Technological Progress and Urbanization Process

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Abstract

I extend the model of cities by Lucas and Rossi-Hansberg (Econometrica 2002) to include an agricultural sector. My aim is to understand how productivity changes in the manufacturing sector and the agricultural sector will impact on the relative size of a city and the surrounding rural area, both in terms of physical area and population. We also examine rural-urban differentials in land rental. Government policies leading to the optimal size of city can be discussed.

Keywords: Land use, urbanization, sectoral labor migration, city size

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1 Introduction

One of the most well-known stylized facts on development is structural change: as national income increases, the production shifts from agricultural sector into manufacturing, and then to service sector (Clark 1940 and Chenery and Syrquin 1975). Multi-sector models have been built to illustrate the driving forces behind the structural change, see for example, Lewis (1954), Matsuyama (1992), Echevarria (1997), Kongsamut, Rebelo and Xie (2001). These models examine the issues on the surplus labor in the countryside, the push and pull effects of productivity changes in the agricultural and manufacturing sectors, and Engel's law that results from lower income elasticity of demand for agricultural goods than for manufacturing goods. Recent contributions on structural changes include Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2006). Both papers explain the structural changes from the "supply-side": Ngai and Pissarides emphasize on differences in sectoral total factor productivity whereas Acemoglu and Guerrieri focus on facor proportion differences and capital deepening. Nevertheless, all these models have taken the relative size of land used for agricultural and manufacturing purposes as fixed and therefore are silent on questions related to urbanization process: What determines the size of the city in a given area where both manufacturing and agricultural activities could potentially take place? How will the border between the city and the surrounding countryside shift as productivities in the two sectors change? What happens to the welfare of an individual and how drastically will labor relocate in response to productivity changes? Will the Rural-urban differentials in land rentals shrink or expand?

The present paper tries to answer these questions. The framework that I use is an extension of Lucas and Rossi-Hansberg (2002). Lucas and Rossi-Hansberg (2002) take the size of a city as given and examine the internal structure of the city as a function of parameters such as commuting cost and degree of externality in industrial production. The conclusion of their study is that the city maps at the symmetrical equilibrium could be radically different from one another for different parameter combinations. Only when the commuting cost is sufficiently small, the city would turn into a Mills city (Mills 1967), namely a central business district (the manufacturing center) surrounded by a ring of residencial area. Similar conclusions have been obtained earlier by Fujita and Ogawa (1982) in a theoretical analysis of a linear city. The extension that I entertain in this paper is to surround the city by another ring of rural area in which the land has mixed use for residence and farming.

In order to focus on urbanization process, I simply assume that the commuting cost is low so that the city map indeed takes Mills' form. This assumption simplifies the analysis enough so I could drop those assumptions in Lucas and Rossi-Hansberg (2002) that are partial equilibrium in nature, for instance, the assumptions of absentee landlords and of the exogenously given reservation utility.

The numerical analysis conducted here suggests that, both in terms of labor relocation and of land usage, the pull effect of productivity improvement in the manufacturing sector is not as nearly as important as the push effect of productivity change in the Agricultural sector. Also, the differential in land rental is sensitive only to productivity change in the Agricaltural sector. The intuition is that productivity change in any sector will worsen the terms of trade against that sector, hence partially cancel out the impact of productivity change in terms of resource re-allocation. On the other hand, any productivity improvement will have an income effect that increases the demand for all goods, manufacturing as well as agricultural. Hence, if the productivity change occurs in the manufacturing sector only, the increased demand for agricultural goods implies that the agricultural labor will have to remain in rural area to meet the demand. If the productivity change is in the agricultural sector, labor could be released from the agricultural sector because of the Engel's law. In this case, labor is "pushed" out of the rural area.

The rest of the paper is organized as follows. Section 2 sets up the model. Sector 3 contains numerical examples and comparative statics analysis. Policy discussion and concluding comments can be found in Section 4 together with suggestions on possible empirical testings of the model.

2 The Model

I keep as much as possible the same notations as in Lucas and Rossi-Hansberg (2002). I consider a circular community of fixed radius S, which is populated by a continuum of agents with a measure of unity who have the same preferences and productive capacity. There are two goods produced in this community, an agricultural good, A, and a manufacturing good, M.

The land is owned at equal share by all the individuals in this community. This assumption is different from Lucas and Rossi-Hansberg (2002), who assume that the land is owned by absentee landlords.

Each individual has one unit of labor to be inelastically supplied to either manufacturing (partly wasted on commuting, as explained below) or farming. I abstract away from labor-leisure tradeoff and the representative individual is assumed to have the following preferences over the manufacturing good, M, the agricultural good, A, and the residential land, l:

$$U(A, M, l) = (A - \bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma}$$

where $\bar{A} > 0$ captures the idea that one needs to consume an amount of the agricultural good greater than $\bar{A}$ in order to survive. This feature of the preferences also implies an income elasticity of demand for agricultural goods lower than for manufacturing goods, leading to the Engel's law that as income increases, the share of expenditure on agricutural goods declines.

The total land area of the community, πS^2, is divided among manufacturing good production use, agricultural use, and residential use. I describe locations within the community by their polar coordinates (r, ϕ), but I will focus on symmetric equilibrium, where nothing depends on ϕ, and a location can be referred simply as "location r". If a worker lives at location s and works at location r, he can only deliver

$$e^{-\kappa|r-s|}$$

units of labor at location r. Namely $1 - e^{-\kappa|r-s|}$ is lost on commuting. In Lucas and Rossi-Hansberg (2002), they study how the value of κ may influence the equilibrium map of the city. One result is that if κ is sufficiently small, the city map will be like the Mills' map: manufacturing at the center, with residential area at the outer part of the city. In our model, we assume that κ is indeed small. Following our assumption on the production function of the agricultural sector (see below), the community at market equilibrium will be a Mills' city, surrounded by rual area, where farming and residential are mixed.

To fix notations, the community is represented as follows: a manufacturing center that is a circle with radius s_1; an urban residential area where workers for the manufacturing center are resided, which is a ring with distance to city center between s_1 and s_2; and a mixed rural area of agricultural production and residence of farmers, located in a ring with distance to city center between s_2 and S (Figure 1). The emergence of the mixed rural area results from the absence of external effect in the agricultural sector assumed below and the existence of commuting cost assumed above.

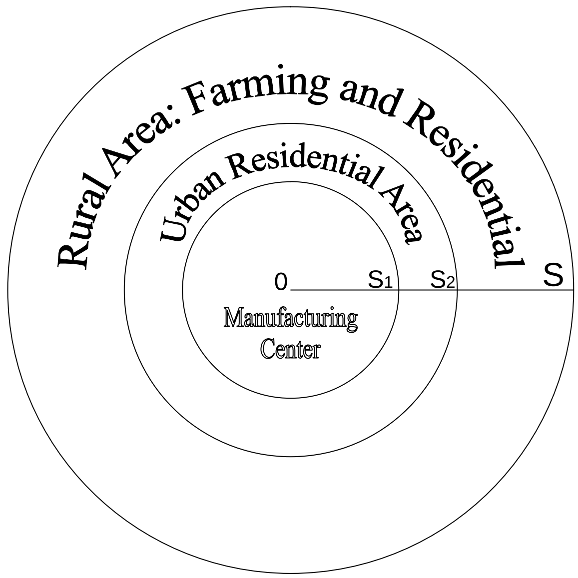

Figure 1. Community Map

2.1 Production Functions

Employment density is defined as employment per unit of production land. Let employment density in manufacturing be denoted $n_m(r)$ at location r; and let employment density in agriculture be denoted $n_a(r)$. Let $N(r)$ denote the number of workers housed at r, per unit of residential land. Then if each such person occupies $l(r)$ units of land, we have $l(r)N(r) = 1$.

Manufacturing production per unit of land at location r is given by,

$$m(r) = z(r)^{\gamma} B_m n_m^{\alpha}(r),$$

where $z(r)$ captures the idea of positive external effect on the productivity of the firm at location r by the employed workers in the other firms:

$$z(r) = 2\pi \int_0^{s_1} \int_0^{2\pi} s n_m(s,\phi) g\left(\rho(r,s,\phi)\right) d\phi ds.$$

In the above expression, function $\rho(.)$ denotes the distance between two firms at location $(r, 0)$ and (s, ϕ),

$$\rho(r,s,\phi) = \left[r^2 - 2\cos(\phi) rs + s^2\right]^{1/2},$$

and function $g(.) > 0$ is assumded to be decreasing, capturing the idea that the further away the distance between the two firms, the lower the external effect.

It is assumed as in Lucas (2001) and in Fujita, Krugman, and Venables (1999) that

$$0 < \gamma < 1 - \alpha$$

to ensure that the production externality does not swamp the effects of land prices, otherwise every firm wants to locate at the city center.

Since allocations are assumed to be symmetric, we can write

$$z(r) = \int_0^{s_1} \psi(r,s) s n_m(s) ds \tag{1}$$

where

$$\psi(r,s) = 2\pi \int_0^{2\pi} g\left(\rho(r,s,\phi)\right) d\phi.$$

For the sake of simplying numerical procedures later, we will assume that there exists a function $g(.)$ such that

$$\psi(r,s) = e^{-\delta(r+s)}$$

Following Lucas and Rossi-Hansberg (2002), let $H(r)$ have the interpretation of the stock of manufacturing workers that remain unhoused at r after employment and housing have been determined for locations $s \in [0, r)$. At equilibrium, we must have: $H(0) = 0$ and $H(s_2) = 0$. The differential equation that governs the evolution of $H(r)$ is given by,

$$\frac{dH(r)}{dr} = 2\pi r n_m(r) + \kappa H(r), \text{ for } r \in [0, s_1)$$

and

$$\frac{dH(r)}{dr} = -2\pi r N(r) + \kappa H(r), \text{ for } r \in [s_1, s_2]$$

where the term $\kappa H(r)$ appears because it takes $e^{\kappa dr} H(r)$ units of labor from location $r+dr$ to arrive at $H(r)$ units of full time equivalent labor at location r. In fact, the existence of non-zero commuting cost makes labor at different locations different in terms of full time equivalent, similar to the idea that non-zero interest rate makes identical goods at different period different from one another in present value terms. Integrating over locations yields:

$$\begin{aligned} H(s_1)e^{-\kappa s_1} - H(0) &= \int_0^{s_1} 2\pi r n_m(r) e^{-\kappa r} dr \\ H(s_2)e^{-\kappa s_2} - H(s_1)e^{-\kappa s_1} &= -\int_{s_1}^{s_2} 2\pi r N(r) e^{-\kappa r} dr \end{aligned}$$

Plugging in equilibrium conditions $H(0) = 0$ and $H(s_2) = 0$, I obtain

$$\int_0^{s_1} 2\pi r n_m(r) e^{-\kappa r} dr = \int_{s_1}^{s_2} 2\pi r N(r) e^{-\kappa r} dr,$$

which states that in the urban area, the full-time equivalent units of labor demand equals the full-time equivalent units of labor supply.

Total supply of manufacturing goods is given by,

$$M^s = 2\pi \int_0^{s_1} r z(r) n_m^{\alpha}(r) dr$$

Let $q_m(r)$ be the profit per unit of land (land rental per unit of land) at a manufacturing location r,

$$q_m(r) = pz(r)^{\gamma} B_m n_m^{\alpha}(r) - w(r)n_m(r) = \max_n pz(r)^{\gamma} B_m n^{\alpha} - w(r)n$$

where p is the relative price of manufacturing goods in terms of agricultural goods which is taken as numeraire.

The first order condition for the firm,

$$n_m(r) = \left[\frac{\alpha pz(r)^{\gamma} B_m}{w(r)}\right]^{1/(1-\alpha)}, \tag{2}$$

gives the demand for labor at location r as a function of $w(r)$, given p and $z(r)$. The resulting profit per unit of land at location r is thus given by,

$$\begin{aligned} q_m(r) &= (1-\alpha)\left[pz(r)^{\gamma} B_m\right]^{1/(1-\alpha)} \left[\frac{\alpha}{w(r)}\right]^{\alpha/(1-\alpha)} \\ &= (1-\alpha)\left[pz(r)^{\gamma} B_m\right] n_m(r)^{\alpha} \end{aligned}$$

Agricultural production per unit of land at location r:

$$a = B_a n_a^{\varphi}(r), \text{ where } \varphi \in (0,1).$$

Thus the agricultural sector is modelled not to invovle any externality. The total supply of agricultural goods is given by,

$$A^s = 2\pi \int_{s_2}^{S} rB_a n_a^{\varphi}(r)\xi(r)dr$$

where $\xi(r)$ denotes the fraction of land used at location r for agricultural production, hence $1-\xi(r)$ is then used for residential purposes in rural area, which, due to the absence of external effect and the existence of commuting cost, is a mixed area and the farmers work wherever they live. Therefore, the total supply of labor is as follows and there is no need to apply "discounting":

$$2\pi \int_{s_2}^{S} rN(r)(1-\xi(r))dr.$$

Let $q_a(r)$ be the profit per unit of land (land rental per unit of land) at a farm location r,

$$q_a(r) = B_a n_a^{\varphi}(r) - w(r)n_a(r) = \max_n B_a n^{\varphi} - w(r)n$$

The demand for labor is hence:

$$n_a(r) = \left[\frac{\varphi B_a}{w(r)}\right]^{1/(1-\varphi)}$$

and the profit per unit of land,

$$\begin{aligned} q_a(r) &= (1-\varphi)B_a^{1/(1-\varphi)}\left[\frac{\varphi}{w(r)}\right]^{\varphi/(1-\varphi)} \\ &= (1-\varphi)B_a n_a^{\varphi} \end{aligned} \tag{3}$$

At equilbrium where labor demand equals labor supply, we have:

$$2\pi \int_{s_2}^{S} rn_a(r)\xi(r)dr = 2\pi \int_{s_2}^{S} rN(r)(1-\xi(r))dr \tag{4}$$

Let $q_R(r)$ denote the rent per unit of urban residential land at r and let

$$q(r) = \begin{cases} q_m(r) & \text{for} \quad r \in [0, s_1] \\ q_R(r) & \text{for} \quad r \in [s_1, s_2] \\ q_a(r) & \text{for} \quad r \in [s_2, S] \end{cases}$$

We will derive the equilibrium value of $q_R(r)$ from utility maximization.

The representative worker or farmer who lives sat r must choose $A^d(r)$, $M^d(r)$ and $l^d(r)$ that solve the following problem:

$$\begin{aligned} & \max (A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma} \\ \text{subject to:} \qquad & A + pM + q(r)l \leq w(r) + \int_0^S q(s) 2\pi s ds \end{aligned}$$

and at the equilibrium, $(A^d(r)-\bar{A})^{\sigma}\left(M^d(r)\right)^{\beta}\left(l^d(r)\right)^{1-\beta-\sigma}$ must be constant for all $r \in [s_1, S]$. We can write down the Lagrangian for the representative agent:

$$L = (A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma} + \lambda(r)\left[w(r) + \int_0^S q(s) 2\pi s ds - (A + pM + q(r)l)\right]$$

First Order Conditions:

$$\sigma \frac{(A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma}}{(A-\bar{A})} = \lambda(r)$$

$$\beta \frac{(A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma}}{M} = \lambda(r)p$$

$$(1-\beta-\sigma)\frac{(A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma}}{l} = \lambda(r)q(r)$$

The three conditions above yield,

$$\lambda(r)(A-\bar{A}) + \lambda(r)pM + \lambda(r)q(r)l = (A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma},$$

which can be used to solve for $\lambda(r)$:

$$\lambda(r) = \frac{(A-\bar{A})^{\sigma} M^{\beta} l^{1-\beta-\sigma}}{w(r) + \int_0^S q(s) 2\pi s ds - \bar{A}}$$

Plugging this back to the three conditions give us the familar results:

$$A^d(r) - \bar{A} = \sigma \left[w(r) + \int_0^S q(s)2\pi s ds - \bar{A} \right], \tag{5}$$

$$M^d(r) = \beta \left[w(r) + \int_0^S q(s)2\pi s ds - \bar{A} \right] /p, \tag{6}$$

$$l^d(r) = (1 - \beta - \sigma) \left[w(r) + \int_0^S q(s)2\pi s ds - \bar{A} \right] /q(r). \tag{7}$$

In the derivations in this section, I follow Lucas and Rossi-Hansberg in using $w(r)$ to denote both the wage rate paid at location r and the earnings of a worker housed at location r. If r is a purely manufacturing location, $w(r)$ denotes the wage paid by firms operating there, and a worker who commutes to r from s has earnings $w(s) = e^{-\kappa|r-s|}w(r)$ available to spend at his place of residence. If r is a purely residential location, then $w(r)$ denotes earnings of people who live there, and a resident at r who works at s must receive $w(s) = e^{\kappa|r-s|}w(r)$ per unit of labor supplied at s. Finally, if r is a mixed farm-residential location, people who live there also work there, and $w(r)$ denotes both the wage rate and the net earnings at r.

2.2 Equilibrium Conditions

Demand for agricultural goods equals its supply:

$$\begin{aligned} &\int_{s_1}^{s_2} 2\pi r N(r) A^d(r) dr + \int_{s_2}^{S} 2\pi r (1 - \xi(r)) N(r) A^d(r) dr \\ = &\int_{s_2}^{S} 2\pi r \xi(r) B_a^{1/(1-\varphi)} \left[\frac{\varphi}{w(r)} \right]^{\varphi/(1-\varphi)} dr \end{aligned}$$

Demand for manufacturing goods equals its supply:

$$\begin{aligned}
& \int_{s_1}^{s_2} 2\pi r N(r) M^d(r) dr + \int_{s_2}^{S} 2\pi r (1-\xi) N(r) M^d(r) dr \\
= & \int_0^{s_1} 2\pi r \left[p z(r)^\gamma B_m \right]^{1/(1-\alpha)} \left[\frac{\alpha}{w(r)} \right]^{\alpha/(1-\alpha)} dr
\end{aligned}$$

Demand for housing equals its supply:

$$l^d(r) = 1/N(r) \text{ for any } r \in [s_1, S],$$

As in Lucas and Rossi-Hansberg (2002), the equilibrium wage structure would appear as follows,

$$w(r) = w_0 e^{-\kappa r}, \text{ for any } r \in [0, s_2],$$

and from the fact that all farmers are identical and they work with the same technology wherever they live, we must have,

$$w(r) \equiv w_a \text{ for any } r \in [s_2, S],$$

and in particular,

$$w_a = w(s_2) = w_0 e^{-\kappa s_2} \tag{8}$$

In addition, the employment density per unit of land $n_a(r)$ and the housing consumption per farmer $l_a(r)$ in the rural area are independent of of location r, and will henceforth be denoted as n_a and l_a.

In this paper, the retail needs for land and labor are assumed away to the keep the analysis manageable. The total population is either employed as a worker or a farmer:

$$2\pi \int_{s_1}^{s_2} r N(r) dr + 2\pi \int_{s_2}^{S} r N(r)(1 - \xi(r)) dr = 1 \tag{9}$$

To arrive at equilibrium $z(r)$, I take the following steps. Given the assumption on $\psi(r,s) = e^{-\delta(r+s)}$, the following equation

$$\begin{aligned} z(r) &= \int_0^S \psi(r,s) s\theta(s) n_m(s) ds \\ &= \int_0^{s_1} e^{-\delta(r+s)} s \left[\frac{\alpha p z(s)^\gamma B_m}{w(s)}\right]^{1/(1-\alpha)} ds \end{aligned}$$

has an explicit solution for the equilibrium $z(r)$:

$$z(r) = v(s_1, w_0/p) e^{-\delta r}. \tag{10}$$

In fact, $v(s_1, w_0/p)$ takes the following form as shown in Appendix A:

$$\begin{aligned} v(s_1, w_0/p) &= \left[\frac{\alpha p B_m}{w_0}\right]^{1/(1-\alpha-\gamma)} \\ &\times \left[\frac{1 - e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{(\delta - (\kappa - \delta\gamma)/(1-\alpha))^2} - \frac{s_1 e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{\delta - (\kappa - \delta\gamma)/(1-\alpha)}\right]^{(1-\alpha)/(1-\alpha-\gamma)} \end{aligned} \tag{11}$$

3 Numerical Examples and Comparative Statics

To obtain numerical solutions for the equilibrum, I first express all other variables in terms of w_0, p, ξ, $s_1$and s_2. Then, I will collect the five equations that pin down these five variables.

To begin, I use (8) to find w_a, which can then be used to find the employment density in rural area, n_a:

$$n_a = \left[\frac{\varphi B_a}{w_a}\right]^{1/(1-\varphi)}.$$

Equation (4) reduces to the following relationship between n_a and the housing consumption per farmer, l_a:

$$\begin{aligned} \xi n_a &= (1-\xi) N_a \\ &= \frac{1-\xi}{l_a}, \end{aligned}$$

which gives l_a. The continuity for housing consumption per person at location s_2 requires,

$$l_a = l^d(s_2),$$

which, together with (7) yields,

$$\begin{aligned} l_a &= (1-\beta-\sigma)\left[w_a + \int_0^S q(s)2\pi s ds - \bar{A}\right]/q(s_2) \\ &= (1-\beta-\sigma)\left[w_a + \int_0^S q(s)2\pi s ds - \bar{A}\right]/\left[(1-\varphi)B_a n_a^{\varphi}\right] \end{aligned}$$

where the last equality makes use of equation (3). Hence, I can solve for $\int_0^S q(s)2\pi s ds$.

Given the total value of the land, $\int_0^S q(s)2\pi s ds$, the constancy of the welfare for any individual imples that for any $r \in [s_1, s_2]$,

$$\begin{aligned} &\left[w_0 e^{-\kappa r} + \int_0^S q(s)2\pi s ds - \bar{A}\right] q(r)^{-(1-\beta-\sigma)} \\ = &\left[w_a + \int_0^S q(s)2\pi s ds - \bar{A}\right]\left[(1-\varphi)B_a n_a^{\varphi}\right]^{-(1-\beta-\sigma)}, \end{aligned}$$

which can be used to compute the housing cost $q(r)$ for urban residential area.

I use (7) to obtain $l^d(r)$ and use (11) to computer $v(s_1, w_0/p)$. I use (5) and (6) to $A^d(r)$ and $M^d(r)$ and A_a^d and M_a^d for $r \in [s_1, s_2]$. Then I use (10) to computer $z(r)$, which then pins down the employment density at each manufacturing location, $n_m(r)$, through equation (2).

- The five equations used to pin down w_0, p, ξ, s_1 and s_2 are as follows:
- population equation:

$$2\pi \int_{s_1}^{s_2} \frac{1}{l^d(r)} r dr + \pi \frac{(1-\xi)}{l_a}(S^2 - s_2^2) = 1$$

- Demand for Agriculture goods equals supply:

$$\int_{s_1}^{s_2} 2\pi r A^d(r) \frac{1}{l^d(r)} dr + \pi(S^2 - s_2^2)(1-\xi)\frac{A_a}{l_a^d} = \pi \xi B_a n_a^{\varphi}(S^2 - s_2^2)$$

- Demand for manufacturing goods equals supply:

$$\int_{s_1}^{s_2} 2\pi r M^d(r) \frac{1}{l^d(r)} dr + \pi(S^2 - s_2^2)(1-\xi)\frac{M_a^d}{l_a^d} = 2\pi \int_0^{s_1} z(r)^{\gamma} B_m \left(n_m(r)\right)^{\alpha} r dr$$

- In the urban area, the full-time equivalent units of labor demand equals the full-time equivalent units of labor supply:

$$\int_0^{s_1} 2\pi r n_m(r) e^{-\kappa r} dr = \int_{s_1}^{s_2} 2\pi r N(r) e^{-\kappa r} dr \qquad (12)$$

- Continuity of land price at the border between the manufacturing sector and the urban residential area:

$$q_m(s_1) = \lim_{r \to s_1 +} q(s_1).$$

The parameter values in the numerical examples are as follows. First, I set $\kappa = 0.001$, $\gamma = 0.04$, $\delta = 5$, $S = 10$, $B_m = 1$, which are the same as in Lucas and Rossi-Hansberg (2002). Next, since there is one more sector, namely, the agricultural sector, some parameters in Lucas and Rossi-Hansberg (2002) have to be modified accordingly and new parameters be introduced. These parameters are: $\alpha = 0.85$, $\sigma = 0.05$, $\beta = 0.85$, $B_a = 1$, $\varphi = 0.7$, and $\bar{A} = 2$.

I focus on how changes in B_m and B_a would affect the equilibrium. In particular, I ask in the first case what happens if B_m increases by 20% and in the second case what happens if B_a increases by 20%. Given the asymmetry built into the preferences and the production functions, the outcomes in the two cases are expected to be different. Nevertheless, some of the results are unexpected.

When B_a is held the same while B_m increases by 20%, the most noticible change in the equilibrium outcome is the relative price p. In fact, p would decline by 16.66 percent in our numerical example so that in terms of the marginal product value of labor, this decline nearly cancels out the 20% increase in productivity. Other things being equal, the net effect of 3.3% of increase in the marginal product value of labor in the manufacturing sector would "pull" out of the agricultural sector. But this "pull" effect is very small in the numerical exercise and almost indistinguishable from zero. The same can be said of the areas of the manufacturing center, the urban residential area, and the countryside. None of the changes are distinguishable from zero. The value of land at the center is slightly higher than before and the value of land in the countryside slightly lower than before. The welfare however increases by 16.73 percent.

When B_m is held to be unity while B_a increases by 20%, it causes changes in many aspects of the equilibrium. First, the relative price increases by 23.21 percent. It changes more than the productivity change because of the smaller income elasticity of demand for agricultural goods than for manufacturing goods. Thus, there is a net effect of 3.21 percent of decrease in the marginal product value of labor in the agricultural sector. According to our previous example, this would not generate any significant labor relocation. Nevertheless, labor in agricultural sector is reduced by 15.70 percent. This large relocation is due to the "push" effect: given the low income elasticity of demand for agricultural goods, the increased productivity in the agricultural sector allows a release of labor into manufacturing while at the same time meeting the increased demand for food. Also, the city expands in a meaningful way, with the radius increased by 9.30 percent and the area increased by 19.46 percent. More specifically, the manufacturing center sees its area increased by 6.94 percent and the urban residential area expands by 23.80 percent. Not only that, the residential housing space increases all around, thanks to the increased productivity in the agricultural sector that releases part of the agricultural land for residential use. The land value at the center increases by 30.45 percent while the land value in the countryside increases by only 14.84 percent. In other words, the productivity increase in the agricultural sector increases the rural-urban differential in land rentals. In terms of utility for the representative agent, the productivity change in the agricultural sector brings a gain of 9 percent, significantly lower than the gain brought by the productivity change in the manufacturing sector.

Obviously, the results of numerical exercises above will be more pronouced if the subsistence level of agriculture, $\bar{A}$, plays an important role, namely if the equilbrium level of agricultural consumption per person is just above the subsistence level. In developed countries where the equilibrium agricultural consumption is far above $\bar{A}$, the "push" effect will no longer be as dramatic as suggested above.

4 Testable equations and policy discussion

The above analysis indicates that the size of the city would change the most in regions where productivity increase mainly comes from the agricultural sector. In China, a natural experiment is to see if the provinces where agricutural reform started the earliest would see their city sizes expand the earliest. Similar test can be done in terms of the rural-urban differential in land rentals.

One has to be careful about underline assumptions in the model here when drawing real world lessons. For example, the model would not be suitable for analysing the pre-reform China when labor mobility is highly limited. Also, the implications from this closed-economy model can be drastically different if the community is relatively open. In latter case, a productivity increase in the manufacturing sector would increase emloyment in that sector because the relative price is given exogenously. So the "pull" effect could indeed be noticible. By the same token, the "push" effect may not be as strong as in the numerical example above.

Given the externality in the manufacturing sector, the market allocation is sub-optimal. The government can restore the optimum by subsidizing the manufacturing sector if lump-sum taxes are available. Suppose for example that the required subsidy is 20% to each manufacturing firm. Would this have the same effect in terms of resource allocation as a 20 percent increase in manufacturing productivity? The answer is no. On the one hand, the 20 percent subsidy is equivalent to 20 percent productivity increase from individual firms' point of view. On the other hand, the 20% subsidy does not really increase the manufacturing output the same way as a 20 percent productivity increase. Hence the relative price p would not drop by 16.66 percent as in the numerical example above. Therefore, the subsidy will indeed move employment toward manufacturing and the city size will expand, which tends to increase the rural-urban differential in land rentals. China during the "planning economy" era had many subsidies in the urban sector such as subsidized land rental and artificially depressed agricutural prices, which in theory should have quickened the urbanization process. Unfortunately, these policies co-existed with formal migration restrictions that limit the rural people to move to urban area. As a result, 30 percent of the cities in China are undersized, according to estimates by Au and Henderson (2004).

The main message from this paper is that for a relatively closed developing country in which the subsistence level of agriculture still plays an important role, the productivity change in the agricultural sector has tremendous implications in labor relocation across sectors, in the size of cities, and in the rural-urban diffferential in land rentals. Rural productivity change is the key to urbanization process. Putting into the context of China, the new agriculture policy should focus on rural productivity change rather than beautification of the countryside. The former will speed up urbanization while the latter could delay the process. Nevertheless, productivity change in the manufacturing sector is also important because it brings higher utility.

In open economy settings, the above conclusions will be weakened to a certain extent, but the main message will probably remain true, given that the transportation cost accross communities is significant larger than within community.

Appendix A

In this appendix, I derive the functional form of $v(s_1, w_0/p)$.

Note that,

$$\begin{aligned} v(s_1, w_0/p)e^{-\delta r} &= \int_0^{s_1} e^{-\delta(r+s)} s \left[\frac{\alpha p v(s_1, w_0/p)^{\gamma} e^{-\delta s \gamma} B_m}{w_0 e^{-\kappa s}}\right]^{1/(1-\alpha)} ds \\ &= \left[\frac{\alpha p v(s_1, w_0/p)^{\gamma} B_m}{w_0}\right]^{1/(1-\alpha)} \int_0^{s_1} e^{-\delta(r+s)} s e^{(\kappa s - \delta s \gamma)/(1-\alpha)} ds \\ &= e^{-\delta r} \left[\frac{\alpha p v(s_1, w_0/p)^{\gamma} B_m}{w_0}\right]^{1/(1-\alpha)} \int_0^{s_1} e^{-\delta s} s e^{(\kappa s - \delta s \gamma)/(1-\alpha)} ds \end{aligned}$$

Integrating by parts,

$$\begin{aligned}
v(s_1, w_0/p)e^{-\delta r} &= e^{-\delta r}\left[\frac{\alpha p v(s_1, w_0/p)^\gamma B_m}{w_0}\right]^{1/(1-\alpha)}\left[\begin{array}{c} -\frac{s e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s}\Big|_0^{s_1}}{\delta-(\kappa-\delta\gamma)/(1-\alpha)} \\ +\int_0^{s_1}\frac{e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s}}{\delta-(\kappa-\delta\gamma)/(1-\alpha)}ds \end{array}\right] \\
&= e^{-\delta r}\left[\frac{\alpha p v(s_1, w_0/p)^\gamma B_m}{w_0}\right]^{1/(1-\alpha)}\left[\begin{array}{c} -\frac{s_1 e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{\delta-(\kappa-\delta\gamma)/(1-\alpha)} \\ +\frac{1-e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{(\delta-(\kappa-\delta\gamma)/(1-\alpha))^2} \end{array}\right]
\end{aligned}$$

Thus

$$\begin{aligned}
v(s_1, w_0/p) &= \left[\frac{\alpha p v(s_1, w_0/p)^\gamma B_m}{w_0}\right]^{1/(1-\alpha)} \\
&\quad \times\left[\frac{1-e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{(\delta-(\kappa-\delta\gamma)/(1-\alpha))^2}-\frac{s_1 e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{\delta-(\kappa-\delta\gamma)/(1-\alpha)}\right]
\end{aligned}$$

Solving for $v(s_1, w_0/p)$, I obtain,

$$\begin{aligned}
v(s_1, w_0/p) &= \left[\frac{\alpha p B_m}{w_0}\right]^{1/(1-\alpha-\gamma)} \\
&\quad \times\left[\frac{1-e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{(\delta-(\kappa-\delta\gamma)/(1-\alpha))^2}-\frac{s_1 e^{-(\delta-(\kappa-\delta\gamma)/(1-\alpha))s_1}}{\delta-(\kappa-\delta\gamma)/(1-\alpha)}\right]^{(1-\alpha)/(1-\alpha-\gamma)}
\end{aligned}$$

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