From Trade-to-Trade in US Treasuries: Durations, Workups and News Effects*

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Abstract

The aim of this paper is to model the trading intensity of the US Treasury bond market, which has a unique expandable limit order book which distinguishes it from other asset markets. An analysis of tick data from the eSpeed database suggests that the US bond market displays a greater degree of clustering in trade durations than is evident in other asset markets. Duration is affected by the presence of news particularly in the hour following the release of news to the markets. Finally, the length of time taken to complete a given transaction, or 'workup', has a measurable impact on the trade duration.

Keywords: US Treasuries, trade duration, workups, news.

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1 Introduction

Information based models of trading distinguish between informed and uninformed traders. While noise traders operate continuously in the market, informed traders only enter the market when they are in possession of private information and will continue to trade for long as this information has value. In such a world, the market will learn about the private information held by informed traders through their transactions. Thus, the trading process acts as an important conduit for information and empirical studies have identified a number of trade characteristics as being important, including the intensity of trading.

Engle and Russell (1998) were the first to focus on modelling the informational content of the time between trades. They found that durations are characterized by a type of autocorrelation that is analogous to the well documented clustering in volatility.[1] To capture this dependence, the authors introduced the Autoregressive Conditional Duration (ACD) model and a literature has since evolved that has focussed on this model and its application to financial market data (see inter alia Engle and Russell, 1997, 1998, Engle, 2000, Bauwens and Giot, 2000, Zhang, Russell, and Tsay, 2001, Bauwens and Veredas, 2004, Xu, Chen and Wu, 2005 and Bauwens, 2006). A survey of the ACD literature may be found in Engle and Russell (2004) and Bauwens and Hautsch (2007).

While the stock and foreign exchange markets have received considerable attention in the duration modelling literature, there has not been any attempt to characterize the intensity of the trading process in bond markets. This represents an important omission given the significance of these markets in the global financial arena (see Goodhart and O'Hara, 1997). This paper will address this gap in the literature and characterize the trading intensity of bond markets. In particular, our focus is on the US market, which provides a benchmark that is referenced by many market participants and is a major provider of global liquidity. Modeling of the duration between trades presents an interesting challenge however, as the US bond market operates under an expandable limit order book which distinguishes it from all other markets. Specifically, US market dealers post 'indicative' price quotes for each bond maturity. The volume associated

[1] The market microstructure literature offers some explanations for the autocorrelation of the duration process. In particular, the contributions of Easley and O'Hara (1992), Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) are most relevant.

with that price however, may be well below the actual quantity the dealer is willing to trade and often only one unit of volume is posted (=US$1million). When a customer hits a passive bid displayed on the system, this initiates a trade with a broker and sets the price and initial volume to be traded. Market participants may subsequently negotiate additional volume at the already agreed price and hence, the volume traded is 'expandable'. This subsequent negotiation over quantity is known as the 'workup' and it is readily observable to other market participants via the market trading information system.[2] The trade ends when either there is no more volume to transact or no participant responds sufficiently fast to the opportunity to trade.

Boni and Leach (2004) argue that upon observing a private signal, informed traders will enter the market in an attempt to profitably exploit this informational advantage. When the order book is expandable, informed traders will use small limit orders to search for trading counter parties. Once they have identified a dealer who has indicated a willingness to trade, they will enter into quantity negotiations, ie. a workup will take place. This process allows more anonymity to informed traders than a conventional order book, which requires the placement of a large limit order[3] that may signal the information to the market, in which case any advantage is lost. Thus, Boni and Leach (2004) assign an informational role to workup insomuch as the average workup time will increase and stay high for as long as the private signal has value.

The purpose of this paper is to model the US bond market in transaction time. Drawing on the previous literature, an ACD model is applied to high frequency US Treasury data sampled over the period January 3, 2006 to October 10, 2006. This model is extended to include the workup period, which potentially constitutes an additional source of transactional based information. Another important source of information in the US bond market is regularly scheduled macroeconomic news announcements (see Fleming and Remolona, 1999, Balduzzi, Elton and Green, 2001, Green, 2004, Simpson and Ramchander, 2004, Pasquariello and Vega, 2007). As such, our model is also extended to incorporate information in the form of news, particularly around the

[2]Boni and Leach (2004) provide details of the nature of the expandable order book under the voice protocol system and most of these features have been retained in the two ECN's that currently dominate the market. It is worth noting that expandable limit orders and the associated workup process are distinct from hidden orders seen in other markets.

[3]An alternative strategy would be to engage in stealth trading, whereby trades are broken up into smaller and less conspicuous lots (see Barclay and Warner, 1993).

8.30am period, when the majority of the important news releases occur.

The results of this paper may be summarized as follows. We find that trade duration for US Treasuries displays different properties to those documented for other asset markets. Specifically, the US bond market displays a far greater degree of clustering in trade durations. Further, duration is affected by the presence of news on any given day and particularly in the hour following the release of news to the markets. Finally, the unique microstructure of the US Treasury market, which operates on the basis of an expandable limit order book, means that the presence of information via the transaction process in the market is directly measurable. The length of time taken to complete a given transaction has a measurable impact on the time between transactions. Specifically, we find that a long workup tends to be associated with a shorter time to the next transaction. Further, the time taken to workup a trade has a greater impact on reducing the expected adjusted duration than does the arrival of news in the form of scheduled macroeconomic news announcements. Our findings have implications for the theoretical modelling of the behavior of market participants and suggests that the microstructure of the market has an important impact on its functioning during times of particularly high activity

The remainder of this paper proceeds as follows. Section 1.1 provides a detailed description of the trading process in the US bond market. Section 2 introduces the dataset and characterizes the data in terms of transaction size, intensity and workups. A comprehensive examination of the data is undertaken and the analysis suggests that the information content of news arrival, volume and workup time each have a role to play in understanding the duration of transactions in the US Treasury market. Building on this preliminary analysis, Section 3 proceeds to develop and estimate a formal model of US bond market durations. Starting with the standard ACD model of Engle and Russell (1998), we modify this model to incorporate the additional sources of transactional based information that are unique to the US Treasury market. Finally, Section 4 concludes our paper and suggests some directions for further research.

1.1 The Trading Process

Trading in US Treasury bonds begins by participants placing orders on the electronic system and this infers a commitment to trade the given volume at the stated price until that order is cancelled. Typically, orders are for a minimum of US$1 million. When a trader hits (takes) a passive bid (ask) displayed on the system, this initiates a trade with a broker and sets the price and initial volume to be traded. The key distinguishing feature of this market exists in the next step of the trading process. Market participants may subsequently negotiate additional volume at the already agreed price and hence, the volume traded is 'expandable'. This subsequent negotiation over quantity is known as the 'workup' and it is readily observable to other market participants via the market trading information system. Note that workup time may be zero, in which case the transaction is only for the volume quoted at the given price. The trade formally 'ends' when sufficient time has elapsed with no further volume being traded.

A representation of this trading process for three hypothetical transactions is shown in Figure 1 (note that we do not allow for overlapping transactions as they are virtually nonexistent in the data). Transaction 1 begins at t_0, when a market participant agrees to transact at a price and quantity posted to the ECN. Additional quantity is then incrementally negotiated and the transaction is completed when a sufficiently small period of time has elapsed without further deals being struck (f_0).[4] The workup, $W_i = f_i - t_i$ is the time between the start and finish of the transaction and may differ for every transaction depending on the length of the negotiations over additional volume. For the hypothetical transactions shown in the figure, the first and third transactions contain workup, while the second transaction is for the posted volume only, with no subsequent workup. Further, the first transaction has a shorter workup than the third transaction. Note that the length of the workup does not necessarily reflect the size of the traded volume and this relationship is considered more fully in Section 2.2.

While the definition of trade duration is fairly straight forward in other markets, this is not true for the US bond market as there are a number of time points associated with a transaction. Duration in this context may be defined in three different ways: duration could be measured as the time between the initiation of consecutive trades, ie. the time between at which participants agree to undertake a transaction. An alternative would be to measure duration as the time between the end point of trades. A third possibility is to measure duration as the time between the end of the previous trade

[4]Note that consecutive trades may be negotiated at the same price as in some circumstances it is advantageous for a market participant to wait to initiate a new trade at the same price.

and the initiation of the next consecutive trade. In this paper, we adopt the first definition of duration, ie. duration is the time between the initiation of consecutive trades. We justify this choice based on the notion that duration modelled for other markets captures the arrival of traders to the market and a trade initiation based duration measure achieves this goal.

2 Descriptive Statistics

One of the most significant difficulties faced by any study of US bond markets, has been obtaining a suitable sample of data. Until recently, the GovPX database was the main source of data. The use of GovPX data however, brought with it a number of problems related to identifying trades, matching the actual bid-ask spread to trades and, most importantly, correctly calculating the volume of trade. Since 2000 however, US bond market trading has changed significantly and is now dominated by the ECNs of Cantor Fitzgerald and ICAP. The eSpeed and BrokerTec databases provide trading information for each of these markets and Mizrach and Neely (2006) find that there are qualitatively few differences between the two. This suggests that our use of the eSpeed database is unlikely to bias the results.[5]

The eSpeed dataset represents 10 millisecond shots of the transaction process, which is the maximum updating frequency that the traders using this platform see in real time. We consider trading in the 2, 5 10 and 30 year benchmark on-the-run Treasuries over the sample period January 3, 2006 to October 10, 2006. Although more recent data is available, this period is specifically chosen to avoid any periods of market turbulence that may bias the results. The trading day is defined as beginning at 7:30am New York time and ending at 5:30pm.

Insights into the nature of our dataset may be obtained by considering the average total trading volume and average trade size for trades in half hour periods across the trading day (see Figures 2 and 3). The total volume traded and average trade size both increase substantially from the open to peak at 8:30am for all maturities and then proceed to decline throughout the rest of the day, with the exception of a smaller and less pronounced peak at 10:00am. It is interesting to note that 8.30am

[5]Mizrach and Neely (2006) provide details of these two US ECN bond markets. They report that the market is split 60/40 in favour of the ICAP ECN. The more recent evidence of Jiang et al (2007) and Dungey et al (2007) however, suggests the market is more evenly split.

and 10.00am are the two scheduled major macroeconomic news announcement times in the US. The previous literature has clearly demonstrated the bond markets interest in these announcements as evidence by the speed and size of the markets response to the release of news, resulting in a higher volume of transactions and possibly even disruptions to the pricing process; see for example Fleming and Remolona (1997), Balduzzi, Elton and Green (2001), Simpson and Ramchander (2004), Pasquariello and Vega (2007), Dungey, McKenzie and Smith (2007) and Andersen, Bollerslev, Diebold and Vega (2007).

2.1 Durations

While the eSpeed database records a great deal of information, our focus is specifically on the time between transactions. A basic description of the trading intensity in this dataset is given in Table 1. In the sample period, the number of trades per day averages between 623 for the 2 year maturity to 1465 for the 10 year.[6] The largest trades are reported in the shorter 2 year maturity, where both the average and the maximum are substantially greater than for other maturities. The smallest average and maximum volumes are reported for the 30 year and average and maximum trade size tends to decrease as the term to maturity increases.

Table 1 also summarizes the raw duration data (x_i) associated with each maturity. The most heavily traded contracts are the 5 and 10 year bond contracts. These contracts have the shortest average duration (around 24 seconds), while the 2 year maturity exhibits the longest average duration (55.86 seconds). The average duration for trades in half hour periods across the trading day is presented in Figure 4. From the beginning of the sample, durations progressively fall to around 8.30am and then progressively lengthen as the trading day progresses, with a slight deviation at 10.00am. As has been previously discussed, the 8.30am and 10.00am periods are the two scheduled major macroeconomic news announcement times in the US.

Compared with previous research on other markets, these bond market durations may seem somewhat perverse. For example, Engle and Russell (1998) examine data for IBM across the 9:30am to 5pm stock market trading day and find that duration

[6] Although Cantor-Fitzgerald's reputation is as a long end specialist, Jiang et al (2007) compares the total turnover by maturity across the two electronic platforms and find that any difference is marginal.

displays a hump shape, which peaks around 2pm. A closer examination of the bond duration data across the same 9:30am to 5pm time period, reveals that this subset of the day has the same characteristic hump shape, ie. relatively low bond durations are observed around the time the stock market opens, higher duration occurs at lunch time and then duration falls toward the end of the stock market trading day. What distinguishes the bond data from the standard trading pattern of stocks is the low trading intensity (longer durations) in the periods prior to the stock market opening and after the stock market closing. These differences are apparent in Figure 4 and give the bond durations plot its unique daily profile.

Intradaily data is typically characterized by strong diurnality (see Engle and Russell, 2004), which may bias any estimation results. Such patterns are a clear feature of the bond durations data. To account for these intradaily effects, we follow the approach of, *inter alia*, Engle and Russell (1998, 2004) and Zhang, Russell and Tsay (2001) by constructing diurnalised estimates of duration (and workup times). Defining the raw duration between the i^{th} and $i-1^{th}$ transactions as $x_i = t_i - t_{i-1}$, then the adjusted duration is:

$$x_i^* = \frac{x_i}{\Phi\left(t_{i-1}\right)}, \tag{1}$$

The deterministic effect on trade durations due to the time of day is defined as the expected duration conditioned on time-of-day $\Phi\left(t_{i-1}\right) = E\left(x_i|t_{i-1}\right)$. This expectation is obtained by averaging the durations over thirty minutes intervals for the trading day. A cubic spline is employed to smooth the time of day function across the thirty minutes intervals. By construction, the mean of x_i^* is approximately 1.

A brief summary of the diurnalised duration for each maturity is presented in Table 1, and x_i^* for the two year maturity is plotted in Figure 5. Table 2 presents a more detailed summary of the adjusted durations for the 2, 5 10 and 30 year maturities including the number of trades (n), sample average (μ), p-value for a test of the null of a sample average of zero (in parenthesis) and standard deviation (σ^2). The results of a battery of Ljung-Box tests for p^{th} order serial correlation in x_i and the corresponding squares are also presented in Table 2 and the results uniformly reject the null hypothesis of no serial correlation. Thus, there is clear evidence of significant persistence in the filtered data and the results show considerable structure in the adjusted durations.

2.2 Workups

For each transaction, there is potentially a period of workup. Table 1 presents a summary of the average workup and, unlike the average of the raw durations which are distinctly higher for the 2 and 30 year maturities, the average of the raw workup times is similar across all maturities (approximately 2 seconds). Differences do exist however, when comparing the range of raw workup times. The 5 and 30 year contract have maximum workup values that are approximately five times larger than those of the 2 and 10 year bonds. These range estimates show that, while the average workup values are similar, the range of observations recorded in the database across all maturities differ substantially. Similar to total traded volume and average trade size, the average workup (Figure 6) increases from the open to peak at 8:30am for all maturities and then proceed to decline throughout the rest of the day, with the exception of a smaller and less pronounced peak at 10:00am. To account for this time dependency, we construct diurnalised workup times, denoted W_i^*, using Equation 1 and a plot of this data is presented in Figure 7.

Table 3 presents a detailed summary of the workup process, distinguishing between transactions with and without a workup. It is interesting to note that the majority of trades occur without any workup - over 60% of the transactions for the 2 and 30 year bonds and around 54% for the 5 and 10 year bonds. Trades without workup however, are typically for small volumes - the average size of a trade with no workup is less than half the average trade size for the 2, 5 and 10 year notes, and under 60% of that for the 30 year bonds. Table 3 shows that up to 76.5% of dollar volume is discovered in trades which involve some workup. This is consistent with Boni and Leach (2004) who report that 56.5% of the dollar volume in the superseded voice protocol trading system, GovPX, is discovered with the workup process. The data also suggests that larger volume does not necessarily result in more steps in the workup process. Recall that the average length of raw workup is two and a half seconds or less across all maturities (Table 1). When the large number of no workup transactions are accounted for, this figure rises by only around 1 second for any given maturity. By way of comparison, the fastest average workup time reported in Boni and Leach (2004) using GovPX data for 1997, is over four times greater.

The faster workup times of the ECNs has implications for the occurrence of simultaneous transactions. Boni and Leach (2004) considered overlapping transactions (Boni and Leach, 2004, Table 2) and found substantial evidence of impatience with the queuing process that may result from the workup period. In their results, they find that one third of all transactions in their 5 year bond sample have overlap with the previous transaction, comprising some 18 percent of the total volume traded. Further, in the event that a new transaction begins before the completion of the previous transaction, a price improvement is likely to result. By way of contrast, we find trivial instances of overlapping trades - there are 2 cases in each of the 2 and 10 year maturities, none in the sample for the 5 year bond and 20 overlapping transactions for the 30 year contract. Thus, the speed at which orders are filled in the ECN is substantially faster when compared to the voice protocol GovPX system, such that there are virtually no overlapping transactions. As such, there can be no evidence of the impatience discussed in Boni and Leach (2004) under the current trading arrangement.

The diurnal pattern of workup time shown in Figure 6 shows a peak at 8:30am, which is the period of greatest market activity - both in times of total volume and average trade size - and the lowest time between trades. Given that there is no evidence of impatience in the market, in that new transactions do not begin before the current workup has expired, these results suggest that workup time is an important indicator of liquidity in the market and may be an appropriate explanatory variable in understanding durations.

One important question that needs to be addressed is the extent to which workup is proxying for volume. It is possible that a transaction that is twice as big may take twice a long to negotiate, in which case workup is not providing any additional information beyond that already provided by volume. To shed some light on this issue, Figure 8 records the number of transactions (log linearized vertical scale) for each trade size (measured in $US million on the horizontal axis) for all maturities. It is immediately obvious that the greatest number of transactions are for relatively small trade sizes and the number of transactions generally declines with as the size of a transaction increases in value. The outstanding feature of the figure however, are the spikes in number of transactions observed at trade sizes of $5m, $10m, $25m, $50m, each $50m increment from $100m to $300m and then at $400m and $500m. For larger volumes there are no further standard package sizes (Figure 8 is truncated at $500m) . These relatively standard package sizes exhibit a workup time that is generally faster than similar sized, yet nonstandard trade volumes. For example, some of the longer workup periods recorded in the sample are for transactions of size $219m and $322m in the 5 year and $417m in the 30 year. In both cases there are higher volume transactions with lower workup times.

To highlight the diversity of the volume and workup relationship, we present some illustrative examples for the 5 year maturity. Figure 9 presents the distribution of observed workup times for transactions of different sizes - $5m, $10m, $25m and $100m (excluding those transactions with no workup), which Figure 8 identified as being common transaction sizes and represent a substantial number of trades in the database. These histograms highlight a surprising diversity of workup times given that each figure represent a homogenous trade size. For example, consider the histogram for $25m transaction presented in the lower left hand panel of Figure 9. There is a clear clustering of transactions at the 2 second mark, but there are many transactions that take less time to negotiate and also a substantial number of $25m dollar transactions that take longer, including a large clustering at the 4 second mark, with some taking over 10 seconds to negotiate. The same broad characterization of the data applies to all maturities and all transaction sizes. Thus, there does not appear to be any systematic relationship between workup time and trade size.

2.3 News Effects

A substantial body of literature exists that has focussed on the effect of prescheduled news releases on the US Treasury market using intra-daily data. Fleming and Remolona (1997, 1999), Balduzzi, Elton and Green (2001), Simpson and Ramchander (2004) and Green (2004) all look directly at the impact of news releases on price in this market. Johannes (2004) and Dungey et al (2007) specifically consider the relationship of pricing discontinuities (jumps) with news releases. The general consensus across these papers is that these prescheduled announcements contain important information which moves the market. Further, the sensitivity of the bond market to an announcement is related to the size of the surprise component of the news, which is usually measured as the difference between the anticipated value of the economic indicator and the actual estimate.

The previous analysis of durations and workups has presented clear evidence of diurnality at 8:30am (and to a lesser extent 10.00am), which coincide with the time at which the majority of news releases occur. This raises questions around (*i*) the extent to which these differences are driven by the existence of news, and (*ii*) whether this seasonality is deterministic and so may be exploited for modeling purposes (we consider this latter issue in the following section).

To begin the analysis, Figures 10 and 11 give the diurnal picture of the average duration and workup time (in seconds) for the 5 year maturity Treasury for all days, no news days and news days (the results for the other maturities are qualitatively consistent with those presented and are not presented to conserve space).[7] The figures highlight the difference between a news release day and a non-news release day in terms of trading intensity, in particular around the 8.30am period. For news release days, the average workup (duration) is greater (lower) at 8.30am than for any other time of the day. Where the day does not contain a regularly scheduled release of news however, the average workup increases at the open and is fairly constant until lunch time, at which point it declines toward the close where the lowest average values are observed. The average time between trades on the other hand, decreases at the open, is fairly constant until lunch time and then progressively increases to record its highest levels at the close.

Insights into the impact of macroeconomic news announcements on bond market trading behaviour can be garnered by considering Figure 12, which presents the distribution of workups for 5 year maturity US Treasury bond transactions on non-news (column 1) and news (column 2) days, grouped by transaction size (rows 1 corresponds to a transaction volume of between \$5m and \$10m, row 2 from \$10m up to \$20m, and row 3, greater than \$20m). The data for the other maturities are qualitatively similar and are not presented to conserve space. This figure illustrates some general characteristics of the data. First, the clustering of workup length at around 2 and 4 seconds is a feature of both news and non-news days. Second, although the number of trades differs across news and non-news days, there is no obvious change in the distribution of the workup. Thus, while the earlier results suggest that news days are characterized by a higher volume and intensity of trading, these results show that this does not translate

[7] See Section 3 for a discussion of these macroeconomic news announcements.

into any discernible difference in the characterization of the workup process across all trades.

3 Duration Modelling

Engle and Russell (1998) specify a standard univariate ACD model to capture the clustering and serial correlation observed in duration in high frequency equity market data. The ACD(p,q) model is given by

$$\begin{aligned} x_i^* &= \psi_i \varepsilon_i, \\ \psi_i &= \mu + \sum_{j=1}^{p} \gamma_j x_{i-j}^* + \sum_{j=1}^{q} \omega_j \psi_{i-j} \end{aligned} \tag{2}$$

where ψ_i is the expected value of duration given the information set, and the error term ε_i, which follows some defined process. The analysis of the previous sections suggests that workup and the information content of news arrival may each have a role to play in understanding the duration of transactions in the US Treasury market. In which case, it is possible to extend the model (2) as:

$$\begin{aligned} x_i^* &= \psi_i \varepsilon_i, \\ \psi_i &= \mu + \beta_1 A_i + \beta_2 N_i + \beta_3 I_{W^*,i-1} + \beta_4 I_{W^*,i-1} W_{i-1}^* + \beta_5 I_{W^*,i-1} N_i \\ &\quad + \sum_{j=1}^{p} \gamma_j x_{i-j}^* + \sum_{j=1}^{q} \omega_j \psi_{i-j} + \sum_{j=1}^{p} \lambda_j \left(N_i x_{i-j}^* \right) \end{aligned} \tag{3}$$

where A_i takes the value 1 if x_i^* occurs on a day with a prescheduled news announcement, and zero otherwise. N_i is a dummy variable that takes the value of 1 if x_i^* occurs between 8:30am and 9:30am on a news day. $I_{W^*,i-1}$ is a dummy variable that identifies the presence of a workup in the previous transaction, while $I_{W^*,i-1}W_{i-1}^*$ will take the value of the last adjusted workup time or zero. Finally the terms in $N_i x_{i-j}$ capture changes in the persistence of a shock to trade intensity around the news announcement time.[8]

The period of 8:30 to 9:30am is specified as it encompasses the period over which the market responds to news - Andersen et al (2007) find that the majority of the

[8] Note that we control for end of day effects in that workups on previous transactions refer to workups on the same day. In preliminary work, terms in $A_i x_{i-j}^*$ and $I_{W^*,i-1} x_{i-j}^*$ were also included but were nowhere significant. In the interests of brevity and clarity we omit these results.

price impact has dissipated within 5 minutes, although volatility impact is present for 30 minutes. Balduzzi, Elton and Green (2001) however, record volume and volatility responding to news releases for up to 60 minutes. Thus, this model allows for durations to have a different dynamic process on days with a scheduled macroeconomic news announcement. The announcements used are the 8:30am releases of the US CPI, PPI, retail sales, housing starts, GDP, durable goods and non-farm payrolls. This set of announcements have been found to have greatest impact on bond markets (see Fleming and Remolona, 1999) and are similar to the set used in Andersen et al (2007).

The threshold ACD model in (3) admits tests of several hypotheses:

$$H_0^1: \text{ The correct model is a simple ACD}(p,q).$$

Under H_0^1 there are no possible impacts on trade durations from either the announcement of news or the presence of a workup or both. This hypothesis is tested using a Wald test of the restriction $H_0^1 : \beta_j = \lambda_j = 0, \forall_j$ and is distributed as $\chi^2(10)$.

$$H_0^2: \text{ News announcement days are irrelevant}$$

Failure to reject H_0^2 implies that the scheduled news announcement days are not unusual. This hypothesis is tested using a t-ratio test of the restriction $H_0^2 : \beta_1 = 0$.

$$H_0^3: \text{ News announcement time is irrelevant}$$

Using a Wald test of the restriction $H_0^3 : \beta_1 = \beta_2 = \lambda_j = 0, \forall_j$, distributed as $\chi^2(7)$ we may examine whether there is evidence that arrival of news impacts upon the time between trades. This tests the significance of the decrease in diurnality effect observed in Figure 4 at the scheduled macroeconomic announcement period of 8:30am.

$$\begin{aligned} H_0^4 &: \text{ The presence of a workup is irrelevant} \\ H_0^5 &: \text{ The length of the workup time is irrelevant} \\ H_0^6 &: \text{ There are no workup effects} \end{aligned}$$

H_0^4 and H_0^5 are t-ratio tests for the significance of β_3 and β_4, respectively. Rejection of the null $H_0^4 : \beta_3 = 0$ implies that the presence of a workup (the actual workup time) impacts on the duration of a trade. Rejection of the null $H_0^5 : \beta_4 = 0$ implies that the actual workup time impacts the duration of a trade. Green (2004) suggests that the delay imposed by the workup process may influence the next transaction. Specifically, traders may be impatient with large workups, in which case a long workup process may result in a relatively shorter duration to the next transaction. Some care needs to be taken with the relative timing here as duration is assumed to be affected by workup from the previous transaction - that is the delay in finishing the previous transaction is associated with a shorter duration to the current transaction. The joint null, $H_0^6 : \beta_3 = \beta_4 = \beta_5 = 0$ may be tested using a Wald test and is distributed as $\chi^2(2)$. This test also allows for the possibility of an interaction between news arrival and workups impacting upon the duration between trades.

4 Results

An ACD(1,1) model is specified as a starting point for the analysis, as it has been found in the previous literature to provide a satisfactory representation of the data (see Hautsch, 2006, for example). Following Engle and Russell (1998), the distribution functions of ε_i on $(0, \infty)$ is specified as a Weibull distribution.[9] Table 4 presents the results of estimating (2), where $p = 1$, $q = 1$ is specified, ie. a WACD(1,1), for the 2, 5, 10 and 30 year US Treasuries.[10] The results in Table 4 indicate that the parameters in the duration model are all significant at the 1% level or better. The expected value of the adjusted duration is given as $E(x_i^*) = \mu/(1 - (\gamma_1 + \omega_1))$ and tends to rise with the maturity up to 10 years, but then declines quite dramatically for the 30 year maturity. The persistence of the adjusted duration is given by the sum of the coefficients in the duration process, $\rho(x_i) = \gamma_1 + \omega_1$. Across all maturities, the sum of the coefficients is high, but less than unity, which is consistent with the findings of the previous literature for other markets (see inter alia Engle and Russell, 1998, Engle, 2000, and Bauwens and Veredas, 2004). The persistence of trade durations is highest in the 10 year contract and lowest in the 2 year contract.

Engle and Russell (1998) suggest the Ljung-Box statistic as an appropriate diag-

[9] A number of alternative distributional functions of ε_i have been considered in the literature including an exponential distribution, Burr distribution (see Grammig and Maurer, 2000) and generalized gamma distribution (see Lunde, 2000).

[10] The Weibull nests the exponential distribution, so that when the shape parameter $\alpha = 1$, the conditional log likelihood of the WACD model collapses to that of an exponential ACD. The null hypothesis $H_0 : \alpha = 1$ was rejected for all maturities, which indicates that the hazard function is not a constant.

nostic for the ACD model. To this end, the Ljung-Box statistics are reported in the middle of the table and they suggest the presence of significant serial correlation in the standardized innovations and their corresponding squares. This is a common feature of ACD modelling as several authors (Engle and Russell, 1998; Engle, 2000; Zhang, Russell, and Tsay, 2001; Fernandes and Grammig, 2006 among others) report substantial difficulties in completely removing dependence in the residual series.

To control for the excess serial correlation in the WACD model, we consider a higher order specification. The use of higher order autocorrelation structures is not unusual as Engle and Russell (1998), for example, specified an ACD(2,2) process when modeling their IBM transactions data. In the case of the bond market data, even longer lag lengths are required to obtain a satisfactory fit. Table 5 presents the estimation results for a WACD(5,5) model. In terms of the coefficient estimates, the WACD(5,5) model shows an increase in the sum of the autoregressive parameters, $\sum_{j=1}^{5} \gamma_j$ when compared to the single AR(1) parameter γ_1 reported in Table 4. This is consistent with the data requiring a model with a higher degree of serial correlation to fully capture the structure present. By way of contrast however, the clustering effects given by $\sum_{j=1}^{5} \omega_j$ in the WACD(5,5) model are generally slightly smaller than the single clustering effects in the WACD(1,1). The expected value of the adjusted duration, given by $E(x_i) = \left(1 - \left(\sum_{j=1}^{5} \gamma_j + \sum_{j=1}^{5} \omega_j\right)\right)^{-1}$ for this higher order model, is about 25% lower than the WACD(1,1). The exception is the 30 year maturity, where the expected adjusted duration has risen by almost 4 times when compared to the previously discussed estimate. In this case, the expected adjusted duration for the 30 year maturity is somewhat greater than that of the other maturities. The persistence of the adjusted duration, given by the sum of all the γ_j and ω_j parameters is not substantially different to the earlier estimates, remaining high and tending to increase with maturity.

An examination of the diagnostics for this WACD(5,5) model reveals that, while the Ljung-Box statistics are improved, at conventional statistical levels they continue to reject the null of no serial correlation. It is interesting to note that despite this rejection of the null, the test scores are nevertheless consistent with those reported in other studies as for models that are deemed to have relatively 'good' performance.

This WACD(5,5) model may be extended to consider the impact of news arrival and the workup process on transaction duration. To this end, Equation (3) is estimated and the results are presented in Table 6 and 7. The estimated intercept $\hat{\mu}$, increases with maturity. The coefficient sums $\sum_{j=1}^{5} \gamma_j$, $\sum_{j=1}^{5} \omega_j$ and $\sum_{j=1}^{5} \lambda_j$ are all significant indicating that the persistence of a shock to trade durations may vary depending on whether the shock arrives between 8:30am and 9:30am on a news announcement day. The Ljung Box statistics presented in Table 4 are vastly improved over those estimated for the standard ACD models and there is no evidence of misspecification.

Turning to the hypothesis tests, the Wald test of the restriction $H_0^1 : \beta_j = \lambda_j = 0, \forall_j$ is strongly statistically significant at all usual levels of confidence for all maturities (marginal significance level 0.0000). This implies that the WACD(5,5) model obtained under the restrictions would represent a misspecificed conditional characterization of the data; durations are affected by the announcement of news or the presence of a workup or both.

The second hypothesis, $H_0^2 : \beta_1 = 0$, that news announcement days are irrelevant in determining durations is satisfied for all maturities considered. There is no evidence that trade durations are different for days with and without a scheduled news announcement. In contrast, there is some evidence that the arrival of news impacts upon the time between trades. The third hypothesis $H_0^3 : \beta_1 = \beta_2 = \lambda_j = 0, \forall_j$ distributed as $\chi^2(7)$, which takes into account the hour immediately following the scheduled news announcements, is rejected for all but the 5 year maturity at the 5% level of confidence.

With the exception of the 2 year maturity where $H_0^4 : \beta_3 = 0$, the presence of a workup in the previous transaction is a significant determinant of the i^{th} trade duration. For the 5, 10 and 30 year maturities a workup in the previous transaction tends to reduce the i^{th} trade duration as $\hat{\beta}_3$ is negative and significant. The null hypothesis $H_0^4 : \beta_4 = 0$ is uniformly rejected. For all maturities considered, there is a negative and significant relationship between the i^{th} trade duration and the workup time in the $i-1^{th}$ transaction. The evidence suggests that a longer time to finish the previous transaction via workup is associated with a shorter duration to the current transaction (recall that duration is time between the start of each transaction). Finally, the null hypothesis $H_0^6 : \beta_3 = \beta_4 = \beta_5 = 0$ is rejected for all usual levels of confidence.

Using these results an estimate for the expected duration can be derived. The baseline estimate for the expected duration between transactions is obtained from $E(x_i^*) = \mu / \left(1 - \left(\sum_{j=1}^{5} \gamma_j + \sum_{j=1}^{5} \omega_j\right)\right)$. $E(x_i^*)$ is the expected duration between transactions with no workup occurring at times other than 8:30-9:30 on a non-news announcement day. At 1.8187 seconds the expected duration between trades is shortest for the 2 year maturity. The expected durations are longer for the longer maturities but it is interesting to note that the expected duration between trades for the 10 year maturity is less than the expected duration for the five year maturity. The 30 year maturity displays the highest expected trade duration of 8.12 seconds.

With the exception of the two year maturity, the expected trade durations between 8:30 am and 9:30 am, $E^N(x_i^*) = (\mu + \beta_1 + \beta_2) / \left(1 - \left(\sum_{j=1}^5 \gamma_j + \sum_{j=1}^5 \omega_j + \sum_{j=1}^5 \lambda_j\right)\right)$, are longer than the baseline expected duration $E(x_i^*)$. Given that $\hat{\beta}_1$ and/or $\hat{\beta}_2$ are statistically significant for the 2 and 30 year maturities this implies that the differences between $E(x_i^*)$ and $E^N(x_i^*)$ are significant. In contrast, the expected adjusted duration for transactions following workup $E^W(x_i^*) = (\omega + \beta_3 + \beta_4) / \left(1 - \left(\sum_{j=1}^5 \gamma_j + \sum_{j=1}^5 \omega_j + \sum_{j=1}^5 \lambda_j\right)\right)$ is shorter for all maturities considered. Table 4 shows that expected durations following workups are 37% (5 year maturity), 27% (10 year maturity) and 45% (30 year maturity) of the corresponding baseline values. Given that the null hypothesis $H_0^6 : \beta_3 = \beta_4 = \beta_5 = 0$ is rejected for all maturities at all usual levels of confidence, we can conclude that the workup of volume in the previous transaction of a given maturity is associated with a shorter duration to the current transaction. Accounting for the information induced by the workup process results in a substantial decrease in the estimate expected adjusted duration. Thus, our results provide support for Boni and Leach (2004), that workups are a source of information to the market. Expected duration in the case of news, announcement times and workup is given by $E^{NW}(x_i^*) = \left(\omega + \sum_{j=1}^5 \beta_j\right) / \left(1 - \left(\sum_{j=1}^5 \gamma_j + \sum_{j=1}^5 \omega_j + \sum_{j=1}^5 \lambda_j\right)\right)$. The dominance of the workup effect over the announcement period effect is evident in all but the 2 year maturity. In that case the statistically insignificant point estimate of the presence of workups β_3 does not act in the same direction as for other maturities. For the 5 to 10 year maturities the expected durations with all effects accounted for are shorter than the benchmark, representing the importance of workup in understanding intensity in these instruments.

5 Conclusion

This paper focuses on the characterization and modelling of the trading process in the US bond market. Unlike other asset markets, the US Treasury market operates under an expandable limit order book. The unique microstructure of this market raises an interesting question about the role of workup as a important conduit of information that may impact on the trading process. Further, this market is also distinguished by the importance of regularly scheduled macroeconomic news announcements, which have been shown to impact on bond market trading intensity. In this paper, we consider the role of both workup and news announcements when modelling bond data in transaction time.

The results of our analysis suggest that the duration for US Treasuries displays a number of unique properties. First, where a WACD(1,1) specification has been found to be adequate for other markets, bond markets require a model with much longer lag specifications; a WACD(5,5) specification provided a superior fit to a first order lag model. Second, news effects are shown to be an important variable for the period immediately surrounding scheduled news release times. Finally, the length of time taken to complete a given transaction has a measurable impact on the time between transactions. A long time to complete tends to be associated with a shorter time to the next transaction. This feature turns out to have a greater impact on reducing the expected adjusted duration than that of the presence of news announcements, and suggests that the microstructure of the market has an important impact on its functioning during times of particularly high activity. These results have implications for our understanding of how bond markets work and the construction of theoretical models of the behaviour of market participants.

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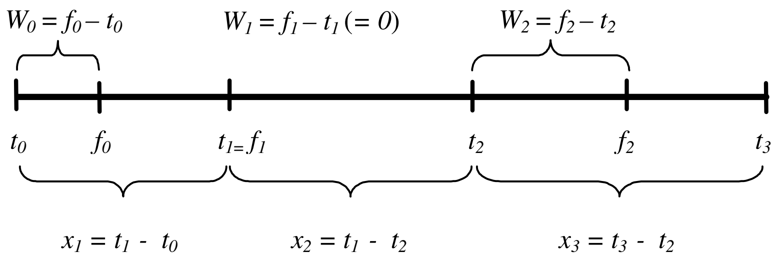

Figure 1: Stylised Representation of the Bond Trading Process

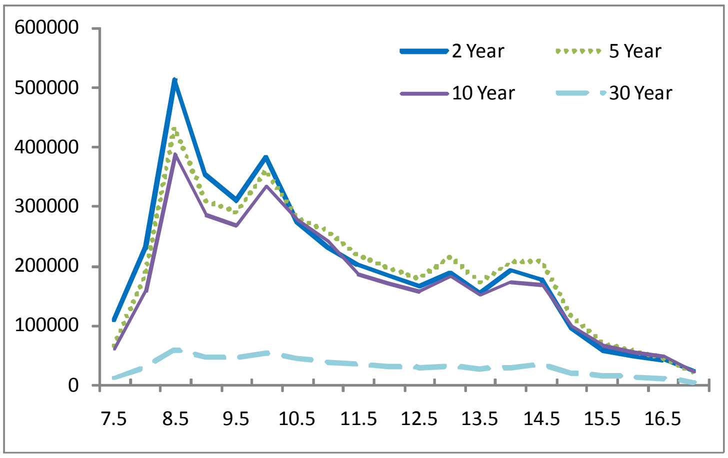

Figure 2: Average Total Trading Volume in a day ($USmil) by half hour interval for each maturity in the period 7:30 am to 5:30 pm for January 3, 2006 to October 6, 2006 inclusive.

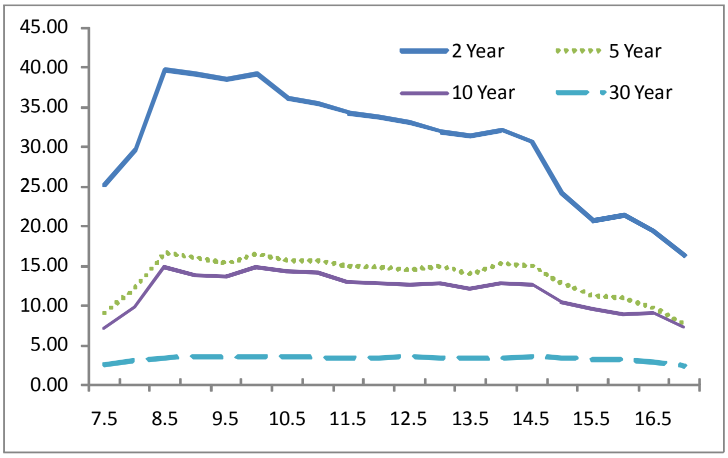

Figure 3: Average Trade Size in $USmillion in a day by half hour interval for each maturity in the period 7:30 am to 5:30 pm for January 3, 2006 to October 6, 2006 inclusive.

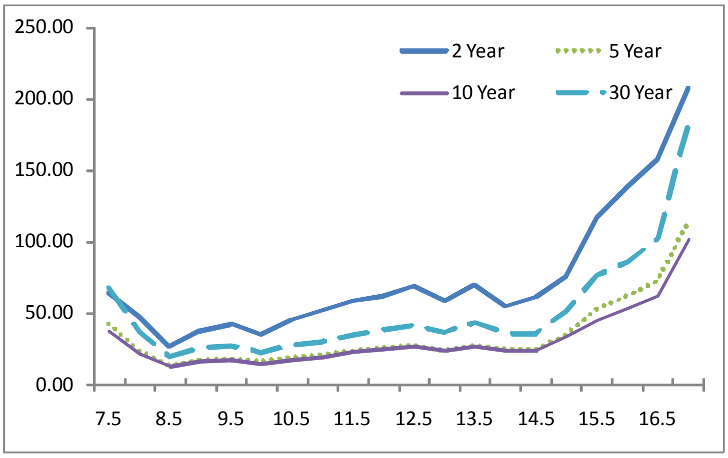

Figure 4: Average Duration in Seconds by half hour intervals in a day for each maturity in the period 7:30 am to 5:30 pm for January 3, 2006 to October 6, 2006 inclusive.

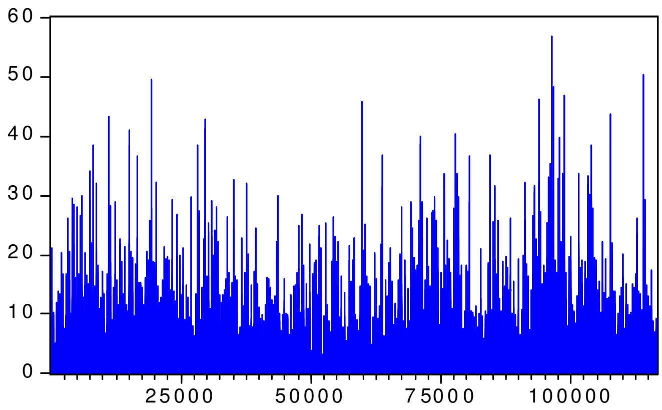

Figure 5: Adjusted duration, x_i^*, for the two year maturity

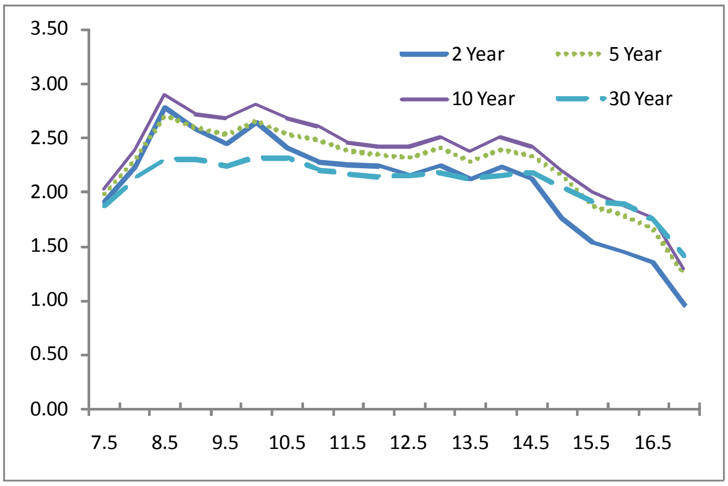

Figure 6: Average Workup Time in Seconds by half hour intervals in a day for each maturity in the period 7:30 am to 5:30 pm for January 3, 2006 to October 6, 2006 inclusive.

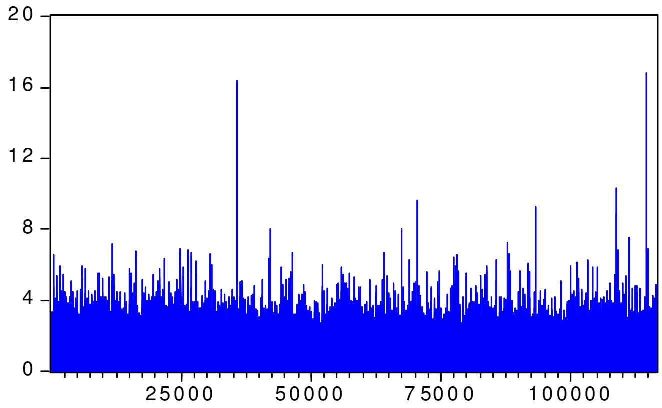

Figure 7: Adjusted workup, W_i^*, for the two year maturity

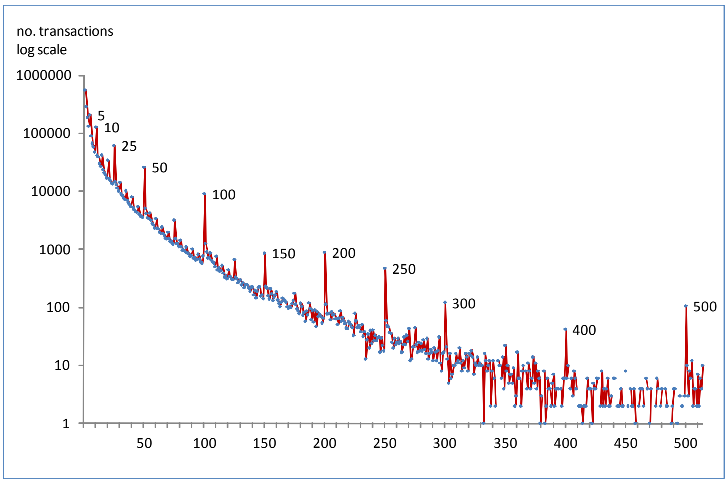

Figure 8: Number of transactions in the 2,5,10 and 30 year maturities transacted by volume of transaction in $USmil, using a logarithmic vertical scale.

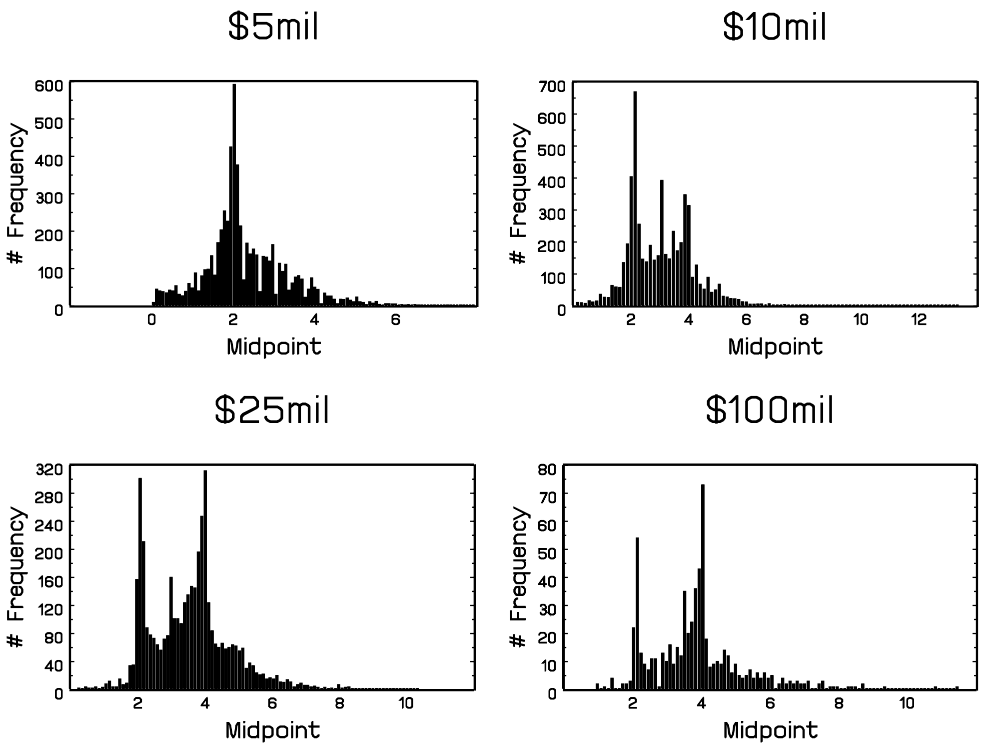

Figure 9: The distribution of workup times for transactions of a given size in the 5 year maturity for the sample period.

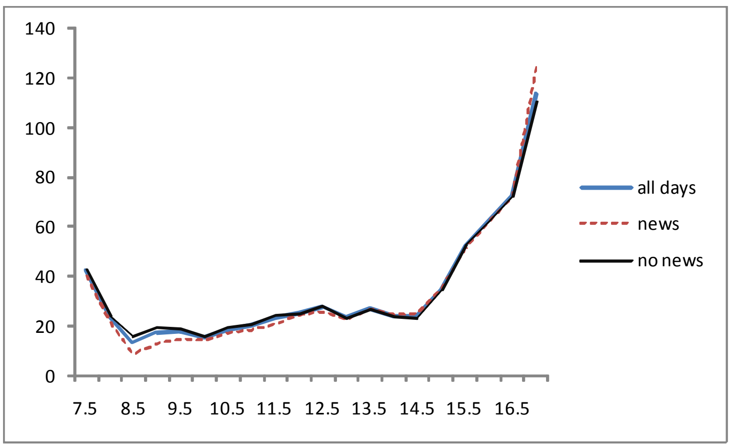

Figure 10: Average duration in seconds for 5 year maturity transactions on all days, news days and no-news days by half hour period.

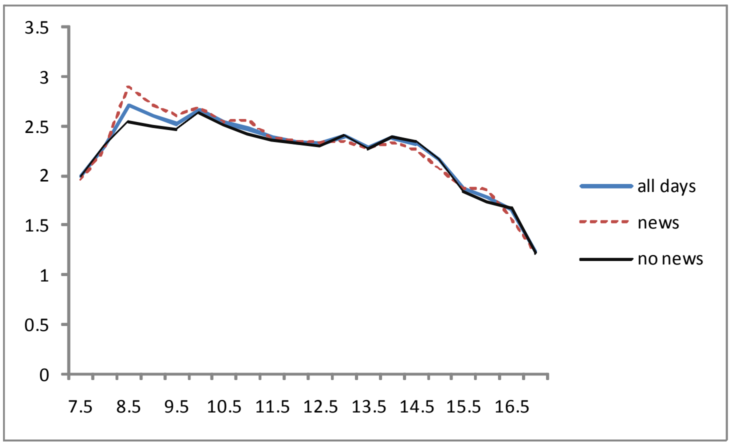

Figure 11: Average workup time in seconds for transactions in 5 year maturity bond for all days, news days and no news days by half hour period.

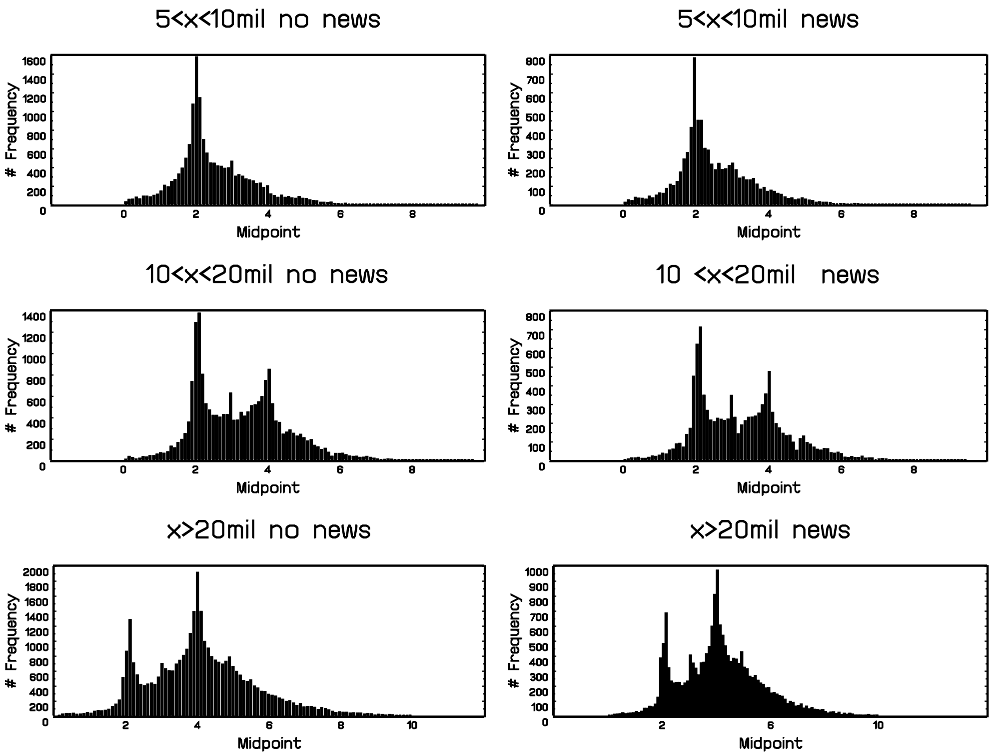

Figure 12: Distribution of workup times for 5 year maturity US Treasury bonds (excluding those with no workup) on no news days and news days by size of transaction in $USmil.

Table 1:
Descriptive statistics of the average trading day for January to September 2006 by maturity

		2 year	5 year	10 year	30 year
average trades per day		623	1417	1465	951
size of trade ($USm)	average	33.50	14.56	12.68	3.35
	max	982	641	464	114
	min	1	1	1	1
workups per day		1128	2976	3101	1494
duration (seconds)	average	55.86	24.80	24.04	36.53
	max	3545.35	1614.46	1883.37	2163.12
	min	0.006	0.006	0.006	0.001
duration (diurnalised)	average	1.14	1.10	1.10	1.10
	max	56.89	57.02	64.96	45.12
	min	0.00	0.00	0.00	0.00
workup (seconds)	average	2.27	2.39	2.51	2.17
	max	36.30	140.01	25.20	137.58
	min	0.00	0.00	0.00	0.00
workup (diurnalised)	average	1.02	1.02	1.02	1.10
	max	16.79	58.05	16.51	45.12
	min	0.00	0.00	0.00	0.00

Table 2:
Summary statistics for the diurnalised durations, x_i^*.

	2 year	5 year	10 year	30 year
n	116,479	264,902	273,898	177,770
μ	1.1414	1.0967	1.0951	1.0951
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
σ^2	5.3739	2.6412	2.7170	2.6130
$Q\,(10)$	6176.3812	20688.621	19466.9705	3348.7136
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q\,(20)$	7792.5831	32684.767	31190.6821	35793.7004
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q\,(30)$	8844.726	41826.372	39487.8677	44018.4993
	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Table 3:
Descriptive statistics for transactions with and without workup by maturity for January to September 2006.

	2 year	5 year	10 year	30 year
Transactions with no workup				
Proportion of total trades (%)	63.4	53.9	54.8	64.0
Average size of trade ($USm)	18.22	6.35	5.80	2.00
Transactions with workup				
Average workup time	3.59	3.36	3.58	2.97
Volume of trade discovered with workup (%)	65.5	76.5	74.9	61.8

Table 4:
WACD(1,1) model estimates by maturity. [11]

$$x_i = \psi_i \varepsilon_i$$
$$\psi_i = \omega_0 + \gamma_1 x_{i-1} + \omega_1 \psi_{i-1}$$

	2 year	5 year	10 year	30 year
ω_0	0.0657	0.0143	0.0140	0.0211
	(0.0095)	(0.0012)	(0.0008)	(0.0010)
γ_1	0.0960	0.0575	0.0567	0.0846
	(0.0082)	(0.0024)	(0.0016)	(0.0021)
ω_1	0.8414	0.9299	0.9310	0.8969
	(0.0168)	(0.0031)	(0.0020)	(0.0027)
α	0.7255	0.9752	0.9555	0.9026
	(0.0016)	(0.0024)	(0.0014)	(0.0016)
$Q(10)$	509.1707	121.6699	609.7800	250.9219
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(20)$	523.8488	691.4006	174.3761	327.5845
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(30)$	560.9178	174.3761	735.9692	343.5661
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q^2(10)$	73.7984	16.6681	48.2440	39.0051
	[0.0000]	[0.0822]	[0.0000]	[0.0000]
$Q^2(20)$	97.0448	28.5084	66.5963	53.9125
	[0.0000]	[0.0964]	[0.0000]	[0.0000]
$Q^2(30)$	153.1990	56.8736	86.8946	59.0098
	[0.0000]	[0.0022]	[0.0000]	[0.0012]
$E(x_i)$	1.0495	1.1349	1.1382	0.4637
$\rho(x_i)$	0.9374	0.9874	0.9877	0.9545

Notes: α is the shape parameter in the Weibull distribution. $Q(p)$ and $Q^2(p)$ are tests for p^{th} order serial correlation in ε_i and ε_i^2, respectively. Standard errors are displayed as (.). Marginal significance levels are displayed as [.] .
$E(x_i) = \omega_0 / (1 - (\gamma_1 + \omega_1))$ is the expected adjusted duration. The expected persistence of the adjusted durations is $\rho(x_i) = \gamma_1 + \omega_1$.

Table 5:
WACD(5,5) model estimates by maturity.[12]

$$x_i = \psi_i \varepsilon_i$$
$$\psi_i = \omega_i + \sum_{j=1}^{5} \gamma_j x_{i-j} + \sum_{j=1}^{5} \omega_j \psi_{i-j}$$

	2 year	5 year	10 year	30 year
ω_0	0.0439	0.0373	0.0343	0.0017
	(0.0033)	(0.0023)	(0.0020)	(0.0023)
$\sum_{j=1}^{5} \gamma_j$	0.1497	0.2702	0.2389	0.0608
	(0.0061)	(0.0085)	(0.0064)	(0.0098)
$\sum_{j=1}^{5} \omega_j$	0.7934	0.6852	0.7186	0.9291
	(0.0087)	(0.0100)	(0.0077)	(0.0117)
α	0.7292	0.9564	0.9561	0.9043
	(0.0018)	(0.0014)	(0.0014)	(0.0013)
$Q(10)$	20.4096	52.5600	65.389	23.373
	[0.0011]	[0.0000]	[0.0000]	[0.0095]
$Q(20)$	39.4280	96.9730	85.223	34.294
	[0.0059]	[0.0000]	[0.0000]	[0.0242]
$Q(30)$	62.5230	119.150	118.910	53.980
	[0.0005]	[0.0000]	[0.0000]	[0.0046]
$Q^2(10)$	57.743	29.4960	36.609	38.041
	[0.0000]	[0.0010]	[0.0001]	[0.0000]
$Q^2(20)$	73.860	69.510	68.382	64.375
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q^2(30)$	86.552	107.593	89.005	71.142
	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$E(x_i)$	0.7715	0.8363	0.8071	1.1584
$\rho(x_i)$	0.9431	0.9554	0.9575	0.9899

Notes: α is the shape parameter in the Weibull distribution. $Q(p)$ and $Q^2(p)$ are tests for p^{th} order serial correlation in ε_i and ε_i^2, respectively. Standard errors are displayed as (.). Marginal significance levels are displayed as [.].
$E(x_i) = \left(1 - \left(\sum_{j=1}^{5} \gamma_j + \sum_{j=1}^{5} \omega_j\right)\right)^{-1}$ is the expected adjusted duration. The expected persistence of the adjusted durations is $\rho(x_i) = \sum_{j=1}^{5} \gamma_j + \sum_{j=1}^{5} \omega_j$

Table 6:
Threshold WACD(5,5) estimates.

	2 year	5 year	10 year	30 year
μ	0.0482	0.0483	0.0522	0.0658
	(0.0081)	(0.0059)	(0.0063)	(0.0054)
β_1	-0.0023	-0.0002	-0.0008	0.0003
	(0.0008)	(0.0004)	(0.0004)	(0.0004)
β_2	-0.0363	0.0006	0.0001	0.0030
	(0.0557)	(0.0015)	(0.0015)	(0.0013)
β_3	0.0022	-0.0025	-0.0023	-0.0073
	(0.0058)	(0.0010)	(0.0010)	(0.0011)
β_4	-0.0218	-0.0239	-0.0248	-0.0454
	(0.0022)	(0.0054)	(0.0057)	(0.0049)
β_5	0.0271	-0.0091	-0.0133	-0.0043
	(0.0470)	(0.0009)	(0.0008)	(0.0008)
$\sum_{j=1}^{5} \gamma_j$	0.0490	0.0336	0.0371	0.0701
	(0.0043)	(0.0013)	(0.0016)	(0.0037)
$\sum_{j=1}^{5} \omega_j$	0.9244	0.9586	0.9498	0.9218
	(0.0071)	(0.0018)	(0.0023)	(0.0047)
$\sum_{j=1}^{5} \lambda_j$	0.0173	0.0021	0.0027	0.0048
	(0.0030)	(0.0016)	(0.0014)	(0.0013)
α	0.7319	0.9604	0.9607	0.9055
	(0.0017)	(0.0034)	(0.0015)	(0.0016)

Notes: α is the shape parameter in the Weibull distribution. Standard errors are displayed as (.).

Table 7:
Hypothesis tests and diagnostic statistics for the Threshold WACD(5,5).

	2 year	5 year	10 year	30 year
$Q(10)$	13.351	14.906	8.251	11.991
	[0.2047]	[0.1355]	[0.6043]	[0.2587]
$Q\,(20)$	27.137	31.353	18.548	24.546
	[0.1315]	[0.0507]	[0.5572]	[0.2193]
H_0^1 :WACD	132.5719	126.930	273.4498	21.2908
$\tilde{}\chi^2\,(10)$	[0.0000]	[0.0000]	[0.0000]	[0.0192]
H_0^3 :NEWS	43.1907	12.9482	16.3875	7.0662
$\tilde{}\chi^2\,(7)$	[0.0000]	[0.0734]	[0.0000]	[0.0422]
H_0^6 :Workup	98.8932	103.2824	227.0574	16.6068
$\tilde{}\chi^2\,(3)$	[0.0000]	[0.0000]	[0.0000]	[0.0008]
$E\,(x_i)$	1.8120	6.1923	3.9847	8.1235
$E^N\,(x_i)$	1.0323	8.5436	4.9519	20.8485
$E^W\,(x_i)$	1.0752	2.8077	1.9160	1.6173
$E^{NW}\,(x_i)$	1.8387	2.3158	1.0674	3.6667
$\rho\,(x_i)$	0.9874	0.9922	0.9869	0.9919
$\rho^N\,(x_i)$	0.9907	0.9943	0.9896	0.9967

Notes: $Q\,(p)$ is a test for p^{th} order serial correlation in ε_i. Marginal significance levels are displayed as [.] . $E\,(x_i)$is the expected adjusted duration. The expected persistence of the adjusted durations is $\rho\,(x_i) = \gamma_1 + \omega_1$.