

# Perils of quantitative easing\*

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## Abstract

Since the financial crisis began, there has been a significant shift from conventional monetary policy to unconventional measures. While central banks including the Bank of England conducted a policy of “quantitative easing”, in contrast, in the US the Federal Reserve conducted a policy of “Credit Easing”. Although the two terms have largely been used interchangeably, we show that these policies have specific and significant implications for the ability of monetary policy to target the rate of inflation. Like conventional monetary policy, Credit Easing leads to determinacy of the path of prices in a stochastic environment. On the other hand, if monetary policy is conducted in a manner akin to quantitative easing, we show that an indeterminacy arises which reduces the central banks control of inflation.

**Key words:** Quantitative easing; credit easing; monetary determinacy; inflation expectations.

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## 1 Introduction

In 2009, Federal Reserve Chairman Bernanke contrasted the Japanese approach of Quantitative Easing (QE) from 2001 to 2006 with the Fed's current approach, which he classified as credit easing (CE). He argued that CE is conceptually distinct from QE, as the focus of CE was on the composition of the Fed's balance sheet, with the size being largely incidental, as opposed to the emphasis on the size under QE. Much of the commentary on unconventional monetary policy has used QE and CE interchangeably, emphasising the effect both policies are designed to have on easing credit and liquidity conditions. Here we show that the distinction between the two policies has significant implications for the ability of monetary policy to target the stochastic path of inflation. Specifically, we show that like conventional policy, CE allows monetary policy to pin down the rate of inflation while under QE, indeterminacy prevails.

Under normal conditions, monetary policy sets a target for the short-term (here one period) interest rates, and conducts open market operations or repo transactions, using as collateral Treasury securities, with various maturities, but to conform to an ex-ante determined overall portfolio composition which has an exclusive focus on Treasuries of short maturity. Unconventional monetary policy expands the balance sheet by increasing the maturity range (and possibly range of assets) of the monetary authority portfolio. As under conventional monetary policy, under CE it is the explicit target for the composition of the balance sheet that allows the monetary authority to target the stochastic path of inflation: the target for the composition of the portfolio guarantees the necessary restrictions to obtain determinacy. The absence of such restrictions under QE manifests nominal (and possibly) real indeterminacy.

To examine this issue, we consider a stochastic cash-in-advance economy, where prices are flexible, though our results are equally valid when they are sticky, and show that indeterminacy is pervasive under QE without adequate targets: short term interest rates does not suffice to determine the stochastic path of inflation. We shall, in our baseline analysis, make the assumption of non-Ricardian seigniorage policy for the central bank. The reason for our decision to start with a non-Ricardian

specification is to emphasise that the indeterminacy that we consider arises in an environment that typically yields a determinate price level. It is worth emphasizing that the type of indeterminacy in our paper does not derive from the stability of a steady state or the infinity of the horizon<sup>1</sup>. Nevertheless, one might want to restrict attention to equilibria that stay in a neighborhood of a steady state. In such a case, its stability would be important, which is related to recent discussions of the Taylor rule: although, as long as fiscal policy is Ricardian, the coefficient in the Taylor rule does not change the degree of indeterminacy, it affects the number of locally bounded equilibria.<sup>2</sup>

There is a vast and important literature on indeterminacy of monetary equilibria. [Sargent and Wallace \(1975\)](#) discussed the indeterminacy of the initial price level under interest rate policy; [Lucas and Stokey \(1987\)](#) derived the condition for the uniqueness of a recursive equilibrium with money supply policy; [Woodford \(1994\)](#) analyzed the dynamic paths of equilibria associated with the indeterminacy of the initial price level under money supply policy. In this paper, we give the exact characterization of the indeterminacy in stochastic economies in terms of the nominal equivalent martingale measure and show that there is a continuum of recursive equilibria under QE with interest rate policy. [Carlstrom and Fuerst \(1998\)](#) discussed the indeterminacy of sticky-price equilibria when the nominal interest rates are zero. Here, we discuss the indeterminacy in more general case. In closely related models, [Dubey and Geanakoplos \(2003\)](#) considered non-Ricardian fiscal policy with no transfers and [Geanakoplos and Tsomocos \(2002\)](#) extended their model to an open economy. [Dreze and Polemarchakis \(2000\)](#) and [Bloise et al. \(2005\)](#) studied the existence and indeterminacy of monetary equilibria with a particular Ricardian fiscal policy, seigniorage distributed contemporaneously as dividend to the private sector.

Our work builds most closely upon [Nakajima and Polemarchakis \(2005\)](#) and [Bloise et al. \(2005\)](#). There, in a Ricardian specification, the indeterminacy in a monetary economy is characterized by the price level and a nominal martingale measure; and it is real or nominal, depending, among other things, on the asset structure. Our point here is similar, but focuses on the role of the balance sheet of the central bank and how it is affected by unconventional policies. As [Reis \(2010\)](#) points out, in normal times the central bank operates using only interest rate policy

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<sup>1</sup>[Benhabib and Farmer \(1999\)](#) is a useful survey of the literature on indeterminacy arising from the stability of a steady state.

<sup>2</sup>[Woodford \(1999\)](#) and [Benhabib et al. \(2001\)](#); [Benhabib et al. \(2002\)](#) examine how non-Ricardian fiscal policy interacts with the Taylor rule to obtain a unique equilibrium.

and the central bank balance sheet is such that they back up the money supply with holdings of government debt. When the central bank moves to unconventional policies such as the outright purchase of assets, the central bank typically has no committed portfolio position. The basic explanation for our finding is that we require the extra restrictions imposed by ex-ante limiting the portfolio of the central bank in order to pin down the price level using conventional monetary policy.

The indeterminacy that we discuss in the baseline model is nominal; while the central bank loses the control of inflation, the indeterminacy does not affect the attainable equilibrium allocations (a real indeterminacy would also affect allocations). If the central bank were to switch to a money supply, rather than interest rate, policy then the indeterminacy becomes real and affects consumption allocations. Moreover, while our baseline model assumes that asset markets are complete and prices are flexible, if prices are sticky or the asset market is incomplete, unconventional monetary policies similarly lead to a real indeterminacy.

The remainder of this paper is structured as follows:

## 2 The Conduct of Monetary Policy in the US and UK

### 2.1 Monetary Policy in the US Federal Reserve System

In addition to the reduction in the target federal funds rate from 5-1/4 percent to effectively zero, the Federal Reserve implemented a number of programs designed to support the liquidity of financial institutions and foster improved conditions in financial markets. These new programs led to significant changes to the Federal Reserve's balance sheet<sup>3</sup>.

The tools can be divided into three groups. The first set of tools, which are

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<sup>3</sup>In light of improved functioning of financial markets, many of the new programs have expired or been closed. These include the Money Market Investor Funding Facility (MMIFF), the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF), the Commercial Paper Funding Facility (CPFF), the Primary Dealer Credit Facility (PDCF), the Term Securities Lending Facility (TSLF), and the Term Auction Facility (TAF). The temporary liquidity swap arrangements between the Federal Reserve and other foreign central banks (FCBs) expired in February 2010. However, in May 2010, temporary dollar liquidity swap lines were re-established with certain FCBs. Subsequently, these arrangements were extended through August 1, 2011, and then through August 1, 2012.

closely tied to the central bank's traditional role as the lender of last resort, involve the provision of short-term liquidity to banks and other depository institutions and other financial institutions. The traditional discount window, Term Auction Facility, PDCF, and TSLF fall into this category. Because bank funding markets are global in scope, the Federal Reserve also approved bilateral currency swap agreements with 14 foreign central banks. These swap arrangements assisted these central banks in their provision of dollar liquidity to banks in their jurisdictions.

### **Large Scale Asset Purchases** <sup>4</sup>

During the crisis of 2008, policymakers took a number of extraordinary steps to improve the functioning of financial markets and stimulate the economy. Among the most important of these measures, in terms of both scale and prominence, were the Federal Reserve's purchases of large quantities of government-backed securities in the secondary market, conventionally known as the Large Scale Asset Purchase—or "LSAP"—programs. The LSAPs included debt obligations of the government-sponsored housing agencies, mortgage-backed securities (MBS) issued by those agencies, and coupon securities issued by the U.S. Treasury, and they collectively amounted to \$1.7 trillion over a period of about 15 months—the single largest government intervention in financial-market history. Given the unprecedented size and nature of these programs and the speed with which they were proposed and implemented, policymakers could have had, at best, only a very rough ex ante sense of their potential impact. The minutes of the December 2008 Federal Open Market Committee meeting summarized the prospects thus: "The available evidence indicated that [LSAP] purchases would reduce yields on those instruments, and lower yields on those securities would tend to reduce borrowing costs for a range of private borrowers, although participants were uncertain as to the likely size of such effects."

When LSAP was established in November 2008, the FOMC intended to acquire up to \$600 billion in agency mortgage-backed securities and agency debt. From

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<sup>4</sup>Unlike the Large Scale Asset Purchase programme (LSAP), the Maturity Extension Program (MEP) explicitly aims at extending the average maturity of the Fed's Treasury holdings without changing the overall size of the central bank's balance sheet. In this regard it is essentially a new version of Operation Twist, implemented in the early 1960s, which sought to "twist" the yield curve by nudging the longer-term yields lower while keeping the short rates at existing levels. Under that programme, the Fed bought about \$8.8 billion of longer-term Treasury securities and reduced its holdings of short-term Treasury bills by \$7.4 billion. The size of purchases was comparable to the LSAP programmes, relative to GDP and to Treasury debt outstanding.

March 2009 to March 2010, it committed an additional \$850 billion to purchases of agency securities, and a further \$300 billion to acquiring longerterm Treasury securities (LSAP1). As the recovery faltered, in November 2010 the FOMC put in place LSAP2, which consisted of further purchases of \$600 billion in longer-term Treasury securities until mid-2011. The Federal Reserve’s asset holdings expanded rapidly as a consequence of these purchases, reaching about 17% of Treasury securities outstanding by mid-2011<sup>5</sup>.

Overall, purchases of nominal securities under the Treasury LSAP program included 160 unique CUSIPs, spanning remaining maturities of about two to thirty years. \$300 billion represented about 3 percent of the total stock of outstanding Treasury debt and about 8 percent of the outstanding coupon securities as of the time of the announcement. Most purchases were concentrated in the 2- to 7-year sectors, although, as a percentage of total outstanding Treasuries within each sector, purchases across maturities were less concentrated. Coupon rates and vintages of securities purchased were roughly similar to the averages of all outstanding Treasuries. The average maturity of securities bought was a bit longer than average, but the yields on purchased securities were notably higher than average—seemingly by too great a margin to be accounted for solely by their slightly longer maturities, especially given that a relatively high fraction (approximately 30 percent) of purchases were on-the-run issues, which generally have lower yields.

## 2.2 The Conduct of Monetary Policy by the Bank of England

The first major event of the financial disruption occurred earlier in the UK compared with other countries; disruptions in the money market beginning April 2007 caused funding problems for a number of commercial banks and led to the run on Northern Rock in September 2007. Figure 1 shows the monetary policy response of the Bank of England in terms of the base interest rate (Bank Rate) which, until January 2009,

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<sup>5</sup>The logistics of the purchase operations were as follows. Every-other Wednesday, the Desk announced the broad maturity sectors in which it would be buying over the subsequent two weeks and the days on which it would be conducting these operations. Auctions took place from Monday of the first week through Friday of the second week and typically settled on the following day.<sup>5</sup> At 10:15 on the morning of each auction, the Desk published a list of CUSIPs that were eligible for purchase, which generally included nearly all securities in the targeted sector, and began accepting propositions. At 11:00 AM, the auction closed. The Desk then determined which securities to buy from among the submitted bids based on a confidential algorithm, and it published the auction results within a few minutes of the auction close. Market participants were not aware in advance of the total amount to be purchased or of the distribution of purchases across CUSIPs.

was the main instrument which the Monetary Policy Committee (MPC) could use in order to try to hit its inflation target.

Figure 1: Bank of England Policy Decisions

Notes: This figure shows decisions on base rate and asset purchases by the Bank of England Monetary Policy Committee. The vertical lines mark the date that of various important economic events and policy decisions. Source: Bank of England.

In that meeting in March 2009, the MPC also decided to start using the Asset Purchase Facility (APF). Following its establishment in January 2009, the APF is designed allow the MPC “to buy high-quality assets financed by the issuance of Treasury Bills” [Bank of England \(2010\)](#). It is the use of this policy that represents the Bank of England’s quantitative easing policy. Until October 2011, the MPC had voted to purchase a stock of £200bn of assets; the first decision in March 2009 was to purchase £75bn worth of assets with additional purchases of £50bn in May 2009, £50bn in August 2009 and £25bn in November 2009.<sup>6</sup> These policy decisions are also shown in [Figure 1](#).

### **The Bank of England Balance Sheet**

As we argue below, a key factor in the control of inflation is the central bank balance sheet. [Figure 2](#) shows the asset side of the Bank of England balance sheet over the 2007-2010 period; for a complete discussion of the management of the Bank’s balance sheet during the crisis see [Cross et al. \(2010\)](#). During this period, the central bank balance sheet grew from around 4-5% of nominal GDP to 17% (meaning that central bank assets are at about the same level (relative to nominal GDP) as the historical peak reached in the aftermath of World War II.

Figure 2: Bank of England Balance Sheet - Assets

Notes: This figure shows the asset side of the Bank of England balance along with important economic events and policy decisions dates marked with vertical lines. Source: Bank of England.

The first issue to consider is how the use of the Asset Purchase Facility affected the Bank’s balance sheet. Actually the assets purchased are not held in accounts

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<sup>6</sup>On October 6 2011, the MPC increased the stock of approved purchases to £275bn.

that are part of the consolidated Bank of England balance sheet. Rather, they are held in separate accounts and the value of the assets is “indemnified for losses by the Government” [Cross et al. \(2010\)](#). However, this fund is financed by loans from the Bank and these loans are included in “Other Assets” in [Figure 2](#) and make up over 90% of this classification following the MPC decision in March 2009 to undertake significant asset purchases through the APF.<sup>7</sup> Quantitative easing has certainly led to a significant change in both the size, and importantly for our analysis, composition of central bank assets.

### 3 Expected Inflation Over the Crisis

#### 3.1 Evidence from the UK

We will not examine the effect of QE, or other schemes, on the wide variety of asset markets and even on economic activity; the interested reader is referred to [Joyce et al. \(2011\)](#) for a recent discussion of the Bank of England’s analysis of these issues. Instead we focus here on the behaviour of the distribution of inflation expectations. [Macallan et al. \(2011\)](#) discuss the behaviour of inflation expectations in the UK. They reveal that there is a wider dispersion of inflation expectations using measures derived from asset prices and from the Survey of Professional forecasters. While they note that the inter-quartile range of expected inflation derived from the quarterly NOP survey of households has remained fairly constant, some measures of dispersion of households expected inflation, such as the percentage of the distribution in the upper and lower tails, have increased consistent with the other inflation expectations distributions.

In [Figures 3 and 4](#),<sup>8</sup> we report some of the measures of the distribution of inflation expectations from “Year-on-Year (YoY)” inflation options using the same underlying data as in [Macallan et al. \(2011\)](#).<sup>9</sup> These options work in the following way: a cap

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<sup>7</sup>[Cross et al. \(2010\)](#) present a similar breakdown of Bank of England assets but they have are able to separate out “Loans to the APF” from other assets whereas this split is not reported in the weekly Bank Returns data that I use.

<sup>8</sup>We are working to update these data to include the whole of 2007 as well as extending the range to include the October 2011 decision to expand the asset purchases by a further £75bn to £275bn.

<sup>9</sup>Price information is available from Bloomberg (RILO <GO>).

(floor) pays out if average inflation in the UK (measured by the Retail Price Index, RPI) over a set number of years is greater (less than) than the agreed strike price. Taking a cross-section of prices for these caps and floors, it is possible to back out an implied probability distribution which measures market expectations of RPI over the period of the derivative.

### Figure 3: Measures of Dispersion of Inflation Expectations I

Notes: This figure shows the standard deviation of the options implied probability distribution of annual RPI outturns for 2-3 years ahead. Important economic events and policy decisions are marked with vertical lines. Source: Bank of England, see [Macallan et al. \(2011\)](#).

### Figure 4: Measures of Dispersion of Inflation Expectations II

Notes: This figure shows the probability attached to greater than 5% inflation and less than 0% inflation as derived from the options implied probability distribution of annual RPI outturns for 2-3 years ahead. Important economic events and policy decisions are marked with vertical lines. Source: Bank of England, see [Macallan et al. \(2011\)](#).

In [Figure 3](#) we show standard deviation of the options implied probability distribution of annual RPI outturns for 2-3 years ahead. It is clear that the distribution of expected inflation has increased and that this increased dispersion followed changes in policies which affected the Bank of England balance sheet or the consolidated Fiscal-Monetary balance sheet. In particular, the standard deviation starts to increase markedly following the introduction of the SLS and then jumps again as the MPC cuts aggressively to the lower bound and quantitative easing becomes an increasingly likely policy; the Figure also shows that Google searches and news articles in the UK using the phrase “quantitative easing” grew steadily during this cutting cycle. The dispersion has increased fourfold since the start of unconventional policy.

In [Figure 4](#), we use an alternative measure of dispersion.<sup>10</sup> Using the same underlying implied pdf for annual RPI outturns 2-3 years ahead, we report the

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<sup>10</sup>A third alternative measure is the ratio of a particular maturity of YoY cap with an equivalent index cap to capture expected RPI volatility. The intuition is that the YoY cap is more likely to pay out than an index cap when RPI is unstable and so the price of a YoY cap relative to an index cap will be higher when inflation is expected to be less stable over the period. Unfortunately Bloomberg only reports index cap prices starting in May of 2009 which misses most of the unconventional policy changes that are of interest.

probability that RPI inflation is greater than 5% (upper tail risk) and the likelihood that inflation is less than 0% (lower tail risk). Though lower tail risk was prominent at the beginning of the crisis and has subsided since the start of quantitative easing (as [Joyce et al. \(2011\)](#) argue is consistent with the aims of QE), the probability attached to deflation remains much higher than it was previously. The latter statement also applies to the upper tail outturns. The overall probability attached to being in the upper or lower tail has increased substantially since 2007 and is consistent with greater dispersion of expected inflation. We now turn to our theoretical model in which we consider the implications of unconventional policies for control of inflation.

## 4 Model

Suppose that shocks follow a Markov chain with transition probabilities  $f(s'|s) > 0$ . The history of shocks up through date  $t$  is denoted by  $s^t = (s_0, \dots, s_t)$ , and called a date-event. The initial shock,  $s_0$ , is given, and the initial date-event is denoted by 0. The probability of date-event  $s^t$  is  $f(s^t)$ . Successors of date-event  $s^t$  is  $s^{t+i}|s^t$ . For  $s^{t+i}|s^t$ , the probability that  $s^{t+i}$  occurs, conditional on  $s^t$ , is  $f(s^{t+i}|s^t)$ .

At each date-event  $s^t$  there are  $J$  nominally risk-free bonds available for trade, each maturing at periods  $j \in J$ . We assume that  $J = S$ : markets are complete. Let the *annualized* interest rate at date-event  $s^t$  of a bond maturing in  $j \in J$  periods be  $r(s^{t+j}|s^t)$ . The term-structure of interest rates at  $s^t$  is then given by the  $j$  dimensional vector  $\{r(s^{t+1}|s^t), \dots, r(s^{t+J}|s^t)\}$ . Furthermore, let the price of a  $j$ -period bond at date-event  $s^t$  be  $q(s^{t+j}|s^t) = \frac{1}{(1+r(s^{t+j}|s^t))^j}$ . Finally let the price of a bond maturing at date-event  $s^t$  be  $q(s^t|s^t) = 1$  with interest rate  $r(s^t|s^t) = 0$ .

Let  $\mu$  denote the nominal equivalent martingale measure. It is a probability measure over the date-event tree with  $\mu(s^t) > 0$ , all  $s^t$ . Let  $\psi(s^{t+1}|s^t)$  denote the no-arbitrage price at  $s^t$  of the (untraded) elementary security that pays off one unit of currency if and only if the date-event  $s^{t+1}$  is reached. Then,

$$\psi(s^{t+1}|s^t) = \frac{\mu(s^{t+1}|s^t)}{1 + r(s^{t+1}|s^t)},$$

where  $r(s^t)$  is the one-period nominal interest rate at  $s^t$ . More generally,

$$\psi(s^i|s^t) = \frac{1}{1+r(s^{t+1}|s^t)} \cdots \frac{1}{1+r(s^i|s^{i-1})} \mu(s^i|s^t).$$

As markets are complete in the sense of there being as many traded securities as states of nature, the no-arbitrage value of the elementary securities is determined entirely by the path of prices of the traded bonds.

For simplicity, consider the flexible-price economy. The representative household has preferences given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u[c(s^t), \bar{y}(s_t) - y(s^t)] f(s^t), \quad (1)$$

where  $c(s^t)$  is consumption at  $s^t$ ;  $\bar{y}(s_t)$  is the endowment;  $y(s^t)$  is production.<sup>11</sup> At each date-event, the asset market opens first, followed by the goods market.

The budget constraint for the household in the asset market is

$$\hat{m}(s^t) + \sum_j q(s^{t+j}|s^t) b(s^{t+j}|s^t) \leq w(s^t) + \tau(s^t), \quad (2)$$

where  $\hat{m}(s^t)$  is the amount of cash obtained by the household and  $b(s^{t+j}|s^t)$  the portfolio of bonds of varying maturities and  $w(s^t)$  is the wealth of the householder entering that date-event.

The market for goods opens next. The purchase of consumption goods is subject to the cash-in-advance constraint

$$P(s^t)c(s^t) \leq \hat{m}(s^t). \quad (3)$$

The household also receives cash by selling its product,  $y(s^t)$ . Hence, the amount of cash that it carries over to the next period,  $m(s^t)$ , is

$$m(s^t) = P(s^t)y(s^t) + \hat{m}(s^t) - P(s^t)c(s^t). \quad (4)$$

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<sup>11</sup>We have assumed that the endowment,  $\bar{y}$ , depends only on the current shock.

Given (4), the cash-in-advance constraint (3) is equivalent to the constraint

$$m(s^t) \geq P(s^t)y(s^t). \quad (5)$$

It turns out that (5) is more convenient than (3) to describe the cash constraint in our economy. This is due to the assumption that the asset market precedes the goods market.

The household enters state  $s^{t+1}|s^t$  in the next period with nominal wealth

$$w(s^{t+1}|s^t) = m(s^t) + \sum_j q(s^{t+1+j-1}|s^{t+1})b(s^{t+j}|s^t). \quad (6)$$

The constraints the household faces are summarized by: (i) the flow budget constraints:

$$\begin{aligned} P(s^t)c(s^t) + m(s^t) + \sum_j q(s^{t+j}|s^t)b(s^{t+j}|s^t) \\ \leq w(s^t) + \tau(s^t) + P(s^t)y(s^t), \end{aligned} \quad (7)$$

(ii) the cash constraints:

$$m(s^t) \geq P(s^t)y(s^t), \quad (8)$$

and (iii) the natural debt limit (Ljungqvist and Sargent, 2000):

$$w(s^t) \geq - \sum_{i=t}^{\infty} \sum_{s^i|s^t} \psi(s^i|s^t) [P(s^i)\bar{y}(s_i) + \tau(s^i)]. \quad (9)$$

Here,  $P(s^t)$  is the price level at  $s^t$ ;  $m(s^t)$  is the nominal balances carried over from  $s^t$  to the next period;  $w(s^{t+1})$  is the nominal value of the financial asset at the beginning of  $s^{t+1}$ ;  $\tau(s^t)$  is the nominal transfer from the fiscal authority at  $s^t$ . The natural debt limit (9) says that the amount the agent can borrow at a given date-event,  $-w(s^t)$ , is bounded by the present discounted value of his future earnings,  $P(s^i)\bar{y}(s_i) + \tau(s^i)$ . It is equivalent to

$$\lim_{i \rightarrow \infty} \sum_{s^i|s^t} \psi(s^i|s^t) w(s^i) \geq 0. \quad (10)$$

Remember that when the goods market follow the asset market, the cash-in-advance constraint is equivalent to (8).

The first-order conditions are given by

$$\frac{u_1[c(s^t), \bar{y}(s_t) - y(s^t)]}{u_2[c(s^t), \bar{y}(s_t) - y(s^t)]} = 1 + r(s^{t+1}|s^t), \quad (11)$$

$$\sum_{s^{t+1}|s^t} \frac{\beta u_1[c(s^{t+1}), \bar{y}(s_{t+1}) - y(s^{t+1})] f(s^{t+1}|s^t)}{u_1[c(s^t), \bar{y}(s_t) - y(s^t)]} \frac{P(s^t)}{P(s^{t+1}|s^t)} q(s^{t+1+j-1}|s^{t+1}) = q(s^{t+j}|s^t); \quad (12)$$

and the transversality condition is

$$\lim_{i \rightarrow \infty} \sum_{s^i|s^t} \psi(s^i|s^t) w(s^i) = 0. \quad (13)$$

As markets are complete, we can construct individual Arrow securities from the Treasury securities available. To see this, note that the value of a bond maturing in  $j$  periods at date-event  $s^t$  is  $q(s^{t+j}|s^t)$  and at  $s^{t+1}$  is  $q(s^{t+1+j-1}|s^{t+1})$ . Given these  $J = S$  prices, it follows that there exists a unique *martingale measure*,  $\mu(s^{t+1}|s^t)$  such that  $q(s^{t+j}|s^t) = \frac{1}{1+r(s^t)} \sum_{s^{t+1}|s^t} \mu(s^t) q(s^{t+1+j-1}|s^{t+1})$ .

Given this debt constraint, and using the state prices, the flow budget constraints (6) and (7) reduce to the single, lifetime budget constraint

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) \left\{ P(s^t) c(s^t) + \frac{r(s^t)}{1+r(s^t)} m(s^t) \right\} \\ & \leq w_0 + \sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) \{ \tau(s^t) + P(s^t) y(s^t) \}. \end{aligned} \quad (14)$$

**Fiscal Policy** Fiscal policy sets a path of transfers,  $\{T(s^t)\}$ , and accommodates this by choosing a portfolio of liabilities,  $\{W(s^t)\}$ , that satisfy the flow budget constraint. Note that the composition of the portfolio is not pre-specified, and only the transfers. The flow budget constraint of the fiscal authority at date-event  $s^t$  is

$$\sum_j q(s^{t+j}|s^t) B_T(s^{t+j}|s^t) + T(s^t) = W_T(s^t), \quad (15)$$

with

$$W_T(s^{t+1}|s^t) = \sum_j q(s^{t+1+j-1}|s^{t+1}) B_T(s^{t+1+j-1}|s^{t+1})$$

and where  $W_T(0) = T_M(0)$  is the initial transfer from the monetary authority to satisfy the present value budget constraint of the fiscal authority.

Using similar arguments as that for the households, a single present-value budget constraint can be obtained for the Fiscal Authority:

$$\sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) T(s^t) \leq W_T(0). \quad (16)$$

**Monetary Policy** Monetary policy sets a path of one-period nominal interest rates,  $\{r(s^t)\}$ , and a rule on which of the  $j > 1$  period bonds are traded. The flow budget constraint of the monetary authority at date 0 is

$$M(0) + \sum_j q(s^j|0) B_M(s^j|0) + T_M(0) = W_M(0), \quad (17)$$

and at date-event  $s^t$

$$M(s^t) + \sum_j q(s^{t+j}|s^t) B_M(s^{t+j}|s^t) = W_M(s^t), \quad (18)$$

with

$$W_M(s^{t+1}|s^t) = M(s^t) + \sum_j q(s^{t+1+j-1}|s^{t+1}) B_M(s^{t+1+j-1}|s^{t+1})$$

and where  $w(0) = W_M(0) - T_M(0)$  and  $W_M(0) > 0$  is given.

It is well known that a Ricardian policy of the combined fiscal-monetary authority results in indeterminacy of degree 1: the price level is indeterminate. We focus on the indeterminacy of the *stochastic distribution* of inflation, and so the distinction between the two regimes is not relevant. For notational convenience, we focus on a Non-Ricardian policy by the Monetary authority, where the initial liability,  $W(0)$  is returned to the monetary authority via seignorage revenue.

In the infinite-horizon economy, fiscal policy is said to be non-Ricardian if it guarantees that the present discounted value of the fiscal authority liability converges to zero and the present discounted value of the monetary-authority liability converges

to the initial liability: Fiscal policy sets transfers so that

$$\lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) w(s^i) = \lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) W_T(s^i) = T_M(0),$$

Monetary policy accommodates money supply so that

$$\lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) W_M(s^i) = W_M(0),$$

for any path of  $q$ ,  $P$ ,  $r$ ,  $M$ , in or out of equilibrium. Of course, these two conditions are simultaneously satisfied at equilibrium because of the transversality condition (13) of the household.

**Definition.** Given initial nominal wealth,  $W_M(0) > 0$ ,  $W_T(0) = T_M(0)$  and  $w(0) = W_M(0) - T_M(0)$ , interest rate policy,  $r(s^{t+1} | s^t)$ , and fiscal policy,  $T(s^t)$ , a competitive equilibrium consists of an allocation,  $\{c(s^t), y(s^t)\}$ , a portfolio of households,  $\{m(s^t), w(s^t)\}$ , a portfolio of the monetary authority,  $\{M(s^t), W_M(s^t)\}$ , a portfolio of the fiscal authority,  $W_T(s^t)$ , transfers,  $T(s^t)$ , goods-market prices,  $P(s^t)$  and bond-market prices,  $\{q(s^{t+2}), \dots, q(s^{t+J})\}$ , such that

1. given  $W_M(0)$  and  $\{r(s^{t+1} | s^t), M(s^t)\}$ , fiscal policy  $T(s^t)$  determines transfers  $\tau(s^t) = T(s^t)$ ,  $s \in \mathcal{S}$  and  $t \in \{0, 1, \dots, \infty\}$ , the monetary authority debt portfolio  $W_M(s^t)$ , and the fiscal authority debt portfolio  $W_T(s^t)$ ;
2. the monetary authority accommodates the money demand,  $M(s^t) = m(s^t)$ ;
3. given interest rates,  $r(s^{t+1} | s^t)$ , goods-market prices,  $p(s^t, h) = P(s^t)$ , all  $h$ , bond-market prices,  $\{q(s^{t+2}), \dots, q(s^{t+J})\}$ , and transfers,  $\tau(s^t)$ , the household's problem is solved by  $c(s^t)$ ,  $y(s^t)$ ,  $m(s^t)$  and  $w(s^t)$ ;
4. all markets clear.

We restrict attention to symmetric equilibria.

**Existence of Equilibrium** The existence of equilibrium requires further restrictions on the flow utility function.

**Assumption 1.** *The flow utility function,  $u$ , satisfies*

$$\lim_{c \rightarrow 0} \frac{u_1(c, y - c)}{u_2(c, y - c)} = \infty,$$

for each  $y > 0$ .

The following proposition shows that  $P_0$  and  $\mu$  are not determined, and hence, there is  $S$ -dimensional indeterminacy.

**Proposition 1.** *Given initial nominal wealth,  $w_0 = W_0$ , interest rate policy,  $\{r(s^{t+1}|s^t)\}$ , and fiscal policy,  $\{T(s^t)\}$ ,*

1. *a competitive equilibrium exists;*
2. *the equilibrium allocation  $\{c(s^t), y(s^t)\}$  is unique;*

**Proof** Given interest rates  $r(s^{t+1}|s^t)$ ,  $s^t = \{0, 1, \dots, s_t\}$ , the first-order condition(11) determines the allocation of resources at each date-event:

$$\frac{u_1[c(s^t), \bar{y}(s_t) - c(s^t)]}{u_2[c(s^t), \bar{y}(s_t) - c(s^t)]} = 1 + r(s^{t+1}|s^t).$$

Our assumptions on  $u$  guarantees the existence and uniqueness of the solutions to these equations. The equilibrium output at each date-event,  $y(s^t)$ ,  $s \in \mathcal{S}$ , is given by

$$y(s^t) = c(s^t).$$

Fiscal policy sets transfers so that

$$\lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) w(s^i) = \lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) W_T(s^i) = 0,$$

Monetary policy accommodates money supply so that

$$\lim_{i \rightarrow \infty} \sum_{s^i | s^t} \psi(s^i | s^t) W_M(s^i) = W_M(0),$$

all  $s, t$ . Hence, the allocation is uniquely determined. □

At equilibrium we can now conclude that there exists a unique state price  $\psi(s^{t+1}|s^t)$

such that  $\psi(s^{t+1}|s^t) = \frac{\beta^t u_1[c(s^{t+1}|s^t), \bar{y}(s_{t+1}) - y(s^{t+1}|s^t)] f(s^{t+1}|s^t)}{u_1[c(s^t), \bar{y}(s^t) - y(s^t)]} \frac{p(s^t)}{p(s^{t+1}|s^t)}$ .

## 5 Conventional Monetary Policy

Under conventional monetary policy, the central bank sets a path of one-period interest rates  $r(s^t)$  by exchanging one-period securities for money. In addition the monetary authority trades, but holds to maturity, bonds of maturities greater than 1. This is equivalent to the monetary authority selecting a portfolio such that the wealth it carries from a date-event  $s^t$  to any immediate successor node  $s^{t+1}|s^t$  being *state-independent*. To see this, consider the marketed, or effective, wealth which the monetary authority carries into successor state  $s^{t+1}|s^t$ . Any bonds not maturing at this date-event would be immediately re-invested so as to not affect the net-wealth position. Bonds maturing at this date-event would be bonds purchased from the previous  $J$  periods. In the same way that one-period bonds purchased at  $s^t$  would have a state-independent pay-off at all  $s^{t+1}|s^t$ , the two period bonds purchased at the node preceding  $s^t$  would have one year left to maturity at  $s^t$  and be also effectively one-period bonds at  $s^t$ , and so on for all  $J$  bonds. It is then straightforward that the effective portfolio of the central bank under conventional monetary policy consists of purchasing all Arrow securities: a safe portfolio.

**Definition.** Conventional monetary policy consists of a portfolio equivalent to the monetary authority purchasing all available Arrow securities.

Under this case, the following proposition holds. The proof is in Appendix.

**Proposition 2.** *Given conventional monetary policy, with non-Ricardian fiscal policy, the equilibrium is unique.*

The uniqueness of equilibrium with the non-Ricardian policy considered here follows from the fact that there is only a unique path of price levels that is consistent with the transversality condition (13).

## 6 Unconventional Monetary Policy

### 6.1 Quantitative Easing

Under unconventional monetary policy, the central bank trades a portfolio of bonds of various maturities, setting the path of one-period interest rates  $r(s^t)$  only. Bonds of maturities longer than 1 are not necessarily held to maturity. This is defined as *unconventional monetary policy under quantitative easing*. This is equivalent to the monetary authority portfolio being state-contingent but not in a pre-specified manner.

**Definition.** Unconventional monetary policy under quantitative easing consists of a monetary authority holding a portfolio without any restrictions on the maturity of bonds purchased at each date-event.

Under this case, the following proposition holds. The proof is in Appendix.

**Proposition 3.** *Given unconventional monetary policy under quantitative easing, with non-Ricardian fiscal policy, the equilibrium is indeterminate.*

### 6.2 Credit Easing

Under unconventional monetary policy, the central bank trades a portfolio of bonds of various maturities, with explicit state-contingent targets for the composition of its portfolio. This is defined as *unconventional monetary policy under credit easing*. This is equivalent to the monetary authority targetting the value of its portfolio in successor nodes.

**Definition.** Unconventional monetary policy under credit easing consists of a monetary authority holding a portfolio with a pre-specified weights on the maturities of bonds purchased at each date-event.

Under this case, the following proposition holds. The proof is in Appendix.

**Proposition 4.** *Given unconventional monetary policy under credit easing, with non-Ricardian fiscal policy, the equilibrium is determinate.*

## 7 Concluding Remarks

### Appendix

#### Proof of Proposition 2

The allocation is found as in Proposition 1. Note that as  $T(s^t) = \tau(s^t)$ , we can combine the household present value budget constraint, 14, with that of the Fiscal Authority, 16, together with market clearing in the goods market,  $c(s^t) = y(s^t)$ , to obtain the consolidated household present value budget constraint:

$$\sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) \frac{r(s^t)}{1+r(s^t)} m(s^t) \quad (19)$$

$$\leq w(0) + W_T(0). \quad (20)$$

Using market clearing in the money market,  $m(s^t) = M(s^t)$  and  $W_M(0) = w(0) + W_T(0)$ , 19 becomes

$$\sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) \frac{r(s^t)}{1+r(s^t)} M(s^t) \quad (21)$$

$$\leq W_M(0).$$

Using the cash-in-advance constraints 8 and the value of the Arrow price from individual optimization,

$$\begin{aligned} W_M(0) &= \sum_{t=0}^{\infty} \sum_{s^t} \psi(s^t|0) \frac{r(s^t)}{1+r(s^t)} P(s^t) y(s^t) \\ \frac{W_M(0)}{P(0)} &= \sum_{t=0}^{\infty} \sum_{s^t} \frac{\psi(s^t|0) P(s^t)}{P(0)} \left\{ \frac{r(s^t)}{1+r(s^t)} y(s^t) \right\} \\ &= \frac{\beta^t u_1[c(s^t), \bar{y}(s^t) - y(s^t)] f(s^t)}{u_1[c(0), \bar{y}(0) - y(0)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\} \\ &> 0 \end{aligned}$$

Since the equilibrium allocation is unique, the right hand side of the second equation is unique. Since  $W_M(0) > 0$ , there is a unique  $P(0) > 0$  that solves the equation. Given  $P(0)$ ,  $\psi(s^t|0)P(s^t)$  are determined uniquely by

$$\psi(s^t|0)P(s^t) = P(0) \frac{\beta^t u_1[c(s^t), \bar{y}(s^t) - y(s^t)] f(s^t)}{u_1[c(0), \bar{y}(0) - y(0)]}. \quad (22)$$

The paths of debt and prices,  $\{W(s^{t+1}), P(s^t), q(s^t|0)\}$  are, then, determined inductively as follows. From the cash-in-advance constraint  $M(0) = P(0)y(0)$ . The monetary authority budget constraint at date 0 gives that

$$W(0) = \sum_j q(s^j|0)B(s^j|0) + M(0).$$

At date 1, the wealth is

$$W(s^1|0) = \sum_j q(s^{j-1}|s^1)B(s^j|s^1) + M(0).$$

However, the commitment to hold bonds to maturity means that the wealth that is marketed is *state-independent* or

$$W(1^1) = W(2^1) = \dots = W(S^1).$$

From market clearing, this is also the *effective* marketed wealth the agent carries into date 1:  $w(s^1) = W(s^1)$ . Importantly, this wealth is not state contingent.

At date 1, state  $s$ , after substituting in market clearing conditions the household

budget constraint becomes

$$w(s^1) = \sum_{t=1}^{\infty} \sum_{s^t} \psi(s^t|s^1) \frac{r(s^t)}{1+r(s^t)} P(s^t) y(s^t) \quad (23)$$

$$\begin{aligned} \frac{W(s^1)}{P(s^1)} &= \sum_{t=1}^{\infty} \sum_{s^t} \frac{\psi(s^t|s^1) P(s^t)}{P(s^1)} \left\{ \frac{r(s^t)}{1+r(s^t)} y(s^t) \right\} \\ &= \sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s_t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\} \end{aligned} \quad (24)$$

$$P(s^1) = \frac{W(s^1)}{\sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s_t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\}}. \quad (25)$$

The final step is to use the first order condition for a one period bond:

$$\begin{aligned} \frac{1}{1+r(s^1|0)} &= \sum_{s^1|0} \frac{\beta u_1[c(s^1), \bar{y}(s_1) - y(s^1)] f(s^1|0)}{u_1[c(0), \bar{y}(0) - y(0)]} \frac{P(0)}{P(s^1|0)} \\ &= \sum_{s^1|0} \frac{\beta u_1[c(s^1), \bar{y}(s_1) - y(s^1)] f(s^1|0)}{u_1[c(0), \bar{y}(0) - y(0)]} \\ &\quad \times \sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s_t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\} \frac{P(0)}{W(s^1)} \\ W(s^1) &= \sum_{s^1|0} \frac{\beta u_1[c(s^1), \bar{y}(s_1) - y(s^1)] f(s^1|0)}{u_1[c(0), \bar{y}(0) - y(0)]} \\ &\quad \times \sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s_t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\} P(0) (1+r(s^1|0)). \end{aligned}$$

As  $P(0)$  was determined in the previous step, then  $W(s^1)$  is also determined. Using this, and 25 the price level in each state can be determined. Using this logic, it should be clear that as the new liabilities of the monetary authority at each date-event are a function of the price level at that date-event, and which is determinate, then recursively all prices in the economy can be determined.

### Proof of Proposition 3

The proof here follows closely that of Proposition 2. The allocation and initial price level are found in the same way. The monetary authority budget constraint at date

0 gives that

$$W(0) = \sum_j q(s^j|0)B(s^j|0) + M(0),$$

where the one period interest rate is set by the monetary authority ( $q(s^1|0)$  is considered fixed). At date 1, the wealth is

$$W(s^1|0) = \sum_j q(s^{j-1}|s^1)B(s^j|s^1) + M(0).$$

However, now no bonds need to be held to maturity. This means that the wealth that is marketed is

$$W(s^1) = \sum_j q(s^{j-1}|s^1)B(s^{j-1}|s^1) + M(0).$$

From market clearing, this is also the *effective* marketed wealth the agent carries into date 1:  $w(s^1) = W(s^1)$ . Now this wealth is state contingent, as it is value subject to the  $J - 1$  interest rates at that date event. As these interest rates are left to market forces to determine, then it follows that any choice of  $q(s^{j-1}|s^1)$  for  $j \in J$  satisfies the properties of equilibrium and corresponds to a unique goods market price at that date event.

This shows that as the net liability of the monetary authority is not determined, then neither is the price level in each state. Only the *average* rate of inflation one period ahead is determined by the policy of quantitative easing. The degrees of indeterminacy are  $S - 1$ : The  $S$  prices at each date have only by one interest rate (the one-period interest rate) to determine them.

#### **Proof of Proposition 4**

The price level in this case is determinate and is proved as in Proposition 2. Let, at date 0, the restriction on the monetary authority portfolio be of the following form. Let there be an  $\alpha(s^j|0)$  such that  $\sum_j \alpha(s^j|0) = 1$  and  $B(s^j|0) = \alpha(s^j|0)\bar{B}(0)$ . The monetary authority budget constraint at date 0 gives that

$$W(0) = \bar{B}(0) \sum_j \alpha(s^j|0)q(s^j|0) + M(0),$$

where the one period interest rate is set by the monetary authority ( $q(s^1|0)$  is considered fixed). At date 1, the wealth is

$$W(s^1|0) = \bar{B}(0) \sum_j q(s^{j-1}|s^1) \alpha(s^j|0) + M(0).$$

We can normalise the wealth at date 1 and find a  $\beta(s^1)$  such that  $\sum_{s^1} \beta(s^1) = 1$  and  $W(s^1|0) = \beta(s^1) \bar{W}(s^1|0)$ , where  $\bar{W}(1^1) = \bar{W}(2^1) = \dots = \bar{W}(S^1)$ .

Thus fixing  $\alpha(s^j|0)$  corresponds (locally) to a  $\beta(s^1)$ , which is easier to work with.

From Proposition 2, at date 1, state  $s$ , after substituting in market clearing conditions the household budget constraint becomes

$$\begin{aligned} P(s^1) &= \frac{W(s^1)}{\sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s^t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\}} \\ &= \frac{\beta(s^1) \bar{W}(s_1|0)}{\sum_{t=1}^{\infty} \sum_{s^t} \frac{\beta^t u_1[c(s^t), \bar{y}(s^t) - y(s^t)] f(s^t)}{u_1[c(s^1), \bar{y}(s^1) - y(s^1)]} \left\{ \frac{r(s^t)}{1+r(s^t)} c(s^t) \right\}}. \end{aligned} \quad (26)$$

Now take the Fisher equation, which gives the no-arbitrage relationship between the fundamental Arrow prices and the one period interest rate at date 0:

$$\begin{aligned} \sum_{s^1} \psi(s^1|0) &= \frac{1}{1+r(0)} \\ \sum_{s^1} \frac{\beta u_1[c(s^1), \bar{y}(s_1) - y(s^1)] f(s^1)}{u_1[c(0), \bar{y}(0) - y(0)]} \frac{P(0)}{P(s^1)} &= \end{aligned} \quad (27)$$

Substituting in the expression for  $P(s^1)$  from (26), we can solve for  $\bar{W}(s_1|0)$  and hence for  $P(s^1)$ . In this way the path of prices can be determined recursively and so a policy of credit easing pins down the stochastic rate of inflation.

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