Commodity currencies: Why are exchange rate futures biased if commodity futures are not?

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March 11, 2003

Abstract

This paper adds to the puzzle of the forward bias of exchange rates by noting that while the exchange rate of a small commodity-exporting economy can be closely tied to commodity prices, a portfolio of commodity futures exhibits little if any bias. Using data for Australia, the bias of exchange rate forwards is shown by the negative slope coefficient from a 'Fama regression'. This paper finds that the slope coefficient from an equivalent regression using a portfolio of commodity futures designed to replicate export prices, and so the exchange rate, is positive. A model of a small open economy is developed from micro foundations in which the exchange rate depends on export prices, as well as import prices, non-traded output and the domestic money supply. This exchange rate model is used to examine whether the domestic monetary supply could cause the bias in exchange rate forwards when there is an absence of bias in commodity futures. Three potential explanations are considered. Systematic expectation errors about the monetary process, while requiring strong assumptions, receive empirical support from the behaviour of the exchange rate. Neither monetary policy nor peso problems seem capable of explaining the puzzle.

*The views expressed in this paper are those of the author and should not be attributed to the Reserve Bank of Australia. This paper is a chapter from my PhD thesis at MIT. I am deeply indebted to my advisors Rudi Dornbusch and Roberto Rigobon. I would also like to thank Jaume Ventura for valuable suggestions. The usual disclaimer applies

1 Introduction

The forward bias of exchange rates has puzzled economists for over two decades. This paper adds to the puzzle by noting that while the exchange rate of a small commodity-exporting economy can be closely tied to commodity prices, a portfolio of commodity futures exhibits little if any bias. This fact is demonstrated using data for Australia, a country for which approximately 60 per cent of exports are commodities.[1] The bias of exchange rate forwards can be seen in that the slope coefficient from a regression of the change in the spot price on the forward premium, the 'Fama regression', is less than unity and often, as is the case for Australia, negative. Yet, as this paper shows, the coefficient from an equivalent regression using a portfolio of commodity futures designed to replicate Australian export prices, and so the exchange rate, is positive and significant.[2] This implies that the bias in exchange rate futures must come from one of the other determinants of the exchange rate.

A small open economy model of a 'commodity currency', along the lines of the new open economy macroeconomic models, is developed from micro foundations. This shows that in addition to the commodity export price, import prices and non-traded output, the exchange rate is also determined by the domestic money stock. Three potential explanations of the bias of exchange rate forwards, despite the absence of bias in commodity futures, that rest on the behaviour of monetary factors are considered. Systematically biased expectations regarding the monetary process, while requiring strong assumptions, receive empirical support from the behaviour of the exchange rate. However, neither the practice of monetary policy nor peso problems seem capable of fully explaining the puzzle.

The remainder of this paper proceeds as follows. Section 2 outlines the puzzle in more detail and discusses possible explanations. A small open economy model is then developed in Section 3. This model is used, in Section 4, to consider the three explanations of the forward bias of exchange rates that depend on the money stock and so could be consistent with unbiased commodity futures. This is followed by some concluding remarks.

[1] Australia is used as it meets the two conditions that its exchange rate has a close relationship with commodity prices and futures and forward contracts exist for a large proportion of its exports. Other 'commodity currencies' considered either had a weaker exchange rate-commodity price link (e.g. Canadian dollar) or fewer of their commodity exports are covered by futures (e.g. New Zealand dollar).

[2] The forward premium regression conducted using an index constructed to replicate Canadian commodity exports also had a positive and significant beta coefficient indicating this result is not peculiar to the Australian commodity index.

2 The puzzle

Nominal exchange rates are notoriously difficult to model as demonstrated by Meese and Rogoff (1983) and numerous researchers in their wake.[3] Yet the price of commodities is often used as a guide to the value of the exchange rate of commodity exporting countries, at least within financial markets. Figure 1 demonstrates this relationship for Australia.[4] This is obviously an incomplete representation of the fundamental value of a currency. But it is a crucial component. In the simplest case, consider a small country that exports one good in exchange for a consumption good, and has no access to international borrowing. An increase in the price of the exported good improves the country's terms of trade and so increases their consumption of the imported good. If monetary policy does not respond to this shock the domestic price of the imported consumption good will fall, which under the Law of One Price (LOP) will imply a depreciation of the exchange rate. An increase in the price of the exported good is then associated with a depreciation of the exchange rate, as Figure 1 shows is the case for Australia. In this example the money supply is a crucial determinant of the domestic price of the imported good and so the exchange rate. It could feasibly alter the relationship between the exchange rate and commodity price. As argued in this paper, the behaviour of money appears to be the most likely explanation of the differing degrees of bias in exchange rate and commodity futures. Section 3 formally develops from micro foundations this model of a 'commodity currency' in which the economy is a price taker in international markets.

So long as covered interest parity holds, which it does for major currencies, the forward bias of exchange rates will be equivalent to the failure of uncovered interest rate parity (UIP). Since UIP is a central tenet of both empirical and theoretical international macro models the forward bias then has significant implications well beyond the efficiency of the exchange rate market. The bias of forward exchange rates has been demonstrated in many studies using a simple regression from Fama (1984).[5] The conditional efficiency of exchange rate futures is tested by the regression

$$s_{t+1} - s_t = \alpha + \beta \left(f_t - s_t \right) + \varepsilon_t \tag{1}$$

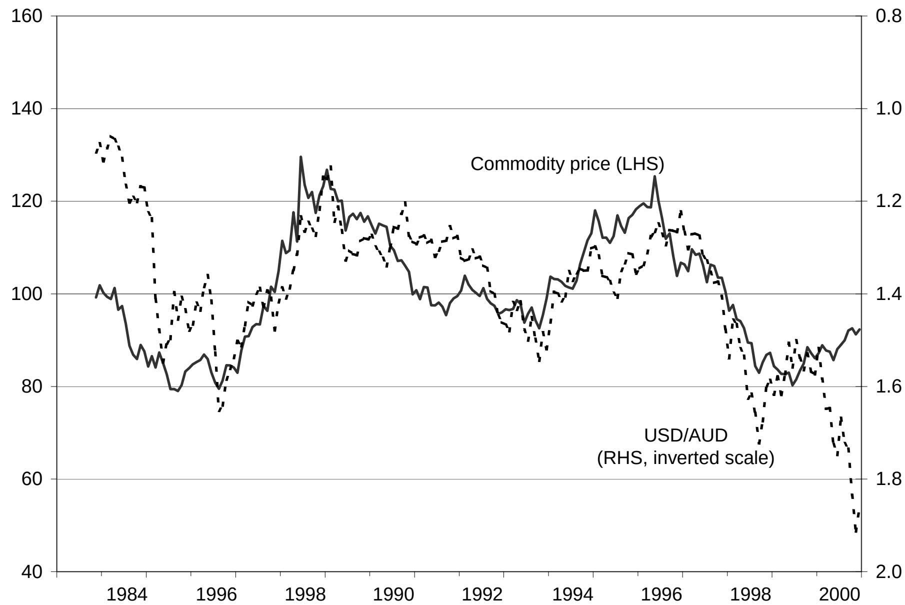
where s_t is the (log) spot exchange rate, and f_t is the (log) one period forward exchange rate.

[3]See for example the forthcoming February 2003 issue of the Journal of International Economics which publishes papers from the September 2001 conference at the University of Wisconsin marking 20 years since the Meese and Rogoff paper.

[4]This graph is drawn using an index of spot prices for commodity futures that is described below.

[5]The literature on the exchange rate forward bias is truly volumous and so only incomplete attention is devoted to it here. For surveys see Hodrick (1987), Lewis (1995) and Engel (1996).

Figure 1: Commodity price index and Australian dollar – US dollar exchange rate.



Conditional efficiency of exchange rate forwards is equivalent to the hypothesis that $\beta = 1$.[6] Almost without exception across country pairs and sample periods, this null hypothesis has been rejected, and typically not by a small margin. In many cases the point estimate of β is negative. The Australian dollar is no different. Over the period during which the Australian dollar has floated, the β from this regression using 3-month forwards is -0.94, as shown in Table 1. It is not only significantly different from unity but, at the 10 per cent level, significantly less than zero. Conversely, the β from an equivalent 'Fama regression' using indices of the spot and futures prices of commodities, which are described below, is positive, 0.55, and significantly different from zero at the five per cent significance level. Given the economic relationship these series should have, and is borne out in Figure 1, this result implies that the forward bias of exchange rates must be attributable to the other determinants of the exchange rate. Three possible explanations are discussed below, but as already suggested, domestic money will be crucial to these explanations. In common with other studies, the point-estimate of β from the

[6]Strictly speaking conditional efficiency also implies $\alpha = 0$ though this is typically omitted from testing.

exchange rate forward premium regression is found to vary substantially within the sample, as seen in Figure 2. The variability in the exchange-rate beta contrasts with the stability of the beta from the commodity regression, which is almost always between zero and two.

Table 1: Forward Premium Regressions: Exchange rates and commodity prices.

	Dependent variable: 3-month change in spot price of:			
	exchange rate	commodity price index: full index	futures only	forwards only
constant	0.02	0.00	0.00	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)
forward premium	-0.94	0.55	0.58	0.75
	(0.58)	(0.21)	(0.19)	(0.29)
$\overline{R}^2$	0.018	0.046	0.070	0.041
Durbin Watson	0.66	0.60	0.65	0.65

Exchange rate sample is 1/1984 - 9/2000
Commodity price samples are 11/1983 - 9/2000

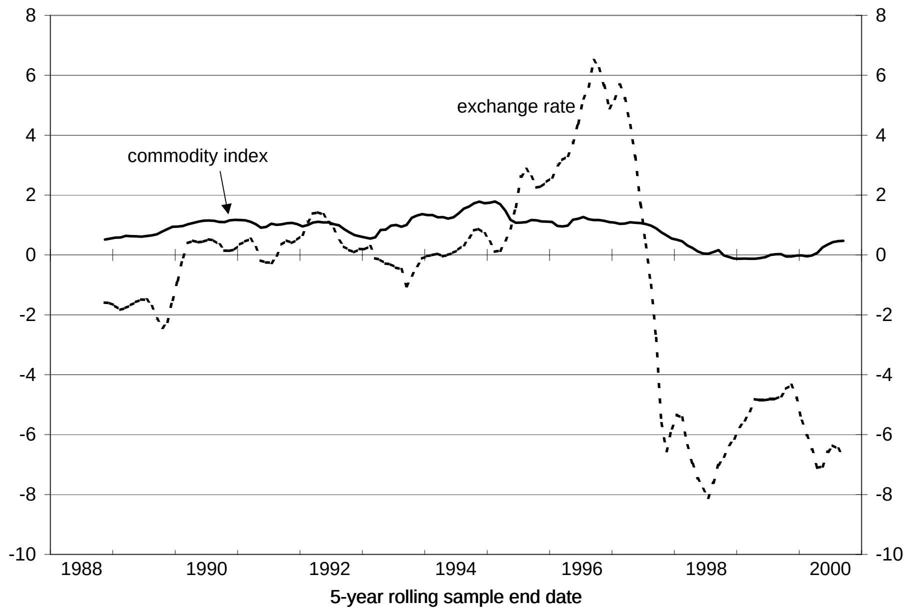
2.1 The commodity indices

The construction of the commodity indices is outlined briefly here, with greater detail provided in Appendix B.[7] The indices are constructed from portfolios of commodity futures and forwards trading in US dollars on the London Metal Exchange (LME) and various commodity futures exchanges in the USA. The index weights correspond to the export-share weights from the Reserve Bank of Australia (RBA) commodity price index. Two indices are constructed, an index of commodity spot prices and an index of 3-month futures prices. A 3-month horizon is chosen as the LME forwards are available for 3-month contracts, as are most futures. There are seventeen traded contracts in the indices, covering just over half of the RBA index.[8] As is typically done when working with commodity futures, spot prices are constructed from expiring contacts to avoid problems of basis risk when comparing the futures and spot indices. Because not all futures contracts expire each month, prices in missing months are extrapolated from longer horizon contracts, following Pindyck (1993). This clearly introduces some noise, but the

[7]There are two traded futures on commodity indices, the CRB futures index and the Goldman Sachs commodity index, but both of these futures have too short a time series for use in this study. Further, their composition does not reflect the export base of Australia and Cashin et al (1999) suggest that index composition does matter for commodity price indices due to commodity specific shocks.

[8]There more futures than forwards in the index and so it is refered to as a 'commodity futures index'.

Figure 2: Betas from rolling 5-year forward premium (Fama) regressions



behaviour of the prices of forwards, for which 3-month contracts are available each month and so no extrapolation is needed, serves as a robustness check. As shown in Table 1, regressions using sub-indices which separate the futures and forwards have almost identical coefficients on the forward premium, 0.56 and 0.75 respectively, and again both are significantly greater than zero. The large and significant coefficient when using just commodity forwards indicates that while the extrapolation used to construct missing months for commodity futures prices likely contributes to attenuation bias, it is not driving the finding that the bias in the commodity futures index is at most small. Notably, the coefficient from the regression using only futures is insignificantly different from unity so that the hypothesis that this subindex is conditionally unbiased cannot be rejected.

Despite its smaller coverage, the spot futures commodity price index does an excellent job of replicating the movements in the comprehensive RBA commodity price index. The spot commodity price index has a 0.95 correlation with the RBA index, and correlation of 0.59 with the exchange rate, as can been seen in Figure 1.[9] The properties of the forward premiums and 3-month spot price returns for the exchange rate and commodity price index are shown in Table 2. The commodity price index is seen to have similar volatility to the exchange rate, but notably the commodity index forward premium, $f_t - s_t$, is more volatile than the exchange rate forward premium. Similarly it is less autocorrelated. These summary statistics are not dissimilar when considering futures and forwards separately.

Table 2: Properties of Forward Premiums and Spot Returns for both the Exchange Rate and Commodity Price.

	exchange rate		commodity price full index	
	change in spot	forward premium	change in spot	forward premium
mean	0.81	0.79	-0.25	0.31
standard deviation	5.37	0.87	4.56	1.88
standard deviation / mean	6.65	1.10	-18.41	6.16
autocorrelation	-0.09	0.87	0.12	0.37

	commodity price futures only		commodity price forwards only	
	change in spot	forward premium	change in spot	forward premium
mean	-0.17	0.34	-0.53	0.47
standard deviation	4.59	2.16	9.78	2.79
standard deviation / mean	-27.01	6.44	-18.46	5.92
autocorrelation	0.02	0.40	0.09	0.42

2.2 Explaining the bias

The rejection of conditional unbiasedness for exchange-rate forwards, $\beta \neq 1$, implies that there are predictable excess returns. This is demonstrated by rewriting equation (1) as

$$s_{t+1} - f_t = \alpha + (\beta - 1)(f_t - s_t) + \varepsilon_t \tag{2}$$

[9]This is only slightly lower than the 0.65 correlation between the RBA index and the exchange rate. The similar movements in the futures index and the broader RBA index are likely in part contributed to by the common determinants of commodity prices (Borensztein and Reinhart (1994)). Pindyck and Rotemberg (1990) suggest that commodity prices display excess comovement, though Cashin et. al. (1999) dispute this.

With $\beta \neq 1$ the forward premium, $f_t - s_t$, which is known at time t, can be used to predict the difference between the realised exchange rate at time $t+1$ and the time t price at which time $t+1$ currency can be bought and sold. In other words, the forward premium can be used to predict the excess return, $er_{t+1} = s_{t+1} - f_t$. The identity in equation (3) decomposes the predicted excess returns, per, as the sum of a time-varying risk premium, and a market expectation error.

$$per_{t+1} = E_t s_{t+1} - f_t = (E_t^m s_{t+1} - f_t) + (E_t s_{t+1} - E_t^m s_{t+1}) \tag{3}$$

where E_t^m is the market expectation formed at time t, and E_t is the rational expectations operator. The first term in parentheses is the risk premium, while the second is the market expectations error. As many authors have shown drawing on Fama, in order for $\beta < 1$ the predicted excess return must be time varying, and negatively correlated with the forward premium. Even more puzzling, a finding of $\beta < 0$ implies that the predicted excess returns must be more volatile than exchange rate returns. Standard models of risk premia are unable to deliver these properties, as summarised by Engel (1996). Efforts to generate these properties with more complex models, such as first-order risk aversion (Bekeart et. al. (1997)) or habit persistence (Backus et. al. (1993)) have met with a similar lack of success.

The alternative avenue of explanations, market expectation errors, appears to hold more promise in finding an answer to the puzzle. These explanations have the common element that the ex post expectations formed by the researcher differ from those formed ex ante by agents, based on their subjective probability distribution. One possibility is that there are 'peso problems', that some event given non-zero probability weight by agents occurred with an unrepresentative frequency in the researcher's sample. Uncertainty as to whether the event occurred, with Bayesian updating of expectations, can also contribute to seemingly systematic expectation errors that may be correlated with the forward premium. Expectations could also be systematically biased possibly resulting in the exchange rate adjusting slowly to changes in the forward premium, as suggested by Froot and Thaler (1990). Gourinchas and Tornell (2002) provide a tractable specification for biased expectations which demonstrates that the slow adaptation of expectations to monetary factors can result in the forward bias of exchange rates. These explanations could potentially explain the bias of exchange rate forwards, but not of commodity futures, if they are applied to one of the other determinants of the exchange rate, notably the domestic money stock.

A third class of explanations stems directly from the practice of monetary policy, and so could also be consistent with the exchange rate–commodity price bias puzzle. McCallum (1994) demonstrates that in the presence of a persistent deviation from UIP (which need not be a risk premium) monetary policy aimed at minimising exchange rate changes can result in a negative covariance between the forward premium and predicted excess returns, and so a downward biased beta. An alternative explanation relating to policy is proposed by Fisher Black in Engel (1996). He is quoted as suggesting that central bank intervention could be responsible for the forward bias of exchange rates. But the consistency of the finding of forward bias across countries and time periods, while the practice of intervention has been anything but consistent, suggests this is unlikely to be the cause. Notably, two similar countries Australia and New Zealand have equivalent degrees of exchange rate forward bias despite the Australian central bank frequently intervening in the foreign exchange market while the New Zealand central bank has never done so.

Commodity markets are in many ways more complex than foreign exchange markets, due to the existence of storage for many commodities, seasonal factors and illiquid markets. In particular because of the physical nature of the goods traded in commodity markets there are different influences on risk premia. An extensive literature exists relating futures risk premia to systematic risk and hedging pressures, for example see Bessembinder (1992) and Hirshleifer (1988, 1989).[10] But while the existence of inventory means that spot and futures prices will be codetermined, as modeled by Hong (2000) and Routledge et al (2000), the existence of inventory does not necessarily infer that commodity futures need be either biased or unbiased predictors of future spot prices. Just as time varying risk premia, expectation errors or peso problems could be responsible for the bias in exchange rate markets, these factors could be present in commodity markets. Fama and French (1987) consider the determinants of futures prices for 21 individual commodities. They find that for each of the commodities, the point estimate of β from a regression of the form of equation (1) is between zero and one, though because of large standard errors it is significantly greater than zero in only seven cases. Despite this evidence of time-varying predictable returns, they find a greater role for the theory of storage in the formation of commodity futures prices than for time varying risk premia.[11] By combining commodities into portfolios many of the idiosyncratic factors that affect the cost of storage for

[10] This is much broader than the question, dating back to Keynes, as to whether futures prices must rise as they approach expiration, that is they demonstrate normal backwardation, to compensate speculators. After comprehsive testing Kolb (1992) suggests normal backwardation isn't so normal. Normal backwardation would imply $\alpha \neq 0$ but not necessarily that $\beta \neq 0$.

[11] The 'theory of storage' of Kaldor (1939), Tesler (1958) and Working (1949) describes the futures price of a storable commodity, for which inventory is held, as a function of the interest foregone, warehousing costs and convenience yield (the benefit from holding the physical commodity rather than a futures contract). The impact storage can have on the dynamics of the spot price is estimated by Deaton and Laroque (1996).

individual commodities can be mitigated to a large extent. That is the approach pursued in this paper.

3 Small open economy model of the exchange rate

Models of small open economy, sometimes called dependent economy models, for example Dornbusch (1980), consider countries that are assumed to be unable to influence conditions in the rest of the world. This framework is especially relevant for small commodity exporting countries, such as Australia, whose export product is homogenous and market share is small.[12]

This section develops a simple small open economy model of the nominal exchange rate using micro foundations. The economy is endowed with a fixed quantity per period, normalised to unity, of a good, X, that is exported and not consumed. The assumption of a fixed supply is appropriate for a commodity exporting country; the large investments for mineral extraction and the delay to harvest of agricultural commodities imply that their supply is inelastic. The world price of the export good, P^*_{Xt}, is taken as given and assumed to be a log normal random walk.

$$\log P^*_{Xt+1} \sim N\left(\log P^*_{Xt}, \sigma^2_{P^*_X}\right)$$

This assumption contradicts the finding in Section 1 that there are predictable changes in commodity prices, but is made to focus the attention on the bias present in exchange rate futures. Standard notation of a star to indicate the value taken by a foreign variable, and lower case variables to indicate the log of their uppercase equivalents, will be used throughout. All variances are assumed to be constant through time. The country imports a traded good for consumption, T, whose world price is also assumed to follow a log normal random walk. Agents also consume a non-traded good, Y_{Nt}, of which the economy receives a random per period endowment. This too follows a log normal random walk. Preferences are a function of aggregate consumption and real money balances and are given by

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\delta} \left(\frac{M_s}{P_s} \right)^{1-\delta} \right] \tag{4}$$

[12]The intuition of this framework has inspired many empirical studies. Freebairn (1990) uses this intuition to motivate his study of the relationship between the Australian dollar and commodity prices. Similar analysis was performed for the real exchange rate in Gruen and Wilkinson (1994). Sjaastad (1998) took the dependent economy intuition beyond commodity exporting countries to examine the behaviour of the Swiss franc.

where C_t is a Cobb-Douglas aggregation over the traded good and the non-traded good, $C = C_N^{\gamma} C_T^{1-\gamma} / \gamma^{\gamma} (1-\gamma)^{1-\gamma}$. The representative consumer maximises the expectation of (4) subject to the budget constraint

$$\begin{aligned} & B_{t+1} P_{Tt} + \tilde{B}_{t+1} + S_t \tilde{B}_{t+1}^* + M_t \\ = \quad & (1+r) B_t P_{Tt} + (1+i_t) \tilde{B}_t + (1+i_t^*) S_t \tilde{B}_t^* + M_{t-1} + P_{Xt} + Y_{Nt} P_{Nt} - C_t P_t - \tau_t P_t \end{aligned}$$

where $P_t = P_{Nt}^{\gamma} P_{Tt}^{1-\gamma}$ is the aggregate price index of the economy; B_t,$\tilde{B}_t$, and $\tilde{B}_t^*$ are holdings at the start of period t of the real bond, denominated in the tradable good and paying a constant rate of return r, the domestic nominal bond, and the foreign nominal bond; S is the exchange rate (price of foreign currency); and, M_{t-1} is the holdings of money at the start of period t. The bonds, of which initial holdings are zero, are the only financial assets; there are not complete markets. Since there is a representative consumer and money has no real effects Ricardian equivalence will hold and so the value of government transfers, $\tau_t P_t$, will cancel with the two nominal money holding terms in the budget constraint. There are no restraints to trade or transport costs so that the LOP holds, $P_T = S P_T^*$, which will define the exchange rate. Prices are flexible in the model, although the implication of sticky prices is discussed.

The assumption of random walk exogenous prices and the real-nominal dichotomy produces a neat closed form for the model without any assumptions on the behaviour of, or the nature of expectations about, the money supply. This basic model then serves as the building block to consider different potential causes of the forward bias of exchange rate futures.

Since the model structure is standard, details of the solution are relegated to Appendix A, and only a sketch is outlined here. All exogenous variables are log normal and so the solution will have a log normal form. The first order conditions imply that the paths of tradable and non-tradable consumption are governed by

$$\gamma (\rho - 1) E_t^m \{c_{Nt+1} - c_{Nt}\} + (\rho (1-\gamma) + \gamma) E_t^m \{c_{Tt+1} - c_{Tt}\} = K_1 \tag{5}$$

where E_t^m is the market expectation operator and the letter K with a subscript is used to indicate constants (that are functions of the exogenous parameters). Consumption of the non-traded good must equal its production, which is a log random walk. This condition then confirms that consumption of the traded good will also be log normally distributed. As a result of the log random walk of exogenous prices, trade will be balanced in each period.[13]

[13]This assumes constraints on the parameter values that ensure a solution exists as described in the Apendix.

The derivation of the exchange rate starts with the first order condition with respect to money balances.

$$1-\chi\left(\frac{M_t}{P_t}\right)^{-\delta}C_t^{\rho}=\beta E_t^m\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\rho}\frac{P_t}{P_{t+1}}\right\} \tag{6}$$

The left hand side of this expression is not log linear, and so is linearised around the non-stochastic steady state. Using the first order condition with respect to domestic nominal bonds, in the steady state $1-\chi\left(\frac{\overline{M}}{\overline{P}}\right)^{-\delta}\overline{C}^{\rho}=\frac{1}{1+\overline{i}}$ where an overbar indicates the value of a variable in equilibrium. The log linearisation of the left hand side of equation (6) is then

$$\log\left(1-\chi\left(\frac{M_t}{P_t}\right)^{-\delta}C_t^{\rho}\right)=\overline{i}\delta\left(m_t-p_t\right)-\overline{i}\rho c_t-\overline{i}\log\left(\frac{\chi}{\overline{i}}\left(1+\overline{i}\right)^{\frac{1+\overline{i}}{\overline{i}}}\right) \tag{7}$$

Equating this to the log of the right hand side of (6) gives

$$\begin{aligned}&\delta\left(m_t-p_t\right)-\rho c_t-\log\left(\frac{\chi}{\overline{i}}\left(1+\overline{i}\right)^{\frac{1+\overline{i}}{\overline{i}}}\right) \\ =\;&\frac{1}{\overline{i}}\log\beta-\frac{1}{2\overline{i}}E_t^m\left\{p_{t+1}-p_t\right\}-\frac{\rho}{\overline{i}}E_t\left\{c_{t+1}-c_t\right\}+\frac{1}{2\overline{i}}var\left(p_{t+1}-\rho c_{t+1}\right)\end{aligned} \tag{8}$$

Using conditions already established it is easily shown that $E_t^m\left\{p_{t+1}-p_t\right\}=E_t^m\left\{s_{t+1}-s_t\right\}$ and $E_t^m\left\{c_{t+1}-c_t\right\}=0$. Substituting these into equation (8) delivers

$$\begin{aligned}&\overline{i}\delta\left(m_t-s_t\right)-\overline{i}\left(\delta\gamma+\rho\left(1-\gamma\right)\right)p_{Xt}^*-\overline{i}\left(\delta-\rho\right)\left(1-\gamma\right)p_{Tt}^*+\overline{i}\gamma\left(\delta-\rho\right)y_{Nt} \\ =\;&-E_t^m\left\{s_{t+1}-s_t\right\}-K_2\end{aligned} \tag{9}$$

where K_2 is a constant.[14] Substituting forward, and imposing that there are no bubbles, the exchange rate is derived as

$$\begin{aligned}s_t \;=\;& E_t^m\left\{\frac{\overline{i}\delta}{1+\overline{i}\delta}\sum_{j=0}^{\infty}\left(\frac{1}{1+\overline{i}\delta}\right)^j m_{t+j}\right\} \\ &-\left[\gamma\left(p_{Xt}^*+y_{Nt}\right)+\left(1-\gamma\right)p_{Tt}^*\right]-\frac{\rho\left(1-\gamma\right)}{\delta}\left(p_{Xt}^*-p_{Tt}^*\right)-\frac{\gamma\rho}{\delta}y_{Nt}+K_2\end{aligned} \tag{10}$$

This equation highlights two channels through which the world price of the exported good can affect the nominal exchange rate. The real wealth effect is represented by the third term.

[14] $K_2=\log\beta+\frac{1}{2}var\left(p+\rho c\right)+\overline{i}\delta\gamma\log\left(\frac{\gamma}{1-\gamma}\right)-\overline{i}\rho\log\left(\gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)+\overline{i}\left(\frac{\chi}{\overline{i}}\left(1+\overline{i}\right)^{\frac{1+\overline{i}}{\overline{i}}}\right)$

An increase in the price of the exported good increases the net present value of expected wealth and so the consumption of the imported good. In order to obtain internal balance the domestic relative price of the imported good must fall, so that by the LOP the domestic currency appreciates. This effect is greater the more open is the economy, $1-\gamma$, and the larger the intertemporal elasticity of substitution of consumption, ρ, relative to the elasticity of money demand, δ.

The second channel occurs through the impact on the general price level and so the real money supply. The second term in equation (10) is the domestic price level, measured in units of the foreign currency. An increase in the price of the exported good increases the domestic price level in foreign currency terms. To restore real money demand equilibrium the exchange rate must appreciate.

The exchange rate depends on the future stream of the domestic money supply, which could potentially respond to the exogenous variables. The exchange rate also appreciates in the output of the non-traded sector both through the relative price change from the substitution effect, and the impact on the aggregate price level.

The possibility for sectoral switch of labour is obscured in this model by the exogenous supplies of the exported and non-traded goods. If output were to respond to relative price changes – through labour switching between the sectors – then the effect on the exchange rate of the world price of the exported good would be reinforced. An increase in its price would lead to an increase in the production of the exported good, and a contraction in output of the non-traded good. The relative price of the traded good would fall even further in the home country resulting in a larger appreciation of the exchange rate.

Sticky prices could weaken the commodity price-exchange rate link by reducing the margin on which prices can adjust to attain internal balance. Similarly, more extensive financial markets which lessen the exposure of domestic agents to the world commodity price could weaken the commodity currency relationship. In practice, neither of these are likely to be strong enough to completely break the correlation of the exchange rate with commodity prices.

4 Explanations of the puzzle

Section 3 showed that in addition to the commodity price, the exchange rate is also determined by the domestic money supply, the non-traded output and the price of imported goods. While there is no futures market for the price of imported goods, given commodity futures prices display little bias it would seem unlikely that the price of the imported good causes the bias in exchange rate forwards. Similarly, there is no evidence of significant expectation errors or peso-type problems with non-traded output. Rather, the behaviour of money seems to be the more likely cause for the bias in exchange rate forwards despite the absence of such bias in commodity futures. Given this, the small open economy model was developed to provide the maximum flexibility to the behaviour of, and expectations about, the money supply. The model of the exchange rate assumed non-monetary exchange rate determinants are random walks, simplifying the model to focus attention on monetary factors. This obscures the fact that a component of the exchange rate fundamental value is predictable and such predictions are virtually unbiased. In reality the factor inducing negative covariance between the interest differential and realised exchange rate return must be even larger to account for the positive covariance that commodity price changes would induce.

This section uses the small open economy model to consider three leading explanations of the forward bias of exchange rates that rely on the money stock and so could be consistent with the lack of bias in commodity futures. Section 4.1 demonstrates how systematically biased expectations about monetary policy could cause the observed relationships using the model of Gourinchas and Tornell (2002). The possibility that peso problems are to blame is considered in Section 4.2 while Section 4.3 considers whether the objective of monetary policy could drive the result.

4.1 Systematic expectation errors

Following Froot and Thaler (1990), Gourinchas and Tornell (2002) attribute the observed forward bias to agents' perception that interest rate shocks contain a large transitory component when in reality they are highly persistent. The initial under estimation, and gradual learning, of the change in the future fundamentals of the exchange rate results in a gradual response to an interest rate shock. The gradual appreciation (depreciation) of the exchange rate as agents learn about the persistence of a positive (negative) interest rate shock results in a downward bias in the 'Fama coefficient'. Gourinchas and Tornell justify their expectations theory by demonstrating that expectations of interest rates from surveys do not reflect the true persistence in interest rates.

Here Gourinchas and Tornell's expectations misperception is adapted to apply directly to money in order to fit in with the model of the exchange rate developed in Section 3. Since Section 1 demonstrated that there is very little bias in commodity futures, in order to focus on explanations of the bias in exchange rate forwards, it is assumed that agents have rational expectations with regard to foreign prices. This is a strong assumption but is shown to be consistent with the behaviour of the exchange rate. Implicit in this assumption is that foreign prices are determined in such a way that any misperception about foreign money shocks does not carry over to foreign prices.

So long as agents understand the relationship between fundamentals and the money supply, changes in the money supply due to fundamentals can not introduce the bias in forward exchange rates. To simplify notation then, the money supply is assumed to be unrelated to economic fundamentals. The money supply is proposed to follow a persistent process

$$\begin{aligned} m_t &= \lambda m_{t-1} + \varepsilon_t && (11) \\ &= \sum_{j=0}^{\infty} \lambda^j \varepsilon_{t-j} && (12) \end{aligned}$$

where the constant term is normalised to zero. Agents believe that there is a larger transitory component to shocks than in reality. Specifically they believe that the money process is

$$\begin{aligned} m_t &= z_t + v_t && (13) \\ z_t &= \lambda z_{t-1} + \varepsilon_t \end{aligned}$$

where their misperception is governed by the value of $\sigma_v^2 = var\,(v)$. If $\sigma_v^2 = 0$ then expectations reflect the true money process. As described in Gourinchas and Tornell, agents will use a Kalman filter to produce forecasts of money so that their expectations are given by

$$E_t^m m_{t+1} = k\lambda m_t + (1-k)\,\lambda E_{t-1}^m m_t \qquad (14)$$

where $k \in [0,1]$ is a constant that depends on the relative sizes of the variances σ_v^2 and σ_ε^2 and other model parameters. The constant k governs the degree of bias in expectations, the larger is k the less is the bias, with rational expectations corresponding to $k = 1$. Expression (14) is derived in Appendix A. From equation (14) it follows that $E_t^m m_{t+j} = \lambda^{j-1} E_t^m m_{t+1}$. This can then be used to simplify the exchange rate given by equation (10) to

$$\begin{aligned} s_t &= \frac{\bar{i}\delta}{1+\bar{i}\delta}\left(m_t + \frac{1}{1+\bar{i}\delta-\lambda} E_t^m m_{t+1}\right) \\ &\quad - \left[\gamma p_{Xt}^* + (1-\gamma)\, p_{Tt}^*\right] - \frac{\rho\,(1-\gamma)}{\delta}\left(p_{Xt}^* - p_{Tt}^*\right) - \frac{\gamma\,(\delta+\rho)}{\delta} y_{Nt} + K_2 && (15) \end{aligned}$$

As shown in Section 1 the bias in β, the 'Fama coefficient', can be calculated from the covariance of the market expectation error with the interest differential. As a first step to calculating these expressions it is useful to first express the expected money stock in terms of past shocks. The expected value of future money will be a discounted sum of past values of the money supply, and so a discounted sum of past shocks. By recursively substituting equation (14) and then substituting in equation (12) the expected money supply is given by

$$\begin{aligned} E_t^m m_{t+1} &= k\lambda \sum_{j=0}^{\infty} \left\{ [k(1-\lambda)]^j \sum_{i=0}^{\infty} \lambda^i \varepsilon_{t-j-i} \right\} \\ &= \sum_{j=0}^{\infty} \lambda^{j+1} \left[1-(1-k)^{j+1}\right] \varepsilon_{t-j} \qquad (16) \end{aligned}$$

The misperception as to the persistence of shocks, $k<1$, leads agents to underestimate the impact of shocks on the future money supply. Variables other than money are expected to follow random walks so the expected depreciation depends only on the current, and next period's expected, money supplies. Both of these variables are a weighted sum of the history of shocks to money. The depreciation expected by the market can then be calculated as

$$\begin{aligned} E_t^m \{s_{t+1}-s_t\} &= \frac{\bar{i}\delta}{1+\bar{i}\delta}\left[\left(E_t^m m_{t+1} + \frac{\lambda E_t^m m_{t+1}}{1+\bar{i}\delta-\lambda}\right)\right. \qquad (17) \\ &\quad \left. -\left(m_t + \frac{1}{1+\bar{i}\delta-\lambda} E_t^m m_{t+1}\right)\right] \\ &= \frac{\bar{i}\delta}{1+\bar{i}\delta}\left[\frac{\bar{i}\delta}{1+\bar{i}\delta-\lambda}\sum_{j=0}^{\infty}\lambda^{j+1}\left[1-(1-k)^{j+1}\right]\varepsilon_{t-j} - \sum_{j=0}^{\infty}\lambda^j\varepsilon_{t-j}\right] \qquad (18) \\ &= \frac{-\bar{i}\delta(1-\lambda)}{1+\bar{i}\delta-\lambda}\sum_{j=0}^{\infty}\lambda^j\varepsilon_{t-j} \\ &\quad -\frac{\left(\bar{i}\delta\right)^2\lambda(1-k)}{\left(1+\bar{i}\delta\right)\left(1+\bar{i}\delta-\lambda\right)}\sum_{j=0}^{\infty}[\lambda(1-k)]^j\varepsilon_{t-j} \qquad (19) \end{aligned}$$

The first term in equation (19) is the expected depreciation in a world with rational expectations and results from the persistence of monetary shocks. The second term introduces the impact of systematic expectation errors. The expression for the expected depreciation can then be substituted into the UIP expression derived from the first order conditions with respect to holdings of domestic and foreign nominal bonds.

$$
\begin{aligned}
i_t - i_t^* &= E_t^m \{s_{t+1} - s_t\} + \frac{1}{2} var(s) + cov(s, \rho c + p) && (20)\\
&= \frac{-\bar{i}\delta (1-\lambda)}{1+\bar{i}\delta - \lambda} \sum_{j=0}^{\infty} \lambda^j \varepsilon_{t-j} - \frac{\left(\bar{i}\delta\right)^2 \lambda (1-k)}{\left(1+\bar{i}\delta\right)\left(1+\bar{i}\delta-\lambda\right)} \sum_{j=0}^{\infty} \left[\lambda (1-k)\right]^j \varepsilon_{t-j} \\
&\quad + \frac{1}{2} var(s) + cov(s, \rho c + p) && (21)
\end{aligned}
$$

When expectations about the money supply aren't rational, market expectations of next periods' exchange rate will be systematically biased. The market expectation error, the difference between the rational expectation and the market expectation of the exchange rate, is calculated as

$$
\begin{aligned}
\zeta_t &= E_t s_{t+1} - E_t^m s_{t+1} \\
&= \frac{\bar{i}\delta}{1+\bar{i}\delta} \left[\lambda m_t + \frac{\lambda^2 k m_t + \lambda (1-k) E_t^m m_{t+1}}{1+\bar{i}\delta - \lambda} - \frac{1+\bar{i}\delta}{1+\bar{i}\delta-\lambda} E_t^m m_{t+1} \right] \\
&= \frac{\bar{i}\delta \left(1+\bar{i}\delta - \lambda (1-k)\right)}{\left(1+\bar{i}\delta\right)\left(1+\bar{i}\delta-\lambda\right)} \left[\lambda m_t - E_t^m m_{t+1}\right] \\
&= \frac{\bar{i}\delta \left(1+\bar{i}\delta - \lambda (1-k)\right)}{\left(1+\bar{i}\delta\right)\left(1+\bar{i}\delta-\lambda\right)} \left[\sum_{j=0}^{\infty} \left[\lambda (1-k)\right]^{j+1} \varepsilon_{t-j} \right] && (22)
\end{aligned}
$$

The second line follows from the definition of market expectations in equation (14). The market expectation error arises because agents place too little weight on more recent shocks as they believe these are in part transitory. The magnitude of the error declines as the degree of misperception, $1-k$, declines.

The 'Fama coefficient' is the coefficient from the regression of the realised change in the exchange rate on the interest differential. Using the fact that the rational expectations error is by definition uncorrelated with time t variables β can be expressed as

$$
\begin{aligned}
\text{plim}\left(\hat{\beta}\right) &= \frac{cov\left(i_t - i_t^*, \Delta s_{t+1}\right)}{var\left(i_t - i_t^*\right)} \\
&= \frac{cov\left(i_t - i_t^*, E_t^m \Delta s_{t+1}\right)}{var\left(i_t - i_t^*\right)} + \frac{cov\left(i_t - i_t^*, E_t s_{t+1} - E_t^m s_{t+1}\right)}{var\left(i_t - i_t^*\right)}
\end{aligned}
$$

By equation (20) the first term is unity. Substituting equations (21) and (22) into the second term, the probability limit of the coefficient β is given by

$$\operatorname{plim}\left(\hat{\beta}\right)=1-\left[\frac{\frac{\lambda(1-k)\left(1+\bar{i}\delta-\lambda(1-k)\right)\left(1+\bar{i}\delta\right)(1-\lambda)}{1-\lambda^2(1-k)}+\frac{\bar{i}\delta\left(1+\bar{i}\delta-\lambda(1-k)\right)\lambda^2(1-k)^2}{1-\lambda^2(1-k)^2}}{\frac{\left(1+\bar{i}\delta\right)^2(1-\lambda)^2}{1-\lambda^2}-\frac{2\bar{i}\delta\lambda(1-\lambda)(1-k)}{1-\lambda^2(1-k)}+\frac{\left(\bar{i}\delta\right)^2\lambda^2(1-k)^2}{1-\lambda^2(1-k)^2}}\right] \tag{23}$$

Since $k,\lambda\in[0,1]$ the term in brackets will always be greater that or equal to zero and so $\operatorname{plim}\left(\hat{\beta}\right)$ will be less than or equal to unity. Equation (23) is somewhat difficult to interpret, and so the value of β, as a function of λ and k, is shown in Figure 3. This is drawn for a Taylor expansion value of $\bar{i}\delta=0.05$ but the value of β is insensitive to this choice, except for large values of λ.[15] Beta is biased downward because agents under-estimate the persistence of monetary shocks and so the exchange rate initially under responds to shocks. The revision to expectations as agents learn the persistence of the shock results in a tendency for the exchange rate to continue its initial trajectory. Figure 3 confirms the intuition of the model that β will be more biased, that is larger negative values, for highly persistent monetary shocks (large λ) and greater bias of expectations (smaller k). For λ very close to unity (larger than the 0.99 shown on the graph) the biased expectations can result in a large negative value for β (even double digit values), though as noted before, for large λ, β is sensitive to the value of $\bar{i}\delta$.

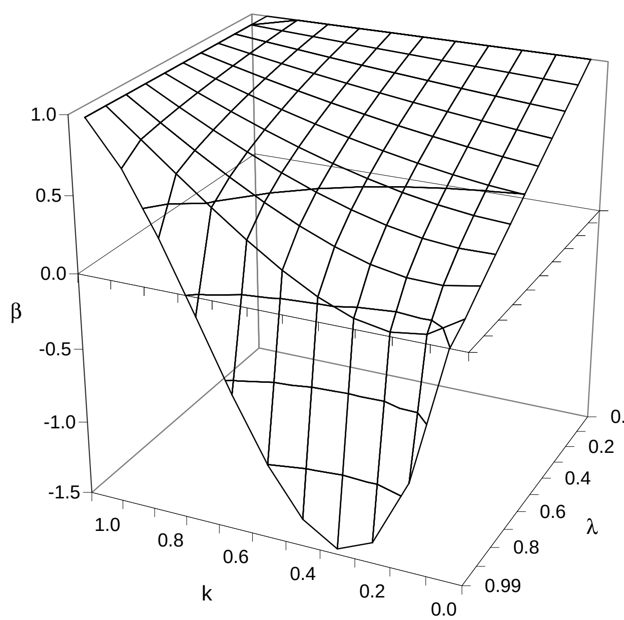
Another result of the biased expectations is that the exchange rate can demonstrate delayed overshooting, of the form found empirically by Eichenbaum and Evans (1995). The impulse response function of the exchange rate to a money shock is

$$s_{t+j}=\frac{-\bar{i}\delta\,(1-\lambda)}{1+\bar{i}\delta-\lambda}\lambda^j\varepsilon_{t-j}-\frac{\left(\bar{i}\delta\right)^2\lambda\,(1-k)}{\left(1+\bar{i}\delta\right)\left(1+\bar{i}\delta-\lambda\right)}\left[\lambda\,(1-k)\right]^j\varepsilon_{t-j} \tag{24}$$

Again the first term represents the rational expectations response of the exchange rate to monetary shocks, and the second the impact of biased expectations. If the second decays slowly, because k is small, then it is possible for the response of the exchange rate to a monetary shock to be larger after several periods than it is immediately. Figure 4 shows the range of parameters for which there is overshooting of various lengths.

[15]This value is selected for the average monthly interest rate is $\bar{i}=0.01$ and the elasticity of money demand $\delta=5$.

Figure 3: Systematic expectation errors: the 'Fama coefficient' from a regression of the exchange rate return on the forward premium

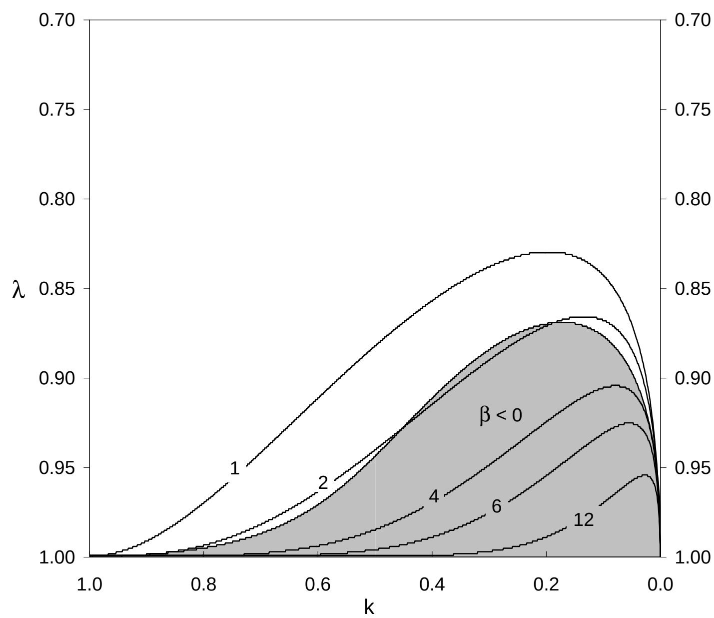


4.1.1 Testing the model

The persistence of monetary shocks, λ, can be estimated using a Kalman Filter. The results of this estimation, in Table 3, show that the money supply is highly persistent, λ is essentially unity, and the variance of transitory shocks is found to be zero, consistent with the money process in equation (12).

The markets' expectation for monetary policy cannot be measured directly for Australia and so it is not possible to estimate k, as Gourinchas and Tornell can do using survey based interest rate expectations. An alternative approach is to infer the values of k that are consistent with the exchange rate overshooting implied by the model. A VAR using monthly data from July 1984 to December 2000 is estimated based on the small open economy model developed in

Figure 4: Number of periods of overshooting and sign of beta for various parameter combinations.



Section 3.[16] A VAR estimation is used here to facilitate comparison with previous research such as Eichengreen and Evans (1995).[17] Import (traded goods) prices are measured by the price of exports of industrialised countries (since monthly import prices are not available for Australia) and the output in the non-traded sector is proxied by the unemployment rate since this is available monthly and a large portion of non-traded output comes from the labour-intensive service sector. The data are described in Appendix B. The VAR ordering reflects the small open economy assumption that foreign prices are exogenous, but the exchange rate depends on these prices, non-traded output, and the money supply. The money supply is allowed to depend on the exogenous variables but because of the causal ordering, not the contemporaneous exchange rate. The ordering used is {commodity prices, import prices, unemployment, money supply,

[16]The Australian dollar floated in December 1983. The sample is constrained to start in January 1984 by the availability of the commodity price data. The VAR was tested with up to 6 lags resulting in a start date of July 1984.

[17]The results for the VAR are consistent with the single equation approach used in Kearns (2002) to estimate commodity currency models.

Table 3: Estimates from Kalman Filter of money supply.

λ	$\overline{m}$	σ_ε	σ_v
0.9999	0.7415	0.0067	0.0000
(0.0002)	(0.4226)	(0.0003)	(0.0005)

Standard errors in brackets.
The estimated model is:
$m_t = \overline{m} + z_t + v_t$
$z_t = \lambda z_{t-1} + \varepsilon_t$

forward premium, exchange rate}. The BIC would select the lag length of the VAR to be one, while the AIC selects a lag length of two. The results using a lag length of two are presented here since they potentially provide for a richer model, but these only differ marginally to those using a lag length of one. The impulse response functions for the exchange rate from shocks to the money supply and commodity price are shown in Figure 5.

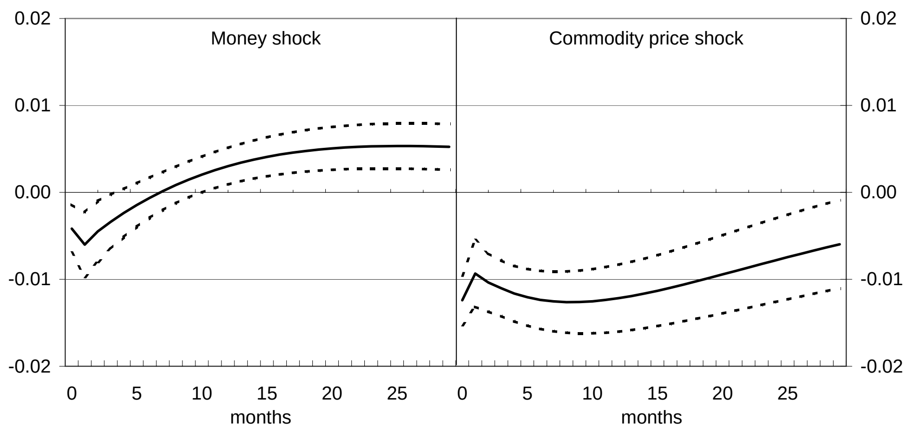
The Australian dollar exchange rate displays delayed overshooting in response to a monetary shock, consistent with the model of biased expectations about money shocks. The maximum response of the exchange rate comes after 2 years. For the value of λ estimated with the Kalman filter, 0.9999, this implies a value of k of approximately 0.3, suggesting agents greatly underestimate the persistence of monetary shocks. This value of k is in the middle of those estimated for five countries by Gourinchas and Tornell.[18] By contrast, consistent with the model assumption that agents don't make systematic expectation errors with regard to commodity prices, the exchange rate response to a commodity price shock is immediate as shown by the second panel of Figure 5. The high persistence of monetary shocks given by the estimated λ, and the implied value of k of 0.3, would generate a large negative β of -5.8, substantially larger in absolute terms than the estimate for the Australian dollar.[19] As noted earlier, the value of β is sensitive to value of $\bar{i}\delta$ around which the Taylor expansion is made, and so the precise magnitude of β implied by the model must be treated with some caution.

Overall the model of biased expectations with regard to monetary shocks, but not commodity price shocks, appears to be consistent with the data. The predictions these assumptions have for a rapid response of the exchange rate to commodity price shocks, but delayed response to monetary shocks, are borne out by the VAR based on the small open economy model of the exchange rate.

[18]Gourinchas and Tornell's sample is: Japan, Germany, France, UK and Canada.

[19]This value is similar in magnitude to the estimate for the more recent period, as shown in Figure 2, although the standard errors for estimates based on such short samples are wide.

Figure 5: Response of exchange rate to one-standard-deviation money supply and commodity price shocks

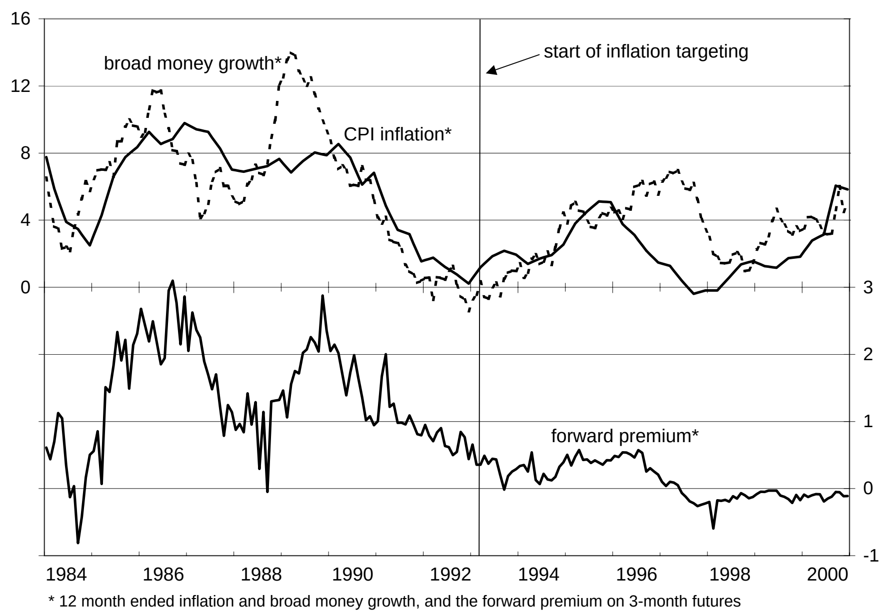


4.2 Peso Problems

The negative correlation of ex post expectation errors and the forward premium, which is implicit in the finding of $\beta < 0$, need not be caused by agents having systematically biased expectations, but rather may be a spurious small sample result. The expectations formed ex post by the researcher based on the whole sample could appear systematically biased if agents had to learn about a regime change, or a particular event occurred with an unrepresentative frequency. Notably if the learning was about a domestic monetary event, or the unrepresentative regime change concerned monetary policy, then exchange rate forwards would be biased while commodity futures would not.

The learning hypothesis, which is developed by Lewis (1988, 1995), initially appears to show promise as an explanation for the relative bias in exchange rates and commodity prices for Australia. In the early 1990s the monetary regime in Australia seemingly changed, as money growth and inflation declined. Further, this 'regime change' was not 'announced' until the first reference to inflation targeting by the central bank in 1993. If agents learned about this change slowly then their expectations over this period would, ex post, be recognised to have been systematically biased. But as Figure 6 shows, agents appear to have displayed rapid learning as the forward premium declined at the same time as the regime change indicated by the fall in money growth. Evans (1996) notes that if agents learn about a change rapidly, or the period of learning is a small portion of the sample, which is also the case in this sample, then the learning process will not cause the substantial ex post expectation errors that are needed to deliver the forward bias of exchange rates. Further, as Figure 2 showed, the β from the exchange rate forward premium regression is negative even for subsamples excluding this period of 'learning'. This evidence implies the apparent change in monetary regime in the sample is not the explanation for the disparate relative degrees of bias.

Figure 6: Monetary policy and the Australian dollar forward premium



The alternative peso problem explanation relates to anticipated regime changes. Evans and Lewis (1995) and Kaminsky (1993) demonstrate that exchange rates in the 1970s and 1980s can be classified as having followed a Markov switching process of appreciating and depreciating regimes. They suggest that agents' rational expectations about these regime changes leads to seemingly systematic expectation errors that display a spurious small sample correlation with the forward premium. The absence of such a spurious small sample correlation in commodity prices indicates that for this small sample correlation to be the cause of the bias of exchange rate forwards it would have to come from some other factor, likely domestic monetary policy. But the consistency of the finding that exchange rate forwards are biased, both across countries and time, strongly suggests that expected changes in one country's domestic monetary policy is unlikely to be the explanation of the puzzle.[20] While peso problems may contribute to particular episodes of bias, the complete answer to the puzzle lies elsewhere.

4.3 Monetary policy

The observed forward bias of exchange rates, but near absence of bias for commodity prices, could potentially result from the practice of domestic monetary policy. McCallum (1994) proposed that monetary policy aimed at smoothing changes in exchange rates can lead to the observed bias in forward exchange rates.[21] This section applies to the small open economy model from Section 3 a monetary rule akin to McCallum's, in which the central bank is assumed to dislike both sharp changes in policy and large changes in the exchange rate. The monetary policy rule, which summarises these goals and a potential dependence on the change in commodity prices, is given by

$$\Delta m_t = A\Delta m_{t-1} - B\Delta s_t + C\Delta p^*_{Xt} + \zeta_t \tag{25}$$

The persistence of monetary policy is governed by the parameter A. The use of monetary policy to counteract changes in the exchange rate is shown by the negative coefficient on exchange rate returns, $-B$.

While expectations are assumed to be rational, two possible deviations from the expected depreciation implied by equation (10) are considered. McCallum's framework requires that there are random deviations from UIP in order for monetary policy to cause the forward bias, although he does not elaborate on the rationale for this strong assumption. These shocks are assumed

[20] Estimation of the Markov switching model of exchange rates showed that when the sample is extended using more recent data the pattern of long swings is no longer present. While this suggests there are more than two regimes, it also diminishes the likelihood that agents rationally expect such changes in regime.

[21] Meredith and Ma (2002) extend McCallum's analysis to show that the central bank's objective with regard to changes in the exchange rate can result from policy objectives with regard to output and inflation.

to be persistent in order to generate the observed persistence in the forward premium. This deviation from the general equilibrium framework is also taken here in order to demonstrate the impact of monetary policy on the exchange rate. The setting used here also allows for the possibility that part of the one period change in commodity prices is predictable, as Section 1 showed is in fact the case.

The expected depreciation then is given by

$$E_t \Delta s_{t+1} = \left\{ \frac{\bar{i}\delta}{1+\bar{i}\delta} \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta\bar{i}} \right)^j E_t \Delta m_{t+1+j} \right\} + \varphi q_t - \eta_t \tag{26}$$

where $\eta_t = D\eta_{t-1} + u_t$ is the persistent deviation from UIP, and φq_t is the discounted sum of expected commodity price changes. For simplicity it is assumed that there is no additional information regarding commodity price changes over a horizon of greater than one period so that the discounted sum of expected commodity price changes is a multiple of the expected one period change $E_t \Delta p^*_{Xt+1} = q_t = \Delta p^*_{Xt+1} - \xi_{t+1}$.

Solving the model involves conjecturing a solution and then demonstrating it is consistent with the system given by (25) and (26). Changes in the exchange rate and money supply are proposed to be a function of the state variables, ζ_t,η_{t-1}, u_t, q_{t-1} and ξ_t:

$$\Delta m_t = \phi_1 \zeta_t + \phi_2 \eta_{t-1} + \phi_3 u_t + \phi_4 q_{t-1} + \phi_5 \xi_t \tag{27}$$

$$\Delta s_t = \phi_6 \zeta_t + \phi_7 \eta_{t-1} + \phi_8 u_t + \phi_9 q_{t-1} + \phi_{10} \xi_t \tag{28}$$

Substituting equations (27) and (28), and their expectations, into equations (25) and (26), and then equating coefficients on the state variables, the expressions for the changes in the money supply and exchange rate are found to be

$$\Delta m_t = \frac{B\theta}{BD\bar{i}\delta - (A-D)\theta} (D\eta_{t-1} + u_t) \tag{29}$$

$$\Delta s_t = \frac{1}{B}\zeta_t + \frac{\theta}{BD\bar{i}\delta - (A-D)\theta} \left[(A-D)\eta_{t-1} - u_t \right] + \varphi (q_{t-1} + \xi_t) \tag{30}$$

where $\theta = 1 + \bar{i}\delta - D$. The reduced form solution for changes in the money supply is a function only of the deviations from UIP, notably it does not depend on commodity prices. Changes in the exchange rate by contrast reflect the shocks to UIP, the shocks to monetary policy and commodity price changes. The 'Fama coefficient' will depend on the covariance between the interest differential and the realised change in the exchange rate and is calculated as

$$\begin{aligned} \operatorname{plim}\left(\hat{\beta}\right) &= \frac{cov\left(i_t - i_t^*, \Delta s_{t+1}\right)}{var\left(i_t - i_t^*\right)} && (31) \\ &= \frac{cov\left(E_t \Delta s_{t+1} + \eta_t, \Delta s_{t+1}\right)}{var\left(E_t \Delta s_{t+1} + \eta_t\right)} && (32) \\ &= \frac{\left(1 + \bar{i}\delta - D\right)\left(A - D\right) + \Theta}{DB\bar{i}\delta + \Theta} && (33) \end{aligned}$$

where $\Theta = \frac{\varphi^2\left[BD\bar{i}\delta - (A-D)\left(1+\bar{i}\delta - D\right)\right]^2}{BD\bar{i}\delta}\frac{\sigma_q^2}{\sigma_\eta^2}$. If there is no variability in the predictable change in commodity prices, so $\sigma_q^2 = 0$, then this reduces to $\operatorname{plim}\left(\hat{\beta}\right) = \frac{\left(1+\bar{i}\delta - D\right)}{\bar{i}\delta}\frac{(A-D)}{B}$. Assuming the UIP shocks are not explosive, $D < 1$, the first term is positive and so β will be negative if the shocks to UIP demonstrates greater persistence than monetary policy, $D > A$.[22] Because of the number of parameters and the endogenous relationship between policy and the exchange rate the model is not identified and so the parameter values can't be estimated. Instead, the consistency of this framework with empirical observations can be gauged by calibrations. For example, if $A = 0.85$, $D = 0.9$ and $B = 0.2$ then the implied value for β is -0.75, close to the empirical estimate.[23] But depending on parameter values, β can take on any value, either positive or negative.

If there are predictable changes in commodity prices $\sigma_q^2 > 0$, and so $\Theta > 0$, the range of parameters for which β will be heavily downward biased is greatly reduced. The value of β will depend on the ratio of the variances of predictable commodity price changes to the UIP shocks, σ_q^2/σ_η^2. While an estimate of the volatility of the commodity forward premium is given in Table 2, because the shocks to UIP are unobserved, and McCallum does not provide a foundation for their existence, their volatility cannot be measured. In the absence of an alternative assumption, suppose these variances are equal. Calibrations then indicate β cannot be negative.[24] For β to be as low as the -0.75 obtained above, the variance of the shocks to UIP must be of the order

[22]This is the opposite result to McCallum, who finds that the UIP shocks must demonstrate greater persistence than the policy function, $D > A$, in order for $\beta < 0$. The difference arises because in the current set up the interest rate is a function of the discounted stream of expected future money growth rather than being the policy variable as in McCallum. In this setting the interest rate moves in the same direction as the change in money, which is being used to offset the UIP shock. Despite this technical difference, the framework still demonstrates McCallum's point that monetary policy can induce negative covariance between interest rates and the exchange rate.

[23]The calibration uses $\varphi = -0.7$, based on estimates in Kearns (2002), and $\bar{i}\delta = 0.05$, as used in Section 4.1.

[24]The minimum occurs for $A = D \to 0$ implying that $\beta \to 0$.

of six times that of the predictable commodity price changes.[25]

This simple model demonstrates that the practice of monetary policy could induce bias in exchange rate forwards despite its absence in commodity futures. But the model requires strong assumptions with little justification, such as the shocks to UIP. Further, calibrations demonstrate that if the unbiased predictability of commodity prices is taken into consideration, the variance of these shocks to UIP must be very large, substantially larger than the variance of the predictable component of commodity prices.

5 Conclusion

This paper observed that a portfolio of commodity futures does not exhibit the same bias as do exchange rate forwards. This may be of only passing interest if it were not for the fact that some exchange rates have a close relationship with commodity prices. The nature of this relationship is developed in the small open economy model. In this model, in addition to being a function of the price of commodity exports, the exchange rate is also a determined by the price of imports, non-traded output and the domestic money supply. The behaviour of, and expectations about, the money supply appears to be the most likely cause of the bias in exchange rate forwards despite the absence of bias in commodity futures.

Three explanations for the exchange rate forward bias that depend on the domestic money supply were considered. Explanations in the 'peso problem' class appear unlikely to be the cause of the puzzle. Learning about an in-sample regime change was rapid, while the existence of forward bias across exchange rates and sample periods suggests it is not sample specific, as would likely be the case with a regime change. Monetary policy aimed at smoothing the exchange rate could bias the estimate of beta downward, but calibrations suggest that if expectations about commodity futures are unbiased, the monetary policy response is unlikely to be able to cause the full extent of the bias.

The explanation that depends on systematic bias in expectations about the monetary process appears consistent with the empirical facts, in particular the rapid response of exchange rates to commodity shocks but delayed response to monetary shocks. However this explanation requires a very strong assumption, that agents are capable of forming rational expectations with regard to commodity prices, but not with regard to the money process. Perhaps the slow adaptation

[25] This value of beta occurs for a range of values of the parameters A, B and D. One such combination is $A = 0.58$, $D = 0.99$, $B = 0.20$.

of expectations to monetary shocks stems from institutional factors or other economic variables, but before this explanation can be treated with any confidence, greater evidence as to the cause of biased monetary expectations must be uncovered.

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Appendix A – Model Solutions

Small open economy model

Consumption

All exogenous variables in the model are log normally distributed. Further, their log levels are random walks – the expected future value is simply the current value.

The three main first order conditions from maximising equation (4) with respect to the budget constraint are

$$\frac{C_{Tt}P_{Tt}}{1-\gamma} = \frac{C_{Nt}P_{Nt}}{\gamma} \tag{34}$$

$$\frac{P_{Tt}}{C_t^{\rho}P_t} = \beta\left(1+r\right)E_t^m\left\{\frac{P_{Tt+1}}{C_{t+1}^{\rho}P_{t+1}}\right\} \tag{35}$$

$$\chi\left(\frac{M_t}{P_t}\right)^{-\delta}C_t^{\rho} = 1-\beta E_t^m\left\{\left(\frac{C_t}{C_{t+1}}\right)^{\rho}\frac{P_t}{P_{t+1}}\right\} \tag{36}$$

All shocks are log normally distributed and so C_T is assumed to be log normally distributed. Since $C_N = Y_N$ is also log normal, this will imply aggregate consumption, C, is also log normal. Domestic prices are also assumed to have log normal distributions. In the solution these assumptions hold. Using the log normality, equation (35) can be expanded as

$$\begin{aligned}
& -\gamma E_t^m\left\{p_{Tt+1}-p_{Tt}\right\}+\rho\gamma E_t^m\left\{c_{Nt+1}-c_{Nt}\right\}+\rho\left(1-\gamma\right)E_t^m\left\{c_{Tt+1}-c_{Tt}\right\} \\
& +\gamma E_t^m\left\{p_{Nt+1}-p_{Nt}\right\} \\
= \; & \frac{1}{2}var\left(p_{Tt+1}-\rho c_{t+1}-p_{t+1}\right)+\ln\left[\beta\left(1+r\right)\right] \\
= \; & K_1
\end{aligned}$$

Substituting in equation (34) this simplifies to equation (5) in the text. Since non-traded consumption must equal non-traded output, which has an exogenously given log random walk distribution, the expectation of traded consumption is given by

$$E_t^m\left\{c_{Tt+1}\right\} = c_{Tt}+\frac{K_1}{\rho\left(1-\gamma\right)+\gamma}$$

Then the expected value of traded good consumption is given by

$$E_t^m \{C_{Ts} P_{Ts}\} = C_{Tt} P_{Tt} \exp(K_3)$$

where $K_3 = \frac{(s-t)K_1}{\rho(1-\gamma)+\gamma} + \frac{(s-t)}{2} var(c_{Tt+1} + p_{Tt+1})$.

The intertemporal budget constraint, that the discounted present values of traded consumption and the exported good must be equal, then implies that

$$\frac{1+r}{1+r-e^{\frac{1}{2}\sigma_X^2}} P_{Xt}^* = \frac{1+r}{1+r-e^{K_3}} C_{Tt} P_{Tt}^*$$

Assuming the parameter values satisfy $K_3 = \frac{1}{2}\sigma_X^2$, ensuring that a steady state solution exists, then tradeables consumption is indeed log normally distributed and follows a random walk governed by

$$c_{Tt} = p_{Xt}^* - p_{Tt}^*$$

Exchange rate

The first order conditions with respect to holdings of the domestic nominal bond and money balances are

$$\frac{1}{C_t^\rho P_t} = \beta(1+i_t) E_t^m \left\{ \frac{1}{C_{t+1}^\rho P_{t+1}} \right\} \tag{37}$$

$$1 - \chi\left(\frac{M_t}{P_t}\right)^{-\delta} C_t^\rho = \beta E_t^m \left\{ \left(\frac{C_t}{C_{t+1}}\right)^\rho \frac{P_t}{P_{t+1}} \right\} \tag{38}$$

Combining these gives

$$\chi\left(\frac{M_t}{P_t}\right)^{-\delta} C_t^\rho = \frac{i_t}{1+i_t} \tag{39}$$

The non-stochastic steady state is defined by a constant rate of growth of money such that equation (39) is constant.

$$\chi\left(\frac{\overline{M}}{\overline{P}}\right)^{-\delta} \overline{C}^\rho = \frac{\overline{i}}{1+\overline{i}} \tag{40}$$

The left hand side of equation (38) can then be log linearised around the non-stochastic steady state, equation (40). Doing so gives equation (7) in the text. The right hand side of (38) is log linear, because all shocks are log normally distributed, and so does not need to be approximated.

Taking the log of the right hand side, and equating it to the log approximation of the left hand side, equation (7), gives equation (8) in the text.

Derivation of biased market expectations

Agents' beliefs of the money process are given by

$$\begin{aligned} m_t &= z_t + v_t \\ z_t &= \lambda z_{t-1} +_t \end{aligned} \tag{41}$$

where $var\,(v) = \sigma_v^2$ and $var\,(\varepsilon) = \sigma_\varepsilon^2$. The Kalman algorithm for updating forecasts, as used by Gourinchas and Tornell and described in Hamilton (1995), is

$$E_t^m z_{t+1} = \lambda E_{t-1}^m z_t + \lambda P \left(P + \sigma_v^2\right)^{-1} \left(m_t - E_{t-1}^m m_t\right) \tag{42}$$

The expectations of money are then given by

$$\begin{aligned} E_t^m m_{t+1} &= E_t^m z_{t+1} \\ &= \lambda E_{t-1}^m z_t + \lambda P \left(P + \sigma_v^2\right)^{-1} \left(m_t - E_{t-1}^m m_t\right) \\ &= \lambda E_{t-1}^m m_t + \lambda k \left(m_t - E_{t-1}^m m_t\right) \\ &= k\lambda m_t + (1-k)\,\lambda E_{t-1}^m m_t \end{aligned}$$

where $k = P\left(P + \sigma_v^2\right)^{-1}$ and P is the mean-squared error of the expectation as given by

$$P_{t+1} = \lambda^2 \left[P_t - P_t^2 \left(P_t + \sigma_v^2\right)^{-1}\right] + \sigma_\varepsilon^2$$

As in Gourinchas and Tornell it is assumed that this estimate of the prediction error variance has converged to a stable value

$$\begin{aligned} P &= \lambda^2 \left[P - P^2 \left(P + \sigma_v^2\right)^{-1}\right] + \sigma_\varepsilon^2 \\ P &= \frac{-\left(\left(1-\lambda^2\right)\theta - 1\right) + \Theta}{2/\sigma_\varepsilon^2} \end{aligned}$$

where $\theta = \sigma_v^2/\sigma_\varepsilon^2$ summarises the agents' belief of the noise-to-signal ratio in the money process, and $\Theta = \left(\left(1-\lambda^2\right) - 1\right) - 4\theta$. Since $P > 0$ the constant k is between zero and one.

Appendix B – Data

Exchange Rate Data

Australian dollar spot and 3-month forward rates for the first business day of each month were obtained from Bloomberg (AUD Curncy, AD3M Curncy). The data are the average of buy and sell rates, however, Bekaert and Hodrick (1993) found this introduces no bias relative to the use of correctly aligned transaction prices. Since futures data often contain occasional measurement error, as shown by Maynard and Phillips (2001), the data were filtered mechanically and manually. For the days used in this study no obvious measurement errors were found in the data.

Commodity Data

The commodity futures indices are constructed using data from three sources. The weights are derived from the export-share based weights used in the Reserve Bank of Australia (RBA) commodity price index. The weights, listed in Table 4 are rescaled to account for the omission of several components for which futures or forwards prices are not available. The commodity price data come from the London metal exchange (LME) and the Futures Industry Association (FIA).

LME forwards

The data from the LME are daily spot and 3-month forward prices for aluminium, copper, nickel, zinc and lead. The FIA data, unlike the LME data, are forwards rather than futures. There is a substantial literature examining the theoretical and empirical differences between forwards and futures. Cox, Ingersoll and Ross (1981), show that forward and futures prices can differ if the interest rate is stochastic. Empirically it is found that there is no discernible difference between the prices of futures and forwards (viz. French (1983), and Cornell and Reinganum (1981)) and so there seems little problem with combining these series. While some of the historical data prior to 1989 had to be converted from British pounds to dollars – using the daily spot and 3-month forward rates – almost all of the LME data are for trades in US dollars. A three year overlap of the LME copper series which traded in pounds was compared with the FIA copper series trading in dollars and found to have almost identical monthly movements, indicating the currency conversion did not introduce significant bias.

Table 4: Composition of Commodity Futures Index.

	Weights: RBA index	Futures indices: Total	Forwards only	Futures only	Contract (Exchange)	weight, period	Correlation with RBA index component
Agricultural							
Wool	11.4						
Beef and Veal	9.0	17.2		22.4			0.79
					Cattle - Feeder (CME)		0.72
					Cattle - Live (CME)		0.76
Wheat	10.8	20.5		26.7			0.92
					Wheat (KCBT)		0.94
					Wheat - White (MINN)		0.91
					Wheat (CBT)		0.86
Barley	1.8	3.4		4.5			0.88
					Corn (CBT)	0.50, 11/83 on	0.82
					Wheat (index*)	0.50, 11/83 on	0.77
Rice	0.7	1.4		1.8			0.39
					Rice (CBT)	1.00, 12/86 on	0.39
					Corn (CBT)	0.50, pre 12/86	0.22
					Wheat (index*)	0.50, pre 12/86	0.27
Sugar	4.3	8.3		10.8	Sugar No.11 (CSCE)		0.96
Cotton	1.9	3.7		4.8	Cotton No. 2 (NYCE)		0.85
Mineral							
Gold	11.7	22.3		29.0	Gold (NYNEX)		0.99
Iron Ore	10.3						
Aluminium	6.4	12.2	52.6		Aluminum (LME)		0.98
Lead	1.3	2.4	10.5				0.89
					Lead (LME)	1.00, 1/89 on	0.98
					Aluminum (LME)	0.33, pre 1/89	0.54
					Copper (LME)	0.33, pre 1/89	0.70
					Zinc (LME)	0.33, pre 1/89	0.56
Copper	2.0	3.8	16.4				0.99
					Copper (LME)	1.00, pre 11/94	0.98
					Copper (NYNEX)	0.50, 11/94 on	0.98
					Copper (LME)	0.50, 11/94 on	0.98
Zinc	1.5	2.8	12.0				0.98
					Zinc (LME)	1.00, 9/84 on	0.98
					Aluminum (LME)	0.50, pre 9/84	0.53
					Copper (LME)	0.50, pre 9/84	0.68
Nickel	1.0	2.0	8.4				0.69
					Nickel (LME)	1.00, 1/89 on & pre 9/86	0.99
					Aluminum (LME)	0.33, 9/86 - 1/89	0.76
					Copper (LME)	0.33, 9/86 - 1/89	0.68
					Zinc (LME)	0.33, 9/86 - 1/89	0.73
Energy							
LNG	0.9						
Coking Coal	16.6						
Steaming Coal	8.3						
Proportion of RBA index	100.0	52.6	12.2	40.4			

* index – indicates the use of an index formed from the three Wheat contacts

FIA futures

The FIA data come from numerous exchanges, as shown in Table 4, all in the USA and trading in US dollars. Cash prices for commodities can be for a different grade commodity, in a different location, or include discounts, and as such they are not necessarily a good measure of the spot price. For this reason, studies using commodity futures data frequently use expiring contracts as the spot price. Most commodity contracts are deliverable in the first 3 weeks of the contract month. Following Fama and French (1987) the price on the first day of the delivery month, when the contract can be delivered and so the contract price should have converged to the implicit spot price, but the contract is still liquid, is used as the spot price.

Constructing observations for missing months For most commodities there are not contracts expiring in each month. In order to construct the commodity index it is necessary to have monthly observations for each series. Following Pindyck (1993) log linear extrapolations of longer horizon futures are used to construct a spot price, or 3-month futures price, in months in which there is no observation. For example, if there is no 3-month futures contract in a given month, but there are 4- and 5-month contracts, these are used to construct an implied 3-month contract, adjusting for the number of days between contracts. Specifically, if F_{0t} is the futures price to be constructed, and F_{1t} and F_{2t} are the two longer contracts, the formula used is

$$F_{0t} = F_{1t} \left(F_{1t}/F_{2t} \right)^{n_{01}/n_{12}} \tag{43}$$

where n_{ij} is the number of days between the expiration of contracts i and j. The accuracy of this method of extracting futures prices was assessed by comparing the implied price with the observed price in the months where there was an actual observation. Table 5 shows that the series of actual and implied prices are highly correlated, and that the mean percentage error is close to zero. The mean percentage squared error is larger for agricultural commodities for which seasonal factors are likely to be relevant, and indicates that this extrapolation method introduces some measurement error. This is unavoidable. Fortunately, the similar behaviour of the futures series and the LME forwards series, for which extrapolations are not needed, suggests the measurement error does not unduly affect the results.

Substitute contracts For several commodities there are no futures markets, or were no active futures markets for the full length of the sample. Where possible contracts for substitute commodities (e.g. for rice) or complement commodities (e.g. for the metals) were used instead,

Table 5: Accuracy of extrapolated values for missing months.

Contract	Exchange	Spot price			3-month futures price		
		Mean % Error	RMS % Error	Correlation with actual series	Mean % Error	RMS % Error	Correlation with actual series
Cattle - Feeder	CME	0.40	2.09	0.992	0.33	1.84	0.994
Cattle - Live	CME	0.11	3.78	0.968	-0.07	2.92	0.979
Wheat	KCBT	-0.06	2.74	0.991	0.47	2.62	0.989
Wheat - White	MINN	-0.53	2.70	0.987	0.46	2.41	0.989
Wheat	CBT	0.76	3.82	0.989	0.66	3.41	0.990
Rice - Rough	CBT	-0.68	4.82	0.983	-0.01	6.00	0.948
Corn	CBT	-0.03	3.19	0.984	-0.12	3.26	0.987
Sugar No. 11	CSCE	1.98	7.85	0.993	0.22	2.83	0.999
Cotton No. 2	NYCE	0.09	6.69	0.937	0.04	3.83	0.981
Gold	NYNEX	-0.09	0.27	1.000	0.00	0.08	1.000
Copper - HG	NYNEX	0.19	0.87	0.999	-0.04	0.60	0.999

as shown in Table 4. In each case the constructed or spliced series was compared to spot price data used in the RBA commodity price index to ensure they accurately represent movements in the given commodity. The correlations with the RBA components are shown in Table 4. The constructed index of futures covers just over half of the weights in the RBA commodity index.

Other data

Money Financial reform in Australia over the 1980s and 1990s resulted in an increasing proportion of funds being captured by narrow monetary aggregates. To mimimise the influence of this measurement issue the broadest monetary measure, broad money (BM), is used. The level of BM is taken from RBA Bulletin table D3 and backcast using monthly growth rates from table D1 which account for the largest of breaks to the series.

Import price A monthly import price series is not available for Australia to use as the tradable good price series. Instead the export price from industrial countries from the IMF IFS database (line 11074..DZF) is used.

Unemployment rate The unemployment rate, taken from RBA Bulletin table G6, is used to proxy non-traded output.