Optimal Interest-rate Smoothing in a Small Open Economy

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Abstract

Optimal monetary policy design in the context of a small open economy is studied in this paper. The monetary-policy design problem for the small open economy need not be isomorphic to the closed-economy problem. In this paper, the existence of endogenous deviations from the law of one price makes achieving the objectives of monetary policy a task fraught with compromises. Specifically, there is a trade off between stabilizing domestic producer prices on the one hand, and stabilizing the output gap, the law-of-one-price gap and interest rate, on the other. It is shown that if the central bank has the incentive to deviate from a commitment policy (a time-inconsistency problem), it may be optimal to delegate policy making to a central banker who not only exhibits the Rogoff inflation conservatism, but who also has a taste for smoother interest-rate movements – an interest rate conservative. We also prove analytically the existence of policy inertia under pre-commitment and provide verifiable propositions about the interest-rate rules that arise from optimal pre-commitment and discretion, with and without an interest-rate smoothing objective.

Keywords: Interest-rate smoothing; sticky prices; small open economy; law-of-one-price gap; stabilization bias.

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1 Introduction

The seminal paper by Kydland & Prescott (1977) and Barro & Gordon (1983) provided an insight into why in the long run average inflation may be too high for society's liking. The story is as follows. While a central bank may announce some target for inflation as its policy, it may not necessarily have an incentive to follow through on that announced policy, in subsequent periods. This is simply because such policy may no longer be optimal in the latter periods. For instance, having announced 2 percent as the targeted inflation rate, workers and firms would have formed their expectations that the inflation rate will be 2 percent and set prices and wages accordingly. If the central bank follows through on the pre-commitment policy of 2 percent inflation rate, it would have to allow the labor market to clear at the given prices. That is, under pre-commitment, workers are not fooled systematically. However, as Kydland & Prescott (1977) showed, such a policy is not time consistent. As soon as the private actors' expectations are fixed on the 2 percent policy, the central bank has the incentive to inflate the economy by more than 2 percent, so that real wages fall, and output can be stimulated towards full-employment output. Thus, by "fooling" the private actors in the economy under discretionary policy, the central bank can, in the short run, reap some benefits in terms of higher output. However, in the long run, the benefit of reaching full employment is more than outweighed by the rising inflation cost of the discretionary policy, since private actors will not be continuously fooled. In the long run, output remains inefficiently low while inflation is too high. In the monetary policy literature, this has come to be known as the average inflation bias.

The temptation to cheat on a pre-commitment policy also results in short run dynamics and volatilities that affect welfare over the business cycle. This is often termed stabilization bias in the literature. Dennis & Söderström (2002) considered this issue using a variety of model specifications. They find that in the context of the models studied, the stabilization bias appears in the form of trading off greater inflation volatility relative to output volatility. While Rogoff (1985) considered delegation as a solution to the average inflation bias problem, Woodford (1999*a*) uses this idea to justify an explicit interest-rate smoothing objective for the central bank.

Woodford (1999*a*) shows that if society or the government delegates policy to a central banker with a taste for interest-rate smoothing, society's welfare over the business cycle can be improved, or the stabilization bias problem can be minimized.

This paper is concerned with the notion of optimal interest-rate smoothing, or what appears to be smoothing done by a central bank, in a small open economy. While interest-rate smoothing has been studied empirically (e.g. Sack & Wieland 2000, Sack 2000, Drew & Plantier 2000), other observers and researchers try to explain the source or motivation for interest-rate smoothing. Some suggest that interest-rate smoothing is the result of the central bank's dislike of interest rate volatility (e.g. Debelle & Stevens 1995, Söderlind 1997), or a desire to prevent large movements in financial-market prices (e.g. Cukierman 1996), or the existence of measurement errors in key macroeconomic variables (e.g. Sack 2000). Sack & Wieland (2000) show that in a vector autoregression framework, policy gradualism can be the result of an optimal interest-rate policy when the central bank is uncertain about the parameters in the economy's law of motion. Similarly, Clarida, Galí & Gertler (1999) show in the context of their model that parameter uncertainty may give rise to interest-rate smoothing behavior by the central bank. Another explanation points to the central bank's exploitation of the forward-looking behavior of the private sector in controlling longer-term interest rates (e.g. Goodfriend 1991, Woodford 1999*b*, Rotemberg & Woodford 1999).

Rotemberg & Woodford (1999) showed that, in the context of simple policy rules in a closed economy, policies where there is a smoothing of the rate of interest-rate change can be optimal under certain classes of policy rules and parameterization. They based their argument on the fact that in their model environment, private agents and firms have rational expectations of future variable, in particular, the real interest rate. Therefore, implementing interest-rate changes in small steps over time has a larger business-cycle stabilizing effect through the rational-expectations-based term structure of the interest rate. Rotemberg & Woodford (1999) termed this as super-inertial interest-rate policy. Woodford (1999*a*) then showed that the notion interest-rate smoothing can be rationalized as the result of optimal monetary policy when the central bank can credibly set policy in a "timeless" fashion under pre-commitment. Effective, the central banker, in acting as Stackelberg leader in a dynamic game between policy maker and the private sector, has first-mover advantage in conditioning the latter's expectations of future outcomes. In doing so, Woodford (1999*a*) showed analytically that an intrinsically inertial interest-rate process or rule is the outcome of pre-commitment policy, and the inertia is independent of the autocorrelation in exogenous stochastic processes. However, these papers have focused on the standard New Keynesian framework for a closed economy.

There has been tremendous growth in the literature on the conduct of monetary policy, specifically in the area of interest-rate policy rules, for small open economies. Often, it can be shown that the monetary-policy design problem for the small open economy is similar to its closed-economy counterpart. This is often obtained under very restrictive assumptions. Specifically, under perfect exchange-rate pass through, Clarida, Galí & Gertler (2001), in using the popular model of Galí & Monacelli (2002), showed that the open- and closed-economy environments are isomorphic and hence monetary policy design ought to be qualitatively similar in terms of targeting domestic inflation.

When there is complete exchange-rate pass through, there is no difference between domestic- and foreign-dollar prices of imports after adjusting for the exchange rate. Clarida et al. (2001) showed that, in such circumstances, it is still optimal for the central bank to target domestic inflation and the output gap. Intuitively, any volatility in the exchange rate gets transmitted to aggregate demand immediately via the terms of trade and is thus captured in the output-gap stabilization objective of the central bank. Thus nominal rigidity in domestic goods prices can be counteracted by the central bank's domestic-goods inflation target.

However, in a small open economy with an incomplete exchange rate pass-through channel there are further complications. Shocks to the economy, among other things, result in gaps between the prices of imports in domestic currency terms and the prices charged for these goods domestically. This is termed the law-of-one-price (LOP) gap in Monacelli (2003). The intuition of this is as follows. One can think of the workhorse Galí & Monacelli (2002) and Clarida et al. (2001) model as the case when the economy's openness to trade affects only the slope of the IS curve. That is, openness only makes output gap more sensitive to the interest rate. In the model here, not only does the slope of the IS curve in the interest-rate-output-gap space become steeper, the IS curve also shifts endogenously to movements in the interest rate or exchange rate as a result of a short-run LOP gap. Furthermore, the LOP gap alters the slope of inflation with respect to output gap and creates endogenous shifts on the aggregate-supply (or Phillips curve) side.

In this paper, the Woodford (1999*a*) idea is extended to the case of the small open economy of Monacelli (2003). Monacelli (2003) showed that in his model, there is a trade off between stabilizing domestic producer prices on the one hand, and stabilizing the output gap or the LOP gap on the other. When one admits a concern for interest-rate stability as well in the central bank's loss function, it will be shown that in the model there is a further trade off between stabilizing interest rate and stabilizing the other goals in the central bank loss function. The paper contributes to the issue of monetary policy design for open economies and specifically to the role of central banks having an interest-rate smoothing objective in the open economy. Adolfson (2002) considers having an exchange rate objective in a model similar to the one in this paper. She also finds that by delegating monetary policy to an interest-rate smoothing central banker in the style of Woodford (1999*a*), the stabilization bias can be reduced and this can stand in for having an exhcange rate stabilization objective. However, there is no explicit role for the labor market, and hence labor supply elasticity, in affecting the real marginal cost and aggregate demand side of her model. This paper also differs from Adolfson (2002) in that a central bank objective is couched in terms of stabilizing the output gap, inflation and the interest rate, an approach consistent with Woodford (1999*a*) and the existing literature. Adolfson (2002) uses output instead of the output gap. Lastly, Adolfson (2002) introduces many ad-hoc shocks to the model, which are not derived from the model's microfoundations, to obtain her numerical-simulation conclusions. This paper focuses just on the case of domestic and foreign technology shocks as the main drivers of the business cycle in the small open economy in the tradition of international Real-Business-Cycle models (e.g. Backus, Kehoe & Kydland 1995). These fundamental and real shocks are linked tightly to the definitions of the natural rate of interest and potential output. The novelty in this paper is as follows. It shows that Woodford's (1999*a*) conclusion about optimal monetary policy inertia still carries through in a small-open-economy setting that breaks the closed–and-open-economy monetary policy isomorphism. It also proves analytically the existence of policy inertia under pre-commitment and provides verifiable propositions about the interest-rate rules that arise from optimal pre-commitment and discretion, with and without an interest-rate smoothing objective.

The remainder of the paper is organized as follows. Section 2 provides the description of the microeconomic foundations of the small open economy model. The problem of optimal time-inconsistent monetary policy is considered and contrasted with the optimal discretionary policy in Section 3. Some analytical results are obtained for the policy rule involved. The optimal delegation of discretionary policy is also considered for the case of interest-rate smoothing. In Section 4, the advantage of having interest-rate smoothing, especially when the central bank cannot pre-commit to an announced policy response, is considered numerically. The effect of such a policy, *vis-à-vis* pre-commitment to society's valuation of the policy and the pure discretion case thereof, is considered in an impulse response example. Finally, Section 5 concludes.

2 Model

The model used in this paper is based on the small-open-economy model of Monacelli (2003) which, in turn, is a modification of the model in Galí & Monacelli (2002), to allow for incomplete exchange-rate pass through to imports prices. The model retains the spirit of the two-country Real-Business-Cycle (RBC) model in that domestic and foreign technology shocks are the only exogenous stochastic processes that drive the business cycle. However, monetary nonneutrality in the model arises as a result of market imperfection and price stickiness.

In this section, the microeconomic foundations of the model are presented. The model consists of the household sector, imperfectly competitive domestic goods firms, foreign goods importers, and the central bank. The central bank can either credibly determine policy in terms of a Stackelberg equilibrium or simply form its Nash-equilibrium best response policy each period, given the expectations and evolution of the economy.

2.1 Household Sector

The small open economy is represented by the household that seeks to maximize its lifetime utility payoff

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

subject to the sequence of constraints given by

$$\int_0^1 \left[P_{H,t}(i) C_{H,t}(i) + P_{F,t}(i) C_{F,t}(i) \right] di + E_t Q_{t,t+1} B_{t+1} \leq B_t + W_t N_t + T_t; \quad t \in \{0, \mathbb{Z}_+\} \tag{2}$$

The notation E_0 denotes the usual mathematical expectations operator, conditioned on the available information set at time 0. The prices of home and foreign goods of type i are respectively given by $P_{H,t}(i)$ and $P_{F,t}(i)$, B_{t+1} is the nominal value of assets held at the end of period t, $W_t N_t$ is the total wage income and T_t is a lump-sum tax or transfer. The stochastic discount factor is $Q_{t,t+1}$ which is defined in the optimality conditions for the household below. The consumption index C_t is linked to a continuum of domestic, $C_{H,t}(i)$, and foreign goods, $C_{F,t}(i)$ defined on the compact interval of $[0,1]$ through the following indexes

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta-1}{\eta}} \tag{3}$$

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{4}$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

Thus the elasticity of substitution between home and foreign goods is given by $\eta > 0$ and the elasticity of substitution between goods within each goods category (home and foreign) is $\varepsilon > 0$.

The choice of consuming goods within each category and within the consumption index (3) can be broken into a static problem for the household. It can be shown that optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{6}$$

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{7}$$

for all $i \in [0,1]$ where

$$P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{8}$$

$$P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{9}$$

and

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{10}$$

$$C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{11}$$

where the consumer price index (CPI) can be solved as

$$P_t = \left[(1-\gamma)\, P_{T,t}^{1-\eta} + \gamma P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{12}$$

Another intratemporal condition relating labor supply to the real wage must also be satisfied

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{13}$$

Finally, intertemporal optimality for the household decision problem must satisfy the stochastic Euler equation

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = Q_{t,t+1} \tag{14}$$

which says that the projected marginal rate of substitution of consumption between two periods, conditional on available information at time t, must equal the relative price of the two bundles, measured by the price of holding an asset for the duration of one period, $Q_{t,t+1}$. Thus $Q_{t,t+1}$ also takes on the interpretation of a stochastic discount factor on the risk-free asset.

2.2 Domestic Production

As is standard in sticky-price models, it is assumed that there is a continuum of monopolistically competitive firms defined on the compact interval $[0, 1]$. Firms utilize a constant returns-to-scale technology

$$Y_t(i) = Z_t N_t(i) \tag{15}$$

where $Z_t = \exp(z_t)$ is a total productivity shift term. Cost minimization leads to the first-order condition

$$MC_{H,t}(i) Z_t = W_t \tag{16}$$

Given (16) it can be seen that nominal marginal cost is common for all firms such that $MC_{H,t}(i) = MC_{H,t}$ for all $i \in [0, 1]$. Furthermore, in subsequent discussions on optimal monetary policy, it will be assumed that fiscal policy provides for an employment subsidy of τ to deliver the first-best allocation under flexible prices. Therefore, (16) can be rewritten, after integrating across all firms, as

$$mc_{H,t} = \frac{(1-\tau) W_t}{Z_t P_{H,t}}. \tag{17}$$

2.3 Domestic pricing

The retail side of the firms producing domestic goods change prices according to a discrete-time version of Calvo's (1983) model. The signal for a price change is a stochastic time-dependent process governed by a geometric distribution. The expected lifetime of price stickiness is $(1-\theta_H)^{-1}$. Recall that the nontraded goods price index was given in (8). In a symmetric equilibrium all firms that get to set their price in the same period choose the same price. Thus prices evolve according to

$$P_{H,t} = \left[(1-\theta_H)\left(P_{H,t}^{new}\right)^{1-\varepsilon} + \theta_H\left(P_{H,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{18}$$

That is, each period a fraction $1-\theta_H$ of all the firms gets to charge a new price and the remaining fraction θ_H must charge the previous period's price.

The price set at time t, $P_{H,t}^{new}$ will be the solution to the following problem where firms face a probability θ_H that a new price commitment, $P_{H,t}^{new}$, in period t will still be charged in period $t+k$. Thus, when setting $P_{H,t}^{new}$, each firm will seek to maximize the value of expected discounted profits:

$$\max_{\left\{P_{H,t}^{new}\right\}_{t\in\{0,\mathbb{Z}_+\}}} E_t\left\{\sum_{k=0}^{K-1} Q_{t,t+k}\theta_H^k\left[P_{H,t}^{new} - MC_{H,t+k}(i)\right]C_{H,t+k}\left(P_{H,t}^{new}, i\right)\right\} \tag{19}$$

and demand is given by

$$C_{H,t+k}(i) = \left(\frac{P_{H,t}^{new}}{P_{H,t+k}}\right)^{-\varepsilon} C_{H,t+k}. \tag{20}$$

The optimal pricing strategy is thus one of choosing an optimal path of price markups as a function of rational expectations forecast of future demand and marginal cost conditions,

$$P_{H,t}^{new} = P_{H,t}\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{E_t\sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{-1-\varepsilon}\left(\frac{MC_{H,t+k}}{P_{H,t+k}}\right)C_{H,t+k}}{E_t\sum_{k=0}^{\infty} Q_{t,t+k}\theta_H^k\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}C_{H,t+k}}. \tag{21}$$

Notice that if the chance for stickiness in price setting is nil, $\theta_H = 0$ for all $k \in \{0, \mathbb{Z}_+\}$, the first order condition in (21) reduces to $mc_{H,t} = (1 - \varepsilon^{-1})$, for all t, which says that the optimal price is a constant markup over marginal cost, or that the real marginal cost is constant over time. This is the same result as that for a static model of a firm with monopoly power. Thus with price-setting behavior, the markup is positive. Straightforward algebra and manipulation of the pricing decision determines the inflation dynamics of nontraded goods as:

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda_H mc_{H,t}. \tag{22}$$

where $\lambda_H = \theta_H^{-1} (1 - \theta_H) (1 - \beta\theta_H)$. This is a forward-looking or New Keynesian Phillips curve for home goods.

2.4 Imports Retailer

Let ϵ_t denote the level of the nominal exchange rate. There exists local firms acting as retailers who purchase imports at the marginal cost equal to the imports price in domestic dollar terms, $\epsilon_t P^*_{F,t}(j)$, and re-sell them domestically at a markup price, $P^{new}_{F,t}$. It is the stickiness in the domestic price of imported goods that will cause a persistent and potentially large gap in what would otherwise be the law of one price. Thus the local retailer importing good j solves

$$\max_{\left\{P^{new}_{F,t}\right\}_{t\in\{0,\mathbb{Z}_+\}}} E_t \left\{ \sum_{k=0}^{K-1} Q_{t,t+k} \theta_F^k \left[P^{new}_{F,t} - \epsilon_{t+k} P^*_{F,t+k}(j)\right] C_{F,t+k}(j) \right\} \tag{23}$$

such that

$$C_{F,t+k}(j) = \left(\frac{P^{new}_{F,t}}{P_{F,t+k}}\right)^{-\varepsilon} C_{F,t+k} \tag{24}$$

The optimal pricing strategy is thus

$$P^{new}_{F,t} = P_{F,t} \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \theta_F^k \left(\frac{P_{F,t}}{P_{F,t+k}}\right)^{-1-\varepsilon} \left(\frac{\epsilon_{t+k} P^*_{F,t+k}}{P_{F,t+k}}\right) C_{F,t+k}}{E_t \sum_{k=0}^{\infty} Q_{t,t+k} \theta_F^k \left(\frac{P_{F,t}}{P_{F,t+k}}\right)^{-\varepsilon} C_{F,t+k}}.$$

and assuming the evolution of the aggregate retail imports price index as

$$P_{F,t} = \left[(1-\theta_F) \left(P_{F,t}^{new} \right)^{1-\varepsilon} + \theta_F \left(P_{F,t-1} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} .$$

Let e_t, $p_{F,t}^{*}$ and $p_{F,t}$ denote the log deviations of the nominal exchange rate, foreign price of imports and domestic retail price of imports respectively. The law-of-one-price gap in log-deviation term is measured as

$$\psi_{F,t} = e_t + p_{F,t}^{*} - p_{F,t}. \tag{25}$$

A first-order approximation to the pricing dynamics will result in a similar aggregate supply schedule

$$\pi_{F,t} = \beta E_t \left\{ \pi_{F,t+1} \right\} + \lambda_F \psi_{F,t}. \tag{26}$$

where $\lambda_F = \theta_F^{-1} (1-\theta_F)(1-\beta\theta_F)$. Notice that if the domestic dollar price of foreign goods exceed the domestic retail price of foreign goods, or $\psi_{F,t} > 0$, ceteris paribus, $\pi_{F,t} > 0$.

2.5 Market Clearing Conditions

In the rest of the world, it is assumed that in the limit of being a closed economy, the home goods price of the rest of the world equals its CPI, or $P_{H,t}^{*} = P_t^{*}$ and consumption equals output, $C_t^{*} = Y_t^{*}$. Market clearing in the small open economy requires that

$$Y_t(i) = C_{H,t}(i) + C_{H,t}^{*}(i) \tag{27}$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[\left(\frac{P_{H,t}}{P_t} \right)^{-\eta} (1-\gamma) C_t + \left(\frac{P_{H,t}}{\epsilon_t P_t^{*}} \right)^{-\eta} \gamma^{*} Y_t^{*} \right] \tag{28}$$

The above expression has made use of (6) and (10) and the analogous counterpart for the rest of the world.

2.6 Linearized first-order conditions

It can be shown that after log-linearizing the various first order conditions, one obtains a set of linear identity and stochastic difference equations. These equations are in terms of the log-deviations from steady state for CPI inflation, domestic inflation, the output gap, retail imports inflation, and the LOP gap respectively.

$$\pi_t = (1-\gamma)\,\pi_{H,t} + \gamma\pi_{F,t} \tag{29}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_y \widetilde{y}_t + \kappa_\psi \psi_{F,t} \tag{30}$$

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \frac{\omega_s}{\sigma}\left(r_t - E_t \pi_{H,t+1} - r_t^n\right) + \Gamma_y E_t \left(\psi_{F,t+1} - \psi_{F,t}\right) \tag{31}$$

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda_F \psi_{F,t} \tag{32}$$

$$\psi_{F,t} = E_t \psi_{F,t+1} - r_t + r_t^* + E_t \pi_{F,t+1} + E_t \pi_{t+1}^* \tag{33}$$

$$E_t e_{t+1} = e_t + r_t - r_t^* \tag{34}$$

where

$$\begin{aligned}
\kappa_y &= \lambda_H \left(\varphi + \frac{\sigma}{\omega_s}\right) > 0 \\
\kappa_\psi &= \lambda_H \left(1 - \frac{\omega_\psi}{\omega_s}\right) \geq 0 \\
\omega_s &= 1 + \gamma\,(2-\gamma)\,(\sigma\eta - 1) \geq \omega_\psi = 1 + \gamma\,(\sigma\eta - 1) > 0 \\
\Gamma_y &= \frac{\gamma\,(1-\gamma)\,(\sigma\eta - 1)}{\sigma} > 0.
\end{aligned}$$

There are only two exogenous stochastic processes given by technology shock in the rest of the world and its counterpart in the small open economy given by the following first-order Markov processes:

$$z_t^* = \rho^* z_{t-1}^* + \upsilon_t^* \tag{35}$$

$$z_t = \rho z_{t-1} + \upsilon_t \tag{36}$$

where

$$\begin{bmatrix} \upsilon_t \\ \upsilon_t^* \end{bmatrix} \xrightarrow{d} i.i.d.N\left(\mathbf{0}_{2\times 1}, \mathbf{I}_2\right).$$

We can append endogenous variables such as the natural rate of interest, the terms of trade, and the real exchange rate to the system. These, respectively, can be solved as identities in the system:

$$r_t^n = -\frac{(1+\varphi)(1-\rho)}{\sigma+\varphi}\left[\frac{\sigma(\omega_s-1)\varphi}{\sigma+\varphi\omega_s}\right]z_t^* - \left[\frac{\sigma(1+\varphi)(1-\rho)}{\sigma+\varphi\omega_s}\right]z_t \tag{37}$$

$$s_t = \frac{\sigma}{\omega_s}\widetilde{y}_t + \left[\frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s}\right]z_t - \left[\frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s}\right]z_t^* + \frac{\sigma}{\omega_s}\left[\frac{\omega_\psi-\omega_s}{\sigma+\varphi\omega_s} - \omega_s\right]\psi_{F,t} \tag{38}$$

$$q_t = \psi_{F,t} + (1-\gamma)s_t \tag{39}$$

The domestic natural interest rate (37) is a function of technology shocks in the rest of the world and also in the small open economy. A positive shock in both cases lowers the natural rate. The log terms of trade (38), is the difference between foreign and domestic prices of exportables. Ceteris paribus, (38) and (39) capture the Samuelson-Balassa effect of productivity differentials between the small economy and the rest of the world. All else being equal, a positive domestic technology shock lowers domestic prices and thus the terms of trade and real exchange rate improve while the converse is true for foreign technology shock. However, a deviation from the law of one price (deviation of foreign goods price from domestic currency price of imports) has a negative effect on the terms of trade since $\omega_\psi - \omega_s < 0$.

A rational expectations equilibrium in the model is defined below.

Definition 2.1 *A rational expectations equilibrium in the small open economy is a set of bounded stochastic processes* $\left\{\pi_t, \pi_{H,t}, \widetilde{y}_t, \pi_{F,t}, \psi_{F,t}, r_t, e_t\right\}_{t\in\{0,\mathbb{Z}_+\}}$ *that satisfies the system of equations (29)-(34) for any given set of processes* $\left\{r_t^n, r_t^*, \pi_{t+1}^*, z_t, z_t^*\right\}_{t\in\{0,\mathbb{Z}_+\}}$.

In fact, in our numerical solutions, we replace the boundedness requirement with a stronger requirement that the solution to the system be stable.

2.7 Dynamics and Policy in the Rest of the World

In solving the rational expectations equilibrium, we assume that monetary and fiscal policy in the rest of the world maintains a first-best flexible price equilibrium. Specifically the aggregate supply equivalent of (22) in the rest of the world, combining with labor supply decisions, yields

$$mc_t^* = (\sigma + \varphi)\, y_t^* - (1 + \varphi)\, z_t^*$$

and under the natural flexible price level of output in the world economy, $mc_t^* = 0$ which implies that markup is constant. Thus output in the rest of the world equals its natural output

$$y_t^* = \left(\frac{1+\varphi}{\sigma+\varphi}\right) z_t^* \tag{40}$$

Since the evolution of output in the rest of the world is given by

$$y_t^* = E_t y_{t+1}^* - \frac{1}{\sigma}\left(r_t^* - E_t \pi_{t+1}^*\right) \tag{41}$$

making use of (40) in the flexible price equilibrium yields the natural rate of interest in the rest of the world as

$$r_t^* = -\left(\frac{1+\varphi}{\sigma+\varphi}\right)(1-\rho)\, z_t^*. \tag{42}$$

3 Optimal Monetary Policy

In this section, the problem of optimal monetary policy is considered. The traditional literature on optimal monetary policy focuses on the problem of the average inflation bias under discretion – when the central bank tends to cause too much long-run inflation in its attempt to bring output beyond potential without actually improving an inefficient level of output (see e.g. Barro & Gordon 1983). However, in this paper the focus is on how the inability of a central bank to commit to maximizing society's payoff (the time-inconsistency problem) affects the evolution and transition of the economy in response to exogenous shocks. This is often termed the "stabilization bias" (see e.g. Clarida et al. 1999, Woodford 1999*a*, Dennis & Söderström 2002). It should be noted that even when the output level in the long run is made efficient, stabilization bias (a short-run business cycle phenomenon) can still exist when the central bank cannot optimally commit to a once-and-for-all policy. To abstract from the problem of an average inflation bias it is assumed, as shown in Galí & Monacelli (2002), that fiscal policy in the long run provides a subsidy to real wage of $\tau = \frac{1}{\varepsilon}$. This yields output at steady state which equals the first-best equilibrium outcome; or output that equals the natural level of output. Having done this, the focus can then be solely on the welfare effects of the stabilization bias problem.

It is assumed that the objective of the monetary policy maker is to minimize the expected value of a loss function in the form of

$$W = E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{43}$$

where $\beta \in (0,1)$ and the loss per period is measured by

$$L_t = \pi_t^2 + b_w \widetilde{y}_t^2 + b_r r_t^2 \tag{44}$$

The weights $b_w > 0$ and $b_r > 0$ should then be interpreted as the concern of the central bank for output gap and interest rate variability respectively, relative to a concern for inflation which has its weight normalized to one. This follows closely the traditional loss function used in the literature. Under certain assumptions on preferences of households, one can derive a second-order accurate approximation of the true household welfare in terms of such a loss function. See Woodford (2001), pp.22-23, for the closed economy case, and Galí & Monacelli (2002) for a small open economy upon which our model is based. Galí & Monacelli (2002) and Clarida et al. (2001) showed that the relevant inflation measure for the typical open economy case is the domestic good inflation. (because the only source of price rigidity is the domestic goods sector). However, in our case, there is further imports-price stickiness. In this case there is no analytical expression linking household preferences to the typical social loss function in terms of inflation and output gap. Monacelli

(2003) justifies an objective such as (44) as a reasonable approximation to the true social loss function, since the CPI measure is a convex combination of both sticky domestic and foreign goods inflation. The inclusion of $b_r > 0$ can be justified either as a desire to maintain financial market stability (e.g. Goodfriend 1991, Cukierman 1996) or a quadratic penalty on interest-rate volatility given a zero bound on nominal interest rates (e.g. Woodford 1999*a*).

3.1 Commitment and the problem of time inconsistency

When the central bank can commit to minimizing (43) and (44) subject to the constraints of the evolution of the economy in (29)-(34), it behaves like a Stackelberg leader. Essentially the central bank solves an approximate Ramsey problem which involves exploiting private-sector expectations of the future, once and for all, and the private sector reacts to the given policy. That is, the private sector behaves like the Stackelberg follower. The first-order conditions for the central bank's problem are then:[1]

$$(1-\gamma)\,\pi_t + \phi_{1,t} - \phi_{1,t-1} - \frac{\omega_s}{\beta\sigma}\phi_{2,t-1} = 0 \tag{45}$$

$$b_w \widetilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t} - \beta^{-1}\phi_{2,t-1} = 0 \tag{46}$$

$$b_r r_t + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{47}$$

$$-\kappa_\psi \phi_{1,t} + \Gamma_y\left(\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\right) - \lambda_F \phi_{3,t} + \beta^{-1}\phi_{4,t-1} - \phi_{4,t} = 0 \tag{48}$$

$$\gamma\pi_t + \phi_{3,t} - \phi_{3,t-1} + \beta^{-1}\phi_{4,t-1} = 0 \tag{49}$$

Furthermore, if the central bank can commit to such a policy for $t \geq 0$, they must be unable to exploit the expectations of the private sector prior to $t = 0$, when the policy is laid down. In other words, the initial conditions

$$\phi_{1,-1} = \phi_{2,-1} = \phi_{3,-1} = \phi_{4,-1} = 0. \tag{50}$$

are required.

[1] The details for these are given in Appendix A.

The existence of lagged Lagrange multipliers in (45)-(49) implies that endogenous variables and in particular the optimal interest-rate instrument under pre-commitment must not only react to current (and expected future) shocks, but also past movements. Specifically, as is shown in Appendix A, one can simplify the first-order conditions (45)-(49) to obtain the implicit policy rule as

$$\begin{aligned} r_t = {} & \frac{b_w}{b_r}\left(\frac{\omega_s}{\sigma} - \Gamma_y\right)\widetilde{y}_t + \frac{1}{b_r}\pi_t \\ & - \left[1 + \kappa_y\left(\frac{\omega_s}{\sigma} - \Gamma_y\right) + \kappa_\psi\right]\phi_{1,t} - \left(1 + \lambda_F\right)\phi_{3,t} + \phi_{3,t-1}. \end{aligned} \tag{51}$$

This can be interpreted as a Taylor-type rule augmented with additional response terms with respect to current and past Lagrange multipliers – the result of the central bank having to carry through its promises or commitment made at some earlier date if it is to credibly influence private-sector expectations. The intrinsic Lagrange multiplier dynamics also introduce some degree of policy inertia, independent of the serial correlation of exogenous stochastic processes, as Woodford (1999*a*) has shown in the case of a typical closed-economy New Keynesian model. Specifically, once the rule (51) is rewritten only in terms of the primitive shocks and the interest rate instrument, it can be shown that there is still intrinsic sluggishness in the process for the interest rate under pre-commitment in this model. This is stated in Proposition 3.1 below. Let the vector of exogenous domestic and foreign technology shocks be defined by $\mathbf{z}_t := (z_t^*, z_t)$ and the transition law of these be

$$\mathbf{M} = \begin{bmatrix} \rho^* & 0 \\ 0 & \rho \end{bmatrix}.$$

Proposition 3.1 *In a rational expectations (RE) equilibrium under the optimal pre-commitment problem of minimizing (43)-(44) subject to (29)-(34), the resulting interest rate rule is backward and forward looking in terms of current and past RE forecasts of domestic and foreign technology shocks:*

$$r_t = \rho_r r_{t-1} - \mathbf{\Theta}_b \sum_{s=0}^{t-1} \mathbf{N}^s \mathbf{C} \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} \mathbf{M}^j \mathbf{z}_{t-s-1} - \mathbf{\Theta}_f \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} \mathbf{M}^j \mathbf{z}_t. \tag{52}$$

where $\mathbf{\Theta}_b$, $\mathbf{\Theta}_f$, $\mathbf{N}$, $\mathbf{C}$, $\mathbf{H}$, *and* $\mathbf{F}$ *are matrices obtain under the RE equilibrium. It is also intrinsically inertial and the inertia coefficient* ρ_r *is independent of the structure of serial correlation in* $\mathbf{z}_t$.

Proof. See Appendix B. ■

It can also be seen in Appendix B that $\mathbf{\Theta}_b$ and $\mathbf{\Theta}_f$ are decreasing in absolute terms with the central banker's preference for interest rate stability, b_r, in the case of pre-commitment. In other words, when the pre-commiting central bank places greater weight on interest-rate variability, it implies smaller elasticities of the policy instrument with respect to current and past forecasts of the technology shocks.

1.1 An analytical limiting case

Suppose, as Woodford (1999*a*) did, that $\lambda_H \to 0$ and $\lambda_F \to 0$ implying that domestic inflation, the LOP gap and thus retail imports inflation are zero for all time periods: $\pi_{H,t} = \psi_{F,t} = \pi_{F,t} = 0$. This implies that the Lagrange multipliers $\phi_{1,t}$, $\phi_{3,t}$ and $\phi_{4,t}$ are no longer binding. Then the first-order condition (46) and (47) become

$$b_w \widetilde{y}_t + \phi_{2,t} - \beta^{-1}\phi_{2,t-1} = 0$$

and

$$b_r r_t + \frac{\omega_s}{\sigma}\phi_{2,t} = 0.$$

Substituting the latter equation into the first yields an expression for the remaining first-order conditions in terms of an interest-rate rule:

$$r_t = \frac{b_w}{b_r}\left(\frac{\omega_s}{\sigma}\right)\widetilde{y}_t + \beta^{-1} r_{t-1}. \tag{53}$$

Remark 3.1 *Equation (53) shows that even in the special limiting case with no price changes, the interest-rate rule under commitment still has the character of inertia given by the coefficient on lagged interest rate of* $\beta^{-1} > 1$.

3.2 Discretionary or time-consistent optimal policy

As Kydland & Prescott (1977) and Barro & Gordon (1983) showed, the pre-commitment rule in the previous section is not time consistent. That is, while the rule was optimal at a time when the announcement of the policy was made, it is no longer so in subsequent periods as the policy maker has an incentive to cheat to take advantage of the given expectations of the private sector at the latter dates. That is, compared to the optimal pre-commitment rule, the central bank under discretion has an incentive to disregard the lagged constraints in (45)-(49) in the conduct of its optimal policy.

Effectively, in each period, the central bank will just minimize (44) subject to the constraints (29)-(34). The resulting first-order conditions now are:

$$(1-\gamma)\,\pi_t + \phi_{1,t} = 0 \tag{54}$$

$$b_w \widetilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t} = 0 \tag{55}$$

$$b_r r_t + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{56}$$

$$-\kappa_\psi \phi_{1,t} + \Gamma_y \phi_{2,t} - \lambda_F \phi_{3,t} - \phi_{4,t} = 0 \tag{57}$$

$$\gamma \pi_t + \phi_{3,t} = 0 \tag{58}$$

The following defines the notion of such a discretionary policy as a Markov-perfect Nash equilibrium.

Definition 3.1 *A Markov-perfect Nash equilibrium is the set of sequences*

$$\left\{\pi_t, \pi_{H,t}, \widetilde{y}_t, \pi_{F,t}, \psi_{F,t}, r_t, e_t, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \phi_{4,t}\right\}_{t\in\{0,\mathbb{Z}_+\}}$$

that satisfies (29)-(34) and (45)-(49) for all $t \in \{0, \mathbb{Z}_+\}$*, for any given set of exogenous stochastic processes* $\left\{r^n_t, r^*_t, \pi^*_{t+1}, z_t, z^*_t\right\}_{t\in\{0,\mathbb{Z}_+\}}$.

Thus, the equilibrium under discretionary policy is a fixed point of the problem where the private sector (29)-(34) forms its best response in terms of its dynamic programming problem taking monetary policy as given and at the same time, the central bank takes private expectations and decisions as given while solving its dynamic program. Furthermore, it has the Markov property in that only state variables at time t are relevant for solving the optimum; initial conditions and history do not constrain the central bank's optimal decision making. While such a discretionary policy is time-consistent – because as each period unfolds the central bank has no incentive to do otherwise but conduct the discretionary optimal policy – it is not necessarily optimal from the social welfare point of view (see e.g. Woodford 1999*a*). Under the case of optimal discretion, one can find an analytical expression for the interest-rate rule in the model. This turns out to be a Taylor-type rule where the rule reacts to both CPI inflation (as opposed to just domestic inflation) and output gap. This is summarized in the proposition below.

Lemma 3.1 *The Markov-perfect Nash equilibrium for the central bank which solves the problem in Definition 3.1 yields an optimal Taylor-type rule of the form*

$$r_t = \Phi_\pi \pi_t + \Phi_y \widetilde{y}_t \tag{59}$$

where

$$\Phi_\pi = b_r^{-1} \left\{ (1-\gamma) \left[\kappa_\psi + \kappa_y \sigma^{-1} \left(1 + \gamma \left(\sigma\eta - 1\right)\right) + \gamma \lambda_F \right] \right\} > 0$$

$$\Phi_y = b_w b_r^{-1} \sigma^{-1} \left[1 + \gamma \left(\sigma\eta - 1\right)\right] > 0.$$

Proof. This follows from solving (54)-(58) simultaneously and eliminating the Lagrange multipliers. ■

Proposition 3.2 *The optimal elasticity of the central-bank interest rate instrument with respect to inflation Φ_π in Lemma 3.1 is decreasing with price stickiness in the domestic and retail imports sector, θ_H and θ_F, while the elasticity of the central-bank interest rate instrument with respect to output gap, Φ_y, is independent of θ_H and θ_F.*

Proof. See Appendix C. ■

Proposition 3.2 suggests that the degree of sluggishness in price changes in domestic goods and retail imports, affects only the central bank's response to CPI inflation in the optimal discretionary rule (59). Intuitively, the more sluggish price changes in both domestic and imported goods are, the less interest-rate policy has to respond to movements in the CPI inflation, since inflation will be less volatile. However, it would be reasonable to expect that the nominal exchange rate will be more volatile in such circumstances. This also implies greater volatility in the interest rate, through the UIP condition (34), which brings about a more volatile LOP gap. With a more volatile LOP gap, a more volatile output gap will result, via the output-gap IS curve (31). Because of the presence of the LOP gap, there will be a trade-off in terms of the policy objective, even if the central bank wishes to further stabilize the output gap and interest rate when prices become more sticky. The following proposition states the trade-off in terms of the optimal discretionary rule (59).

Proposition 3.3 *Given the optimal discretionary rule (59),there is a trade-off between maintaining interest rate stability, inflation (domestic and imports) and output-gap stability.*

Proof. See Appendix D. ■

It is interesting to note that in the case of discretionary policy above, there are no intrinsic inertia in the optimal interest-rate policy (59). That is, no lagged Lagrange multiplier terms appear in the first-order condition for the central bank's optimal choice. This is simply because the central banker in each period has no incentive to be bound by the constraints from the past periods. Therefore, policy under discretion would be suboptimal in contrast with the approximate Ramsey problem in the pre-commitment case, since the former has no influence on private expectations.

3.3 Interest-rate smoothing and the Rogoff Conservative

In this section, interest-rate smoothing under discretionary policy is considered as a second-best solution to the inability of the central bank to commit. Intuitively, this ought to approximate the intrinsic inertia in policy found in the case of pre-commitment. Suppose now, instead of attempting to force the central bank to commit to minimizing society's lifetime loss function, and realizing that the central bank will cheat by acting in discretion anyway, society delegates policy making to a central banker with an additional preference for interest rate smoothing (see e.g. Woodford 1999*a*).

Specifically, assume that the central banker (under discretion) is one who minimizes

$$L_t^{CB} = \pi_t^2 + b_w \widetilde{y}_t^2 + b_r \left(r_t - r_{t-1}\right)^2 \tag{60}$$

subject to the constraints of private variables in (29)-(34). Notice that rather than having an objective with an interest-rate target or variability term, $b_r > 0$ involves a a concern for changes in the interest rate. Since this is also given in quadratic form, it means that the larger the changes in interest rate between two periods, the more the central bank is penalized in terms of its loss per period, L_t^{CB}. Now the first-order conditions become

$$(1-\gamma)\,\pi_t + \phi_{1,t} = 0 \tag{61}$$

$$b_w \widetilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t} = 0 \tag{62}$$

$$b_r \left(r_t - r_{t-1}\right) + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{63}$$

$$-\kappa_\psi \phi_{1,t} + \Gamma_y \phi_{2,t} - \lambda_F \phi_{3,t} - \phi_{4,t} = 0 \tag{64}$$

$$\gamma \pi_t + \phi_{3,t} = 0 \tag{65}$$

Lemma 3.2 *The Markov-perfect Nash equilibrium for the central bank in the case of interest-rate smoothing yields an optimal difference-type rule of the form*

$$r_t = r_{t-1} + \Phi_\pi \pi_t + \Phi_y \widetilde{y}_t \tag{66}$$

where

$$\Phi_\pi = b_r^{-1}\left\{(1-\gamma)\left[\kappa_\psi + \kappa_y \sigma^{-1}\left(1+\gamma\left(\sigma\eta - 1\right)\right) + \gamma\lambda_F\right]\right\} > 0$$

$$\Phi_y = b_w b_r^{-1} \sigma^{-1} \left[1 + \gamma \left(\sigma\eta - 1\right)\right] > 0.$$

Proof. This is a straightforward result from amending Lemma 3.1 for interest-rate growth in the first-order conditions; specifically in (63). ■

However, notice that now an additional pre-determined state variable r_{t-1} enters the Markov-perfect Nash equilibrium characterization. This is simply an artefact of the central banker's explicit interest rate smoothing objective which constrains the optimal time-consistent policy. In this case, with interest-rate smoothing, trade-offs in terms of policy targets, as in Proposition 3.3, still carry through. However, the trade off now is with respect to stabilizing interest-rate changes. This is summarized as follows.

Proposition 3.4 *Given the optimal discretionary rule (66), there is a trade-off between stabilizing interest-rate changes, inflation (domestic and imports) and output-gap stability.*

4 Numerical Simulation Results

In this section, the welfare and business cycle effect of delegating monetary policy to a central banker with an explicit taste for interest-rate smoothing is considered. This is considered alongside equilibrium outcomes under the original social loss function (44). Stabilization bias will be measured as the difference in society's loss function value under a given discretionary policy and society's loss function value under the theoretical assumption that a central bank can commit to minimizing society's true social loss. For instance, the benchmark stabilization bias will be measured as the difference in society's loss function value under problem (54)-(58) and society's loss function value under problem (45)-(49), for given benchmark parameterization of private and policy parameters.

The benchmark parameter values are set out as follows. The private sector parameters are retained from Monacelli (2003). The common rate of time preference is set as $\beta = 0.99$. The coefficient of relative risk aversion is set as $\sigma = 1$, implying a log period utility in consumption. The elasticity of substitution between home and foreign goods is given by $\eta = 1.5$. Labor supply elasticity is given by $\varphi = 3$

while price stickiness in both domestic and retail imports sectors are assumed equal, and they take on the standard value of $\theta_H = \theta_F = 0.75$. This implies average price-stickiness of 4 quarters. The degree of openness in the economy, governed by the imports share in the consumption basket is given by $\gamma = 0.4$. There are only two exogenous stochastic processes given by technology shocks domestically and abroad. The persistence parameter for both processes are $\rho = \rho^* = 0.9$ and their standard deviations are assumed to be one. Finally society's loss function (44) is parameterized as $b_w = 0.5$ and $b_r = 0.2$.

4.1 Business-cycle volatility and social loss

Table 1 summarizes the effect on the volatility (standard deviation) of the variables in the model under the different policy settings. The variables are the nominal one-period interest rate, r_t, the LOP gap, $\psi_{F,t}$, output gap, $\widetilde{y}_t$, nominal exchange rate, e_t, CPI inflation, π_t, the terms of trade, s_t, and the real exchange rate, q_t. Where applicable, the last two rows of the table refer to society's loss function value and the measure of stabilization bias, respectively.

A few results merit comment in Table 1. Firstly, suppose the central bank's loss function is indeed society's loss function. This is given as the first two columns labeled "Commitment" and "Discretion". If the central bank is unable to uphold the pre-commitment rule and ends up acting in discretion, this results in greater volatility for all variables except the LOP gap and the real exchange rate, which is driven in part by movements in the LOP gap, as shown in equation (39). Thus it appears that the central bank under discretion trades off inflation, output and interest rate variability for less variability in the LOP gap. The resulting loss is about 10 times larger than under pre-commitment or the stabilization bias in terms of social loss is 1.38.

The third and fourth columns of Table 1 has the central bank solve an alternative or delegated problem which involves the loss function with interest-rate smoothing (60). For the sake of comparison, the loss function values are kept the same as society's, although the b_r term now refers to a concern for interest-rate-change variability. Again, the case under pre-commitment will do better than its discretionary counterpart. However, since the pre-commitment case with interest rate smoothing (column 3) is not shared by society's true loss function, it registers a higher loss value than in the first column. Nevertheless this is much lower than the discretionary outcome in column 2.

Furthermore, even if it is accepted that the central banker under delegation will never have the incentive to commit – i.e. it will instead choose to act in discretion albeit with a smoothing preference now – it turns out that the variability of almost all variables are dampened compared to discretion in column 2. This is shown in column 4 and the stabilization bias is shown to have been reduced from 1.38 to 0.134, a ten times reduction in social loss. Finally, the last column, labeled "Delegation" refers to the case of optimal delegation. Optimal delegation in this context is defined as the choice of the policy weights on output gap and interest rate changes than minimizes social loss. This choice was obtained numerically using grid search over feasible spaces for the parameter pair (b_w, b_r) in the delegated central bank loss function. These were determined to be $(0.4, 0.17)$ respectively, as shown in Figure 1. Interestingly, this retains the Rogoff conservative central banker result. That is, the relative output concern of the central banker is much less than society's concern, or that the central bank cares more about inflation than society. A positive weight $b_r = 0.17$ shows that the difference rule (66) which results from discretion with interest-rate smoothing does improve welfare in the sense of further reducing the stabilization bias to 0.129.

These numerical results corroborate the intuition of Woodford (1999a) that by hiring a central banker who cares about interest-rate smoothing even when society does not value it, one can reduce the stabilization bias, it is accepted that the central bank is going to act in discretion anyway. More importantly, this conclusion is maintained in the case of a small open economy where monetary policy faces a trade off in terms of stabilizing domestic inflation and the LOP gap on the one hand and output gap and interest rate on the other.

Table 1: Business cycle volatility and welfare loss

	Commitment	Discretion	Commitment 2	Discretion 2	Delegation
r	0.056	0.189	0.092	0.106	0.107
ψ_F	0.429	0.161	0.170	0.103	0.101
$\widetilde{y}_t$	0.025	0.025	0.027	0.024	0.024
e	2.060	3.074	2.357	2.337	2.344
π	0.026	0.089	0.008	0.021	0.019
s	0.901	0.927	0.782	0.911	0.909
q	0.513	0.443	0.499	0.445	0.446
Loss	0.1636	1.5411	0.2104	0.2979	0.2930
Bias	-	1.38	-	0.134	0.129

4.2 Optimal Policy Inertia and Business-Cycle Dynamics

It is also interesting to consider the stabilization bias problem in terms of the magnitude, direction and persistence of dynamic adjustments in the model. Since the model identifies two exogenous serially correlated shocks in the form of domestic and foreign shocks, it would be interesting to analyze the individual effects of these shock on the volatility and persistence of the endogenous variables. In this section, the example of a domestic technology shock is taken up as one of the two cases. This is shown in Figure 2. The circled lines correspond to the case when optimal policy is conducted with a supposedly credible pre-commitment to society's loss function. The lines marked with diamonds are given by the case when optimal policy is operated under discretion but is still concerned with society's loss function. Finally the solid lines represent discretionary policy under optimal delegation (with interest-rate smoothing central bank).

Consider first the case of pre-commitment to society's lifetime loss. Generally, the amplitude of the impulse responses under this case are much smaller than both cases of discretion. Under a positive domestic technology shock, the direct effect through the production function (15) should increase the level of output gap. However, for a given level of nominal interest rate, the technology shock lowers the natural interest rate, resulting in a larger gap between the nominal and natural interest rates. This has a tendency to depress the output gap initially. This can be seen by inspecting the IS equation in (31). However, since policy responds to output-gap deviations, the nominal interest rate falls, and this creates a positive output gap eventually. Given a fall in the nominal interest rate, there is a currency depreciation resulting in the nominal exchange rate deviation being positive under uncovered interest rate parity. Alternatively, one can observe from (38) that a positive technology shock has the Samuelson-Balassa effect of improving the small open economy's terms of trade and therefore creating a depreciation of its currency. A depreciation of the domestic currency which is persistent, creates an expectation that future imports prices will be falling as demand switches from imports to domestic goods. This causes domestic inflation to rise (which is also boosted by the rise in output gap) while imports inflation falls negating the tendency of a depreciation to create a positive LOP gap. In fact a negative LOP gap is obtained which reinforces the fall in imports inflation. This can be verified by inspecting equations (32)-(34).

When the central bank has the incentive to cheat by creating too much stabilization, since it ignores the promises which would have been made in some distant past by a pre-commiting central bank, it creates a much larger disinflation in domestic and imports inflation processes. Furthermore, it has the tendency to lower nominal interest rate by a very large amplitude relative to the natural interest rate such that it engineers a negative output gap response. This can be thought of as the reverse case of the stabilization-bias analogue to the average inflation bias problem. Here a positive domestic technology shock is countered by a discretionary policy that attempts to prevent too much "overheating" in output that it actually causes a negative output gap response.

However, when one considers allowing for the case of optimally delegating discretionary policy to a policy maker who has the additional preference for interest-rate smoothing, it can be seen that the impulse responses tend to track those of the outcomes under pre-commitment much better; and this is a desirable property if the central bank were to act with discretion. Essentially the story of interest-rate smoothing as an approximate means of constraining the central banker in its optimal policy (as though it is following through on some pre-commitment policy) is corroborated by the impulse responses.

5 Conclusion

In this paper, the role of optimal interest-rate smoothing is considered in the context of a small open economy model that faces incomplete exchange-rate pass through to import prices. This gave rise to a law-of-one-price gap, that acted as an endogenous shifter to the forward-looking IS curve (representing aggregate demand) and the Phillips curves for domestic and imports inflation. In such a context, domestic monetary policy can no longer target just domestic inflation, as proposed by Clarida et al. (2001). In this case, monetary policy was assumed to target several variables, namely the CPI inflation measure, which is a convex combination of domestic and imports inflation, the output gap and the interest rate.

It was shown that Woodford's (1999*a*) conclusion about optimal monetary policy inertia still carries through in a small-open-economy setting that breaks the closed–and-open-economy monetary policy isomorphism. The paper proceeded from the benchmark of assuming that the central bank can solve a pre-commitment policy problem. However, if it is accepted that the central bank cannot credibly commit to that policy and thus acts in discretion, this creates too much stabilization on the central bank's part. Such a stabilization bias manisfested itself in the form of greater uncertainty around the macroeconomic variables in the model. The bias is also measured as a larger loss in terms of society's common loss function.

A possible solution, as was proposed by Woodford (1999*a*), is to hire a central banker whose preferences include interest-rate smoothing even though this is not shared by society's preferences. It was shown in the paper that allowing for interest-rate smoothing results in a difference rule for the interest rate. That is, optimal (discretionary) policy in such a case involves setting the change in interest rate in response to CPI inflation and output gap. The reason for having an interest-rate conservative central banker is that it tends to introduce intrinsic inertia into the interest rate process, thus approximating the desired pre-commitment outcome. While pure discretionary policy without interest-rate smoothing carries a large trade off between stabilizing domestic inflation, output gap, interest rate and the LOP gap, optimally delegating policy to an interest-rate smoother is seen to dampen such a trade off. Therefore, it was found that the stabilization bias under discretion was greatly reduced when one delegates policy to a central banker with a taste for interest-rate smoothing.

Continuing work on this paper includes comparing the performance of simple interest-rate smoothing policy rules with optimal policy. The relationship between the degree of exchange-rate pass through and the concern for smoothing interest rates will also be examined. Finally, robustness of the conclusion will also be checked with regard to alternative aggregate demand and aggegrate supply dynamic specifications.

APPENDIX

A Optimal Plan with Pre-commitment

When the central banker is assumed to be able to commit at time 0 to minimize the present discounted value of all future losses subject to the constraint of the evolution of the private sector. Specifically the central banker solves the following linear-quadratic dynamic programming problem:

$$\max_{\left\{\pi_{H,t},\pi_{F,t},r_t,\widetilde{y}_t,\psi_{F,t}\right\}_{t\in\{0,\mathbb{Z}_+\}}} -\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left[\left(\pi_t-\pi^{opt}\right)^2+b_w\left(\widetilde{y}_t-\widetilde{y}^{opt}\right)^2+b_r r_t^2\right] \quad (67)$$

such that

$$\pi_t=(1-\gamma)\,\pi_{H,t}+\gamma\pi_{F,t} \quad (68)$$

$$\pi_{H,t}=\beta E_t\pi_{H,t+1}+\kappa_y\widetilde{y}_t+\kappa_\psi\psi_{F,t} \quad (69)$$

$$\widetilde{y}_t=E_t\widetilde{y}_{t+1}-\frac{\omega_s}{\sigma}\left(r_t-E_t\pi_{H,t+1}-r_t^n\right)+\Gamma_y E_t\left(\psi_{F,t+1}-\psi_{F,t}\right) \quad (70)$$

$$\pi_{F,t}=\beta E_t\pi_{F,t+1}+\lambda_F\psi_{F,t} \quad (71)$$

$$\psi_{F,t}=E_t\psi_{F,t+1}-r_t+r_t^*+E_t\pi_{F,t+1}+E_t\pi_{t+1}^* \quad (72)$$

In our steady state, we have $\pi^{opt}=\widetilde{y}^{opt}=0$. Folding constraint (68) directly into the objective (67) the problem can be written as a Lagrangian:

$$\begin{aligned}
&\max_{\left\{\pi_{H,t},\pi_{F,t},r_t,\widetilde{y}_t,\psi_{F,t}\right\}_{t\in\{0,\mathbb{Z}_+\}}} -\frac{1}{2}E_0\{\sum_{j=0}^{\infty}\beta^j\left[\left((1-\gamma)\,\pi_{H,t}+\gamma\pi_{F,t}\right)^2+b_w\widetilde{y}_t^{\,2}+b_r r_t^2\right]\\
&+2\phi_{1,t}\left[\pi_{H,t}-\beta E_t\pi_{H,t+1}-\kappa_y\widetilde{y}_t-\kappa_\psi\psi_{F,t}\right]\\
&+2\phi_{2,t}\left[\widetilde{y}_t-E_t\widetilde{y}_{t+1}+\frac{\omega_s}{\sigma}\left(r_t-E_t\pi_{H,t+1}-r_t^n\right)-\Gamma_y E_t\left(\psi_{F,t+1}-\psi_{F,t}\right)\right]\\
&+2\phi_{3,t}\left[\pi_{F,t}-\beta E_t\pi_{F,t+1}-\lambda_F\psi_{F,t}\right]\\
&+2\phi_{4,t}\left[\psi_{F,t}-E_t\psi_{F,t+1}-r_t+r_t^*+E_t\pi_{F,t+1}-E_t\pi_{t+1}^*\right]\}
\end{aligned}$$

The first-order conditions are then given by (45)-(49). These are repeated here for the reader's convenience:

$$(1-\gamma)\,\pi_t + \phi_{1,t} - \phi_{1,t-1} - \frac{\omega_s}{\beta\sigma}\phi_{2,t-1} = 0 \tag{73}$$

$$b_w \widetilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t} - \beta^{-1}\phi_{2,t-1} = 0 \tag{74}$$

$$b_r r_t + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0 \tag{75}$$

$$-\kappa_\psi \phi_{1,t} + \Gamma_y\left(\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\right) - \lambda_F \phi_{3,t} + \beta^{-1}\phi_{4,t-1} - \phi_{4,t} = 0 \tag{76}$$

$$\gamma\pi_t + \phi_{3,t} - \phi_{3,t-1} + \beta^{-1}\phi_{4,t-1} = 0 \tag{77}$$

In the following steps, it will be shown that the implied interest-rate rule under pre-commitment yields an Taylor-type interest-rate process which is intrinsically inertial. First, substitute (77) into (76) to eliminate the $\beta^{-1}\phi_{4,t-1}$ term:

$$-\kappa_\psi \phi_{1,t} + \Gamma_y\left(\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\right) - (1+\lambda_F)\,\phi_{3,t} + \phi_{3,t-1} - \phi_{4,t} = \gamma\pi_t \tag{78}$$

And substituting (74) into (78) to eliminate $\phi_{2,t} - \beta^{-1}\phi_{2,t-1}$:

$$-\kappa_\psi \phi_{1,t} - \Gamma_y\left(b_w \widetilde{y}_t - \kappa_y \phi_{1,t}\right) - (1+\lambda_F)\,\phi_{3,t} + \phi_{3,t-1} - \phi_{4,t} = \gamma\pi_t \tag{79}$$

and further substitution of (75) into (79) returns

$$(\kappa_y \Gamma_y - \kappa_\psi)\,\phi_{1,t} - b_w \Gamma_y \widetilde{y}_t - (1+\lambda_F)\,\phi_{3,t} + \phi_{3,t-1} - \gamma\pi_t - b_r r_t - \frac{\omega_s}{\sigma}\phi_{2,t} = 0. \tag{80}$$

Now substitute (74) into (73) to gain

$$(1-\gamma)\,\pi_t + \phi_{1,t} - \phi_{1,t-1} - \frac{\omega_s}{\sigma}\left(b_w \widetilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t}\right) = 0$$

and finally, using (80) in the last equation gives (51).

B Proof of Proposition 3.1

The system of forward-looking private-sector variables (29)-(34) together with the predetermined Lagrange multipliers can be written in the canonical form

$$\begin{bmatrix} E_t\mathbf{x}_{t+1} \\ \boldsymbol{\phi}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\phi}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{F} \\ \mathbf{0}_4 \end{bmatrix} \mathbf{z}_t \tag{81}$$

where

$$\mathbf{x}_t = \begin{bmatrix} \pi_{H,t} \\ \widetilde{y}_t \\ \pi_{F,t} \\ \psi_{F,t} \\ e_t \end{bmatrix}, \ \boldsymbol{\phi}_t = \begin{bmatrix} \phi_{1,t} \\ \phi_{2,t} \\ \phi_{3,t} \\ \phi_{4,t} \end{bmatrix}, \ \mathbf{z}_t = \begin{bmatrix} z_t^* \\ z_t \end{bmatrix}.$$

In a rational expectations (RE) equilibrium given by Definition 2.1, the unique and bounded solution for the forward-looking part of the model can be found by "solving forward" to yield

$$\mathbf{x}_t = \mathbf{G}\boldsymbol{\phi}_{t-1} - \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)}\mathbf{F}E_t\mathbf{z}_{t+j} \tag{82}$$

where $\mathbf{G}$ and $\mathbf{H}$ contain coefficients that are determined in the RE equilibrium. The predetermined Lagrange multipliers can be solved backward to obtain

$$\boldsymbol{\phi}_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}\boldsymbol{\phi}_{t-1} = \mathbf{N}\boldsymbol{\phi}_{t-1} - \mathbf{C}\sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)}\mathbf{F}E_t\mathbf{z}_{t+j} \tag{83}$$

where $\mathbf{N} = \mathbf{CG} + \mathbf{D}$. By recursive backward substitution of (83), one obtains

$$\begin{aligned}
\boldsymbol{\phi}_t &= \mathbf{N}\left(\mathbf{N}\boldsymbol{\phi}_{t-2} - \mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_t\mathbf{z}_{t+j-1}\right) - \mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_t\mathbf{z}_{t+j} \\
&\vdots \\
&= \mathbf{N}^{t+1}\boldsymbol{\phi}_{-1} - \sum_{s=0}^{t}\mathbf{N}^s\left(\mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_{t-s}\mathbf{z}_{t-s+j}\right).
\end{aligned}$$

Given the initial conditions $\boldsymbol{\phi}_{-1} = \mathbf{0}_{4\times 1}$, this can be written as

$$\boldsymbol{\phi}_t = -\sum_{s=0}^{t}\mathbf{N}^s\left(\mathbf{C}\sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_{t-s}\mathbf{z}_{t-s+j}\right). \tag{84}$$

From the first-order condition (47) of the central bank's problem, we have

$$b_r r_t + \frac{\omega_s}{\sigma}\phi_{2,t} - \phi_{4,t} = 0$$

which, in matrix notation and making use of the solution (84), can be re-written as

$$\begin{aligned}
b_r r_t = \begin{bmatrix} 1 \\ -\frac{\omega_s}{\sigma} \end{bmatrix}' \Bigg\{ &\begin{bmatrix} \mathbf{N}_{21}\phi_{1,t-1} + \mathbf{N}_{22}\phi_{2,t-1} + \mathbf{N}_{23}\phi_{3,t-1} + \mathbf{N}_{24}\phi_{4,t-1} \\ \mathbf{N}_{41}\phi_{1,t-1} + \mathbf{N}_{42}\phi_{2,t-1} + \mathbf{N}_{43}\phi_{3,t-1} + \mathbf{N}_{44}\phi_{4,t-1} \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{C}_2 \sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F} \\ \mathbf{C}_4 \sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F} \end{bmatrix}\Bigg\} E_t\mathbf{z}_{t+j}
\end{aligned}$$

where $\mathbf{N}_{ij}$ refers to the (i,j)-th element of the matrix $\mathbf{N}$ and $\mathbf{C}_i$ refers to the i-th row of matrix $\mathbf{C}$.

Now use can be made of (47) again, by lagging it one period, to replace the $\phi_{4,t-1}$ terms above to yield

$$\begin{aligned}
b_r r_t &= \begin{bmatrix} 1 \\ -\frac{\omega_s}{\sigma} \end{bmatrix}' \Bigg\{ \begin{bmatrix} \mathbf{N}_{21}\phi_{1,t-1} + \mathbf{N}_{22}\phi_{2,t-1} + \mathbf{N}_{23}\phi_{3,t-1} + \mathbf{N}_{24}\left(b_r r_{t-1} + \frac{\omega_s}{\sigma}\phi_{2,t-1}\right) \\ \mathbf{N}_{41}\phi_{1,t-1} + \mathbf{N}_{42}\phi_{2,t-1} + \mathbf{N}_{43}\phi_{3,t-1} + \mathbf{N}_{44}\left(b_r r_{t-1} + \frac{\omega_s}{\sigma}\phi_{2,t-1}\right) \end{bmatrix} \\
&\quad + \begin{bmatrix} \mathbf{C}_2 \\ \mathbf{C}_4 \end{bmatrix}\Bigg\} \sum_{j=0}^{\infty}\mathbf{H}^{-(j+1)}\mathbf{F}E_t\mathbf{z}_{t+j}.
\end{aligned}$$

Using (84) we can re-write this as

$$r_t = \rho_r r_{t-1} - \mathbf{\Theta}_b \sum_{s=0}^{t-1} \mathbf{N}^s \mathbf{C} \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} E_{t-s} \mathbf{z}_{t+j-s-1} - \mathbf{\Theta}_f \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} E_t \mathbf{z}_{t+j}$$

where

$$\begin{aligned}
\rho_r &= N_{44} - \frac{\omega_s}{\sigma} N_{24}, \\
\mathbf{\Theta}_b &= b_r^{-1} \begin{bmatrix} 1 \\ -\frac{\omega_s}{\sigma} \end{bmatrix}' \begin{bmatrix} \mathbf{N}_{21} & \left(\mathbf{N}_{22} + \frac{\omega_s}{\sigma} N_{24}\right) & \mathbf{N}_{23} \\ \mathbf{N}_{41} & \left(\mathbf{N}_{42} + \frac{\omega_s}{\sigma} N_{44}\right) & \mathbf{N}_{43} \end{bmatrix} \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times 1} \end{bmatrix}, \\
\mathbf{\Theta}_f &= b_r^{-1} \begin{bmatrix} 1 \\ -\frac{\omega_s}{\sigma} \end{bmatrix}' \begin{bmatrix} \mathbf{C}_2 \\ \mathbf{C}_4 \end{bmatrix}.
\end{aligned}$$

In the model, it was assumed that $\mathbf{z}_t$ follows a first-order Markov process given by the transition matrix

$$\mathbf{M} = \begin{bmatrix} \rho_z^* & 0 \\ 0 & \rho_z \end{bmatrix}$$

where $\mathbf{M}$ is stable. Therefore, the interest rate process can be further solved in terms of the primitive shocks as (52), given below:

$$r_t = \rho_r r_{t-1} - \mathbf{\Theta}_b \sum_{s=0}^{t-1} \mathbf{N}^s \mathbf{C} \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} \mathbf{M}^j \mathbf{z}_{t-s-1} - \mathbf{\Theta}_f \sum_{j=0}^{\infty} \mathbf{H}^{-(j+1)} \mathbf{F} \mathbf{M}^j \mathbf{z}_t.$$

The optimal interest-rate process under the pre-commitment policy results in an inertial rule so long as $N_{44} - \frac{\omega_s}{\sigma} N_{24} \neq 0$. This rule is both forward and backward looking in terms of past and current forecasts of foreign and domestic technology shocks, since $\mathbf{\Theta}_b \neq 0$ and $\mathbf{\Theta}_f \neq 0$. Finally, since the coefficient on lag interest rate, ρ_r, is independent of $\mathbf{M}$, the inertia in policy under pre-commitment in this small open-economy-model is not an artefact of serial correlation in the exogenous stochastic processes $\mathbf{z}_t$.

C Proof of Proposition 3.2

Since

$$\Phi_\pi = b_r^{-1}\left\{(1-\gamma)\left[\kappa_\psi + \kappa_y\sigma^{-1}\left(1+\gamma\left(\sigma\eta-1\right)\right)+\gamma\lambda_F\right]\right\} > 0$$

$$\Phi_y = b_w b_r^{-1}\sigma^{-1}\left[1+\gamma\left(\sigma\eta-1\right)\right] > 0.$$

and given that

$$\begin{aligned}\kappa_y &= \lambda_H\left(\varphi+\frac{\sigma}{\omega_s}\right) > 0\\ \kappa_\psi &= \lambda_H\left(1-\frac{\omega_\psi}{\omega_s}\right) \geq 0\\ \omega_s &= 1+\gamma\left(2-\gamma\right)\left(\sigma\eta-1\right) \geq \omega_\psi = 1+\gamma\left(\sigma\eta-1\right) > 0\end{aligned}$$

where

$$\begin{aligned}\lambda_H &= \frac{\left(1-\theta_H\right)\left(1-\beta\theta_H\right)}{\theta_H}\\ \lambda_F &= \frac{\left(1-\theta_F\right)\left(1-\beta\theta_F\right)}{\theta_F}\end{aligned}$$

Using the chain rule of differentiation, since $\theta_H \in [0,1]$ and $\beta \in (0,1)$,

$$\begin{aligned}\frac{\partial\Phi_\pi}{\partial\theta_H} &= \frac{\partial\Phi_\pi}{\partial\kappa_y}\cdot\frac{\partial\kappa_y}{\partial\lambda_H}\cdot\frac{\partial\lambda_H}{\partial\theta_H}+\frac{\partial\Phi_\pi}{\partial\kappa_\psi}\cdot\frac{\partial\kappa_\psi}{\partial\lambda_H}\cdot\frac{\partial\lambda_H}{\partial\theta_H}\\ &= \left(\frac{\partial\Phi_\pi}{\partial\kappa_y}\cdot\frac{\partial\kappa_y}{\partial\lambda_H}+\frac{\partial\Phi_\pi}{\partial\kappa_\psi}\cdot\frac{\partial\kappa_\psi}{\partial\lambda_H}\right)\times\left(\beta-\theta_H^{-2}\right) < 0\end{aligned}$$

given the functions κ_y, κ_ψ and λ_H above such that the first term on the RHS is positive. Therefore the interest-rate instrument elasticity with respect to CPI inflation, Φ_π, is lower the greater the price stickiness in domestic goods, θ_H.

Similarly,

$$\frac{\partial\Phi_\pi}{\partial\theta_F} = \frac{\partial\Phi_\pi}{\partial\lambda_F}\cdot\frac{\partial\lambda_F}{\partial\theta_F} = \frac{\partial\Phi_\pi}{\partial\lambda_F}\cdot\left(\beta-\theta_F^{-2}\right) < 0$$

meaning that the interest-rate instrument elasticity with respect to CPI inflation,

Φ_π, is lower the greater the price stickiness in retail imported goods, θ_F.

It is straightforward to see that neither θ_H nor θ_F appear in the optimal response weight Φ_y.

D Proof of Proposition 3.3

Under the optimal discretionary rule (59) we have

$$\Phi_\pi = b_r^{-1}\left\{(1-\gamma)\left[\kappa_\psi + \kappa_y\sigma^{-1}\left(1+\gamma\left(\sigma\eta-1\right)\right)+\gamma\lambda_F\right]\right\} > 0$$

$$\Phi_y = b_w b_r^{-1}\sigma^{-1}\left[1+\gamma\left(\sigma\eta-1\right)\right] > 0.$$

Because $\partial\Phi_y/\partial b_r < 0$ and $\partial\Phi_\pi/\partial b_r < 0$ in (59), the greater the concern for interest rate volatility, the greater the volatility on output gap and inflation, all else being equal, since the responses of the policy instrument to inflation and output gap become weaker.

Alternatively, the first-order conditions in (54)-(58) can be reduced to

$$\pi_t = \Phi_r r_t - \Phi_{y\pi}\widetilde{y}_t$$

where

$$\Phi_r = \frac{b_r}{\left\{(1-\gamma)\left[\kappa_\psi + \kappa_y\sigma^{-1}\left(1+\gamma\left(\sigma\eta-1\right)\right)+\gamma\lambda_F\right]\right\}} > 0$$

$$\Phi_{y\pi} = \frac{b_w\sigma^{-1}\left[1+\gamma\left(\sigma\eta-1\right)\right]}{\left\{(1-\gamma)\left[\kappa_\psi + \kappa_y\sigma^{-1}\left(1+\gamma\left(\sigma\eta-1\right)\right)+\gamma\lambda_F\right]\right\}} > 0.$$

Since $\partial\Phi_r/\partial b_r > 0$ and $\partial\Phi_{y\pi}/\partial b_w > 0$, this implies, *ceteris paribus*, the greater the concern for interest-rate or output gap variability, the greater is inflation volatility.

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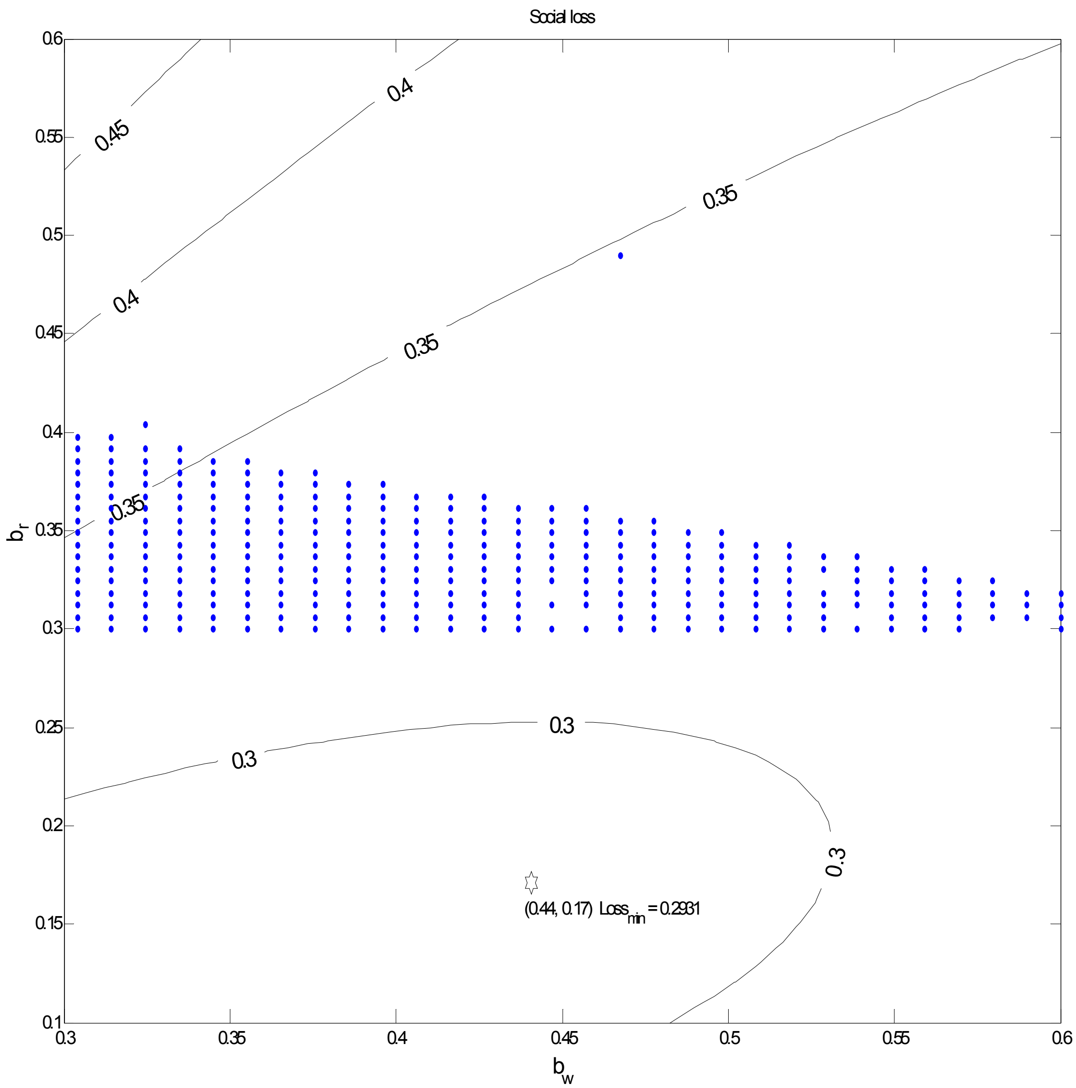

Figure 1: Optimal delegation. Social loss as a function of b_w and b_r in the delegate central banker's loss function with interest-rate smoothing objective. The optimal delegation is $b_w = 0.44$ and $b_r = 0.17$. (The dotted patch represents multiple equilibria outcomes.)

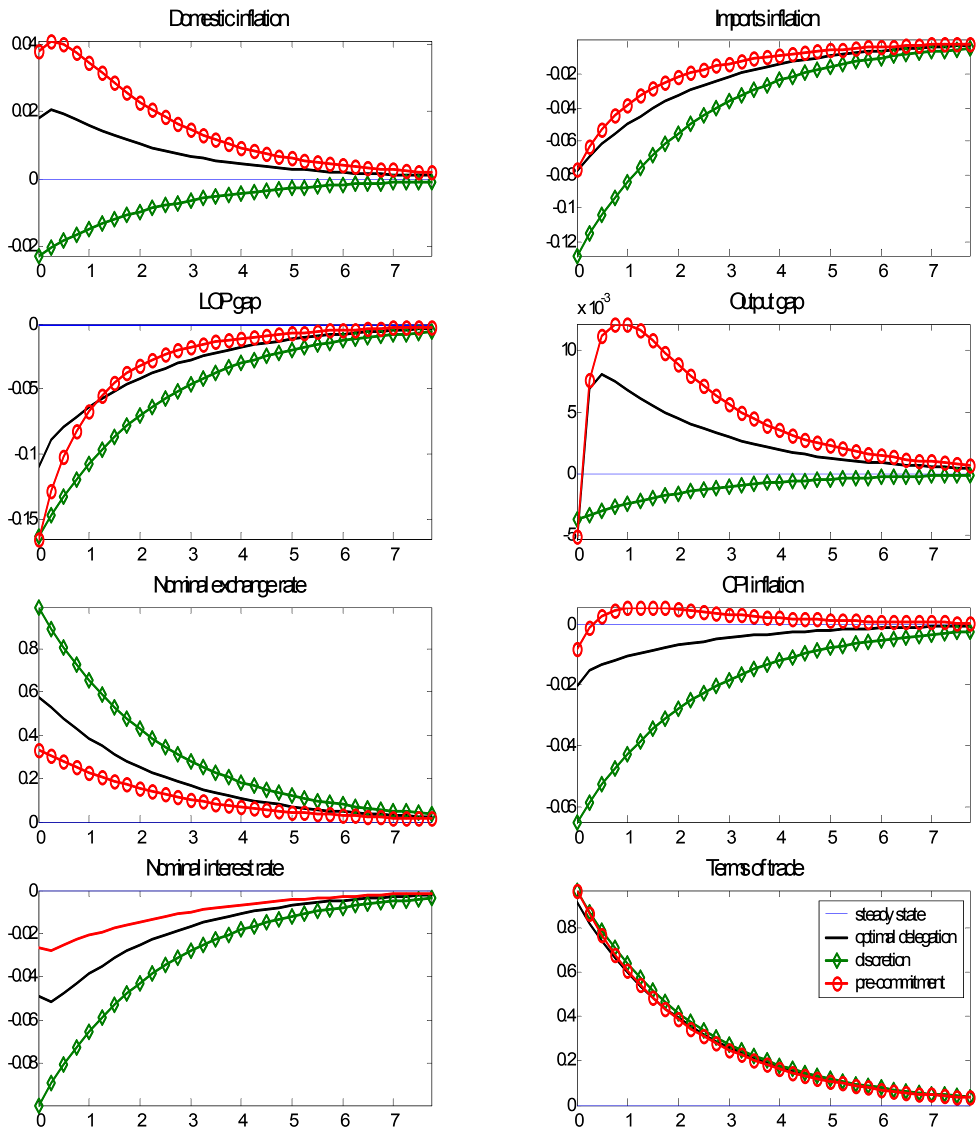

Figure 2: Impulse responses to domestic technology shock