A Single-Index Dynamic Factor Model for Current-Quarter Estimates of Economic Activity in Hong Kong

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Abstract

This paper applies the single-index dynamic factor model, developed by Stock and Watson (1991), to produce current-quarter estimates of economic activity in Hong Kong. The Hang Seng index, a residential property price index, retail sales and total exports are used as coincident indicators in the model. Principal Component Analysis is first used to obtain an impression of the common component of the indicator series. This component and the dynamic factor identified by the Stock-Watson methodology are strongly correlated and seem to account for economic fluctuations in Hong Kong reasonably well.

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1. Introduction

To conduct economic policy, some degree of foresight is needed, as many policy actions require a long lag to have an effect. In light of this need, government agencies, central banks and economic research institutes across the world are routinely producing macro economic forecasts for policy purposes. Much research has been devoted to the issue of how to construct more accurate and more timely economic forecasts. In particularly, one approach, using the fact that economic activity evolves in cycles, attempts to find reliable forecasting tools for business cycle turning points. This approach has led to an extensive literature developing, from the early landmark study by Burns and Mitchell (1946) to the much more formalised approach of Stock and Watson (1989, 1991 and 1993).

Burns and Mitchell (1946) develop a list of composite leading, coincident, and lagging indexes of business cycles, using a large numbers of economic variables (so-called indicator variables). These indexes have played an important role for summarising the state of macroeconomic activity in the United States. Burns and Mitchell's definition of business cycles has two key features. The first is the comovements among individual indicator variables. The second is their division of business cycles into separate phases or regimes.

Using Burns and Mitchell's notion of business cycles, Stock and Watson (1989, 1991 and 1998) formulate a modern statistical framework to study business fluctuations. They assume that the comovements have a common element that can be captured by a single underlying, unobservable variable and that this unobservable variable represents the general "state of the economy." They call their model, which provides a formal definition of the unobservable state of the economy, the single-index model. Using this model, they compute a composite index of coincident indicators.

Since the seminal work of Stock and Watson, the single-index model, also called the one-factor model, has been widely used by many other researchers. Recent literature includes Camba-Mendez (2001) and Garcia-Ferrer and Poncela (2002), who modify the model to forecast GDP growth for European countries. For Germany in particular, Bandholz and Funke (2003) use the model to develop leading indicators of economic activity in the country. In Asia, Fukuda and Onodera (2001) and Chen and Lin (2000) apply the model to improve forecasts of the Japanese economy and to identify turning points and business cycles in Taiwan respectively.

In the case of Hong Kong, real GDP figures are published almost three months after the reference quarter. This arrangement may not be prompt enough to monitor the economy on an ongoing basis. Thus, an indication of the state of the economy appears warranted.

The APEC Study Centre of the Hong Kong University has developed a high frequency macroeconomic forecasting model for Hong Kong[1] in collaboration with Lawrence Klein of the Wharton School of the University of Pennsylvania. Since the first quarter of 2000, the APEC Study Centre has been regularly producing quarterly forecasts of the growth of Hong Kong GDP using a large number of high frequency financial and macroeconomic variables, Principal Component Analysis (PCA) and the bridge equation approach.

In this paper, we apply Stock and Watson's single-index model to construct a coincident indicator of economic activity in Hong Kong. For computational efficiency, we only use four monthly indicator series—two financial series and two macroeconomic series—for the analysis. Applying the single-index model on the selected indictor series and using the Kalman filter for the estimation of model parameters and state vectors, we generate estimates of the growth rate of the real GDP in the same quarter before it is published.

This paper proceeds as follows. Section 2 describes the data used in the empirical studies in this paper. Section 3 presents the results of applying PCA on obtaining an impression of the unobservable comovement component. Section 4 describes the single-index model. Section 5 presents the empirical results from using the model and an index of coincident indicators of economic activity in Hong Kong. The last section concludes.

[1] High frequency macroeconomic forecasting models for Hong Kong, APEC Study Centre, Hong Kong Institute of Economic and Business Strategy University of Hong Kong. The forecasts are available on the following website link: http://www.hku.hk/apec/.

2. The Data for Empirical Analysis

The purpose of this section is to explain the choice of indicator series used in the single-index model estimated below. A large number of monthly financial and macroeconomic variables may contain useful information about real economic activity in Hong Kong.

Given the range of possible indicator series, we first reduce the dimensionality of the problem by selecting a subset of the indicator series: the Hang Seng index, the 3-month Hong Kong interbank offer rate, a residential property price index, total exports, retained imports, retail sales, tourist arrivals and electricity consumption.[2] The sample period is from January 1990 to December 2002. Our choice of these eight series is based on the assumption that they may be more relevant to be used as contemporaneous indicators.

The eight indicator variables, except the HIBOR series which is in percentage points, are shown in logarithmic scale in Figure 1. Furthermore, the five macroeconomic variables have been seasonally adjusted by the X-12 method. The figure suggests that all variables can be characterised as having a stochastic trend, except the interest rate series.

For better computational efficiency, it would be desirable to use a subset of these eight series. In order to select the series that are more informative for real activity, we first look at the cross correlation between their growth rates and the seasonally adjusted quarterly growth rates of real GDP.

Figure 2 shows the cross correlates between the growth rates of each of the eight series (for interest rates we use the level of the series) and the quarterly real growth of Hong Kong from lead 8 quarters to lag 8 quarters.[3] It is clear that all of them except the electricity consumption display considerable correlation with GDP. The Hang Seng Index, property price index, total exports, retail sales and retained imports series have strong positive correlation with the GDP, whereas the HIBOR series shows negative correlation as expected. Because the first four series have the largest contemporaneous correlation, we select them as coincident indicators for the empirical work.

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[2] The data and the Hong Kong real GDP growth come from the CEIC Data Ltd database.

[3] The cross-correlation function of *GDP* and indicator series (*I*) is estimated by the following equation:

$$\rho_{GDP,I}(k) = \Sigma\left(GDP_t - \overline{GDP}\right)\left(I_{t-k} - \bar{I}\right) \Big/ \sqrt{\Sigma\left(GDP_t - \overline{GDP}\right)^2 \Sigma\left(I_t - \bar{I}\right)^2}$$

Regarding the time series properties, the selected four indicator series all have a unit root, confirmed by the augmented Dickey-Fuller test. Following the Stock and Watson's methodology, we assume that the four indicator series are not cointegrated. Different modelling strategies are needed, however, for unobservable-component models with cointegrated variables[4].

3. Principal Components Analysis

To obtain an impression of the unobservable comovement component of the four indicator series, we first apply Principal Components Analysis (PCA) to extract the first principal component from the data. It is well known that the general objectives of PCA are data reduction and interpretation.[5] It explains the variance-covariance structure of the data using linear combinations of the original variables.

Figure 3 shows the monthly growth rates of the four indicator series. We find that if we apply PCA to these series, the first and second principal components (PCs) all work strongly correlated with the quarterly GDP growth. It is because the principal components extracted from the monthly growth rates will lose the orthogonal property when they are converted into quarterly frequency for the calculations of the correlation with the quarterly GDP growth.

However, if we first convert the monthly growth rates into quarterly rates and then apply PCA on them, the first PC is highly correlated with the quarterly real GDP growth. Table 1 indicates that the first and second PCs capture about 53% and 22% of the variance for the data.

The correlation between the first PC and the quarterly GDP growth rate is 0.59, much larger than that, 0.09, of the second PC. The third and fourth PCs are even less correlated with the quarterly GDP growth rate. One interpretation of this finding that there is an unobservable underlying component among the quarterly growth rates of the

[4] Harvey, Fernandez-Macho, and Stock (1987) discuss modelling strategies for unobservable-component models with cointegrated variables.

[5] For a good introduction to the theory and the applications of Principal Component Analysis, see Jolliffe (1986).

indicator four series that is highly correlated with the overall economic conditions in Hong Kong.

4. The Single-Index Dynamic Factor Model

In this section we present the single-index model we use to construct a coincident index for the Hong Kong economy. We first outline the model, show how it can be written in state-space form and discuss how to estimate it using Kalman filtering.

4.1 Specification

The single-index model we use follows Stock and Watson (1991). It can be formulated in terms of the first differences of the four indicator variables, as follows:

$$(1)\qquad \Delta Y_{it} = D_i + \gamma_i \Delta C_t + u_{it}, \qquad\qquad i = 1,\ldots,4$$

$$(2)\qquad (\Delta C_t - \delta) = \phi_1(\Delta C_{t-1} - \delta) + \ldots \phi_p(\Delta C_{t-p} - \delta) + \eta_t, \qquad \eta_t \sim i.i.d.N(0,\sigma_\eta^2)$$

$$(3)\qquad u_{it} = d_{i1}u_{it-1} + \ldots + d_{iq}u_{it-q} + v_{it}, \qquad v_{it} \sim i.i.d.N(0,\sigma_v^2)\, and\; i = 1,\ldots,4$$

where Y_{it} denotes the logarithm of series i.

In the above model, ΔY_{it} consists of two stochastic components: an unobservable common component, ΔC_t and an idiosyncratic component, u_{it}. Both of these components are modelled as autoregressive stochastic processes, AR(p) and AR(q), respectively. For a normalisation, the scale of ΔC_t, is identified by setting σ_η^2 to unity. In addition, all shocks are assumed to be independent

The main identifying assumption in the above model is that the comovements in the indicator series arise from the single source C_t, i.e. ΔC_t enters each indicators with a different weights, γ_i, $i = 1,\ldots,4$. This is made precise by assuming that u_{it} and ΔC_t are mutually uncorrelated at all leads and lags for all series.

Note that, as the parameters D_i and δ are not separately identified, Stock and Watson (1991) suggest writing the model in deviation from means, thus concentrating the D_i and $\gamma_i\delta$ terms out of the likelihood function:

(4) $$\Delta y_{it} = \gamma_i \Delta c_t + u_{it}, \qquad i = 1,...,4$$

(5) $$\Delta c_t = \phi_1 \Delta c_{t-1} + ...\phi_p \Delta c_{t-p} + \eta_t, \qquad \eta_t \sim i.i.d.N(0,1)$$

(6) $$u_{it} = d_{i1} u_{it-1} + ... + d_{iq} u_{it-q} + v_{it}, \qquad v_{it} \sim i.i.d.N(0,\sigma_i^2)\ and\ i = 1,...,4$$

where $\Delta y_{it} = \Delta Y_{it} - \Delta \overline{Y}_i$ and $\Delta c_t = \Delta C_t - \delta$.

As the single-index model in deviation from means is linear in the unobservable components, we can use the Kalman filter to construct the Gaussian likelihood function and to estimate the unknown parameters of the model by maximising the likelihood function. However, to use the Kalman filter, we have to transform the above three equations into state space form.[6]

4.2 State-Space Representation

The state space form of the system is comprised of a measurement equation and a transition (or called state) equation. The measurement equation, which relates the observed variables to the elements of the state vector, is given by:

(7) $$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \\ \Delta y_{4t} \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \gamma_4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ \Delta c_{t-2} \\ u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ C_{t-1} \end{bmatrix}$$

[6] For a discussion of state-space models and the Kalman filter, see Harvey (1989, 1990) or Hamilton (1994).

The transition equation, which describes the evolution of the unobservable state vector, which in our case contains Δc_t and u_{it} and their lags can be written:

$$(8)\qquad \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ \Delta c_{t-2} \\ u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ C_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{31} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_{41} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \Delta c_{t-1} \\ \Delta c_{t-2} \\ \Delta c_{t-3} \\ u_{1t-1} \\ u_{2t-1} \\ u_{3t-1} \\ u_{4t-1} \\ C_{t-2} \end{bmatrix} + \begin{bmatrix} \eta_t \\ 0 \\ 0 \\ v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \\ 0 \end{bmatrix}$$

where $[\eta_t \quad 0 \quad 0 \quad v_{1t} \quad v_{2t} \quad v_{3t} \quad v_{4t} \quad 0]$ is the vector of disturbances, which we assume is diagonal. This assumption implies that shocks to the unobservable common component and the idiosyncratic components are mutually uncorrelated at all leads and lags.

4.3 Estimation

As mentioned before, to estimate the model, which is given by the *measurement* equation and the *transition* equation, we form the likelihood function:

$$(9)\qquad \log L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_1^T \log|F_t| - \frac{1}{2}\sum_1^T v_t^T F_t^{-1} v_t ,$$

where T, v_t and F_t denote the sample size, the prediction errors and the mean square matrix of the prediction errors, respectively. Estimates of the model can then be obtained by numerically maximising the likelihood function, using the Kalman filter. Since the indicator variables are non-stationary, in estimating the model we follow the suggestions of Harvey (1989) and assume that the prior state vector is a random variable and has a diffuse distribution, that is, we assume that its covariance matrix is given by κI with $\kappa \to \infty$. This is tantamount to assuming that nothing is known about the initial state.

5. Empirical Results

Table 2 presents the estimates of ϕ, γ and the variances of the disturbances in the single-index model on the four indicator series (in the order of the Hang Seng index, the residential property price index, retail sales and total exports).

All parameters are consistent with that predicted by theory and significant at the 5 percent level, except γ_4[7].

With regard to the estimated autoregressive coefficients, the roots of $\phi(B)$ all lie outside the unit circle, with two of them complex conjugates. Thus, the estimated AR(3) process of Δc_t is stationary and exhibits a cyclical pattern. Regarding the idiosyncratic components, we cannot reject the hypothesis that u_{11} and u_{21} equal zero. Thus, the growth rates of the two financial indicator series are an AR(3) process plus white noise.

Figures 4 shows the estimated unobservable common component, Δc_t, against the standardised growth rates of each of the four indicator series. The unobservable common component looks similar to the two financial series because the factor loadings, γ_1 and γ_2, are large. Although the factor loadings in the two macroeconomic series are smaller, they also play an important role in generating the unobservable common component. As discussed above, if we discard the total exports series, the estimated Δc_t will be correlated much less with the quarterly GDP growth.

We convert the Δc_t into quarterly frequency and, with an adjustment on its scale, plot it against quarterly real GDP growth in Figure 5, with its two standard error confidence bands. The contemporaneous correlation between the two series is reasonably high, at around 0.58. Our model, however, somewhat underestimated the GDP growth in 2002. The underestimation may due to a few reasons. First, the property market and retail sales were still depressed by the lack of confidence in the Hong Kong economy and

[7] We have also tried another model without the fourth series (total exports). For that model, although all the estimated parameters are significant at the 5 percent level, the estimated series, Δc_t, has a much smaller correlation with real GDP growth than that of the original model. As our main purpose is to provide an estimate of the current state of the economy, we decided to keep the fourth indicator series and use the original model, despite that γ_4 is only significant at around the 30 percent level.

the high unemployment rate. Second, the stock market performed much less satisfactorily in the last quarter of 2002 because of the 11th of September incident. Lastly, the growth rebound in 2002 was mainly due to the good export performance of the Hong Kong economy.

In order to construct an index of coincident indicators of economic activity in Hong Kong, we need to estimate the mean growth rate for the comovement component ΔC_t. This mean is calculated as a weighted average of the growth rates of the indicator series. The weights are those implicitly used to construct Δc_t from the indicator variables and can be estimated from the Kalman Filter algorithm. In our model, the estimated mean growth rate for ΔC_t is 2.98 percentage points. Figure 6 shows the index of coincident indicators of economic activity in Hong Kong, constructed by applying the above single-index dynamic factor model on the four indicator series.

6. Conclusion

We have shown that the single-index dynamic factor model is useful in assessing current-quarter real economic activity in Hong Kong. As the Hong Kong GDP figures are published with a time lag of two to three months, our approach can produce estimates of the GDP growth of the same quarter, particularly on showing any turning points before the dissemination.

We have also constructed an index of coincident indicators of economic activity in Hong Kong from the single-index dynamic model. With the availability of this index, we can adopt the methodology of state-space models with regime switching[8], developed by Kim (1994), to identify the business cycle fluctuations in Hong Kong in future work.

[8] For the details of the methodology, see Kim and Nelson (1999).

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Table 1. PC Analysis

	first principal component	Second Principal Component	Third Principal Component	Fourth Principal component
Eigenvalue	2.129	0.879	0.529	0.462
Variance Proportion	0.532	0.220	0.201	0.147
Cumulative Proportion	0.349	0.659	0.853	1.000
Eigenvector				
Retail Sales	0.566	0.752	0.885	0.804
Property Price	0.542	-0.100	-0.810	-0.202
Hang Seng Index	0.450	-0.665	0.474	-0.361
Total Exports	0.428	0.737	0.302	-0.428

Table 2
Maximum Likelihood Estimates
(Sample period: January 1990 to December 2002)

Parameters	Estimates	Std. Error	Asymptotic t-values
ϕ_1	2.017	0.215	9.37
ϕ_2	-1.781	0.314	-5.672
ϕ_3	0.702	0.148	4.755
$d_{3,1}$	-0.461	0.069	-6.662
$d_{4,1}$	-0.564	0.052	-10.802
γ_1	0.092	0.032	2.871
γ_2	0.151	0.053	2.868
γ_3	0.047	0.020	2.354
γ_4	0.017	0.017	1.020
σ_{u1}^2	0.837		
σ_{u2}^2	0.563		
σ_{u3}^2	0.730		
σ_{u4}^2	0.646		
Log Likelihood	-811.04		

Figure 1: Indicator Series
(in logarithm scale, except interest rate)

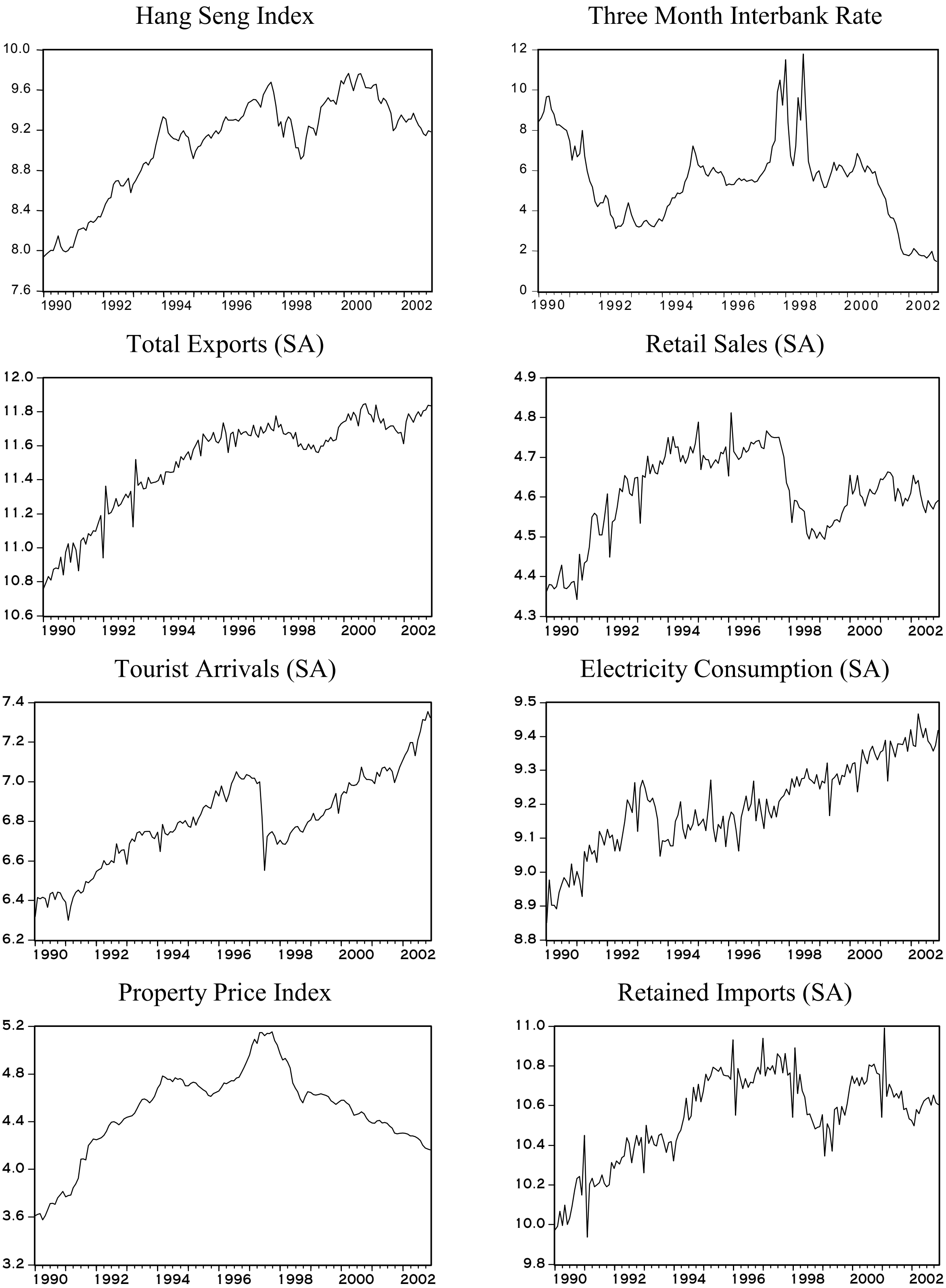
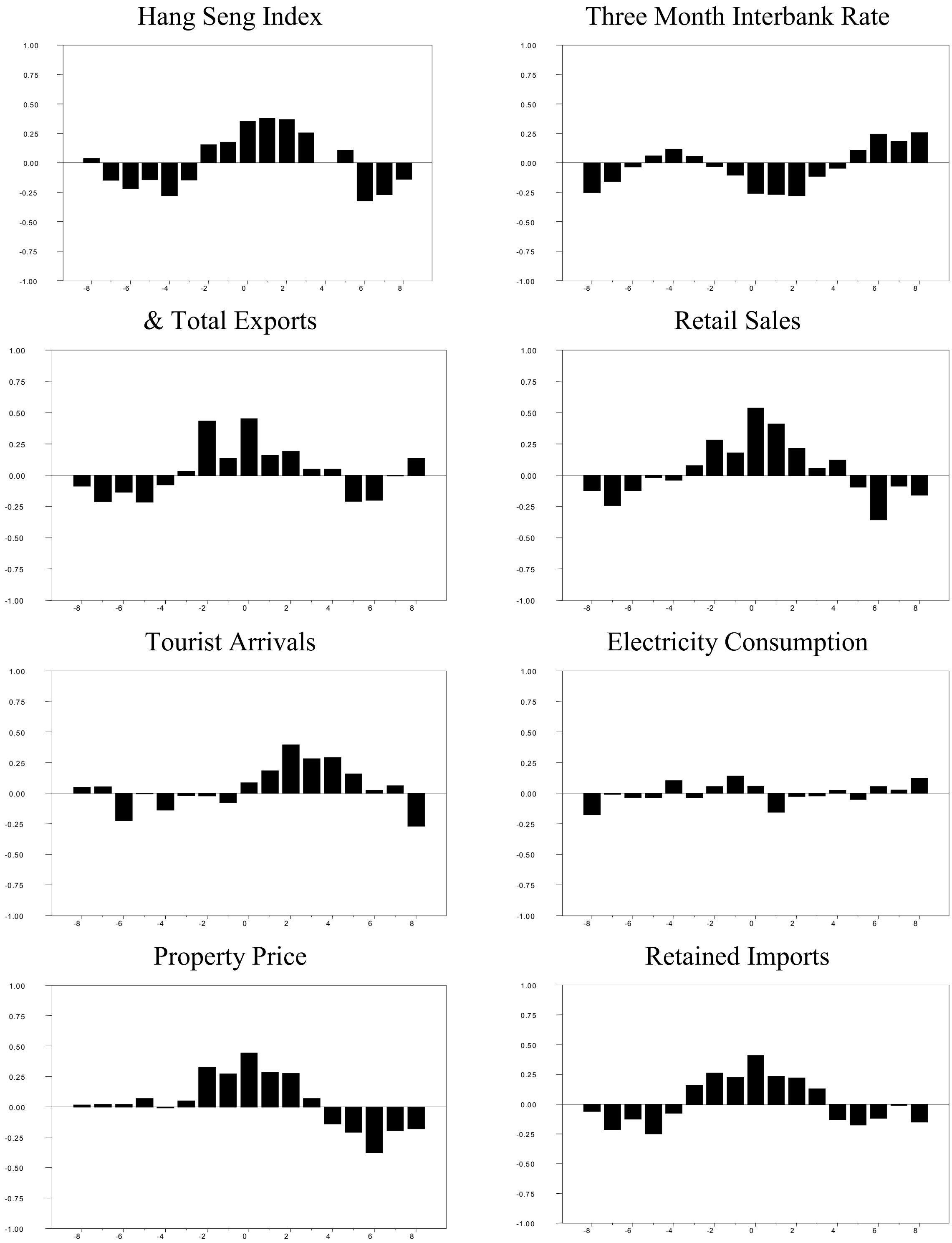


Figure 2: Cross Correlation between Quarterly Real GDP Growth and Indicator Series[9]



[9] The cross-correlation function of *GDP* and indicator series (*I*) is estimated by the following equation:
$\rho_{GDP,I}(k) = \Sigma(GDP_t-\overline{GDP})(I_{t-k}-\bar{I}) \Big/ \sqrt{\Sigma(GDP_t-\overline{GDP})^2\Sigma(I_t-\bar{I})^2}$

Figure 3: Indicators (Standardised Growth Rate)

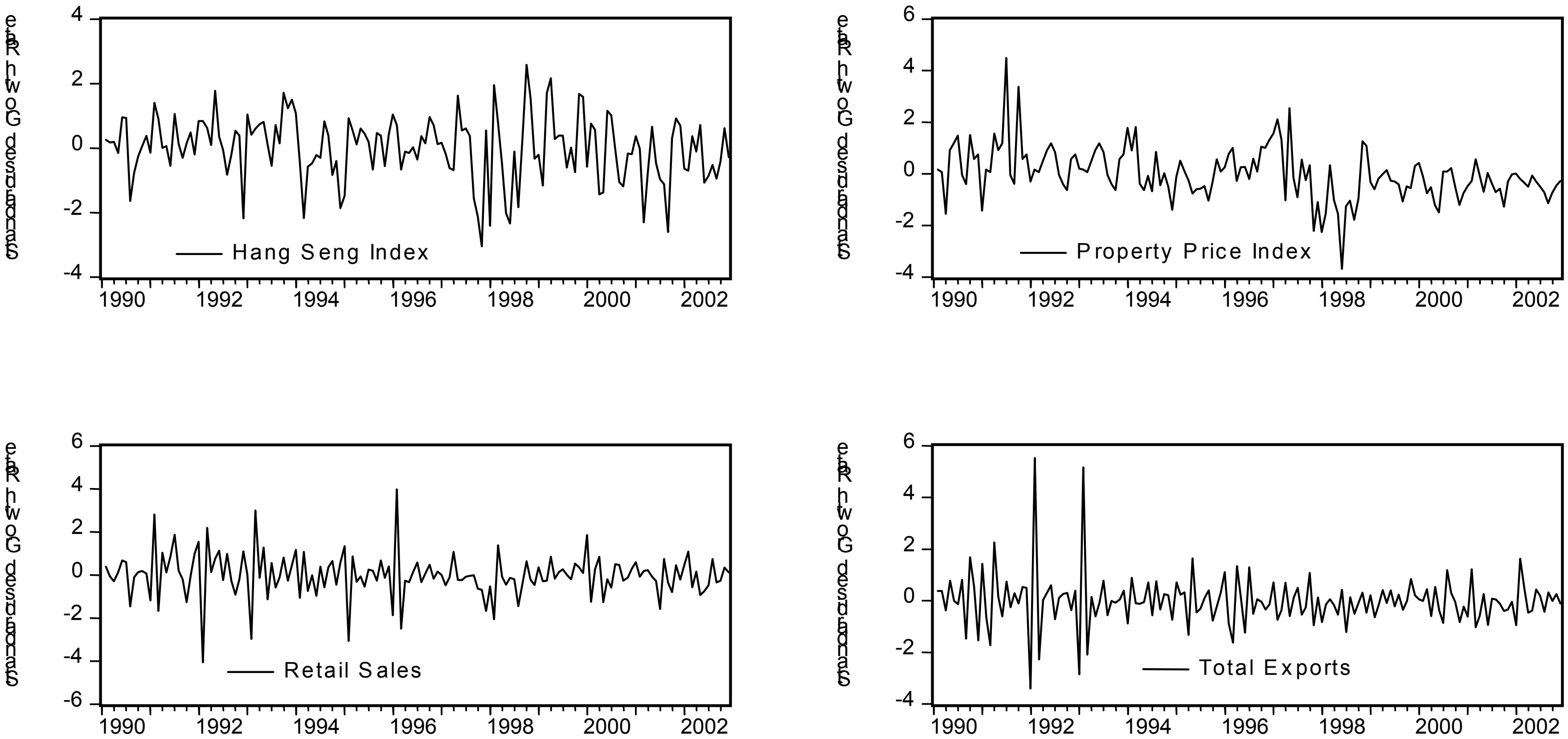


Figure 4: Indicator Series and Comovement Component

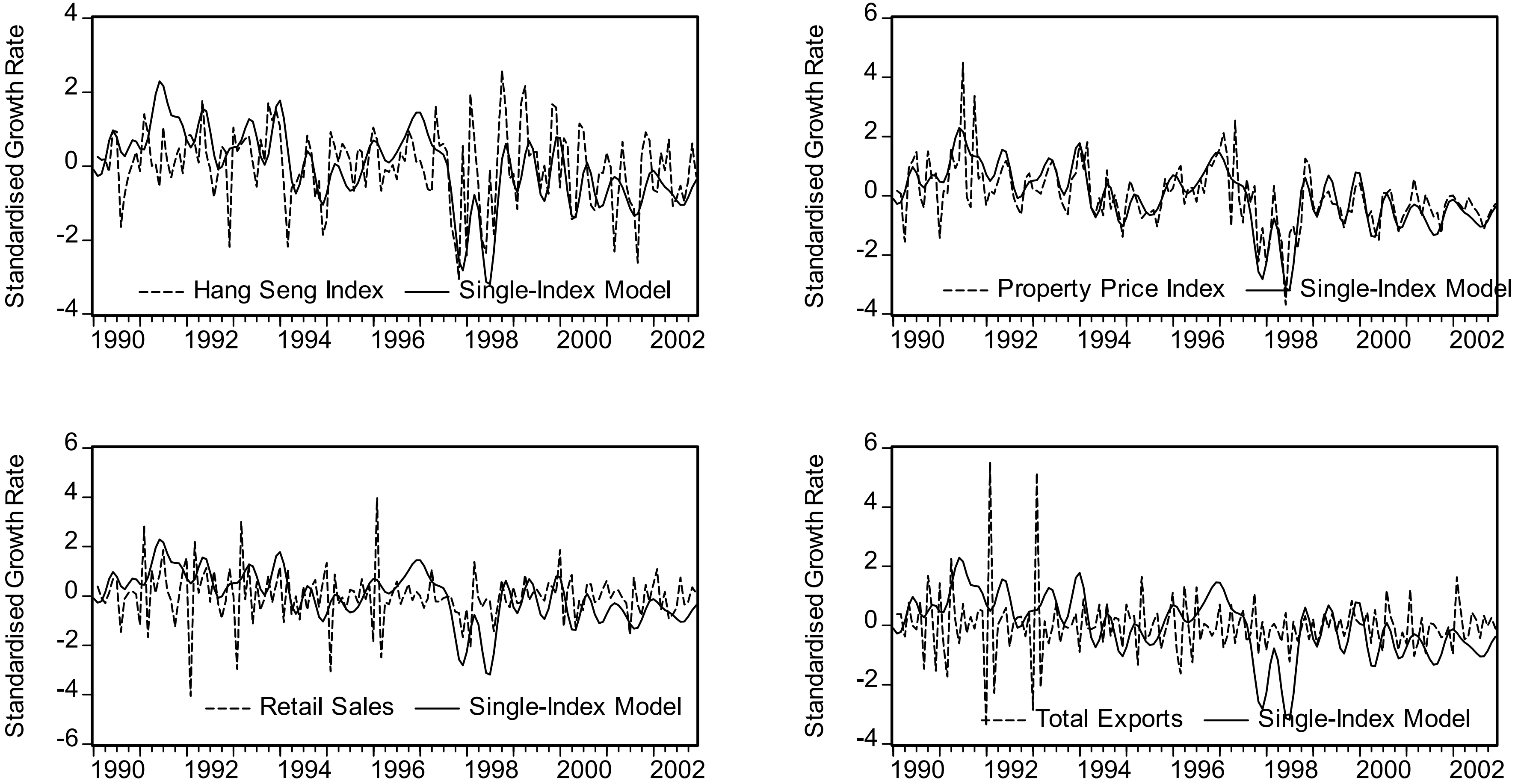


Figure 5: Comovement Component and GDP Growth

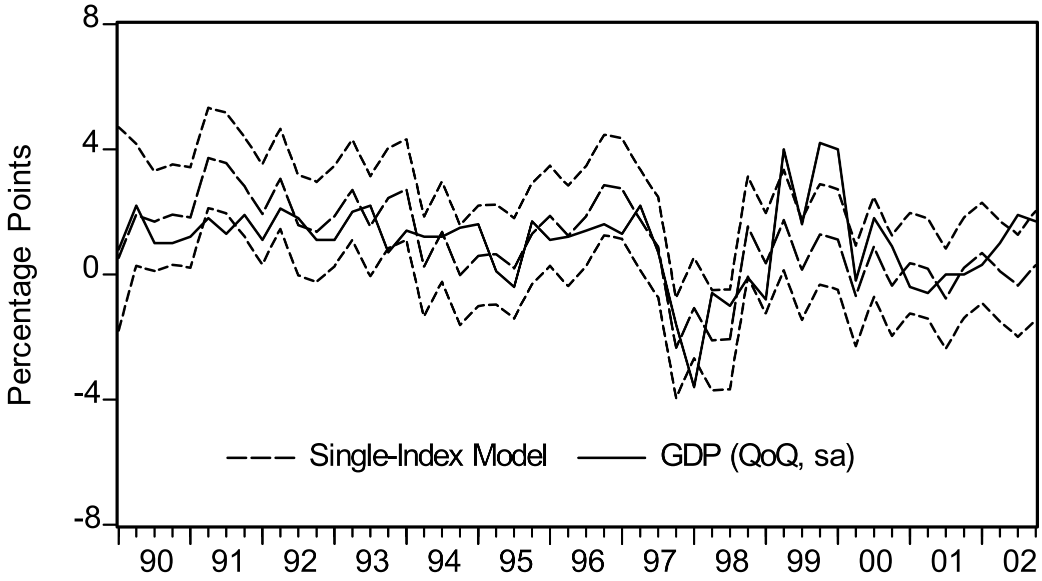


Figure 6: Index of Coincident Indicators in Hong Kong