Let A_m be the maximum amount of money present in the economy and let $\mathbf{1}$ denote the unit vector in R^{S^*}. Then $\exists Q > 0 \ni u^b(0, ..., Q, ..., 0) > u(A_m\mathbf{1})$ for Q in an arbitrary component.

(i.e. strict monotonicity in every component).[4] Also, continuity and concavity are assumed.

2.2 Central Bank and the Regulator

The Central Bank conducts open market operations in the interbank credit market (though it could also do so by buying, or selling, its own debt instruments).[5]

Formally, the following vector gives the Central Bank's action

$$(M^{CB}, \mu^{CB}, m^{CB}) \equiv (M^{CB}, \mu^{CB}, (m^{CB}_{bs})_{b \in B, s \in S^*})$$

where,
M^{CB} = OMOs on behalf of the Central Bank,
μ^{CB} = bond sales by the Central Bank,
m^{CB} =money financed Emergency Liquidity assistance to commercial banks.

Note that the Central Bank is not required to spend less than it borrows; the existence of equilibrium is compatible with the Central Bank printing money to finance its expenditures. All the results hold for both cases (i.e. with or without money financing) except where otherwise stated. Also, the Central Bank may fix the interbank interest rate ρ and then accommodate the ensuing money demand.

Similarly, the following vector gives the regulator's actions

$$(k, \lambda, \omega) \equiv ((\bar{k}_t)_{t \in T}, (\lambda^h_{sn})_{h \in H \cup B, s \in S, n \in N}; (\omega_{lj})_{b \in B, l \in T, j \in Z})$$

[3]The results remain unaffected if, instead of the previous condition, we assume smoothness of u^h.
[4]The results remain unaltered if, instead of the previous condition, we assume smoothness of u^b.
[5]LOLR assistance to commercial banks could also be modelled in this context, a subject for future analysis.

where

k_t = time-dependent capital requirements $\forall b \in B$,

λ^h_{sz} = bankruptcy (or capital requirements violation) penalties imposed upon $h \in H \cup B$ when contractual obligations are abrogated

$z \in Z = \{N\} \cup \{J\} \cup \{k_1, k_2\} = \{0^*, 0^1, ..., 0^B\} \cup \{1, ..., J\} \cup \{k_1, k_2\}$

Z is the set of all credit markets (i.e. loan market $(0^1, ..., 0^B)$ and interbank markets (0^*)), secondary asset markets and time dependent capital requirements. (see section 2.4-2.6)

ω_{tj} = risk-weights $\forall b \in B, \forall t \in T = \{0, 1\}$ and $j \in \{0, 1, ..., J\}$.

The risk weights may be functions of other macroeconomic variables such as aggregate default levels, interest rates, volumes of trade, prices, etc. Consumer loans are bank specific (see section 2.5) whereas the interbank credit market is an aggregate market where the Central Bank and all the commercial banks participate. Finally, since our focus is on formulating a framework for financial stability and not monetary policy, we have collapsed the interbank and the repo markets into one.

2.3 The Time Structure of Markets

At $t = 0$, the commodity, asset, equity, credit and interbank markets meet. At the end of the first period consumption and settlement (including any bankruptcy and capital requirements' violation penalties) take places.

At $t = 1$, commodity and equity markets meet again, loans and assets are delivered. At the end of the second period consumption and settlement for default and second period capital requirements' violations takes place. Also, commercial banks are liquidated. Figure 2 makes the time line of the model explicit.

2.4 Asset Markets

The set of assets is $J = \{1, ..., J\}$. Assets are promises sold by the seller in exchange for a price paid by the buyer today. They are traded at $t = 0$ and the contractual obligations are delivered at $t = 1$ for a particular state $s \in S$. An asset $j \in J$ is denoted by a vector $A^j \in R_+^{S(L+1)}$ indicating the collection of goods deliverable plus the money at any future state $s \in S$. Therefore, the asset market is summarised by an $((L + 1)S) \times J$ matrix A.

All the deliveries are made in money (outside cash or inside deposits/loans). When the assets promise commodities the seller delivers the money equivalent of the value of the agreed commodities at their spot prices in the relevant state. Whenever rank $|J|$ =rank $|S|$ the capital markets are said to be complete whereas when rank $|J|$ <rank $|S|$ the markets are said to be incomplete.

Furthermore, we assume

(A5) $A^j \neq 0, \forall j \in J$

(i.e. no asset makes zero promises).

(A6) $A^j \geq 0, \forall j \in J$

(i.e. asset payoffs are non-negative).

Finally, note that agents do not hold positive endowments of assets and thus all sales of assets are effectively short sales.

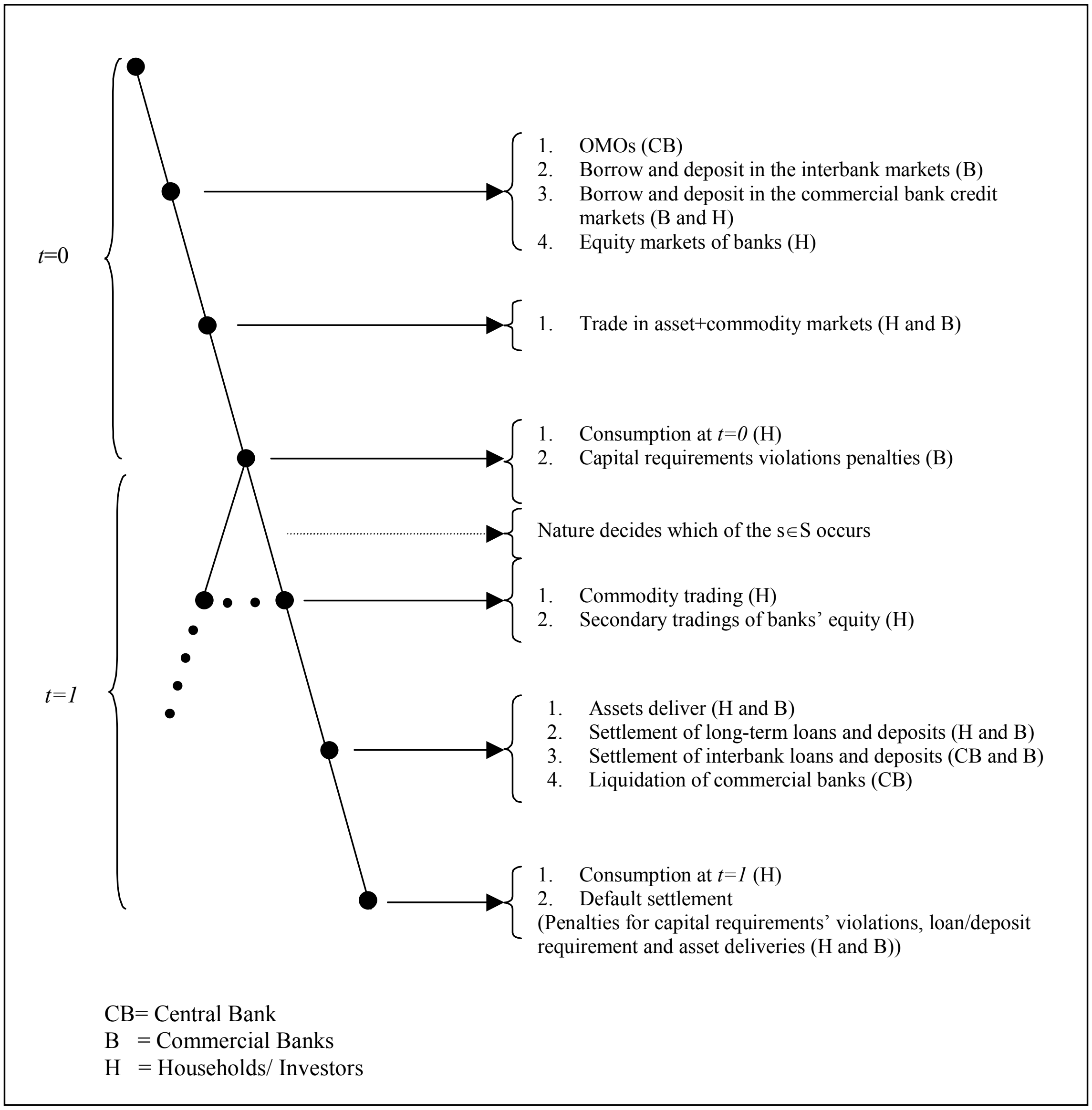

Figure 2: The Time Structure of the Model

Individuals are price takers in asset markets, where θ_j represent asset prices. Let $b_j^h \equiv$ amount of money sent by $h \in H \cup B$ in the market of asset j. Also, let $q_j^h \equiv$ promises sold of asset j by agent $h \in H \cup B$. In equilibrium, at positive levels of trade, $0 < \theta_j < \infty$,

$$\theta_j = \frac{\sum\limits_{h \in H} b_j^h + \sum\limits_{b \in B} b_j^b}{\sum\limits_{h \in H} q_j^h + \sum\limits_{b \in B} q_j^b}$$

for $j \in J, h \in H, b \in B$. All the asset markets meet contemporaneously; hence cash obtained from the sale of asset j cannot be used for the purchase of another asset $j^{'} \neq j$. Thus, the volume of trade in the asset market is affected by the overall liquidity of the economy. This way monetary policy interacts with asset markets and influences asset prices (i.e. asset price inflation channel).

2.5 Money and Credit Markets

Fiat money is the stipulated means of exchange. All commodities can be traded for money, and (as noted) all asset delivery is exclusively in money. Money can be either *inside* or *outside*. At the outset some individuals and banks hold *net* monetary assets-outside money-(which includes Central Bank liabilities). Inside money is credit created by the banking sector through the credit markets in period 0, in part depending on current monetary policy, and is matched by the individual borrowers' debt obligation to the banks. When the Central Bank undertakes expansionary OMO, in the interbank market, the commercial banks gain cash reserve assets matched by an interbank deposit liability to the Central Bank. In turn, commercial banks lend to borrowers. This represents an asset of the commercial bank and thus a liability to investors. The *net* assets of the private sector as a whole remain unchanged. Cash-in-advance is required for any purchase.

A market involves a symmetric exchange between two instruments. Just as agents cannot sell money which they do not have in a market, so in the model agents cannot sell commodities they do not have. The only exceptions are assets and credit markets, where we allow agents to write their own promises (bonds).

Money enters the economy in three ways. First, it may be present at the outset ($t = 0$) in the private endowments of agents and commercial banks. Agent $h \in H$ has an endowment m^h of money, $\forall s \in S^*$ and commercial banks have initial capital endowment e_s^b, $\forall s \in S^*$, part of which may be held in cash or deposits at the central bank. Second, when the Central Bank lends on the interbank market, or purchases bonds with currency or the government engages in money financed fiscal transfers, then the money stock increases. Third, previously issued Central Bank bonds, or interbank loans, are repaid; money then exits the system via redemptions of debt from investors and from commercial banks.

Agents are permitted to borrow from, and deposit with, each particular commercial bank. However, borrowers are distributed initially to a particular bank from which they borrow. This may be a result of relationship banking, or some other informational advantage that a commercial bank has with particular borrowers. This restricted participation assumption generates different interest rates charged by commercial banks, but could be relaxed in more complicated versions of the model.

So, let us partition $H = \{1, ..., H\}$ to $\{h_1^\alpha, ..., h_k^\alpha\} \cup \{h_1^b, ..., h_l^b\} \cup ... \cup \{h_1^B, ..., h_m^B\}$ disjoint sets whose union is H. Let the generic element of a subset H^b be h^b indicating the particular agent who can borrow from $b \in B$. Note that $h \in H$ can deposit in any commercial bank she wishes. Let μ^{h^b} be the amount of fiat money agent $h^b \in H^b$ chooses to owe on the loan market of bank b. If all agents repay exactly what they owe, then $\forall b \in B$ we must have that,

$$1 + r^b = \frac{\sum\limits_{h^b \in H^b} \mu^{h^b}}{\overline{m}^b}$$

where $\overline{m}^b$ is the amount of credit that commercial banks extend which is also subject to their capital requirements set by the regulator (see section 2.6).

Thus, the ratio of nominal value of loans over loans supply (i.e., commercial banks credit extension) determines the gross *nominal* interest rate. We add that r^b is the *ex ante* nominal interest rate that incorporates both the liquidity and default premium of loans since default is permitted in equilibrium. The effective (*ex post*) interest is suitably adjusted to account for default on loans and deposits.

The asset, bank loans, bank deposits, and bank equities (but *not* commodities) can be inventoried; they are the only stores of value in our model.

2.6 Capital Requirements

As already mentioned in section 2.1, the regulator sets the banks' minimum capital requirements. Given that the assets of commercial banks consist of loans (including interest rate payments), investments in marketable assets and some initial distribution of government bonds, the capital requirements constraint becomes,

$$\overline{k}_t \leq \frac{e_s^b + \sum\limits_{h \in H} \psi_{bs}^h}{\overline{\omega}(\eta,\delta)\overline{R}_s^b \overline{m}^b (1+r^b) + \sum\limits_{j \in J} \omega_{ij}(\eta,\delta) R_{sj}(p_s A_s^j) \left(\frac{b_j^b}{\theta_j}\right) + \omega(\eta,\delta) R_s^b d^b (1+\rho)}, \quad \forall s \in S^*, b \in B$$

The variables are defined as follows:

$\eta \equiv$ set of macro variables,
$\delta \equiv$ choice variables of the investors and commercial banks,
$\overline{\omega}(\cdot) \equiv$ risk-weights for long-term loans,
$\omega_{ij}(\cdot) \equiv$ risk-weights for marketable assets,
$\omega(\cdot) \equiv$ risk weights for interbank market deposits,
$\psi_{bs}^h \equiv$ equity of commercial banks,
$p_s \equiv$ commodity prices, $\forall l \in L, s \in S$,
$e_s^b \equiv$ commercial banks' initial capital endowment $\forall s \in S^*$,
R's $\equiv$ expected rates of delivery of various instruments (see also section 2.7)

Capital requirements are set by the regulators at each $l \in T, \forall b \in B$. However, evaluation of the capital and the risk-weighted assets occur at each state's, $s \in S$, prices, delivery rates, initial capital endowments and capital adjustments. Banks may not necessarily hold the same capital since precautionary capital over and above the regulatory minimum can vary across banks. In addition, as we will describe in the next section, banks are allowed to violate the capital requirements constraints, subject to a penalty payment.

Note that credit requirements in the first period are calculated with respect to the realised asset deliveries in the *ex post* equilibrium and not to the expected *ex ante* ones as in $t = 0$. The impact of regulatory policy, since it affects credit extension and banks' portfolio composition, is akin to the workings of monetary policy.

2.7 Default

Default can be either strategic or due to ill fortune. Lenders cannot distinguish whether default (or equivalently capital requirements constraints' violation) occurs because the debtors are unable to honour their contractual obligations or they choose not to do so even though they have the necessary resources. The default (or capital requirements' violation) penalties are proportional to the level of default (or violation). Their purpose is to induce debtors to honour their obligation when they are able to do so and to refrain from making promises that they know they will not honour in the future.

Let us define $D^h_{sz} = (1-v^h_{sz})\mu^{h^b}$ ($D^b_{sz} = (1-v^b_{sz})\mu^b$) where v^h_{sz} (v^b_{sz}) is the rate of repayment by households (banks). D^h_{sz} is the nominal value of debt under default in the credit markets (analogously in the asset market, or on deposit and interest rate obligations). In practice, default penalties and the bankruptcy code depend normally on the *nominal* values of debt and are only adjusted at discrete intervals as the general level of prices increases. In the model, nominal values are deflated so that penalties are real. Finally, note that households are not allowed to default on their obligations either in the primary or secondary equity market of commercial banks.

Let the parameters λ^h_{sz} (λ^b_{sz}) represent the marginal disutility of defaulting for each 'real' dollar on liabilities in state s. Therefore, the payoffs to investors and commercial banks will be respectively $\forall s \in S^*$,

$$\Pi^h_s(\chi^h_s, (D^h_{sz})_{z\in Z}, p_s) = u^h_s(\chi^h_s) - \frac{\sum_{z\in Z} \lambda^h_{sz}[D^h_{sz}]^+}{p_s g_s}$$

and

$$\Pi^b_s(\pi^b_s, (D^b_{sz})_{z\in Z}, p_s) = u^b_s(\pi^b_s) - \frac{\sum_{z\in Z} \lambda^b_{sz}[D^b_{sz}]^+}{p_s g_s}$$

where g_s is the base basket of goods which serves as a price deflator with respect to which the bankruptcy penalty is measured and

$$[c]^+ \equiv \max[0, c]$$

Also, since we allow commercial banks to violate their capital requirement constraints, $\lambda^b_{k_t}$ represents the marginal disutility of violating their capital requirement constraint for each 'real' dollar. Thus, we need to subtract from the payoff of commercial banks the additional term:

$$\frac{\lambda^b_{k_t} \max[0, \overline{k}_t - k^b_t]}{p_s g_s}$$

Note that, since we allow for capital requirements' violations, these do not appear in the budget set of banks, but only in the objective function (see section 2.9). Since capital requirements constraints do not enter the commercial banks' optimisation problem as constraints, it is only the possibility of incurring the penalty for any violation that provides an incentive to banks to fulfil their capital requirements. If, for example, $\lambda^b_{k_t} = +\infty$, $\forall b \in B$, then commercial banks would never violate their capital adequacy ratios.

This specification of default captures the idea (first introduced by Shubik and Wilson (1997)) that utility decreases monotonically in the level of default. In equilibrium, agents equalise the marginal utility of defaulting with the marginal disutility of the bankruptcy penalty. Thus, the expected rates of delivery of interbank, long-term loans, assets and deposits $R = (R^b_s, \overline{R}^b_s, R_{sj}, \overline{R}^b_{ds})$ $\forall s \in S, j \in J$ and $b \in B$ are equal to actual rates of delivery in equilibrium. This is a crucial ingredient of this model. It allows us to establish default as an equilibrium phenomenon and produces different interest rates in each credit market. Different interest rates are produced because different risk attitudes and initial capital endowments of both individual borrowers and commercial banks induce different default levels that in turn generate different default risk premia in each credit market.

2.8 Commodity Markets

Commodity prices p_{sl} are taken as exogenously given by the agents. Let $b^h_{sl} \equiv$ amount of fiat money spent by $h \in H$ to trade in the market of commodity $sl \in L$. In addition, let $q^h_{sl} \equiv$ amount of good $l \in L$ offered for sales at state $s \in S^*$ by $h \in H$. Agents cannot sell commodities they do not own, so $q^h_{sl} \leq e^h_{sl}$. In equilibrium, at positive levels of trade $0 < p_{sl} < \infty$,

$$p_{sl} = \frac{\sum\limits_{h \in H} b^h_{sl}}{\sum\limits_{h \in H} q^h_{sl}}$$

All markets meet simultaneously; hence cash obtained from the sale of commodity l at state s cannot be used for the purchase of another commodity $l \in L$ at some $s \in S^*$. This institutional arrangement is a fundamental feature of a model that captures the importance of liquidity constraints and the transaction demand for cash. Cash-in-advance constraints should be viewed as liquidity constraints that distinguish commodities from liquid wealth. Without loss of generality one could extend the present model to accommodate different liquidity characteristics of commodities, or of other assets, by introducing liquidity parameters for each commodity that would be determined in equilibrium.[6]

2.9 Commercial Banks

Commercial banks enter the model because of their importance both for enabling agents to smooth consumption, for the transmission of monetary policy, and for contagion of financial crises during periods of financial fragility.

Let $b \in B = \{1, ..., B\}$ be the set of commercial banks. We assume:

(A7) perfectly competitive banking sector (i.e. commercial banks take interest rates and asset prices as exogenously given)

(A8) perfect financial intermediation (i.e. no market imperfections with respect to information and participation in the capital and credit markets)

Taken together (A7) and (A8) imply that there is no margin between borrowing and lending rates in each credit market, $\forall b \in B$ and in the interbank market as well.

[6] See section 3.3 for further discussion.

The balance sheet of commercial banks is as follows:

A	L
Loans to individual agents	Deposits from individual agents
Interbank deposits	Interbank borrowing
Asset investments	Equity

Similarly, the balance sheet of the Central Bank,

A	L
Interbank loans	Currency (Fiat money)
Residual[7]	

The modelling of banking behaviour here is akin to the portfolio balance approach of the banking firm introduced by Tobin (1963 and 1982).

Shares of ownership of commercial banks are determined on a prorated manner as follows:

$$s_b^h = \frac{\psi_b^h}{\sum\limits_{h \in H} \psi_b^h}, \forall b \in B$$

where $\psi_b^h \equiv$ amount of fiat money offered by h for ownership shares of banks $b \in B$.

$s_b^h \equiv$ percentage of shares of ownership by $h \in H$ acquired at $t = 0$ at the initial public offering of $b \in B$

As can be seen from the time structure of the model, at $t = 1$, $\forall s \in S$, retrading occurs and the secondary bank equity market clears as follows:

$$\theta_s^b = \frac{\sum\limits_{h \in H} b_{sb}^h}{\sum\limits_{h \in H} s_{sb}^h V_s^b}, \forall s \in S, b \in B$$

where $b_{sb}^h \equiv$ amount of fiat money offered by $h \in H$ at $s \in S$ for shares of ownership at the secondary bank equity market of $b \in B$,

$s_{sb}^h \equiv$ percentage shares of ownership by $h \in H$ sold at $t = 1$ at the secondary bank equity market of $b \in B$, and finally $V_s^b \equiv e_s^b + \triangle(2^b)$.

Also, $s_{sb}^h \leq s_b^h$ (i.e. no short sales of bank equity, without loss of generality). Note that dividends are not distributed at the end of $t = 0$, since our focus is not on the capital structure of commercial banks. At $t = 1$, the profits (if any) of commercial banks are liquidated and distributed back to the individual owners according to their ownership shares, (and for default penalties the same rule applies). This way we close the model. We also remark that, because of the capital requirements' violation penalty, banks will never go bankrupt and therefore bank equity prices will be non-zero. The formal argument will be presented in section 6. In cases where the book value of commercial banks is negative due to penalty payments, the share holders bear the responsibility of paying the penalty (i.e. unlimited liability).

Finally, as will be discussed in section 6, we analyse not only the default channel and liquidity trap for the banking system, but also the effect on financial fragility of a collapse in bank equity values.

[7] The residual entry in the balance sheet of the Central Bank accounts for the endogenous default in the interbank market.

2.10 Interbank Credit Market

The Central Bank conducts its monetary policy through OMOs in the interbank market, (though other routes for OMO are also possible in practice). Also, interbank lending and borrowing occurs in this market. Alternatively, the central bank could set the interbank interest rate and accommodate the ensuing excess demand (or supply) of liquidity.

The interbank interest rate is established in equilibrium at positive levels of trade,

$$(1+\rho) = \frac{\sum_{b\in B} \mu^b + \mu^{CB}}{\sum_{b\in B} d^b + M^{CB}}$$

where $\mu^b \equiv$ amount of zero coupon bonds issued by $b \in B$, or equivalently the amount of money b chooses to owe in the interbank credit market, $d^b =$ amount of money that b deposits. Similarly, $\mu^{CB} \equiv$ amount of zero-coupon bonds issued by the central bank and $M \equiv$ central bank money supply.

Note that monetary policy is *not* symmetric since default can lead to varying responses to the central bank's OMO actions. Also, the central bank could determine the interest rate instead and let borrowing and lending equilibrate the market, as the current practice of implementing monetary policy is nowadays.

As the notations used in this paper is extensive, we summarise them for quick reference in Appendix II.

3 The Budget Set

It is assumed that commodities are perishable lasting only one period, and that each market meets once in each period. In order to ensure that agents have the necessary liquidity before they spend (i.e. cash-in-advance) the order in which markets meet should be carefully chosen. Accordingly, the interbank market meets first to enable commercial banks to acquire funds to supply in the credit markets which in turn meet before commodity markets meet to allow investors to borrow, if necessary, for their expenditures. However, if the time horizon is extended to a large T the order does not very much matter as long as receipts from sales cannot be used contemporaneously for the purchase of commodities.

Unlike Tsomocos (2003a, and 2003b), we assume that asset markets (as well as the banks' equity market) clear automatically via a giant clearing house. Thus,we attempt to capture the fact that financial markets clear faster than commodity markets.

3.1 Investors

Macro variables ($\eta = (p, \rho, r, \theta, s, R)$) are determined in equilibrium and every agent takes them as given. Agents are perfect competitors and therefore are price takers. The choice of investors, $h \in H$, are determined by $\sigma^h \in \sum^h(\eta)$ where,

$\sigma^h = (\chi^h, \mu^{h^b}, d^{h^b}, b_j^b, q_j^b, \psi^h, b_{sb}^h, s_{sb}^h, v^h) \in R_+^{LS^*} \times R_+ \times R_+ \times R_+^{LS^*+J} \times R_+^{LS^*+J} \times R_+^B \times R_+^{SB} \times R_+^{SB} \times R_+^{S^*+J}$ is the vector of all of investors' decisions.

$B^h(\eta) = \{\sigma^h \in \Sigma^h(\eta) : (1^h) - (7^h) \text{ below}\}$ is the budget set where $\triangle(i)$ represents the difference between RHS and LHS of inequality (i).

For $t = 0$,

$$\sum_{j\in J} b_j^h + \sum_{b\in B} \psi_b^h + \sum_{l\in L} b_{0l}^h + \sum_{b\in B} d_b^h \leq \left(\frac{\mu^{h^b}}{1+r^b}\right) + \sum_{j\in J} \theta_j q_j^h + m_0^h \tag{1^h}$$

(i.e. expenditures for assets, banks' equity in the primary market, and commodities + bank deposits ≤ borrowed money at the credit markets + receipts from sales of assets + initial private monetary endowments)

$$q_{0l}^h \leq e_{0l}^h, \ \forall l \in L \tag{2^h}$$

(i.e. sales of commodities ≤ endowments of commodities)

$$\chi_{0l}^h \leq e_{0l}^h - q_{0l}^h + \frac{b_{0l}^h}{p_{0l}}, \quad \forall l \in L \tag{3^h}$$

(i.e. consumption ≤ initial endowment - sales + purchases)
$\forall s \in S$,

$$\sum_{l\in L} b_{sl}^h + \sum_{b\in B} b_{sb}^h \leq \triangle(3^h) + \sum_{l\in L} p_{0l} q_{0l}^h + \sum_{b\in B} s_{bs}^h \theta_s^b V^b + m_s^h \tag{4^h}$$

(i.e. expenditures for commodities and banks' equity in the secondary market ≤ money at hand + receipts from sales of commodities + receipts from sales of banks' equity in the secondary market + initial private monetary endowment in states s)

$$\begin{aligned}\sum_{j\in J} (v_{sj}^h p_s A_s^j) q_j^h + \sum_{b\in B} v_{sb}^h \mu^{h^b} \leq \triangle(4^h) + \sum_{l\in L} p_{sl} q_{sl} + \sum_{b\in B} (\frac{b_{sb}^h}{\theta_s^b V_s^b} + s_b^h - s_{sb}^h) \pi_s^b \\ + \sum_{b\in B} \overline{R}_{sd}^b d_{sb}^h (1+r^b) + \sum_{j\in J} (R_{sj} p_s A_s^j) \left(\frac{b_j^h}{\theta_j}\right)\end{aligned} \tag{5^h}$$

(i.e. asset and loan deliveries ≤ money at hand + receipts from commodities sales + distribution of commercial banks' profits + deposit and interest payment + asset deliveries)

$$q_{sl}^h \leq e_{sl}^h, \ \forall l \in L, \forall s \in S \tag{6^h}$$

(i.e. sales of commodities ≤ endowments of commodities)

$$\chi_{sl}^h \leq e_{sl}^h - q_{sl}^h + \frac{b_{sl}^h}{p_{sl}}, \quad \forall l \in L, \forall s \in S \tag{7^h}$$

(i.e. consumption ≤ initial endowment - sales + purchases)

3.2 Commercial banks

Denote the choices of commercial banks $b \in B, \sigma^b \in \Sigma^b(\eta)$ where $\sigma^b = (\mu^b, d^b, \overline{m}^b, b_j^b, q_j^b, v_s^b, \pi_s^b) \in R_+ \times R_+ \times R_+ \times R_+^J \times R_+^J \times R_+^{J+2} \times R_+^{S^*}$ is the vector of all of their choices.

$B^b(\eta) = \{\sigma^b \in \Sigma^b(\eta) : (1^b) - (3^b) \text{ below}\}$ is the budget set where $\triangle(i)$ represents the difference between RHS and LHS of inequality (i).

For $t = 0$,

$$d^b \leq \sum_{h\in H} \psi_b^h + e_0^b \tag{1^b}$$

(i.e. deposits in the interbank market $\leq$ initial capital endowment)

$$\overline{m}^b + \sum_{j\in J} b_j^b \leq \triangle(1^b) + \frac{\mu^b}{(1+\rho)} + \sum_{j\in J}\theta_j q_j^b + \sum_{h\in H} d^h \tag{2^b}$$

(i.e. credit extension + expenditures for equities $\leq$ money at hand + interbank loans + receipts from banks' primary equity market and asset sales + consumer deposits)

$\forall s \in S$,

$$\sum_{h\in H}\overline{v}_s^b(1+r^b)d^h + \widetilde{v}_s^b\mu^b + \sum_{j\in J} v_{sj}^b p_s A_j^j q_j^b \leq \triangle(2^b) + \sum_{h\in H}\overline{R}_s\mu^{h^b} + \sum_{j\in J}(R_{sj}p_s A_s^j)(\frac{b_j^b}{\theta_j}) + \overline{R}_{ds}d^b(1+\rho)\Psi + e_s^b \tag{3^b}$$

(i.e. deposits and interest repayment + interbank loan repayment + expenditures for asset deliveries $\leq$ money at hand + loan repayments + money received from asset payoffs + interbank deposits and interest repayment + initial capital endowment in state s)

where

$\Psi = d^b/(\sum_{b\in B} d^b + M^{CB})$

$\pi_0^b \equiv \triangle(2^b)$ and $\pi_s^b \equiv \triangle(3^b)$

Note that since the interbank market is perfectly competitive, in cases where there has been default, deposit repayments are made proportional to the deposits made by each commercial bank relative to the aggregate supply of credit by the entire commercial banking sector and central bank.

3.3 A Remark on Cash-In-Advance

A usual criticism of the cash-in-advance (C-I-A) models is that these constraints are *ad hoc* and do not adequately capture liquidity. Our view is that C-I-A constraints are the simplest form of liquidity constraints and can be straightforwardly generalised to model more complicated liquidity constraints. The main intuition of these constraints is that the different instruments and commodities of the economy are not equally liquid. Put differently, not all receipts from sales can be contemporaneously used for other purchases. As long as there exist some liquidity parameters for the commodity endowment which are less than 1 (otherwise, the budget constraints collapse to the standard Arrow-Debreu constraints), money (or liquidity or credit) demand is positive in order to bridge the gap between expenditures and receipts. Indeed, Grandmont and Younes (1972) have used these liquidity parameters. However, the pure C-I-A constraint offers accounting clarity and ease of exposition. (see section 7)

4 Equilibrium

We say that[8] $(\eta, (\sigma^h)_{h\in H}, (\sigma^b)_{b\in B})$ is a Monetary Equilibrium with Commercial Banks and Default (MECBD) for the economy

$$E\{(u^h, e^h, m^h)_{h\in H}; (u^b, e^b)_{b\in B}; A, M^{CB}, \mu^{CB}, m^{CB}, k, \lambda, \omega\}$$

iff:

(i) $p_{sl} = \dfrac{\sum_{h\in H} b^h_{sl}}{\sum_{h\in H} q^h_{sl}}, \ \forall s \in S^*, l \in L;$

Condition (i) shows that all commodity markets clear (or equivalently that price expectations are rational).

(ii) $1+\rho = \dfrac{\sum_{b\in B} \mu^b + \mu^{CB}}{\sum_{b\in B} d^b + M^{CB}}$

Condition (ii) shows that the interbank credit market clears (or equivalently that interbank interest rate forecasts are rational).

(iii) $1+r^b = \dfrac{\sum_{h^b\in H^b} \mu^{h^b}}{\overline{m}^b}, \ \forall b \in B, h^b \in H^b;$

Condition (iii) shows that the long-term credit markets clear (or equivalently that prediction of the long-term interest rate is rational).

(iv) $\theta_j = \dfrac{\sum_{h\in H} b^h_j + \sum_{b\in B} b^b_j}{\sum_{h\in H} q^h_j + \sum_{b\in B} q^b_j}, \forall j \in J$

Condition (iv) shows that every asset market clears (or equivalently, asset price expectations are rational).

(v) $\sum_{h\in H} s^h_b = 1, \forall b \in B;$

Condition (v) shows that the primary equity market for the bank ownership clears (or equivalently bank equity shareholding expectation are rational).

(vi) $\theta^b_s = \dfrac{\sum_{h\in H} b^h_{sb}}{\sum_{h\in H} s^h_{sb} V^b}, \forall b \in B, s \in S;$

Condition (vi) shows that the secondary equity market of commercial banks clears (or equivalently secondary market bank equity price expectations are rational.)

(vii) $R_{sj} = \left\{ \begin{array}{c} \dfrac{\sum_{h\in H\cup B} [v^h_{sj} p_s q^h_j A^j]}{\sum_{h\in H\cup B} [p_j q^h_j A^j]}, \text{ if } \sum_{h\in H\cup B} [p_j q^h_j A^j] > 0 \\ \text{arbitrary, if } \sum_{h\in H\cup B} [p_j q^h_j A^j] = 0 \end{array} \right\}$

[8]Recall that by assumption p, ρ, r^b, θ, R are different from 0 and ∞ in each component.

Condition (vii) shows that each asset buyer is correct in his expectations about the fraction of assets that will be delivered to him.

$$\text{(viii)-(xi) } \overline{R}_s(R_{sd}, \widetilde{R}_{sd}, \widetilde{R}_s) = \left\{ \begin{array}{c} \dfrac{\sum\limits_{h \in H \cup B} [\overline{v}_s \mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)]}{\sum\limits_{h \in H \cup B} [\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)]}, \\ \text{if} \sum\limits_{h \in H \cup B} [\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)] > 0, \forall s \in S \\ \text{arbitrary, if} \sum\limits_{h \in H \cup B} [\mu^{h^b}(d^{h^b}(1+r^b), d^b(1+\rho), \mu^b)] = 0 \end{array} \right\}$$

Conditions (viii)-(xi) show that the Central Bank and commercial banks are correct in their expectations about the fraction of loans that will be delivered to them. Similarly, investors and commercial banks are correct in their expectations about the fraction of deposits and interest rate payment that will be delivered to them.

(xii) (a) $\sigma^h \in \underset{\sigma^h \in B^h(\eta)}{\text{Argmax}} \Pi^h(\chi^h)$

(b) $\sigma^b \in \underset{\sigma^b \in B^b(\eta)}{\text{Argmax}} \Pi^b(\pi^b)$

Condition (xii) shows that all agents optimise.

In sum, all markets clear and agents optimise given their budget sets. These are the defining properties of a competitive equilibrium.

5 Orderly Functioning of Markets: Existence of a Monetary Equilibrium with Commercial Banks and Default

If a MECBD exists, then default and financial instability manifest themselves as equilibrium phenomena entirely consistent with the proper-functioning of markets. Thus, if any of these phenomena are deemed detrimental for the economy and for the welfare of the society, then regulatory intervention may be justified. Moreover, active crisis management and prevention can become necessary.

As can be seen from conditions (viii)-(xi) of section 4, expected deliveries of assets, loans and deposits are equal to realised deliveries in equilibrium. However, the specification of expectations for inactive markets is arbitrary. Thus, we need a hypothesis to rule out trivial equilibrium (in which trade in the corresponding markets collapses). Following Tsomocos (2003a) we impose the Inactive Market Hypothesis.

Inactive Market Hypothesis (IMH): Whenever credit or asset markets are inactive the corresponding rates of delivery are set equal to 1.

This hypothesis follows closely Dubey, Geanakoplos, and Shubik (2000), and Dubey and Shubik (1978) that allows an external agent to be added in these markets that always supplies an ε amounts and never abrogates his contractual obligations. It may be thought as the FDIC or an analogous institution.

Economic agents in our model are not required to trade and they always have the option to consume their own endowment. This situation arises when interest rates are prohibitively high and thus there is no demand for credit. Then, it also becomes uncertain whether the interbank market will be active as well. This happens whenever the marginal cost of borrowing (i.e. interest rate payments) is higher than the marginal benefit of the extra consumption.

We are thus naturally led to adopt a condition that guarantees sufficient gain from trade. For an extensive discussion on this issue see Dubey and Geanakoplos (1992), (2003a). Geanakoplos and Tsomocos (2002) who extends the gain from trade condition to a model related to the present one. The crucial insight of this condition is that, even if transaction costs are high, utility from extra consumption would still make such transaction attractive. We use the definition of gaining from trade from Tsomocos (2003a) which is given as follows:

Definition:

Let $(\chi^h, \pi^b) \in R_+^{S^* \times (L+1)} \quad \forall h \in H,\, b \in B$. $\forall \delta > 0$, we will say that $(\chi^1, ..., \chi^h; \pi^1, ..., \pi^h) \in (R_+^{S^* \times L})^H \times R_+^{S^* \times B}$ permits at least $\delta-$*gain-from-trades* $\tau^1, ..., \tau^H; \tau^1, ..., \tau^B$ in R^{L+1} such that

1. $\sum_{h \in H} \tau^h + \sum_{b \in B} \tau^b = 0$
2. (a) $\chi_s^h + \tau^h \in R_+^L$, $\forall h \in H$
   (b) $\pi_s^b + \tau^h \in R$, $\forall b \in B$
3. (a) $u^h(\overline{x}^h) > u^h(x^h)$, $\forall h \in H$
   (b) $u^b(\overline{\pi}^b) > u^b(\pi^b)$, $\forall b \in B$

where,

$$\overline{x}_{tl}^h = \left\{ \begin{array}{c} \chi_{tl}^h,\ t \in S^* \backslash \{s\} \\ \chi_{tl}^h + \min\{\tau_l^h, \tau_l^h/(1+\delta)\} \ \text{ for } l \in L \text{ and } t = s \end{array} \right\}$$

$$\overline{\pi}_t^b = \left\{ \begin{array}{c} \pi_t^b,\ t \in S^* \backslash \{s\} \\ \pi_t^b + \min\{\tau^b, \tau^b/(1+\delta)\} \ \text{ for } t = s \end{array} \right\}$$

Note that when $\delta > 0$, $\overline{x}_l^h < \chi_l^h + \tau_l^h$, if $\tau_l^h > 0$ and $\overline{x}_{tl}^h = x_{tl}^h + \tau_l^h$ if $\tau_l^h \leq 0$. Also, $\overline{\pi}_t^b < \pi_t^b + \tau^b$, if $\tau^b > 0$ and $\overline{\pi}_t^b = \pi_t^b + \tau^b$, if $\tau^b \leq 0$

Formally, the hypothesis that we impose on the economy for sufficient gains from trades is:

G from T:
$\forall s \in S$, the initial endowment $(e^h, e^b)_{h \in H \cup B}$ permits at least δ_s-gains to trade in state s, where

$$\delta_s = \frac{\sum_{h \in H} m_0^h + \sum_{h \in H} m_s^h + \sum_{b \in B} e^b + \sum_{b \in B} e_s^b}{M^{CB}}$$

We are now ready to state the existence theorem that establishes default and financial fragility compatible with equilibrium and the orderly functioning of markets.

Theorem:
If in the economy $E = \{(u^h, e^h, m^h)_{h \in H}; (u^b, e^b)_{b \in B}; A, M^{CB}, \mu^{CB}, m^{CB}, \lambda, \omega\}$
1. G from T and IMH hold,
2. $M^{CB} > 0$,
3. $\forall s \in S^*$, $\sum_{h \in H} m_s^h + \sum_{b \in B} e_s^b > 0$ and
4. $\lambda >> 0, \forall h \in H, b \in B$

then a MECBD exists.[9]

[9] For an extensive discussion of the theorem in such a model see Tsocomocos (2003a). See also the proof of theorem in the appendix where the diference from the arguments of Tsomocos (2003a and 2003b) are discussed.

6 Financial Fragility, Default and the Liquidity Trap

6.1 Financial Fragility and Contagion: Concepts and Definitions

We adopt a definition of financial fragility introduced in Tsomocos (2003a), where a MECBD is financially fragile whenever a substantial 'number' of households and commercial banks default on some of their obligations (i.e. a liquidity 'crisis'), without necessarily becoming bankrupt, and the aggregate profitability of the banking sector decreases significantly (i.e. a banking 'crisis').

The formal definition of financial fragility is as follows;

Definition: A MECBD $(\eta(\sigma^h)_{h\in H}, (\sigma^b)_{b\in B})$ is financially fragile at s whenever $D^{h^*}_{sz}, D^{b^*}_{sz} \geq \overline{D}$, $\sum\limits_{b\in B} \pi^b_s \leq \overline{\Pi}$, for $|H^*| + |B^*| \geq \overline{Z}$, and $s \in S^*$ where $\overline{Z} \in (0, |H| + |B|)$ and $\overline{\Pi}, \overline{D} \in R_{++}$.

This definition requires both increased default *and* reduced aggregate profitability. Increased default by itself might indicate excessive risk taking without necessarily engendering a serious strain on the financial sector of the economy, whereas a decrease in profitability by itself might indicate the onset of a recession in the real economy and not of financial vulnerability. Moreover, the authorities may seek to define the threshold that is commensurate with the onset of a financially fragile regime. Also, with heterogenous agents, the welfare of society depends not only on aggregate outcomes, but also on their distribution over agents.

The interaction of investors and commercial banks in the various markets of this model allows us to precisely trace the different channels of contagion given an adverse shock. The first channel of contagion is the one generated by increased default in a specific sector of the economy. For example, if a specific bank charges exobitantly high interest rates on its clients then their subsequent default impacts upon the rest of the economy. Commercial banks reduce their repayment rates in the interbank market and investors and/or commercial banks abrogate their obligation in the asset markets. Alternatively, the commodity markets may be affected either through reduced supply (or demand) which in turn affects expected income of the household sector (or the supplier). The upshot of this chain of contagion is that reduced liquidity hurts the lenders whose income (or equivalently their expenditures) is reduced, thus decreasing their consumption and welfare. We note that this chain may be broken, for example, with emergency liquidity assistance that neutralises the reduced loan repayment rate to the initial commercial bank. The same reasoning applies for contagion through the interbank market's increased default.

Second, contagion may commence through the collapse of the banking sector's equity value in the secondary market. Since the distribution of profits to investors is determined by the shares of ownership as they are specified in the secondary banks' equity market, weakness of the banking sector is translated to investors' income. Reduced expected profitability of the banking sector will be reflected in a reduced value of the shares of ownership of banks' equity and thus the reduced income will lower such agents' repayment rates of loans and asset deliveries. For example, if bank b's equity drops in value then its investors will increase their default in the rest of the economy which will adversely affect other agents' welfare as well, who transact with them in the asset market. Finally, the last channel of contagion which will be discussed in section 6.2 is generated by a possible ineffectiveness of monetary policy. As monetary policy eases without affecting the real side of the economy (i.e. we enter a liquidity trap), the extra liquidity inflates activity in the asset markets. This in turn leads commercial banks to violate excessively their capital requirements which adversely affects their profitability and subsequently their equity value. Through the investor sector's ownership of bank shares contagion spreads outside the banking sector and may reduce welfare of in the rest of the economy.

Finally, due to limited liability and active default in equilibrium, the Modigliani-Miller irrelevance proposition does not hold in our model for the commercial banking sector. Equity is default free, whereas debt (either through interbank or credit market loans) is defaultable.

6.2 Liquidity Trap

The Keynesian liquidity trap describes a situation in which monetary policy would not affect real expenditures in the economy. If interest rates are sufficiently low and investors expect them to rise in the future, then they do not invest into assets like bonds whose value is expected to fall. Thus, they hold the extra money balances due to expansionary monetary policy for speculative purposes without affecting commodity prices. Various authors provide models that allow for the occurrence of a liquidity trap (e.g. Tobin (1963, 1982), Grandmont and Laroque (1973), and Hool (1976)).

Dubey and Geanakoplos (2003) provide a novel interpretation of this phenomenon. They argue that in a monetary *GEI* model, as monetary policy eases, then commodity prices remain unaffected whereas the extra liquidity is channeled into asset market(s) where trading activity becomes large. However, in the aggregate there is almost no new *net* trading activity in such asset markets. This possibility is non-generic, and occurs only in an equilibrium where the corresponding real *GEI* economy possesses no equilibrium (i.e. the case of the Hart (1975) counter example).

In the present model, the same phenomenon reappears, (coupled with financial instability), only when capital requirements are non-binding. Otherwise, such increased trading activity would lead commercial banks to increase their risk-weighted assets and thus violate their capital requirements even more so. Moreover, the liquidity trap originates from the interbank market and may propagate to the asset markets via investments of the banks only. However, note that this is a non-generic case and occurs only when equilibrium fails to exist in the underlying economy. Of course, the assets most commonly bought by commercial banks in a liquidity trap are government bonds, and these usually bear a zero risk-weight. This analysis therefore provides a rationale for imposing some positive risk-weighting on these assets as well.

However, when capital requirements are binding, the liquidity trap may still be present via excessive trading activity in equity markets. Banks now switch to credit extension and consumers spend in the primary bank equity market (thus helping to satisfy the capital requirements of banks): Or, alternatively, in anticipation of a higher liquidation value of commercial banks, consumers restructure their portfolio of banks' equity in the secondary market. The next proposition summarises this intuition.

Proposition 1

Suppose that the economy has a riskless asset $A^j_{sm} = (1, ..., 1)$ (i.e. monetary payoffs in every state are equal to one) and $A^j_{sl} = 0, \forall s \in S$ and $l \in L$ for $k^b_t = 0$. Also, consider the case in which the underlying economy has no *general equilibrium with incomplete market* (*GEI*). Then as $M^{CB} \to \infty$, then

(i) $\overline{\omega}(\eta, \delta)\overline{R}^b_s \overline{m}^b(1 + r^b) + \sum_{j \in J} \omega_{ij}(\eta, \delta) R_{sj}(p_s A^j_s) \left(\frac{b^b_j}{\theta_j} \right) + \omega(\eta, \delta) R^b_s d^b \to \infty, \forall s \in S^*$, from some $b \in B$, $\dfrac{M^{CB}}{\|p_{0l}\|} \to \infty$ and $(\sum_{h \in H} q^h_j + \sum_{b \in B} q^b_j) \to \infty$.

(ii) There exists $\overline{D}, \overline{\Pi}$ such that $D_{sz}^{h^*}, D_{sz}^{b^*} > \overline{D} > 0$ for some $b^* \in B, h^* \in H$, and $\sum_{b \in B} \pi_s^b \leq \overline{\Pi}$.

(iii) Suppose that $\lambda_{k_t}^b = +\infty, \forall b \in B$ and $k_t^b > 0$. Then, $\frac{M^{CB}}{\|p_{0l}\|} \to \infty$ and $(\sum_{h \in H} q_j^h + \sum_{b \in B} q_j^b) < K$, for some $K \in (0, +\infty)$, but $\sum_{h \in H} s_{sb}^h V^b \to \infty$.

Note that regulatory policy may 'break' this liquidity trap by imposing harsh bank capital requirements (or equivalently higher ω's) and so blocking excessive trading in asset markets, since these latter would in most cases increase the risk-weighted assets of the banks that engage in excessive activity in the asset markets. Ultimately, the capital requirements' violation penalties stop further investment in such asset markets. Thus, commercial banks switch to credit extension that provides extra liquidity to households and generates further *gains from trade* due to lower interest rates.

6.3 Regulatory Policy and Default

Since both default and capital requirements' violations incur a cost, consumers and banks weight the marginal costs and benefits of abrogating their contractual or regulatory obligation. Thus, for sufficiently high penalties, default and capital requirements' violations vanish in equilibrium. We therefore observe the importance of capital requirements for financial stability. For example, whenever credit is fully collateralised, the regulator guarantees future financial stability. This, however, has an opportunity cost since the resulting higher interest rates due to stricter capital requirements would reduce efficient trade.

In sum, we note that at least one aspect of the well-known trade-off between financial stability and efficiency is present and this indicates the interconnectedness of monetary and regulatory policies.

Proposition 2
There exist $\overline{\lambda}_{k_t}^b$ and $\overline{\lambda}_{sz}^h, \overline{\lambda}_{sz}^b$ such that $D_{sz}^h = D_{sz}^b = 0$ and $\overline{k}_t - k_t^b = 0, \forall b \in B, h \in H$

Since agents may opt to default or violate their capital requirements in equilibrium, changes in regulatory practice changes their marginal rates of substitution among various choices and thus produce equilibria with different allocations, as the following proposition indicates. The model is liquidity based with well-defined transaction technology and settlement processes; therefore both real changes as well as changes of the nominal constraints have necessarily non-neutral effect.

Proposition 3
Suppose that u^h, u^b are differentiable and $m_s^h, e_s^b > 0$ or $\lambda^b < \overline{\lambda}^b$ and $\lambda^h < \overline{\lambda}^h$ for all $h \in H, b \in B$ and $s \in S^*$. Then any change by the regulator of λ, k or ω results in a different MECBD in which for some $b \in B$ the payoff is different.

6.4 Modigliani-Miller Proposition

The modeling of commercial banks that have diverse financing and investment opportunities sheds light on the Modigliani-Miller proposition. The traditional argument for the validity of the irrelevance of financing rests on perfect and frictionless capital markets. Various arguments such as limited liability, bankruptcy costs and differential taxation between debt and equity have been offered to invalidate this proposition.

In the present model, only when (i) markets are complete, (ii) limited participation does not produce different borrowing behaviour, and (iii) banks' risk taking behaviour, and capital requirements are identical, then financing does not matter. Put differently, only when we remove all the frictions of the model and also impose homogeneity across banks, do we recover the Modigliani-Miller proposition. When one models active banks with diversified portfolios, not only lack of frictions but also identical investment guarantees the validity of the irrelevance proposition.

As our next proposition shows, any deviation from these principles destroys the symmetry of debt-equity financing. First, limited participation, even for banks with identical financing, creates different returns from credit extension, and thus changes the value of banks in $t = 1$. Second, even if all other variables remain the same across banks, different risk-appetites lead them to form different portfolios. Given differential returns amongst the various investments (i.e. credit extension, investment in the interbank and asset markets), banks' preferences towards risk generate different terminal values for banks' portfolios. Third, different capital requirements and/or risk-weights provide different incentives to banks when forming their portfolios. Thus, the forces of demand and supply will typically equilibrate the markets so that banks' equity will trade at different prices in the secondary equity market. This may be relevant for analysing the impact of the New Basel Accord.

Finally, even in the absence of the previous frictions but with incomplete markets, i.e. $|J| < |S|$, different financing alters the space of marketed assets and therefore produces different equilibria and consequently different values for banks. In other words, this is akin to the distinction of comparing *within* an equilibrium two different financing structures and *across* different equilibria of a bank that changes its financing. In the first instance, value is not affected, whereas in the second it is. This distinction holds only when markets are incomplete; otherwise since the space of marketed assets does not change, the two cases are equivalent.

Proposition 4

Let $b_1, b_2 \in B$ with $\mu^{b_1} = \mu^{b_2}$ and $\sum_{h \in H} \psi_{b_1}^h = \sum_{h \in H} \psi_{b_2}^h$. Moreover, suppose $\overline{\lambda}_{k_t}^{b_1} = \overline{\lambda}_{k_t}^{b_2}, \lambda_{sz}^{b_1} = \lambda_{sz}^{b_2}$ and $e_s^{b_1} = e_s^{b_2}$, $\forall s \in S^*$ (i.e., identical financing, default and initial capital endowments)

(i) *Limited participation*: If $u^{b_1} = u^{b_2}$ but $\exists h \in H_1$ and $\widehat{h} \in H_2$, $h \neq \widehat{h}$ with respect to their endowments of preferences, then $\theta_s^{b_1} \neq \theta_s^{b_2}$ for some $s \in S$.

(ii) *Risk Preferences*: If $h = \widehat{h}$ for all $h \in H_1$ and $\widehat{h} \in H_2$ with respect to their endowments and preferences but $u^{b_1} \neq u^{b_2}$ then $\theta_s^{b_1} \neq \theta_s^{b_2}$.

(iii) *Regulation*: If both $u^{b_1} = u^{b_2}, h = \widehat{h} \quad \forall h \in H_1$ and $\widehat{h} \in H_2$ but either $k_t^{b_1} \neq k_t^{b_2}$ or $\omega_{tj}^{b_1} \neq \omega_{tj}^{b_2}$ for some $j \in J$ or $t \in T$ then $\theta_s^{b_1} \neq \theta_s^{b_2}$.

(iv) *Incomplete markets*: Suppose $|J| < |S|, u^{b_1} = u^{b_2}, h = \widehat{h} \quad \forall h \in H_1$ and $\widehat{h} \in H_2$, $k_t^{b_1} = k_t^{b_2}$ or $\omega_{tj}^{b_1} = \omega_{tj}^{b_2}, \forall j \in J$. Now let $\widehat{\mu}^{b_1} \neq \mu^{b_2}$ but $\sum_{h \in H_1} \hat{\psi}_{b_1}^h + \widehat{\mu}^{b_1} = \sum_{h \in H_1} \psi_{b_2}^h + \mu^{b_2}$. Then $\theta_s^{b_1} \neq \theta_s^{b_2}$ for some $s \in S$ in the new MECBD.

(v) *Complete markets*: If $|J| = |S|, u^{b_1} = u^{b_2}, h = \widehat{h} \quad \forall h \in H_1$ and $\widehat{h} \in H_2$, $k_t^{b_1} = k_t^{b_2}$ or $\omega_{tj}^{b_1} = \omega_{tj}^{b_2}$ then $\theta_s^{b_1} = \theta_s^{b_2}$.

In sum, the Modigliani-Miller principle is violated primarily from two sources. First, when structural frictions such as limited participation and market incompleteness are present. Second, when investment behaviour of *active* banks is affected by different incentives or different attitudes towards risk.

7 Money Demand, Interest Rates, and the Non-Neutrality of Monetary Policy

The monetary/financial sector of our simple specification of the economy, coupled with the transaction technology, produces the traditional motives for holding money. Thus, we observe that the standard Hicksian determinants of money demand is present in the model.

In particular, liquidity provision by banks and default by both banks and households produce an intricate relationship among interest rates. Since money is fiat and the horizon is finite[10], in the end money exits the system. This means that both central bank money, M^{CB} (i.e. inside money) and money and liquidity present in the initial endowments of banks and households (i.e. outside money) would exit the system either via loan repayments to commercial banks or to the Central Bank by the commercial banks. Thus, the overall liquidity of the economy affects the determination of interest rates. Moreover, endogenous default is possible in equilibrium and inevitably affects interest rates as well. In sum, both a liquidity and default premium affects interest rates. However, further structural assumptions are needed to be able to disentangle the term premia.

Liquidity Structure of Interest Rates Proposition:
In any MECBD, $\forall s \in S$

$$\sum_{b \in B} \overline{m}^b r^b = \sum_{h \in H} (m_0^h + m_s^h)(\frac{1}{v_{sb}^h}) + \sum_{b \in B} (e_0^b + e_s^b)(\frac{1}{\overline{v}_s^b})$$

(The analogous equation holds with weak inequality for $s = 0$)

In our multi-period setting, if $|B| > |S|+1$ then there are more interest rates than equations. Thus, they depend on the real data of the economy and are subject to policy intervention. The only exception is when $m_s^h = e_s^b = 0, \forall s \in S^*, h \in H, b \in B$ and $v_{sb}^h = \overline{v}_s^b = 1, \forall s \in S^*, h \in H, b \in B$. In such a case, all interest rates are zero (including ρ) and money is essentially a veil.

If all interest rates are positive, then all the available liquidity will be channeled in the commodity markets $\forall s \in S$. However, this is not the case at $s = 0$ because of uncertainty and incomplete markets, investors may opt to spend it in the asset markets or hold some precautionary reserves.

Quantity Theory of Money Proposition:
In any MECBD with $\rho > 0$,

$$\sum_{h \in H} \sum_{l \in L} p_{sl} q_{sl}^h = \sum_{b \in B} m_s^b + \sum_{h \in HUB} \sum_{j \in J} v_{sj}^h p_s q_j^h A^j + \sum_{h \in H} \sum_{b \in B} s_{bs}^h \theta_s^b V^b + \Delta(1^h)$$

$\forall s \in S$.

For $s = 0$,

$$\sum_{h \in H} \sum_{l \in L} p_{0l} q_{0l}^h = M^{CB} + \sum_{b \in B} d^b - \sum_{h \in HUB} b_j^h - \sum_{h \in H} \sum_{b \in B} \psi_b^h - \Delta(1^h) - \sum_{b \in B} \pi_0^b$$

[10] Had we used an infinite horizon model, as long as there is settlement and liquidation in regular time intervals, similar results would hold.

We hasten to add that this is no 'crude' quantity theory of money. Velocity will always be less than or equal to 1 (one if all interest rates are positive). However, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's *sell-all* assumption), the real velocity of money, that is how many real transactions can be moved by money per unit time, is endogenous.

The interest rates determined in equilibrium are in nominal terms. Thus, they depend not only on the intertemporal marginal rate of substitution, but also on the inflation rate of the economy. So, the well known Fisher relation holds, as it is argued in the next proposition.

Fisher Effect Proposition:

Suppose that for some $h \in H^b, b^h_{0l}$ and $b^h_{sl} > 0$ for $l \in L$ and $s \in S$. Suppose further that $\Delta(4^h) > 0$. Then in a MECBD,

$$(1+r^b) = \left(\frac{\left(\frac{\partial u^h(\chi)}{\partial \chi_{0l}} \right)}{\left(\frac{\partial u^h(\chi)}{\partial \chi_{sl}} \right)} \right) \left(\frac{p_{sl}}{p_{0l}} \right)$$

Taking the logarithm of both sides and interpreting loosely, this says that the nominal rate of interest is equal to the rate of interest plus the (expected) rate of inflation.

As in the case with regulatory policy, monetary policy also has non-neutral effects. As it is proved in Tsomocos (2001), MECBD are finite with respect to both real allocations and nominal variables. Thus, any monetary change (except the one mentioned in the remark after proposition 5) affects interest rates and therefore economic agents' decisions. Finally, since MECBD are typically constrained inefficient, policy changes do not necessarily affect welfare and financial stability monotonically.

Proposition 5

Suppose that all u^h, u^b are differentiable and $m^h_s, e^b_s > 0$ or $\lambda^b < \overline{\lambda}^b$ and $\lambda^h < \overline{\lambda}^h$ for all $h \in H, b \in B$ and $s \in S^*$. Suppose at an indecomposable MECBD at every $s \in S^*$ all $h \in H$ consume positive amounts of all goods $l \in L$ and that some $h \in H$ carries over money from period 0 to 1. Then any change by the Central Bank or the regulator (except the one described in the remark) results into a different MECBD in which for some $h \in H$ consumption is different.

Remark: The 'no money illusion' property holds in the model. A proportional increase of all the nominal endowments of the consumers and commercial banks while k stays fixed and penalties are scaled down proportionally does not affect the real variables of MECBD.

8 Applications

Our theoretical framework is computationally tractable in practice. Given a change in a policy parameter, or the physical data for the economy, we can calculate the effects on all the variables in the economy. We do not need to resort to a stationary state, or to a representative agent, or to ignore some of the equilibrium conditions. Moreover, the comparative statics can be interpreted and usually predicted, on the basis of principles that we derive through our propositions in sections 6 and 7. The calculations have been done on a home computer using a version of Newton's method in *Mathematica*.

We are most interested in comparative statics that involve the interaction of the real and the banking sectors of the economy. In particular, we are interested on how changes in the consumer sector of the economy, or in the regulatory policy, affect the behaviour of the banking sector and the overall financial fragility of the economy, which is interpreted, broadly speaking, as changes in aggregate consumer default, capital requirement violation and bank profitability. For example, if monetary policy changes what happens to the economy? How is the investment behaviour of the households and banks affected? What are the repercussion on aggregate consumer default, and bank profitability? Would banks that are in a precarious capital requirement condition improve or worsen their position? Can regulatory policy substitute monetary policy? Is the response to higher risk-weights procyclical or counter-cyclical? How are different banks, which differ in size and investment behaviour, affected by a change in a policy parameter?

8.1 Comparative Statics I

We first specialise the general model presented earlier to the case of three households and two banks, where the time horizon extends over two periods ($t \in \{0, 1\}$) and three possible states ($s \in \{1, 2, 3\}$) in the first period. One bank, bank γ, is relatively poor at $t = 0$ and therefore has to seek external funds to finance its loans to its *nature-selected* borrower, Mr. α. Bank γ can raise its funds either by borrowing from the *default free* interbank market or selling its Arrow securities which only pay in state 1, the good state. Thus, this bank can be thought of as a typical straightforward small commercial bank. Bank γ's assets comprise only of its credit extension to the consumer loan market. This way we can focus on the effects of policies on banks that cannot quickly restructure their portfolios, perhaps due to inaccessibility of capital and asset markets, during periods of financial adversity. The other bank, bank δ, is a large and relatively rich investment bank which in addition to its lending activities to its *nature-selected* borrower, Mr. β, its portfolio consists of deposits in the interbank market and investment in the asset market (i.e. purchasing bank γ's Arrow security). Bank δ which is relatively richer compared to bank γ, has also alternative investment opportunities such as depositing in the interbank market. Its richer portfolio allows it to diversify quickly and more efficiently than bank γ. As we shall see later, this extended opportunity set enables bank δ to transfer the negative impact of adverse shocks to the rest of the economy without necessarily reducing its profitability. Mr. α and Mr. β are poor in terms of their commodity endowment at $t = 0$, and therefore have to borrow money from banks γ and δ, respectively, to buy commodities. As they are rich at $t = 1$, they sell commodities in the three states of period 1. On the other hand, Mr. ϕ is rich in $t = 0$ and poor in $t = 1$ (both in terms of commodity and monetary endowments). Therefore, he invests by depositing his money in bank γ, which in equilibrium offers the highest *default free* deposit rate, and buying Arrow securities to transfer wealth from $t = 0$ to $t = 1$, and thus smooth his consumption. In sum, Mr. α and β represent the household sector of the economy in which the main activity is borrowing for present consumption in view of future expected income. On the other hand, Mr. ϕ represents the investor sector with a more diversified portfolio consisting of deposits and investments in the asset market in order to smooth his intertemporal consumption. Consistent with the general model (see Assumption 8), since there is no asymmetric information in the banking sector, the deposit rate is equal to the lending rate offered by bank γ.

As a model with a primary aim to study financial fragility, this specification is the simplest version possible since at least two *heterogenous* banks are needed to analyse the *intra-sector* contagion effect within the banking sector via their interaction in the interbank and asset markets and the possible *inter-sector* contagion effect to the real sector via the credit, deposit, asset and commodity markets. Furthermore, by allowing two separate *defaultable* consumer loan markets, default in one market can produce an additional source of contagion channel to the other and the rest of the economy. Finally, an additional household who is relatively rich at $t = 0$, Mr. ϕ, is needed since we want to have an active deposit market in equilibrium.

At this stage, we simplify the model by assuming away the bank equity market since we want to focus on an adverse economic environment in which banks do not have easy access to the capital markets to raise new equities.[11] Thus, as a technical matter, we assume that commodity trading and the settlement in the asset, loan and interbank markets occur simultaneously, i.e. equations 4^h and 5^h collapse to one.

Formally, consider a two-period model with three future states with the following utility and profit functions of the five agents.

$$\begin{aligned}
U^{\alpha} &= [\chi_0^{\alpha} - c_0^{\alpha}(\chi_0^{\alpha})^2] + \sum_{s=1}^{3}[\chi_s^{\alpha} - c_s^{\alpha}(\chi_s^{\alpha})^2] - \sum_{s=1}^{3}\lambda_{s\gamma}^{\alpha}\max[0, \mu^{\alpha^{\gamma}} - v_{s\gamma}^{\alpha}\mu^{\alpha^{\gamma}}] \\
U^{\beta} &= [\chi_0^{\beta} - c_0^{\beta}(\chi_0^{\beta})^2] + \sum_{s=1}^{3}[\chi_s^{\beta} - c_s^{\beta}(\chi_s^{\beta})^2] - \sum_{s=1}^{3}\lambda_{s\delta}^{\beta}\max[0, \mu^{\beta^{\delta}} - v_{s\delta}^{\alpha}\mu^{\beta^{\delta}}] \\
U^{\phi} &= [\chi_0^{\phi} - c_0^{\phi}(\chi_0^{\phi})^2] + \sum_{s=1}^{3}[\chi_s^{\phi} - c_s^{\phi}(\chi_s^{\phi})^2] \\
U^{\gamma} &= \sum_{s=1}^{3}\pi_s^{\gamma} - \sum_{s=1}^{3}\lambda_{ks}^{\gamma}\max[0, \overline{k} - k_s^{\gamma}] \\
U^{\delta} &= \sum_{s=1}^{3}\pi_s^{\delta} - \sum_{s=1}^{3}\lambda_{ks}^{\delta}\max[0, \overline{k} - k_s^{\delta}]
\end{aligned}$$

The value of the exogenous variables are summarised in table I of appendix III. Basically, the value of commodity and monetary endowments of households are chosen so that Mr. α and β (Mr. ϕ) are poor (rich) at $t = 0$, and therefore are the net borrowers (lender) in equilibrium. Similarly, the selected value of capital endowments of banks ensures that bank γ is relatively poor at $t = 0$ and has to borrow from the interbank and asset markets in equilibrium, and vice versa for bank δ. Furthermore, the value of regulatory capital adequacy requirement is chosen to be sufficiently high (0.4) in order to ensure that all banks violate their capital requirement in equilibrium. The risk weight for consumer loans is set to 1, while that of interbank loans and assets are set to 0.5, to reflect the fact that loans are defaultable and therefore riskier than the other two types of assets. The rest of the exogenous variables/parameters are chosen to ensure a reasonable initial MECBD equilibrium which is summarised in table II of appendix III. In particular, they are chosen to ensure that the values of all the repayment rates are realistic, i.e. less than 1, and the interbank interest rate is lower than both the interest rates charged by both banks since interbank loans are assumed to be default free and thus do *not* include a default premium and finally, the loan rate of bank γ is higher than that of bank δ to ensure that Mr. ϕ who deposits chooses bank γ to do so, since deposits are assumed to be default free. Finally, the sources and uses of funds for households and banks as well as the market clearing conditions are given in tables IIIa, IIIb and IIIc of appendix III, respectively.

Notice that the marginal utilities of buying either commodities or assets are higher, (than the marginal utilities of selling), since agents do not incur interest rate payments. This is due

[11] We plan to incorporate an active equity market in the next comparative static exercise.

to the price wedge that is introduced by the liquidity constraints between selling and buying prices. Also, the Fisher relation holds with respect to the intertemporal transfer of wealth. Finally, observe that even though repayment rates are higher for Mr. α, still the interest rates set by bank γ are higher than those of bank δ. This reflects the high liquidity premium of bank γ which is relatively poorer than bank δ, as the liquidity structure would predict (see discussion in section 7). Recall that our liquidity structure incorporates both a liquidity as well as a default premium.

Now we consider the effects of changes in the exogenous variables/parameters of the model. Table IV of appendix III describes the directional effects on endogenous variables of increasing various parameters listed in the first column. In what follows, we elaborate on three comparative statics which we believe capture various important results in detail (i.e. an increase in money supply, an increase in endowment of the poor (Mr. α) and an increase in capital requirements loan risk weights)). However, the detailed explanation for changes in several other exogenous parameters are given in Appendix IV. We conclude this section by highlighting the key results that we obtain from this numerical exercise.

8.1.1 An Increase in Money Supply

Let the central bank engage in expansionary monetary policy by increasing the money supply (M^{CB}) in the interbank market (or equivalently lowering the interbank interest rate(ρ)). Lowering the interbank rate induces bank γ to borrow more from the interbank market and therefore to increase its supply of loans to Mr. α, pushing down the corresponding lending rate r^{γ}. Consequently, agent ϕ reduces his deposits in bank γ and switches his investment to the asset market, pushing up slightly the asset price. Given lower expected rates of return from investing in the interbank and asset markets, bank δ supplies more loans to Mr. β, causing the corresponding lending rate r^{δ} to decline. However, paradoxically at first sight, bank δ also switches part of its investment from the asset market to the interbank market even though the rate of return in the latter market falls relatively more precipitously. This is due to the *'intra-state diversification effect'*. To illustrate, since Mr. ϕ increases his investment in the asset market, his expected income would be relatively more favourable in state 1 where the Arrow security pays. This in turn implies that he would demand relatively more commodities in this particular state, pushing up the corresponding prices and therefore Mr. α and β's expected income from selling commodities in state 1 relative to the other two states. Consequently, they default relatively less in state 1, increasing bank δ's relative expected revenue from lending in this particular state. In order to smooth its income across the three states, it therefore invests more in the interbank market which pays relatively less in state 1, but more evenly across all states. In sum, the marginal cost from increasing its investment in the interbank market in terms of lowering rate of return is outweighed by the corresponding marginal benefit in terms of its being able to smooth its expected income stream across different states of the world.

Since more money chases the same amount of goods, by the quantity theory proposition, prices in both periods and all states increase. The price in state 1 increases the most since Mr. ϕ purchases more Arrow securities in equilibrium and therefore has more income to buy relatively more commodities in state 1 when the Arrow security pays. Lower interest rates make trade more efficient and the liquidity increase results in lower default rates by both Mr. α and Mr. β, again especially in state 1. Thus aggregate consumer default falls.

Turning now to capital requirements' violation, both banks violate their capital requirement constraints more than before, particularly bank δ. Higher repayment rates and credit extension over-compensate for the decrease in interest rates and thus, for given capital, risk weighted total assets increase. Bank δ, which is relatively richer than bank γ, violates its requirements even more so, since the marginal benefit of the increased profits is greater than the marginal cost of the capital requirement violation. The only state in which bank δ reduces its total risk weighted assets is state 1, when the Arrow security pays. It does so by switching its investment from the Arrow security to the credit market and the interbank market. Thus, given an adverse capital requirement position (and also banks' inability to access capital markets to raise new equity), expansionary monetary policy worsens their capital adequacy condition. The reason is that the extra profit effect dominates the capital requirement violation cost.

Both regulatory and monetary policies affect credit extension, and default and capital requirements' violation have different marginal costs (due to the different penalties). So, there exists a trade-off between excess return through interest payments and the cost of capital requirements' violation. Thus, the interaction of the capital adequacy ratio and credit extension should be analysed contemporaneously in order to determine the optimal composition of banks' asset. We also remark that lower consumer default does not necessarily have 'positive' contagion effects on the total risk weighted assets since banks which primarily care for profits 'gamble to resurrect' with the extra liquidity provided by the higher repayment rates.

As far as the welfare of the agents is concerned, the utility of Mr. α and the profit of bank γ improve whereas profits of bank δ deteriorate. The welfare of Mr. β and Mr. ϕ remain almost unaffected (slight improvement). The welfare improvement of Mr. α results from lower interest rates, (and consequently a higher repayment rate on his loans and thus lower default penalties). The higher expected prices in period 1 also contribute to the higher repayment rates since higher prices imply higher expected income from selling commodities. Profitability of bank γ increases mainly due to lower consumer default which dominates the higher cost of capital requirements' violations. However, the positive spillover effect of lower consumer default fails to dominate the lower revenue due to lower interest rates for bank δ whose profitability therefore decreases along with higher capital requirements violations.

In sum, expansionary monetary policy in an adverse economic environment, particularly in the banking industry, even though it improves aggregate consumer default rates does not necessarily induce less financial fragility. Higher liquidity provides an incentive for investment oriented commercial banks to 'gamble to resurrect' without necessarily improving their precarious capital requirements condition.

8.1.2 An Increase in the Monetary Endowment of the Poor (Mr. α)

Increasing the private monetary endowment of Mr. α in state 1 (perhaps by a fiscal transfer) will have some similar effects with monetary policy, but also with noticeable differences with respect to interest rates, prices and welfare effects. Given more liquidity in state 1, Mr. α repays more to bank γ. This increases the average repayment rate of Mr. α, though, as we shall see, default increases relatively weakly in the other two states. Thus, bank γ's expected revenue from lending increases, it extends more credits, pushing down r^{γ}. Because the deposit rate is lower, Mr. ϕ then switches its investment portfolio from the deposit market to the asset market. Since deposits from Mr. ϕ are lower, while bank γ wants to extend more credit, bank γ has to borrow more from the interbank market, pushing up the interbank rate.

Since Mr. ϕ invests more in the asset market, and less in the interbank market, and that the Arrow security pays only in state 1, his expected income increases in state 1 and decreases in the other two states. Thus, he purchases more commodities in state 1 and less so in states 2 and 3, causing prices in state 1 to rise and in states 2 and 3 to fall. Given these changes in prices, Mr. α and β sell more of their commodities endowment in state 1 and less so in states

2 and 3. Given higher income from selling commodities in state 1, they repay more in this particular state (the opposite is true for states 2 and 3). Since extra liquidity is injected only to Mr. α, the negative effect of higher default in states 2 and 3 dominates the positive effect of lower default in state 1, thus on average Mr. β defaults more. Anticipating lower expected revenue from lending to Mr. β, bank δ reallocates its investment from the credit market to the interbank market where the rate of return is higher. Thus r^{δ} rises.

Although Mr. α demands more commodities as he benefits from a lower cost of borrowing, the opposite is true for Mr. β. Thus, prices in period 0 remain unaffected. Moreover, even though, the repayment rates for both households decline in states 2 and 3, the average consumer default decreases because of the substantial increase of Mr. α's repayment rate in state 1.

Turning now to the banks' capital requirement condition, we observe a deterioration of both banks and in particular of bank γ. Bank γ's capital position in state 1 worsens substantially owing to the significant increase in the repayment rate of Mr. α in this particular state. The extent of deterioration in this particular state is so large that it adversely affects bank γ's capital position on average. Although bank δ's capital position is also negatively affected on average, since the repayment rate in state 1 of Mr. β does not increase as much compared to that of Mr. α (recall that the increase in the monetary endowment in state 1 only applies to Mr. α who therefore has relatively more money to repay his debt), the extent of deterioration is comparably less.

As expected, welfare of Mr. α and bank γ increases due to the extra liquidity of Mr. α. The interesting point is that although bank δ is initially negatively affected by the shock since the repayment rates from Mr. β is lower on average, its *flexibility* in reallocating its portfolio of investment enables it to mitigate the negative effect by passing the negative externalities onto its borrower and therefore to increase its profitability slightly. To illustrate, because bank δ has alternative sources of investment besides the credit market, as the shock affects the expected rate of return on the credit market, it could contemporaneously reduce its credit supply to Mr. β, switch its investment to the interbank market, and ultimately be able to pass the negative externalities to Mr. β via charging a higher lending rate. Unlike Mr. β whose utility is worsen, the utility of Mr. ϕ increases slightly because he limits the fall of income due to lower interest rates by switching to the asset market. This reinforces the same message: agents which have more investment opportunities can use their flexibility in reallocating their investment portfolio to switch themselves out from the adverse condition.

In sum, increases in the outside money causes distributional effects that have negative effects in some part of the economy and partly reduces the efficiency of the economy since some of the interest rates are bound to rise.

8.1.3 An Increase in Capital Requirements Loan Risk Weights

An increase in the risk weights on loans for both banks ($\overline{\omega}$) will underline even further the argument which we have already mentioned; namely those agents who have more investment opportunities, and therefore greater flexibility, can mitigate the effect of a negative shock by restructuring their portfolios. Given that the initial condition of the economy is adverse where capital requirements are binding and there is no access to the capital markets to raise new equity, the impact would be procyclical. Bank δ will further reduce credit extension to avoid the extra cost of additional capital requirements' violation penalty, and bank γ in particular will increase its violation since it cannot switch its investments to support its profitability. Consequently, its payoff will be severely affected both from reduced interest rate payments and also the higher penalties for capital requirements' violation. In contrast, bank δ reduces investments in both the loan market and the asset market and increases its deposits in the interbank market.

Bank γ, anticipating the higher expected capital requirement violation penalty, will increase its credit extension to lessen its profit reduction, by borrowing more from the interbank market and thus raising the interbank market interest rate. Since bank γ will charge lower interest rates in order to increase credit extension, deposits from Mr. ϕ decreases and he switches to asset investments. In contrast, bank δ which diversifies away from the loan market increases its interest rates. Moreover, reduced investments in the asset market lower the asset price. Tighter credit condition reduce commodity prices in all periods except state 1 where the Arrow security pays and there is extra liquidity in the economy. Finally, default of both agents increases on average, (even though both of them maintain higher repayment rates in state 1), because of tighter credit market conditions for Mr. α and lower expected income for both Mr. α and Mr. β.

Profitability for bank γ is reduced substantially whereas bank δ's ability to restructure its portfolio generates slightly positive profits, even though the aggregate profit of the banking industry is reduced. Paradoxically, though, Mr. α's welfare is improved. Because in effect bank γ follows a countercyclical policy with respect to the higher risk weights, so lower interest rates help Mr.α to borrow more cheaply and increase his consumption in period 0 and therefore slightly improves his utility, unlike Mr. β who is hurt by the higher interest rate charged by bank δ. Finally, Mr. ϕ's utility is almost unchanged (with an ambiguous direction) since he compensates his reduced purchasing power from lower interest rate payments on deposits in the asset market whose return is increased.

Regulatory policy may be seen as the mirror image of monetary policy since it directly affects credit extension via the capital requirements constraint. Moreover, banks without well-diversified portfolios and thus not so many investment opportunities follow a countercyclical credit extension policy that hurts them but benefits their respective clients. The countercyclical credit extension policy of not well diversified banks may also be thought of as a built-in-stabilizer in the economy when regulatory policy becomes tighter and the economy faces the danger of multiplicative credit contraction. On the contrary, banks that can quickly restructure their portfolios transfer the negative externalities of higher risk weights to their clients. Thus, restrictive regulatory policy in periods of economic adversity may enhance financial fragility by inducing lower profitability, higher default and further capital requirement violations.

8.1.4 Conclusion

Here we seek to draw out and recapitulate the key results obtained from the comparative statics. First, in an adverse economic environment, expansionary monetary policy can aggravate financial fragility since the extra liquidity injected by the Central Bank may be used by certain banks to gamble for resurrection, worsening their capital position, and therefore the overall financial stability of the economy. Thus, a trade-off between efficiency and financial stability need not exist only for regulatory policies, but also for monetary policy.

Second, agents which have more investment opportunities can deal with negative shocks more effectively by using their flexibility in quickly restructuring their investment portfolios as a means to transfer 'negative externalities' to other agents with a more restricted set of investment opportunities. This result has various implications. Among others, banks which have no well-diversified portfolios tend to follow a countercyclical credit extension policy in face of a negative regulatory shock in the loan market (e.g. tighter loan risk weights). In contrast, banks which can quickly restructure their portfolio tend to reallocate their portfolio away from the loan market, thus following a procyclical credit extension policy. Moreover, regulatory policies which are selectively targeted at different groups of banks can produce very *non-symmetric* results. When the policy is aimed at banks which have more investment opportunities, much less contagion effect to the rest of the economy is observed since those banks simply restructure their portfolios between interbank and asset markets without greatly perturbing the credit market, and therefore interest rates and prices in the economy. On the contrary, when the same policy is targeted at banks which have relatively limited investment opportunities, they are forced to 'bite the bullet' by altering their credit extension. This produces changes in a series of interest rates, and therefore the cost of borrowing for agents. This in turn produces a contagion effect to the *real* sector in the economy.

Thirdly, an improvement such as a positive productivity shock, which is concentrated in one part of the economy, does not necessarily improve overall welfare and profitability of the economy. The key reason for this lies in the fact that our model has *heterogenous* agents and therefore possesses various feedback channels among various sectors in the economy which all are active in equilibrium. Thus, a positive shock in one specific sector can produce a negative contagion effect in others, causing the welfare and profitability of the whole economy to fall.

Finally, increasing the endowment of banks produces much the same result as increasing their corresponding capital violation penalties, particularly in the light of its contagion effect to the rest of the economy. Thus, a direct injection of emergency recapitalisation funds to banks is, to a certain extent, substitutable by increasing the banks' capital violation penalties.

8.2 Comparative Statics II: Interbank Contagion

We turn now to another comparative statics which aims at illuminating how the effect of various shocks can generate contagion effect via the interbank and deposit markets. To that end, compared with the previous comparative statics, we modify the structure of the simplified model as follows;

First, we allow endogenous defaults in the interbank market, i.e. bank γ can default on its interbank loans in which it borrows from bank δ. Second, we make a more realistic assumption by allowing separated deposit markets.[12] Moreover, Mr. ϕ has a choice to deposit his money with either bank. To distinguish between bank γ's and bank δ's deposits, we assume that the relatively more risky bank, bank γ, can default on its borrowing from the deposit market. Third, in order to incorporate these additional complexities while retaining the model tractability, we simplify the model by assuming away the Arrow security market. Finally, we assume that the cost of default in the interbank and deposit market is quadratic. This in turn implies that the marginal cost of default in these markets is greater as the size of borrowing is larger. Thus the modified profit function of bank γ is given as follows;

$$U^{\gamma} = \sum_{s=1}^{3} \pi_s^{\gamma} - \sum_{s=1}^{3} \left(\lambda_{ks}^{\gamma} \max[0, \overline{k} - k_s^{\gamma}] + \lambda_s^{\gamma} [\mu^{\gamma} - v_s^{\gamma} \mu^{\gamma}]^2 + \lambda_{s\phi}^{\gamma} [\mu_d^{\gamma} - v_s^{\gamma} \mu_d^{\gamma}]^2 \right)$$

As before, we summarise the value of exogenous variables/parameters and the initial equilibrium in tables I and II of Appendix V, respectively. Tables IIIa, IIIb, IIIc of Appendix V explain the sources/uses of funds for households and banks and market clearing conditions, respectively. Table IV of Appendix V then describes the directional effects on endogenous variables of increasing various parameters listed in the first column.

[12] Recall that in the previous comparative static we assume no separated deposit market, i.e. a perfectly elastic demand for deposits by bank γ at the rate of interest equal to its lending rate.

9 Concluding Remarks

In reality, the economic system is both complex and heterogenous. In order to model it in a way that is both mathematically tractable, rigorous, and yet simple enough to be illuminating, economists have tended to assume homogeneity amongst agents in the sectors involved. Unfortunately that prevents analysis of certain key features of the real world, especially those relating to interbank interactions and financial fragility.

We have sought to focus on such inter-active channels. That has inevitably raised the complexity of our modelling; however we have tried to limit such complexity by adopting an endowment economy with just endowed consumers and banks, (no firms, no external sector, no other financial intermediaries, a black-box official sector).

Even so, as the results in section 8, and Appendix VI, show there are numerous interacting channels, and some of the outcomes involve complex and subtle interactions and feedback mechanisms. Given the sizeable number of agents, assets and markets, the only way to solve such a system is numerically. There will, no doubt, be a question how robust the results shown here as a function of the particular model structure and initial conditions used. We intend to explore this further, beyond the comparative statics already documented in section 8 and Appendix VI.

What have we learnt already? One clear result is that increasing bank flexibility, by switching between asset (and liability) markets, is highly beneficial. Disadvantaged bank γ, who was restricted to a limited set of markets, often ran into trouble, while its richer and more flexible companion, bank δ, could trade its way out of problems.

For the rest, the main result, perhaps not surprisingly, is that in a complex world, the full response to a shock is often not easy to see in advance. Thus, as shown in section 8.1, expansionary monetary policy may cause such asset expansion that capital ratios fall relative to requirements. Again, when default penalties rise in one sector of the economy, even though it improves efficiency, it generates a negative externality to the rest of the economy which is relatively inefficient. However, financial fragility of the economy is reduced because the first order effect of the increased efficiency dominates the second order effect of the negative externality to the rest of the economy.

We see this paper as the start of a major programme to use models such as this for the analysis of financial fragility. We shall stick to two main principles; first, heterogeneity is essential; second our approach to modelling default is the best available modelling strategy. Otherwise we hope to examine a wide range of alternative structures.

The main challenges ahead will be, first to represent a complex reality in a manner that is both illuminating and yet reflects that reality, and second to be able to draw general conclusions from an array of models that may, each individually, be sensitive to their individual particularities. This paper is the first step in our planned programme.

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Appendix I: Proofs

[to be inserted]

Appendix II: Summary of Notation

This appendix summarises the notations used in the paper.
$t \equiv$ time period
$s \equiv$ future states ($s = 0$ refers to the initial period $t = 0$)
$h \equiv$ households/investors
$b \equiv$ commercial banks
$l \equiv$ commodities
$e^h \equiv$ household endowments
$m^h \equiv$ monetary endowments of households
$e^b \equiv$ capital endowments of commercial banks
$u^h \equiv$ utility functions of households
$\pi^b_s \equiv$ monetary holdings of b at $s \in S^*$
$u^b(\cdot) \equiv$ objective function of commercial banks
$M^{CB} \equiv$ central bank money supply
$\mu^{CB} \equiv$ bond sales by the central bank
$m^{CB} \equiv$ emergency liquidity assistance to commercial banks
$k^b_t \equiv$ capital to asset ratio of commercial banks
$\overline{k}_t \equiv$ capital requirements
$z \equiv$ credit markets, secondary asset markets or capital requirements
$\lambda^h_{sz} \equiv$ penalties
$\omega_{tj} \equiv$ risk-weights
$j \equiv$ assets traded in the secondary asset markets
$A^j \equiv$ asset payoffs
$\theta^j \equiv$asset prices
$b^h_{\cdot} \equiv$ amount of money sent by h in a market
$q^h_{\cdot} \equiv$ quantity of commodities (or promises of assets) sold in the corresponding market
$r^b \equiv$ nominal interest rate charged by b
$\mu^{h^b} \equiv$ nominal loans demanded by $h^b \in H^b$
$\overline{m}^b \equiv$ credit extension by b
$\psi^h_{sb} \equiv$ equity of commercial bank $b \in B$ owned by $h \in H$, at $s \in S^*$
$R \equiv$ set of expected rates of deliveries of various instruments
$D^h_{sz} \equiv$ debt
$v^h_{sz} \equiv$ repayment rates by agents
$\Pi^h_s(\cdot) \equiv$ payoffs to households
$\Pi^b_s(\cdot) \equiv$ payoffs to commercial banks
$p_{sl} \equiv$ commodity prices at $s \in S^*, l \in L$
$s^h_b \equiv$ quantity of shares of ownership by $h \in H$, at $t = 0$ at the initial public offering of $b \in B$
$\chi^h \equiv$ households' consumption
$\pi^b_s \equiv$ banks' profit
$\theta^b_s \equiv$ prices of commercial bank equity at the secondary market
$\rho \equiv$ interbank interest rate
$d \equiv$ deposits at various markets

Appendix III

Table I: Exogenous variables

Coefficient of risk aversion	Endowment			Penalty		Others
	Commodities	Money	Capital	Default	CAR violation	
$c_0^\alpha = 0.0319$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 0.1$	$\lambda_{1\gamma}^\alpha = 0.2$	$\lambda_{1k}^\gamma = 5$	$M^{CB} = 0.193$
$c_1^\alpha = 0.0458$	$e_1^\alpha = 18$	$m_1^\alpha = 0.028$	$e_1^\gamma = 2$	$\lambda_{2\gamma}^\alpha = 0.1$	$\lambda_{2k}^\gamma = 4$	$\overline{\omega} = 1$
$c_2^\alpha = 0.048$	$e_2^\alpha = 18$	$m_2^\alpha = 0.0071$	$e_2^\gamma = 2$	$\lambda_{3\gamma}^\alpha = 0.05$	$\lambda_{3k}^\gamma = 3.7064$	$\omega = 0.5$
$c_3^\alpha = 0.0453$	$e_3^\alpha = 18$	$m_3^\alpha = 0.0373$	$e_3^\gamma = 2$	$\lambda_{1\delta}^\beta = 0.2$	$\lambda_{1k}^\delta = 2.4856$	$\widetilde{\omega} = 0.5$
$c_0^\beta = 0.0264$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 16$	$\lambda_{2\delta}^\beta = 0.1$	$\lambda_{2k}^\delta = 0.9318$	$A_j = [1\ 0\ 0]'$
$c_1^\beta = 0.0371$	$e_1^\beta = 20$	$m_1^\beta = 1.65$	$e_1^\delta = 1.5$	$\lambda_{3\delta}^\beta = 0.05$	$\lambda_{3k}^\delta = 0.9318$	$\overline{k} = 0.4$
$c_2^\beta = 0.0374$	$e_2^\beta = 20$	$m_2^\beta = 2.3679$	$e_2^\delta = 1.5$			
$c_3^\beta = 0.0367$	$e_3^\beta = 20$	$m_3^\beta = 2.1468$	$e_3^\delta = 1.5$			
$c_0^\phi = 0.0347$	$e_0^\phi = 25$	$m_0^\phi = 1.7066$				
$c_1^\phi = 0.0205$	$e_1^\phi = 0$	$m_1^\phi = 0$				
$c_2^\phi = 0.019$	$e_2^\phi = 0$	$m_2^\phi = 0$				
$c_3^\phi = 0.0327$	$e_3^\phi = 0$	$m_3^\phi = 0$				

Table II: Initial Equilibrium

Prices	Loans/deposits	Capital/Asset ratio	Repayment rate	Commodities	Assets
$p_0 = 1$	$\overline{m}^\gamma = 8$	$k_1^\gamma = 0.19$	$v_{1\gamma}^\alpha = 0.938$	$b_0^\alpha = 8$	$b_j^\phi = 0.257$
$p_1 = 1.103$	$\overline{m}^\delta = 10$	$k_2^\gamma = 0.193$	$v_{2\gamma}^\alpha = 0.927$	$q_1^\alpha = 9.492$	$b_j^\delta = 1.626$
$p_2 = 1.177$	$d_\gamma^\phi = 1.449$	$k_3^\gamma = 0.194$	$v_{3\gamma}^\alpha = 0.923$	$q_2^\alpha = 8.809$	$q_j^\gamma = 6.773$
$p_3 = 1.336$	$d^\delta = 4.374$	$k_1^\delta = 0.085$	$v_{1\delta}^\beta = 0.899$	$q_3^\alpha = 7.7$	
$r^\gamma = 0.399$	$\mu^{\alpha^\gamma} = 11.19$	$k_2^\delta = 0.102$	$v_{2\delta}^\beta = 0.892$	$b_0^\beta = 10$	
$r^\delta = 0.349$	$\mu^{\beta^\delta} = 13.48$	$k_3^\delta = 0.103$	$v_{3\delta}^\beta = 0.881$	$q_1^\beta = 9.497$	
$\rho = 0.198$	$\mu^\gamma = 5.474$			$q_2^\beta = 8.205$	
$\theta = 0.278$				$q_3^\beta = 7.286$	
				$q_0^\phi = 18.017$	
				$b_1^\phi = 20.952$	
				$b_2^\phi = 20.027$	
				$b_3^\phi = 20.027$	

Table IIIa: Sources and Uses of Funds for Households

period/state	α		β		ϕ	
	Sources	Uses	Sources	Uses	Sources	Uses
0	$\mu^{\alpha^\gamma}/(1+r^\gamma)$ [11.19/(1.399)]	b_0^α [8]	$\mu^{\beta^\delta}/(1+r^\delta)$ [13.48/1.349]	b_0^β [10]	m_0^ϕ [1.7066]	b_j^ϕ [0.257]
						d_γ^ϕ [1.449]
1/1	$p_1q_1^\alpha$ [(1.103)(9.492)]	$v_{1\gamma}^\alpha\mu^{\alpha^\gamma}$ [(0.938)(11.19)]	$p_1q_1^\beta$ (1.103)(9.497)	$v_{1\delta}^\beta\mu^{\beta^\delta}$ [(0.899)(13.48)]	$p_0q_0^\phi$ (1)(18.017)	b_1^ϕ [20.952]
	m_1^α [0.028]		m_1^β [1.65]		$d_\gamma^\phi(1+r^\gamma)$ (1.449)(1.399)	
					b_j^ϕ/θ [0.257/0.278]	
1/2	$p_2q_2^\alpha$ [(1.177)(8.809)]	$v_{2\gamma}^\alpha\mu^{\alpha^\gamma}$ [(0.927)(11.19)]	$p_2q_2^\beta$ [(1.177)(8.205)]	$v_{2\delta}^\beta\mu^{\beta^\delta}$ [(0.892)(13.48)]	$p_0q_0^\phi$ [(1)(18.017)]	b_2^ϕ [20.027]
	m_2^α [0.0071]		m_2^β [2.3679]		$d_\gamma^\phi(1+r^\gamma)$ [(1.449)(1.399)]	
1/3	$p_3q_3^\alpha$ [(1.336)(7.7)]	$v_{3\gamma}^\alpha\mu^{\alpha^\gamma}$ [(0.923)(11.19)]	$p_3q_3^\beta$ [(1.336)(7.286)]	$v_{3\delta}^\beta\mu^{\beta^\delta}$ [(0.881)(13.48)]	$p_0q_0^\phi$ [(1)(18.017)]	b_3^ϕ [20.027]
	m_3^α [0.0373]		m_3^β [2.1468]		$d_\gamma^\phi(1+r^\gamma)$ [(1.449)(1.399)]	

Table IIIb: Sources and Uses of Funds for Banks

period/state	γ		δ	
	Sources	Uses	Sources	Uses
0	$\mu^\gamma/(1+\rho)$ [5.474/1.198]	$\overline{m}^\gamma$ [8]	e_0^δ [16]	$\overline{m}^\delta$ [10]
	d_γ^ϕ [1.499]			d^δ [4.374]
	θq_j^γ [(0.278)(6.773)]			b_j^δ [1.626]
	e_0^γ [0.1]			
1/1	$v_{\gamma 1}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.938)(1.399)(8)]	μ^γ [5.474]	$v_{1\delta}^\beta \overline{m}^\delta(1+r^\delta)$ [[(0.899)(10)(1.349)]]	π_1^δ [24.5]
	e_1^γ [2]	q_j^γ [6.773]	e_1^δ [1.5]	
		$(1+r^\gamma)d_\gamma^\phi$ [(1.399)(1.449)]	b_j^δ/θ [1.626/0.278]	
		π_1^γ [−1.776]	$d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(4.374)(1.198)(4.374/(4.374+0.193))]	
1/2	$v_{\gamma 2}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.927)(1.399)(8)]	μ^γ [5.474]	$v_{2\delta}^\beta \overline{m}^\delta(1+r^\delta)$ [(0.892)(10)(1.349)]	π_2^δ [18.552]
	e_2^γ [2]	$(1+r^\gamma)d_\gamma^\phi$ [(1.399)(1.449)]	e_2^δ [1.5]	
		π_2^γ [4.874]	$d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(4.374)(1.198)(4.374/(4.374+0.193))]	
1/3	$v_{\gamma 3}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.923)(1.399)(8)]	μ^γ [5.474]	$v_{3\delta}^\beta \overline{m}^\delta(1+r^\delta)$ [(0.881)(10)(1.349)]	π_3^δ [18.403]
	e_3^γ [2]	$(1+r^\gamma)d_\gamma^\phi$ [(1.399)(1.449)]	e_3^δ [1.5]	
		π_3^γ [4.829]	$d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(4.374)(1.198)(4.374/(4.374+0.193))]	

Table IIIc: Market Clearing Conditions

Commodity Market	Financial Market
$p_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi}$	$1+\rho = \frac{\mu^\gamma}{M^{CB}+d^\delta}$
$p_1 = \frac{b_1^\phi}{q_1^\alpha + q_1^\beta}$	$1+r^\gamma = \frac{{\mu^\alpha}^\gamma}{\overline{m}^\gamma}$
$p_2 = \frac{b_2^\phi}{q_2^\alpha + q_2^\beta}$	$1+r^\delta = \frac{{\mu^\beta}^\delta}{\overline{m}^\delta}$
$p_3 = \frac{b_3^\phi}{q_3^\alpha + q_3^\beta}$	$\theta = \frac{b_j^\delta + b_j^\phi}{q_j^\gamma}$

Table IV: Directional Effects of an increase in exogenous parameters on endogenous variables

	p_0	p_1	p_2	p_3	r^γ	r^δ	ρ	θ	$\overline{m}^\gamma$	$\overline{m}^\delta$	d_γ^ϕ	d^δ	μ^{α^γ}	μ^{β^δ}	μ^γ
M	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$+$	$\approx$	$\approx$	$+$
m_1^α	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$\approx$	$\approx$	$+$
ω	$\approx$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$\approx$	$\underset{\approx}{+}$	$+$
$\lambda_{1\gamma}^\alpha$	$\approx$	$\underset{\approx}{+}$	$-$	$-$	$-$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$\approx$	$+$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$-$	$\approx$	$-$	$\underset{\approx}{-}$
λ_{1k}^γ	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$
λ_{1k}^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$
e_0^ϕ	$-$	$+$	$-$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$+$
e_1^α	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$
e_1^β	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$
e_1^γ	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$
e_1^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$
c_1^α	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$

Note: $+(-)$: substantial increase (decrease)

$\underset{\approx}{+}(\underset{\approx}{-})$:weak increase (decrease), $\approx$: approximately equal

$+/-$:ambiguous effect

Table IV (continue)

	k_1^γ	k_2^γ	k_3^γ	k^γ	k_1^δ	k_2^δ	k_3^δ	k^δ	k	$v_{1\gamma}^\alpha$	$v_{2\gamma}^\alpha$	$v_{3\gamma}^\alpha$	v_γ^α	$v_{1\delta}^\beta$	$v_{2\delta}^\beta$	$v_{3\delta}^\beta$	v_δ^β	v
M	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$-$	$-$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_1^α	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\underset{\approx}{+}$
ω	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$\underset{\approx}{-}$	$+$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
$\lambda_{1\gamma}^\alpha$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$\underset{\approx}{+}$	$-$	$-$	$-$	$+$
$\lambda_{1\delta}^\beta$	$+$	$-$	$-$	$\underset{\approx}{+}$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$+$	$+$	$+$	$+$
λ_{1k}^γ	$+$	$-$	$-$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\approx$	$-$	$+$	$+$	$\underset{\approx}{+}$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
λ_{1k}^δ	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
e_0^ϕ	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
e_1^α	$-$	$-$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$\approx$
e_1^β	$+$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$\underset{\approx}{-}$	$\approx$	$+$	$+$	$+$	$\approx$
e_1^γ	$+$	$-$	$-$	$+$	$+$	$\approx$	$\approx$	$\underset{\approx}{+}$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
e_1^δ	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
c_1^α	$-$	$-$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$\approx$

Note: $k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3, k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3, k \equiv (k^\gamma + k^\delta)/2$

$v_\gamma^\alpha \equiv (v_{1\gamma}^\alpha + v_{2\gamma}^\alpha + v_{3\gamma}^\alpha)/3, v_\delta^\beta \equiv (v_{1\delta}^\beta + v_{2\delta}^\beta + v_{3\delta}^\beta)/3, v \equiv (v_\gamma^\alpha + v_\delta^\beta)/2$

Table IV (continue)

	b_0^α	q_1^α	q_2^α	q_3^α	b_0^β	q_1^β	q_2^β	q_3^β	q_0^ϕ	b_1^ϕ	b_2^ϕ	b_3^ϕ	b_j^ϕ	b_j^δ	q_j^γ
M	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$-$	$-$
m_1^α	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$+$
ω	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$+$	$-$	$-$	$+$	$-$	$+/-$
$\lambda_{1\gamma}^\alpha$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$+$	$-$	$-$	$+$	$-$	$+$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$+$	$-$	$+/-$	$-$
λ_{1k}^γ	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$-$	$+$	$+$	$-$	$+$	$-$
λ_{1k}^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$
e_0^ϕ	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$+$	$-$	$-$	$+$	$-$	$+$
e_1^α	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$+$	$+$	$-$	$+$	$+$
e_1^β	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$+$	$+$	$-$	$+/-$	$-$
e_1^γ	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$-$	$+$	$+$	$-$	$+$	$-$
e_1^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$
c_1^α	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$+$	$+$	$-$	$+$	$+$

Table IV (continue)							
	U^{α}	U^{β}	U^{ϕ}	U^{γ}	U^{δ}	U^{H}	U^{B}
M	$\underset{\approx}{+}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\underset{\approx}{-}$
m_1^{α}	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$\underset{\approx}{+}$
ω	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+/-$	$-$	$\underset{\approx}{+}$	$\approx$	$-$
$\lambda_{1\gamma}^{\alpha}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\approx$
$\lambda_{1\delta}^{\beta}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$
λ_{1k}^{γ}	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+/-$	$-$	$-$	$\approx$	$-$
λ_{1k}^{δ}	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$-$
e_0^{ϕ}	$+$	$+$	$+$	$+$	$\underset{\approx}{-}$	$+$	$\approx$
e_1^{α}	$+$	$-$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
e_1^{β}	$-$	$\underset{\approx}{+}$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$
e_1^{γ}	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\underset{\approx}{+}$
e_1^{δ}	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\approx$	$\underset{\approx}{+}$
c_1^{α}	$-$	$-$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
Note: $U^H \equiv (U^{\alpha} + U^{\beta} + U^{\phi})$ $U^B \equiv U^{\gamma} + U^{\delta}$							

Appendix IV

This appendix explains the results of cnanges in various other exogenous variables obtained from comparative static I which have not been explained in section 8.

A. Increases in Consumer Default Penalties

The next experiment of regulatory policy we consider is the increase of consumer default penalties for Mr. α in state 1 ($\lambda_{1\gamma}^{\alpha}$). As expected, repayment rates of Mr. α increase when his default penalty increases. Moreover, his borrowing decreases, loan interest rates fall and thus bank γ deposits from Mr. ϕ fall as well. The interbank interest rate falls since bank γ borrows less due to lower demand for loans. Prices in state 1 increase since Mr. ϕ switches from deposits to spending more on the asset market, thus increasing his expected income in state 1. Prices in $t = 0$ remain constant whereas those in states 2 and 3 fall since Mr. ϕ reduces his expenditures there due to lower income (recall that the Arrow security does not pay in states 2 and 3). Moreover, the diversification of Mr. ϕ pushes upwards asset prices. Given that commodity prices increase in state 1, Mr. β sells more in this particular state, thereby increasing his expected revenue from selling commodities and repaying more to bank γ. In contrast, since commodities prices in states 2 and 3 declines, so is the expected revenue of Mr. β and his repayment rates. On average, the repayment rate of Mr. β declines, the expected rate of return from lending of bank δ therefore also falls. Although a similar effect also applies to Mr. α, he defaults less in all states. This is because Mr. α borrows substantially less from bank γ since the cost of default is significantly greater in state 1. Given a smaller size of his obligation to bank γ, he can repay more even though his income is lower in states 2 and 3 due to lower commodity prices.

Because bank δ's expected rate of return from lending to Mr. β decreases, it reallocates its investment from the credit market to the interbank market, causing the interbank rate to fall and r^{δ} to rise. Thus bank δ's value of total risk weighted assets decreases (since interbank loans carry lower risk weight). Similarly for bank γ, its expected capital requirements improve on average, even though it worsens in state 1 where Mr. α's repayment rate increases drastically. So, in sum, the aggregate capital adequacy position of the banking industry improves mainly because of the switching of bank δ's investments to lower risk-weighted assets.

The welfare of Mr. α improves due to lower default penalty payments and likewise the profitability of bank γ improves because of higher repayment rates and its slightly better capital adequacy position. The utility of Mr. β slightly falls because of his higher default due to the increased default premium he incurs in the loan market. Similarly, the profitability of bank δ deteriorates mildly due to lower return on its investments that outweigh its improved capital adequacy situation. Finally, the utility of Mr. ϕ slightly improves because lower prices in states 2 and 3 increase his purchasing power and also his income in state 1 rises because of the payoff of his asset investment.

Recapitulating, increases in default penalties in one sector of the economy, even though it improves efficiency, generate a negative externality to the rest of the economy which is relatively inefficient. However, financial fragility of the economy is reduced because the first order effect of the increased efficiency dominates the second order effect of the negative externality to the rest of the economy.

Turning to the experiment in which we increase the default penalty for household β. The main difference of this change compared to the previous one lies to the fact that the interbank interest rate now increases and also prices move, surprisingly at first sight, the opposite way than before. The reason for the increase in the interbank rate is that bank δ switches to investments in the loan market. Moreover, since now bank γ's interest rate increases for the equivalent reason as before, Mr. ϕ increases his deposits to bank γ and thus has more income to spend at $t = 1$. However, commodity prices fall in state 1 because Mr. β increases his supply (sales) to generate more income to reduce his default level since it is more costly for him to default at this state. Finally, even though the channels are slightly different now the welfare effects are the same as before.

B. Increases in the Commodity Endowment of Households

We now turn to an experiment which can be analogously thought of as a positive productivity shock (i.e. an increase in the commodity endowment of households). Let the commodity endowment of Mr. α in state 1 (e_1^{α}) increase. Mr. α then supplies more of the commodities, pushing down the state-1 commodity price. Since Mr. α's revenue from selling commodities in this particular state increases, the corresponding default rate falls. As we shall see, Mr. α also repays more in the other two states, implying that bank γ's expected return from lending to Mr. α increases. It therefore extends more credits and charges a lower interest rate (r^{γ}). As the commodity price in state 1 declines, Mr. β decreases his supply in the commodity market, thus receiving lower revenue and defaults more. The extent of default in state 1 by Mr. β is so high that it drives down the overall repayment rate, and therefore decreases bank δ's expected return from lending. Consequently, bank δ reduces its credit supply to Mr. β, and charges a higher interest rate (r^{δ}). As state 1 is the state in which bank δ is hurt the most (recall that default in state 1 by Mr. β increases drastically) and that the rate of return from investing in the Arrow security is the highest in this particular state, it reallocates its portfolio from the credit, and interbank markets to the asset market. Thus the interbank rate also increases.

The asset price falls, although slightly. This is because, as we shall see, Mr. ϕ also reduces his demand for assets. Thus, bank γ borrows more from the asset market, and less from the interbank market.

Other things constant, the reduction in r^γ implies a lower deposit income for Mr. ϕ, particularly in states 2 and 3 where the Arrow security does not pay. In order to smooth his income, he therefore re-allocates part of his investment from the asset market to the deposit market. This *intra-state diversification effect* increases his income so much that it dominates the negative effect from declining deposit rate. Consequently, he demands more commodities in states 2 and 3, causing the prices in these two states to increase. Because of higher prices, Mr. α and β increase slightly their supplies in the commodity market in states 2 and 3, increasing their revenue and therefore their loan repayment rates in these two states. Anticipating higher expected income in period 1, Mr. ϕ consumes more and sells less of his endowment in period 0, pushing up the period 0 price. Similarly, Mr. α who anticipates higher endowment in state 1 demands more commodities in period 0. In contrast, the opposite is true for Mr. β as he anticipates lower expected revenue from selling commodities in period 1 (recall that commodities in state 1 drops precipitously).

A point worth emphasising is that Mr. ϕ is able to transfer his negative externalities from a lower deposit rate to bank γ by switching his investment from the asset market to the deposit market. Bank γ in turn is forced to lend more to Mr. α as that is the only investment opportunity that he has. This crucially highlights the point stressed before that banks which have more investment opportunities are more capable of coping with negative shocks as they have more flexibility in readjusting their portfolio in a way that the negative externalities are at least partially transferred to those which have relatively more restrictive choices of investments. This argument is confirmed by observing the changes in the profitability of banks and the welfare of agents. While the utility of Mr. α weakly increases since his increased endowment allows him to consume more in equilibrium and therefore defaults less, the profitability of bank γ from which Mr. α borrows his money surprisingly decreases. This is so because the benefit from higher repayment rate is outweighed by the negative externalities transferred from Mr. ϕ via the deposit market, forcing the bank to overlend to Mr. α and thereby pushing down the lending rate significantly. The utility of Mr. β falls as he suffers from a substantial decrease in revenue in state 1 due to the reduction in the commodity price. However, the ability of bank δ to switch its portfolio to the asset market allows it to mitigate the negative effect from lower repayment rates from Mr. β and therefore to be able to maintain its prior level of profitability. Similarly, Mr. ϕ's utility weakly increases as he is able to transfer his negative externalities to bank γ through transferring his investment from the asset market to the deposit market.

We also observe that, while an increase in Mr. α's endowment improves the financial stability of the economy via increasing the repayment rate of Mr. α, such benefit is attenuated as Mr. β on average defaults more. Thus the average economy-wide repayment rate is nearly unchanged. Moreover, the extent of capital requirement violation for bank γ worsens due to the negative externalities transferred from Mr. ϕ, as well as its inflexibility to readjust his investment portfolio. Although the capital to asset position of bank δ improves on average due to its ability to readjust its portfolio from the credit market to asset market, the extent of improvement is not sufficient to bring about a greater capital to asset ratio of the whole banking industry. In sum, a positive shock on the endowment of Mr. α appears to worsen the overall financial fragility of the economy. The primary reason for this result is that the improvement of Mr. α relative to bank γ causes an adverse shock to the financial sector of the economy because the corresponding financial intermediary is not flexible enough to absorb the adverse distributional effect through adjustments in its portfolio.

Let now turn to the next experiment in which the endowment of Mr. β is increased. The results are generally symmetric to those of the previous case, with certain noticeable exceptions. These exceptions arise from the fact that banks γ and δ are different; one being a straightforward bank which is relative poor at $t = 0$, takes deposits and has limited investment opportunity and the other being more like an investment bank which relatively rich at $t = 1$, and has more investment opportunities. Because bank δ has more flexibility to adjust its portfolio of assets, its extent of capital requirement violation on average is not as big compared to that of bank γ in the previous case. Thus, unlike the previous case, an endowment shock to Mr. β tends to improve the overall financial stability of the economy. We remark that in the presence of incomplete asset markets and therefore constrained inefficient allocation (see section 8.3), an improvement in the economy such as a productivity improvement does not necessarily implies welfare and profitability improvements. Thus, policy intervention needs to be context specific in each case and carefully designed to avoid adverse equilibrium adjustments as is the case here.

Finally, it is worth pointing out that although deposits from Mr. ϕ to bank γ increase as in the previous case, they do so through a different channel. While in the previous case it increases because Mr. ϕ switches from the asset market to the deposit market to maintain his income (recall that r^{γ} falls) and therefore transfer the negative externalities to bank γ, here he does so in order to reap an extra benefit from higher deposit rate. Thus, unlike bank γ who suffers from the negative externalities, bank δ here does not. This together with its flexibility to reallocate its investment to the asset and interbank markets are the main the reason why bank δ's profit increases slightly while that of bank γ in the previous case falls rather precipitously.

Let now consider the effect of an increase in the endowment of Mr. ϕ in period 0 (e_0^{ϕ}). This directly increases the supply of commodities in period 0 by Mr. ϕ, which in turn markedly decreases the price in that period. This produces a *positive spill over effect* to Mr. α and Mr. β who can now purchase commodities in period 0 cheaply. They, therefore, demand less credit from their banks, causing r^{γ} and r^{δ} to decline. Mr. ϕ then reallocates part of his investment from the deposit market, which now offers lower return, to the asset market. Asset prices therefore rise, though weakly since bank γ also sells more of its asset to substitute for the reduction in Mr. ϕ's deposits. This in turn implies that bank δ would restructure part of his investment from the asset market to the interbank market. This brings about a decline in the interbank market rate.

Because Mr. ϕ invests more in the asset market, and less in the deposit market, his income in state 1 where the Arrow security pays increases while the opposite is true for the other two states. This justifies why he purchases more commodities in state 1 and less in states 2 and 3, causing the price to increase in state 1 and to decrease in states 2 and 3.

Both Mr. α and β default less in all states. This is because they borrow less in period 0 and therefore can repay more in period 1, given that their income from selling commodities changes slightly. The repayment rate increases the most in state 1 where commodity price and quantity sold increase which in turn produce a substantial increase in their income. Although the significant increase in the repayment rate in state 1 implies that banks γ and δ violate more of their capital requirement in that state, the overall financial stability of the banking industry on average improves. In sum, a positive endowment shock to the rich household who functions as an ultimate lender (i.e. investor) in equilibrium tends to reduce the extent of financial fragility in the economy.

The welfare of Mr. α and β improves greatly as they benefit from lower cost of borrowing for their respective banks. Mr. ϕ remains better off from higher endowment, though the positive effect is offset by the lower rate of return from asset and deposit investments. Bank γ, which is the net borrower in the asset and interbank markets, also benefits from a lower cost of borrowing. In contrast, the profitability of bank δ declines as the rates of return from investing in the asset and interbank markets decline.

C. Preference change

Let the preference in state 1 of Mr. α changes. In particular, let us consider a sudden drop of Mr. α's taste for consumption in state 1 (a higher c_1^{ϕ}) . In other words, suppose that Mr. α switches his preference in favour of current consumption, and moreover regarding future consumption he prefers states 2 and 3 consumption relatively more than state 1. We observe that the directional changes of all variables here are exactly the same as those of the previous exercise where the endowment of Mr. α in state 1 is increased, with the only exception on the directional change of Mr. α's utility. This is so because a higher c_1^{ϕ} implies a lower marginal utility from consuming in state 1. Thus, he reduces his consumption and sells more goods in this particular state. Equivalently, an increase in Mr. α's endowment of commodities in state 1 also lowers the marginal utility of his consumption in this particular state. We also see the Fisher relation operating since a lower marginal utility of future consumption decreases the lending rate offered by bank γ from which Mr. α borrows. However, unlike the case of increasing endowments where Mr. α benefits directly from being endowed with more commodities at $t = 0$, and therefore can still consume more even though he also sells more, now he is worse off since he ends up consuming less.

D. Increases in Capital Requirement Violation Penalties

We now turn to another set of regulatory policy related experiments. Let the capital violation penalty of bank γ in state 1 increase, implying that the marginal cost of expanding the bank's assets in state 1 rises. As credit extended to Mr. α is the only type of asset that bank γ has, it responds by cutting its supply of credit, pushing up the interest rate r^{γ}. As the deposit rate increases, Mr. ϕ switches part of his investment from the asset market to the deposit market. Since bank γ has more deposits but is now willing to supply less credit, it lowers its demand from the interbank and asset markets. Thus the interbank market rate decrease while the asset price increases, though the latter does so slightly as Mr.ϕ also buy fewer securities. Bank δ therefore reallocates its portfolio from the interbank and asset markets, both of which offer lower return, to the credit market, pressing r^{δ} to go down.

The commodity price in period 0 remains unchanged because Mr. α, whose cost of borrowing is higher, demands more goods while the opposite is true for Mr. β. Because Mr. ϕ invests more now in the deposit market and less in the asset market, his expected income declines in state 1 (where the Arrow security pays) and rise in states 2 and 3. Thus he demands less commodities in state 1 and more in states 2 and 3, causing the prices to decrease in state 1 and increase in the other two states. Consequently, Mr. α and β supply less commodities in state 1 and more in states 2 and 3, though the magnitude of changes in quantities supplied are relatively minor. Given less (more) revenue from commodities sold in state 1 (states 2 and 3), they default more (less) in state 1 (states 2 and 3). However, the repayment rate of the household sector increases on average.

Owing to a lower repayment rate in state 1, the capital to asset ratios of both banks increase in this state. However, both banks violate more of their capital requirement condition in states 2 and 3 due to higher repayment rates of Mr. α and β, (thus increasing the value of its risk-weighted assets) though the extent of violation is much less for bank δ as it also reduces its investment in the interbank market. On average, the situation of the capital to asset ratio of the whole banking industry remains nearly unaffected.

Concerning changes in the welfare, the utility of Mr. α (β) slightly decreases (increases) as his cost of borrowing rises (falls). The directional change in the utility of Mr. ϕ is ambiguous as there is a trade off between the benefit from a higher deposit rate and the cost in terms of a higher asset price. Bank δ's profitability declines slightly as the rates of return from investing in the asset, interbank and credit markets all go down. Finally, the profitability of bank γ declines precipitously as it is negatively affected by a higher marginal cost of expanding the size of its asset book. In sum, the result shows that implementing this type of regulatory policy tends to improves slightly the overall financial stability of the whole economy via higher average repayment rates, though it has to be stressed that the overall capital position of banks remains largely unaffected. However, such improvement comes at the cost in terms of efficiency since the profitability of the banking industry also declines.

Now consider instead an experiment in which bank δ faces a higher capital violation penalty in state 1. Interestingly, the results are *very non-symmetric* compared to the previous case. Given a higher penalty, the marginal cost of expanding its assets in state 1 rise. However, unlike bank γ whose asset is restricted to loans to Mr. α, bank δ has greater flexibility in reallocating its holding of assets. Thus, in order to mitigate the effect of a negative shock which is specific to state 1, it holds more of the Arrow security which pays the most in that particular state and reduces its investment in the interbank market which pays relatively less but more evenly across all states. Thus the interbank market rate rises and the asset price falls slightly. Bank γ then borrows less from the interbank market and more from the asset market, leaving its asset side approximately unchanged. Moreover, the ability of bank δ to switch its portfolio from the interbank market to the asset market implies that its consumer credit market is almost unaffected. Since the cost of borrowing for households remains nearly the same, equilibrium in the commodity market is also left unaffected. Therefore, a regulatory shock to the bank which has more *flexibility* in adjusting its portfolio of asset produces a substantially weaker '*spill over effect*' to the rest of the economy.

While bank δ's capital position worsens in state 1 since it invests more in the asset market, the situation improves in the other two states since it reduces its investment in the interbank market. This in turn causes the overall capital position of bank δ to improve. Since the capital position of bank γ remains unaffected, that of the overall banking industry improves. This gain in terms of lower overall financial fragility comes at the cost of efficiency since the profitability of bank δ declines, although the utility and profitability of the other agents remains almost unaffected.

E. Increases in the Capital Endowment of Banks

The last set of experiments concerns changes in the capital endowment of banks. This can be analogously viewed as an injection of emergency recapitalisation funds to banks by the regulators. Interestingly, we observe that increasing the capital endowment of banks produces nearly the same result as increasing their corresponding capital violation penalties. This is because, an increase in the capital endowment of banks also increases the *marginal* cost of expanding the size of banks' asset. In other words, raising more capital increases the capital adequacy ratios, *ceteris paribus*. This implies that the marginal cost of violating the capital requirements increases too since we penalise violation by subtracting a linear term from the objective function. Thus the directional changes of all variables in the commodity, asset, credit, and interbank markets for this and the previous experiments are exactly the same. The upshot of this is that the effect is nearly the same as if we had increased the penalty parameter.

However, the exceptions lies in the changes in the capital position of banks and their profitability. An increase in the capital endowment of bank γ (δ) in state 1 produces a direct positive income effect on the bank's capital position and therefore its profitability in state 1. This outweighs the negative effect from the fact that the bank incurs greater marginal cost of expanding the size of its assets and therefore the extent of its capital requirements' violation. Thus, the capital position and welfare of the bank in state 1 improves. This improvement is so large that it ultimately improves the overall financial stability as well as the profitability of the entire banking industry.

Appendix V

Table I: Exogenous variables

Coefficient of	Endowment			Penalty		Others
risk aversion	Commodities	Money	Capital	Default	CAR violation	
$c_0^\alpha = 0.011118$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 9$	$\lambda_{1\gamma}^\alpha = 0.2$	$\lambda_{1k}^\gamma = 3$	$M^{CB} = 0.5$
$c_1^\alpha = 0.111429$	$e_1^\alpha = 27$	$m_1^\alpha = 2.9472$	$e_1^\gamma = 1.7278$	$\lambda_{2\gamma}^\alpha = 0.1$	$\lambda_{2k}^\gamma = 2$	$\overline{\omega} = 1$
$c_2^\alpha = 0.073333$	$e_2^\alpha = 27$	$m_2^\alpha = 3.015$	$e_2^\gamma = 1.4108$	$\lambda_{3\gamma}^\alpha = 0.05$	$\lambda_{3k}^\gamma = 1$	$\omega = 0.5$
$c_3^\alpha = 0.066786$	$e_3^\alpha = 27$	$m_3^\alpha = 1.9015$	$e_3^\gamma = 1.1161$	$\lambda_{1\delta}^\beta = 0.2$	$\lambda_{1k}^\delta = 3$	$\overline{k} = 0.1$
$c_0^\beta = 0.010476$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 21.5$	$\lambda_{2\delta}^\beta = 0.1$	$\lambda_{2k}^\delta = 2$	
$c_1^\beta = 0.014740$	$e_1^\beta = 27$	$m_1^\beta = 3.3973$	$e_1^\delta = 2.0474$	$\lambda_{3\delta}^\beta = 0.05$	$\lambda_{3k}^\delta = 1$	
$c_2^\beta = 0.105184$	$e_2^\beta = 27$	$m_2^\beta = 1.8518$	$e_2^\delta = 1.6555$	$\lambda_1^\gamma = 1.576$		
$c_3^\beta = 0.070840$	$e_3^\beta = 27$	$m_3^\beta = 2.5431$	$e_3^\delta = 1.3160$	$\lambda_2^\gamma = 1.123$		
$c_0^\phi = 0.040000$	$e_0^\phi = 45$	$m_0^\phi = 9$		$\lambda_3^\gamma = 0.936$		
$c_1^\phi = 0.007000$	$e_1^\phi = 0$	$m_1^\phi = 0$		$\lambda_{1\phi}^\gamma = 1.685$		
$c_2^\phi = 0.008673$	$e_2^\phi = 0$	$m_2^\phi = 0$		$\lambda_{2\phi}^\gamma = 1.404$		
$c_3^\phi = 0.010767$	$e_3^\phi = 0$	$m_3^\phi = 0$		$\lambda_{3\phi}^\gamma = 1.203$		

Table II: Initial Equilibrium

Prices	Loans/deposits	Capital/Asset ratio	Repayment rate	Commodities
$p_0 = 1$	$\overline{m}^\gamma = 19$	$k_1^\gamma = 0.06$	$v_{1\gamma}^\alpha = 0.919$	$b_0^\alpha = 19$
$p_1 = 1.1$	$\overline{m}^\delta = 21$	$k_2^\gamma = 0.05$	$v_{2\gamma}^\alpha = 0.9$	$q_1^\alpha = 23.5$
$p_2 = 1.2$	$d_\gamma^\phi = 4$	$k_3^\gamma = 0.04$	$v_{3\gamma}^\alpha = 0.89$	$q_2^\alpha = 21$
$p_3 = 1.3$	$d_\delta^\phi = 5$	$k_1^\delta = 0.06$	$v_{1\delta}^\beta = 0.898$	$q_3^\alpha = 20$
$r^\gamma = 0.65$	$d^\delta = 5.5$	$k_2^\delta = 0.05$	$v_{2\delta}^\beta = 0.87$	$b_0^\beta = 21$
$r^\delta = 0.6$	$\mu^{\alpha^\gamma} = 31.35$	$k_3^\delta = 0.04$	$v_{3\delta}^\beta = 0.865$	$q_1^\beta = 24.354$
$\rho = 0.48$	$\mu^{\beta^\delta} = 33.6$		$v_1^\gamma = 0.964$	$q_2^\beta = 22.817$
$r_d^\gamma = 0.48$	$\mu^\gamma = 8.905$		$v_2^\gamma = 0.95$	$q_3^\beta = 20.401$
$r_d^\delta = 0.4$	$\mu_d^\gamma = 5.936$		$v_3^\gamma = 0.94$	$q_0^\phi = 40$
	$\mu_d^\delta = 7$		$v_{1\phi}^\gamma = 0.95$	$b_1^\phi = 52.640$
			$v_{2\phi}^\gamma = 0.94$	$b_2^\phi = 52.580$
			$v_{3\phi}^\gamma = 0.93$	$b_3^\phi = 52.521$

Table IIIa: Sources and Uses of Funds for Households						
period/state	α		β		ϕ	
	Sources	Uses	Sources	Uses	Sources	Uses
0	$\mu^{\alpha^\gamma}/(1+r^\gamma)$ [31.35/(1.65)]	b_0^α [19]	$\mu^{\beta^\delta}/(1+r^\delta)$ [33.6/1.6]	b_0^β [21]	m_0^ϕ [9]	d_γ^ϕ [4]
						d_δ^ϕ [5]
1/1	$p_1 q_1^\alpha$ [(1.1)(23.5)]	$v_{1\gamma}^\alpha \mu^{\alpha^\gamma}$ [(0.919)(31.35)]	$p_1 q_1^\beta$ (1.1)(24.3542)	$v_{1\delta}^\beta \mu^{\beta^\delta}$ [(0.898)(33.6)]	$p_0 q_0^\phi$ (1)(40)	b_1^ϕ [52.64]
	m_1^α [2.947]		m_1^β [3.397]		$v_{1\phi}^\gamma d_\gamma^\phi (1+r_d^\gamma)$ (0.95)(4)(1.4841)	
					$d_\delta^\phi (1+r_d^\delta)$ (5)(1.4)	
1/2	$p_2 q_2^\alpha$ [(1.2)(21)]	$v_{2\gamma}^\alpha \mu^{\alpha^\gamma}$ [(0.9)(31.35)]	$p_2 q_2^\beta$ [(1.2)(22.8169)]	$v_{2\delta}^\beta \mu^{\beta^\delta}$ [(0.87)(33.6)]	$p_0 q_0^\phi$ (1)(40)	b_2^ϕ [52.58]
	m_2^α [3.015]		m_2^β [1.852]		$v_{2\phi}^\gamma d_\gamma^\phi (1+r_d^\gamma)$ (0.94)(4)(1.4841)	
					$d_\delta^\phi (1+r_d^\delta)$ (5)(1.4)	
1/3	$p_3 q_3^\alpha$ [(1.3)(20)]	$v_{3\gamma}^\alpha \mu^{\alpha^\gamma}$ [(0.89)(31.35)]	$p_3 q_3^\beta$ [(1.3)(20.401)]	$v_{3\delta}^\beta \mu^{\beta^\delta}$ [(0.865)(33.6)]	$p_0 q_0^\phi$ (1)(40)	b_3^ϕ [52.521]
	m_3^α [1.9015]		m_3^β [2.543]		$v_{3\phi}^\gamma d_\gamma^\phi (1+r_d^\gamma)$ (0.93)(4)(1.4841)	
					$d_\delta^\phi (1+r_d^\delta)$ (5)(1.4)	

Table IIIb: Sources and Uses of Funds for Banks				
period/state	γ		δ	
	Sources	Uses	Sources	Uses
0	$\mu^\gamma/(1+\rho)$ [8.9046/1.4841]	$\overline{m}^\gamma$ [19]	e_0^δ [21.5]	$\overline{m}^\delta$ [21]
	$\mu_d^\gamma/(1+r_d^\gamma)$ [5.936/1.4841]		$\mu_d^\delta/(1+r_d^\delta)$ [7/1.4]	d^δ [5.5]
	e_0^γ [9]			
1/1	$v_{1\gamma}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.919)(1.65)(19)]	$v_1^\gamma\mu^\gamma$ [(0.964)(8.905)]	$v_{1\delta}^\beta\overline{m}^\delta(1+r^\delta)$ [[(0.898)(21)(1.6)]]	μ_d^δ [7]
	e_1^γ [1.7278]	$v_{1\phi}^\gamma\mu_d^\gamma$ [(0.95)(5.936)]	e_1^δ [2.0474]	π_1^δ [32.433]
		π_1^γ [16.315]	$v_1^\gamma d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(0.964)(5.5)(1.484)(5.5/(5.5+0.5))]	
1/2	$v_{2\gamma}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.9)(1.65)(19)]	$v_2^\gamma\mu^\gamma$ [(0.95)(8.905)]	$v_{2\delta}^\beta\overline{m}^\delta(1+r^\delta)$ [[(0.87)(21)(1.6)]]	μ_d^δ [7]
	e_2^γ [1.41075]	$v_{2\phi}^\gamma\mu_d^\gamma$ [(0.94)(5.936)]	e_2^δ [1.65546]	π_2^δ [30.955]
		π_2^γ [15.586]	$v_2^\gamma d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(0.95)(5.5)(1.484)(5.5/(5.5+0.5))]	
1/3	$v_{3\gamma}^\alpha(1+r^\gamma)\overline{m}^\gamma$ [(0.89)(1.65)(19)]	$v_3^\gamma\mu^\gamma$ [(0.94)(8.905)]	$v_{3\delta}^\beta\overline{m}^\delta(1+r^\delta)$ [[(0.865)(21)(1.6)]]	μ_d^δ [7]
	e_3^γ [1.11606]	$v_{3\phi}^\gamma\mu_d^\gamma$ [(0.93)(5.936)]	e_3^δ [1.316]	π_3^δ [30.413]
		π_3^γ [15.126]	$v_3^\gamma d^\delta(1+\rho)\left(d^\delta/(d^\delta+M^{CB})\right)$ [(0.94)(5.5)(1.484)(5.5/(5.5+0.5))]	

Table IIIc: Market Clearing Conditions	
Commodity Market	Financial Market
$p_0=\frac{b_0^\alpha+b_0^\beta}{q_0^\phi}$	$1+\rho=\frac{\mu^\gamma}{M^{CB}+d^\delta}$
$p_1=\frac{b_1^\phi}{q_1^\alpha+q_1^\beta}$	$1+r^\gamma=\frac{\mu^{\alpha^\gamma}}{\overline{m}^\gamma}$
$p_2=\frac{b_2^\phi}{q_2^\alpha+q_2^\beta}$	$1+r^\delta=\frac{\mu^{\beta^\delta}}{\overline{m}^\delta}$
$p_3=\frac{b_3^\phi}{q_3^\alpha+q_3^\beta}$	$1+r_d^\gamma=\frac{\mu_d^\gamma}{d_\gamma^\phi}$
	$1+r_d^\delta=\frac{\mu_d^\delta}{d_\delta^\phi}$

Table IV: Directional Effects of an increase in exogenous parameters on endogenous variables

	p_0	p_1	p_2	p_3	r^γ	r^δ	ρ	r_d^γ	r_d^δ	$\overline{m}^\gamma$	$\overline{m}^\delta$	d_γ^ϕ	d_δ^ϕ	μ_d^γ	μ_d^δ	d^δ	μ^{α^γ}	μ^{β^δ}	μ^γ
M	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{+}$	$+$	$-$	$+$	$-$	$+$	$+$	$\approx$	$\approx$	$+$
m_1^α	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$-$	$+$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
ω	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$-$	$\approx$	$\approx$	$-$
$\lambda_{1\gamma}^\alpha$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$
λ_{1k}^γ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$
λ_{1k}^δ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
e_0^ϕ	$-$	$\approx$	$\approx$	$\approx$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$\approx$
e_1^α	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
e_1^β	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$
e_1^γ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$
e_1^δ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
c_1^α	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
e_0^δ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\approx$	$+$
e_0^γ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$
λ_1^γ	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$
$\lambda_{1\phi}^\gamma$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$
m_1^β	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
m_0^ϕ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\approx$	$+$

Note: $+(-)$: substantial increase (decrease)

$\underset{\approx}{+}(\underset{\approx}{-})$:weak increase (decrease), $\approx$: approximately equal

$+/-$:ambiguous effect

Table IV (continue)

	$v_{1\gamma}^{\alpha}$	$v_{2\gamma}^{\alpha}$	$v_{3\gamma}^{\alpha}$	v_{γ}^{α}	$v_{1\delta}^{\beta}$	$v_{2\delta}^{\beta}$	$v_{3\delta}^{\beta}$	v_{δ}^{β}	v	v_1^{γ}	v_2^{γ}	v_3^{γ}	v^{γ}	$v_{1\phi}^{\gamma}$	$v_{2\phi}^{\gamma}$	$v_{3\phi}^{\gamma}$	v_{ϕ}^{γ}
M	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
m_1^{α}	$+$	$\approx$	$\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
ω	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\gamma}^{\alpha}$	$+$	$+$	$+$	$+$	$-$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\delta}^{\beta}$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$+$	$+$	$+$	$+$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
λ_{1k}^{γ}	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
λ_{1k}^{δ}	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
e_0^{ϕ}	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
e_1^{α}	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
e_1^{β}	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
e_1^{γ}	$\approx$	$\approx$	$\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
e_1^{δ}	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
c_1^{α}	$+$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
e_0^{δ}	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
e_0^{γ}	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
λ_1^{γ}	$\approx$	$\approx$	$\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\phi}^{\gamma}$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$
m_1^{β}	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$+$ $\approx$	$+$ $\approx$	$+$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
m_0^{ϕ}	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$

Note: $v_{\gamma}^{\alpha} \equiv (v_{1\gamma}^{\alpha} + v_{2\gamma}^{\alpha} + v_{3\gamma}^{\alpha})/3, v_{\delta}^{\beta} \equiv (v_{1\delta}^{\beta} + v_{2\delta}^{\beta} + v_{3\delta}^{\beta})/3, v \equiv (v_{\gamma}^{\alpha} + v_{\delta}^{\beta})/2$

$v^{\gamma} \equiv (v_1^{\gamma} + v_2^{\gamma} + v_3^{\gamma})/3, v_{\phi}^{\gamma} \equiv (v_{1\phi}^{\gamma} + v_{2\phi}^{\gamma} + v_{3\phi}^{\gamma})/3$

Table IV (continue)

	b_0^α	q_1^α	q_2^α	q_3^α	b_0^β	q_1^β	q_2^β	q_3^β	q_0^ϕ	b_1^ϕ	b_2^ϕ	b_3^ϕ
M	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_1^α	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
ω	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
$\lambda_{1\gamma}^\alpha$	$\underset{\approx}{+}$	$+$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$+$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$
λ_{1k}^γ	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
λ_{1k}^δ	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
e_0^ϕ	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$
e_1^α	$+$	$+$	$\approx$	$\approx$	$-$	$\underset{\approx}{-}$	$\approx$	$\approx$	$-$	$-$	$+$	$+$
e_1^β	$-$	$\underset{\approx}{-}$	$\approx$	$\approx$	$+$	$+$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$
e_1^γ	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
e_1^δ	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
c_1^α	$+$	$+$	$\approx$	$\approx$	$-$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$
e_0^δ	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$
e_0^γ	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$
λ_1^γ	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
$\lambda_{1\phi}^\gamma$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
m_1^β	$-$	$\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_0^ϕ	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$

Table IV (continue)																
	U^α	U^β	U^ϕ	U^γ	U^δ	U^H	U^B	k_1^γ	k_2^γ	k_3^γ	k^γ	k_1^δ	k_2^δ	k_3^δ	k^δ	k
M	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$ $\approx$	$-$	$\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$
m_1^α	$+$	$-$ $\approx$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$
ω	$+$ $\approx$	$\approx$	$\approx$	$-$	$-$ $\approx$	$\approx$	$-$ $\approx$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$\lambda_{1\gamma}^\alpha$	$+$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$\approx$	$+$	$\approx$	$-$ $\approx$	$\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$
$\lambda_{1\delta}^\beta$	$-$ $\approx$	$+$	$+$ $\approx$	$\approx$	$\approx$	$+$	$\approx$	$+$ $\approx$	$\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$
λ_{1k}^γ	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$
λ_{1k}^δ	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$	$+$	$+$	$+$	$\approx$
e_0^ϕ	$+$	$+$	$+$ $\approx$	$\approx$	$\approx$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
e_1^α	$+$	$-$	$+$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$-$ $\approx$
e_1^β	$-$	$+$	$+$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$	$\approx$	$\approx$	$+$	$-$	$+$ $\approx$	$+$ $\approx$	$-$	$+$ $\approx$
e_1^γ	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$	$-$ $\approx$	$\approx$	$+$	$+$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$
e_1^δ	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$	$\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$+$	$+$
c_1^α	$-$	$-$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$+$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$-$ $\approx$
e_0^δ	$+$	$+$	$\approx$	$-$	$+$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
e_0^γ	$+$ $\approx$	$+$ $\approx$	$\approx$	$+$	$-$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$
λ_1^γ	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$	$+$ $\approx$
$\lambda_{1\phi}^\gamma$	$+$ $\approx$	$-$ $\approx$	$\approx$	$-$ $\approx$	$+$ $\approx$	$\approx$	$+$ $\approx$	$-$ $\approx$	$+$ $\approx$	$+$ $\approx$	$-$ $\approx$	$-$	$-$	$-$	$-$	$-$
m_1^β	$-$ $\approx$	$+$	$\approx$	$+$	$+$	$+$ $\approx$	$+$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$	$-$ $\approx$	$-$ $\approx$	$-$ $\approx$	$-$
m_0^ϕ	$+$	$+$	$-$ $\approx$	$-$	$+$	$+$ $\approx$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$

Note: $U^H \equiv (U^\alpha + U^\beta + U^\phi),\ U^B \equiv U^\gamma + U^\delta$

$k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3, k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3, k \equiv (k^\gamma + k^\delta)/2$