How well do Markov switching models describe actual business cycles? The case of synchronization

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Abstract

The objective of this paper is to evaluate the effectiveness of using Markov switching models to measure the strength of synchronization between business cycles. Synchronization is defined by the fraction of time in which two country's business cycles are in the same state (concordance) and by the degree to which turning points cluster together (correlation).

We use a Bayesian, Gibbs sampling approach to estimate a univariate Markov switching model of GDP growth for several countries. We obtain posterior distributions of business cycle states by simulating data from the posterior distributions of the model parameters, then censoring the simulated data using the Bry-Boschan algorithm. The business cycle states are then mapped into posterior distributions of concordance and correlation statistics. We compare these posterior distributions with the point estimates of concordance and correlation of cycles in the data. As a point of reference, we repeat this exercise using simulated data from a heteroskedastic AR(1) model. Posterior odds ratios overwhelmingly favor the Markov switching model.

While the model performs well in describing the concordance of the data, it performs quite poorly in capturing the correlation of business cycles across countries; the actual correlations lie far in the tails of the posterior distributions. Mapping the priors on the parameters of the univariate Markov switching model into priors on the synchronization statistics, we find that the implied priors on the concordance statistics are quite flat. However, the priors on the correlation statistics are fairly tight around zero. This may be a consequence of the diffuseness of the concordance prior but may also be attributable to our univariate modeling framework.

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1 Introduction and motivation

Since James Hamilton's seminal 1989 *Econometrica* paper, numerous studies have applied Markov switching models to the study of business cycles. Among many others, see McCulloch and Tsay (1994), Albert and Chib (1993), Durland and McCurdy (1994), Filardo (1994), Goodwin (1993), Kim and Nelson (1999a), Kim and Nelson (1999b), and McConnell and Perez-Quiros (2000). The popularity of these models stems both from their intuitive appeal and their ability to fit the data well. However, the ability to produce plausible business cycle features is an important test of any model that purports to explain the business cycle.

Harding and Pagan (2002b) and Hess and Iwata (1997) evaluate several popular models of real GDP growth, including Markov switching models, and find that they often do not match cyclical characteristics of the observed data. Harding and Pagan develop a new set of nonparametric tools for analyzing business cycle characteristics and use them to assess the fit of various models of the cycle, including Hamilton's basic Markov switching model. Amongst their findings is that Markov switching models perform quite poorly relative to a simple AR(1) model. A consequence of this is that Markov-switching non-linear effects do not appear to be very important for describing actual business cycles, despite their popularity and intuitive appeal. In previous work (Smith and Summers (2002)), we investigated this issue by computing the posterior distributions of the Harding-Pagan statistics for several variants of Markov switching models for a group of six countries.

This paper uses a similar procedure to assess the ability of Markov switching models to account for the observed synchronization of business cycles across pairs of countries. The nature of what 'synchronization' means in the context of binary time series has been investigated by Harding and Pagan (2002a), who draw a distinction between *concordance* (the fraction of time that two series are in the same state) and *correlation* (the extent to which turning points in the two series occur near each other). We use Bayesian methods to investigate the posterior distributions of both of these measures implied by one type of Markov switching model. For comparison, we also investigate the posterior distributions of these statistics generated from a heteroskedastic AR(1) model. Finally, we compute the posterior odds ratios in favor of the Markov switching and heteroskedastic AR(1) models, relative to a homoskedastic AR(1). The posterior odds ratios overwhelmingly favor the Markov switching model.

1.1 Previous evidence on synchronization of business cycles

Artis et al. (1997) establish business cycle dates for industrial production for the G7 plus several European countries through the Bry and Boschan (1971) method. They then investigate the degree of concordance between cycles by using a version of a chi-squared test for independence. The evidence of the existence of regional cycles is found to be the strongest amongst North American and European economies. They find weaker evidence to suggest that the cycles of the regions are linked through the major international economies of the US, Japan and Germany.

Phillips (1991) estimates a bivariate version of Hamilton's (1989) regime switching model in which the unconditional means of real GDP growth for a pair of economies are driven by a four state Markov process, where the states that generate changes in the unconditional means are unobserved. He then imposes restrictions on the transition matrix of the states to test whether the state vectors for each country can be modelled as independent Markov chains. This amounts to a test of correlation between the states. Phillips is unable to reject the hypothesis of the perfect correlation of the US business cycle with that of either Germany, Canada, or the UK. Interestingly, he also does not reject the hypothesis of *independence* of the US and UK cycles.

Bodman and Crosby (2000) find that the method used to construct business cycle chronologies has implications for conclusions about the synchronization of cycles across countries. When these authors make use of the NBER-type dating methods to define cycles, they find evidence of synchronization of business cycles across the G7 countries. However when using a simple Okun rule (defining a recession as two successive quarters of negative growth) or a two state Markov switching model, the data seem to be more supportive of a regional and an English-speaking cycle, rather than a common G7 cycle.

In summary, investigations into the synchronization of cycles in the levels of economic activity across countries appear to show fairly clear evidence of distinct European and North American cycles. On the other hand, empirical evidence regarding the existence of a global business cycle is rather mixed, with the balance of information appearing to support the idea that the regional cycles are linked by the major international economies of the US, Germany and Japan. This is in reasonably close agreement with the wide body of evidence on the international synchronization of growth cycles (for example Backus and Kehoe (1992) or Backus et al. (1992)).

2 Dating Cycles

2.1 The model

Our basic Markov switching model is the same as that of McConnell and Perez-Quiros (2000) who augment the prototypical two-state Markov switching model to also allow for the possibility of switching in the residual variance as well as in the conditional mean:

$$\begin{aligned} \phi(L)(\Delta y_t - \mu(S_t, D_t)) &= e_t, \qquad (1) \\ e_t &\sim iidN(0, \sigma^2 (D_t)). \end{aligned}$$

Here, Δy_t is the first difference of the log of real GDP and $\mu(S_t, D_t)$ is the mean of Δy_t conditional on the unobserved state vectors S_t and D_t. Specifically, $\mu(S_t)$ switches between high and low growth states depending on whether the economy is in a period of expansion ($S_t = 0$) or contraction ($S_t = 1$) and according to whether the residual, e_t, is in a high variance ($D_t = 0$) or low variance ($D_t = 1$) state. Here:

$$\mu(S_t, D_t) = \mu_0 + \mu_{00} D_t + (\mu_1 + \mu_{11} D_t)\, S_t \qquad (2)$$

and

$$\begin{aligned}\sigma^2(D_t) &= \sigma_0^2(1-D_t)+\sigma_1^2 D_t \\ &= \sigma_0^2(1+h_1 D_t),\end{aligned} \tag{3}$$

with $h_1 = \left(\frac{\sigma_1^2}{\sigma_0^2}-1\right)$. We identify the low-growth state with the event $S_t = 1$ by restricting the mean growth rates in this state, $(\mu_0+\mu_1)$ and $(\mu_0+\mu_1+\mu_{00}+\mu_{11})$, to be negative and identify the low variance state with the event $D_t = 1$ by restricting $h_1 < 0$.

In order to obtain a probabilistic statement about the growth state of an economy at a given point in time, an assumption must be made about the process governing the state variable. To achieve this we assume that the latent state variables S_t and D_t are generated by independent first-order hidden Markov chains with transition probabilities $\Pr[S_t = 1|S_{t-1} = 1] = p_{11}$, $\Pr[S_t = 0|S_{t-1} = 0] = p_{00}$, $\Pr[D_t = 1|D_{t-1} = 1] = q_{11}$, and $\Pr[D_t = 0|D_{t-1} = 0] = q_{00}$.This is the same specification as McConnell and Perez-Quiros (2000) but differs slightly different from Kim and Nelson (1999a) who assume that $D_t = 1$ is an absorbing state and so restrict $q_{11}$to 1. Following Kim and Nelson (1999a) and McConnell and Perez-Quiros (2000), we specify a first-order autoregression for deviations around the Markov trend, so that $\phi(L) = 1-\phi L$. Versions of this model were also estimated in Smith and Summers (2002).

We estimate equation (1) in a Bayesian framework, which offers an alternative method for making inferences about the state vector. Bayesian analysis treats both the parameters of the model and the unobserved states as random variables, with inference about S_t drawn from their joint distribution conditional upon the data, $p(S_t,\theta|y_t)$ rather than the conditional distribution, $P(S_t = j|y_t;\widehat{\theta})$.

Recent work by Albert and Chib (1993) and McCulloch and Tsay (1994) has demonstrated that Bayesian estimation of Markov switching models is relatively simple to implement using the Gibbs sampler. Gibbs sampling is a Markov chain Monte Carlo (MCMC) method of simulating complex joint and marginal distributions by drawing repeatedly from the conditional distributions, which are much simpler in many cases. As noted by Albert and Chib (1993), the Bayesian approach allows us to treat the unobserved states, $\{S_t, D_t\}_{t=1}^{T}$, as additional parameters to be estimated (through simulation), along with the unknown parameters, θ.

2.2 Prior specification and starting values

The transition probabilities contain important information about the expected duration of regimes. It can be easily shown that the expected duration of a high-growth regime is $(1-p_{00})^{-1}$, while the expected duration of a low-growth regime is $(1-p_{11})^{-1}$. Estimates are generally considered to consistent with business cycle frequencies if transition probabilities lie in the interval 0.75-0.95 for quarterly data, which implies business cycle durations in the range of 1-5 years (see Phillips (1991)). This information is employed and the prior distributions of p_{11} and p_{00} are accordingly set to have means of 0.8 and standard deviations of 0.16. In view of the fact that most explanations of a reduction in the variance of the growth rate of real GDP since the early

1980s are structural in nature, it seems reasonable to assume priors about the transition probabilities of switches in the residual variance in (3) which imply nearly absorbing states. Therefore, we set the prior mean and standard deviation of q_{00} to be 0.999 and 0.004 respectively and the prior mean and standard deviation of q_{11} to be 0.9 and 0.09. We specify relatively non-informative prior distributions for the other parameters with means of 0 and standard deviations of 1, with the exception of the prior mean for the high growth, high variance state, μ_0, which is set to have a mean of -0.5.

Starting values for the MCMC simulation of (1) are obtained in an approach which is similar to the method described by Albert and Chib (1993, p. 8). This involves setting the initial values of p_{00}, p, q_{00} and q_{11} to be 0.9, 0.76, 0.9 and 0.76 respectively and using the implied Markov process to construct the initial state vectors D_t^0 and S_t^0. Least squares estimates of a regression of y_t on a constant, y_{t-1}, D_t^0 and S_t^0 are then used to determine the starting values of the the other parameters. We generate 11,000 iterations and of the Gibbs sampler and use the final 10,000 for inference. Further details of the implementation of the Gibbs sampling algorithm for this model and conditional distributions may be found in Kim and Nelson (1999a)[1].

2.3 Censoring of Markov states

Once we have identified an initial set of Markov states S_t, we need to convert them into business cycle states W_t. This is necessary because we wish to assess the ability of the Markov switching model to generate 'model business cycles' that look like the actual ones that we observe. Our ultimate functions of interest therefore concern the W_t's, not the S_t's.

A common method of converting the former to the latter is to set $W_t = 1$ if $\Pr(S_t = 1)$ is at least 0.5. However, the process by which business cycle chronologies such as those of the NBER are constructed is much more complex. In particular, various censoring rules are imposed in order to enforce minimum lengths of expansions and recessions, to ensure that peaks and troughs alternate, etc. These censoring rules were formalized in an algorithm by Bry and Boschan (1971). Harding and Pagan (2002b) discuss these issues and suggest a version of the Bry-Boschan algorithm that is applicable to quarterly data, which they denote 'BBQ.' We follow Harding and Pagan and use their BBQ algorithm to map the S_t into W_t.This procedure ensures that the simulated data from the Markov switching model is treated in the same way as the actual data, insofar as the location of turning points is concerned.

3 Assessing synchronization

Recent papers by Artis et al (1997) and Bodman and Crosby (2000) employ nonparametric methods to test for the independence of business cycles in the G7 countries (the United States, Japan, Germany, the United Kingdom, France, Italy, and Canada). Both papers rely (implicitly or explicitly) on a binary indicator variable, taking the value one in expansions and zero in recessions. The indicator variable is constructed from

[1]With the exception that in our model, the state variable, D_t, may be treated in exactly same way as S_t : unlike Kim and Nelson (1999a), we do not assume that D_t has an absorbing state.

a business cycle chronology for each country.[2] Artis et al (1997) use a version of the Bry-Boschan algorithm to obtain their chronologies. Once the peak and trough dates (i.e., the beginning dates of recessions and expansions, respectively) have been identified for each country, these authors use Pearson's contingency coefficient to test the null hypothesis of independence of the G7 business cycles.

Alternatively, Bodman and Crosby (2000) treat the product $(1 - W_{xt})(1 - W_{yt})$ as an independent Bernoulli random variable for each country. These authors are thus specifically testing the independence of *recessions* in each country.

Harding and Pagan (2002a) point out that the assumption that the state (indicator) variables are serially uncorrelated is particularly important (and problematic) for the concordance measures discussed above. In particular, they argue that this assumption is likely to be violated in general for business cycle chronologies generated via Markov switching models. They show that, for a wide class of dating rules, the binary state variable W_t can be written as a first-order autoregression:

$$W_t = 1 - p_{11} + (p_{00} + p_{11} - 1) W_{t-1} + \eta_t$$

where p_{ij} is the probability that state i will follow state j. (also see Hamilton (1994, p. 684)). Therefore, except in the case where $p_{11} + p_{00} = 1$, the statistics used by Artis et al (1997) and Bodman and Crosby (2000) will both suffer from serial correlation (as well as conditional heteroskedasticity). We illustrate the importance of Harding and Pagan's point in table 1. The table presents the mean, median and mode of the posterior distribution of $\rho = p_{11} + p_{00} - 1$ for each of the countries we study. To compute these posteriors, we first simulated data from the posterior distribution of the parameters in the Markov switching models (one data series for each posterior draw), then passed each series through the 'BBQ' dating algorithm described in Harding and Pagan (2002b). We are thus measuring the serial correlation in the *business cycle states* W_t, rather than the *Markov states* S_t. The summary statistics in table 1 indicate substantial evidence of autocorrelation in the W_t series for each country.

A third measure of synchronization of two or more business cycles is the concordance index described in Harding and Pagan (2002a) . They define the concordance index, $\hat{I}$, as the covariance between two binary series as the average fraction of the sample for which the two series are in the same state:

$$\hat{I} = T^{-1} \left[\sum_{t=1}^{T} W_{xt} W_{yt} + \sum_{t=1}^{T} (1 - W_{xt})(1 - W_{yt}) \right], \tag{4}$$

where T is the sample size. Harding and Pagan note that this can be re-written as

$$\hat{I} = 1 + 2 \left(\hat{\rho}_W \hat{\sigma}_{Wx} \hat{\sigma}_{Wy} + \hat{\mu}_{Wx} \hat{\mu}_{Wy} \right) - \hat{\mu}_{Wx} - \hat{\mu}_{Wy}. \tag{5}$$

In this expression, $\hat{\mu}_{Wx}$ and $\hat{\sigma}_{Wx}$ are the estimated mean and standard deviation of W_x, etc., while $\hat{\rho}_W$ is the correlation between W_x and W_y. As Harding and Pagan point out, the usefulness of equation (5) is that it makes clear that the value of $\hat{I}$ depends on both

[2] In studies that employ Markov switching models, W_t is typically derived from the state variable S_t described above, by a rule such as $W_t = 1$ if $\Pr(S_t = 0) > 0.5$ (recall that our parameterization identifies $S_t = 0$ with periods of high growth).

the correlation between the two series and on how often they are in the expansion state $W = 1$. In measuring synchronization between cycles in industrial production for several countries, Harding and Pagan note that it is often the case that a high value of $\hat{I}$ coincides with a low value of $\hat{\rho}$ (for a given pair of series). We find a similar phenomenon when we examine cycles in GDP across the six countries listed above. For most of the country pairs that we examine, the posterior distribution of $\hat{I}$ is a much better description of the actual data than is the posterior of $\hat{\rho}$.

In this paper, we assess the extent to which the United States business cycle is synchronized with those of the other G7 countries, by presenting finite-sample distributions of the mean-corrected concordance indexes $\hat{I}_{mc}$. We construct this index for the US and each of the other countries.

4 Results

4.1 Parameter Estimates

Seasonally adjusted real GDP data for Australia Canada, Japan, Germany the U.K. and the U.S. were taken from Datastream for the period 1961:I to 2001:IV.

The results of estimation of equation (1) are presented in table 2, along with the means and standard deviations of our prior. The estimates of the posterior means of the transition probabilities of the Markov states, S_t imply average business cycle durations of of 5 to 10 years, which seems to be too long relative to the data. The most extreme example is that of Canada, for which the posterior distribution of turning points implies an average length of expansion of approximately 10 years (39 quarters) and an average length of contraction of 1 year.

Figure 1 depicts the posterior probability of the business cycle sates, W_t, being in a high growth state at each point in time, calculated by taking the mean of the 10000 draws of the business cycle state vector at each time t. These probabilities are also quite informative about the average duration of cycles implied by the posterior estimates of business cycle states, W_t . Only the 1990s recession displays a recession probability of greater than 0.5, and even then only for the United Kingdom and Australia. This may be taken as further evidence that the business cycle states, W_t implied by the estimates of (1) , appear to produce cycles for which the average duration of expansions is too high relative to the data.

The posterior distributions of the average growth rates in the high volatility states, μ_0 and $\mu_0 + \mu_1$, appear plausible both in size and in the sense that most of the posterior mass of the distributions lies away from zero. However there appears to be little evidence that there has been any change in the severity of contractions or in the vigor of expansions associated with a reduction in volatility. A large proportion of the probability mass of the posterior distributions of μ_0 and μ_{00} and μ_{11} lies above zero for each of the countries. Finally, it is worth noting that there does appear to be significant evidence of switching in the residual variance of the model , σ_0^2 and σ_1^2 in table 2. The English speaking countries in the sample all exhibit what appears to be a more or less permanent reduction in the residual variance since the early 1980s. However the nature of switching in the residual variance for Japan and Germany appears to be more complex. These estimates and the implications for inference of changes in GDP growth and volatility since the early 1980s are discussed in greater detail in Smith and Summers (2002).

4.2 Posterior odds

For comparison purposes, we also simulated business cycle states implied by estimating a heteroskedastic AR(1) model for each country. As described in Geweke (1993), this model can be written as a scale mixture of normals:

$$\begin{aligned} \Delta y_t &= \beta_0 + \beta_1 \Delta y_{t-1} + e_t, \\ & \quad e_t \tilde{\ } N\left(0, \sigma^2 V\right), \\ V &= diag\left(v_1, v_2, ..., v_T\right) \\ & \quad r/v_i \tilde{\ } \chi^2 \left(r\right). \end{aligned}$$

Our priors for this model are: $\beta_i \tilde{\ } N\left(0, 100^2\right)$; $r = 4$; $\sigma \propto \sigma^{-1}$. We employ this model as a benchmark linear model. A by-product of our estimation procedure for both the Markov switching and the AR(1) scale mixture models is an estimate of the posterior odds ratio in favor of each of these models, relative to a homoskedastic AR(1). By using these estimates, we can assess the degree of support in the data for both the non-linear features of the Markov switching model and the time-varying nature of the variance of GDP growth. These estimated posterior odds are reported in table 4, and show overwhelming support for the Markov switching model for each country *vis-à-vis* either version of the AR(1) model. There is also virtually no support for the homoskedastic AR(1) model relative to the scale mixture model for any country except Canada.

4.3 Synchronization of G7 business cycles

4.3.1 Concordance

The results of our analysis of concordance among business cycles in the six countries are shown below the diagonal in table 3 and also in figure 2. In each figure, we present the prior (dotted lines) and posterior (unbroken lines) distributions of the concordance index from the Markov switching model, along with the actual value computed from the business cycle chronology of each country (the vertical dashed lines) and the posterior estimates from the AR(1) mixture model (dash-dot lines).[3]

We constructed the prior and posterior distributions of the concordance measures using the corresponding distribution of the parameters of the Markov switching model. For each draw of the parameters, we simulated a data series of the same length as our

[3] To compute the actual values, we use the NBER business cycle chronology for the United States (http://www.nber.org/cycles/), and the Melbourne Institute's chronology for Australia (http://wff2.ecom.unimelb.edu.au/iaesrwww/bcf/bdates5197.html). All other chronologies are from the Economic Cycle Research Institute (ECRI; http://www.businesscycle.com/research/intlcycledates.php).

observed sample, then filtered these series using the BBQ dating algorithm and NBER-style censoring rules. This resulted in 10,000 simulated business cycle chronologies for each country. We have used the same procedure in earlier work (Smith and Summers 2002) to examine the posteriors of various nonparametric business cycle measures, but to our knowledge this is the first attempt to relate Bayesian priors on model parameters to implied priors on observable features of real-world business cycles.

For all the country pairs, the prior distribution of the concordance index is very diffuse relative to the posterior. This indicates that the data are quite informative about the amount of time any given pair of countries could be expected to be in the same business cycle state. Furthermore, the actual value of the concordances lie well within the mass of the posterior distributions, suggesting that the underlying Markov switching model provides a good framework for studying this aspect of actual business cycles.

4.3.2 Correlations

Posterior distributions of the bilateral correlations of censored business cycle states are presented in table 3 above the diagonal and in figure 3. As with the figures displaying the concordance statistics, the simulated prior distributions (the dotted lines) and actual data values (the vertical broken lines) for these correlations have also been plotted. Again, the heteroskedastic AR(1) model posterior (dash-dot lines) is included for comparison. There are two striking features of these distributions: first , the posterior distributions of the correlation statistics appear to suggest a much smaller degree of synchronization than the posterior distributions of the concordance statistic and second, the data appear to be far less informative about the correlations between business cycle states than it is for the concordance between them. The Markov switching model is often noticeably better than the AR(1) model at capturing the actual correlations, although there are several cases where the two models perform equally well (or poorly).

Harding and Pagan (2002, p. 14) present a table similar to table 3 for industrial production in several OECD countries. While they also find quite small correlations relative to the estimated concordance statistics, their correlations are generally much larger in absolute value than the modes of our posterior distributions. As described by Harding and Pagan, the estimated correlations between states are effectively a measure of the clustering of turning points. It would be interesting to repeat our exercise for industrial production series so a better comparison could be made.

The smaller than expected magnitude of our posterior distributions for the correlation statistics is made even more obvious by inspecting the posterior distributions, in figure 3. In these figures, most of the probability mass is centered around zero in contrast with the correlation between countries' business cycles in the data, as represented by the correlation between ECRI and NBER business cycle states. For every country pair for which the actual correlation in the data lies some distance from zero, most of the posterior probability mass of the distributions lies well to the left of the data value.

The discrepancy between the posterior and data values can be largely be explained by the corresponding prior distributions. These prior distributions, as measured by the pale dotted lines in figure 3, are all centered quite tightly around zero. So, it appears that the data are not very informative about the correlations between business cycle states, in the sense that the posterior distributions seem to be strongly influenced by the priors. This is in stark contrast to the relatively flat distributions of the prior on the concordance indexes depicted in figure 2.

This result may well be an unintended consequence of our univariate modeling framework. Recall that in generating the business cycle states, we simulated data for each country separately. This essentially means that the simulated data are drawn from a multivariate normal distribution with a diagonal covariance matrix, under both the prior and the posterior. The departures of the pairwise business cycle correlations from zero would then be due merely to sampling error. A multivariate analysis of these data would certainly be appealing, however we do not pursue this here for the following reasons. First, it is not straightforward to generalise the version of the Markov switching model that we use (with independently switching variance) to the vector case. Second, one would also need to address the possible existence of any cointegrating relationships between the series, and their implications for the synchronization of business cycles. The modeling framework of Paap (1997) and Paap and Dijk (2001) seems particularly useful for addressing these issues, but we leave them for future research.

This possible explanation notwithstanding, it is worth reemphasizing Harding and Pagan's (2002) point that the correlation statistics measure the clustering of turning points, while concordance statistics measure time spent in the same phase. Correlations and the concordance index thus measure complementary phenomena. The flatness of the prior distribution of the concordance indices implies a belief that the business cycle states across countries are just as likely to be out of phase as they are to be in phase. Therefore there will be very little clustering of turning points and the prior distribution of the correlations between the business cycle states of country pairs will be centered around zero.

5 Conclusions and future directions

This paper has presented further evidence on the ability of Markov switching models to generate business cycle features that compare with those that are observed in the data. Our results suggest that such models are quite good at capturing the *concordance* of business cycles across pairs of countries. For all the country pairs we study, the concordance indexes computed from actual business cycle chronologies fall in high-mass regions of the posterior distributions implied by the Markov switching model. Moreover, the prior distributions of these indexes (implied by the priors on the model parameters) is quite reasonable in the sense that it is non-informative across the allowable parameter space. We believe this is a further argument in favour of Bayesian analysis of these models.

The model does less well, however, in describing the *correlation* of business cycles (i.e., the clustering of turning points). Here, the posterior distributions tend to lie far from the actual data values. This phenomenon seems to be at least partially due to our univariate modeling framework. Due to the interrelatedness of the concordance and correlation measures however, the tightness of the correlation prior may also be a consequence of the diffuseness of the concordance prior.

On the whole, we believe that this research can provide useful information regarding the types of business cycle models that best describe actual business cycles. Further work is needed, both in the analysis of the most interesting (measurable) features of business cycles, and on the interactions between these features and the parameters of the model being employed. A multivariate generalization of the model presented here seems especially promising.

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Table 1: Posterior quantiles of ξ = p+q-1

Country	mean	median	mode	standard deviation	data
United States	-0.0529	-0.0491	-0.0491	0.0201	-0.0674
Japan	-0.0562	-0.0552	-0.0547	0.0192	-0.043
Canada	-0.0413	-0.0368	-0.036	0.019	-0.0368
Germany	-0.1223	-0.1227	-0.1221	0.0271	-0.0552
Australia	-0.0848	-0.0859	-0.0848	0.0244	-0.0552
United Kingdom	-0.0868	-0.0859	-0.0852	0.0232	-0.0368

Table 2: Prior and posterior distributions of model parameters

		Prior	Australia	Canada	Germany	Japan	UK	US
p_{00}	mean	0.8	0.9667	0.9742	0.9506	0.9677	0.9690	0.9648
	s.d.	0.16	0.0269	0.0181	0.0518	0.0197	0.0227	0.0234
p_{11}	mean	0.8	0.7211	0.7762	0.6678	0.8827	0.7695	0.7547
	s.d.	0.16	0.1497	0.1314	0.1748	0.1071	0.1353	0.1343
q_{00}	mean	0.999	0.9926	0.9923	0.9722	0.9911	0.9863	0.9910
	s.d.	0.004	0.0072	0.0077	0.0183	0.0084	0.0129	0.0088
q_{11}	mean	0.9	0.9836	0.9712	0.9748	0.9810	0.9733	0.9763
	s.d.	0.09	0.0160	0.0284	0.0156	0.0169	0.0224	0.0237
μ_0	mean	0	0.9451	1.0652	1.0012	2.002	0.5892	0.9349
	s.d.	1	0.1784	0.1466	0.5301	0.2286	0.1700	0.1875
μ_1	mean	-0.5	-1.2461	-1.3346	-1.3793	-2.1488	-0.9132	-1.1734
	s.d.	1	0.3278	0.2807	0.5276	0.2690	0.3249	0.2728
μ_{00}	mean	0	0.0008	-0.2954	-0.3444	-1.0538	0.1378	-0.0880
	s.d.	1	0.2018	0.2045	0.5634	0.2679	0.1880	0.2194
μ_{11}	mean	0	-0.0385	-0.2232	0.2865	0.8183	-0.2333	-0.1604
	s.d.	1	0.5302	0.8287	0.6774	0.7562	0.6080	0.6527
ϕ_1	mean	0	-0.0146	0.2406	-0.0994	-0.0617	0.0317	0.1526
	s.d.	0.5	0.0909	0.1074	0.0932	0.1100	0.0966	0.1256
σ_0^2	mean	0.33	2.8590	0.9782	8.7798	1.6242	1.5738	1.0195
	s.d.	0.40	0.4618	0.1567	4.3577	0.3653	0.4439	0.2185
σ_1^2	mean	0.33	0.4620	0.2362	1.5794	0.6608	0.2623	0.2720
	s.d.	0.40	0.1051	0.0890	0.6217	0.2020	0.0999	0.0798

Table 3: Posterior quantiles of ρ and I

	United States	Australia	Canada	Japan	Germany	United Kingdom
United States		0.069[a]	0.091	-0.014	0.015	0.078
		(0.135)	(0.160)	(0.111)	(0.118)	(0.136)
Australia	0.809		0.064	-0.055	0.003	0.074
	(0.052)		(0.139)	(0.097)	(0.111)	(0.131)
Canada	0.868	0.818		-0.035	0.009	0.060
	(0.046)	(0.054)		(0.096)	(0.116)	(0.138)
Japan	0.807	0.755	0.815		0.008	-0.048
	(0.055)	(0.061)	(0.054)		(0.119)	(0.105)
Germany	0.728	0.695	0.735	0.706		0.006
	(0.067)	(0.066)	(0.068)	(0.067)		(0.116)
United Kingdom	0.794	0.759	0.799	0.740	0.685	
	(0.053)	(0.056)	(0.056)	(0.063)	(0.066)	

[a] Entries above the diagonal are posterior means (standard deviations) of correlations between business cycle states. Statistics below the diagonal are posterior means (standard deviations) of the concordance indices.

Table 4: Posterior odds ratios

	$AR(1)$/mixture	$AR(1)/MS$	MS/mixture
US	0.0137	$1.6e-10$	$1.1e8$
Australia	$1.8e-7$	$3.7e-17$	$4.4e8$
Canada	0.0713	$4.6e-14$	$1.6e12$
Japan	7.5813	$6.1e-5$	$1.3e5$
Germany	$9.3e-18$	$1.2e-25$	$7.7e7$
UK	$6.1e-4$	$5.9e-41$	$1.0e37$

Figure 1. Posterior probabilities of being in an expansion, $P(W_t = 1)$

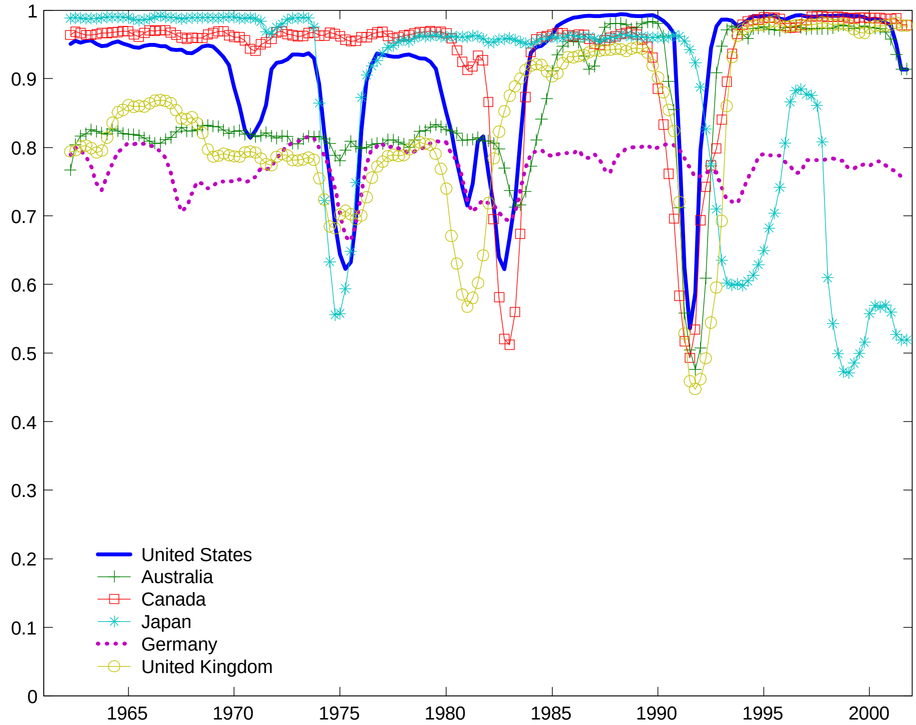

Figure 2. Posterior distributions of the pairwise concordance index, $\widehat{I}$, between business cycle states

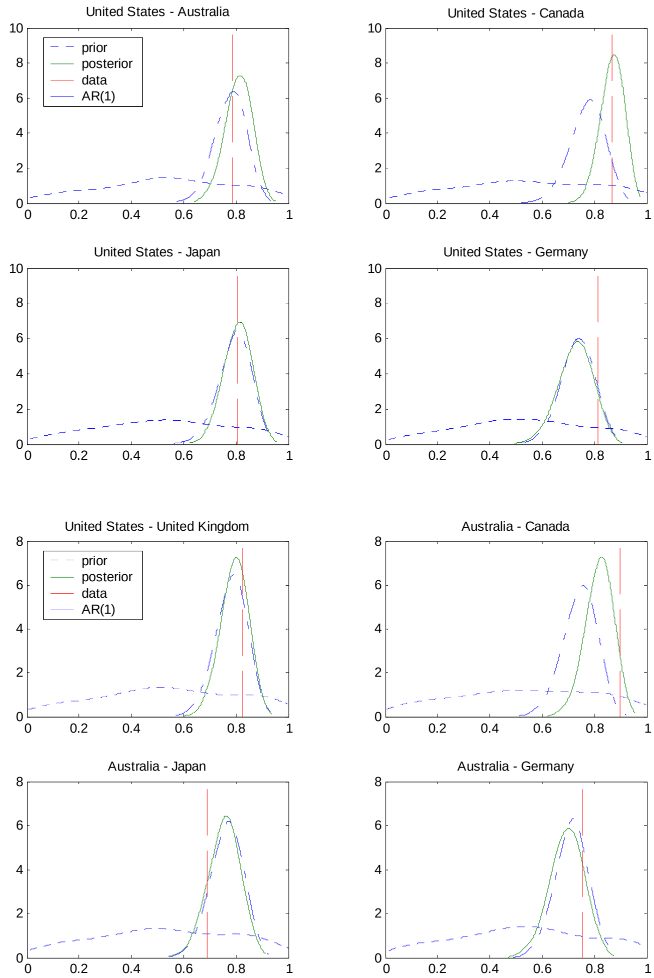

Figure 2. continued

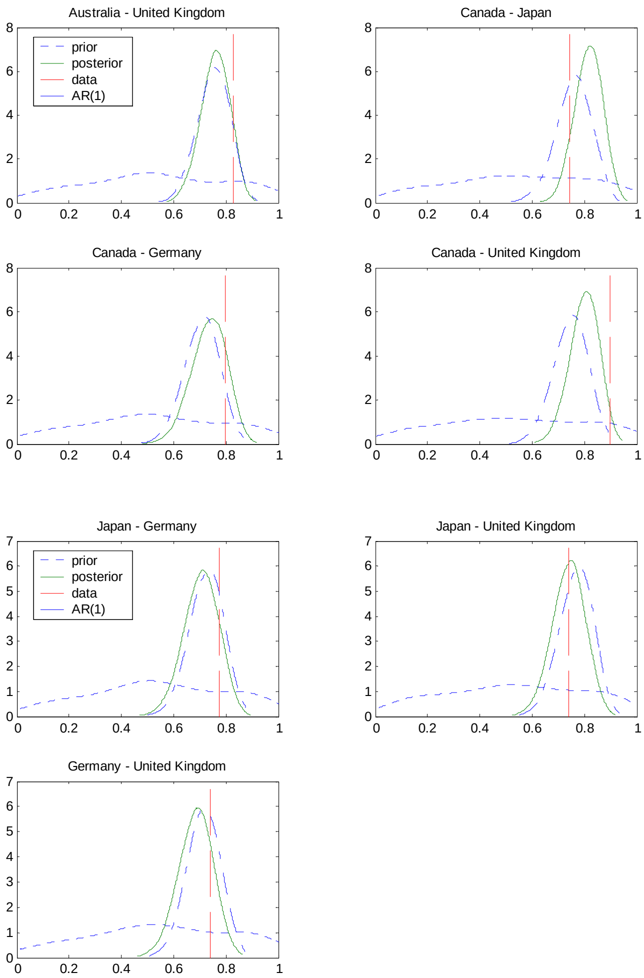

Figure 3. Posterior distributions of the pairwise correlations, ρ, between business cycle states

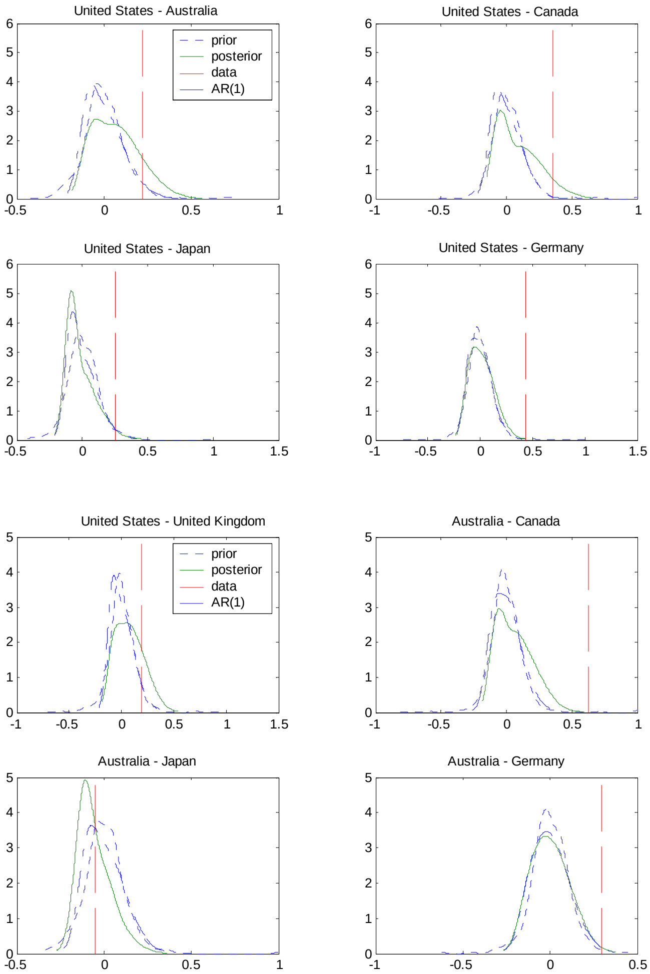

Figure 3. continued