Credit Risk of "Exotic" Mortgage Products

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Abstract

This paper offers a consistent framework to quantify relative credit risks of various non-traditional mortgage products under a stochastic economy. Reflecting cash flows of different adjustable-rate and fixed-rate mortgages (ARMs and FRMs), we estimate their probabilities of default that are caused by two underlying determinants - the willingness-to-pay problem due to a drop in the collateral value, measured via probability of negative equity, and the ability-to-pay problem due to a shock in consumer liquidity, measured via probability of liquidity shortage. The abnormally high default rates observed from our model for various "exotic" mortgage products that have traded in the subprime mortgage market confirm the importance of the probability of liquidity shortage as a critical default risk indicator. We apply Monte Carlo simulations with three correlated stochastic variables, mortgage interest rate, home price, and household income, to assess the default risk of various ARM and FRM contracts under normal as well as stressed economies. Our results confirm the general intuition that the default risk of Option ARM contract with a minimum monthly interest payment can be extreme: that is, its probability of default surpasses all other products with a great margin under both stressed and normal-economic scenarios. We further show that the additional credit risks of these exotic mortgage products are mostly systematic. This indicates that such mortgage products should require substantially higher risk-based capital under the Basle II framework and can be more costly to offer by the lenders.

Key words: Payment shock; Default, Credit risk; Default option; Affordability, systematic risk, risk-based capital, Basel II.

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1. Introduction

During the past five years or so, various "exotic" mortgage products have flourished in the US, mainly due to the rise of the subprime mortgage market since 2003. These products generally entail various new creative features, such as hybrid fixed-rate and adjustable-rate periods and different payment options for borrowers. In general, those products have made home purchase more affordable for numerous borrowers confronted with borrowing-constrained due to rapidly-rising home prices in different parts of the country. But, at the same time, as we are currently witnessing, the observed delinquent and defaulting rates of these non-traditional mortgage products (NMP) are much higher than the market originally anticipated. As most defaults out of NMP were caused by the shock in the scheduled repayments of principal and interest, rather than the volatility of the collateral value, the conventional put-option valuation framework is no longer adequate to analyze them. That is, valuing the riskiness of these products requires considering the payment shock, which is in turn influenced not only by home price trend but also by the interest rate and household income processes.

On the consumer welfare point of view, the choice between fixed-rate mortgage (FRM) and adjustable-rate mortgage (ARM) is a seasoned research topic. (See the next section for a brief summary of findings from this strand of literature.) In recent year, Campbell and Cocco (2003), Miles (2004), Miles and Pillonca (2007) also demonstrate the key trade-offs involved with these two types of mortgage contract in the context of a life-cycle consumer utility maximization. Among various findings from these studies, while ARM contracts generally enhance the affordability of home purchase for borrowers, they are riskier than FRM, particularly for those borrowers who purchase homes whose values are relatively high given their income. Therefore, as the argument goes, considering the stochastic processes of economic variables involved the rational choice by those marginal borrowers should be FRM over ARM.

On the viewpoint of mortgage pricing, however, there is a paucity of relevant studies that show a consistent measurement framework of NMP by considering all the key economic risk factors involved. While there are numerous option-theoretic mortgage-pricing studies (in particular, Foster and Van Order (1985), Deng, Quigley and Van Order (2000) and Calhoun and Deng (2002)), Yang, Buist, and Megbolugbe (1998) and Buist and Yang (1998) are about the only two studies that explicitly deals with the income process along with home price and interest rate trends in analyzing the embedded default option in mortgage contracts. This paper attempts toto fill that gap by applying the simulation framework put forth by Buist and Yangto quantify and compare various NPMs with their counterparts in the conventional mortgage market.

There are three main research objectives of this paper. First, we assess different ARM and FRM products traded in the market place both in terms of their cash flow attributes and in terms of two drivers of default risk – the probability of negative equity (PnegQ) and the probability of liquidity shortage in mortgage payment (PSHORT). Following Yang, Buist, and Megbolugbe (1998), we also analyze the probability of default as the probability of both events occurring simultaneously. Second, we apply Monte Carlo simulations with three correlated stochastic variables – mortgage interest rate, home price, and household income - to determine the default risk of various ARM and FRM contracts under normal as well as stress economies. Third, we examine implications of our findings in terms of mortgage portfolio management and risk-based capital requirements.

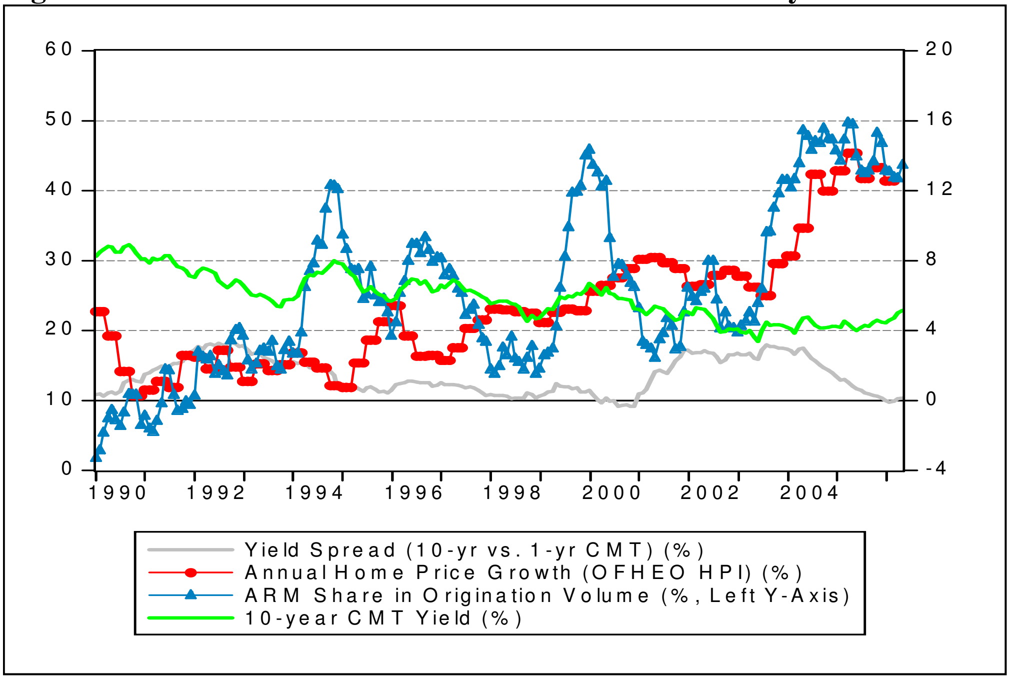
The remaining of the paper is organized as follows. Section 2 describes background of the study. Section 3 presents model framework and products being analyzed. Section 4 discusses the results of our simulations under a variety of economic conditions and separation of income risk. Section 5 discusses the risk management implications of our findings in terms of Basel-II like risk-based capital requirements. Section 6 concludes the paper.

2. Background

The ARM share in the US has been on the rise during the first half of the decade of 2000's, from about 20 percent of the total origination in 2001 to close to 50% in 2005 (see Figure 1). This rapid increase is largely caused by the worsening affordability in owning homes during that time period, rather than by the consumers' rational choice between ARM and FRM. That is, as shown in Figure 1, the ramp-up in ARM share since 2003 closely coincides with the explosive home price growth in the US since the late 1990s. On the other hand, the recent rise in that product shows a contradicting sign of correlation with the two conventional drivers of consumers' ARM choices: that is, the long rate represented by the yields on the 10-year Constant Maturity Treasuries (CMT) rate has been declining during this period; and, the yield spread between the long and short rates (the 10-year vs. 1-year CMT rates) has been flat or inverted. These outcomes are contrary to the earlier mortgage choice literature.[1] Prior studies (cite) indicate, ceteris paribus, the lower the long-term FRM rate and the flatter the yield curve, the less likely borrowers choose ARM products.

[1] See Nothaft and Kogut (2001).

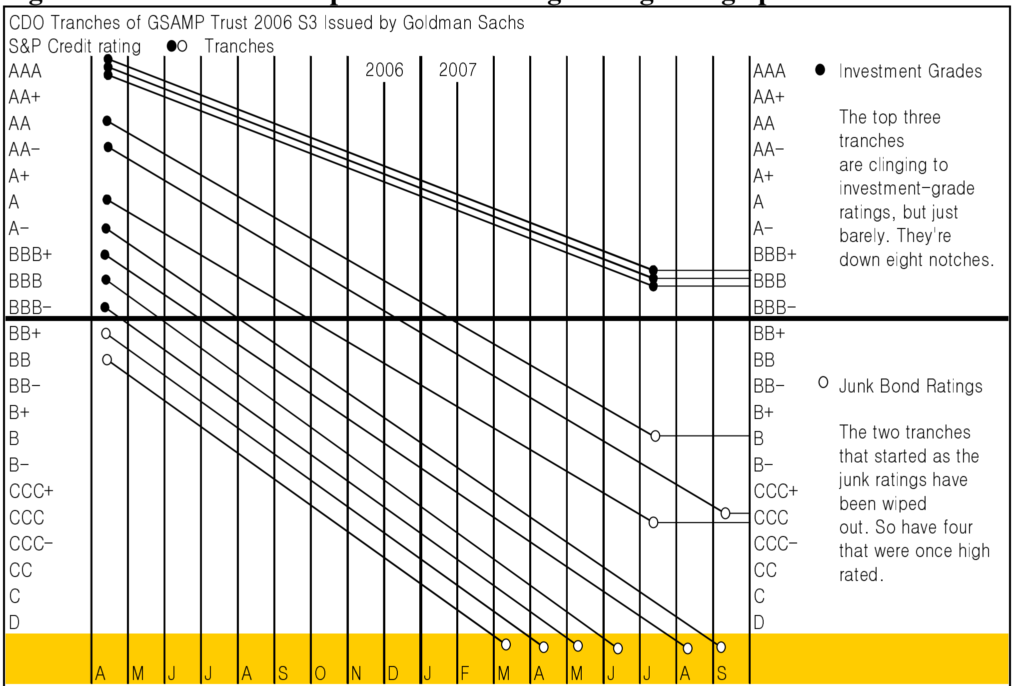
Figure 1: ARM Share and Economic Variables of Relevancy



As the home affordability-enhancement vehicles, we also see the rise of various specialized ARM products in the recent years, such as 2/28 Hybrid ARMs, Negative Amortization Mortgages, Option ARMs, along with more conventional 3/1 or 5/1 ARMs with a cap structure. These so-called "exotic" mortgage products pose a challenge both in terms of assessing their ex ante risk at origination and at point-of-sale to the secondary mortgage market. Due to the newness of those products, there is lack of performance data such that one can gauge the extent of the risks posed by these products in different stages of economic cycle. Furthermore, the existing mortgage default literature is in large part not adequate in properly measuring the level of credit risk borne by these products.

The current market outcomes in terms of the worsening subprime mortgage and Mortgage-Backed Security (MBS) performance indicate that the capital market fails to understand and price for the credit risk involved in these products. As one such evidence of the failure of ex ante pricing, Figure 2 shows the rapid rating downgrading for one sample subprime MBS deal issued by Goldman Sachs (GSAMP Trust 2006-S3) in April 2006. Initially, the deal was designed as 12 tranches including three senior pieces along with two tranches in the junk bond ratings (BB+ and BB). After fifteen months, six lowest grade segments were written off as credit losses; three mezzanine tranches were downgraded as the junk-bond level; and only three AAA segments barely made the investment grade with the BBB ratings. Historically, the probability of AAA bond being downgraded to BBB (within a three-year period) was only 0.04 percent as reported by Moody's, indicating that that is an extremely rare event. But similar downgrades in the case of subprime CDOs do not seem to be rare, as evidenced by the fact that more than 3000 subprime CDO tranches were downgraded in the month of October 2007 alone.

Figure 2: A Particular Subprime MBS Rating Downgrading Speed



SOURCE : Reproduced from Allan Slaon's column in Washington Post (October 16, 2007)

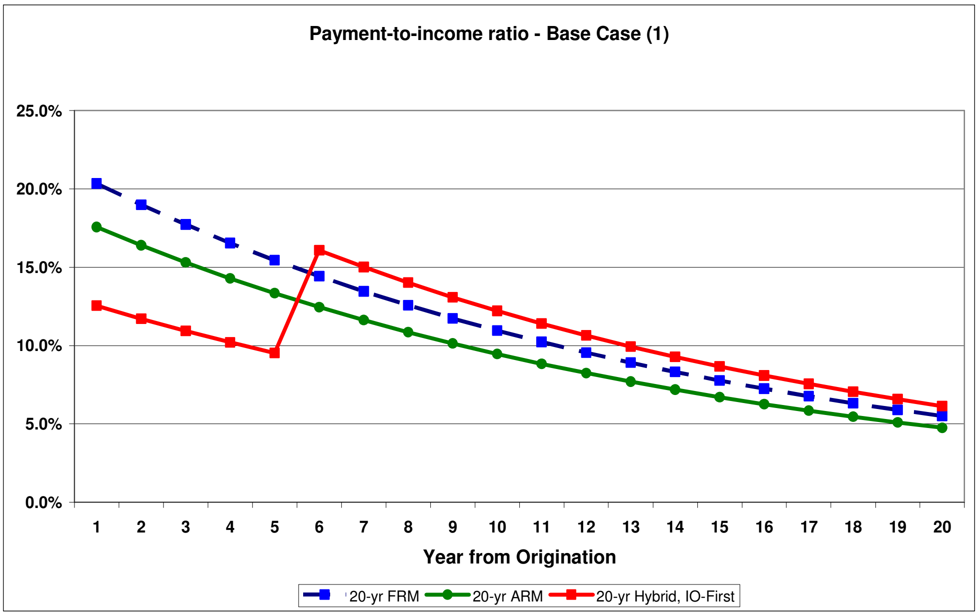
From household's choice point of view, selection between ARM and FRM has been the topic of numerous studies since the late 1980s. The first strand of those mortgage choice studies focus on consumers' self-selection between the two product categories and the impacts of various economic and demographic factors on that decision (Dillon, Shilling and Sirman (1987), Linneman and Wachter (1989), Bruekner and Follain (1988), Brueckner (1993), Cho et al. (1996), and Vanderhoff (1996)). Employing a cross-sectional econometric model, these studies report that the propensity of the ARM increases as the yield curve is steeper (or the larger the spread between FRM and ARM rates), the longer-term interest rate is higher, the borrower is more mobile, the borrower has higher income, and the mortgage underwriting standard is more stringent (the wealth and income constraints in particular). Most these studies utilize the generic categorizations of ARM and FRM in their analyses and do not differentiate sub-product types within each category.

The second strand of studies link the mortgage choice issue to the macroeconomic outcomes, either by employing a life-cycle consumer choice model (Campbell and Cocco (2003), and Miles (2003) and (2004)) or by examining the mortgage choice issue in an industrial organization perspective (Vickery (2006) and Mercer Oliver Wyman (2005)). ARM contracts involve with an income (or consumption) risk for borrowers caused by short-term variability in the real payments required each period, corresponding to the changes in the indexed interest rate, which can cause significant payment shocks in an up-rate environment and an increase in mortgage default risk particularly for income- and/or wealth-constrained households. Consumer myopic plays an important role in mortgage choice. Typical borrowers have poor understanding of interest rate risk and risk profiles of different mortgage products. The initial teaser rates in ARM contracts may be viewed very attractive to borrowers at the initial stage, but is subject to subsequent increase in the likelihood of payment shock and default risk. Market segmentation also has a role in consumer mortgage choice in that Vickery (2006) reports a big and discrete drop by about 40% in the FRM share around the GSE loan size limit.

The third strand of studies, which are most relevant to the current research, examine the mortgage product choice in the context of loan termination and default-prepayment risks involved in the process. (Cunningham and Capone (1990); Stanton and Wallace (1995); Ambrose and LaCour-Little (2001); Calhoun and Deng (2002); Ambrose, LaCour-Little and Huszar (2005)) Findings from these studies indicate that hybrid AMRs are associated with higher prepayment rates , particularly at time of initial adjustment. They also experience relatively high default rates, as would be expected outcome with the payment shock at the time of rate adjustment.

Figure 3 below shows the main differences between three particular mortgage contracts – FRM, ARM, and 5/1 Hybrid – in terms of debt-to-income (DTI) ratios over time assuming a constant interest rate along with a constant growth of household income. As shown in the figure, ARM contract incurs a lower DTI than FRM due to the usually lower interest rate. The 5/1 Hybrid ARM is the most affordable mortgage product with the lowest initial DTI. However, its DTI jumps up after the IO period which makes it the least affordable product later on. In real world, interest rate and household income volatilities can change the payment burden dramatically and, hence, the default risk. In this study, we offer a consistent measurement framework for the default risk embedded in these alternative products, by extending the third strand of literature.

Figure 3: Payment-to-Income Ratios of Different Products

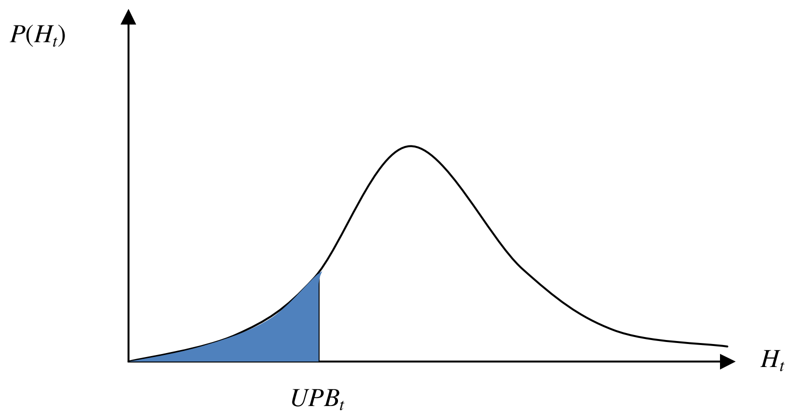


3. Model Framework and Product designs

Mortgage defaults are generally caused by two factors: ability-to-pay problem and willingness-to-pay problem. The former refers to the situation where the borrower does not have enough income to afford and is forced to miss the monthly mortgage payment. The later refers to the situation where the value of the underlying house falls below the mortgage UPB. Under such a circumstance, the borrower has the incentive to quit making the monthly payment and let the lender take over the collateral house. Due to the data limitation, most literature focuses on the analysis of the impact of the willingness-to-pay problem.[2] Yang, Buist, and Megbolugbe (1998) and Elmer and Seelip (1999) argued that the willingness-to-pay problem is only a necessary condition, but not the sufficient condition of mortgage defaults. With the severe consequence of default and the potential deficiency judgment, most borrowers would not default on a mortgage unless are subject to both problems. Buist and Yang (1998) showed that the default rates estimated by such argument are in line with the empirical evidences and generate reasonable value for mortgages. Following the analytical framework of Buist and Yang (1998), we expand the risk factors and use it to investigate the default risk embedded in various mortgage product types.

Following Calhoun and Deng (2002), IFE Group (2007), and Yang, Lin and Cho (2008), the probability of negative equity (PnegQ) indicates the probability that the UPB of the mortgage is greater than the value of the underlying collateral house (H). The PnegQ can be measured at any point in time, either historical or forward-looking, for a specific mortgage loan. In Figure 4, PnegQ can be measured by the shaded area under the probability density curve. Its value is bounded between zero and one.

Figure 4: Probability Density of House Price at time *t*



This PnegQ is closely related to the current loan-to-value (CLTV) ratio of a mortgage. As the CLTV ratio exceeds 1.0, negative equity is observed. Similarly, the probability of payment shortage (PSHORT) refers the probability of the borrower's income is not enough to support the minimum living expense and the mortgage monthly payments at any point in time. When

[2] See Foster and Van Order (1984), Epperson et al (1985), Vandell and Thibodeau (1985), Kau and Keenan (1988), Vandell (1993), and Kau, Keenan, and Kim (1994).

normalized, the PSHORT is closely related to the current payment-to-income ratio of a mortgage. As the mortgage payment becomes a large portion of the borrower's gross income, payment shortage (ability-to-pay problem) occurs. Finally, similar to Buist and Yang (1998), we assume default occurs if and only if the two events happen simultaneously.

The mortgage products being investigated in this paper include the followings:

1. 30-year FRM, fully amortized
2. 30-year ARM with annual reset frequency, no cap, and fully amortizing
3. 30-year ARM with annual reset frequency, with 5/1/1 cap structure, and fully-amortizing
4. 30-year ARM with annual reset frequency, with 5/1/1 cap structure, and initial teaser of 2.75% below indexed rate
5. 30-year Option ARM with minimum payment, with 7.5% annual payment cap, with recast period 60 months, and initial contract rate of 3.75%

PnegQ and PSHORT are determined by three stochastic state variables – mortgage interest rate, home price growth rate, and household income growth rate – or their intertemporal changes summarized as [*r, h, y*]. We assume these three risk factors follow the following stochastic processes.

$$dr = \alpha(\mu_r - r)dt + \sigma_r \sqrt{r} dz_r \qquad (1)$$

$$\begin{aligned} h &= h_1 + h_2 \\ dh_1 &= \mu_{h_1} dt + \sigma_{h_1} dz_{h_1} ; dh_2 = \sigma_{h_2} dz_{h_2} \end{aligned} \qquad (2)$$

$$\begin{aligned} y &= y_1 + y_2 \\ dy_1 &= \mu_{y_1} dt + \sigma_{y_1} dz_{y_1} ; dy_2 = \sigma_{y_2} dz_{y_2} \end{aligned} \qquad (3)$$

The interest rate follows a Cox-Ingersoll-Ross mean-reverting square root process, where α is the mean-reverting speed, μ_r is the long term mean reverting level, and σ_r is the volatility. It is a macro-economic variable that affects all mortgage loans. It involves no idiosyncratic risk. Following Yang, Lin and Cho (2008), we assume the house price growth rate follows a two factor process. The first factor h_1, which captures the variability of the local market's house price index appreciation rate (HPA), follows a geometric Brownian motion (log-normal process) with mean μ_{h_1} and volatility σ_{h_1}. The second factor h_2, which captures the dispersion of individual house appreciation rate from local HPA, also follows a geometric Brownian motion with mean zero and volatility σ_{h2}. Similarly, the household income growth rates are modeled in the same manner to capture the variability of the local market's income growth rate (y_1) and the dispersion of a particular household's income growth rate from the local market average (y_2).

The following $z_r, z_{h_1}, z_{y_1}, z_{h_2}, z_{y_2}$ are five standard Weiner processes with the following correlation matrix.

$$\Sigma = \begin{bmatrix} 1 & \rho_{r,h_1} & \rho_{r,y_1} & 0 & 0 \\ \rho_{r,h_1} & 1 & \rho_{h_1,y_1} & 0 & 0 \\ \rho_{r,y_1} & \rho_{h_1,y_1} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho_{h_2,y_2} \\ 0 & 0 & 0 & \rho_{h_2,y_2} & 1 \end{bmatrix} \tag{4}$$

This correlation matrix assumes that there exist correlations among the systematic risks z_r, z_{h_1}, and z_{y_1}, the interest rate, the local HPA, and the average household income growth rate in the area, respectively. The two risks z_{h_2} and z_{y_2} are independent of idiosyncratic risks. However, there exists a correlation between the dispersion of a single house's appreciation rate from the local HPA and the dispersion of the particular household income growth rate from the local average income growth rate. Intuitively, they are positively correlated because a lower income household is likely to under-maintain the house, causing the house to suffer abnormal physical deterioration. As a result, the appreciation rate of the house would be lower than other houses in the same local market.

Under discrete time framework, the three stochastic risk factors can be rewritten as the following orthogonal processes:

$$\Delta r_t = (\alpha + q)(\mu_r - r_{t-1}) + \xi_r \sigma_r \sqrt{r_{t-1}} \tag{5}$$

$$\Delta h_t = \mu_{h_1} + \xi_{h1} \sigma_{h_1} + \xi_{h_2} \sigma_{h_2} \tag{6}$$

$$\Delta y_t = \mu_{y_1} + \xi_{y_1} \sigma_{y_1} + \xi_{y_2} \sigma_{y_2} \tag{7}$$

where q is the individual's risk preference parameter and the correlated shocks (ξ) are specified as:

$$\xi_r = \eta_r \tag{8}$$

$$\xi_{h_1} = \rho_{r,h_1} \eta_r + \sqrt{1 - \rho_{r,h_1}^2}\, \eta_{h_1} \tag{9}$$

$$\xi_{y_1} = \rho_{r,h_1} \eta_r + \frac{(\rho_{h_1,y_1} - \rho_{r,h_1} \rho_{r,y_1})}{\sqrt{1 - \rho^2{}_{r,h_1}}} \eta_{h_1} + \frac{\sqrt{1 - \rho_{r,y_1}^2}\sqrt{1 - \rho_{r,h_1}^2}}{(\rho_{h_1,y_1} - \rho_{r,h_1} \rho_{r,y_1})} \eta_y \tag{10}$$

$$\xi_{h_2} = \eta_{h_2} \tag{11}$$

$$\xi_{y_2} = \rho_{h_2,y_2} \eta_{h_2} + \sqrt{1 - \rho_{h_2,y_{21}}^2}\, \eta_{y_2} \tag{12}$$

and η's represent independent random normal deviate for each state variable.

Parameters used in our simulations are described as follows (see Buist and Yang (1998) for justification of specific values):

- Long-term means and volatilities: $\mu_r = 0.065$, $\sigma_r = 0.15$; $\mu_{h_1} = 0.05$, $\sigma_{h_1} = 0.06$, $\sigma_{h2} = 0.08$; $\mu_{y_1} = 0.035$, $\sigma_{y_1} = 0.05$, $\sigma_{y_2} = 0.07$;
- Pair-wise correlations: $\rho_{r,h_1} = 0.4$, $\rho_{r,y_1} = 0.7$, $\rho_{h_1,y_1} = 0.6$, and $\rho_{h_2,y_2} = 0.1$
- Initial loan characteristics: LTV = 95, Contract rate = 7%, Teaser rate = 2.75%, Periodic Cap =1%, Lifetime Cap=5%, Adjustment Interval = 12 months, Monthly discount rate = -0.3% α = 0.25; and
- Initial Option ARM characteristics: Initial Contract rate = 3.75%, Annual Cap = 7.5%, Loan Recast Period = 60 months, Minimal Payment Rest Frequency = 12 months

The above assumptions and estimates provide initial values for our model's parameters and enable us to construct future economic shocks by Monte Carlo simulation. The parameter values represented as [$\mu_{h_1}, \sigma_{h_1}, \sigma_{h2}$] for house appreciate rate were imputed based on the annualized estimates by Yang, Lin and Man (2008).[3] Related to the parameter values represented as [$\mu_{y_1}, \sigma_{y_1}, \sigma_{y_2}$] for household income growth rate, the former two values μ_{y_1} and σ_{y_1} are able to be observed from the market, but σ_{y_2} is unobserved and thus an assumed value is taken in this study.[4]

4. Results

[3] In Yang, Lin, and Cho (2008) estimates the quarterly μ_{h1}, σ_{h1}, and σ_{h2} to be 0.0139, 0.0303, and 0.0409, respectively.

[4] Similar to Yang, Lin and Man (2008), we also compute the parameter values for household income growth rate and also its variability using data of state household income, published by U.S. Census Bureau.

We simulate the correlated state variable paths, as specified in Equations (5) through (7), with 10,000 iterations. We also define a stress economy in terms of the interest rate and home price as well as household income and compare the results with those from a normal (or non-stress) economy. The simulation results computed using parameters outlined above serve as the base outcomes in our analyses. However, under a stress economy, the shock assumptions we adapted are based on an accepted industry common practice. The interest rate shock requires testing an increase in the mean growth rate of interest rates by its own standard deviation during a 2-year event window initiating from origination. Likewise, equipped with the same 2-year duration from origination, the home price and household income shocks both require testing a decrease in mean growth rates of their respective series by each of their standard deviations. The shock of each series will revert to their respective normal economy level after 2 years from origination and stay constant throughout the loan life. The following four graphs summarize our results by showing the average values of PSHORT and PBOTH (probability of default) over loan life across the simulation paths:

4.1. Findings for PSHORT Trends

As reported in Figures 5 and 6, the PSHORT trends for different loan products are expressed under a normal and stress economy, respectively. As can be seen in Figure 5, all loan products show the same PSHORT value in the initial year thanks to product features since an annual payment adjustment interval is assumed for all ARMs in our study. After that period, two ARM products and the Option ARM show PSHORT value has a series of discrete jumps. Among these trends, Option ARM moves close to that of such ARM without consumer protection mechanism until around the 5th year after origination. From about six years after origination, Option ARM stands out as the product with the highest PSHORT value and stays as such until the maturity. The impact is obviously significant caused by negatively amortization principal. That is, it represents the gap between the contract itself and versus actual interest payments being added onto the unpaid outstanding principal. As to other loan products, FRM shows the lowest PSHORT, followed by ARM with 5/1/1 cap structure and ARM 5/1/1 with no such protection feature imposed. Therefore, the order of risk rankings is Option ARM, ARM 5/1/1 with teaser rates, ARM without cap structure, ARM 5/1/1, and FRM. The first two has relatively lower payment since origination and thus the impact of payment shock is significant once triggered. Besides, the ARM with cap structure is less sensitive to payment shock relative to its counterpart without caps constrained. Finally, there is no payment shock for FRM because its payment stays the same till maturity.

The stress economy dramatically increases PSHORT values for all loan products, but the impact is less on both ARM 5/1/1 with the teaser rate and Option ARM due to lower initial payment and minimum interest payment within loan recast period, accordingly. Comparing the peak values, ARM 5/1/1 with cap structure shows about 3.2 times increase in PSHORT value, followed by ARM without cap structure with approximately 2.6 times increase in PSHORT value, and FRM around 2.2 times increase in PSHORT value. Both Option ARM and ARM 5/1/1 with teaser show approximately 1.9 times increase in PSHORT values. Likewise, with comparisons in peak values, the order of risk ranking is ARM without cap structure, Option ARM, ARM 511 with teaser, ARM511, and FRM.

Figure 5. PSHORT Trends, Non-Stress Economy

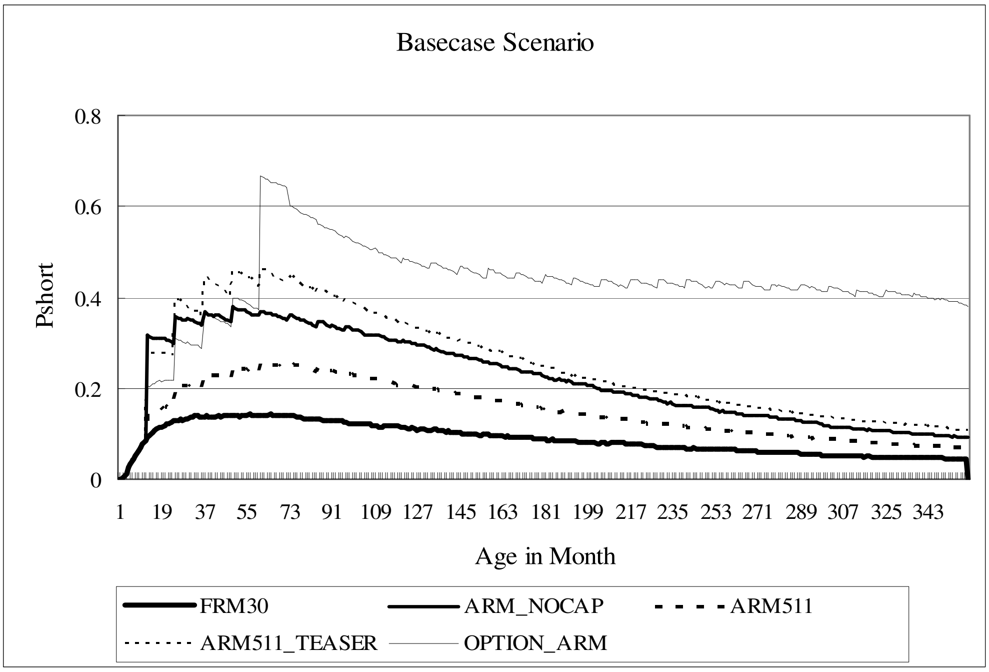
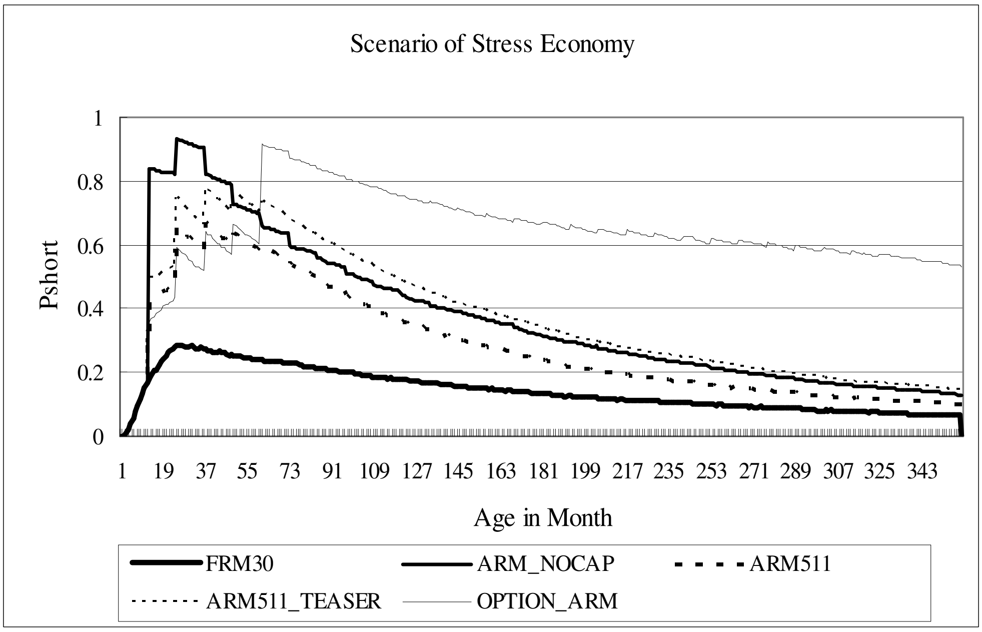


Figure 6. PSHORT Trends, Stress Economy



4.2. Findings for PD Trends

As mentioned earlier, the probability of default is defined to be triggered by both events, the negative equity and the liquidity shock. As displayed in Figure 7, the PD trends for different loan products under a non-stress economy tend to be clustered with the peak shown in about two year from origination. After that period, Option ARM shows a higher likelihood of default in comparison to other products over the course of the loan life. This outcome is mainly due to a special feature of negative amortization meaning that the unpaid principal balance exceeds loan origination amount since the interest repayment is more than the contract payment. ARM with 5/1/1 cap structure and teaser rate shows a consistently higher PD value during the time span between year one and year six after origination, compared to its counterpart without teaser rate. Across all products examined, the PD value is short of 10 percent even at the peak except for Option ARM. In terms of PD values, Option ARM is still the riskiest product, ARM 5/1/1 with teaser rates is the second, with the other three cross over in tandem, but FRM shows the lowest degree of riskiness.

Comparing to the peak values under a normal economy level, the PD values under a stress economy show a longer initial ramping period with the peak shown in around second year from origination. The exception is Option ARM which climbs to its peak in about 5th year since origination with PD values close to 60 percent as the maximum, but it is greater than the one about 12 percent under a non-stress economy. Likewise, other loan products also show higher PD values at the peak compared to the values within the base economy, with ARM 5/1/1 no cap structure recording about 30 percent as the riskiest product. The order of risk ranking is Option ARM, ARM without consumer protection mechanism, ARM 5/1/1 with teaser, ARM 5/1/1, and FRM.

Figure 7. PD Trends, Non-Stress Economy

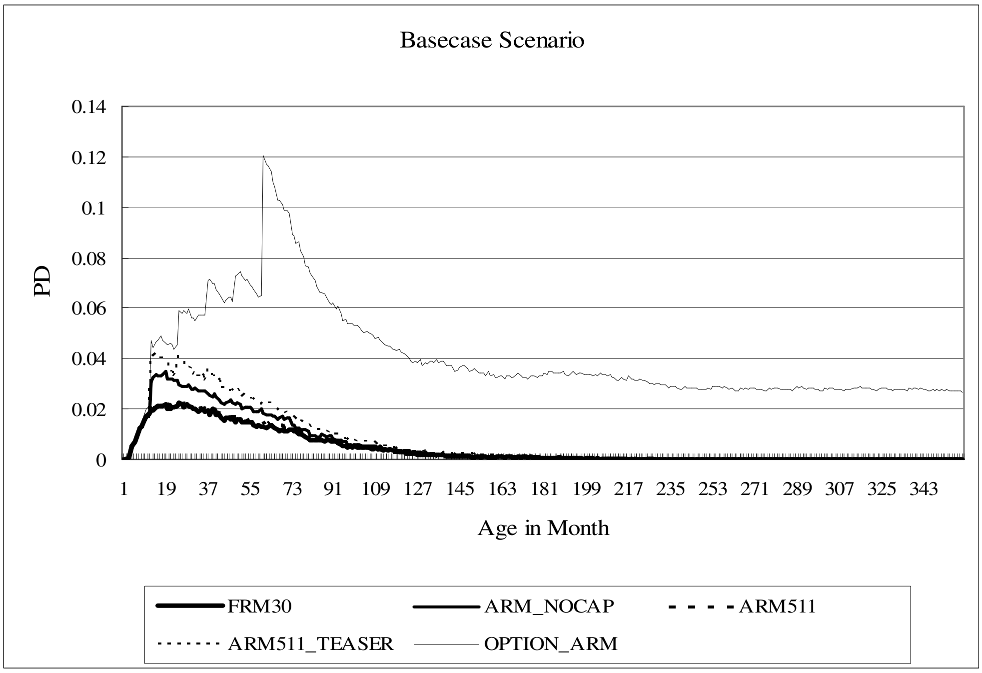
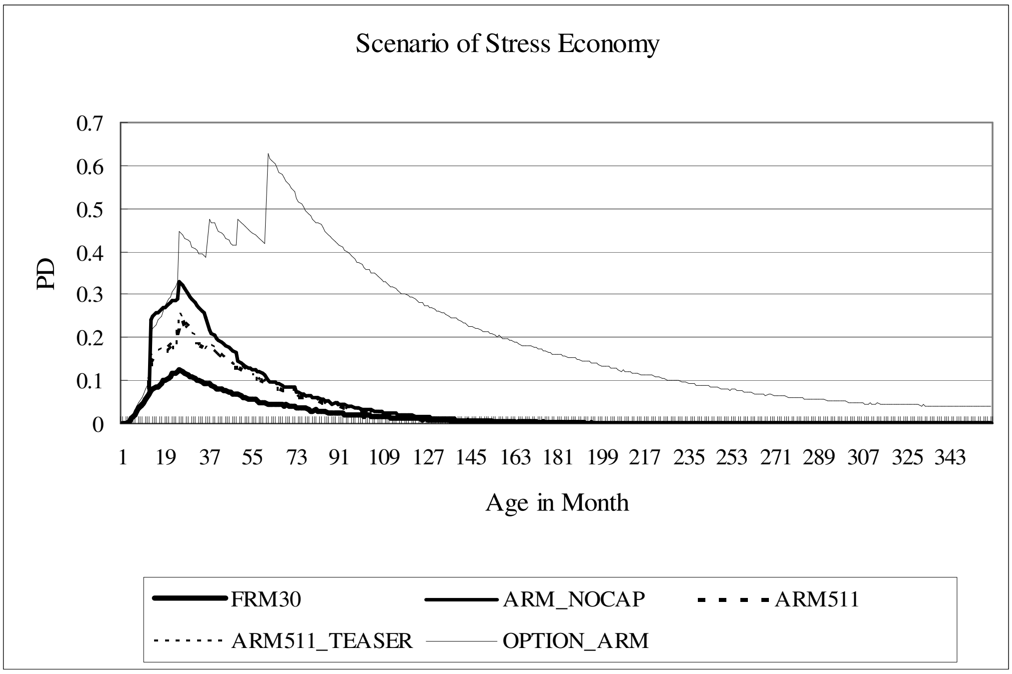


Figure 8. PD Trends, Stress Economy



4.3. Idiosyncratic Risk versus Systematic Risk

Yang, Lin and Cho (2008) propose one logistics to decompose home price risk, the so-called collateral risk, into the idiosyncratic risk and systematic risk.[5] The modeling framework introduced in Section 3 includes the interest rate, home price and the household income. The interest rates only face the systematic risk, while the other two series' risks are able to be segmented into two components: the idiosyncratic risk and systematic risk. Similar to the work proposed by Yang, Lin and Cho (2008), the income risk, the so-called affordability risk, is also separated into the systematic risk and idiosyncratic risk. From Equation (3), the total income risk is comprised of systematic and idiosyncratic risks summarized as $[\sigma_{y_1}, \sigma_{y_2}]$. Table 2 summarizes the μ_y and σ_y of individual state income lognormal process.

By definition, diversification benefit means that one can fully take away the idiosyncratic risk and face with only the systematic risk. With the idiosyncratic risk as part of affordability risk, one way to eliminate this risk is to form a mortgage portfolio. For a large mortgage portfolio, the key is completely to eliminate the idiosyncratic risk. That is, under a deterministic income growth path, the probability of ability-to-pay default becomes an expected outcome without uncertainty. One can consider that each individual mortgage loan is a Bernoulli trial faced with only two outcomes, i.e., default probability of PD and non-default probability of (1-PD). When pooling a bunch of homogeneous mortgages, the pool will follow a Binominal distribution with the number of actual defaults to be:

$$\pi(x) = \frac{n!}{x!(n-x)!} PD^x (1-PD)^{n-x}, x = 0, 1, 2, ..., n \qquad (10)$$

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[5] Their contribution is the first one to quantify collateral risk in terms of the systematic and idiosyncratic risks.

where $\pi(x)$ is the probability density function of realizing x defaults, n is the total number of loans in the pool, and PD is the expected default probability of individual loans. This distribution has expected value of (n PD) and standard deviation as $\sqrt{\frac{PD(1-PD)}{n}}$. Thus, as the number of loans in the pool becomes large, i.e., as $n \to \infty$, the percentage of realized defaults in the pool will converge to $\frac{n \cdot PD}{n} = PD$ with zero standard deviation.

Indeed, it results in an important policy implication to gauge various credit risks across different loan products borne by a mortgage portfolio under the Basel II framework. Specifically, this regulation requires lenders to charge higher interest rate to cover the expected credit losses and to use the capital reserve to cover the unexpected default losses. All else being equal, borrowers of multiple loan products face the same income risk, but the payment shocks are not equal due to different features of product design. The payment shock stems from the changes in interest rates and it is being considered as the systematic risk. Therefore, the bank should reserve a higher capital for ARMs than similar FRMs to cover the unexpected credit losses.

5. Policy Implications of Basel-II Risk-based Capital Rules

Table 1 below shows the average PD during the first 5 years of individual mortgages. The PD Multiplier (Base) shows that the Option ARM is about 3 times riskier than a standard FRM. Expected PD of ARM511 is marginally higher than the one with FRMs. The PD Multiplier (Stress) shows that the deviation of PDs among products under the stressed economy environment gets larger. Even the ARM511 without teasers is about 1.8 times riskier than the standard FRMs. However, such an effect would not be captured by default models with no payment shock variables.

Table1. PD and Capital among Products

Product	PD (Base)	PD Multiplier (Base)	PD (stress)	PD Multiplier (Stress)	Economic Capital	Basel II Regulatory Capital	RC Multiplier	EC as % of RC
FRM30	1.63%	1.00	7.35%	1.00	3.68%	6.19%	1.00	0.59
ARM_ NOCAPS	2.27%	1.39	17.95%	2.44	9.75%	7.59%	1.23	1.28
ARM511	1.69%	1.04	13.00%	1.77	7.04%	6.34%	1.02	1.11
ARM511_ TEASER	2.74%	1.68	13.67%	1.86	6.97%	8.50%	1.37	0.82
OPTION ARM	4.98%	3.06	32.10%	4.37	17.02%	11.83%	1.91	1.44

Note: Economic capital is computed assuming LGD 60 percent for PD (Stress), and LGD 45 percent for PD (Base Case)

The economic capitals are calculated by taking the difference of the default loss under the stressed economy and the base economy. It represents the measure of unexpected default losses. From Table 1, we find that the unexpected default losses of ARMs are much higher than similar FRMs, mainly due to payment shock risk. The central spirit of Basel II suggests that the banks should charge premium for expected default loss and use the capital reserve as a buffer against the unexpected credit risk. As a result, a bank should reserve much higher risk based economic capital for ARMs to cover the payment shock risks.

The Basel II regulatory capital in Table 1 shows the capital requirement adopting the Basel II advanced internal rating based (AIRB) risk-based capital formula with the base economy PD (expected PD) of individual products. The results show that the Basel II's capital rule is not a fairly reasonable reflection of the unexpected default loss of different loan products. This is because it substantially understates the default risk associated with all loan products.

6. Concluding Remarks

This paper documents the results of our analysis on the relative risk of different mortgage products. Our simulation results indicate that: As expected, Option ARM with the minimum monthly payments is by far the most risky product regardless of which risk measure to use, while FRM with the amortization throughout the loan life is the least risky product; ARM with 5/1/1 cap structure and teaser rate shows a consistently higher PSHORT under both economic scenarios, and also relatively higher PD under a normal economy level, but about comparable PD, compared to its counterpart without teaser rate under a stress economy.

For a bank to cover payment shock risks, a larger risk-based economic capital should be reserved for ARMs compared to standard FRMs. For examining the capital requirements under advanced internal risk-based capital framework, our findings indicate that the new Basel II rule does not reflect a reasonable capital measure serves as a cushion against the unexpected default loss of various mortgage loans. The outcomes show that it substantially understates the scale of default risk associated with different loan products. Put it differently, the Basel II rule does not fully recognize credit risk modeling on mortgage sector. To accurately reflect the capital changes on mortgage products within AIRB formulas, one of the solutions is to relax the assumption of a fixed rule of 15 percent correlation values used. Not only it is the key driver of Basel II capital charges, but also it is an important policy lever in adjusting capital requirements. To achieve it successfully, banks should be allowed to generate their own internal estimates of correlation to compute capital reserves.

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Table 2. Household Income μ, σ among Individual States and USA

State Name	HHI μ	HHI σ
Alaska	0.0357	0.0496
Alabama	0.0253	0.0560
Arkansas	0.0354	0.0449
Arizona	0.0391	0.0452
California	0.0356	0.0281
Colorado	0.0350	0.0444
Connecticut	0.0334	0.0521
District of Columbia	0.0322	0.0629
Delaware	0.0393	0.0646
Florida	0.0380	0.0311
Georgia	0.0411	0.0519
Hawaii	0.0336	0.0680
Iowa	0.0357	0.0448
Idaho	0.0326	0.0376
Illinois	0.0314	0.0540
Indiana	0.0402	0.0468
Kansas	0.0280	0.0526
Kentucky	0.0365	0.0612
Louisiana	0.0298	0.0557
Massachusetts	0.0361	0.0588
Maryland	0.0346	0.0472
Maine	0.0327	0.0411
Michigan	0.0341	0.0463
Minnesota	0.0379	0.0553
Missouri	0.0369	0.0588
Mississippi	0.0347	0.0518
Montana	0.0338	0.0390
North Carolina	0.0369	0.0284
North Dakota	0.0321	0.0499
Nebraska	0.0396	0.0619
New Hampshire	0.0407	0.0389
New Jersey	0.0301	0.0663
New Mexico	0.0356	0.0299
Nevada	0.0300	0.0373
New York	0.0310	0.0402
Ohio	0.0312	0.0354
Oklahoma	0.0276	0.0460
Oregon	0.0359	0.0470
Pennsylvania	0.0395	0.0371
Rhode Island	0.0414	0.0600
South Carolina	0.0304	0.0721
South Dakota	0.0387	0.0430
Tennessee	0.0403	0.0489
Texas	0.0287	0.0299
Utah	0.0392	0.0606
Virginia	0.0379	0.0687
Vermont	0.0349	0.0453

Washington	0.0356	0.0648
Wisconsin	0.0375	0.0591
West Virginia	0.0415	0.0614
Wyoming	0.0309	0.0582
	HHI μ	HHI σ
Mean	0.0350	0.0498
Standard Deviation	0.0039	0.0113
Max	0.0415	0.0721
Min	0.0253	0.0281
USA	0.0348	0.0163