A Model of Bank Runs: Imperfect Competition and Feedback Effect

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Abstract

This paper extends Diamond-Dybvig's model of bank runs in order to analyze some special phenomena observed during financial crises. By introducing imperfect competition, negative shocks and feedback effect, this model can explain a range of apparently contradictory phenomena observed in recent financial crises more sucessfully than existing bank run models. In this framework, the relationship between financial liberalization and financial crises is discussed and some policy measures to prevent crisis are suggested.

1 Introduction

There were several serious currency crises in 1990s: Britain and Italy in 1991-1992, Mexico and Argentina in 1994-1995, East Asia in 1997-1998, and Russia in 1998. Except for Britain and Italy, all the other countries involved were developing countries. Moreover, Britain and Italy were hit less hard than the others, because after currency devaluation, foreign investment flowed in, exports rose, macroeconomic situation improved, and subsequently their economies grew strongly. However, in the developing countries in East Asia and Latin America, serious banking crises occurred almost immediately after currency crises (Kaminsky and Reinhart (1999)), and they had to fight against capital outflow and economic recession. Clearly, there were significant differences between these crises.

An obvious phenomenon in banking crises is "bank run", i.e., depositors withdraw their funds en masse from banks before bank assets mature. Diamond and Dybvig (1983) (here after DD for short) develop a classic analytical framework for understanding bank runs. In their model, a bank invests deposits in a long term illiquid asset. If no depositor withdraws deposits before the asset matures, there is no bank run. However, if every depositor believes all other depositors will withdraw early, his best strategy is to withdraw early too, thus triggering a bank run. When many

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depositors withdraw their deposits before the long term asset matures, the bank becomes insolvent and without external help will become bankrupt. In this model with multiple equilibria, which equilibrium realizes depends on each depositor's belief about other depositors' actions. But the model does not explain why depositors change their expectations suddenly. In contrast, Allen and Gale (1998) provide a unique equilibrium model which predicts that during a recession the interest rate is too low to attract depositors, thus leading to a bank run. Chang and Velasco (1997) extend the DD model to an open economy. Radelet and Sachs (1998) apply the DD model to explain the Asian financial crisis and argue that it was a self-fulfilling liquidity crisis.

A phenomenon inconsistent with the prediction of these models is the high interest rates actually observed during banking crises. Those countries in crises often had to raise interest rates to very high levels. Since those countries had already given up their initial fixed exchange rates, the phenomenon of high interest rates, which were harmful during and after bank runs, cannot be explained in the DD framework.

Another problem with the DD and related models is that the banks' optimal responses to a potential crisis are not discussed at all because the above papers invariably assume that the banking sector is perfectly competitive. Under perfect competition, the banks' profits are zero and their strategies are simply to maximize the depositors' utilities. However, this assumption does not hold in reality. In countries where banking crises occurred, typically a few banks had most of the market shares, and the banking sector usually did not exhibit free entry. In addition, the probability of bankruptcy, i.e., the ex post return to the long term asset is so low that a bank cannot repay the promised return to its depositors, is also ignored.

Feedback effects are usually present in bank runs. In a banking crisis, as some depositors start to withdraw their deposits, other depositors' confidence is adversely affected, causing them to follow suit. Zhu (2000) provides an explanation of this phenomenon. However, his explanation relies on the questionable assumption that depositors make withdrawal decisions sequentially.

In this paper, I extend the DD model by introducing imperfect competition, negative shocks and feedback effect. In my model, banks are active players in an oligopoly game. Both the banks and agents recognize the probability of bankruptcy. With these extensions, we can explain the interest rate movement observed during financial crises. I will show that under some situations raising interest rate is the best strategy for the banks, and explain why high interest rate can prevent bank runs in some economies (e.g. Britain, Italy and Hong Kong), but not in others. By assuming that agents have different opportunity costs for their deposits, this paper can explain the feedback effect. For example, if foreign depositors have higher opportunity costs, they will withdraw their deposits first after a negative shock. Their actions will force the banks to liquidate part of the long term investment, which worsens the returns to the remaining investment. As a result, local depositors with lower opportunity costs may also begin to withdraw their deposits.

The model developed in this paper helps to answer three important questions. First, it offers an explanation of the phenomenon of "twin crises", i.e., currency and banking crises occur in close proximity. When a negative shock occurs, the central bank may bail out the affected banks by extending credit. However, this action will trigger the balance of payment (BOP) problem and cause a currency crisis. Moreover, the currency devaluation may act as a strong negative shock to the banking sector and exacerbate the banking crisis. Second, this paper explains why banking crises happened more frequently in developing countries than in developed countries. When different economies were hit by shocks with a similar magnitude, the consequences differed. The developed countries were less susceptible to banking crises because their banking systems were stronger. Third, my model shows in a country without a strong banking sector, the probability of banking crisis will rise if its financial market is too open. This helps to explain why the "Asia miracle" became a nightmare all of sudden and why China could avoid the crisis.

The rest of this paper is organized as follows. Section 2 introduces the basic framework. Section 3 analyzes whether a banking crisis occurs after an unexpected shock. Section 4 extends the model to the case where agents are not identical and discusses feedback effect. In section 5, I analyze the relationship between financial liberalization and banking crises, and explain the occurrence of twin crises. Section 6 reports the results of numerical simulations. Section 7 provides some policy recommendations and section 8 concludes.

2 Model of A Closed Banking Sector

In this section we construct a three-period ($t = 0, 1, 2$) banking model for an economy with closed banking sector. There are only domestic banks and depositors.

2.1 Depositors, banks and investment returns

There are I_L identical local agents. All of them are risk neutral. At $t = 0$, each agent is endowed with one unit of a good. This good can be deposited his good in a bank in period 0, or be invested in a risk free asset in period 0 or period 1. The risk free asset produces R_{F1} units return in period 1 and R_{F2} units return in period 2, so one unit of the long term risk free asset in period 0 produces $R_F = R_{F1}R_{F2}$ units of return in period 2, where $R_F > 1$.

We assume the entry barrier of the banking sector is high, so the number of banks is fixed, and denote them by bank 1, bank 2, ..., bank n. Each bank has its own initial capital. All banks have similar properties. For simplicity we analyze a representative bank i.

Suppose bank i has initial capital S_i. It can invest its own capital and the agents' deposits in a long term risky asset in period 0. Each unit of the asset produces a fixed return $r_i < 1$ in period 1 and a random average return R_i in period 2. That is to say, if the asset is liquidated in period 1, the bank suffers a net loss $1-r_i$ for each unit of investment. However, if the asset is held till period 2, its random return is R_i. We assume that the expected value of R_i exceeds R_F if the scale of investment is not too large. More formally, let $R_i(I)$ denote the average return of the asset, where I is the total size of investment, $R_i(I) \in [\underline{R}, \overline{R}]$ for all I. Let the cdf of $R_i(I)$ be denoted by $F_i(R; I)$. $F_i(R; I) \leq F_i(R; I')$ if $I < I'$ and the equality holds only when $F_i(R; I) = F_i(R; I') = 0$ or 1. So $E(R_i(I)) > E(R_i(I'))$ if $I < I'$, i.e., the expected average return is decreasing in I. Moreover, we assume that $F_i(R; I)$ is continuous in R and I, and $\lim_{I\to\infty} E(R_i(I)) = 1$. For simplicity, we assume the distribution of $R_i(I)$ will not change if the bank liquidates a part of investment in period 1. Finally, we assume that there is $I_{Ci} > 0$ such that $E(R_i(I)) \geq R_F$ if and only if $I \leq I_{Ci}$. To give the bank an incentive to accept deposits, we assume $S_i < I_{Ci}$.

2.2 Deposit Contract, bankruptcy and bank runs

Bank i provides a contract to agents in period 0. It promises to repay a fixed return P_i for each unit of deposit in period 2. If agents accept this contract, they deposit in bank i. Let I_i be the total deposits taken by the bank.

In period 1, depositors have the right to withdraw a fixed claim of R_{F1} per unit deposited in period 0.[1] They can only invest the withdrawn good in the risk free asset and get risk free return R_{F2}, so the total return is $R_{F1}R_{F2} = R_F$. The bank, if it likes, may raise the promised return to retain deposits from period 1 to period 2.

If a bank run occurs because too many depositors decide to withdraw in period 1, then bank i respects the principle of sequential service (first come-first served) and liquidates some long term asset to repay them. If the bank's entire asset is liquidated but still cannot satisfy the depositors' request for withdrawal, those depositors who have not withdrawn their deposits get nothing.

If the bank run does not occur in period 1, R_i is realized in period 2. If bank i can repay the promised return P_i to depositors, the contract is over; otherwise, the bank goes into bankruptcy. When that happens, the bank's entire revenue will be repaid to depositors less a bankruptcy cost. Then each agent will get $\frac{\theta W_i}{I_i}$, where I_i is the bank's outstanding deposits, W_i is bank i's total wealth after liquidation and $\theta < 1$ captures the need to pay a bankruptcy cost.

Each bank provides its contract to agents, who are assumed to be risk neutral and only care about the expected return of the contracts. They always deposit in the bank that provides the highest expected return. The expected return of bank i's contract depends on P_i and I_i, so we denote it by $R_{Mi}(P_i, I_i)$ .In equilibrium, the expected returns of all banks' contracts must be the same if they have positive deposits. So the market equilibrium expected return (if it exists) $R_M = R_{M1}(P_1, I_1) = R_{M2}(P_2, I_2) = ... = R_{Mn}(P_n, I_n)$. R_M is not directly observed because the banks only announce their promised returns P_i, but it can be calculated if P_i is given. R_M is a function of the equilibrium P_i, i.e., $R_M(P_1, P_2, .., P_n)$.

[1] In Diamond and Dybvig (1983), they explained why the bank gives this right to agents when there are some impatient agents, who only consume in period 1. For simplicity, I ignore the impatient agents, just assume this condition must be included in the contract.

2.3 Bank i's constrained optimization problem

We analyze bank i's optimization problem BP1.

$$\underset{I_i,P_i}{Max}\int_{R_{Li}}^{\overline{R}}[(S_i+I_i)\cdot R_i-I_iP_i]dF_i(R_i;S_i+I_i)$$

s.t.

$$R_M\le(1-F_i(R_{Li};S_i+I_i))P_i+\int_{\underline{R}}^{R_{Li}}\frac{(S_i+I_i)\theta R_i}{I_i}dF_i(R_i;S_i+I_i)$$

where $R_{Li}=\frac{I_iP_i}{S_i+I_i}$ denotes "bankruptcy threshold rate of return", meaning that if the realized return is less than this level, bank i will go into bankruptcy. Bank i's constraint is that the expected return to its depositors including the event of bankruptcy is not less than the prevailing expected return offered by other banks. Because we assume $S_i<I_{Ci}$, bank i can earn more profit by accepting some deposits than by investing its own capital alone.

We consider the depositors' expected return function. Define

$$U(P_i,I_i,S_i)=(1-F_i(R_{Li};S_i+I_i))P_i+\int_{\underline{R}}^{R_{Li}}\frac{(S_i+I_i)\theta R_i}{I_i}dF_i(R_i;S_i+I_i)$$

Taking second order derivative with respect to P_i, we get

$$\frac{\partial^2U}{\partial P_i^2}=\frac{I_i(\theta-2)}{S_i+I_i}f(\frac{I_iP_i}{S_i+I_i};S_i+I_i)+\frac{I_i^2(\theta-1)}{(S_i+I_i)^2}P_if_1(\frac{I_iP_i}{S_i+I_i};S_i+I_i) \tag{1}$$

where $f=\frac{\partial F_i}{\partial R_i}$ is the pdf of R_i and $f_1=\frac{\partial^2F_i}{\partial R_i^2}$.

Because $f\ge 0$ and $\theta<1$, the first term of the RHS of (1) isnot positive, but the second term is uncertain because its sign depends on f_1, the derivative of f. If the distribution of R_i is relatively "uniform", then f_1 is very close to 0, and thus $\frac{\partial^2U}{\partial P_i^2}\le 0$. Although concavity of U in R is not a necessary condition of the following results, for convenience we assume it holds for all $P_i\le\overline{R}$.

Lemma 1 describes some properties of U.

Lemma 1 *If* $0\le P_i\le\frac{(S_i+I_i)\underline{R}}{I_i}$, $U(P_i,I_i,S_i)=P_i$; *if* $\frac{(S_i+I_i)\underline{R}}{I_i}<P_i\le\frac{(S_i+I_i)\overline{R}}{I_i}$, $U(P_i,I_i,S_i)<P_i$; *and if* $P_i>\frac{(S_i+I_i)\overline{R}}{I_i}$, $U(P_i,I_i,S_i)=\frac{\theta(S_i+I_i)}{I_i}E(R_i(S_i+I_i))$. *In all cases* $U(P_i,I_i,S_i)\le P_i$. *For all* $I'>I''$, $U(P_i,I',S_i)\le U(P_i,I'',S_i)$ *for all* P_i *and* S_i, *the equality holds only if* $\frac{S_i+I'}{I'}\underline{R}\ge P_i$. *For any given* I_i *and* S_i, *there exists* $P_{iH}\in[\underline{R},\overline{R}]$ *such that* $U(P_{iH},I_i,S_i)=\underset{P_i}{Max}U(P_i,I_i,S_i)$.

That is, if $P_i \in [0.\frac{(S_i+I_i)\underline{R}}{I_i}]$, there is no bankruptcy risk, so that actual return is equal to the promised return. If $P_i \in (\frac{(S_i+I_i)\underline{R}}{I_i}, \frac{(S_i+I_i)\overline{R}}{I_i}]$, there is a chance of bankruptcy, so depositors' expected return is less than P_i. And if $P_i > \frac{(S_i+I_i)\overline{R}}{I_i}$, bankruptcy is inevitable, so depositors will only get the bank's salvage value. P_{iH} is not the bank's optimal promised rate of return but depositors can get the highest utilities at this point, i.e. $U(P_{iH}, I_i, S_i)$ is the highest return that the bank can give.

These properties are depicted in Figure 1.

In Figure 1, $P_1 = \frac{(S_i+I_i)\underline{R}}{I_i}$ and $P_2 = \frac{(S_i+I_i)\overline{R}}{I_i}$. For $P_i < P_1$, the curve of U is a straight line segment $U(P_i, I_i, S_i) = P_i$. For $P_i > P_2$, the curve is the horizontal line $U(P_i, I_i, S_i) = \frac{\theta(S_i+I_i)}{I_i}E(R_i(S_i + I_i))$. For $P_i \in (P_1, P_2)$, the shape of curve is unknown, but if f_1 is very close to 0, it is concave over the region. When $P_i = P_{iH}$, the function $U(P_i, I_i, S_i)$ takes the global maximal value for given I_i and S_i. In this figure, an interesting observation is that if the promised return is higher than P_{iH}, the probability of bankruptcy will be so high that an increases in P_i lowers instead of raising the depositors' expected return.

Next, let us analyze the relationship between R_M, S_i, I_i and P_i.

Consider the bank's objective function $\int_{R_{Li}}^{\overline{R}}[(S_i+I_i)\cdot R - I_iP_i]dF_i(R; S_i+I_i)$ given S_i and I_i. It is decreasing in P_i. So if S_i and I_i are given, the optimal P_i must be the minimal value that satisfies the constraint condition of BP1. Given R_M, we can define a function $P(R_M, S_i, I_i) = \min(P_i)$ such that $U(P_i, I_i, S_i) \geq R_M$.[2] Let us define $P(R_M, S, I_1)$ by $U(P(R_M, S_i, I_i), I_i, S_i) = R_M$ and assume $P(R_M, S, I_1)$ is continuously differentiable.

Lemma 2 *$P(R'_M, S_i, I_i) > P_i(R''_M, S_i, I_i)$ for all S_i, I_i and $R'_M > R''_M$. $P(R_M, S', I_i) \leq P(R_M, S'', I_i)$ for all R_M, I_i and $S' > S'' > 0$. $P(R_M, S_i, I_1) \geq P(R_M, S_i, I_2)$ for all R_M, S_i and $I' > I''$. The two weak inequalities hold as equalities only when $F_i(\frac{I_iR_M}{S''+I_i}; S''+I_i) = 0$ and $F_i(\frac{I'R_M}{S_i+I'}; S_i+I') = 0$ respectively.*

The results contained in Lemma 2 can be explained intuitively. If the market return rises, the bank has to raise the promised return. If the bank's capital increases, the bankruptcy risk

[2]In equilibrium, R_M is not exogenous but depends on P_i. Here I take R_M as exogenous for analyzing the properties of P_i for bank i.

decreases, so the bank can decrease the promised return. If deposits increase, the bankruptcy risk increases, so the bank needs to raise the promised return. The last two conditions indicate that if there is no risk of bankruptcy, then the promised return is equal to the expected return.

2.4 Market equilibrium

In this model, agents are supplier of deposits and banks are bidders for their deposits. Each suppliers is too small to affect the expected return, so the suppliers are perfectly competitive. Each bank bids by offering a promised return to agents, but the expected return takes into account the probability of bankruptcy. As oligopolists, the banks compete against each other by setting the promised returns. In equilibrium, all banks provide the same expected return $R_M(P_1, P_2, ...P_n)$.

We consider three possible market structures.

2.4.1 Monopoly: $n = 1$.

If there is only a monopolist, it can set $R_M = R_F$ and its optimal $I_1^* \leq I$ is given by the solution to BP1.

2.4.2 Oligopoly: $n > 1$ but not very large.

In this case, no single bank can set the market expected return R_M but each can affect it, i.e., $\frac{\partial R_M(P_1,P_2,...P_n)}{\partial P_i} \neq 0$. This case is very complicated. For simplicity, we analyze the case of $n = 2$. The main results should hold for the case of $n > 2$.

When $n = 2$, bank 1 and bank 2 provide their promised return P_1, P_2 and attract deposits $I_1 > 0$, $I_2 > 0$ respectively. Since both banks take some deposits, depositors must get the same expected return, i.e., $U(P_1, I_1, S_1) = U(P_2, I_2, S_2) = R_M(P_1, P_2) \geq R_F$. From Lemma 1, if $\frac{S_i+I_i}{I_i}\underline{R} < P_i$, then $U(P_i, I_i, S_i)$ is strictly decreasing in I_i. If S_i is small and $\underline{R}$ is not large, $\frac{S_i+I_i}{I_i}\underline{R} < P_i$, i.e., bankruptcy is always a possibility. Let us focus on this case.

Since U is strictly decreasing in I_i, so for given P_1 and P_2, there is only a pair (I_1, I_2) that satisfies $U(P_1, I_1, S_1) = U(P_2, I_2, S_2) \geq R_F$. The last inequity holds if $I_1 + I_2 = I$; $I_1 + I_2 \leq I$ if the inequality holds. We can define two functions $I_1(P_1, P_2)$ and $I_2(P_1, P_2)$ that satisfy the above equation $U(P_1, I_1, S_1) = U(P_2, I_2, S_2)$.

Denote bank i's profit function by

$$\Pi_i(P_1, P_2) = \int_{R_{Li}}^{\overline{R}} [(S_i + I_i) \cdot R_i - I_i P_i] dF_i(R_i; S_i + I_i)$$

where $R_{Li}(P_1, P_2) = \frac{I_i(P_1,P_2)P_i}{(S_i+I_i(P_1,P_2))}$ and $I_i = I_i(P_1, P_2)$

Bank i's problem is to maximize $\Pi_i(P_1, P_2)$, which is continuous in P_1 and P_2. We assume that bank i's best response function $P_i^*(P_j)$ (where $i \neq j$) exists and is continuous.

No matter what value P_j takes, $R_F \leq P_i^*(P_j) \leq \overline{R}$ always holds. Then $P_i^*(P_j^*(0)) \geq 0$ and $P_i^*(P_j^*(\overline{R})) \leq \overline{R}$. So when $P_1^*(P_2)$ and $P_2^*(P_1)$ are continuous, for any given I, there exists at least one equilibrium, i.e., a pair (P_1^E, P_2^E) satisfying $P_1^*(P_2^E) = P_1^E$ and $P_2^*(P_1^E) = P_2^E$.

Unfortunately, we cannot guarantee the uniqueness of equilibrium without stronger assumptions. However, in the numerical simulations reported in section 6, the equilibrium is unique under rather general conditions. In this paper, we only consider the case of hte unique equilibrium.

2.4.3 Nearly perfect competition: n is very large.

Although R_M continues to be decided by all banks together, the impact of each single bank on R_M is negligible, i.e., $\frac{\partial R_M(P_1,\ldots P_n)}{\partial P_i} \approx 0$. By approximation, each bank faces an exogenous R_M as in the case of perfect competition. However, because n is fixed, the banks may still earn some small positive profits.

When R_M is given exogenously to bank i, $(I_i^*(R_M), P_i^*(R_M))$ are the solution to BP1, where $I_i^* \geq 0$. We assume for each given R_M, the optimal solution to BP1, $(I_i^*(R_M), P_i^*(R_M))$ is unique. Since each bank's demand for deposits decreases as the cost of deposits rises, it follows that I_i^* is continuous and decreasing in R_M.

Given the fixed market rate of expected return R_M, bank i's optimal deposits is I_i^* and its optimal promised return is P_i^*. The market equilibrium is characterized by the following:

1. (I_i^*, P_i^*) is a solution of bank i's constrained optimization problem for given R_M;

2. If $R_M = R_F$, then $\sum_{i=1}^n I_i^* \leq I_L$, where I_L is the number of local agents; if $R_M > R_F$, then $\sum_{i=1}^n I_i^* = I_L$.

Since the case $I_i^* = 0$ is meaningless to the model, we assume $I_i^* > 0$ in the remainder of this paper.

We can get the following proposition about the market clearing:

Proposition 1 *In the case of nearly perfect competition, for given I and R_F, there exists a unique $(R_M, (I_1^*, I_2^*, ..., I_n^*), (P_1^*, P_2^*, ..., P_n^*))$ that satisfies the market clearing condition 1 and 2.*

We demonstrate the market equilibrium under nearly perfect competition in Figure 2. The solid curve is the supply curve. If $R_M < R_F$, no agent makes deposits in the banks, so it is a vertical line. If $R_F = R_M$, it is indifferent for agents to deposit in the banks or to invest in the risk free asset, so I^S can be any value between $[0, I_L]$, a horizontal line segment. If $R_M > R_F$, all agents deposit in the banks, so $I^S = I_L$. The two dashed curves represent two different aggregate demand curves under different initial conditions. If the demand curve is D1, point (I_1, R_F) is the market equilibrium. Since $\sum_{i=1}^n I_i^* < I_L$, so $R_M = R_F$. If the demand curve moves to D2, the market equilibrium moves to (I_L, R'_M). The demand curves depend on $(S_1, S_2, ...S_n)$ and the distribution functions $(F_1, F_2, ...F_n)$.

3 Unexpected shock, interest rate increase and bank run

We analyze what will happen if an unexpected negative shock occurs in period 1. The shock can be war, earthquakes, epidemics or any other events that affect the payoff of the long term investment.

As discussed above, if the market equilibrium rate of return is R_M in period 0, bank i attracts I_i^* units of deposits with a promised return P_i^*. Thus the distribution of the return of the risky asset is $F_i(R_i; I_i^* + S_i)$.

In period 1, an unexpected negative shock occurs. The distribution of the return becomes $F_{i-\sigma}(R_i; I_i^* + S_i) \geq F_i(R_i; I_i^* + S_i)$, and the equality holds if and only if $F_{i-\sigma}(R_i; I_i^* + S_i) = 0$ or $F_i(R_i; I_i^* + S_i) = 1$, where $\sigma > 0$ captures the strength of the shock. That is to say, the original distribution $F_i(R_i; I_i^* + S_i)$ first order stochastically dominates the new one $F_{i-\sigma}(R_i; I_i^* + S_i)$.

Obviously, $U(P_i^*, I_i^*, S_i | -\sigma) \leq U(P_i^*, I_i^*, S_i)$. The equality holds if and only if $F_{i-\sigma}(\frac{I_i^* P_i^*}{I_i^* + S_i}; I_i^* +$

$S_i) = 0$. That is to say, only if the probability of bankruptcy is 0 after the negative shock has occurred, will the depositors' expected return in period 2 not decrease. In that case, bank i does not need to do anything although its expected profit goes down.

If $U(P_i^*, I_i^*, S_i| - \sigma) < R_M$, there are three possible cases:

(a) $R_F \leq U(P_i^*, I_i^*, S_i| - \sigma) < R_M$.

(b) $U(P_i^*, I_i^*, S_i| - \sigma) < R_F$ but there exists $P_H > P_i^*$ such that $U(P_H, I_i^*, S_i| - \sigma) \geq R_F$.

(c) $U(P_i^*, I_i^*, S_i| - \sigma) < R_F$ for all $P \geq P_i^*$.

Figure 3 illustrates these three cases under the assumption that $R_M > R_F$. In the figure the solid curve and the dashed curve are depositors' expected return before and after the shock occurs, respectively.

Figure 3.a illustrates case (a). The horizontal line $y = R_M$ intersects the solid curve at point (P_i^*, R_M), so the bank provide the promised return P_i^* that satisfies $U(P_i^*, I_i^*, S_i) = R_M$ before the crisis. The line $x = P_i^*$ intersects the two curves at $(P_i^*, U(P_i^*, I_i^*, S_i))$ and $(P_i^*, U(P_i^*, I_i^*, S_i| - \sigma))$. In this case, depositors' expected return decreases but they still have no incentive to withdraw. It is because that they only can get return R_F if they do so.

Under what conditions $U(P_i^*, I_i^*, S_i| - \sigma) \geq R_F$ holds? If σ and I_i^* are relatively small or S_i is relatively large, then $U(P_i^*, I_i^*, S_i| - \sigma) \geq R_F$. That is, if the negative shock is small, or deposit/equity and deposit/asset ratios are relatively low, then the bank can keep the promised return unchanged.

I believe this case (a) illustrates the experience of Britain and Italy, or more recently the case of Japan. These developed countries had abundant capital and relatively low debt. Moreover, the banking systems were stable and the governments could provide support to banks if necessary. So they did not need to raise interest rates during financial crises.

Figure 3.b illustrates case (b). The horizontal line R_F intersects the dashed curve at point $(P_i^{*\prime}, R_F)$ and $P_i^* < P_i^{*\prime}$. Since $U(P_i^*, I_i^*, S_i| - \sigma) < U(P_i^{*\prime}, I_i^*, S_i| - \sigma) = R_F$, we know if the bank does not raise the promised return after the shock, then all depositors will withdraw their deposits, leading to a bank run. So the bank has to raise the promised return to $P_i^{*\prime}$.

A question is whether raising the promised return can always prevent a bank run. From Lemma

1, there exists $P_H^{-\sigma}$ that maximizes $U(P_i, I_i^*, S_i|-\sigma)$. However, if $U(P_H^{-\sigma}, I_i^*, S_i|-\sigma) < R_F$, a bank run is inevitable. Case (c) is illustrated in Figure 3.c. The horizontal line R_F has no intersection with the dashed curve, meaning that the bank cannot provide any promised return to prevent the depositors from withdrawing after the shock.

Another question is whether a higher promised return is always beneficial to agents? The answer is no. As pointed out in section 2, when P_i is higher than a certain level, the risk of bankruptcy is so large that $U(P_i, I_i^*, S_i| - \sigma)$ begins to decrease. So too high a promised return will worsen the banking crisis rather than stabilizing it.

A remaining question is whether it is possible for the bank not to raise its promised return to prevent the bank run when it is able to do so? The answer is no. Because if a bank run occurs, the bank loses everything, whereas if the bank run is prevented, the bank's expected revenue is positive. Thus, if possible, the bank always tries to prevent a bank run by raising the promised rate of return.

Is bank run the best outcome for depositors? When r_i is small, the answer is no. Let us compare the two different outcomes, namely (a) depositors withdraw and force a bank run, and (b) depositors do not withdraw their deposits thus avoiding a bank run when the bank raises the promised return to the $P_{iH}^{-\sigma}$ which satisfies $U(P_{iH}^{-\sigma}, I_i^*, S_i|-\sigma) = \max_{Pi} U(P_i, I_i^*, S_i|-\sigma)$ depositors get the highest utility.

In situation (a), those depositors who arrive the bank early can get R_{F1}, but the others get nothing. So their expected revenue in period 1 is $\frac{(I_i^*+S_i)r_i}{I_i^*}$. Then, they can invest what they get in the risk free asset, so their expected revenue is $\frac{(I_i^*+S_i)r_i R_{F2}}{I_i^*}$ in period 2.

In situation (b), no depositor withdraws his/her deposit even the expected return is less than R_F. The expected return is $U(P_{iH}^{-\sigma}, I_i^*, S_i| - \sigma)$ in period 2.

Upon comparison, it is clear that if r_i is small, $U(P_{iH}^{-\sigma}, I_i^*, S_i| - \sigma) > \frac{(I_i^*+S_i)r_i R_{F2}}{I_i^*}$ may hold. However, this is a "Prisoner's dilemma" game because every depositor's dominant strategy is to withdraw.

The above discussion attempts to provide an explanation of the interest rate hike observed during the Asia financial crisis and Mexican crisis. Those countries let their domestic currencies devaluate under the pressure of the BOP problem.[3] The conventional wisdom suggests that a currency devaluation will decrease the cost of exports and improve trade balance, just like what happened in Britain and Italy. However, there is an important difference between the developing countries and the developed countries. Because the former's foreign debt/exchange reserve ratio was very high, a devaluation amounted to a big negative shock. Moreover, the banks in the developing countries invested most of their foreign loans in real estates before the crises hit. The expected return measured in US$ dropped significantly, so they had to raise interest rate to prevent foreign capital outflow— capital outflow is just like that agents withdraw their deposits and invest in the risk free asset in my model. Several countries with weak fundamentals were unable to prevent capital outflow despite very high interest rates, making bank runs inevitable. As Kaminsky and Reinhart (1999) point out, the peaks of banking crises usually arrive after BOP crises.

4 An Open Banking Sector and the Feedback Effect in Banking Crises

We assume all agents are identical in section 2 and 3. In this section, we extend the analysis by considering two types of agents: I^1 local agents and I^2 foreign agents. All of them are risk neutral. We assume that I^2 is very large so that there is practically unlimited foreign capital.

We assume the risk free asset is an international asset, with fixed rate of return R_{F1} in period 1 and R_{F2} in period 2. As before we denote $R_F = R_{F1}R_{F2}$, and the local risky asset has the same properties as described in section 2. In addition, we assume that there is a proportional capital flow cost for local agents to invest in the international risk free asset and for foreign agents to invest in local risky asset in period 1 and period 2. For convenience, we assume the rate of capital flow cost is 0 in period 1 and $\tau \in (0,1)$ in period 2.

The local agents can get risk free return $R_{FL} = (1-\tau)R_F$ if they invest in the risk free asset, and foreign agents will deposit in local banks only if the expected return is not less than $R_{FF} = (1+\tau)R_F$. Obviously, $R_{FF} > R_{FL}$ indicating that local and international agents have different opportunity costs.

Assume all banks know I^1, I^2, R_{FL} and R_{FF} but do not know each agent's type. So if the

[3] In some countries, the BOP problems were caused by their banking sectors. I will discuss that in section 5

local banks do take some foreign deposits, $R_M = R_{FF}$ must hold in equilibrium. If depositors withdraw in period 1, they can get fixed return R_{F1}.

If there is a market equilibrium $(R_M^*, (I_{11}^*, ..., I_{n1}^*), (P_1^*, ..., P_n^*))$, where $R_M^* = R_M(P_1^*, ..., P_n^*)) \leq R_{FF}$ and $\sum_{i=1}^n I_{i1}^* \leq I^1$, then the banks do not take foreign deposits. The results are the same as that in section 2 and section 3. In particular, there may be a bank run if a strongly negative shock occurs, making it impossible to attract depositors even with a very high promised return. We only consider the case in which the banks actually take some foreign deposits.

Bank i's problem (BP2) is as follows:

$\underset{I_i, P_i}{Max} \int_{R_{Li}}^{\overline{R}} [(S_i + I_i)R - I_i P_i] dF_i(R; S_i + I_i)$

s.t. $R_{FF} \leq (1 - F_i(R_{Li}; S_i + I_i))P_i + \int_{\underline{R}}^{R_{Li}} \frac{(S_i+I_i)\theta R}{I_i} dF_i(R; S_i + I_i)$

$R_{Li} = \frac{I_i P_i}{S_i + I_i}$ and $I_i = I_{i1} + I_{i2}$

The market clearing condition is:$\sum_{i=1}^n I_{i1}^* = I^1$ and $\sum_{i=1}^n I_{i2}^* \geq 0$.

Since the supply of foreign capital is unlimited, the banks can satisfy their deposits demand at $R_M = R_{FF}$.

The market equilibrium with many banks is illustrated in Figure 4, where the solid curve is the supply curve. If $R_M \leq R_{FF}$, the supply curve is the same as the supply curve in the closed economy. R_M can never exceed R_{FF} because there are infinitely many foreign depositors ready to supply any quantity of funds, i.e., the supply curve is horizontal at $R_M = R_{FF}$ for all $I \geq I^1$. The two dashed curves are two different demand curves with different initial conditions. If the demand curve is D1, local deposits can satisfy the banks' demand, so (I_1, R_{FL}) is the market equilibrium point and the banks do not need foreign deposits. If the demand curve is D2, the market equilibrium point is (I^*, R_{FF}), and the banks will take I^1 of local deposits and $I^* - I^1$ of foreign deposits. This case will occur if S_i is small but the long term investment is sufficiently profitable.

Similar to Lemma 2, there is an optimal (I_i^*, P_i^*), where $I_i^* = I_{i1}^* + I_{i2}^*$. and the inequality constraint of BP2 holds as a strict equality.

Like in section 2, denote $U(P_i, I_i, S_i) = (1 - F_i(R_{Li}; S_i + I_i))P_i + \int_{\underline{R}}^{R_{Li}} \frac{\theta R(S_i+I_i)}{I_i} dF_i(R_i; S_i + I_i)$, where $R_{Li} = \frac{I_i}{S_i+I_i} P_i$, and $U(P_i^*, I_i^*, S_i) = R_{FF} > R_{FL}$.

In this model, what will happen after an unexpected shock in period 1? Depending on the strength of the shock, there are four possible outcomes. I show them in Figure 5. The solid curve is the curve of $U\left(P_i, I_i^*, S_i\right)$ and the dotted curve is the curve of $U\left(P_i, I_i^*, S_i| -\sigma\right)$, the expected return before and after the negative shock, respectively. Before the shock occurs, bank i sets the promised return $P_i = P_{i1}$ such that $U\left(P_{i1}, I_i^*, S_i\right) = R_{FF}$.

If the shock is very strong such that $\underset{P>P_{i1}}{Max} U\left(P, I_i^*, S_i| -\sigma\right) < R_{FL} < R_{FF}$, then a bank run is inevitable. This is the case illustrated in Figure 5.a. No matter how high the bank's promised return is after the shock, both local and foreign depositors choose to withdraw because they know the expected return is lower than their opportunity costs.

If the shock is mild, then $R_{FL} < R_{FF} \leq \underset{P>P_{i1}}{Max} U\left(P, I_i^*, S_i| -\sigma\right)$. The bank can raise interest rate to prevent a bank run. Figure 5.b illustrates this case. After the shock, the bank has to raise the promised return to $P_{i2} > P_{i1}$ such that $U\left(P_{i2}, I_i^*, S_i| -\sigma\right) = R_{FF}$. Although the bank suffers some loss, no bank run will occur.

If the shock is medium, then $R_{FL} \leq \underset{P>P_{i1}}{Max} U\left(P, I_i^*, S_i| -\sigma\right) < R_{FF}$. This case is more complicated than the previous two. In this case, bank i is unable to retain foreign deposits but maybe able to retain local deposits by raising the promised return to an appropriate level.

Suppose bank i raises the promised return to $P^1(-\sigma)$ such that $U(P^1\left(-\sigma\right), I_i^*, S_i| -\sigma) \geq R_{FL}$. If local depositors only consider that, they do not withdraw, but foreign depositors do so. Thus, that is not the end of the story. Since bank i has to liquidate $\frac{I_{i2}^* R_{F1}}{r_i}$ units of the long term asset to satisfy the foreign depositors' withdrawal, local depositors will see that their expected return in period 2 is $U_A = (1 - F_{i-\sigma}(R_{Li}^2(-\sigma); S_i + I_i))P_i^1(-\sigma) + \int_{\underline{R}}^{R_{Li}^2(-\sigma)} \frac{\theta RA}{I_{i1}^*} dF_{i-\sigma}(R_i, S + I_i)$ instead of $U(P^1\left(-\sigma\right), I_i^*, S_i| -\sigma)$, where $A = S_i + I_i^* - \frac{I_{i2}^* R_{F1}}{r}$ stands for bank i's asset holdings after liquidation of investment to meet the withdrawal by foreign depositors, and $R_{Li}^2(-\sigma) = \frac{AP^1(-\sigma)}{I_{i1}^*}$.

Because $r < 1 < \frac{S_i + I_{i1}^* + I_{i2}^*}{I_{i1}^* + I_{i2}^*}$, $U_A < U(P^1\left(-\sigma\right), I_i^*, S_i| -\sigma)$. The reason is that the foreign depositors' withdrawal forces bank i to liquidate a part of the long term asset at a loss, thus depressing its expected revenue further.

If I_{i2}^* is not large, after the bank satisfies the foreign depositors' withdrawal, it may retain local depositors with $P^2(-\sigma)$, thus avoiding a bank run. This situation is illustrated in Figure 5.c. The dashed-dotted curve is the local depositors' expected return after all foreign depositors withdraw. The horizontal line R_{FL} intersects the dashed-dotted curve at point (P_{i2}, R_{FL}), meaning that the bank can raise the promised return to prevent local depositors from withdrawing, thus avoiding a bank run.

In contrast, if I_{i2}^* is large, local agents may find it unworthy to keep their deposits in bank i and also begin to withdraw, making a bank run inevitable. This case is illustrated in Figure 5.d. There is no intersection between the horizontal line R_{FL} and the dashed curve. So bank i cannot provide a promised return such that the local depositors' expected return is at least equal to R_{FL}. In this case, a bank run is inevitable.

So whether a bank run occurs depends on I_{i1}^*, I_{i2}^* and S_i. In the real world, depositors may not know the exact value of I_{i1}^* and I_{i2}^*. During a crisis, some local depositors who believe I_{i2}^* is large will also withdraw their deposits from banks even when it is unnecessary, and their actions will exacerbate the crisis. That is often called a "confidence crisis". So in some situations, there are also multiple equilibria and which equilibrium is realized depends on depositors' beliefs about I_{i1}^* and I_{i2}^*.

We have thus identified a channel of contagion and a sequence of events: a negative shock leads some depositors to withdraw their deposits. These actions hurt the banks' ability to pay the remaining depositors, causing more depositors to withdraw and eventually a bank run is unavoidable.

5 Financial liberalization and Twin crises

It would be necessary to answer several questions before applying this framework to the Asian crisis.

First, why had these Asia countries developed so smoothly for such a long time (the "Asia miracle") before the crisis struck. A related question is: if these countries' fundamentals were so weak and a negative shock could cause so serious a crisis, why had not these Asia countries crashed earlier? In the Latin American countries, there were a lot of crises in 1980s, but how could Asian countries avoid them before the Asian crisis struck?

Second, was there any relationship between financial liberalization and the Asian crisis? The crisis countries' foreign debt increased fast while they liberalized their financial markets. How can we explain this? It is often claimed that China avoided this crisis because of its closed financial market. What is the impact of the openess of the local financial market?

In order to answer these questions, we need to know how financial liberalization affected the banking sector. By definition financial liberalization means lower barriers to cross-country capital flows. In my model, that means the rate of capital flow cost τ decreases. I consider below the banks' response if τ decreases from τ_1 to τ_2 ($\tau_1 > \tau_2$) as a result of financial liberalization.

After financial liberalization, the opportunity cost of local agents rises but that of foreign agents falls. If the local banks take foreign deposits after liberalization then the market equilibrium return must have decreased and foreign deposits must have increased.

The situation is illustrated in Figure 6. The dashed-dotted and the solid curves are the supply curves before and after the financial liberalization, respectively. By definition, $R_{FF} = (1+\tau_1)R_F > (1+\tau_2)R_F = R'_{FF}$ and $R_{LL} = (1-\tau_1)R_F < (1-\tau_2)R_F = R'_{FL}$. D is the demand curve. So after financial liberalization, the market equilibrium moves from $(I^1 + I_2, R_{FF})$ to $(I^1 + I'_2, R'_{FF})$, and $I_2 < I'_2$.

If all other conditions do not change, the local banking sector becomes more vulnerable to negative shocks after liberalization for two reasons. First, the banks accept more foreign deposits, so if a mild negative shock causes foreign depositors to withdraw their deposits, the banks have to liquidate more long term asset at a loss to meet their demand. Second, the local agents' opportunity cost rises, so it is more difficult to prevent them from withdrawing after a negative shock.

Of course, if the banking sector is strong (as in developed countries), S_i is very large and the risk of bank runs may remain low after financial liberalization.

As Kaminsky and Reinhart (1999) say: "the probability of BOP crisis increases the probability that a country will fall prey to a currency crisis.... a currency crisis does help to predict the probability that the banking crisis will worsen". I explained why a currency crisis can worsen a banking crisis in section 2, but did not explain why a banking crisis can trigger a currency crisis.

In section 3, we know that a bank run is Pareto inefficient to both of the bank and depositors. So if there is a central bank, it has an incentive to bail out the bank in trouble to prevent a bank run. This action can improve the social welfare if it works. However, it also causes the BOP problem. After financial liberalization foreign and total deposits will rise. When a negative shock occurs, the central bank needs to bail out banks more to prevent bank runs. That will increase the government's deficit which may trigger a BOP crisis if its foreign reserves are limited and the BOP crisis will exacerbate the banking crisis further. Finally, the twin crises occur.

6 Simulation

To obtain quantitative information about the working of the bank run model developed in the above section, I carried out a simulation exercise. The parameters used and results obtained are reported in the following.

6.1 Parameters Used

Assume the risk free return $R_{F1} = 1$ and $R_{F2} = 1.07$, so $R_F = 1.07$. Also assume the salvage factor of bankruptcy $\theta = 0.9$, the long term project return $R(I+S) \sim Uniform(0.7, 1.3 + \frac{0.6}{1+S+I})$, where $S+I$ is the total long-term investment and the cash value of liquidating a unit of the long-term project $r = 1/1.2$. If a negative shock occurs, suppose the upper bound remains unchanged at $1.3 + \frac{0.6}{1+I+S}$, but the lower bound decreases to $0.7 - \sigma$. That is, the long term return after a negative shock $R_{-\sigma}(I+S) \sim Uniform(0.7 - \sigma, 1.3 + \frac{0.6}{1+I+S})$.

6.2 Results without feedback effect ($n = 1$)

First, we simulate the case of $n = 1$ with a closed banking sector. In this case, $R_M = R_F = 1.07$. We assume the total available deposits are 0.5.

We assume that S takes on values between 0.001 to 0.4 and that the strength of shock $\sigma = 0.21$ (that means the lower bound of the return of the risky asset decreases by 30%.).

Some results are reported in Table 1. P and $P(-\sigma)$ are promised returns, Re and $Re(-\sigma)$ are the expected revenues before and after the shock, respectively. $P(-\sigma) = \phi$ means the bank cannot provide a promised return after the shock to convince depositors not to withdraw. In this case a bank run occurs. R_H is the highest expected return that the bank can offer to depositors when the promised return is P_H. We do not report R_M because $R_M = R_F = 1.07$ for all S.

From Table 1 we can highlight following properties:

1. The bank's revenue rises but average revenue decreases as S increases. The latter occurs because of decreasing returns to scale.

2. The promised return decreases as S increases, the reason being that as S increases, the bankruptcy risk in period 2 decreases. When S is large enough ($S > 0.3$), the bankruptcy risk is eliminated fully so that the bank can set $P = R_F$.

3. As S increases, R_H rises, meaning that the bank is able to promise higher expected return to depositors if it has to do that. As expected, a large bank equity gives the bank a stronger ability to withstand external risks.

4. If S is small, a negative shock can cause a bank run. If S is greater than a certain value (in the example, this value is 0.04) , the bank is able to raise the promised rate of return to a certain level to prevent a bank run. Although the bank still suffers a loss, the result is better than having a bank run. As S increases, the promised return after the negative shock decreases.

Next, we simulate the case of $n = 2$.

6.3 Results without feedback effect ($n = 2$)

For simplicity, we assume the two banks have the same initial capital S, the aggregate available deposits are 0.5 and S takes on value between 0.001 and 0.1 We calculate the market equilibrium and the banks' responses if a negative shock $\sigma = 0.21$ hits. The results are reported in Table 2. In the table, R_M is the market equilibrium return.

No matter which value S takes from $[0.001, 0.1]$, there is always a unique market equilibrium. The results of P_H, P_H^*, $P(-\sigma)$ and $Re(-\sigma)$ are similar to the results of $n = 1$, but the market equilibrium return is higher than that when $n = 1$, i.e., the risk free return $R_F = 1.07$.

6.4 Unexpected negative shock with feedback effect

Because foreign capital is unlimited, the banks do not need to compete against each other for deposits. Thus, we focus on the case of $n = 1$, which yields information on the case of $n > 1$.

We assume the return of international risk free asset is 1.06 and the capital flow cost rate $\tau = 1\%$. Foreign agents deposit their money in the bank only if the expected return is no less than 1.07, and local agents do that if the expected return is no less than 1.05. For simplicity, we only simulate a case that the ratio of foreign deposits to local deposits is 1:9. We assume there are three shocks with different strengths: $\sigma_1 = 0.28$, $\sigma_2 = 0.21$ and $\sigma_3 = 0.14$. Denote the promised return before the shock by P. If the bank can raise the promised return to prevent foreign agents from withdrawing after the shock, denote the promised rate of return by $P_{|-\sigma}$; if the bank cannot prevent foreign depositors from withdrawing their deposits but it can keep local agents' deposits after foreign agents leave, denote the promised return by $P'_{|-\sigma}$. Calculate $P_{|-\sigma}$ and $P'_{|-\sigma}$ when S and σ take different values. The results are reported in Table 3.

The table shows that if the shock is mild (in our simulation is $\sigma = 0.14$), even if a bank with small S can prevent foreign depositors from withdrawing with a higher promised rate of return. If the shock is stronger($\sigma = 0.21$), a bank with $S \leq 0.002$ cannot prevent a bank run, a bank with $S \in (0.002, 0.003)$ cannot prevent foreign agents from withdrawing but can avoid a bank run by keeping local agents' deposits, and a bank with $S \geq 0.004$ can keep all agents' deposits. Similar results are obtained if $\sigma = 0.28$, but the differences between this and the case of $\sigma = 0.21$ are that the banks need larger equity to withstand a stronger shock.

7 Policy Implications

From the above analysis, we can provide some policy implications about banking crises.

1. The effectiveness of an interest rate hike depends on fundamental conditions.

As shown in section 3, a high interest rate can prevent a bank run in a relatively mild crisis. But if the interest rate is higher than a certain level (P_H in Figure 1), then a rate hike not only fails to prevent but also accelerates a bank run.

2. Strong fundamentals are a prerequisite of financial liberalization.

Because that financial liberalization will decrease the cost of capital flow, banks will accept more deposits and become more vulnerable after liberalization. Banking crises usually cause very great social welfare losses and some losses may not be taken by the banks but by other people. So the banks tend to accept more deposits than the social optimal level. For improving social welfare, the country should not liberalize the financial sector and open the capital account until the local bank's equity base is sufficient strong equity base (i.e., a large S).

3. Capital control.

When foreign and local depositors withdraw their money from the banks threatening a bank run, capital control may be a feasible choice to the government. In section 4, when a medium negative shock hits, foreign depositors withdraw their money, causing damages to the banks. If the government taxes the outflow capital with rate ν in period 1 but not in period 2, the additional tax makes withdrawing in period 1 less attractive. A bank run may be prevented. Of course, capital control hurts the foreign agents' confidence. Society has to pay a cost in the long term.

8 Conclusions and Future Work

This paper provides a model of bank runs in which banks have the option of adjusting their interest rates. The model shows that a bank run is neither a "sunspot" (i.e., absolutely unnecessary), nor a "first best result" (i.e., socially optimal), but an outcome that depends on certain initial conditions. It helps to explains why banking crises often happen in developing countries, and why these countries had increased foreign debt before the crises. It also identifies a channel of contagious withdrawal of deposits to help us understand why a small shock may cause a bank run in a developing country.

Further work can be done in several other directions. All bank run models allow for only three periods. If we extend this to multiple periods, in each period new agents appear as old agents disappear, will the results change? I assume the negative shock is unexpected, perhaps this assumption can be relaxed. And in this paper, we do not model the real economy, so we are unable to analyze the effect of the economic structure. Some less developed and less opened countries (e.g. Mexico, Russia and the Philippines) rebounded faster than more developed countries (e.g. Korea and Malaysia). These facts suggest that the economic structure may be an important factor and that would be another direction of future research.

Appendix

Proof. Lemma 1

Since most results of this lemma can be proved easily, we only prove the case of $I' > I''$, then $U(P_i, I', S_i) \leq U(P_i, I'', S_i)$, and the equality holds if and only if $\frac{S_i+I'}{I'}\underline{R} \geq P_i$.

For all $I' > I''$, if $\frac{S_i+I'}{I'}\underline{R} \geq P_i$, then $U(P_i, I', S_i) = U(P_i, I'', S_i) = P_i$. If $\frac{S_i+I'}{I'}\underline{R} < P_i$,

$$
\begin{aligned}
& U(P_i, I', S_i) \\
& = (1 - F_i(\frac{I'P_i}{S_i+I'}; S_i + I'))P_i + \int_{\underline{R}}^{\frac{I'P_i}{S_i+I'}} \frac{(S_i+I')\theta R_i}{I'} dF_i(R_i; S_i + I') \\
& < (1 - F_i(\frac{I''P_i}{S_i+I''}; S_i + I''))P_i + \int_{\underline{R}}^{\frac{I''P_i}{S_i+I''}} \frac{(S_i+I')\theta R_i}{I'} dF_i(R_i; S_i + I'') \\
& < (1 - F_i(\frac{I''P_i}{S_i+I''}; S_i + I''))P_i + \int_{\underline{R}}^{\frac{I''P_i}{S_i+I''}} \frac{(S_i+I'')\theta R_i}{I''} dF_i(R_i; S_i + I'') \\
& = U(P_i, I'', S_i)
\end{aligned}
$$

■

Proof. Lemma 2

I. Given the assumptions, we have for all $R'_M > R''_M$,

$$U(P(R'_M, S_i, I_i), I_i, S_i) = R'_M > R''_M = U(P(R''_M, S_i, I_i), I_i, S_i)$$

By definition, we have $P(R'_M, S_i, I_i) > P(R''_M, S_i, I_i)$.

II. For all $S' > S'' > 0$, denote $P_1 = P(R_M, S', I_i)$ and $P_2 = P(R_M, S'', I_i)$. Consider

$$
\begin{aligned}
U(P, I_i, S_i) &= (1 - F_i(\frac{I_iP_i}{S_i+I_i}; S_i + I_i))P_i + \int_{\underline{R}}^{\frac{I_iP_i}{S_i+I_i}} \frac{(S_i+I_i)\theta R_i}{I_i} dF_i(R_i; S_i + I_i) \\
&= P - P(1-\theta)F_i(\frac{I_iP}{S_i+I_i}; S_i + I_i) - \frac{\theta(S_i+I_i)}{I_i} \int_{\underline{R}}^{\frac{I_iP}{S_i+I_i}} F_i(R; S_i + I_i) dR
\end{aligned}
$$

Denote $R_1 = \frac{S_i+I_i}{I_i}R$. Since $F_i(\frac{I_iP}{S_i+I_i}; S_i+I_i) = \Pr((S_i+I_i)\cdot R_i(S_i+I_i) < I_iP)$, we get

$$U(P, I_i, S_i) = P - P(1-\theta)F_i(\frac{I_iP}{S_i+I_i}; S_i+I_i) - \theta\int_{\frac{S_i+I_i}{I_i}\underline{R}}^{P} F_i(\frac{I_iR_1}{S_i+I_i}; S_i+I_i)dR_1$$

We know $F_i(\frac{I_iP}{S'+I_i}; S'+I_i) \le F_i(\frac{I_iP}{S''+I_i}; S''+I_i)$ and $\Pr((S'+I_i)R_i(S'+I_i) < I_iP) \le \Pr((S''+I_i)R_i(S''+I_i) < I_iP)$. If $P > \frac{S'+I_i}{I_i}\underline{R}$, then $\Pr((S'+I_i)R_i(S'+I_i) < I_iP) > 0$, so we can have

$$\begin{aligned} &P(1-\theta)F_i(\frac{I_iR}{S'+I_i}; S'+I_i) + \theta\int_{\frac{S'+I_i}{I_i}\underline{R}}^{P} F_i(\frac{I_iR_1}{S'+I_i}; S'+I_i)dR_1 \\ &< P(1-\theta)F_i(\frac{I_iR}{S''+I_i}; S''+I_i) + \theta\int_{\frac{S''+I_i}{I_i}\underline{R}}^{P} F_i(\frac{I_iR_1}{S''+I_i}; S''+I_i)dR_1 \end{aligned}$$

From that, we get $U(P, I_i, S') \ge U(P, I_i, S'')$ for all P and I_i, and the equality holds only if $P \le \frac{S''+I_i}{I_i}\underline{R}$. So we have $U(P_2, I_i, S') \ge U(P_2, I_i, S'') = R_M$, just like Part I, we have $P_1 \le P_2$, and the equality holds only if $R_M \le P_2 \le \frac{S''+I_i}{I_i}\underline{R}$, i.e., $F_i(\frac{I_iR_M}{S''+I_i}; S''+I_i) = 0$.

III. For all $I' > I''$, denote $P^1 = P(R_M, S_i, I')$, $P^2 = P(R_M, S_i, I'')$. From part II, we know $U(P, I_i, S_i) = P - P(1-\theta)F_i(\frac{I_iP}{S_i+I_i}; S_i+I_i) - \theta\int_{\frac{S_i+I_i}{I_i}\underline{R}}^{P} F_i(R_1; S_i+I_i)dR_1$. Since $I' > I''$, then $\frac{I'}{S_i+I'} > \frac{I''}{S_i+I''}$, so we can get $F_i(\frac{I'P}{S_i+I'}; S_i+I') \ge F_i(\frac{I''P}{S_i+I''}; S_i+I'')$ for all P. Then, we have $P(1-\theta)F_i(\frac{I'P}{S_i+I'}; S_i+I') + \theta\int_{\frac{S_i+I'}{I'}\underline{R}}^{P} F_i(R_1; S_i+I')dR_1 \ge P(1-\theta)F_i(\frac{I''P}{S_i+I''}; S_i+I'') + \theta\int_{\frac{S_i+I''}{I''}\underline{R}}^{P} F_i(R_1; S_i+I'')dR_1$, so we get $P^1 \ge P^2$. And we can verify, the equality holds only if $R_M \le P^1 \le \frac{S_i+I'}{I'}\underline{R}$, i.e., $F_i(\frac{I'R_M}{S_i+I'}; S_i+I') = 0$. ■

Proof. Proposition 1

Consider $\sum_{i=1}^{n} I_i^*(R_F)$. If $\sum_{i=1}^{n} I_i^*(R_F) \le I_L$, then

$$(R_F, (I_1^*(R_F), ..., I_n^*(R_F)), (P_1^*(R_F), ..., P_n^*(R_F)))$$

is an equilibrium. Otherwise, $\sum_{i=1}^{n} I_i^*(R_F) > I_L$. When $R_M \ge \overline{R}$, no bank has any incentive to absorb deposits, so $I_i^*(\overline{R}) = 0$. So we have $\sum_{i=1}^{n} I_i^*(\overline{R}) = 0 < I_L < \sum_{i=1}^{n} I_i^*(R_F)$. Because $I_i^*(R_M)$ is continuously decreasing in R_M for each i, $\sum_{i=1}^{n} I_i^*(R_M)$ is also continuously decreasing in R_M. So there is a unique $R_M^* > R_F$ such that $\sum_{i=1}^{n} I_i^*(R_M^*) = I_L$ and $(R_M^*, I_1^*, ..., I_n^*), (P_1^*, ..., P_n^*))$ satisfy the equilibrium condition. ■

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Table 1 Market Equilibrium without Feed Back Effect (n=1)

S	Re	P	I	P_H	R_H	$P(-\delta)$	$Re(-\delta)$
0.001	0.0354	1.3086	0.4106	1.572	1.1072	ϕ	0
0.01	0.0488	1.2699	0.4273	1.5979	1.1268	ϕ	0
0.02	0.0629	1.2391	0.4406	1.6259	1.1479	ϕ	0
0.03	0.0765	1.2154	0.4507	1.6534	1.1684	ϕ	0
0.04	0.0898	1.1962	0.4587	1.6805	1.1886	1.5015	0.0204
0.05	0.1027	1.1803	0.4652	1.7075	1.2086	1.4313	0.0344
0.06	0.1155	1.1667	0.4705	1.7344	1.2285	1.3857	0.0471
0.07	0.1281	1.1549	0.4748	1.7612	1.2483	1.3513	0.0591
0.08	0.1406	1.1446	0.4784	1.788	1.268	1.3236	0.0709
0.09	0.153	1.1355	0.4813	1.8149	1.2878	1.3004	0.0823
0.1	0.1653	1.1274	0.4836	1.8418	1.3076	1.2805	0.0937
0.2	0.2846	1.0804	0.4879	2.1219	1.5129	1.1653	0.2028
0.3	0.3983	1.07	0.5	2.2529	1.6165	1.1178	0.3057
0.4	0.5071	1.07	0.5	2.3079	1.666	1.0909	0.4075

Table 2 Market Equilibrium without Feed Back Effect (n=2)

S	P	I	R_M	Re	P_H	R_H	$P(-\delta)$	$Re(-\delta)$
0.001	1.32	0.25	1.0867	0.0251	1.6243	1.1337	ϕ	0
0.002	1.3096	0.25	1.086	0.0269	1.6304	1.138	ϕ	0
0.005	1.2826	0.25	1.084	0.0321	1.6488	1.151	ϕ	0
0.009	1.2535	0.25	1.0815	0.0386	1.6732	1.1684	ϕ	0
0.01	1.2472	0.25	1.081	0.0402	1.6793	1.1727	1.6066	0.0054
0.02	1.1982	0.25	1.0765	0.0553	1.7402	1.216	1.4167	0.0223
0.05	1.1267	0.25	1.0723	0.0956	1.9217	1.3452	1.2572	0.0601
0.09	1.0964	0.25	1.0787	0.1438	2.1609	1.5161	1.1718	0.1061
0.1	1.0942	0.25	1.0816	0.1553	2.2202	1.5585	1.1583	0.1173

Table 3 Market Equilibrium with Feed Back Effect (n=1)

S	P	I	$P_{\|-0.28}$	$P'_{\|-0.28}$	$P_{\|-0.21}$	$P'_{\|-0.21}$	$P_{\|-0.14}$	$P'_{\|-0.14}$
0.001	1.3086	0.4106	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
0.002	1.3035	0.4128	ϕ	ϕ	ϕ	ϕ	ϕ	1.5214
0.01	1.2699	0.4273	ϕ	ϕ	ϕ	ϕ	ϕ	1.4018
0.015	1.2534	0.4344	ϕ	ϕ	ϕ	ϕ	1.5763	1.3649
0.02	1.2391	0.4406	ϕ	ϕ	ϕ	1.4945	1.4756	1.3367
0.03	1.2154	0.4507	ϕ	ϕ	ϕ	1.406	1.3974	1.2944
0.04	1.1962	0.4587	ϕ	1.5007	1.5015	1.3544	1.3503	1.2628
0.05	1.1803	0.4652	ϕ	1.4304	1.4313	1.317	1.3159	1.2377
0.06	1.1667	0.4705	1.5492	1.3837	1.3857	1.2877	1.2886	1.217
0.1	1.1274	0.4836	1.3657	1.2751	1.2805	1.2101	1.2158	1.1589
0.2	1.0804	0.4879	1.2127	1.1559	1.1653	1.1173	1.1281	1.0876
0.3	1.07	0.5676	1.173	1.1232	1.1337	1.0915	1.1039	1.0685
0.4	1.07	0.6702	1.1571	1.1099	1.121	1.0812	1.0943	1.0612

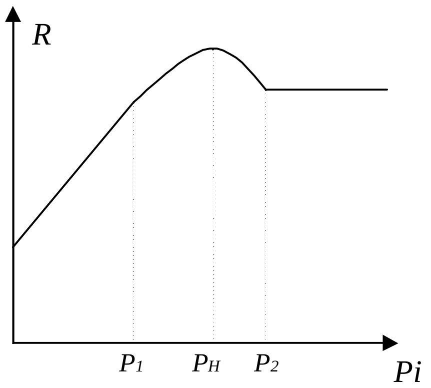


Figure 1

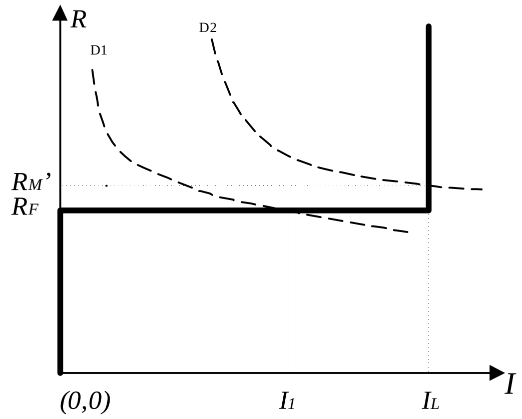


Figure 2

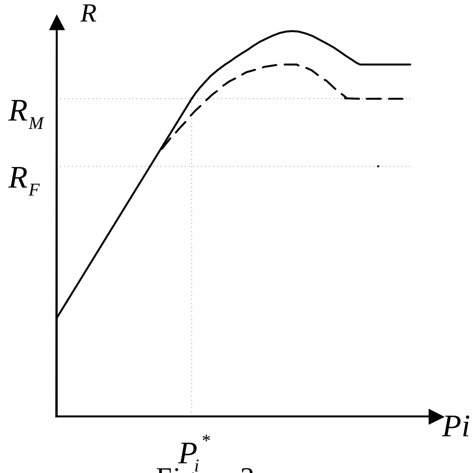
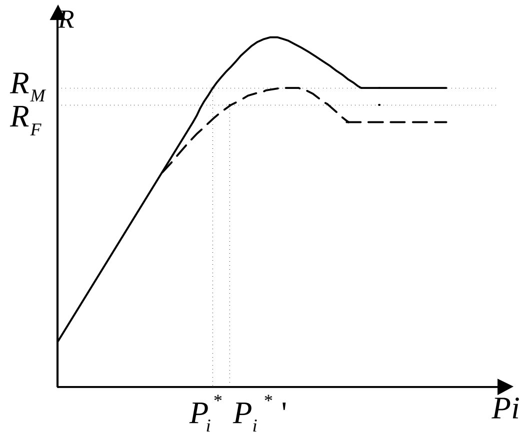


Figure 3.a



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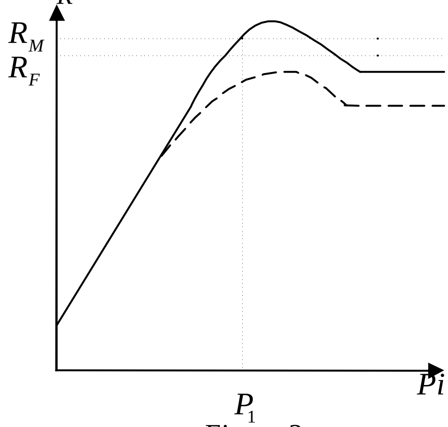
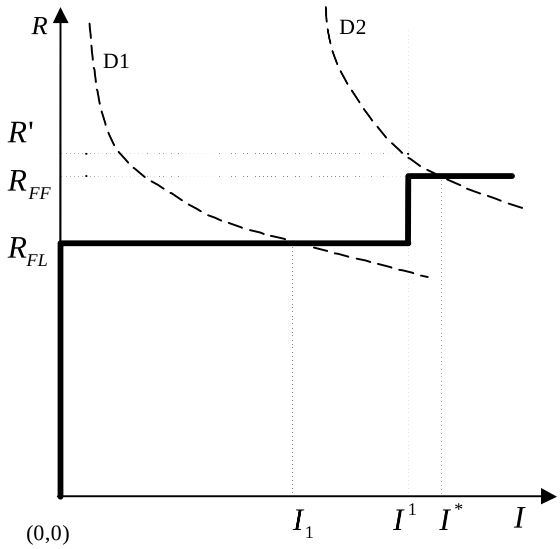


Figure 3.c



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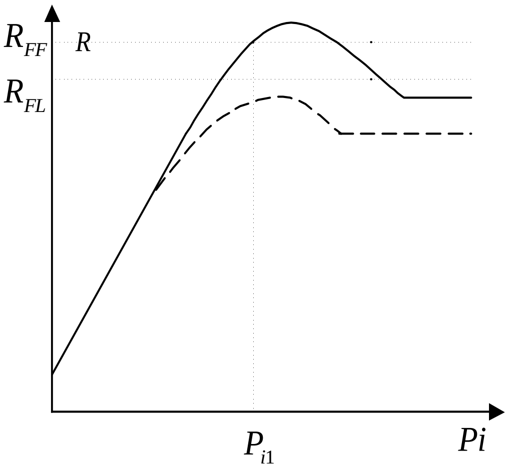
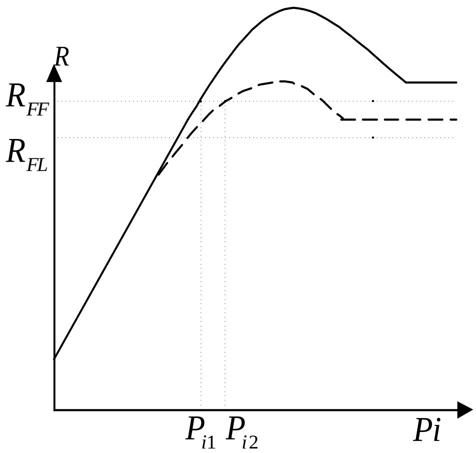


Figure 5.a



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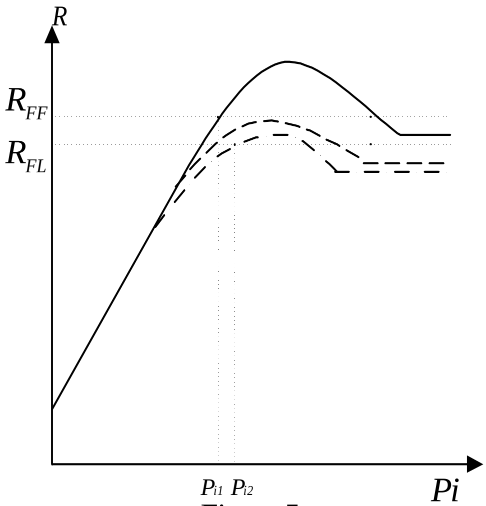

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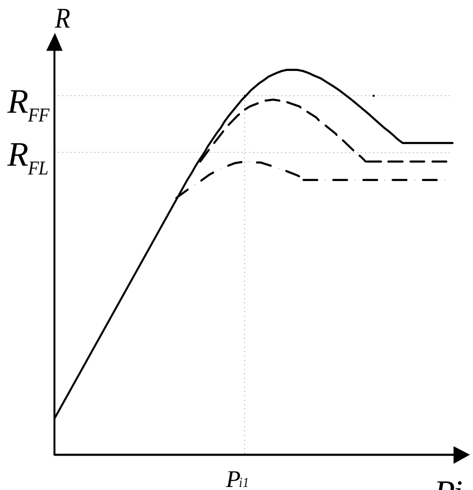

Figure 5.d

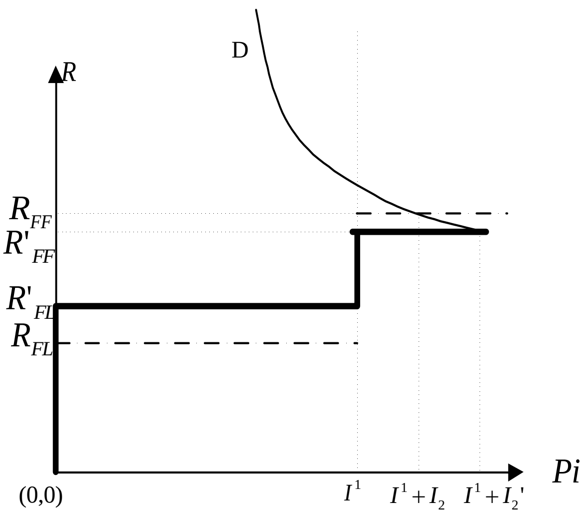

Figure 6