Goods Market Frictions and Real Exchange Rate Puzzles[*]

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Abstract

It is a well-known fact that real exchange rates are highly volatile and persistent in the business cycle. Moreover, there are substantial and systematic differences in the behavior of real exchange rates under fixed versus floating exchange rate regimes. These facts have posed a non-trivial challenge for international business cycle models. Traditional explanations for these features of exchange rates rely on the interaction of the nominal price rigidities and monetary shocks. In this paper we deviate substantially from the literature by developing a two-country dynamic search model to examine the behavior of exchange rates in an environment where prices are fully flexible. In contrast to traditional models, even without any nominal rigidity, our model can generate large fluctuations in real exchange rates. The reason is that fluctuations in the real exchange rate arise mainly from the deviations from the law of one price, which are generated by search frictions rather than nominal price rigidities. In addition, the deviations from the law of one price depend critically on the differential between the country's valuations of the two currencies and the relative effective money holdings across countries. Because monetary shocks directly affect these two factors, they account for a larger part of the movement of exchange rates than productivity shocks. We also compare the behavior of exchange rates under different exchange regimes. The simulated results are consistent with the empirical findings.

JEL Classifications: F41, F31, E30

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1. Introduction

There are several well-known facts that characterize the behavior of real exchange rates in the business cycle data: i) Real exchange rates are highly volatile relative to other macroeconomic variables such as aggregate output and consumption; ii) Changes in real exchange rates are very persistent and the real exchange rate follows approximately a random walk; iii) There are substantial and systematic differences in the behavior of real exchange rates under fixed versus floating exchange rate regimes, while fluctuations of output and consumption do not seem to change systematically with the regimes. As an international relative price, the real exchange rate is expected to play an important role in the real allocation across countries. Thus, the real exchange rate movement should be strongly correlated with fluctuations in real variables. However, the above mentioned empirical facts suggest only a weak link between the behavior of the real exchange rate and other macroeconomic variables, which poses a non-trivial challenge for international business cycle models.

The conventional explanation for real exchange rate puzzles usually rests on the interaction of the nominal price rigidities and monetary shocks. The basic idea is simple: Monetary shocks can induce an immediate change in the nominal exchange rate; Since prices are sticky in the short run, fluctuations in the nominal exchange rate translate one for one into the real exchange rate movement. Under this explanation, the volatility of exchange rates can be much higher than their underlying fundamentals because of the sticky price and high capital mobility. The earliest version of the sticky price model goes back to the celebrated Mundell-Fleming-Dornbusch framework. With the ad hoc link between money demand and saving-investment decision, as well as the lack of an explicit theory of price setting, this framework is inadequate for understanding the real exchange rate. The similar idea can also be embedded in the general equilibrium framework. Following Obstfeld and Rogoff's (1995) pioneering work, a series of papers have applied the general equilibrium model with microfounded price setting to explore the features of exchange rates. Chari, Kehoe and McGrattan (2002) evaluate such models quantitatively and show that sticky price models are potentially capable of accounting for the highly volatile and persistent real exchange rates.[1] To generate enough volatility, however, they have to assume an implausibly high risk aversion.[2] Moreover, their model typically needs 11 quarters or longer of price stickiness

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[1]Bergin and Feenstra (2001) obtain similar results.

[2]Their model sets up a link between real exchange rates and the ratio of the marginal utility of consumption in

to replicate the persistence in the data.[3] Recent evidence in Bils and Klenow (2005), however, seriously calls into question the assumption of such a long-lived price stickiness.[4]

An alternative approach to explore behavior of exchange rates is setting the model in a flexible price framework, an idea that has been implemented firstly in Stockman (1988). Later, Backus, Kehoe and Kydland (1992,1994), and Stockman and Tesar (1995) advance the so-called "international real business cycle models" in a series of influential papers and try to use such models to match the international business data. Quantitatively, however, these models fail to explain the facts above that characterize the behavior of real exchange rates. Moreover, fluctuations of real exchange rates in such models are driven entirely by real disturbances, and persistence in real exchange rates is due to persistence in the underlying real shocks. These results are inconsistent with empirical findings which support the view that monetary shocks, rather than real shocks, account for a substantial fraction of the variability of the real exchange rate.[5]

As demonstrated by Engel (1993, 1999) and Betts and Kehoe (2001, 2006, 2008), the decomposition of real exchange rate movements in the data suggests that most of the movements come from the international deviations of the relative price of tradable goods. Alessandria and Kaboski (2004) in an empirical work also show that the deviations from the law of one price are an important source of violations of absolute PPP across countries. Given these empirical evidence, it is not surprising that models with frictionless Walrasian goods market and flexible prices fail to account for the volatility of real exchange rates in business cycle, since the law of one price always hold in such a frictionless environment.

In order to understand the real exchange rate volatility, it seems clear that we need to introduce at least some type of goods market frictions into the flexible-price model to generate the deviations from the LOP. Monetary search theory is a natural framework to capture these frictions. In this

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the two countries. With separable utility function, the volatility of real exchange rates is essentially determined by the risk aversion parameter and the volatility of consumption. Such a model can easily generates highly volatile exchange rates by imposing a high value of risk aversion parameter.

[3]Duarte (2003) applies the new open-macroeconomics models to explain the systematic changes in the behavior of real exchange rates across regimes. Using the assumption that firms set prices one period in advance, her model shows no persistence in real exchange rates and much less volatile exchange rates than in the data. To get better quantitative results, a longer-term price setting in such sticky price models seems also necessary.

[4]Bils and Klenow (2005) find that half of all posted prices last less than 4.3 months.

[5]Rogers (1999) studies the variation in the real pound-dollar exchange rate and estimates the contribution of various shocks. He finds that monetary shocks account for up to 60 percent of the variations in real exchange rate, while the contribution of fiscal and productivity shocks combined is only 4 to 26 percent. Faust and Rogers (2003) also show that the contribution of monetary shocks can be more than 50 percent of the variance shares.

paper, we integrate the monetary search theory into international economics to derive the role of money in a two-country, two-currency economy. All goods are tradable between the two countries and prices are fully flexible. In contrast to the Walrasian market in standard models, the goods market is decentralized and modelled as randomly and bilaterally matching. Search frictions generate the differential in a household's valuations of the two currencies and create the deviations from the LOP, which contribute to the majority of fluctuations of exchange rates in our model. Even without any nominal rigidity, quantitatively the model can produce enough volatility of exchange rates to be consistent with the data.

For simplicity, we assume that the two countries have the same size, identical preferences and production technologies. Both consumption and production are specialized. Because of the lack-of-double-coincidence-of-wants problem as well as the absence of a perfect record-keeping device, money is essential in the economy as a medium of exchange. Both countries can issue their own currencies and inject money into the economy as a lump-sum transfer to their domestic residents. Thus, money injection is asymmetric in the sense that households receive only the transfer of domestic currency. Both currencies are acceptable in the goods market for any transaction, and there is a centralized currency market where households can exchange currencies. Although a currency does not directly generate utility or facilitate production in the model, it obtains positive value in an equilibrium by alleviating the difficulty of exchange. The relative role of each currency in the trade determines the nominal exchange rate in the currency market.

The valuation of the same currency differs for households in different countries because of the presence of search frictions in the goods market. The centralized currency market only equalizes the relative valuations of the two currencies but not the levels of the valuations. The gap between the two countries' valuations of a currency does not disappear unless households can trade goods directly for currency in the currency market, which is impossible since transactions in the goods market are formed in random and bilateral matches. The inability to instantaneously arbitrage between the goods market and the currency market allows this differential in the valuations of currencies to persist. Therefore, even for the same goods, buyers from different countries face different prices (pricing-to-market behavior) and buyers with different types of currency pay different prices. Thus, there are deviations from the LOP, despite the fact that prices are fully flexible and all goods are tradable between the two countries. We decompose the fluctuations in real exchange rates and illustrate that fluctuations in the deviations from the LOP are the main

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component of the real exchange rate movement.

We calibrate the model to U.S. and European aggregate data and evaluate it quantitatively. The model closely replicates the feature of exchange rates listed earlier as stylized facts. The real exchange rate is much more volatile than output and consumption, exchange rates are highly persistent as in the data, and the cross-correlations between most variables are close to those observed in the data.

The deviations from the LOP come from the relative valuation of the two currencies and the relative effective money holdings across countries, both of which depend critically on the differential between the two countries' money growth rates. Since monetary shocks directly affect this differential, they account for most of the fluctuations in the real exchange rate in our model. This result is consistent with the empirical evidence that suggests monetary shocks, rather than real shocks, cause more real exchange rate fluctuations.

Real shocks, on the other hand, do play a role, albeit a less important one, in explaining the fluctuations of real exchange rates in our model. Real shocks do not change the relative valuation of the two currencies; instead, they change the quantities exchanged in a trade match and hence the level of valuations of the currencies. However, since the goods are sold for both currencies, real shocks affect the valuation of both currencies in the same way and hence the differential in the valuation of the two currencies remains the same.

Search frictions in the goods market are the key feature in the model to derive the main results. The simulated results suggest that the volatility of exchange rates increases with the degree of frictions. Moreover, to illustrate the importance of search frictions in generating volatility and the persistence of exchange rates, we construct a Walrasian model with cash-in-advance constraints in a similar environment.[6] We apply the same preference and technology as those applied in the benchmark model. The only difference is that the goods market is now Walrasian and frictionless. The LOP always holds in this CIA model. Using the same parameter values and shock processes, the simulated results show that neither monetary shocks nor real shocks are capable of generating much volatility of exchange rates without the presence of search frictions in the goods market. The persistence in exchange rates in this CIA model comes entirely from the persistence in the underlying shocks.

The behavior of exchange rates in different regimes is also examined in this paper. The model

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[6]This model is a variant of Helpman's (1981) model with elastic labor supply as in our model.

shows a sharp increase in the volatility of the real exchange rate when a country moves from a pegged to a floating regime, while fluctuations of output and consumption do not change systematically with exchange rate regimes. Moreover, the co-movements of output and consumption across countries are higher under a fixed rate regime than under a flexible rate regime. These results are consistent with the empirical findings. The results are intuitive: To maintain the pegged rate, one country has to follow the other country's monetary policy and keep the money growth at the same rate as the other country. Since the driving force of the deviations of the LOP, the differential in money growth rates across countries, does not exist in this case, the LOP holds and, as a result, there is hardly much fluctuation in the real exchange rate. The movements of other macroeconomic variables, such as output and consumption, do not rely on the two countries' money growth rate differential. Therefore, the switch of exchange rate regimes does not have a systematic effect on these variables.

Many researchers have investigated the deviations from the LOP as a factor generating the fluctuations in the real exchange rate. The explanations of these persistent deviations from the LOP in previous models either rely on the assumption of pricing-to-market or simply assume sticky prices with local-currency-pricing (see the work of Dornbusch (1987), Krugman (1987), Knetter(1989), Goldberg and Knetter (1997), Betts and Devereux (1996, 2000), Chari, Kehoe and McGrattan (2002).). In our paper, however, we focus on the search frictions as the factor that produces the deviations from the LOP. This feature of the model yields new insights into the behavior of exchange rates in the business cycle.

How a country's welfare varies under different exchange rate systems is an interesting issue. Our model is especially suitable to deal with such questions since the role of each currency is derived endogenously from detailed descriptions of preferences and technologies. While exchange-rate stability is often viewed as favorable to trade and therefore welfare improving, whether such a view can be supported with a rigorous model still needs to be examined. Although this examination is interesting, it is outside the scope of this paper and hence is left for future research.

As mentioned earlier, our model is built on the recent development in monetary search theory. Early applications of this theory to multiple currencies and exchange rates include Matsuyama, et al. (1993), Shi (1995), Trejos and Wright (1996) and Zhou (1997). These applications have assumed money or goods, or both, to be indivisible. As a result, these models are not suitable for analyzing issues related to money growth and inflation. The current model eliminates this

restriction by using the construct of large households in Shi (1997).[7]

The papers closest to ours are Head and Shi (2003) and Liu and Shi (2010). Using a similar setup, Head and Shi focus on the determination of the nominal exchange rate while Liu and Shi model currency areas, examining the welfare effects of long-run money growth. Both papers work in a deterministic environment and focus on stationary equilibrium. In this paper, however, we are interested in the dynamics of exchange rates and hence go beyond the steady-state analysis, introducing monetary shocks and technology shocks into the model. Moreover, in contrast to the theoretical analysis in the search literature, we evaluate the model quantitatively and use the search theoretical model to capture the features of exchange rates observed in the data.

2. The Model

2.1. Countries and households

Consider a world economy consisting of country 1 and country 2. To keep the problem simple, the two countries are assumed to have the same size, the same preference and production technology. Each country is populated by a large number of infinitely-lived households, which belong to different types. All the households are specialized in both consumption and production. Specifically, a type k household produces only good k and consumes only good $k+1$ (mod J, $J \geq 3$). Therefore, the problem of lack-of-double-coincidence-of-wants exists in this environment, which generates a need for the use of a medium in the goods exchange.

All goods are tradeable between the two countries and households consume both local and foreign produced goods. The same type of goods produced in the two countries, however, are imperfect substitutes in consumption, with constant elasticity of substitution $\eta > 1$. Let $c_{ih,t}$ and $c_{if,t}$ denote date t country i ($i = \{1, 2\}$) household's consumption of local and imported goods, respectively. The consumption index of a household in country i is defined as

$$\mathcal{C}_{i,t} = \left[\rho^{1/\eta} c_{id,t}^{(\eta-1)/\eta} + (1-\rho)^{1/\eta} c_{if,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}; \tag{2.1}$$

where the parameter η represents the elasticity of substitution between local and imported goods and the weight ρ determines the agents's bias for the consumption of local goods, c_{id}.

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[7]The construct of large households keeps the analysis tractable: By smoothing the matching risks within a household, the assumption makes the distribution of money holdings across households degenerate. Lagos and Wright (2005) use a different device to make the distribution of money holdings degenerate.

Each household consists of a continuum of infinitely-lived members, who are divided into two groups according to their roles in the exchange. Those who carry money and purchase goods are called buyers and those who produce and sell goods are called sellers. The individual member of a household does not make decisions. Instead, he simply follows the rules prescribed by the household.[8] For simplicity, the composition of buyers and sellers in each household is set to be fixed and thus all households in the economy have the same number of buyers and sellers. We normalize the number of buyers in a household to one and use s to denote the number of sellers.

Each country issues fiat money which does not generate direct utility or facilitate the production. The monies are indexed as currency 1 and currency 2, corresponding to the issuing countries. In contrast to the cash-in-advance constraints in traditional models, we do not impose any restrictions on the use of currency in a purchase and therefore both currencies are acceptable in the goods market.[9] There is a restriction on carrying currency, however, that a buyer can bring only one type of currency into a trade at a time. Thus, although buyers can use either of the two currencies to buy the goods, they cannot use both currencies in one purchase.[10]

The buyers and sellers of the households in the two countries are sent to the goods market, meeting with each other bilaterally and randomly. Meetings in which a successful transaction occurs we call a *trade match*. For simplicity, we exclude the possibility of barter. Therefore, all the trade matches involve money-goods transaction, where a buyer who carries money meets with a seller who produces the type of goods that the buyer's family prefers. There are in total eight different types of matches which result in trade: a country i buyer holding domestic currency meets with a country $i\,(i')$ seller, and a country i buyer holding foreign currency meets with a country $i\,(i')$ seller.[11] We use T_{ij}^k to denote the aggregate number of trade matches between a country i buyer who carries currency k and a country j seller, where $i, j, k \in \{1, 2\}$.

The aggregate matching function is assumed to take the form of the commonly used Cobb-

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[8]This large household setting is a device that makes the distribution of money holdings degenerate across housholds and so that allows for a tractable analysis for issues related to money growth, see Shi (1997, 1998, 1999).

[9]Note that a buyer can buy local goods with foreign currency. In our model, if imposing symmetry, the volume of transactions of local goods by domestic buyers with domestic currency is the same as that by domestic buyers with foreign currency. Looking at the real world, however, most transactions of local goods by domestic buyers involve domestic currency rather than foreign currency. The share of trade is poorly matched with the data because we are capturing only traded goods in the model. By adding non-traded goods that is only sold for domestic currency, the model will then characterize the world economy much better and bring the share of trade closer to the data.

[10]If buyers can carry both currencies into the trade and make arbitrage between trades, there is the problem of indeterminancy in the model. Head and Shi (2003) make a similar argument and provide a detailed discussion.

[11]In this chapter, i and i' denote the index of a country, where $i, i' \in \{1, 2\}$, and $i' \neq i$.

Buyer's country	Seller's country	Trading arrangement	Number of matches
i	i	$x_{ik} \longleftrightarrow q_{ik}$	T_{ii}^k
i	i'	$x_{ik}^f \longleftrightarrow q_{ik}^f$	$T_{ii'}^k$
i'	i	$x_{i'k}^f \longleftrightarrow q_{i'k}^f$	$T_{i'i}^k$
i'	i'	$x_{i'k} \longleftrightarrow q_{i'k}$	$T_{i'i'}^k$

Table 2.1: Trading arrangements and trade matches in the goods market

Douglas functions:

$$T_{ii}^k = T_0 N_{ik}^\psi s_i^{1-\psi}, \quad T_{ii'}^k = \alpha T_0 N_{ik}^\psi s_{i'}^{1-\psi}; \tag{2.2}$$

where $\alpha, \psi \in (0,1)$. N_{ik} is the number of country i buyers who carry currency k. s_i is the number of sellers from country i and $s_{i'}$ is the number of sellers from country i'.

An important feature of this matching function is that the matching frequencies within and across countries are asymmetric. Since $\alpha < 1$, according to the matching function, agents are more likely to meet domestic people than foreign people $(T_{ii'}^k/N_{ik} = \alpha T_{ii}^k/N_{ik})$. This assumption is reasonable since trades across countries are generally much more difficult than within a country. Thus, a country can be interpreted as a group of households that have a higher possibility of meeting with each other.

Buyers are assumed to take all the bargaining power in the transactions.[12] They propose an offer of (x_{ik}, q_{ik}) in the trade matches with domestic sellers and $\left(x_{ik}^f, q_{ik}^f\right)$ with foreign sellers. x's denote the amount of money paid by the buyers and q's denote the quantity of goods sold by the sellers. The first subscript of x and q indicates the index of the buyer's country and the second subscript indicates the type of currency used in the trade. Superscript f indicates that the purchase is made between agents from different countries. Table 2.1 lists the trading arrangements in all the types of trade matches. We will describe in details about how x's and q's are determined in section 2.3.

Besides the decentralized goods market, there is also a currency market where households can exchange currencies with each other. In contrast to the goods market, the currency market is centralized and frictionless and a nominal exchange rate clears the market in equilibrium. Note that the goods market and the currency market are separated from each other, and there is no possibility of arbitrage between the two market.

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[12]The assumption of buyers' taking all the bargaining power is not necessarily needed to obtain the main results. In fact, since the deviations from the LOP are the driven force of the fluctuations in exchange rates, the model can generate even larger volatility of exchange rates if sellers obtain some market power.

2.2. Aggregate Shocks and Timing of Events

Two aggregate shocks are considered in this economy: One to monetary growth rates and the other to productivity. Money is injected into the economy at the beginning of each period. Let $M_{i,t}$ denote the aggregate stock of currency i per household in period t. Each household in country i receives a lump-sum transfer of domestic currency $(\gamma_{i,t+1} - 1) M_{i,t}$ at the beginning of period $t+1$. Therefore, $\gamma_{i,t+1} = M_{i,t+1}/M_{i,t}$ is the gross growth rate of currency i between period t and $t + 1$. The money growth rate $\gamma_{i,t+1}$ follows a stochastic process which we will describe later. Productivity shocks are captured by the disutility function of production. By producing the amount of q of goods, each household suffers a cost of $A_i q^\sigma$, measured in terms of utility. σ is greater than 1 so that the cost function is convex. A_i is a stochastic variable that represents the deviation of innovations to productivity in country i.[13]

The events in an arbitrary period t unfold as in Figure 2.1, with the subscript t suppressed. At the beginning of period t, the two aggregate shocks are realized and each household receives a lump-sum monetary transfer of only domestic currency. Then, aggregate money stock, M_i, and each household's money holdings are measured. Denote the money holdings of currency k in country i household as m_{ik}.[14]

We assume the currency market opens before the goods market.[15] A country i household sells f_{ii} unit of domestic currency for foreign currency at an exchange rate e in the currency market. After the currency market closes, the household divides its buyers into two groups and allocates domestic and foreign currencies to these two groups, respectively. In particular, in a country i household, there are a number of n_i buyers holding domestic currency and a number of $(1 - n_i)$ buyers holding foreign currency. Meanwhile, the household also determines the trading arrangement, (q, x), for his buyers to offer in the trade matches. After these decisions are made, all the buyers and sellers are sent to the goods market. Matches are formed and exchanges then follow. The agents carry out trades according to the rules described by their households. After

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[13]Labor supply is assumed to be elastic. The disutility of production can be derived from disutility of labor and the production function. Suppose production function takes the form of $Q = zF(l)$, where z captures technology shocks. Therefore, to produce quantity Q of goods, the labor input required is $l = F^{-1}(Q/z)$. Substituting it into the function of disutility of labor $c(l)$, we obtain the disutility of production. A denotes the part that involves z and hence represents the technology shocks in our model.

[14]We are interested in the stationary equilibrium. To make the economy stationary, we deflate all the nominal variables $(m_{ik}, x_{ik}, x_{ik}^f, e, f_{ii})$ with the level of corresponding money stock. For example, m_{ik} is normalized by M_k and represents the relative measure of money holdings across countries.

[15]This assumption is just to simplify the algebra. As long as the two markets are separated and agents cannot arbitrage between the markets, the sequence of market openning does not affect the main results.

Shocks are t realized	Money holdings are measured	Currency market opens	Household's decision	Goods markets open	Markets close	$t+1$
Receiving monetary transfer	(m_{i1}, m_{i2})	Sell f_{ii}	n_i , (x_{ik}, q_{ik}) (x_{ik}^f, q_{ik}^f)	Matches and exchanges	Household allocates consumption goods	
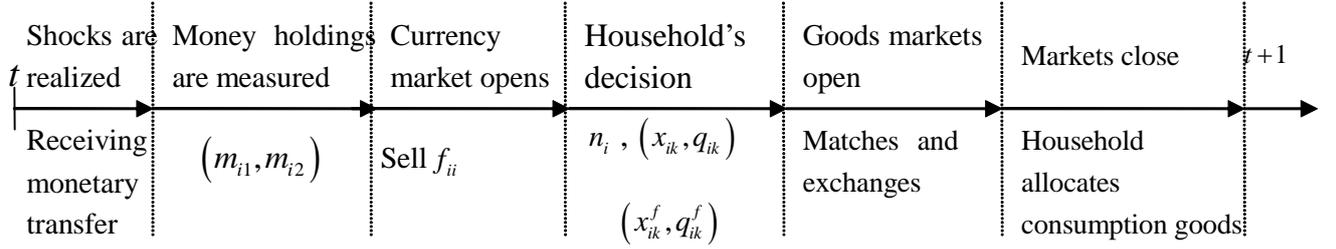
Figure 2.1: Timing of Events

matches and exchanges are completed, all buyers and sellers return home and each household pools the receipts from trade, sharing the goods among all the members to consume until the next period arrives.

2.3. Trading Arrangement in the Goods Market

Before describing a household's trading arrangement in the goods market, we first need to define four variables that represent a household's valuations of the two currencies. Let $V(m_{i1}, m_{i2})$ be the value function of a representative household in country i. Define:[16]

$$\omega_{ik} = E\left[\frac{\beta}{\gamma_{k,+1}} V_k\left(m_{i1,+1}, m_{i2,+1}\right)\right], \ k = 1, 2.$$

where V_k denotes the derivative of the value function with respect to the kth argument. Expectations in these equations are conditional on the information available in the period following the realization of the shocks. By this definition, ω_{ik} refers to a country i household's marginal value of money holdings of currency k in the next period, discounted to the current period.[17] We can regard ω_{ik} as country i household's valuation of money holdings of currency k in current period.

Consider a trade match that involves a country i buyer holding currency k. As we described in the previous section, the buyer proposes (x_{ik}, q_{ik}) when meeting with a domestic seller and

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[16]To simplify notation, in this paper, we suppress the time subscript t and shorten the time subscript $t \pm t'$ to $\pm t'$, where $t' > 0$.

[17]Note that the next period value of money is discounted by the money growth rate, $\gamma_{k,+1}$, as well as by β, because we normalize the money holdings m_{ik} by the aggregate money stock M_k.

$\left(x_{ik}^f, q_{ik}^f\right)$ with a foreign seller. Since the buyer obtains all the bargaining power in a trade match, the trading arrangement (x, q) is set in the way that exhausts all the seller's surplus. For example, a country i seller receives x_{ik} unit of currency k in the trade match, whose value to the seller's household is $\Omega_{ik} x_{ik}$.[18] Meanwhile, the seller has to produce q_{ik} unit of goods and suffers a cost of $A_i (q_{ik})^\sigma$. Thus, the surplus to a country i seller in the trade match is captured by the term $[\Omega_{ik} x_{ik} - A_i (q_{ik})^\sigma]$. Similarly, a country i' seller's surplus in the trade match is denoted by the term $\left[\Omega_{i'k} x_{ik}^f - A_{i'} \left(q_{ik}^f\right)^\sigma\right]$. Setting these surpluses to zero, we have

$$\Omega_{ik} x_{ik} - A_i (q_{ik})^\sigma = 0; \tag{2.3}$$

$$\Omega_{i'k} x_{ik}^f - A_{i'} \left(q_{ik}^f\right)^\sigma = 0. \tag{2.4}$$

In fact, these constraints are the participation constraints for sellers, which prohibit buyers from proposing offers in the trade matches that leave sellers worse off than if they were not to trade. Otherwise, sellers will refuse to produce.

Moreover, all the buyers in the goods market are temporarily separated in the sense that it is impossible for buyers to borrow money across matches. Therefore, the payment in a trade match, x, should not exceed the amount of money that a buyer carries into the trade. After currency exchange, a country i household holds $(m_{ii} - f_{ii})$ unit of domestic money and $\left(m_{ii'} + e^{i'-i} f_{ii}\right)$ unit of foreign money. As we described in the previous section, the household allocates domestic currency to a number of n_i buyers and foreign currency to the other buyers. Then, in a country i household, each buyer either holds $(m_{ii} - f_{ii})/n_i$ of domestic money, or holds $\left(m_{ii'} + e^{i'-i} f_{ii}\right)/(1 - n_i)$ foreign money. Therefore, the following money-goods constraints must be satisfied in a trade match,

$$\frac{m_{ii} - f_{ii}}{n_i} \geq x_{ii}; \frac{m_{ii} - f_{ii}}{n_i} \geq x_{ii}^f; \tag{2.5}$$

$$\frac{m_{ii'} + e^{i'-i} f_{ii}}{(1 - n_i)} \geq x_{ii'}; \frac{m_{ii'} + e^{i'-i} f_{ii}}{(1 - n_i)} \geq x_{ii'}^f. \tag{2.6}$$

Sellers in country i will agree to trade if their trading partners offer (X_{ik}, Q_{ik}) or $\left(X_{ik}^f, Q_{ik}^f\right)$ which follows the similar constraints as (2.3) and (2.4).

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[18]In this chapter, all the lowercase variables denote individual variables, which can be chosen by a household when making the decisions, and all the uppercase variables denote aggregate variables, which are determined by the representative household and taken as given when the household makes the decisions.

2.4. Household's Decision Problem

Pick an arbitrary household in country i as the representative household. In this section, we use dynamic programming to formulate the representative household's decision problem. In each period, the endogenous state variables for the representative household are current period money holdings $(m_{ik})_{k=1,2}$, and the choice variables are $h_i = \left[n_i, f_{ii}, \left(x_{ik}, q_{ik}, x_{ik}^f, q_{ik}^f, m_{ik,+1}\right)_{k=1,2}\right]$. Taking the aggregate variables as given, the representative household in country i faces the following maximization problem:

$$(PH) \qquad V\left(m_{i1}, m_{i2}\right) = \max\left\{u\left(\mathcal{C}_i\right) - \mathcal{P}_i + \beta E\left[V\left(m_{i1,+1}, m_{i2,+1}\right)\right]\right\}.$$

The constraints are (2.3)-(2.6) and the following conditions:

$$\mathcal{C}_i = \left[\rho^{1/\eta} c_{id}^{(\eta-1)/\eta} + (1-\rho)^{1/\eta} c_{if}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}; \tag{2.7}$$

$$c_{id} = n_i \frac{T_{ii}^i}{N_i} q_{ii} + (1-n_i) \frac{T_{ii}^{i'}}{(1-N_i)} q_{ii'}; \tag{2.8}$$

$$c_{if} = n_i \frac{T_{ii'}^i}{N_i} q_{ii}^f + (1-n_i) \frac{T_{ii'}^{i'}}{(1-N_i)} q_{ii'}^f; \tag{2.9}$$

$$\mathcal{P}_i = \left[T_{ii}^i A_i \left(Q_{ii}\right)^\sigma + T_{ii}^{i'} A_i \left(Q_{ii'}\right)^\sigma\right] + \left[T_{i'i}^i A_i \left(Q_{i'i}^f\right)^\sigma + T_{i'i}^{i'} A_i \left(Q_{i'i'}^f\right)^\sigma\right]; \tag{2.10}$$

$$\gamma_{i,+1} m_{ii,+1} = m_{ii} - f_{ii} - n_i \left[\frac{T_{ii}^i}{N_i} x_{ii} + \frac{T_{ii'}^i}{N_i} x_{ii}^f\right] + \left[T_{ii}^i X_{ii} + T_{i'i}^i X_{i'i}^f\right] + (\gamma_{i,+1} - 1); \tag{2.11}$$

$$\gamma_{i',+1} m_{ii',+1} = m_{ii'} + f_{ii} e^{i'-i} - (1-n_{i1}) \left[\frac{T_{ii}^{i'}}{(1-N_i)} x_{ii'} + \frac{T_{ii'}^{i'}}{(1-N_i)} x_{ii'}^f\right] + \left(T_{ii}^{i'} X_{ii'} + T_{i'i}^{i'} X_{i'i'}^f\right). \tag{2.12}$$

The constraints (2.3)-(2.6) come from the earlier discussion on the household's trading arrangement. The expected consumption of a country i household is given by constraints (2.7)-(2.9). As described earlier, $\mathcal{C}_i$ is consumption indices of local goods, c_{id}, and imported goods, c_{if}. Note that both consumption of local goods and imported goods consist of two parts because goods can be purchased with both currencies. The first terms on the right-hand side of (2.8) and (2.9) are the expected amount of goods purchased with domestic currency and the second terms are the expected amount of goods purchased with foreign currency.

The household's disutility of production is given by constraint (2.10). The terms in the first bracket are the costs incurred in trade matches with domestic buyers and the terms in the second

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bracket are the costs incurred in trade matches with foreign buyers. There are two terms inside each set of brackets because each seller has the positive possibility of trading with a buyer holding either currency. Note that here the quantities of goods produced are aggregate variables, since prices are posted by buyers and the household takes it as given when making the decisions.

The last two constraints are the laws of motion of the household's money holdings. We explain the first one here.[19] The left-hand side of the constraint (2.11) is the money holdings of currency i in the next period, where $\gamma_{i,+1}$ appears here because the money holdings are normalized by current period money stock M_i. The right-hand side describes the changes in the money holdings in the current period, which comes from the selling in the currency market (f_{ii}), the buying and selling in the goods market (the two terms with bracket)[20] and monetary transfer at the beginning of the next period ($\gamma_{i,+1} - 1$). The laws of motion of money holdings describe the changes in the household's money holdings between two adjacent periods.

2.5. Equilibrium

Note that all of the money constraints (2.5) and (2.6) hold with equality, provided that ω_{ik} are positive. Let λ_{ik} and λ_{ik}^f be the shadow price of the money constraints that contain x_{ik} and x_{ik}^f. These shadow prices imply the non-pecuniary returns of the currencies that come from their role in relaxing trading restrictions. The solution to the household's problem (PH) is characterized by the following conditions, in addition to the binding money constrains (2.3)-(2.6), and (2.7)-(2.9):

$$\rho^{1/\eta} u'\left(\mathcal{C}_i\right) \mathcal{C}_i^{\frac{1}{\eta}} c_{id}^{-\frac{1}{\eta}} = \left(\lambda_{ik} + \omega_{ik}\right) \frac{A_i \sigma \left(q_{ik}\right)^{\sigma-1}}{\Omega_{ik}}; \tag{2.13}$$

$$(1-\rho)^{1/\eta} u'\left(\mathcal{C}_i\right) \mathcal{C}_i^{\frac{1}{\eta}} c_{if}^{-\frac{1}{\eta}} = \left(\lambda_{ik}^f + \omega_{ik}\right) \frac{A_{i'} \sigma \left(q_{ik}^f\right)^{\sigma-1}}{\Omega_{i'k}}; \tag{2.14}$$

$$\rho^{1/\eta} c_{id}^{-\frac{1}{\eta}} \frac{T_{ii}^i}{N_i} q_{ii} + (1-\rho)^{1/\eta} c_{if}^{-\frac{1}{\eta}} \frac{T_{ii'}^i}{N_i} q_{ii}^f = \rho^{1/\eta} c_{id}^{-\frac{1}{\eta}} \frac{T_{ii}^{i'}}{(1-N_i)} q_{ii'} + (1-\rho)^{1/\eta} c_{if}^{-\frac{1}{\eta}} \frac{T_{ii'}^{i'}}{(1-N_i)} q_{ii'}^f; \tag{2.15}$$

$$\omega_{11} + \frac{T_{11}^1}{N_1} \lambda_{11} + \frac{T_{12}^1}{N_1} \lambda_{11}^f = e \left[\omega_{12} + \frac{T_{11}^2}{(1-N_1)} \lambda_{12} + \frac{T_{12}^2}{(1-N_1)} \lambda_{12}^f\right]; \tag{2.16}$$

$$\omega_{21} + \frac{T_{22}^1}{(1-N_2)} \lambda_{21} + \frac{T_{21}^1}{(1-N_2)} \lambda_{21}^f = e \left[\omega_{22} + \frac{T_{22}^2}{N_2} \lambda_{22} + \frac{T_{21}^2}{N_2} \lambda_{22}^f\right]; \tag{2.17}$$

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[19]The explanation for the second one is similar, except that the household does not receive the monetary transfer of foreign currency.

[20]Note the difference between the buying (choice varibles) and selling (aggregate varibles).

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$$\omega_{ii} = E\left[\frac{\beta}{\gamma_{i,+1}}\left(\omega_{ii,+1} + \frac{T^i_{ii,+1}}{N_{i,+1}}\lambda_{ii,+1} + \frac{T^i_{ii',+1}}{N_{i,+1}}\lambda^f_{ii,+1}\right)\right]; \tag{2.18}$$

$$\omega_{ii'} = E\left[\frac{\beta}{\gamma_{i',+1}}\left(\omega_{ii',+1} + \frac{T^{i'}_{ii,+1}}{(1-N_{i,+1})}\lambda_{ii',+1} + \frac{T^{i'}_{ii',+1}}{(1-N_{i,+1})}\lambda^f_{ii',+1}\right)\right]. \tag{2.19}$$

Constraints (2.13) and (2.14) prescribe the household's optimal choices of trading quantities (x_{ik}, q_{ik}) and $\left(x^f_{ik}, q^f_{ik}\right)$, respectively. These conditions require that the marginal utility of consumption (the left-hand side of the equations) must be equal to the marginal value of corresponding currencies spent on purchasing the consumption goods (the right-hand side of the equations).[21] Note that the cost of giving up one unit of currency k is not fully measured by a household's valuation of this amount of currency, ω_{ik}. By losing one unit of currency k, the buyer also gives up the non-pecuniary return of holding the currency k, which is captured by λ_{ik} and λ^f_{ik}.

The optimal allocation of money holders, n_i, has to satisfy the condition (2.15). Suppose the household decides to allocate one additional buyer to carry domestic currency. There is a possibility of T^i_{ii}/N_i for this buyer to purchase from a domestic seller, thereby increasing the household's expected utility by $\rho^{1/\eta}c_{id}^{-\frac{1}{\eta}}q_{ii}T^i_{ii}/N_i$. The possibility of trading with a foreign seller for him is $T^i_{ii'}/N_i$, which increases the household's expected utility by $(1-\rho)^{1/\eta}c_{if}^{-\frac{1}{\eta}}q^f_{ii}T^i_{ii'}/N_i$. Thus, the left-hand side of the equation is the expected gain due to allocating an additional buyer carrying domestic currency. Similarly, the right-hand side can be interpreted as the expected loss due to the removal of one buyer carrying foreign currency. The optimal value of n_i must equalize the expected marginal benefit and cost.

Conditions (2.16) and (2.17) are the optimal choices of currency exchange of the households in country 1 and 2, f_{ii}, respectively. These two conditions are similar and we explain the first one here. A household in country 1 can sell one unit of domestic currency for e unit of foreign currency. By reducing one unit of domestic currency, the household not only loses the expected future value of currency, ω_{11}, but also the expected non-pecuniary returns of the currency in goods exchange, $\left(T^1_{11}/N_1\right)\lambda_{11} + \left(T^1_{12}/N_1\right)\lambda^f_{11}$. On the other hand, the increased e units of foreign currency increase household's expected utility by its own value, $\omega_{12}e$, as well as the expected non-pecuniary returns of the foreign currency, $\left\{\left[T^2_{11}/(1-N_1)\right]\lambda_{12} + \left[T^2_{12}/(1-N_1)\right]\lambda^f_{12}\right\}e$. In

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[21]According to (2.3), to get one unit of consumption goods, the amount of $A_i\sigma\left(q_{ik}\right)^{\sigma-1}/\Omega_{ik}$ of currency k is needed for the purchase.

equilibrium, the marginal costs of selling currencies should be equal to the marginal benefits the exchanged currencies bring to the household.

The last two conditions, (2.18) and (2.19), are the envelop conditions. Note that we move the time index of the two conditions forward by one period. The expectations are conditional on the information available in the period following the realization of the shocks. Since the two envelop conditions have similar interpretations, we only explain (2.18) here. The left-hand side of (2.18) is the marginal value of currency i to a country i household. The right-hand side is the expected value of the currency in the next period, plus the non-pecuniary return of holding the currency in the next period, discounted to the current period. Envelop conditions maintain that the expected return of money should be equal to the nominal interest rate.

Now we define a stationary and symmetric equilibrium for this economy as follows:

Definition 2.1. *An equilibrium in this economy consists of each household's choice variables h_i and other household's choices H_i, where $h_i = \left[n_i, f_{ii}, \left(x_{ik}, q_{ik}, x_{ik}^f, q_{ik}^f, m_{ik,+1} \right)_{k=1,2} \right]$, and a nominal exchange rate e such that, for any given initial state $(m_{ik,0})_{i,k=1,2}$ and the exogenous shock processes, the following requirements are satisfied: (i) optimality: h_i solves country i household's decision problem (PH) given H_i and (m_{i1}, m_{i2}), $i = 1, 2$; (ii) symmetry: $h_i = H_i$; (iii) the currency market clears: $f_{22} = ef_{11}$; (iv) money holdings add up: $m_{1k} + m_{2k} = 1$, $k = 1, 2$; and (v) the value of $(\omega_{1k}m_{1k} + \omega_{2k}m_{2k})_{k=1,2}$ lies in $(0, \infty)$.*

3. Linearized Dynamic System

3.1. Solution technique

The whole dynamic system of the solution to the model in the previous section consists of 32 equations, five of which involve period $t+1$ variables.[22] To solve the model, we first compute the non-stochastic symmetric steady state. Then, log-linearize around the steady state and derive a system of linear difference equations. Let lower case letters with a hat represent log deviations from an initial steady state, i.e., $\hat{y} = \ln(Y) - \ln\left(\overline{Y}\right)$. We choose to keep two predetermined variables $(\widehat{m}_{11}, \widehat{m}_{12})$ and three jump variables $(\widehat{\omega}_{11}, \widehat{\omega}_{12}, \widehat{\omega}_{22})$, eliminating all the other variables using those 27 equations that contain only current period variables. Then, we obtain a much

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[22] In fact, there are six equations that involve next period variables: the four envelop conditions as well as the two laws of motion of money holdings. However, one of the envelop conditions is redundant and we can derive a linear relationship between $\widehat{\omega}_{ik}$: $\widehat{\omega}_{11} - \widehat{\omega}_{12} = \widehat{\omega}_{21} - \widehat{\omega}_{22}$.

simpler dynamic system that only consists of those five choice variables $(\widehat{m}_{11}, \widehat{m}_{12}, \widehat{\omega}_{11}, \widehat{\omega}_{12}, \widehat{\omega}_{22})$ and four exogenous state variables. Now, the model can be easily solved with the method similar to Blanchard and Khan's (1980). The detailed solution method of the model is described in the Appendix B.. Here we limit the discussion to some key equations.

3.2. Differentials in the Valuation of the Two Currencies

Combining the equilibrium conditions of currency exchange, (2.16) and (2.17), with envelop conditions, (2.18) and (2.19), we can easily derive the relationship between a household's valuations of currencies, $\widehat{\omega}_{ik}$. The following Lemma describes this relationship:

Lemma 3.1. *In an equilibrium, the changes in a household's valuations of currencies must satisfy the following conditions for $i = 1, 2$:*

$$\widehat{\omega}_{i1} - \widehat{\omega}_{i2} = E\left[\widehat{e}_{+1} - (\widehat{\gamma}_{1,+1} - \widehat{\gamma}_{2,+1})\right]. \tag{3.1}$$

$(\widehat{\omega}_{i1} - \widehat{\omega}_{i2})$ is the differential in country i household's valuations of the two currencies. As we will discuss later, this variable is important to the interpretation of our results. We use θ to represent this differential, i.e.,

$$\theta \equiv \widehat{\omega}_{11} - \widehat{\omega}_{12} = \widehat{\omega}_{21} - \widehat{\omega}_{22}.$$

This lemma shows that although households in different countries value the two currencies at different worths, the relative valuations of the two currencies are the same in the two countries. Moreover, the differential in the valuations depends on the expected change in the next period nominal exchange rate and the differential in the two currencies' growth rates. In particular, θ is increasing in $\widehat{e}_{+1}$ and decreasing in $(\widehat{\gamma}_{1,+1} - \widehat{\gamma}_{2,+1})$. This result is quite intuitive: an increase in the nominal exchange rate means more currency 2 can be exchanged by currency 1 and, hence, the value of currency 1 to a household increases. On the other hand, a higher inflation rate of a currency reduces the purchasing power of that currency. Thus, if a currency has a higher growth rate, its value to a household is lower compared to the value of the other currency.

Note that (3.1) also implies that the cross-country differentials in the valuations of currencies are the same, i.e., $\widehat{\omega}_{11} - \widehat{\omega}_{21} = \widehat{\omega}_{12} - \widehat{\omega}_{22}$. Thus, if a country values a currency higher than the other country does, it values the other currency higher as well. Keeping in mind that money injection is asymmetric and only domestic households receive the increased monetary transfer of domestic

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currency, suppose $\gamma_1 > 1$ and country 1 increases injection of currency 1. There is a potential imbalance in the portfolio of each household's money holdings after receiving the transfer. The households then sell part of the increased transfer for currency 2. As a result, the asymmetric money injection of currency 1 increases country 1 households' relative holdings of both currencies. Accordingly, the country 1 households value both currencies less than the country 2 households do.

The differential in the valuations of the two currencies comes from search frictions in the goods market. The centralized currency market only equalizes the relative valuation of currencies in the two countries. Arbitrage in the currency market cannot eliminate the difference in the absolute values of currencies. The gap between valuations of the two currencies does not disappear unless households can trade goods directly for currencies in the currency market. However, transactions in the goods market are formed in bilateral matches and there are different prices for different types of matches. There is no way to arbitrage between the goods market and the currency market to eliminate this differential in valuations of currencies.

3.3. Quantity Differentials across Matches

As we have described, there are eight different types of trade matches in the goods market. Since agents from different countries have different valuations of each currency, the quantity traded in each type of trade match is not the same. However, the centralized currency market equalizes the relative valuation of currencies across countries and as a result, there are certain rules the trading quantities in different types of matches follow. The following lemma summarizes the findings:

Lemma 3.2. *In an equilibrium, the quantities of goods exchanged in the goods market satisfy the following conditions for $i = 1, 2$ and $k = 1, 2$:*

$$\widehat{q}_{1k} - \widehat{q}_{1k}^f = \widehat{q}_{2k}^f - \widehat{q}_{2k} = \frac{1}{\sigma}\left(\widehat{\omega}_{11} - \widehat{\omega}_{21}\right) - \frac{1}{\sigma}\widehat{a}_1 + \frac{1}{\sigma}\widehat{a}_2; \tag{3.2}$$

$$\widehat{q}_{i1} - \widehat{q}_{i2} = \widehat{q}_{i1}^f - \widehat{q}_{i2}^f = \frac{1-\psi}{1+\sigma\left(1-\psi\right)}\theta. \tag{3.3}$$

(3.2) is derived from the binding money constraints. It says that a buyer receives different quantities of goods when trading with sellers from different countries, even though the buyer pays the same amount of currency. This is easy to understand since agents from different countries have different valuations of the currencies. For example, if a country 1 household values both

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currencies higher than a country 2 household does, sellers from country 1 will provide more goods than sellers from country 2 when dealing with the same buyer. Moreover, this quantity differential is always the same regardless of which country the buyer is from or which currency is used in the trade. This result is derived from the fact that the quantity differential depends on the differential in the two countries' valuations of currencies and that the cross-country valuations of the currency are equal.

Now, consider two trade matches that involve the same buyer and seller but different types of currencies. Since households value the two currencies at different values, even in a trade with the same seller, the quantities of goods a buyer obtains when paying with different types of currency are not the same. (3.3) states that the quantity differential between such trade matches is always the same and relies on the relative valuation of the two currencies, θ. The higher the relative value of a currency, the more goods a buyer can purchase in a trade with that currency compared to using the other currency.

Note that search frictions in the goods market generate households' different valuations of currencies, and are responsible for the quantity differentials across trade matches. The quantity differentials imply price differentials and the deviations from the LOP. We will later discuss how this is significant for explaining the fluctuations in exchange rates.

3.4. Division of Money Holders and the Nominal Exchange Rate

The nominal exchange rate is determinate in our model. As we described, in each period, households divide their buyers into two groups and allocate money stocks of the two currencies to the two groups, respectively. The division of money holders generates a relative valuation of the two currencies to the households in each country, and this relative valuation of currencies determines the nominal exchange rate.

Log-linearizing the condition for the optimal division of money holders, (2.15), and rearranging it, we get

$$2\left(1+\mu_1\right)\left(1-\psi\right)\widehat{n}_i = \widehat{q}_{ii} - \widehat{q}_{ii'} + \mu_1\left(\widehat{q}_{ii}^f - \widehat{q}_{ii'}^f\right), \quad \mu_1 = \alpha^{1+\frac{1}{\eta}}\left(\frac{1-\rho}{\rho}\right)^{1/\eta}. \tag{3.4}$$

Therefore, the optimal decision on division of money holders relies on the quantity differential that a buyer can purchase with different types of currencies. According to lemma 3.2, the quantity differentials are proportional to θ. Therefore, we immediately obtain the following proposition:

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Proposition 3.3. *The changes in the division of money holders in a country i household, $\widehat{n}_i$, only depend on the relative valuation of the two currencies, and*

$$\widehat{n}_1 = -\widehat{n}_2 = \frac{1}{2\left[1 + \sigma\left(1 - \psi\right)\right]}\theta. \tag{3.5}$$

$\widehat{n}_i$ denotes the change in the number of buyers holding domestic currency in country i. Proposition 3.3 determines that the changes in the number of buyers who carry the same currency are exactly the same in the two countries. This is not a surprising result since the allocation of money holders depends on a household's relative valuation of the two currencies, and the relative valuations are equalized across countries. Note that $\widehat{n}_1$ responds positively to the differential in the valuations of the two currencies. The higher a household values a currency, the more buyers are assigned to hold the currency.

The nominal exchange rate is determined by the relative role of the two currencies in the trade. Log-linearizing (2.16) and rearranging it, we obtain,[23]

$$\frac{\overline{\gamma}}{\beta}\widehat{e} = (\widehat{\omega}_{11} - \widehat{\omega}_{12}) + \mu_0\left[\frac{\overline{\lambda}}{\overline{\Omega}}\left(\widehat{\lambda}_{11} - \widehat{\lambda}_{12}\right) + \frac{\alpha\overline{\lambda}^f}{\overline{\Omega}}\left(\widehat{\lambda}_{11}^f - \widehat{\lambda}_{12}^f\right)\right] - \frac{2\mu_0\left(1 - \psi\right)\left(\overline{\lambda} + \alpha\overline{\lambda}^f\right)}{\overline{\Omega}}\widehat{n}_1, \tag{3.6}$$

where μ_0 is the steady-state value of matching rate of a buyer meeting with domestic sellers.

The changes in the nominal exchange rate come from three sources: i) changes in the relative value of the two currencies; ii) changes in relative non-pecuniary returns of the two currencies; and iii) changes in the division of money holders. The first two sources are easy to understand since the nominal exchange rate is the relative price of the two currencies. Thus, the changes in the nominal exchange rate should reflect the changes in the relative valuations, as well as the non-pecuniary returns of holding the two currencies. The effect of $\widehat{n}_1$ on $\widehat{e}$, however, is less obvious. Changes in the number of money holders affect the matching rate of buyers, as well as the total number of trade matches. Thus, it changes expected returns of currencies in the trades and then has an influence on the nominal exchange rate.

We can show that the differentials in non-pecuniary returns to the two currencies depend on the relative valuation of the two currencies, as well as quantity differentials in trades with different currencies:

$$\widehat{\lambda}_{11} - \widehat{\lambda}_{12} = (\widehat{\omega}_{11} - \widehat{\omega}_{12}) - (\sigma - 1)\frac{\left(\overline{\lambda} + \overline{\Omega}\right)}{\overline{\lambda}}\left(\widehat{q}_{11} - \widehat{q}_{12}\right);$$

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[23]In the paper, a varible with a bar represents the steady state value of this variable.

$$\widehat{\lambda}_{11}^f - \widehat{\lambda}_{12}^f = (\widehat{\omega}_{11} - \widehat{\omega}_{12}) - (\sigma - 1) \frac{\left(\overline{\lambda}^f + \overline{\Omega}\right)}{\overline{\lambda}^f} \left(\widehat{q}_{11}^f - \widehat{q}_{12}^f\right).$$

Since the changes in both the quantity differentials and the division of money holders depend only on the relative valuation of the two currencies, θ, equilibrium condition (3.6) implies that the change in the nominal exchange rate also relies solely on θ.

Proposition 3.4. *The changes in the nominal exchange rate come entirely from the differential in a household's valuations of the two currencies, and*

$$\widehat{e} = \frac{\overline{\gamma} + \beta \left(1 - \psi\right) \left[\sigma - \mu_0 \left(1 + \alpha\right) \left(\sigma - 1\right)\right]}{\overline{\gamma} + \sigma \overline{\gamma} \left(1 - \psi\right)} \theta. \tag{3.7}$$

Note that lemma 3.1 states that θ depends on $E\left[\widehat{e}_{+1}\right]$ and $E\left[\left(\widehat{\gamma}_{1,+1} - \widehat{\gamma}_{2,+1}\right)\right]$. Thus, we can derive the evolution rule of θ as,

$$\theta = \mu_2 E\left[\theta_{+1}\right] - E\left[\left(\widehat{\gamma}_{1,+1} - \widehat{\gamma}_{2,+1}\right)\right], \tag{3.8}$$

where $\mu_2 \equiv \frac{\overline{\gamma} + \beta(1-\psi)[\sigma - \mu_0(1+\alpha)(\sigma-1)]}{\overline{\gamma} + \sigma \overline{\gamma}(1-\psi)}$.

The formation of θ only involves the differential in the two countries' money growth rates.[24] This result is important for the interpretation of the relative role of monetary and real shocks which we will discussed later. Thus, the difference between the two countries' money growth rates is the only factor that affects the fluctuations of the nominal exchange rate in our model. A higher growth rate of a currency causes a lower valuation of this currency by households, thereby resulting the depreciation of the currency.

3.5. Net Currency Trades and Relative Money Holdings across Countries

The net amount of currency exchange and the relative money holdings per household across countries are two important factors affecting the price levels in the goods market. In this subsection, we examine how these variables respond to shocks in the model.

The following condition for net currency trade, $\widehat{f}_{11}$, can be derived from the binding money constraints:

$$2\widehat{f}_{11} = \frac{1}{(\overline{\gamma} - 1)} \left[(2\overline{\gamma} - 1) \widehat{m}_{11} - \widehat{m}_{12}\right] - \widehat{e}. \tag{3.9}$$

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[24]Note that this result depends crucially on the assumption that goods can be purchased by either currency. If assuming domestic buyers only use domestic currency to buy local goods, then real shocks will affect θ too.

Thus, the changes in net currency exchange depend on the changes in the relative money holdings of the two currencies, $\widehat{m}_{11}$ and $\widehat{m}_{12}$, as well as the changes in the nominal exchange rate, $\widehat{e}$. We have shown in the previous subsections that the changes in the nominal exchange rate depend exclusively on the differential in valuations of the two currencies, θ, which is driven entirely by the difference between the two currencies' gross growth rates. Therefore, equation (3.9) suggests that the currency trade is affected by the relative money holdings across countries and the money growth rate differential across countries.

Log-linearizing the two laws of motion of money holdings, (2.11) and (2.12), together with the quantity differentials and equations for $\widehat{n}_1$ (3.5), we can rewrite the two equations as,

$$\overline{\gamma}\widehat{m}_{11,+1} = (1 - 2\alpha\mu_0)\,\widehat{m}_{11} - (1 - 2\alpha\mu_0)\,\frac{(\overline{\gamma}-1)}{(2\overline{\gamma}-1)}\,\widehat{f}_{11} + \frac{\overline{\gamma}}{(2\overline{\gamma}-1)}\widehat{\gamma}_{1,+1}; \tag{3.10}$$

$$\overline{\gamma}\widehat{m}_{12,+1} = (1 - 2\alpha\mu_0)\,\widehat{m}_{12} + (1 - 2\alpha\mu_0)\,(\overline{\gamma}-1)\left(\widehat{f}_{11} + \widehat{e}\right) - \overline{\gamma}\widehat{\gamma}_{2,+1}. \tag{3.11}$$

These two equations describe the evolution rules for the relative money holdings of the two currencies. Note that $\widehat{f}_{11}$ is a function of $\widehat{m}_{11}$, $\widehat{m}_{12}$, and θ. Thus, the next period money holdings depend on the current period money holdings, $\widehat{m}_{11}$ and $\widehat{m}_{12}$, the relative valuation of currencies, θ, and the exogenous money growth shocks, $\widehat{\gamma}_{1,+1}$ and $\widehat{\gamma}_{2,+1}$. According to (3.8), the formation of θ only involves the differential in the growth rates of the two currencies. Therefore, the two laws of motion of money holdings imply that the relative money holdings of the two currencies can only be affected by monetary shocks. Real shocks play no role in the evolutionary path of the relative money holdings in our model.

Note that $\widehat{m}_{11}$ and $\widehat{m}_{12}$ are the measure of money holdings of a country 1 household at the beginning of each period. The actual amount of money brought into the goods market is the amount of currency a household has after the currency trades. We call the amount of money an individual buyer carries into the goods market *effective money holdings* and denote it as m_{ik}^e. For example, $m_{11}^e = (m_{11} - f_{11})/n_1$ and $m_{12}^e = (m_{12} + ef_{11})/(1 - n_1)$. The effective money holdings of currencies are what really matter for the price levels in the trade matches.

Let $\widehat{m}_k^e = \widehat{m}_{1k}^e - \widehat{m}_{2k}^e$ denote the changes in the relative effective money holdings of currency k across countries. We can show that:

$$\widehat{m}^e \equiv \widehat{m}_k^e = \frac{(2\overline{\gamma}-1)}{\overline{\gamma}}\widehat{m}_{11} + \frac{1}{\overline{\gamma}}\widehat{m}_{12} + \frac{(\overline{\gamma}-1)\,\mu_2}{\overline{\gamma}}\theta;$$

Therefore, the changes in relative effective money holdings across countries are the same for both currencies. If, after the currency trade, a household in one country holds more currency 1 than a household in the other country does, the household in the first country also holds more currency 2.

Combining the two laws of motion of money holdings and using equation (3.1), we obtain the evolutionary rule for the relative effective money holdings across countries:

$$\overline{\gamma}\widehat{m}^e_{+1} = (1 - 2\alpha\mu_0)\,\widehat{m}^e + (\overline{\gamma} - 1)\,\theta + \overline{\gamma}\,(\widehat{\gamma}_{1,+1} - \widehat{\gamma}_{2,+1})\,. \tag{3.12}$$

Since θ is driven entirely by the growth rate differential of the two currencies, $(\widehat{\gamma}_1 - \widehat{\gamma}_2)$, the above equation implies that the changes in relative effective money holdings across countries, $\widehat{m}^e$, can only be affected by the difference between the two countries' money growth rates.

3.6. Price Differentials and Deviations from the Law of One Price

Consider a country i buyer who is carrying currency k. (X_{ik}, Q_{ik}) is the trading arrangement in a trade match with a domestic seller, and $\left(X^f_{ik}, Q^f_{ik}\right)$ is the trading arrangement in a trade match with a foreign seller. Thus, the prices implied in these trade matches are:

$$P_{ik} = \frac{X_{ik}}{Q_{ik}};\ \ P^f_{ik} = \frac{X^f_{ik}}{Q^f_{ik}}.$$

For the goods produced by a country i household, there are four market prices: P_{ik} is the price faced by domestic buyers carrying currency k and $P^f_{i'k}$ is the price faced by foreign buyers carrying currency k. The quantity differentials we discussed in section 3.3 imply following price differentials.

Corollary 3.5. *If $\gamma_1 \neq \gamma_2$, the four prices of goods produced by country i households, $\widehat{p}_{ik}$ and $\widehat{p}^f_{i'k}$, are not equal to each other. Moreover, the price differentials must satisfy:*

$$\widehat{p}_{i1} - \widehat{p}_{i2} = \widehat{p}^f_{i'1} - \widehat{p}^f_{i'2} = -\frac{2 - \psi}{1 + \sigma\,(1 - \psi)}\theta;$$

$$\widehat{p}_{i1} - \widehat{p}^f_{i'1} = \widehat{p}_{i2} - \widehat{p}^f_{i'2} = \frac{(\sigma - 1)}{\sigma}\widehat{m}^e.$$

This corollary suggests a strong violation of the LOP. Even after converting to the same currency, the same goods are sold at different prices in different trade matches. The differential in valuations of the two currencies, θ, and the relative effective money holdings across countries,

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$\widehat{m}^e$, are the two important components of the price differentials. Note that both θ and $\widehat{m}^e$ appear because of the differential in the two countries' money growth rates. Thus, as long as the money growth rates differ in the two countries, the above price differentials always exist and the LOP is violated.

We can further determine whether the LOP holds at the aggregate price level. Let P_{id} denotes the domestic price of local goods in country i and P_{if} denotes the domestic price of imported goods in country i. Since local (imported) goods can be purchased by either domestic currency at the price P_{ii} (P_{ii}^f) or foreign currency at the price $P_{ii'}$ ($P_{ii'}^f$), we define P_{id} and P_{if} as follows:

$$P_{id} = \upsilon_{id} P_{ii} + (1 - \upsilon_{id}) P_{ii'} e^{i-i'};$$

$$P_{if} = \upsilon_{if} P_{ii}^f + (1 - \upsilon_{if}) P_{ii'}^f e^{i-i'},$$

where $\upsilon_{id} = T_{ii}^i Q_{ii} / \left(T_{ii}^i Q_{ii} + T_{ii}^{i'} Q_{ii'} \right)$ is the consumption share of local goods purchased by domestic money, and $\upsilon_{if} = T_{ii'}^i Q_{ii}^f / \left(T_{ii'}^i Q_{ii}^f + T_{ii'}^{i'} Q_{ii'}^f \right)$ is the consumption share of imported goods purchased by domestic money.

The LOP requires the price of country i's goods sold in country i (P_{id}) be equal to the price sold in country i' ($P_{i'f}$) after converting to the same currency. However, we can show that:

$$\widehat{p}_{id} + \widehat{e} - \widehat{p}_{i'f} = \overline{\upsilon}_{id} \left(\widehat{p}_{i1} - \widehat{p}_{i2} + \widehat{e} \right) + \overline{\upsilon}_{if} \left(\widehat{p}_{i'1}^f - \widehat{p}_{i'2}^f + \widehat{e} \right) + \left(\widehat{p}_{i2} - \widehat{p}_{i'1}^f - \widehat{e} \right);$$

where $\overline{\upsilon}_{id}$ and $\overline{\upsilon}_{if}$ are the steady state values of υ_{id} and υ_{if}, respectively.

The difference between the two countries' price levels of goods i can be broken down into three 3 parts: i) the price differential faced by domestic buyers holding different currencies; ii) the price differential faced by foreign buyers holding different currencies; iii) the price differential faced by buyers from different countries. Search frictions cause market segment and generate differential in valuations of currencies across countries. Price differentials across trade matches always exist since the valuations of currencies to households from different countries are different. The LOP is violated in the model as long as two countries have different money growth rates.[25]

3.7. Real Exchange Rate

The real exchange rate is usually defined as the relative price of the common basket of goods where prices are converted into a common numeraire. According to the CES consumption index,

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[25]There are empirical evidences that support this result. Cheung and Fujii (2008), for example, use 25 years of monthly data on individual retail prices to study the behavior of the behavior of product-specific LOP deviations. They find that the deviations from the LOP are positively related with the inflation rate differentials.

the consumption-based price index P_i in country i is derived as

$$P_i = \left[\rho P_{id}^{1-\eta} + (1-\rho) P_{if}^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where P_{id} is the domestic money price of local goods in country i, and P_{if} is the domestic money price of imported goods in country i.

Thus, the real exchange rate can be defined as

$$RER = \frac{eP_1}{P_2}.$$

We can decompose the changes in the real exchange rate as follows:

$$
\begin{aligned}
\widehat{rer} &= \widehat{e} + \widehat{p}_1 - \widehat{p}_2 \\
&= \rho \left[\widehat{p}_{1d} + \widehat{e} - \widehat{p}_{2f} \right] + (1-\rho) \left[\widehat{p}_{1f} + \widehat{e} - \widehat{p}_{2d} \right] + (2\rho - 1) \left[\widehat{p}_{2f} - \widehat{p}_{2d} \right].
\end{aligned}
$$

The first two terms on the right-hand side of the equation indicate the deviations from the LOP. The last term appears because of the difference in the consumption bundles of the households in the two countries, determined by

$$\widehat{p}_{2f} - \widehat{p}_{2d} = \frac{1}{\sigma} \left(\widehat{\omega}_{22} - \widehat{\omega}_{12} \right) + \frac{1}{\sigma} \widehat{a}_1 - \frac{1}{\sigma} \widehat{a}_2. \tag{3.13}$$

Thus, this price differential comes from the two countries' different valuations of the currency, $\left(\widehat{\omega}_{22} - \widehat{\omega}_{12} \right) / \sigma$, as well as the real disturbances, $\left(\widehat{a}_1 - \widehat{a}_2 \right) / \sigma$.

The decomposition suggests that a large part of variations in the real exchange rate are generated by the search frictions in the goods market. Without search frictions, households value all the currencies at the same level, goods are sold at the same price and the LOP holds. The primary driven force of price differential in imported and local goods disappears, causing all of the variations in the real exchange rate to come merely from the real disturbance.

4. Calibration

This section describes how we choose functional forms and benchmark parameter values. The discount factor, β, is set equal to 0.99 to get an annual real interest rate of 4%. The utility of consumption takes the form of CRRA function,

$$U\left(\mathcal{C}\right) = \frac{1}{1-\epsilon} \mathcal{C}^{1-\epsilon};$$

where ϵ is the coefficient of risk aversion. We set $\epsilon = 2$, which is a standard value in the literature.[26] For the consumption index $\mathcal{C}$ defined as (2.1), the elasticity of substitution between local goods and imported goods is set to 1.5, since empirical studies suggest a value between 1 and 2 for U.S. data. The home bias parameter, ρ, is chosen in such a way that the standard deviation of consumption relative to the standard deviation of output is consistent with data. For our benchmark economy, ρ is chosen to be 0.89.

Then consider the parameters in the matching functions (2.2). In the steady state, $\alpha = \overline{c}_f / \overline{c}_d$, which can be interpreted as the import share in a country. We set $\alpha = 0.15$ in the model to match U.S. data. The constant T_0 indicates the degree of search frictions in the goods market. The lower the value of T_0 is, the lower possibility for an agent to find a trade match. In the computation, we choose the value of T_0 so that the standard deviation of output generated from the model matches the data. The buyer's share in the formation of matches, ψ, however, cannot be identified. We choose $\psi = 0.5$ for the benchmark economy and examine how sensitive the results are to this parameter later.

The ratio of sellers to buyers in a household, s, can be interpreted as the ratio of working time to shopping time. According to Juster and Stafford (1991), the shopping time of the population is 11.17% of the working time, thus, we set $s = 8.9$.

The disutility of production, $A_i Q^\sigma$, is derived from the production function and disutility of labor. We assume that production takes place according to a decreasing returns to scale production function,

$$F(l) = z l^\varphi, 0 < \varphi < 1,$$

where z captures technology shocks and follows a stochastic process. We set $\varphi = 2/3$. Therefore, the labor input required to produce $\mathcal{Q}$ units of goods can be found by inverting the above production function. That is, $l = (\mathcal{Q}/z)^{1/\varphi}$. In addition, the disutility of labor takes the form of the function,

$$c(l) = \phi_0 \frac{l^{1+\phi}}{(1+\phi)},$$

where $\phi_0 > 0$ and $\phi > 0$. According to the recent work by Rogerson and Wallenius (2007), the intertemporal elasticity of substitution in aggregate labor supply is high and therefore we

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[26]In sticky price models such as ones put forth by Chari, Kehoe and McGrattan (2001), a high risk aversion is needed to generate enough volatility for exchange rates. Our model, however, does not need such a requirement.

set $\phi = 0.33$. Then, the cost of producing $\mathcal{Q}$ unit of goods, measured in terms of utility, is $c(\mathcal{Q}) = A\mathcal{Q}^{\sigma}$, where $\sigma = (1 + \phi)/\varphi = 1.995$, and $A = \phi_0/(1 + \phi) z^{-\sigma}$. We normalize the steady state value of A to 1.

A in our model represents technology shocks and obviously it follows

$$\log A_i = -\sigma \log z_i,$$

We assume the stochastic process of $\log z$ obeys the following vector autoregressive (VAR) process:

$$\log z = \Gamma \log z_{-1} + \varepsilon_z, \quad \varepsilon_z \sim N(0, \Sigma),$$

where $z = (z_1, z_2)$ and $\varepsilon_z = (\varepsilon_{z1}, \varepsilon_{z2})$. Following the estimation in most of the international business cycle literature, for example, Backus, Kehoe and Kydland (1992, 1995), the real shocks are very persistent, and the autocorrelation is set equal to 0.95, i.e.,

$$\widehat{\Gamma} = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}.$$

The standard deviation of innovations is set to 0.007. The shocks are positively cross-correlated and the cross-correlation is set to 0.25.

The details of the monetary rules followed in the U.S. and Europe are extensively debated, and there is no a widely-accepted way existed to capture the monetary shock process. To make a comparison of our results with those from sticky price model by Chari, Kehoe, and McGrattan (2003), we follow their paper and assume the growth rates of money stocks in both countries follow the same simple rule in the following form:[27]

$$\log \gamma = \rho_{\gamma} \log \gamma_{-1} + \varepsilon_{\gamma}, \quad \varepsilon_{\gamma} \sim N(0, \sigma_{\gamma}). \tag{4.1}$$

where ε_{γ} is a normally distributed, mean-zero shock. We follow the estimation of Chari, Kehoe, and McGrattan (2003) and run a regression of the above equation on quarterly U.S. data for M1 from $1973:1$ to $2007:1$ and set $\rho_{\gamma} = 0.69$ and $\sigma_{\gamma} = 0.014$.[28]

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[27]Another popular way to estimate the shock processes is using VAR estimation, based on the idea that the monetary authorities may respond to technology shocks. We also estimate the shock processes using VAR estimation. The main results still stand and the model can generate excess volatility of exchange rates.

[28]M1 data comes from the Board of Governors of the Federal Reserve System.

	Data	Model		
		Both shocks	Monetary shocks	Real shocks
Standard deviations				
θ	-	0.0843	0.0840	0.0
Output	0.0182	0.0182	0.01268	0.0128
Consumption	0.0171	0.0171	0.01117	0.0123
Price ratio	0.013	0.107	0.107	-
Nominal exchange rate	0.085	0.0787	0.0776	0.0
Real exchange rate	0.080	0.089	0.0867	0.0213
Autocorrelations				
θ	-	0.6871	0.6779	-
Output	0.88	0.8440	0.7193	0.9212
Consumption	0.89	0.8164	0.6186	0.9211
Price ratio	0.87	0.8901	0.890	-
Nominal exchange rate	0.86	0.6987	0.6778	-
Real exchange rate	0.84	0.9643	0.9671	0.9217
Cross-correlations				
Real and nominal exchange rate	0.99	-0.5405	-0.5553	-
RER and output	0.08	0.25	0.5356	-0.5751
RER and relative consumption	-0.35	-0.34	-0.2239	-1.0
foreign and domestic output	0.6	0.11	-0.1008	0.2486
foreign and domestic consumption	0.38	0.27	0.1510	0.3440

Table 5.1: Simulated moments in the benchmark model

5. Results

In this section, we evaluate the quantitative performance of the model. The central interest is the dynamics of exchange rates which is generated by the model. Note that all variables that we discuss in this section are measured by the percentage deviation from their steady state values. Table 5.1 reports the second moments of key variables generated by the model. Three cases are considered: (i) both money growth shocks and technology shocks are present; (ii) only money growth shocks are considered; (iii) only technology shocks are considered.

5.1. Volatility, Persistence and Cross-correlation

Our model reaches some successes in accounting for the main properties of the international business cycle, at least qualitatively. Compared to output, both the nominal and real exchange rate are highly volatile. The volatility of the nominal and real exchange rate are 0.079 and 0.089,

respectively, which are more than 4 times that of output. Both values are closed to the data. Moreover, the model also produces substantial persistence for the real exchange rate (0.96), which is even higher than that in the data. The persistence of nominal exchange rate (0.69) falls below the level in the data. As discussed earlier, the nominal exchange rate relies entirely on the differential in valuations of currencies, which is affected only by monetary disturbances. Persistence in nominal exchange rate inherits roughly the same persistence in the underlying shocks to money growth rates, which is 0.69 in our model. Autocorrelations of output and consumption are all close to the data.

It is well-known that the international business cycle models with complete asset market set up a link between the real exchange rate and the ratio of marginal utility of consumption, which suggests a high correlation between the real exchange rate and relative consumption. For example, Chari, Kehoe and McGrattan (2002) construct a sticky price model with LCP (local currency pricing) and their quantitative exercise shows that such correlation is actually 1. While this correlations in the data are zero or negative, such anomaly remains an interesting puzzle in international economics.[29] The asset market is incomplete in our model in the sense that no state-contigent bonds are provided in current environment. Therefore, the link between the real exchange rate and the ratio of marginal utility of consumption is absent in our model. In fact, our model replicates successfully the negative relationship between the real exchange rate and relative consumption and the correlation between these two variables is close to the data.

The relationships between the real exchange rate and output, domestic and foreign outputs, and domestic and foreign consumptions, suggested by the model, are all consistent with those in the data.

Empirical studies show that the nominal and real exchange rate are highly correlated. Our model, however, fails to generate this property of exchange rates. Instead, the model shows a negative correlation between the nominal and real exchange rate. To explain this, consider a positive monetary shock to currency 1. Households value currency 1 less and then the nominal exchange rate of currency 1 depreciates. However, the relative valuation of currencies across countries changes too. As we discussed earlier, country 1 households value currency 1 less than country 2 households do and this improves terms of trade for country 1 households. As the result, the real exchange rate of currency 1 appreciates.

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[29]This anomaly was first documented by Backus and Smith (1993).

Although the volatility of real and nominal exchange rates generated by the benchmark model is about right, it does less successfully in accounting for the volatility in price ratio across countries. In fact, our model generates too much volatility of price ratio relative to that observed in the data.

5.2. The Roles of the Two Shocks

Many empirical studies support the view that monetary shocks account for a substantial fraction of the variability of the real exchange rate (e.g., Clarida and Gali (1994), Rogers (1999), etc.). In this section, we investigate the roles of money growth shocks and technology shocks in our model. To illustrate this point, we consider the world economy is affected by both monetary and real shocks and take the model with both shocks as a benchmark case. Then, we examine two highly abstracted cases: i) the real shocks are taken away and only monetary shocks are considered; ii) the monetary shocks are taken away and only real shocks are considered. The results from these two cases give us the clue about how much variations in the real exchange rate are generated by the two shocks, respectively. The results are reported in Table 5.1.

It is clear that monetary shocks in our model account for most of the volatility of exchange rates. With only technology shocks presented, there is no variability at all in the nominal exchange rate and volatility of the real exchange rate drops dramatically from 0.089 to 0.021. The main reason that monetary shocks play a relatively more important role in the model is that the main components of the variability of exchange rates, θ and $\widehat{m}^e$, only respond to the changes in the two countries' money growth rate differential, which can only be directly affected by monetary shocks. As discussed in section 3.4, the changes in the nominal exchange rate depend exclusively on θ, while the evolution rule of θ, (3.8), shows that the formation of θ only involves the differential in growth rates of the two currencies. Technology shocks, $\widehat{a}_i$, also affect a household's valuation of currencies. However, they affect the valuations of both currencies in the same way, and thus leave the differential unchanged. For example, suppose that a positive technology shock occurs in country 1. Sellers from country 1 can produce more because of technology improvement and hence buyers may purchase more goods in a trade match with a country 1 seller. Therefore, households value the currency involved in the transaction with country 1's goods more. Since country 1's goods are sold for both currencies, households value both currencies higher. The differential in the valuations, however, does not change. Without monetary shocks, θ does not

change. As a result, the nominal exchange rate remains the same all the time.

Our model suggests that monetary shocks contribute significantly to the variations in the real exchange rate.[30] As we have shown in subsection 3.7, the changes in the real exchange rate can be decomposed into the deviations from the LOP and the price differential caused by the difference in consumption bundles across countries. The deviations from the LOP are caused by the changes in θ and the relative money holdings across countries, both of which can only be affected by the differential in the two countries' money growth rates. In the absence of monetary shocks, the LOP holds. All deviations from PPP arise from the differentials in consumption bundles between the two countries.

Technology shocks, however, help to explain some basic features of output and consumption in the data. Money growth shocks alone cannot generate adequate volatility of output and consumption. The standard deviations of output and consumption are only 65% of that in the data when real shocks are taken away. Moreover, in the absence of real shocks, the persistence of output and consumption is too low compared to that in the data. The correlations between the domestic and the foreign output and consumption match the data more closely if we incorporate the real shocks.

5.3. The Role of Search Frictions in the Goods Market

The non-Walrasian feature in the goods market plays an important role in generating high volatility in the exchange rates. Agents meet bilaterally and randomly in the goods market. They cannot arbitrage across matches and markets. Households form different valuations of the two currencies and the prices differ in different types of trade matches. Search frictions generate the deviations from the LOP and, hence, the main part of variations in the real exchange rates.

The parameter, T_0, can be interpreted as the degree of frictions in the goods market. The lower the value T_0 is, the lower the matching rates for the agents in the goods market are, and the harder it is to find a trade partner in the market. Figure 5.1 shows the relationship between T_0 and the volatility of the real exchange rate (horizontal axis denotes T_0 and vertical axis denotes the volatility of the real exchange rate).

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[30]The assumption that goods can be purchased by either currency is crucial to generate this result. If assuming domestic buyers only use domestic currency to buy local goods, real shocks will affect θ and hence, contribute more to the variations of the real exchange rate. The main conclusion that search frictions generate excess volatility of exchange rates, however, does not change. As long as search frictions exist in the goods market, the shocks can produce the fluctuations in the deviations from the LOP and the excess volatility of real exchange rates.
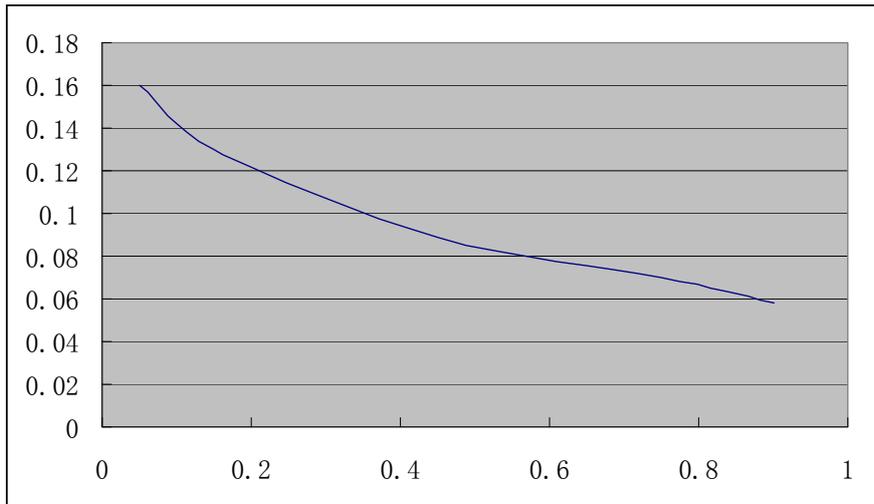
Figure 5.1: Volatility of the real exchange rate at different value of T_0

Clearly, the volatility of the real exchange rate increases with the degree of search frictions in the goods market. With more difficulty in finding a successful transaction in the goods market, the differences in the household's valuations of currencies are larger, which means higher deviations from the LOP and larger variations in the real exchange rate.

To illustrate the importance of the decentralized goods market, we construct a standard Walrasian model with the cash-in-advance constraints.[31] We apply the same preference and the same technology function. Using the same shock processes, we simulate this simple CIA model and report the results in Table 5.2.

Just as we discussed in section 3.7, without any friction in the goods market, the model cannot generate much volatility of both the nominal and real exchange rate. All the persistence of exchange rates comes from the underlying disturbances.

5.4. Flexible versus Fixed Regime

Empirical works have shown that when countries move from a pegged to a floating exchange rate system, the volatility of the real exchange rate increases dramatically, while the behaviors of other macroeconomic variables do not seem to change systematically.[32] Moreover, the co-movement

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[31]The model is a variation of Helpman's (1981) model with elastic labor supply.
[32]See Mussa (1986), Baxter and Stockman (1989).

	Data	Model with Walrasian goods Market		
		Both shocks	Monetary shock	Real shock
Standard deviations				
Output	0.0182	0.0154	0.008	0.013
Consumption	0.0171	0.0152	0.008	0.013
Price ratio	0.013	0.0198	0.011	0.015
Nominal exchange rate	0.085	0.0098	0.0058	0.0077
Real exchange rate	0.080	0.0096	0.0056	0.00075
Autocorrelations				
Output	0.88	0.845	0.68	0.92
Consumption	0.89	0.845	0.68	0.92
Price ratio	0.87	0.845	0.68	0.92
Nominal exchange rate	0.86	0.831	0.68	0.92
Real exchange rate	0.84	0.831	0.68	0.92
Cross-correlations				
Real and nominal exchange rate	0.99	-1.0	-1.0	-1.0
RER and output	0.08	0.635	0.72	0.5813
RER and relative consumption	-0.35	1.0	1.0	1.0
foreign and domestic output	0.6	0.15	-0.03	0.2248
foreign and domestic consumption	0.38	0.18	0.0005	0.2522

Table 5.2: Simulated moments in a CIA model

of output, consumption and investment are usually higher under the fixed rate regime.[33] In this subsection, we examine the model's performance when the nominal exchange rate is fixed. The simulated results are consistent with those empirical evidences appearing in the switches of exchange rate regimes.

Under the fixed rate regime, country 2's monetary authority is assumed to unilaterally peg the nominal exchange rate at a constant level. The model is otherwise identical to the benchmark model described in the previous section, where the nominal exchange rate is allowed to change. Therefore, to maintain the fixed exchange rate, monetary policy in country 2 loses independence and responds according to country 1's monetary policy. More specifically, $\gamma_{1,t}$ is still exogenous and follows the stochastic process (4.1), but $\gamma_{2,t}$ has to be endogenous to maintain the fixed exchange rate.

In our model, to peg the nominal exchange rate at the level $\overline{e}$, country 2 has to follow the same monetary policy as country 1 and maintain $\gamma_{2,t} = \gamma_{1,t}$ in all periods. To make a comparison between fixed and flexible rate regimes, we report the simulated results under different exchange rate systems in table 5.3.

The model does pretty well to replicate the evidence suggested by those empirical studies comparing fixed rate regime with flexible rate regime. The real exchange rate is the variable that is most affected by the change of exchange rate regime: it drops from 0.089 to 0.026 after changing to a fixed regime. Moreover, the real exchange rate is also highly persistent under a fixed rate regime (0.92). The behavior of the other variables, however, does not seem to be sensitive to the exchange rate regime: the standard deviations of output and consumption barely change after the switch of regimes.

The absence of the differential in valuation of currencies, θ, as well as the changes in the relative effective money holdings, $\widehat{m}^e$, in the fixed regime is the main reason for the substantial reduction in the variation of the real exchange rate. Under a fixed regime, the two countries have to apply the same monetary policy to maintain the fixed nominal exchange rate. The same money growth rates across countries eliminate the differential in valuations of currencies, as well as the changes in the relative effective money holdings, which removes the two key components of the fluctuations in the real exchange rate. Volatilities of output and consumption do not depend

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[33]Sopraseuth (2000) studies the data of countries participating and not paticipating in the ERM and finds that the EMS seems to favor a greater degree of synchronization among EMS countries.

	Fixed Rate Regime	Flexible Rate Regime
Standard deviations		
θ	0.0	0.0843
Output	0.0175	0.0182
Consumption	0.0171	0.0171
Nominal exchange rate	-	0.0787
Real exchange rate	0.026	0.089
Autocorrelations		
θ	-	0.68
Output	0.80	0.84
Consumption	0.79	0.82
Nominal exchange rate	-	0.68
Real exchange rate	0.92	0.96
Cross-correlations		
Real and nominal exchange rate	-	-0.52
foreign and domestic output	0.59	0.11
foreign and domestic consumption	0.64	0.27

Table 5.3: Simulated results in fixed and flexible rate regimes

on θ or $\widehat{m}^e$, and hence are not affected by the change of the exchange rate regime.

In accordance with the evidence suggested by the empirical works, the cross-correlations between domestic and foreign output and consumption increase when moving from pegged to flexible exchange rate regime. This is not a surprising result since the two countries have to coordinate their monetary policies to peg the nominal exchange rate. Therefore, the monetary policies of the two countries are correlated under a fixed rate regime instead of being independent under a flexible rate regime. As a result, the cross-country correlations of the variables in the model are higher under fixed rate regime than under flexible rate regime.

6. Conclusion

It has been a well known fact in the international business cycle that the real exchange rate movements disconnect with other macroeconomic aggregates. The behavior of the real exchange rate poses a non-trivial challenge to international business cycle models in which real exchange rates play an important role in real allocation. Traditional explanation for this feature of exchange rate relies on the interaction of the nominal price rigidities and monetary shocks. However, to generate the volatility and persistence of exchange rates that are high enough to match the

data, such models with nominal price rigidities need to assume unrealistically long-lived price stickiness. Given the above-mentioned problem in the sticky price model, in this paper we deviate substantially from the literature by developing a two-country dynamic search model to examine the behavior of exchange rates. The prices in our model are fully flexible and the most important feature of the model is the presence of search frictions in the goods market.

Our model successfully replicates the behavior of exchange rates shown by empirical studies: both the nominal and real exchange rates are highly volatile and persistent, and the behavior of real exchange rates changes systematically across different exchange rate systems. In sharp contrast to previous models which rely on nominal rigidities, the model in this paper focuses on the role of search frictions in the goods market and shows that such a model is capable of capturing the main features of exchange rates in the international business cycle.

The deviations from the LOP are the key components of fluctuations of exchange rates in our model. Search frictions in the goods market induce a differential between the two countries' valuations of a currency, thus creating price differentials across different types of trade matches. The numerical results suggest that the deviations from the LOP contribute about 60 percent to the fluctuations of the real exchange rate in our model.

Monetary shocks play an important role in explaining the fluctuations of exchange rates in our model. The main reason lies in the fact that the deviations from the LOP is driven exclusively by the differential in the two currencies' growth rates, which is directly affected by monetary shocks. Since monetary shocks are responsible for the deviations from the LOP, they account for a large part of the volatility of exchange rates. This result is consistent with the empirical findings which show that monetary shocks play an more important role in generating fluctuations in exchange rates.

Search frictions in the goods market are the key feature that generates high volatility of exchange rates. The simulated results show that the volatility of exchange rates increases with the degree of frictions in the goods market. In addition, we construct a CIA model with a Walrasian goods market. To make a comparison, the other features of the model are set up alike except that the goods market is frictionless. Without search frictions, neither monetary shocks nor real shocks can generate much volatility of real exchange rates, and the persistence of real exchange rates comes entirely from the persistence in the underlying shocks.

We are also interested in the behavior of exchange rates under different regimes. Our results

show that a country experiences a dramatic increase in the volatility of the real exchange rate when moving from a pegged to a floating exchange rate regime. The change in the exchange rate regime, however, does not affect the behavior of the other variables such as output and consumption. Moreover, a higher co-movement of output and consumption across countries is found under a fixed rate regime than under a flexible rate regime. These results are consistent with the empirical studies.

The main discrepancy between the model and the data is that the model generates a negative correlation between the nominal and real exchange rates, while the data suggests they are highly positively correlated. This result arises from the highly abstracted description of the world economy. To sharpen our focus on the role of search frictions, the model is set in the simplest way that the main difference between the two countries comes from their money growth rates. Such a highly abstracted model prevents us from providing a satisfactory explanation for the correlation between variables. To reconcile this mismatch generated by the model, more differences between the two countries should be incorporated into the model. The next step of our research is to introduce the non-traded good sector. With the non-traded good sector, both the monetary shocks and real shocks play a role in generating the deviations from the LOP, and hence the fluctuations of nominal and real exchange rates. The correlation between the nominal and real exchange rate may then be consistent with the data.

Appendix

A. Log-linearized System

Here we derive the results that obtained in Section 3 of the paper. The equilibrium is characterized by the equations described in section 2.5. To solve this system, we take a linear approximation around the initial symmetric steady state, and get following 32 linear equations:

$$\widehat{C}_i = \phi_1 \widehat{c}_{ih} + (1 - \phi_1) \widehat{c}_{if}; \quad \phi_1 = \rho^{1/\eta} \left(\frac{\overline{c}_h}{\overline{C}} \right)^{(\eta-1)/\eta}; \tag{A.1}$$

$$2\widehat{c}_{ih} = \widehat{q}_{ii} + \widehat{q}_{ii'}; \tag{A.2}$$

$$2\widehat{c}_{if} = \widehat{q}_{ii}^f + \widehat{q}_{ii'}^f; \tag{A.3}$$

$$\phi_2 \widehat{\lambda}_{ii} = \left(\frac{1}{\eta} - \epsilon \right) \widehat{C}_i - \frac{1}{\eta} \widehat{c}_{ih} + \phi_2 \widehat{\omega}_{ii} - \widehat{a}_i - (\sigma - 1) \widehat{q}_{ii}; \phi_2 = \frac{\overline{\lambda}}{\left(\overline{\lambda} + \overline{\Omega} \right)}; \tag{A.4}$$

$$\phi_3 \widehat{\lambda}_{ii}^f = \left(\frac{1}{\eta} - \epsilon \right) \widehat{C}_i - \frac{1}{\eta} \widehat{c}_{if} + \widehat{\omega}_{i'i} - (1 - \phi_3) \widehat{\omega}_{ii} - \widehat{a}_{i'} - (\sigma - 1) \widehat{q}_{ii}^f; \phi_3 = \frac{\overline{\lambda}^f}{\overline{\lambda}^f + \overline{\Omega}}; \tag{A.5}$$

$$\phi_2 \widehat{\lambda}_{ii'} = \left(\frac{1}{\eta} - \epsilon \right) \widehat{C}_i - \frac{1}{\eta} \widehat{c}_{ih} + \phi_2 \widehat{\omega}_{ii'} - \widehat{a}_i - (\sigma - 1) \widehat{q}_{ii'}; \tag{A.6}$$

$$\phi_3 \widehat{\lambda}_{ii'}^f = \left(\frac{1}{\eta} - \epsilon \right) \widehat{C}_i - \frac{1}{\eta} \widehat{c}_{if} + \widehat{\omega}_{i'i'} - (1 - \phi_3) \widehat{\omega}_{ii'} - \widehat{a}_{i'} - (\sigma - 1) \widehat{q}_{ii'}^f; \tag{A.7}$$

$$\left(2 - \frac{1}{\overline{\gamma}} \right) \widehat{m}_{11} - \left(1 - \frac{1}{\overline{\gamma}} \right) \widehat{f}_{11} = \sigma \widehat{q}_{11} + \widehat{a}_1 + \widehat{n}_1 - \widehat{\omega}_{11}; \tag{A.8}$$

$$\left(2 - \frac{1}{\overline{\gamma}} \right) \widehat{m}_{11} - \left(1 - \frac{1}{\overline{\gamma}} \right) \widehat{f}_{11} = \sigma \widehat{q}_{11}^f + \widehat{a}_2 + \widehat{n}_1 - \widehat{\omega}_{21}; \tag{A.9}$$

$$\frac{1}{\overline{\gamma}} \widehat{m}_{12} + \left(1 - \frac{1}{\overline{\gamma}} \right) \left(\widehat{f}_{11} + \widehat{e} \right) = \sigma \widehat{q}_{12} + \widehat{a}_1 - \widehat{\omega}_{12} - \widehat{n}_1; \tag{A.10}$$

$$\frac{1}{\overline{\gamma}} \widehat{m}_{12} + \left(1 - \frac{1}{\overline{\gamma}} \right) \left(\widehat{f}_{11} + \widehat{e} \right) = \sigma \widehat{q}_{12}^f + \widehat{a}_2 - \widehat{\omega}_{22} - \widehat{n}_1; \tag{A.11}$$

$$-\frac{1}{\overline{\gamma}} \widehat{m}_{12} - \left(1 - \frac{1}{\overline{\gamma}} \right) \left(\widehat{f}_{11} + \widehat{e} \right) = \sigma \widehat{q}_{22} + \widehat{a}_2 + \widehat{n}_2 - \widehat{\omega}_{22}; \tag{A.12}$$

$$-\frac{1}{\overline{\gamma}} \widehat{m}_{12} - \left(1 - \frac{1}{\overline{\gamma}} \right) \left(\widehat{f}_{11} + \widehat{e} \right) = \sigma \widehat{q}_{22}^f + \widehat{a}_1 + \widehat{n}_2 - \widehat{\omega}_{12}; \tag{A.13}$$

$$\left(1 - \frac{1}{\overline{\gamma}} \right) \widehat{f}_{11} - \left(2 - \frac{1}{\overline{\gamma}} \right) \widehat{m}_{11} = \sigma \widehat{q}_{21} + \widehat{a}_2 - \widehat{n}_2 - \widehat{\omega}_{21}; \tag{A.14}$$

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$$\left(1 - \frac{1}{\gamma}\right)\widehat{f}_{11} - \left(2 - \frac{1}{\gamma}\right)\widehat{m}_{11} = \sigma\widehat{q}^f_{21} + \widehat{a}_1 - \widehat{n}_2 - \widehat{\omega}_{11}; \tag{A.15}$$

$$2\left(1 + \phi_4\right)\left(1 - \psi\right)\widehat{n}_1 = \widehat{q}_{11} - \widehat{q}_{12} + \phi_4\left(\widehat{q}^f_{11} - \widehat{q}^f_{12}\right); \phi_4 = \alpha^{1+\frac{1}{\eta}}\left(\frac{1-\rho}{\rho}\right)^{1/\eta} \tag{A.16}$$

$$2\left(1 + \phi_4\right)\left(1 - \psi\right)\widehat{n}_2 = \widehat{q}_{22} - \widehat{q}_{21} + \phi_4\left(\widehat{q}^f_{22} - \widehat{q}^f_{21}\right); \tag{A.17}$$

$$\begin{aligned}
\frac{\overline{\gamma}}{\beta}\widehat{e} &= \left(\widehat{\omega}_{11} - \widehat{\omega}_{12}\right) + \mu_0\frac{\phi_2}{1-\phi_2}\left(\widehat{\lambda}_{11} - \widehat{\lambda}_{12}\right) + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\left(\widehat{\lambda}^f_{11} - \widehat{\lambda}^f_{12}\right) \\
&\quad -2\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_1;
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
\frac{\overline{\gamma}}{\beta}\widehat{e} &= \left(\widehat{\omega}_{21} - \widehat{\omega}_{22}\right) + \mu_0\frac{\phi_2}{1-\phi_2}\left(\widehat{\lambda}_{21} - \widehat{\lambda}_{22}\right) + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\left(\widehat{\lambda}^f_{21} - \widehat{\lambda}^f_{22}\right) \\
&\quad +2\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_2;
\end{aligned} \tag{A.19}$$

$$\frac{\overline{\gamma}}{\beta}\widehat{\omega}_{11} = E\left[\begin{array}{c} -\frac{\overline{\gamma}}{\beta}\widehat{\gamma}_{1,+1} + \widehat{\omega}_{11,+1} + \mu_0\frac{\phi_2}{1-\phi_2}\widehat{\lambda}_{11,+1} + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\widehat{\lambda}^f_{11,+1} \\ -\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_{1,+1} \end{array}\right]; \tag{A.20}$$

$$\frac{\overline{\gamma}}{\beta}\widehat{\omega}_{12} = E\left[\begin{array}{c} -\frac{\overline{\gamma}}{\beta}\widehat{\gamma}_{2,+1} + \widehat{\omega}_{12,+1} + \mu_0\frac{\phi_2}{1-\phi_2}\widehat{\lambda}_{12,+1} + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\widehat{\lambda}^f_{12,+1} \\ +\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_{1,+1} \end{array}\right]; \tag{A.21}$$

$$\frac{\overline{\gamma}}{\beta}\widehat{\omega}_{22} = E\left[\begin{array}{c} -\frac{\overline{\gamma}}{\beta}\widehat{\gamma}_{2,+1} + \widehat{\omega}_{22,+1} + \mu_0\frac{\phi_2}{1-\phi_2}\widehat{\lambda}_{22,+1} + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\widehat{\lambda}^f_{22,+1} \\ -\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_{2,+1} \end{array}\right]; \tag{A.22}$$

$$\frac{\overline{\gamma}}{\beta}\widehat{\omega}_{21} = E\left[\begin{array}{c} -\frac{\overline{\gamma}}{\beta}\widehat{\gamma}_{1,+1} + \widehat{\omega}_{21,+1} + \mu_0\frac{\phi_2}{1-\phi_2}\widehat{\lambda}_{21,+1} + \alpha\mu_0\frac{\phi_3}{1-\phi_3}\widehat{\lambda}^f_{21,+1} \\ +\left(\overline{\gamma}/\beta - 1\right)\left(1 - \psi\right)\widehat{n}_{2,+1} \end{array}\right]; \tag{A.23}$$

$$\begin{aligned}
\left(2\overline{\gamma} - 1\right)\widehat{m}_{11,+1} &= \alpha\mu_0\left[\sigma\left(\widehat{q}^f_{21} - \widehat{q}^f_{11}\right) - \psi\left(\widehat{n}_1 + \widehat{n}_2\right) + \widehat{a}_1 - \widehat{a}_2 + \widehat{\omega}_{21} - \widehat{\omega}_{11}\right] \\
&\quad + \left(2 - \frac{1}{\gamma}\right)\widehat{m}_{11} - \left(1 - \frac{1}{\gamma}\right)\widehat{f}_{11} + \widehat{\gamma}_{1,+1};
\end{aligned} \tag{A.24}$$

$$\begin{aligned}
\widehat{m}_{12,+1} &= \alpha\mu_0\left[\sigma\left(\widehat{q}^f_{22} - \widehat{q}^f_{12}\right) + \psi\left(\widehat{n}_1 + \widehat{n}_2\right) + \widehat{a}_1 - \widehat{a}_2 + \widehat{\omega}_{22} - \widehat{\omega}_{12}\right] \\
&\quad + \frac{1}{\gamma}\widehat{m}_{12} + \left(1 - \frac{1}{\gamma}\right)\left(\widehat{f}_{11} + \widehat{e}\right) - \widehat{\gamma}_{2,+1};
\end{aligned} \tag{A.25}$$

Combine equation (A.18), (A.20) and (A.21), we get

$$\widehat{\omega}_{11} - \widehat{\omega}_{12} = E\left[\widehat{e}_{+1} - \widehat{\gamma}_{1,+1} + \widehat{\gamma}_{2,+1}\right];$$

Similarly, $\widehat{\omega}_{21} - \widehat{\omega}_{22} = E\left[\widehat{e}_{+1} - \widehat{\gamma}_{1,+1} + \widehat{\gamma}_{2,+1}\right]$ can be derived from (A.19), (A.22) and (A.23). Thus, we obtain the relationship between valuations of the two currencies (3.1). The quantity

differentials (3.2) and (3.3) can be easily derived from equations (A.8)-(A.15). Using these quantity differentials and equation (A.16) and (A.17), we can immediately get proposition 3.3. Note that variables $\left(\widehat{C}_i, \widehat{c}_{ih}, \widehat{c}_{if}, \widehat{\lambda}_{ii}, \widehat{\lambda}_{ii}^f\right)$ can be written as functions of $\left(\widehat{q}_{ik}, \widehat{q}_{ik}^f, \widehat{\omega}_{ik}, \widehat{a}_k\right)$ by (A.1)-(A.7). Substitute $\widehat{\lambda}_{ik}$, $\widehat{\lambda}_{ik}^f$, and $\widehat{n}_1$ into (A.18), we can show that the nominal exchange rate $\widehat{e}$ is proportional to θ, i.e. (3.7). $\widehat{f}_{11}$ can be solved by (A.8) and (A.10), which leads to (3.9). Since both $\widehat{e}$ and $\widehat{n}_1$ are function of θ, it's not hard to show $\widehat{f}_{11}$ is a function of θ, $\widehat{m}_{11}$ and $\widehat{m}_{12}$. Thus, money-goods constraints (A.8)-(A.15) imply that all the $\widehat{q}_{ik}$ and $\widehat{q}_{ik}^f$ can be written as functions of $(\widehat{m}_{11}, \widehat{m}_{12}, \widehat{\omega}_{11}, \widehat{\omega}_{12}, \widehat{\omega}_{22}, \widehat{a}_1, \widehat{a}_2)$.

Above results can further simply the 2 laws of motion of money holdings, (A.24) and (A.25). Substitute $\widehat{q}_{ik}^f$ and $\widehat{n}_i$ into the two equations, we obtain,

$$\overline{\gamma}\widehat{m}_{11,+1} = (1 - 2\alpha\mu_0)\,\widehat{m}_{11} - (1 - 2\alpha\mu_0)\,\frac{(\overline{\gamma}-1)}{(2\overline{\gamma}-1)}\,\widehat{f}_{11} + \frac{\overline{\gamma}}{(2\overline{\gamma}-1)}\widehat{\gamma}_{1,+1}; \tag{A.26}$$

$$\overline{\gamma}\widehat{m}_{12,+1} = (1 - 2\alpha\mu_0)\,\widehat{m}_{12} + (1 - 2\alpha\mu_0)\,(\overline{\gamma}-1)\left(\widehat{f}_{11} + \widehat{e}\right) - \overline{\gamma}\widehat{\gamma}_{2,+1}; \tag{A.27}$$

Note that all the variables $\left\{\widehat{C}_i, \widehat{c}_{if}, \widehat{c}_{if}, \widehat{\lambda}_{ii}, \widehat{\lambda}_{ii}^f, \widehat{q}_{ik}, \widehat{q}_{ik}^f, \widehat{n}_i, \widehat{e}\right\}$ can be expressed as a function of $(\widehat{m}_{11}, \widehat{m}_{12}, \widehat{\omega}_{11}, \widehat{\omega}_{12}, \widehat{\omega}_{22})$ and exogenous state variables. Therefore, above 2 laws of motion of money holdings and 3 envelop conditions (A.20)-(A.22) constitute a much simpler dynamic system, which only involves 2 predetermined variables, $\widehat{m}_{11}$ and $\widehat{m}_{12}$, 3 jump variables, $\widehat{\omega}_{11}$, $\widehat{\omega}_{12}$, and $\widehat{\omega}_{22}$, and exogenous state variables. Once we solve this dynamic system, all the other variables can then be derived accordingly.

B. Solution Technique[34]

We apply Blanchard and Khan's method to solve the dynamic system. Let $x = [\widehat{m}_{11}, \widehat{m}_{12}, \widehat{\omega}_{11}, \widehat{\omega}_{12}, \widehat{\omega}_{22}]^T$, where $\widehat{m}_{11}$ and $\widehat{m}_{12}$ are predetermined variables and the other 3 variables are jump variables. Let $z = [\widehat{\gamma}_1, \widehat{\gamma}_2, \widehat{a}_1, \widehat{a}_2]^T$ denote the vector of exogenous variables. Therefore, the dynamic system can be rewritten as,

$$E_t\left[x_{t+1}\right] = A \cdot x_t + R \cdot E_t\left[z_{t+1}\right];$$

First, we transform A into the Jordan canonical form,

$$A = C^{-1}JC;$$

---

[34]Note that some of the symbols used here don't have the same meaning as the ones used in the text.

where the diagonal elements of J, which are the eigenvalues of A, are ordered by increasing absolute value.

For the system to have a stable solution, there should be exactly two eigenvalues of A outside the unit circle. The calibrated parameter values in our model indeed generate such eigenvalues. Decompose J further as,

$$
J = \left[
\begin{array}{cc}
\begin{array}{c} J_1 \\ (2 \times 2) \end{array} & 0 \\
0 & \begin{array}{c} J_2 \\ (3 \times 3) \end{array}
\end{array}
\right]
$$

where all eigenvalues of J_1 are on or inside the unit circle, all eigenvalues of J_2 are outside the unit circle.

Matrix C, C^{-1}, and R are decomposed accordingly:

$$
C = \left[
\begin{array}{cc}
\begin{array}{c} C_{11} \\ (2 \times 2) \end{array} & \begin{array}{c} C_{12} \\ (2 \times 3) \end{array} \\
\begin{array}{c} C_{21} \\ (3 \times 2) \end{array} & \begin{array}{c} C_{22} \\ (3 \times 3) \end{array}
\end{array}
\right] ; \quad
C^{-1} = \left[
\begin{array}{cc}
\begin{array}{c} B_{11} \\ (2 \times 2) \end{array} & \begin{array}{c} B_{12} \\ (2 \times 3) \end{array} \\
\begin{array}{c} B_{21} \\ (3 \times 2) \end{array} & \begin{array}{c} B_{22} \\ (3 \times 3) \end{array}
\end{array}
\right] ; \quad
R = \left[
\begin{array}{c}
\begin{array}{c} R_1 \\ (2 \times 4) \end{array} \\
\begin{array}{c} R_2 \\ (3 \times 4) \end{array}
\end{array}
\right] ;
$$

For given initial value x_0, the solution to above system is given by,

$$
\begin{aligned}
x_t\left(1:2\right) \;=\; & B_{11} J_1 B_{11}^{-1} x_{t-1}\left(1:2\right) + R_1 E_{t-1} z_t \\
& - \left(B_{11} J_1 C_{12} + B_{12} J_2 C_{22}\right) C_{22}^{-1} \sum_{i=0}^{\infty} J_2^{-i-1} \left[\left(C_{21} R_1 + C_{22} R_2\right) E_{t-1} z_{t+i}\right], \text{ for } t > 0;
\end{aligned}
$$

$$
x_t\left(3:5\right) = -C_{22}^{-1} C_{21} x_t\left(1:2\right) - C_{22}^{-1} \sum_{i=0}^{\infty} J_2^{-i-1} \left[\left(C_{21} R_1 + C_{22} R_2\right) E_t z_{t+i+1}\right], \text{ for } t \geq 0;
$$

Given the exogenous process of z_t, we can calculate the time path of x_t recursively.

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