A Macro-Finance Approach to Exchange Rate Determination*

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Abstract. The nominal exchange rate is both a macroeconomic variable equilibrating international markets and a financial asset that embodies expectations and prices risks associated with cross border currency holdings. Recognizing this, we adopt a joint macro-finance strategy to model the exchange rate. We incorporate into a monetary exchange rate model macroeconomic stabilization through Taylor-rule monetary policy on one hand, and on the other, market expectations and perceived risks embodied in the cross-country yield curves. Using monthly data between 1985 and 2005 for Canada, Japan, the UK and the US, we employ a state-space system to model the relative yield curves between country-pairs using the Nelson and Siegel (1987) latent factors, and combine them with monetary policy targets (output gap and inflation) into a vector autoregression (VAR) for bilateral exchange rate changes. We find strong evidence that both the financial and macro variables are important for explaining exchange rate dynamics and excess currency returns, especially for the yen and the pound rates relative to the dollar. Moreover, by decomposing the yield curves into expected future yields and bond market term premiums, we show that both expectations about future macroeconomic conditions and perceived risks are priced into the currencies. These findings provide support for the view that the nominal exchange rate is determined by both macroeconomic as well as financial forces.

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1 Introduction

This paper proposes to model nominal exchange rates by incorporating both macroeconomic determinants and latent financial factors, bridging the gap between two important strands of recent research. First, against decades of negative findings in testing exchange rate models, recent work by Engel et al (2007), Molodtsova and Papell (2009) among others, shows that models in which monetary policy follows an explicit Taylor (1993) interest rate rule deliver improved empirical performance, both in in-sample fits and in out-of-sample forecasts.¹ These papers emphasize the importance of expectations, and argue that the nominal exchange rate should be viewed as an asset price embodying the net present value of its expected future fundamentals.² While recognizing the presence of risk, in empirical testing, this literature largely ignores risk, rendering it an "unobservable".³ On the finance side, recent research shows that systematic sources of financial risk, as captured by latent factors, drive excess currency returns both across currency portfolios and over time.⁴ Bekaert et al (2007), for instance, further advocate that risk factors driving the premiums in the term structure of interest rates may also drive the risk premium in currency returns.⁵ These papers firmly establish the role of risks but are silent on the role of macroeconomic conditions, including monetary policy actions, in determining exchange rate. They thus fall short on capturing the potential feedback between macroeconomic forces, expectation formation, and perceived risk in exchange rate dynamics. We argue that the macro and the finance approaches should be combined, and propose a joint framework to capture intuition from both bodies of literature.

We present an open economy model where central banks follow a Taylor-type interest rate rule that stabilizes expected inflation, output gap, and the real exchange rate.⁶ The international

¹This approach works well for modeling exchange rates of countries that have credible inflation control policies.

 $^{^{2}}$ Since the Taylor-rule fundamentals – measures of inflation and output gap – affect expectations about future monetary policy actions, changes in these variables induce nominal exchange rate responses.

³Engel, Mark, and West (2007), for example, establish a link between exchange rates and fundamentals in a present value framework. After explicitly recognizing the possibility that risk premiums may be important in explaining exchange rates, they "do not explore that avenue in this paper, but treat it as an 'unobserved fundamental." Molodstova and Papell (2009), show that Taylor rule fundamentals (interest rates, inflation rates, output gaps and the real exchange rate) forecasts better than the commonly used interest rate fundamentals, monetary fundamentals and PPP fundamentals. Again, they explain exchange rate using only observed fundamentals and do not account for risk premium. This is an obvious shortcoming in modeling short-run exchange rate dynamics. Faust and Rogers (2003) for instance argue that monetary policy accounts for very little of the exchange rate volatility.

⁴See Inci and Lu (2004), Lustig et al (2009), and Farhi et al (2009), and references therein for the connection between risk factors and currency portfolio returns.

⁵In addition, Clarida and Taylor (1997) uses the term structure of forward exchange premiums to forecast spot rates. de los Rios (2009) and Krippner (2006) connect the interest rate term structure factors and exchange rate behavior. These papers do not examine the role of macroeconomic fundamentals or monetary policy.

⁶Note that following Clarida, Gali, and Gertler (1998), the incorporation of the exchange rate term to an otherwise

asset market efficiency condition - the risk-adjusted uncovered interest parity (UIP) - implies that nominal exchange rate is the net present value of expected future paths of interest differentials and risk premiums between the country pair. This framework establishes a direct link between the exchange rate and its current and expected future macroeconomic fundamentals; it also allows country-specific risk premiums over different horizons to affect exchange rate dynamics. Since exchange rate in this formulation relies more on expectations about the future than on current fundamentals, properly measuring expectations and time-varying risk becomes especially important in empirical testing.⁷ Previous papers largely fail to address this appropriately.⁸ We propose to use the Nelson-Siegel (1987) latent factors extracted from cross-country yield curves to capture expectations about future macroeconomic conditions and systematic risks in the currency markets. We combine the latent factors with monetary policy targets (output gap and inflation) into a vector autoregression (VAR) to study their dynamic interactions with bilateral exchange rate changes.⁹

The joint macro-finance strategy has proven fruitful in modeling other financial assets such as the term structure of interest rates.¹⁰ As stated in Diebold et al (2005), the joint approach to model the yield curve captures both the macroeconomic perspective that the short rate is a monetary policy instrument used to stabilize the economy, as well as the financial perspective that yields of all maturities are risk-adjusted averages of expected future short rates. Our exchange rate model is a natural extension of this idea into the international context. First, the no-arbitrage condition for international asset markets explicitly links exchange rate dynamics to cross-country yield differences at the corresponding maturities and a time-varying currency risk premium. Yields at different maturities - the shape of the yield curve - are in turn determined by the expected future path of short rates and perceived future uncertainty (the "bond term premiums"). The link with the macroeconomy comes from noticing that the short rates are monetary policy instruments which

standard Taylor rule has become commonplace in recent literature, especially for modeling monetary policy in non-US countries. See, for example, Engel and West (2006) and Molodtsova and Papell (2009).

⁷See Engel and West (2005), Engel et al (2007) for a more detailed presentation and discussion.

⁸Previous literature often ignores risk or makes overly simplistic assumptions about these expectations, such as by using simple VAR forecasts of macro fundamentals as proxies for expectations. For instance, Engel and West (2006) and Mark (1995) fit VARs to construct forecasts of the present value expression. Engel et al (2007) note that the VAR forecasts may be a poor measure of actual market expectations and use surveyed expectations of market forecasters as an alternative. See discussion in Chen and Tsang (2009).

⁹Chen and Tsang (2009) show that the Nelson-Siegel factors between two countries can help predict movements in their exchange rates and excess returns. It does not, however, consider the dynamic interactions between the factors and macroeconomic conditions.

¹⁰Ang and Piazzesi (2003), among others, illustrate that a joint macro-finance modeling strategy provides the most comprehensive description of the term structure of interest rates.

react to macroeconomic fundamentals. Longer yields therefore contain market expectations about future macroeconomic conditions. On the other hand, bond term premiums in the yield curve measure the market pricing of systematic risk of various origins over different future horizons.¹¹ Under the reasonable assumption that a small number of underlying risk factors affect all asset prices, currency risk premium would then be correlated with the bond term premiums across countries. From a theoretical point of view, the yield curves thus serve as a natural measure to both the macro- and the finance-aspect of the exchange rates. From a practical standpoint, the shape and movements of the yield curves have long been used to provide continuous readings of market expectations; they are a common indicator for central banks to receive timely feedback to their policy actions. Recent empirical literature, such as Diebold et al (2006), also demonstrates strong dynamic interactions between the macroeconomy and the yield curves. These characteristics suggest that empirically, the yield curves are also a robust candidate for capturing the two "asset price" attributes of nominal exchange rates: expectations on future macroeconomic conditions and perceived time-varying risks.

For our empirical analyses, we look at monthly exchange rate changes for three currency pairs - the Canadian dollar, the British pound, and the Japanese yen relative to the US dollar over the period from August 1985 to July 2005.¹² For each country pair, we extract three Nelson-Siegel (1987) factors from the zero-coupon yield *differences* between them, using yield data with maturities ranging from one month to five years. These three latent risk factors, which we refer to as the *relative level, relative slope,* and *relative curvature*, capture movements at the long, short, and medium part of the *relative* yield curves between the two countries. We use the Nelson-Siegel factors as they are well known to provide excellent empirical fit for the yield curves, providing a succinct summary of the systematic sources of risk that may underlie the pricing of financial assets. To model the joint dynamics of exchange rates, the macroeconomy, and the latent factors, we set up a state-space system where the measurement equation relates individual yields to timevarying Nelson-Siegel factors, and the transition equation is a six-variable VAR that combines the three relative factors, one-month exchange rate changes, and the relative output gap and inflation

¹¹Kim and Orphanides (2007) and Wright (2009), for example, provide a comprehensive discussion of the bond market term premium, covering both systematic risks associated with macroeconomic conditions, variations in investors' risk-aversion over time, as well as liquidity considerations and geopolitical risky events.

 $^{^{12}}$ We present results based on the dollar cross rates, though the qualitative conclusions extend to other pair-wise combinations of currencies.

differences between each country-pairs. The system is estimated using maximum likelihood under Kalman filtering.

To evaluate the overall performance of this macro-finance model, we compare exchange rate predictions at horizons between 3 months and 2 years using four model set-ups: a VAR with only macro variables, a VAR with only the yield curve factors, a VAR with both (our proposed macro-finance model), and a random walk benchmark. Since our short sample size and overlapping observations preclude accurate estimates of long-horizon regressions, we test for long-horizon exchange rate predictability using the rolling iterated VAR approach proposed in Campbell (1991), Hodrick (1992), and more recently in Lettau and Ludvigson (2005) and others.¹³ We iterate the full-sample estimated VAR(1) to generate exchange rate predictions at horizons beyond one month, and compare the mean squared prediction errors for each of the four models above. We also compute the implied long-horizon R^2 statistics to assess our model fit at different horizons. Next, under the assumption that the same country-specific time-varying latent risks are priced into both the bond and the currency markets, we model the currency risk premium (or excess currency returns) as a linear function of the bond term premiums between the two countries.¹⁴ Using our estimated VAR system which allows for dynamic interactions between the macro variables and the yield curve factors, we construct measures of *expected relative yields* for different maturities between each country-pair, incorporating expectations about future macro conditions. We then take the difference between the actual relative yields and these fitted ones to separate out the time-varying relative bond term premiums.¹⁵ These two variables allow us to test how expectations and risk measures embodied in the bond markets may have differential impact on exchange rate changes and excess currency returns.

Our main results are as follows: 1) empirical exchange rate equations based on only macrofundamentals can miss out on two crucial elements that drive currency dynamics: expectations

¹³While it is more common in the macro-exchange rate literature to compare models using out-of-sample forecasts (Meese and Rogoff 1983), we adopt this iterated VAR procedure used in recent finance literature to evaluate long horizon predictability. Out-of-sample forecast evaluation can be an unnecessarily stringent test to impose upon a model. For both theoretical and econometric reasons, it is not the most appropriate test for the validity of a model (see Engel, Mark, West 2007).

 $^{^{14}}$ Bekaert et al (2007) examines the relationship between deviations from uncovered interest parity condition in the currency markets and deviations from the expectations hypothesis in the bond markets at different horizons. They emphasized in their conclusion the potential interactions between monetary policy and the risk premiums, but did not explore it empirically.

¹⁵That is, the bond term premium at time t for maturity m is the difference between the actual maturity-m yield and the predicted yield. See Diebold, Rudebusch and Aruoba (2006) and Cochrane and Piazzesi (2006), for more discussions.

and risk, both of these elements are reflected in the latent factors extracted from the cross-country yield curves; 2) the macro-finance model delivers the best performance, especially for predicting the yen and pound rates relative to the dollar; the Canadian rates appear to be determined mainly by macroeconomic variables; 3) while most of the very short-term exchange rate variability remains difficult to account for, macro variables and finance factors can explain between 20-40% of the exchange rate changes a year ahead; 4) decomposing the yield curves into expectations for future rates versus bond term premiums, we show that both are relevant for explaining future exchange rate changes and excess currency returns. Overall, these findings support the view that exchange rates should be modeled using a joint macro-finance framework.

2 Theoretical Framework

2.1 Taylor Rule and the Exchange Rate

We present the basic setup of a Taylor-rule based exchange rate model below while emphasizing our proposal for addressing the issues previous papers tend to ignore. Consider a two-country model where the home country sets its interest rate, i_t , and the foreign country sets a corresponding i_t^* . Since our main results in the empirical section below are based on exchange rates relative to the dollar, one can view the foreign country here as the United States. We assume that the U.S. central bank follows a standard Taylor rule, reacting to inflation and output deviations from their target levels, but the other country also targets the real exchange rate, or the purchasing power parity, in addition. This captures the notion that central banks often raise interest rates when their currency depreciates, as supported the empirical findings in Clarida, Gali, and Gertler (1998) and previous work.¹⁶ The monetary policy rules can be expressed as:

$$i_t = \mu_t + \beta_y \widetilde{y}_t + \beta_\pi \pi_t^e + \delta q_t + u_t$$

$$i_t^* = \mu_t^* + \beta_y \widetilde{y}_t^* + \beta_\pi \pi_t^{*e} + u_t^*$$
(1)

where in the home country, \tilde{y}_t is the output gap, π_t^e is the expected inflation, and $q_t(=s_t - p_t + p_t^*)$ is the real exchange rate, defined as the nominal exchange rate, s_t , adjusted by the CPI-price level

¹⁶It is common in the literature to assume that the Fed reacts only to inflation and output gap, yet other central banks put a small weight on the real exchange rate. See Clarida, Gali, and Gertler (1998), Engel, West, and Mark (2007), and Molodtsova and Papell (2009), among many others.

difference between home and abroad, $p_t - p_t^*$. μ_t absorbs the inflation and output targets and the equilibrium real interest rate, and the stochastic shock u_t represents policy errors, which we assume to be white noise. All variables except for the interest rates in these equations are in logged form, and the corresponding foreign variables are denoted with a "*". We assume β_y , $\delta > 0$ and $\beta_\pi > 1$, and for notation simplicity, we assume the home and foreign central banks to have the same β policy weights.¹⁷

Under rational expectations, the efficient market condition for the foreign exchange markets equates cross-border differentials in interest rates of maturity m, with the expected rate of home currency depreciation and the currency risk premium over the same horizon:¹⁸

$$i_t^m - i_t^{m,*} = E_t \Delta s_{t+m} - \widetilde{\rho}_t^m, \forall m$$
(2)

Here $\Delta s_{t+m} \equiv s_{t+m} - s_t$ and $\tilde{\rho}_t^m$ denotes the risk premium of holding foreign relative to home currency investment between time t and t+m. We assume that $\tilde{\rho}_t^m$ depends on the general latent risk factors associated with asset-holding within each country over the period, and that these latent risks are also embedded in the bond-holding term premiums, ρ_t^m and $\rho_t^{m,*}$, at home and abroad:

$$\widetilde{\rho}_t^m = a_0 + a_{m,F} \rho_t^{m,*} - a_{m,H} \rho_t^m + \varsigma_t \tag{3}$$

To simplify notations, we set $a_0 = 0$ and consider the symmetric case where $a_{m,H} = a_{m,F} = a_m$. Combining the above equations and letting m = 1, we can express the exchange rate in the following differenced expectation equation:

$$s_t = \gamma f_t^{TR} + \kappa \widetilde{\rho}_t^1 + \psi E_t s_{t+1} + v_t \tag{4}$$

where $f_t^{TR} = [p_t - p_t^*, \tilde{y}_t - \tilde{y}_t^*, \pi_t^e - \pi_t^{*e}]'$, v_t is a function of policy error shocks u_t and u_t^* , and coefficient vectors, γ , κ , and ψ are functions of structural parameters defined above.¹⁹ Iterating the equation forward, we show that the Taylor-rule based model can deliver a net present value equation

 $^{^{17}}$ Our setup is what Papell et al (2009) term "asymmetric homogenous" in their comparisons of several variations of the Taylor-rule based forecasting equations.

¹⁸By assuming rational expectations, we rule out the role of expectation errors in $\tilde{\rho}$.

¹⁹Since these derivations are by now standard, we do not provide detailed expressions here but refer readers to e.g. Engel and West (2005) for more details.

where exchange rate is determined by the current and the expected future values of cross-country differences in macro fundamentals and risks:

$$s_{t} = \lambda \sum_{j=0}^{\infty} \psi^{j} E_{t}(f_{t+j}^{TR}|I_{t}) + \zeta \sum_{j=0}^{\infty} \psi^{j} E_{t}(\widetilde{\rho}_{t}^{j}|I_{t}) + \varepsilon_{t}$$

$$= \lambda \sum_{j=0}^{\infty} \psi^{j} E_{t}(f_{t+j}^{TR}|I_{t}) + \zeta \sum_{j=0}^{\infty} \psi^{j} a_{j}(\rho_{t}^{j} - \rho_{t}^{j,*}) + \varepsilon_{t}$$
(5)

where ε_t incorporates shocks, such as that to the currency risk (ς_t) , and is assumed to be uncorrelated with the macro and bond risk variables. Note that the second equality follows from eq.(3) and the definition of the risk premium: the perceived risk *at time* t about investment over future horizon j.

This formulation shows that the exchange rate depends on both expected future macro fundamentals and differences in the perceived risks between the two countries over future horizons. From this standard present value expression, we deviate from previous literature in deriving our exchange rate estimation equations; we emphasize the use of latent factors extracted from the yield curves of the two countries to proxy the two present-value terms on the right-hand side of eq.(5). We show in the next section that the Taylor-rule fundamentals are exactly the macroeconomic indicators the yield curves appear to embody information for, and of course, the bond term premiums ρ_t^j and $\rho_t^{j,*}$ are by definition a component of each country's yield curves. Exploiting these observations, we do not need to make explicitly assumptions about the statistical processes driving the Taylor-rule macro fundamentals to estimate eq.(5), as previous papers tend to do. Instead, we allow them to interact dynamically with the latent yield curve factors as we justify below.²⁰ Since nominal exchange rate is best approximated by a unit root process empirically, we focus our analyses on exchange rate change, Δs_{t+m} , as well as excess currency returns, which we define as:

$$XR_{t+m} = i_t^{m,*} - i_t^m + \Delta s_{t+m} (= \tilde{\rho}_t^m) \tag{6}$$

Note that according to our exchange rate definition, XR measures the excess return from dollar investment.

²⁰The use of the yield curves to proxy expectations about future macro dynamics and risks makes our model differ from the traditional approach in international finance, which commonly assume that the macro-fundamentals evolve according to a univariate VAR (e.g. Mark (1995) or Engel and West (2005), among others). See Chen and Tsang (2009) for a more detailed discussions.

2.2 The Yield Curve, Latent Factors, and the Macroeconomy

The yield curve or the term structure of interest rates describes the relationship between yields and their time to maturity. Traditional models of the yield curve posit that the shape of the yield curve is determined by the expected future paths of interest rates and perceived future uncertainty (the bond term premiums). While the classic expectations hypothesis is rejected frequently in empirical analyses, a large body of recent research has convincingly demonstrated that the yield curve contains information about expected future economic conditions, such as output growth and inflation.²¹ The underlying framework for our analysis builds upon the recent macro-finance models of the yield curve, which expresses a large set of yields of various maturities as a function of just a small set of unobserved factors, while allowing them to interact with macroeconomic variables. Below we briefly discuss this latent-factor literature and its connection with the macroeconomy.

2.2.1 The Nelson-Siegel Factors

Diebold, Piazzesi and Rudebusch (2005) advocate the factor approach for yield curve modeling as it provides a succinct summary of the few sources of systematic risks that underlie the pricing of various tradable financial assets. Among the alternative model choices, we adopt the Nelson-Siegel latent factor framework without imposing the no-arbitrage condition.²² The classic Nelson-Siegel (1987) model summarizes the shape of the yield curve using three factors: L_t (level), S_t (slope), and C_t (curvature). Compared to the no-arbitrage affine or quadratic factor models, these factors are easy to estimate, can capture the various shapes of the empirically observed yield curves, and have simple intuitive interpretations.²³ The three factors typically account for most of the information in a yield curve, with the R^2 for cross-sectional fits around 0.99. While the more structural noarbitrage factor models also fit cross-sectional data well, they do not provide as good a description

²¹Briefly, the expectations hypothesis says that a long yield of maturity m can be written as the average of the current one-period yield and the expected one-period yields for the coming m-1 periods, plus a term premium. See Thornton (2006) for a recent example on the empirical failure of the expectations hypothesis.

 $^{^{22}}$ Since the Nelson-Siegel framework is by now well-known, we refer interested readers to Chen and Tsang (2009) and references therein for a more detailed presentation of it.

²³The level factor L_t , with its loading of unity, has equal impact on the entire yield curve, shifting it up or down. The loading on the slope factor S_t equals 1 when m = 0 and decreases down to zero as maturity m increases. The slope factor thus mainly affects yields on the short end of the curve; an increase in the slope factor means the yield curve becomes flatter, holding the long end of the yield curve fixed. The curvature factor C_t is a "medium" term factor, as its loading is zero at the short end, increases in the middle maturity range, and finally decays back to zero. It captures the curvature of the yield curve is at medium maturities. See Chen and Tsang (2009) and references therein.

of the dynamics of the yield curve over time.²⁴ As our focus is to connect the dynamics of the yield curves with the evolution of macroeconomy and the exchange rate, our model extends the dynamic Nelson-Siegel model proposed in Diebold et al (2006) to the international setting, as presented in Section 3.2 below.²⁵

2.2.2 The Macro-Finance Connection

The recent macro-finance literature connects the observation that the short rate is a monetary policy instrument with the idea that yields of all maturities are risk-adjusted averages of expected short rates. This more structural framework offers deeper insight into the relationship between the yield curve factors and macroeconomic dynamics. Two empirical strategies are typically adopted in the literature. The first more atheoretical approach does not provide a structural modeling of the macroeconomic fundamentals but capture their dynamics using a general VAR. Ang, Piazzesi and Wei (2006), for example, estimate a VAR model for the US yield curve and GDP growth.²⁶ By imposing non-arbitrage condition on the yields, they show that the yield curve predicts GDP growth better than an unconstrained regression of GDP growth on the term spread.²⁷ Another body of studies model the macroeconomic variables structurally, such as using a New Keynesian model. Using this approach, Rudebusch and Wu (2007, 2008) contend that the level factor incorporates long-term inflation expectations, and the slope factor captures the central bank's dual mandate of stabilizing the real economy and keeping inflation close to its target. They provide macroeconomic underpinnings for the factors, and show that when agents perceive an increase in the long-run inflation target, the level factor will rise and the whole yield curve will shift up. They model the slope factor as behaving like a Taylor-rule, reacting to the output gap and inflation. When the central bank tightens monetary policy, the slope factor rises, forecasting lower growth in the future.²⁸

 $^{^{24}}$ See, e.g. Diebold et al (2006) and Duffee (2002).

 $^{^{25}}$ As discussed in Diebold et al (2006), this framework is flexible enough to match the data should they reflect the absence of arbitrage opportunities, but should transitory arbitrage opportunities actually exist, we then avoid the mis-specification problem.

²⁶Diebold, Rudebusch and Aruoba (2006) took a similar approach using the Nelson-Siegel framework instead of a no-arbitrage affine model.

 $^{^{27}}$ More specifically, they find that the term spread (the slope factor) and the short rate (the sum of level and slope factor) outperform a simple AR(1) model in forecasting GDP growth 4 to 12 quarters ahead.

 $^{^{28}}$ Dewachter and Lyrio (2006) and Bekaert et al (2006) are two other examples taking the structural approach. Dewachter and Lyrio (2006), using an affine model for the yield curve with macroeconomic variables, find that the level factor reflects agents' long run inflation expectation, the slope factor captures the business cycle, and the curvature represents the monetary stance of the central bank. Bekaert, Cho and Moreno (2006) demonstrate that the level

The above body of literature demonstrates the dynamic connection between latent yield curve factors and macroeconomic indicators - specifically the Taylor rule fundamentals - and thereby justifying their potential usefulness for proxying at least the first present value term in the right hand side of eq.(5). Extending the analysis into an international setting, we follow a similar approach as in Diebold et al (2006) and Ang et al (2006) to jointly estimate a dynamics Nelson-Siegel model of the yield curve and a VAR system of the latent yield factors, Taylor rule variables, and the exchange rate.

2.3 Bond Term Premium and Currency Risk Premium

Empirically, both the currency market and the bond market exhibit significant deviations from their respective risk-neutral efficient market conditions - the UIP and the expectation hypothesis with the presence of time-varying risk being the leading explanation for both empirical patterns.²⁹ As such, another measure of interest in our exchange rate model, eq.(5), is the bond term premiums ρ_t^m and $\rho_t^{m,*}$ embodied in the home and foreign yield curves. Based on the expectations hypothesis, the term risk premium perceived at t associated with holding a long bond until t + m (ρ_t^m) is the difference between the current long yield of maturity m and the average of the current one-period yield and its expected value in the upcoming m - 1 periods:³⁰

$$\rho_t^m \equiv i_t^m - \frac{1}{m} \sum_{j=0}^{m-1} E_t \left[i_{t+j}^1 \right]$$
(7)

The typically upward-sloping yield curves reflect the positive term premiums required to compensate investors for holding bonds of longer maturity. As mentioned earlier, these risks may include systematic inflation, liquidity, and other consumption risks over the maturity of the bond. While previous research has documented these premiums to be substantial and volatile (Campbell and Shiller 1991; Wright 2009), there appears to be less consensus on their empirical or structural

factor is mainly moved by changes in the central bank's inflation target, and monetary policy shocks dominate the movements in the slope and curvature factors.

 $^{^{29}}$ Fama (1984) and subsequent literature documented significant deviations from uncovered interest parity. In the bond markets, the failure of the expectation hypothesis is well-established; Wright (2009) and Rudebusch and Swanson (2009) are recent examples of research that studies how market information about future real and nominal risks are embedded in the bond term premiums.

 $^{^{30}}$ We note that as horizon *m* increases, the average of future short rate forecasts (the summation term) will approach the sample mean. So when *m* is large, the relative term premium of maturity *m* will roughly equal to the relative yields of maturity *m* minus a constant.

relationship with the macroeconomy.³¹ For our purposes, we use the difference between the term premiums across countries to measure the difference in the underlying risks perceived by investors over the investment horizon (see eq. 3); we do not explicitly motivate term premium movements beyond eq.(7) and expectation errors. Note that under the rational expectation paradigm, ρ_t^m will be model-dependent. In the empirical section below, we derive a measure of the time-varying term premiums based on our proposed macro-finance model, and study their linkage with exchange rate dynamics and currency risk premiums.³²

3 Estimation Strategy

3.1 Data Description

The main data we examine consists of monthly observations from August 1985 to July 2005 for the US, Canada, and Japan, and from October 1992 to July 2005 for the United Kingdom on account of the ERM regime change.³³ All rates are annualized.

- Yield data: Our zero-coupon bond yield include maturities 3, 6, 9, 12, 24, 36, 48 and 60 months, where the yields are computed using the Fama-Bliss (1987) methodology.³⁴ We filled in some missing data for 3 month yields using data from the Global Financial Data, and in cases where one-month yield differences cannot be obtained, we use the one-month forward exchange rate (end-of-period) from the same source. To complement our main analyses, we also look at yield data for the UK and the US over the period October 1992 July 2009, which are provided by the Bank of England.
- Macro data: Taylor-rule macroeconomic fundamentals are inflation and output gap relative to those in the U.S. We use inflation and industrial production obtained from the IMF's

³¹A common view among practitioners is that a drop in term premium, which reduces the spread between short and long rates, is expansionary and predicts an increase in real activity. Bernanke (2006) agrees with this view. However, based on the canonical New Keynesian framework, movements in the term premium do not have such implications. For example, Rudebusch, Sack, and Swanson (2007) point out that only the expected path of short rate matters in the dynamic output Euler equation, and the term premium should not predict changes in real activity in the future.

 $^{^{32}}$ The linkage between the bond and currency premiums is also explored in Bekaert et al (2007), though our model further incorporates dynamics of the macroeconomy fundamentals into the expectation formation process.

³³For the period October 1990 - September 1992, the UK was a participant of the Exchange Rate Mechanism (ERM), where the UK pound was effectively pegged within a small margin to countries in the European Community.

 $^{^{34}}$ We thank Vivian Yue for providing us with the yield data from Diebold, Li and Yue (2008), and refer readers to it for details on the dataset. To match with the timing of the monthly macroeconomic variables, we use yields at the second trading day of *the following month*. (That is, the May 2001 yield observations are yields quoted on the second trading day of June 2001.)

International Financial Statistics. Relative inflation, $\pi_t^R (= \pi_t^e - \pi_t^{*e})$, is defined as the difference in the annualized 3-month percentage change of the logged seasonally-adjusted CPI. The logged industrial production index of each country is fitted to a quadratic trend, and the residuals are used to compute the relative output gap, $\tilde{y}_t^R (= \tilde{y}_t - \tilde{y}_t^*)$.

• Exchange rate data: End-of-period monthly exchange rates are obtained from the FRED database. We express first-differenced (d) logged exchange rate as $ds_t = s_t - s_{t-1}$. (We note that we only report results based on the per-dollar rates below, but found qualitatively similar results using the non-dollar currency pairs.)

Table 1 presents the summary statistics for the representative relative yields and the relative Taylor rule fundamentals. Over the twenty year sample, we see that Japan's average interest rates at all maturities were lower than that of the U.S., and its coefficients of variation for the longer relative yields are much lower than those for the two other country pairs. All variables show high degrees of persistence.³⁵

INSERT TABLE 1 HERE

3.2 A Dynamic Macro-Yield Model of Nominal Exchange Rate

To implement the framework discussed in Section 2, we present a dynamic factor model which is an international extension of the Diebold et al (2006) yield curve-macro model. We refer readers to that paper for details of the modeling choice, and focus the below presentation on our extensions. The model has at its core a state-space system, with the dynamic Nelson-Siegel factor model as the measurement equation, and the state vector includes the latent yield factors, Taylor rule fundamentals, and the nominal exchange rate. Following previous work in both the international macro and finance literature, we do not structurally estimate a Taylor rule, nor impose any structural restrictions in our VAR estimations.³⁶ We use the atheoretical forecasting equations to capture any endogenous feedback among the variables.

³⁵Unit root tests on the extracted relative latent yield factors mostly reject the null of a unit root.

 $^{^{36}}$ This non-structural VAR approach follows from Engel and West (2006), Molodtsova and Papell (2009) and so forth on the exchange rate side, and Diebold et al (2006), among others, on the finance side.

3.2.1 A Dynamic Relative Factor Model

Noting that the exchange rate fundamentals discussed above are in cross-country differences, we measure the discounted sums in eq.(5) with the cross-country *differences* in their yield curves. From the panel of yields, we estimate the yield curve factors as latent variables that follow a firstorder vector autoregression. Specifically, assuming symmetry and exploiting the linearity in the factor-loadings, we fit three Nelson-Siegel factors of relative level (L_t^R) , relative slope (S_t^R) , and relative curvature (C_t^R) following the classical Nelson-Siegel formulation:³⁷

$$i_t^m - i_t^{m,*} = L_t^R + S_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m}\right) + C_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)\right) + \epsilon_t^m \tag{8}$$

As the number of yields is larger than the number of factors, eq.(8) cannot fit all the yields perfectly, so an error term ϵ_t^m is appended for each maturity as a measure of the goodness of fit.³⁸ The typical application of the Nelson-Siegel model involves estimating eq.(8) period by period without concerning how the yield curve evolves over time. We instead follow the dynamic approach first proposed by Diebold and Li (2006) and model the three relative factors together as a VAR(1) system.³⁹ The dynamic system can be expressed as:

$$f_t - \mu = A(f_{t-1} - \mu) + \eta_t \tag{9}$$

where

$$f_t - \mu = \begin{pmatrix} L_t^R - \mu_L \\ S_t^R - \mu_S \\ C_t^R - \mu_C \end{pmatrix}$$

The term η_t is a vector of disturbances, μ is a vector of constants, and A is a matrix of coefficients describing the dynamics of the three factors. To complement eq.(9), we express the relative Nelson-Siegel curve described by eq.(8) in vector form as well, with y_t representing the set of m relative

³⁷The parameter λ , which we estimate, controls the particular maturity the loading on the curvature is maximized.

³⁸The interpretation of the relative factors extends readily from the straightforward. For example, an increase in the relative level factor means the vertical difference between the entire home (e.g. Canadian, Japan, or UK) yield curve and the foreign (U.S.) one becomes more positive (or less negative).

 $^{^{39}}$ In Figures A1 - A3 in the Appendix, we show that the two approaches produce estimated factors that are highly correlated.

yields $i_t^m - i_t^{m,*}$ at time t and Λ the Nelson-Siegel factor loadings:

$$y_t = \Lambda f_t + \epsilon_t \tag{10}$$

Equations eq.(9) and eq.(10) form a state-space system that can be estimated by maximum likelihood using Kalman filtering. This dynamic *relative* Nelson-Siegel factor model corresponds to the closed-economy "yields-only model" proposed in Diebold et al (2006). As pointed out there, for the estimation to be feasible, the two sets of error terms are assumed to be uncorrelated:⁴⁰

$$\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.N \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \end{bmatrix}.$$
 (11)

We maintain this restriction throughout the rest of the paper under variations of model specification.

3.2.2 Macro and Yields-based Exchange Rate Models

Augmenting the dynamic relative factor model, we set up the following four exchange rate models for empirical comparisons:

1. The Macro-Yields model. This is our proposed model that incorporate both macro and financial variables into modeling exchange rate dynamics, allowing for joint interaction between the relative term structure and the macroeconomy:

$$f_t^{MY} - \mu = A(f_{t-1}^{MY} - \mu) + \eta_t \text{ and } y_t = \Lambda f_t^{MY} + \epsilon_t$$
(12)
where $f_t^{MY} = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)'$

The dimensions of the parameter matrices (μ, A, Λ, Q) and the disturbance term η_t adjust as appropriate from eqs.(9) and (10) above. In the measurement equation, we set the first three columns of matrix Λ to be zero so that the yields load only on the latent Nelson-Siegel factors as in the dynamic relative factor model.⁴¹ This restriction is consistent with the view that the latent factors are sufficient in summarizing information in the yield curves, which in

⁴⁰The disturbances are also assumed to be orthogonal to the initial state: $E(f_0\eta'_t) = E(f_0\epsilon'_t) = 0$. See Diebold et al (2006) for more details about this state-space setup and estimations.

⁴¹We note that Diebold et al (2006) makes the same assumption in their footnote 14.

turn embody expectations about macro dynamics and risks, as discussed above.

- 2. The Yields model. Dropping the two macroeconomic fundamentals in eq.(12), we reduce the state space system above to a latent factor-based model that explains exchange rate dynamics jointly with the term structure. That is, the yield model replaces f_t^{MY} above with $f_t^Y = (ds_t, L_t^R, S_t^R, C_t^R)'$. This setup reflects approaches presented in Clarida and Taylor (1997), Bekaert et al (2007), and Chen and Tsang (2009).
- 3. The Macro model. Eliminating yield curve factors in eq.(12), the state-space system reduces to a simple VAR model of the exchange rate, relative output gap, and inflation differences, which is similar to the standard monetary exchange rate model:

$$f_t^M - \mu = A(f_{t-1}^M - \mu) + \eta_t$$
where $f_t^M = (\tilde{y}_t^R, \pi_t^R, ds_t)'$

$$(13)$$

4. The Random Walk model. This is the standard benchmark motivated by the post-Meese-Rogoff (1983) literature, where $ds_t = \eta_t$.

3.2.3 Kalman Filter Estimation

We estimate the state space models above using maximum likelihood using Kalman filtering.⁴² To ensure that the variances in the model are positive, we estimate log variances and obtain standard errors by the delta method. Since we have a large number of parameters, choosing the initial values for the optimization problem is an important issue. We consider two sets of initial values. First, we set the variances to 1, λ to 0.0609 (the value commonly imposed for the Nelson-Siegel curve), and all other parameters to 0. The model takes some time to converge under these initial values. As an alternative, we adopt the two-step procedure in Diebold and Li (2005) to first obtain the relative factors using period-by-period OLS regressions. We then estimate the state transition VAR equation in eq.(12) using the OLS factors. The VAR coefficient estimates are then used to initialize the Kalman filter. The model converges faster under this approach, but the final results are almost identical to the previous ones. We use the Marquart algorithm for the optimization,

 $^{^{42}}$ See Kim and Nelson (1999) or Harvey (1981) for a discussion on estimating a state-space model by maximum likelihood.

and set the convergence criterion to 10^{-6} .

We report in Appendix C estimates of \hat{A} and \hat{Q} under the full Macro-Yield Model eq.(12) for each of the three country pairs. Despite the large number of estimated parameters, we see some significant off-diagonal estimates in \hat{A} , indicating the dynamic interactions among the variables. Foreshadowing our findings below, we see significant estimates from among both the set of macro and the set of financial variables in the 3rd row of \hat{A} (ds_t) for Japan and the UK. Figures A1-A3 plot the estimated relative term structure factors $\hat{L}_t^R, \hat{S}_t^R, \hat{C}_t^R$ against OLS estimates that do not impose any dynamic linkage (eq.8). We see the two are highly correlated, but the Kalman filtering process produces generally smoother estimates. Since the focus of this paper is on exchange rates, we present more focused tests and results below.

4 Longer Horizon Exchange Rate Predictability

4.1 Model Comparison

We evaluate the performance of the four exchange rate models presented above by comparing their exchange rate predictions from 3 months to 2 years ahead. We test for longer-horizon exchange rate predictability using the iterated-VAR approach, which has been widely used in the finance literature for testing stock return predictability.⁴³ We note that while long-horizon regressions and recursive out-of-sample forecasting are more common model evaluation procedures in the international macro literature, our short sample size and state-space estimation procedure preclude meaningful estimations under these tests.

We first estimate the models using the full-sample of monthly data; at each time t, we then iterate the estimated transition VAR(1) equation to generate predictions for horizons beyond one month. For example, under the Macro-Yield model (eq.12), the time-t forecast of f_{t+k}^{MY} , for k > 1, is:

$$E_t(f_{t+k}^{MY}) = (\widehat{A})^k (f_t^{MY} - \widehat{\mu}), \tag{14}$$

where $\hat{\mu}$ and \hat{A} are the full-sample estimates (as in Appendix C). The forecast error is then: $f_{t+k}^{MY} - E_t(f_{t+k}^{MY})$. Specifically for the exchange rate, since 1-month exchange rate change ($ds_t = s_t - s_{t-1}$) is the third variable in vector f_t^{MY} , which we denote as $[f_t^{MY}]_3$, the time-t forecast of

⁴³See Campbell (1991), Hodrick (1992), Patelis (1997), and more recently Lettau and Ludvigson (2005) as well.

1-month exchange rate change k-periods later would then be: $E_t(ds_{t+k}) = E_t(s_{t+k} - s_{t+k-1}) = (\widehat{A})^k [f_t^{MY} - \widehat{\mu}]_3$. The k-horizon exchange rate forecast error, FE_{t+k} , is therefore:

$$FE_{t+k} = \Delta s_{t+k} - (\widehat{A})^k [f_t^{MY} - \widehat{\mu}]_3 - (\widehat{A})^{k-1} [f_t^{MY} - \widehat{\mu}]_3 - \dots - \widehat{A} [f_t^{MY} - \widehat{\mu}]_3.$$
(15)

To compare the long-horizon predictive performance of the four models, we generate time-series of forecast errors and compute their root mean squared prediction errors (RMSE).

Table 2 reports the RMSEs and the p-values (in parentheses) for the Diebold-Mariano (1995) test that compares the model forecast with the RW benchmark. The bolded numbers in each row indicate the model with the smallest forecast errors (best performance) for the particular currency and horizon. First, we note that for all exchange rates, the forecast performance of the models improves as the horizon increases, and *some* fundamental-based model – be it macro, yield, or both - always outperforms the random walk statistically. For the year and the pound rates, the Macro-Yields model delivers the smallest forecast errors among all models over all forecast horizons, despite having the most parameters to estimate. The Diebold-Mariano test also picks its forecasts over the random walk ones, even at shorter horizons of less than a year. For the Canadian dollar, the Macro model has a slight edge, but the joint Macro-Yields model performs very similarly. Comparing the macro versus the finance approach, we see that while the Macro model does well for Canada and the UK in terms of significantly outperforming the random walk, the yield-factors are the ones that work in the case of Japan. We see this as a strong support for the more comprehensive approach we propose: the joint Macro-Yields model, encompassing elements from both, indeed stands out in its overall performance in these comparisons.

INSERT TABLE 2

We note that while the forecasts are made using only current variables, these forecasts are not true out of sample as $\hat{\mu}$ and \hat{A} are estimated using the whole sample.⁴⁴ However, despite the practical attractiveness of out-sample forecasting, Engel, Mark and West (2007) point out that it is not a reliable criterion for model evaluation. In addition, because our VAR uses only one-month exchange rate change, ds_t , and not the overlapping variable Δs_{t+m} , our approach thereby avoids the small-sample bias problem that plagues traditional long-horizon predictive regressions using

⁴⁴Our short sample and the large number of parameters prevent us from forecasting out of sample.

overlapping data.

4.2 More on the Macro-Finance Approach

Table 3 provides a closer look at the Macro-Yields model, where we compute the implied longhorizon R^2 statistics to assess the model's overall fit. We adopt the method proposed by Hodrick (1992) to calculate the contribution of each variable in the VAR system for predicting exchange rate change. Appendix A describes the procedure for computing each variable's individual as well as their joint $R^2s.^{45}$ This procedure avoids the small-sample bias in long-horizon regressions using overlapping data; it also allows for dynamic interactions between the exchange rate, macroeconomic fundamentals, and yield factors. In Panel 3a of Table 3, we see that, consistent with earlier results, the macro variables show more explanatory power across all forecast horizons for the Canadian dollar; however, the three factors still play a small role. For Japan, the relative factors have very high R^2s (up to 23% for C_t^R at the one year horizon), yet the macro variables also explain a significant share of the exchange rate variations. Patterns for the pound-dollar rate lean towards the factors as well, with the relative output gap offering some minute contribution.

Turning to the overall fit of the model, the last column of Table 3 shows that for predicting exchange rate changes one-month ahead, the 6-variables together explain between 5 to 10% of the variation. This is consistent with the view that much of the short-term exchange rate volatility is driven by noise. As the forecast horizon increases, the fit of the Macro-Yields model improves significantly. At the one-year horizon, for instance, the model can explain 40% of the movements in the Yen-dollar rate. Figures 1 to 3 plot the three-month and one-year currency forecasts based on the Macro-Yields model, along with the actual exchange rate changes over the same horizon. We see that while most of the high frequency exchange rate volatility remains unaccounted for, the model is successful in capturing the general movements of the currencies, especially at the longer-horizon.

INSERT TABLE 3 & Figures 1-3 HERE

 $^{^{45}{\}rm Since}$ the variables are correlated, the total R^2 is not the sum of individual.

4.3 Two Robustness Checks

Table 4a provides another test for the joint macro-finance approach using non-overlapping data and multivariate OLS regressions. Here we regress one-month and three-month exchange rate changes on both the macro variables and the latent yield factors, and test for the joint significance of each group using the Wald statistics. For the term structure factors, we use the smoothed estimates from the state space model above, although the results are similar if we use period-by-period Nelson-Siegel regressions to extract them. Table 4a shows that the results confirm findings from the state space estimations presented above. For Canada, the latent yield factors do not explain exchange rate changes, but the null hypothesis that the macro variables have no contribution ("No Macro") are rejected. For Japan and the UK, both the "No Macro" and the "No Factors" null hypotheses are strongly rejected. The macroeconomic fundamentals and term structure factors together explain between 5 to 7% of the movements in 1-month exchange rate change rate changes, and 9% and 15% of the movements in 3-month exchange rate changes.⁴⁶ In Appendix B, we report results from similar regressions using quarterly data. We see that while the fit of the model isn't as good, both macro and term structure factors show up as relevant.

INSERT TABLE 4 HERE

We conduct another robustness test using a more recent US-UK dataset from the Bank of England, covering yields over the period between October 1992 and July 2009.⁴⁷ We extract the three relative term structure factors by fitting eq.(8) period-by-period, and combine them into a six-variable VAR(1) with the changes in the dollar-pound rate and the two relative macroeconomic fundamentals. Long-horizon exchange rate predictions are produced using the iteration method as above, and in Table 5, we report the fit of the model computed based on Hodrick's method. We see that the new results using updated data and a variation in the estimation method confirm our findings in Table 3c. Even though the overall longer horizon fit is not as good here, we see both the macro and the term structure factors playing a role in explaining exchange rate dynamics. The model is able to explain about 10% of the exchange rare movement for all forecast horizons,

 $^{^{46}}$ The R^2s for the regressions in Table 4 are lower than those in Table 3 for three reasons: 1) the 3-month results in Table 4 discard data to avoid overlapping data, while the iterated VAR approach does not, 2) the VAR allows of feedback from exchange rate change to the explanatory variables, and 3) the results in Table 4 preclude the (tiny) predictive power of lagged exchange rate.

⁴⁷See Anderson and Sleath (1999) for the details on construction of these yield curve data.

and the slope factor appears to be contributing the most. Figure 4 provides a graphical view of the predicted values against actual pound-dollar movements. Overall, the model prediction tracks the actual series relatively well, except in the early 2000's. Note that in both Table 3c and Table 5, inflation differentials appear to contain no information; this may be related to UK's inflation targeting policy.⁴⁸

INSERT TABLE 5 and Figure 4 HERE

4.4 Currency Risk and the Bond Market Risk

We now look at how the currency risk premium, $\tilde{\rho}_t^m$, or excess currency returns XR_t , relate to the macro variables and the term structure factors. As discussed in Section 2.1, the risk premium demanded by investors for holding one currency over another should depend on the general risk climate between the two countries. We expect these risk to reflect the overall macroeconomic conditions perceived and observed in these countries, and they should be incorporated into the pricing of other assets as well, such as in the bond premiums. As a first cut, we use non-overlapping data and regress one-month and three-month excess currency returns on both the set of macro variables and the set of yield factors. As with the exchange rate changes above, we test for the joint significance of each group using the Wald statistics; Table 4b reports the results. We see very clearly that both the macro variables and the latent yield factors play a role in explaining currency risk premium. For Canada, the null hypothesis of "no Macro" is clearly rejected, though the factors also come in as significance in determining one-month excess return. For Japan, perhaps due to the relative quietness of their macro fundamentals over the last couple of decades, we see the yield factors containing most of the information that explain Yen-dollar risk premium. For the UK, both sets of variables come in as strongly significant. Together, the macroeconomic fundamentals and term structure factors explain 4% to 7% of the movements in 1-month excess return, and 14% to 28% of the movements in 3-month excess return.

We next look more explicitly at the linkage between currency risk premiums and the bond term premiums, as discussed in relation to eq.(3). First, we construct a measure for the cross-

⁴⁸While Canada also adopted an inflation target in 1991, the Canadian sample started in 1985. We use the post-ERM (1992) data for the UK.

country relative bond term premium that follows directly from eq.(7):⁴⁹

$$\rho_t^{R,m} = \rho_t^{m,*} - \rho_t^m \equiv i_t^{m,*} - i_t^m - \frac{1}{m} \sum_{j=0}^{m-1} E_t \left[i_{t+j}^{1,*} - i_{t+j}^1 \right]$$
(16)

To capture the expectation term on the right hand side, we make use of our Macro-Yields model to generate expected relative yields that are model-consistent.⁵⁰ The procedure is similar to the exchange rate forecast calculation discussed above (eq.14), but the variables of focus are the three yield curve factors: $E_t(L_{t+m}^R)$, $E_t(S_{t+m}^R)$, and $E_t(C_{t+m}^R)$. Using these forecasted factors, we calculate expected future relative short rates using the Nelson-Siegel eq.(8).

The relative term premium, $\rho_t^{R,m}$, captures the difference in the level of risk investors perceived in the home and foreign bond markets over investment horizon m. It measures the amount of compensation required for bearing the relative risk of holding longer-term foreign debt till maturity. Given our convention, an increase in ρ_t^R means higher perceived risk in the US market relative to that in the other country. We postulate that ρ_t^R captures the same latent relative risks that affect the currency risk premium $\tilde{\rho}_t^m = X R_t$ (again, measured as US dollar risk over the other currency). We regress the 9-month and 12-month excess currency returns on the current macroeconomic fundamentals and the constructed term premiums for the corresponding maturity.⁵¹ Newey-West standard errors are used to correct for serial correlations due to overlapping data, and we report the results in Table 6. We see that the ex-post realized excess currency return at t+m is positively correlated with the time-t term premiums of maturity m, conditional on the macroeconomic conditions. In other words, a rise in the *m*-period relative term premium, which can be interpreted as higher perceived risk in the US over the next m periods, predicts higher excess return in dollar investment over the same period. This pattern shows up strongly in all three currency pairs, supporting the view that yield curves embody latent risks that also drive currency returns.⁵² To illustrate this connection, we plot the currency risk premium, $\tilde{\rho}_t^m$, with the relative bond term premium, ρ_t^R , for 9 and 12-month horizons in Figures 5 and 6.

INSERT TABLE 6 and Figures 5&6 HERE

⁴⁹Note that the relative term premium is defined as foreign over home, matching our definition of the currency risk premium and excess return.

⁵⁰The VAR approach is proposed in Diebold, et al (2006) and Cochrane and Piazzesi (2005), among others.

⁵¹We do not consider shorter horizons as some of the 3 and 6-months yields are missing.

 $^{^{52}}$ This finding is consistent with discussions in Chen and Tsang (2009) on the yield curve and deviation from the uncovered interest parity puzzle.

5 Conclusions

This paper incorporates both monetary and financial elements into exchange rate modeling. It allows macroeconomic fundamentals targeted in Taylor-rule monetary policy to interact with latent risk factors embedded in cross-country yield curves to jointly determine exchange rate dynamics. As the term structure factors capture expectations and perceived risks about the future economic conditions, they fit naturally into the present-value framework of nominal exchange rate models. Our state-space model fits the data well, especially at longer horizons, and provides strong evidence that both macro fundamentals and latent financial factors matter for exchange rate dynamics. Separating out the bond term premiums from the yields, we further show that investors' expectation about the future path of monetary policy and their perceived risk both drive exchange rate dynamics.

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	Mean	St.Dev.	Max.	Min.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{ ho}(60)$
Can-US							
$i^3 - i^{3,*}$	1.197	1.904	6.718	-4.653	_	_	_
$i^{12} - i^{12,*}$	1.196	1.536	4.517	-2.428	0.926	0.693	-0.027
$i^{60} - i^{60,*}$	1.064	0.936	3.467	-0.849	0.912	0.542	-0.076
$i^{120} - i^{120,*}$	0.668	1.267	6.278	-2.112	0.885	0.447	-0.051
$y - y^*$	0.14	2.581	4.594	-5.356	0.961	0.61	-0.281
$\pi - \pi^*$	-0.126	0.482	1.32	-1.2	0.943	0.171	0
JP-US							
$i^3 - i^{3,*}$	-2.444	2.011	1.911	-6.211	—	_	_
$i^{12} - i^{12,*}$	-2.825	1.954	0.978	-6.296	0.98	0.715	-0.252
$i^{60} - i^{60,*}$	-3.071	1.332	-0.367	-5.723	0.944	0.598	-0.311
$i^{120} - i^{120,*}$	-3.499	1.099	-1.233	-6.321	0.826	0.452	-0.151
$y - y^*$	-0.852	7.788	18.174	-12.536	0.981	0.757	-0.249
$\pi - \pi^*$	-0.993	0.478	0.39	-1.98	0.914	0.215	0.097
UK-US							
$i^3 - i^{3,*}$	2.549	1.795	6.892	-0.489	—	—	_
$i^{12} - i^{12,*}$	1.861	1.889	6.062	-1.865	0.878	0.524	0.059
$i^{60} - i^{60,*}$	1.287	1.292	4.705	-1.893	0.898	0.52	0.093
$i^{120} - i^{120,*}$	0.394	1.706	5.862	-3.338	0.918	0.479	0.02
$y - y^*$	-0.169	2.889	5.078	-6.957	0.928	0.645	-0.301
$\pi - \pi^*$	0.274	0.592	2.18	-0.82	0.954	0.335	0.052

Table 1. Descriptive Statistics for Relative Bond Yields and Macro Fundamentals

Note: Our data sample is monthly from August 1985 to July 2005, of the *relative* variables between Canada, Japan, and the UK with the United States. $\hat{\rho}(\#)$ reports the sample autocorrelation at displacement #. Due to missing data on 3-month bond yields, we do not report $\hat{\rho}(\#)$ for $i^3 - i^{3,*}$.

RMSEs of Model and Random Walk Forecasts of Δs_{t+k}							
Horizon	Macro + Yields	Macro	Yields	RandomWalk			
$f_t =$	$\left(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R\right)$	$\left(\tilde{y}_t^R, \pi_t^R, ds_t \right)$	$\left(ds_t, L_t^R, S_t^R, C_t^R\right)$	$\Delta s_{t+k} = \epsilon_t$			
		Canada-US					
3	$10.46\ (0.11)$	$10.44 \ (0.09)$	$10.96 \ (0.38)$	11.20			
6	7.10(0.17)	$6.98 \ (0.10)$	7.74(0.74)	7.84			
12	5.14(0.10)	$4.92 \ (0.03)$	5.94(0.97)	5.95			
24	4.22(0.04)	$3.85 \ (0.00)$	5.18(0.53)	5.02			
		Japan-US					
3	$22.82 \ (0.04)$	24.21(0.40)	23.62(0.07)	24.83			
6	$15.11 \ (0.01)$	17.22(0.40)	16.24(0.02)	17.91			
12	7.48(0.00)	10.98(0.34)	9.58(0.00)	11.77			
24	$5.90 \ (0.00)$	$9.53\ (0.31)$	$6.95\ (0.01)$	8.65			
		UK-US					
3	$11.99 \ (0.08)$	$13.12 \ (0.63)$	13.19(0.62)	13.26			
6	$7.76 \ (0.06)$	8.48(0.10)	8.92(0.94)	8.94			
12	$5.61 \ (0.06)$	5.88(0.03)	6.45(0.59)	6.40			
24	4.26(0.13)	4.54(0.04)	4.87(0.64)	4.88			
	· · ·		· · · · ·				

Table 2. Predicting Exchange Rates: Model Comparisons

	f_t	$-\mu = A(f_{t-1} - \mu) + \eta_t$
$[i_t^m$	_	$i_t^{m*}] = \Lambda \left(L_t^R, S_t^R, C_t^R \right) + \epsilon_t$

Note: We estimated the state space model using Kalman filter. The state equation $f_t - \mu = A(f_{t-1}-\mu)+\eta_t$, is a VAR(1) with a model-dependent vector f_t , as defined in the table. In the measurement equation, $[i_t^m - i_t^{m*}]$ is the vector of relative yields of maturities m = 3, 6, 9, 12, 24, 36, 48 and 60 months at time t, and matrix Λ is the Nelson-Siegel factor loadings. We iterate the estimated VARs forward to generate predicted exchange rate changes, $E_t(\Delta s_{t+k})$, for future horizons from 3 to 24 months and calculate the root mean square prediction errors (RMSEs). The p-values for the Diebold-Mariano (1995) test comparing the model's prediction and that of the random walk are reported in the parentheses. Note that the sample for the UK starts after the ERM crisis (1992M10).

Table 3. Explaining Exchange Rate Changes Δs_{t+k} with Macroeconomic Fundamentals and Yield Curve Factors

$$\begin{split} f_t^{MY} - \mu &= A(f_{t-1}^{MY} - \mu) + \eta_t. \text{ where } f_t^{MY} = \left(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R\right) \\ &[i_t^m - i_t^{m*}] = \Lambda \left(L_t^R, S_t^R, C_t^R\right) + \epsilon_t \end{split}$$

Table 3a: Partial R^2 of Each Variable in the VAR (US-Canada)							
Horizon	Output Gap	Inflation	Ex. Rate	Level	Slope	Curvature	Total \mathbb{R}^2
1	0.02	0.03	0.00	0.01	0.00	0.00	0.05
3	0.05	0.07	0.00	0.02	0.01	0.01	0.13
6	0.08	0.10	0.01	0.04	0.02	0.01	0.21
12	0.10	0.14	0.01	0.05	0.03	0.01	0.28

Table 3b: Partial R^2 of Each Variable in the VAR (US-Japan)							
Horizon	Output Gap	Inflation	Ex. Rate	Level	Slope	Curvature	Total R^2
1	0.01	0.01	0.00	0.00	0.03	0.02	0.08
3	0.03	0.03	0.01	0.01	0.08	0.08	0.19
6	0.04	0.06	0.01	0.01	0.13	0.15	0.30
12	0.04	0.09	0.02	0.01	0.18	0.23	0.40

r -	Table 3c: Partial R^2 of Each Variable in the VAR (US-UK)							
Horizon	Output Gap	Inflation	Ex. Rate	Level	Slope	Curvature	Total \mathbb{R}^2	
1	0.01	0.00	0.00	0.02	0.01	0.01	0.10	
3	0.01	0.00	0.00	0.05	0.03	0.02	0.19	
6	0.01	0.00	0.00	0.04	0.04	0.01	0.22	
12	0.01	0.00	0.00	0.05	0.06	0.01	0.19	

Note: We iterate the estimated \hat{A} forward to generate forecasts for k-period exchange rate changes, Δs_{t+k} . The partial R^2 reports the contribution of each variable in explaining Δs_{t+k} . It is calculated using \hat{A} and the estimated covariance matrix of the VAR, \hat{Q} , based on the Hodrick (1992) method. Please refer to Appendix B for details.

Table 4a: Explaining Exchange Rate Changes Macroeconomic Fundamentals, Yield Factors, or Both?

	Wald test	<i>p</i> -values					
	No Macro	No Factors	R^2				
	\mathbf{Can}	ada					
Δs_{t+1}	0.01^{**}	0.62	0.03				
Δs_{t+3}	0.09^{*}	0.93	0.06				
	Jap	an					
Δs_{t+1}	0.01^{**}	0.00^{***}	0.05				
Δs_{t+3}	0.05^{*}	0.03^{**}	0.09				
UK							
Δs_{t+1}	0.00***	0.00***	0.07				
Δs_{t+3}	0.01^{**}	0.01^{**}	0.15				

$$\Delta s_{t+k} = a_0 + a_1 \tilde{y}_t^R + a_2 \pi_t^R + a_3 L_t^R + a_4 S_t^R + a_5 C_t^R + \varepsilon_t$$

4b: Explaining Excess Currency Returns

$XR_{t+k} = a_0 + a_1\tilde{y}_t^R + a_2\pi_t^R + a_3L_t^R + a_4S_t^R + a_5C_t^R + $	$-\varepsilon_t$
---	------------------

	Wald tost	n velue						
	wald test	<i>p</i> -values						
	No Macro	No Factors	R^2					
	Can	ada						
XR_{t+1}	0.01^{**}	0.05^{*}	0.06					
XR_{t+3}	0.04**	0.22	0.14					
	Japan							
XR_{t+1}	0.13	0.10^{*}	0.04					
XR_{t+3}	0.53	0.00^{***}	0.28					
UK								
XR_{t+1}	0.00^{***}	0.00***	0.07					
XR_{t+3}	0.01^{**}	0.00***	0.27					

Note: We use the Newey-West standard errors in the Δs_{t+k} and XR_{t+k} OLS regressions. The "No Macro" column reports the *p*-values of the Wald tests for the null hypothesis that macroeconomic fundamentals have no explanatory power ($a_1 = a_2 = 0$), and the "No Factors" column tests the null hypothesis that the relative factors do not matter ($a_3 = a_4 = a_5 = 0$). We use the last month of each quarter to create non-overlapping samples for the 3-month regressions. One-month excess return, XR_{t+1} , is calculated using the forward premium. The sample for the UK starts after the ERM crisis (1992M10). For Japan, the XR_{t+1} regression starts on October 1998 due to the limited availability of 1-month forward rate data.

Table 5: Explaining Exchange Rate Changes Δs_{t+k} with Macroeconomic Fundamentals and Yield Curve Factors More Recent UK Data:Oct 1992 - Jul 2009

$f_t - \mu =$	$= A(f_{t-1} - \mu) + \eta$	η_t .
where $f_t = ($	$(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S)$	C_t^R, C_t^R

Partial R^2 of Each Variable in the VAR								
Horizon	Output Gap	Inflation	Ex. Rate	Level	Slope	Curvature	Total \mathbb{R}^2	
1	0.02	0.00	0.03	0.00	0.00	0.00	0.08	
3	0.01	0.00	0.02	0.00	0.01	0.00	0.11	
6	0.01	0.00	0.01	0.00	0.03	0.00	0.11	
12	0.01	0.00	0.00	0.01	0.05	0.01	0.11	

Note: The yield curve factors are obtained by running the Nelson-Siegel model: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ period by period. We then estimate the VAR(1) above and iterate the estimated \hat{A} forward to generate forecasts for the k-period exchange rate changes, Δs_{t+k} . The partial R^2 reports the contribution of each variable in explaining Δs_{t+k} . It is calculated using \hat{A} and the estimated covariance matrix of the VAR, \hat{Q} , based on the Hodrick (1992) method. Please refer to Appendix B for details.

Table 6: Predicting 9-Month and 12-Month Excess-Returns with Macro Fundamentals and Relative Term Premium

	Output Gap	Inflation	Term Premium	\mathbb{R}^2
$9 ext{-Month}$	Excess Return			
Canada	$1.13(0.29^{***})$	$5.87(1.35^{***})$	$3.70(1.35^{***})$	0.40
Japan	-0.15(0.15)	$9.93(3.01^{***})$	$27.35(3.60^{***})$	0.51
UK	$1.20(0.32^{***})$	$10.16(3.38^{**})$	$11.82(3.65^{**})$	0.28
12-Month	h Excess Return			
Canada	$1.05 \ (0.26^{***})$	$5.55 (1.22^{***})$	$4.15 (1.19^{***})$	0.47
Japan	-0.12 (0.13)	10.94(2.34)	$22.63(2.74^{***})$	0.58
UK	$1.04(0.24^{***})$	$9.65(2.80^{***})$	11.72 (2.93***)	0.40

$$XR_{t+k} = a_0 + a_1\tilde{y}_t^R + a_2\pi_t^R + a_3\rho_t^{R,m} + \varepsilon_t, \ k = 9,12$$

Note: The regressions are estimated with Newey-West standard errors . Refer to the text for the calculation of the relative bond term premium $\rho_t^{R,m}$. We have also estimated the same regression using non-overlapping 9-month and 12-month data and obtained similar results.



Note: Predicted exchange rate changes $E_t(\Delta s_{t+k})$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix Λ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward k-periods to generate predicted exchange rate changes for k = 3 and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.



Note: Predicted exchange rate changes $E_t(\Delta s_{t+k})$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix Λ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward k-periods to generate predicted exchange rate changes for k = 3 and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons



Note: Predicted exchange rate changes $E_t(\Delta s_{t+k})$ are generated as follows: We first estimate a state space model with a VAR (1) state equation: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$, where $f_t = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)$, and a measurement equation: $[i_t^m - i_t^{m*}] = \Lambda (L_t^R, S_t^R, C_t^R) + \epsilon_t$ where matrix Λ is defined by the Nelson-Siegel factor loadings. The estimated VAR(1) is then iterated forward k-periods to generate predicted exchange rate changes for k = 3 and 12 months ahead. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons



Note: Using data provided by the Bank of England, we first obtain the relative yield curve factors by running period-by-period OLS regressions of the Nelson-Siegel model. We then estimate a VAR(1) model: $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$ where $f_t = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)$, and iterate it forward to generate predicted exchange rate changes for different future horizons. The model-generated predictions are plotted against the actual exchange rate changes over the corresponding horizons.







6 Appendix

6.1 Appendix A: VAR Multi-Period Predictions

To compute the partial R^2 for each variable and their total contribution in the VAR, we follow the procedure as described in Hodrick (1992). The method is also adopted in Campbell and Shiller (1988), Kandel and Stambaugh (1988) and Campbell (1991), among others. The VAR models described in Section 3.2.2 can be written as:

$$f_t = Af_{t-1} + \eta_t$$

where the constant term μ is omitted for notational convenience. Denote the information set at time t as I_t , which includes all current and past values of f_t . A forecast of horizon m can be written as $E_t(f_{t+m}|I_t) = A^m f_t$. By repeated substitution, first-order VAR can be expressed in its $MA(\infty)$ representation:

$$f_t = \sum_{j=0}^{\infty} A^j \eta_{t+j}$$

The unconditional variance of f_t can then be expressed as:

$$C\left(0\right) = \sum_{j=0}^{\infty} A^{j} Q A^{j}$$

Denoting C(j) as the *j*th-order covariance of f_t , which is calculated as $C(j) = A^j C(0)$, the variance of the sum, denoted as V_m , is then:

$$V_m = mC(0) + \sum_{j=1}^{m-1} (k-j) \left[C(j) + C(j)' \right]$$

We are not interested in the variance of the whole vector but only that of the long-horizon exchange rate change, ds_t , which is the third element in the vector f_t . We can define $e'_3 = (0, 0, 1, 0, 0, 0)$, and express the variance of the *m*-period exchange rate change as $e'_3V_me_3$.

To assess whether a variable in f_t , say the level factor L_t^R , explains exchange rate change $\Delta s_{t+m} = s_{t+m} - s_t$, we run a long-horizon regression of Δs_{t+m} on L_t^R . The VAR model for f_t allows us to calculate the coefficient from this regression based on only the VAR coefficient estimates. Since the level factor is the fourth element in f_t , the coefficient is defined as:

$$\beta_{4}(m) = \frac{e_{3}'[C(1) + \ldots + C(m)]e_{4}}{e_{4}'C(0)e_{4}}$$

where vector e_4 is defined as $e_4 = (0, 0, 0, 1, 0, 0)$. The numerator is the covariance between Δs_{t+m} and L_t^R , and the denominator is the variance of L_t^R . Finally, the R^2 as reported in the paper is calculated as:

$$R_4^2(m) = \beta_4 (m)^2 \frac{e_4' C(0) e_4}{e_3' V_m e_3}$$

The R^2 for all other variables in the vector f_t can be suitably obtained by replacing e_4 with e_1, e_2, e_3, e_5, e_6 .

To calculate the total R^2 for all explanatory variables, we calculate the innovation variance

of the exchange rate change as $e'_1 W_m e_1$, where

$$W_m = \sum_{j=1}^m (I - A)^{-1} \left(I - A^j \right) Q \left(I - A^j \right)' (I - A)^{-1'}$$

The total \mathbb{R}^2 is then:

$$R^{2}(m) = 1 - \frac{e_{1}'W_{m}e_{1}}{e_{m}'V_{m}e_{m}}$$

6.2 Appendix B: VAR with Quarterly Data

We pick the last month of each quarter over our monthly sample to create a quarterly sample, and we have 80 observations. Since the original model as described has more parameters than the observations, we cannot estimate the model using the state-space model using maximum likelihood. As a compromise (with some loss of efficiency), we first obtain the level, slope and curvature factors by an OLS regression for the Nelson-Siegel curve in every period, as in Chen and Tsang (2009). We then estimate a VAR for the extracted factors, output gap, inflation and 3-month exchange rate change. Only the estimated equation for the 3-month exchange rate is reported below.

Table A1: VAR Estimates with Quarterly Data for $s_{t+3} - s_t$									
Country	\widetilde{y}_t^R	π^R_t	$s_t - s_{t-3}$	L_t^R	S^R_t	C_t^R	R^2		
Canada	1.400	7.176	-0.119	-0.015	-0.128	0.224	0.126		
	(0.554)	(2.753)	(0.117)	(1.704)	(0.653)	(0.482)			
Japan	-1.276	-1.837	-0.084	8.942	5.127	-0.209	0.076		
	(0.494)	(7.055)	(0.116)	(4.573)	(1.858)	(1.144)			
UK	2.238	2.688	0.009	-9.027	-1.762	-1.963	0.020		
	(1.345)	(6.152)	(0.137)	(3.803)	(1.114)	(0.769)			

The sample for the UK is again after the ERM crisis (1992Q3-2005Q2), and the VAR is of order one as in the main text.

6.3 Appendix C: Estimates for the 6-Variable VAR in the Full Model

Below we report the estimates for A and Q from the Macro-Yield model eq.(12) for each of our country pairs:

$$y_t = \Lambda f_t^{MY} + \epsilon_t$$

$$f_t^{MY} - \mu = A(f_{t-1}^{MY} - \mu) + \eta_t$$

$$\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right]$$

where $f_t^{MY} = (\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R)'$. We also plot the estimated latent factors from the state space system against the ones obtained from period-by-period OLS regressions of eq.(8) with no dynamic linkage imposed. In the OLS regressions, we fix the coefficient λ at the value estimated by the state-space model.

Canada-US: VAR Coefficient Estimates from eq.(12), $f_t^{MY} = \left(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R\right)'$

	(\tilde{y}_{t-1}^R	π^R_{t-1}	ds_{t-1}	L_{t-1}^R	S_{t-1}^R	C_{t-1}^R
	$ ilde{y}_t^R$	0.975	-0.040	0.022	-0.026	0.022	0.040
		(0.035)	(0.140)	(0.043)	(0.119)	(0.032)	(0.026)
	π_t^R	0.000	0.932	0.005	0.004	-0.004	0.001
<u> </u>	1	(0.007)	(0.031)	(0.011)	(0.027)	(0.008)	(0.006)
	ds_t	0.0822	0.475	0.014	0.090	0.015	0.039
$\widehat{A} =$		(0.076)	(0.347)	(0.082)	(0.281)	(0.093)	(0.064)
	L_t^R	0.065	-0.065	0.003	0.745	0.015	0.069
		(0.036)	(0.145)	(0.053)	(0.112)	(0.029)	(0.026)
	S_t^R	-0.038	0.097	-0.032	0.157	0.817	0.053
		(0.075)	(0.250)	(0.095)	(0.210)	(0.067)	(0.044)
	C_t^R	-0.189	0.371	0.114	0.570	0.168	0.577
		(0.161)	(0.648)	(0.236)	(0.524)	(0.146)	(0.120)
	(\widetilde{y}_t^R	π^R_t	ds_t	L_t^R	S^R_t	C^R_t
	$\left(\begin{array}{c} \tilde{y}_t^R \end{array} \right)$	\widetilde{y}_t^R 0.487	$\begin{array}{c} \pi^R_t \\ -0.003 \end{array}$	$\frac{ds_t}{-0.133}$	L_t^R 0.104	$\begin{array}{c} S^R_t \\ -0.005 \end{array}$	$\begin{pmatrix} C_t^R \\ -0.369 \end{pmatrix}$
	$\left(\begin{array}{c} \tilde{y}_t^R \end{array} \right)$	\widetilde{y}_t^R 0.487 (0.055)	$\pi_t^R -0.003$ (0.010)	ds_t -0.133 (0.090)	L_t^R 0.104 (0.058)	$S_t^R -0.005$ (0.082)	$\begin{pmatrix} C_t^R \\ -0.369 \\ (0.227) \end{pmatrix}$
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	\widetilde{y}_t^R 0.487 (0.055)	$\begin{array}{c} \pi^R_t \\ -0.003 \\ (0.010) \\ 0.025 \end{array}$	ds_t -0.133 (0.090) -0.017	L_t^R 0.104 (0.058) -0.007	S_t^R -0.005 (0.082) 0.002	$\begin{array}{c} C_t^R \\ -0.369 \\ (0.227) \\ 0.040 \end{array}$
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\pi_t^R \\ -0.003 \\ (0.010) \\ 0.025 \\ (0.002)$	$ds_t -0.133 (0.090) -0.017 (0.021)$	$\begin{array}{c} L_t^R \\ 0.104 \\ (0.058) \\ -0.007 \\ (0.014) \end{array}$	S_t^R -0.005 (0.082) 0.002 (0.021)	$\begin{array}{c} C_t^R \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \end{array}$
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\pi_t^R -0.003 \\ (0.010) \\ 0.025 \\ (0.002)$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263$	$\begin{array}{c} L_t^R \\ 0.104 \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \end{array}$	$S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115$	$\begin{array}{c} C_t^R \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \end{array}$
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\begin{array}{c} \pi^R_t \\ -0.003 \\ (0.010) \\ 0.025 \\ (0.002) \end{array}$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \end{array}$	$\begin{array}{c} S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \end{array}$	$\begin{array}{c} C_t^R \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \end{array}$
$\widehat{Q} =$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\begin{aligned} &\pi^R_t \\ -0.003 \\ &(0.010) \\ &0.025 \\ &(0.002) \end{aligned}$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \\ \textbf{0.291} \end{array}$	$\begin{array}{c} S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \\ 0.054 \end{array}$	$\begin{array}{c} C^R_t \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \\ -1.215 \end{array}$
$\widehat{Q} =$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\pi_t^R -0.003 \\ (0.010) \\ 0.025 \\ (0.002)$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \\ \textbf{0.291} \\ (0.129) \end{array}$	$\begin{array}{c} S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \\ 0.054 \\ (0.082) \end{array}$	$\begin{array}{c} C^R_t \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \\ -1.215 \\ (0.447) \end{array}$
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \end{array}\right)$		$\begin{array}{c} \pi^R_t \\ -0.003 \\ (0.010) \\ 0.025 \\ (0.002) \end{array}$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \\ \textbf{0.291} \\ (0.129) \end{array}$	$S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \\ 0.054 \\ (0.082) \\ \textbf{1.208}$	$\begin{array}{c} C^R_t \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \\ -1.215 \\ (0.447) \\ -1.390 \end{array}$
$\widehat{Q} =$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\pi_t^R -0.003 \\ (0.010) \\ 0.025 \\ (0.002)$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \\ \textbf{0.291} \\ (0.129) \end{array}$	$S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \\ 0.054 \\ (0.082) \\ 1.208 \\ (0.226) \\ \end{cases}$	$\begin{array}{c} C^R_t \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \\ -1.215 \\ (0.447) \\ -1.390 \\ (0.300) \end{array}$
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \\ C_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.487 (0.055)	$\pi_t^R -0.003 \\ (0.010) \\ 0.025 \\ (0.002)$	$ds_t -0.133 (0.090) -0.017 (0.021) 2.263 (0.250)$	$\begin{array}{c} L^R_t \\ \textbf{0.104} \\ (0.058) \\ -0.007 \\ (0.014) \\ -0.033 \\ (0.136) \\ \textbf{0.291} \\ (0.129) \end{array}$	$S^R_t \\ -0.005 \\ (0.082) \\ 0.002 \\ (0.021) \\ 0.115 \\ (0.208) \\ 0.054 \\ (0.082) \\ \textbf{1.208} \\ (0.226) \end{cases}$	$\begin{array}{c} C^R_t \\ -0.369 \\ (0.227) \\ 0.040 \\ (0.063) \\ 0.464 \\ (0.604) \\ -1.215 \\ (0.447) \\ -1.390 \\ (0.300) \\ 7.391 \end{array}$



Japan-US: VAR Coefficient Estimates from eq.(12): $f_t^{MY} = \left(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R\right)'$

	(\tilde{y}_{t-1}^R	π^R_{t-1}	ds_{t-1}	L_{t-1}^R	S_{t-1}^R	C_{t-1}^R	
	$ ilde{y}^R_t$	0.972	-0.648	-0.041	0.217	0.056	-0.028	
		(0.019)	(0.293)	(0.036)	(0.157)	(0.088)	(0.052)	
	π^R_t	0.004	0.919	-0.004	-0.008	-0.001	0.002	
		(0.002)	(0.035)	(0.004)	(0.015)	(0.013)	(0.007)	
$\widehat{A} =$	ds_t	0.113	0.303	-0.052	-0.582	-0.399	-0.052	
		(0.049)	(0.599)	(0.080)	(0.353)	(0.212)	(0.128)	
	L_t^R	0.011	-0.011	-0.004	0.857	-0.023	0.035	
		(0.007)	(0.090)	(0.014)	(0.043)	(0.033)	(0.198)	
	S_t^R	0.005	0.145	-0.014	0.001	0.773	0.104	
		(0.013)	(0.124)	(0.020)	(0.103)	(0.051)	(0.025)	
	C_t^R	-0.016	-0.146	0.038	0.158	0.278	0.805	
		(0.025)	(0.249)	(0.044)	(0.159)	(0.093)	(0.057)	
	,	D	Ð		D	Ð		
	(\tilde{y}_t^R	π^R_t	ds_t	L_t^R	S^R_t	C_t^R	
	$\left(\begin{array}{c} \tilde{y}_t^R \end{array} \right)$	$ ilde{y}^R_t$ 1.829	$\begin{array}{c} \pi^R_t \\ 0.002 \end{array}$	ds_t -0.181	$\begin{array}{c} L_t^R \\ 0.065 \end{array}$	$S_t^R \\ -0.073$	$\begin{array}{c} C_t^R \\ 0.093 \end{array}$	
	$\left(\begin{array}{c} \tilde{y}_t^R \\ 0 \end{array}\right)$	$ ilde{y}^R_t$ 1.829 (0.202)	π^R_t 0.002 (0.023)	ds_t -0.181 (0.417)	L_t^R 0.065 (0.070)	$S_t^R - 0.073 \ (0.095)$	$\begin{array}{c} C_t^R \\ 0.093 \\ (0.223) \end{array}$	
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	$ ilde{y}_{t}^{R}$ 1.829 (0.202)	$\begin{array}{c} \pi^R_t \\ 0.002 \\ (0.023) \\ 0.033 \end{array}$	ds_t -0.181 (0.417) 0.024	L_t^R 0.065 (0.070) 0.003	$S_t^R - 0.073 \ (0.095) - 0.004$	$\begin{array}{c} C^R_t \\ 0.093 \\ (0.223) \\ 0.032 \end{array}$	
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	$egin{array}{c} ilde{y}^R_t \ {f 1.829} \ (0.202) \end{array}$	$\begin{aligned} \pi^R_t \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003) \end{aligned}$	$ ds_t -0.181 (0.417) 0.024 (0.051) $	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \end{array}$	S_t^R -0.073 (0.095) -0.004 (0.013)	$\begin{array}{c} C_t^R \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \end{array}$	
~	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \end{array}\right)$	$ ilde{y}_t^R$ 1.829 (0.202)	$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \end{array}$	$S_t^R \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096$	$\begin{array}{c} C^R_t \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \end{array}$	
$\widehat{Q} =$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \end{array}\right)$	$ ilde{y}_t^R$ 1.829 (0.202)	$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \end{array}$	$\begin{array}{c} S^R_t \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \end{array}$	$\begin{array}{c} C^R_t \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \end{array}$	
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \end{array}\right)$		$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \\ \textbf{0.179} \end{array}$	$\begin{array}{c} S^R_t \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \\ -0.131 \end{array}$	$\begin{array}{c} C_t^R \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \\ -0.123 \end{array}$	
$\hat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ \dots \\ \mu_t^R \end{array}\right)$	$ ilde{y}_t^R$ 1.829 (0.202)	$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \\ \textbf{0.179} \\ (0.035) \end{array}$	$\begin{array}{c} S^R_t \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \\ -\textbf{0.131} \\ (0.043) \end{array}$	$\begin{array}{c} C^R_t \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \\ -0.123 \\ (0.100) \end{array}$	
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \end{array}\right)$		$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \\ \textbf{0.179} \\ (0.035) \end{array}$	$S_t^R \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \\ -0.131 \\ (0.043) \\ 0.508 \\ 0.508$	$\begin{array}{c} C_t^R \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \\ -0.123 \\ (0.100) \\ -0.428 \end{array}$	
$\widehat{Q}=$	$\begin{pmatrix} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \\ \dots \\ D \\ $	$ ilde{y}_t^R$ 1.829 (0.202)	$\begin{aligned} \pi^R_t \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003) \end{aligned}$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \\ \textbf{0.179} \\ (0.035) \end{array}$	$S_t^R \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \\ -0.131 \\ (0.043) \\ 0.508 \\ (0.072) \\ \end{cases}$	$\begin{array}{c} C^R_t \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \\ -0.123 \\ (0.100) \\ -0.428 \\ (0.080) \end{array}$	
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \\ C_t^R \end{array}\right)$	$ ilde{y}_t^R$ 1.829 (0.202)	$\pi_t^R \\ 0.002 \\ (0.023) \\ 0.033 \\ (0.003)$	$ds_t -0.181 (0.417) 0.024 (0.051) 11.171 (1.069)$	$\begin{array}{c} L^R_t \\ 0.065 \\ (0.070) \\ 0.003 \\ (0.010) \\ -0.113 \\ (0.171) \\ \textbf{0.179} \\ (0.035) \end{array}$	$S_t^R \\ -0.073 \\ (0.095) \\ -0.004 \\ (0.013) \\ 0.096 \\ (0.236) \\ -0.131 \\ (0.043) \\ 0.508 \\ (0.072) \\ \end{cases}$	$\begin{array}{c} C_t^R \\ 0.093 \\ (0.223) \\ 0.032 \\ (0.034) \\ 0.426 \\ (0.651) \\ -0.123 \\ (0.100) \\ -0.428 \\ (0.080) \\ 1.540 \end{array}$	



UK-US: VAR Coefficient Estimates from eq.(12): $f_t^{MY} = \left(\tilde{y}_t^R, \pi_t^R, ds_t, L_t^R, S_t^R, C_t^R\right)'$

	(\tilde{y}_{t-1}^R	π^R_{t-1}	ds_{t-1}	L_{t-1}^R	S_{t-1}^R	C_{t-1}^R	
	$ ilde{y}_t^R$	0.874	-0.397	-0.087	0.379	0.160	0.082	
		(0.042)	(0.274)	(0.050)	(0.222)	(0.099)	(0.062)	
	π_t^R	0.004	0.928	-0.001	-0.009	-0.008	0.000	
		(0.009)	(0.052)	(0.009)	(0.038)	(0.015)	(0.011)	
~	ds_t	-0.273	0.456	-0.078	1.207	0.355	0.275	
$\widehat{A} =$		(0.155)	(0.790)	(0.117)	(0.627)	(0.272)	(0.161)	
	L_t^R	0.021	0.021	-0.016	0.999	0.031	0.031	
		(0.115)	(0.466)	(0.117)	(0.537)	(0.198)	(0.131)	
	S_t^R	-0.032	0.198	-0.016	0.099	0.938	0.047	
		(0.094)	(0.557)	(0.106)	(0.435)	(0.162)	(0.111)	
	C_t^R	0.026	-0.182	0.063	-0.572	-0.197	0.720	
		(0.384)	(1.615)	(0.369)	(1.806)	(0.626)	(0.445))
	/	$\sim D$	D		T D	aP	QP	、
	(~ P	\tilde{y}_t^R	π^R_t	ds_t	L_t^R	S_t^R	C_t^R	
	$\left(\begin{array}{c} \tilde{y}_t^R \end{array} \right)$	$ ilde{y}^R_t$ 0.862	$\begin{array}{c} \pi^R_t \\ 0.002 \end{array}$	ds_t 0.083	L_t^R -0.038	S_t^R -0.017	C_t^R 0.236)
	$\left(\begin{array}{c} \tilde{y}_t^R \\ B \end{array}\right)$	\widetilde{y}_t^R 0.862 (0.122)	π_t^R 0.002 (0.020)	ds_t 0.083 (0.373)	$L_t^R - 0.038$ (0.262)	$S_t^R - 0.017$ (0.236)	C_t^R 0.236 (0.758))
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	$ ilde{y}_{t}^{R}$ 0.862 (0.122)	$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ ((100)) \\ 0.022 \\ ((100)) \\ (($	ds_t 0.083 (0.373) 0.013	L_t^R -0.038 (0.262) -0.006	S_t^R -0.017 (0.236) 0.005	C_t^R 0.236 (0.758) 0.005)
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.862 (0.122)	$\begin{array}{c} \pi^R_t \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003) \end{array}$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040)$	$ \begin{array}{c} L_t^R \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \end{array} $	S_t^R -0.017 (0.236) 0.005 (0.036)	$\begin{array}{c} C_t^R \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \end{array}$)
	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \end{array}\right)$		$\begin{aligned} \pi^R_t \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003) \end{aligned}$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.2517) \\$	$ \begin{array}{c} L_t^R \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.031) \end{array} $	S_t^R -0.017 (0.236) 0.005 (0.036) 0.006	$\begin{array}{c} C_t^R \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \end{array}$	
$\hat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ \pi_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.862 (0.122)	$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003)$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.747) \\ \end{cases}$	$ \begin{array}{c} L_t^R \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \end{array} $	$\begin{array}{c} S^R_t \\ -0.017 \\ (0.236) \\ 0.005 \\ (0.036) \\ 0.006 \\ (0.632) \end{array}$	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \end{array}$	
$\hat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \end{array}\right)$	$ ilde{y}_t^R$ 0.862 (0.122)	$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003)$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.747) \\ \end{cases}$	$\begin{array}{c} L_t^R \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \\ 1.187 \end{array}$	$\begin{array}{c} S^R_t \\ -0.017 \\ (0.236) \\ 0.005 \\ (0.036) \\ 0.006 \\ (0.632) \\ -0.627 \end{array}$	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \\ \mathbf{-3.493} \end{array}$	
$\hat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ \tilde{\alpha}_t^R \end{array}\right)$		$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003)$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.747)$	$\begin{array}{c} L^R_t \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \\ 1.187 \\ (0.502) \end{array}$	$\begin{array}{c} S^R_t \\ -0.017 \\ (0.236) \\ 0.005 \\ (0.036) \\ 0.006 \\ (0.632) \\ -0.627 \\ (0.386) \end{array}$	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \\ \textbf{-3.493} \\ (1.360) \end{array}$	
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \end{array}\right)$		$\begin{array}{c} \pi^R_t \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003) \end{array}$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.747)$	$\begin{array}{c} L^R_t \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \\ 1.187 \\ (0.502) \end{array}$	S_t^R -0.017 (0.236) 0.005 (0.036) 0.006 (0.632) -0.627 (0.386) 1.263	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \\ \textbf{-3.493} \\ (1.360) \\ 0.619 \\ (4.933) \end{array}$	
$\widehat{Q} =$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \\ \end{array}\right)$	$ ilde{y}_t^R$ 0.862 (0.122)	$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003)$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ \textbf{5.237} \\ (0.747)$	$\begin{array}{c} L^R_t \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \\ 1.187 \\ (0.502) \end{array}$	$\begin{array}{c} S^R_t \\ -0.017 \\ (0.236) \\ 0.005 \\ (0.036) \\ 0.006 \\ (0.632) \\ -0.627 \\ (0.386) \\ 1.263 \\ (0.457) \end{array}$	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \\ \textbf{-3.493} \\ (1.360) \\ 0.619 \\ (1.035) \end{array}$	
$\widehat{Q}=$	$\left(\begin{array}{c} \tilde{y}_t^R \\ \pi_t^R \\ ds_t \\ L_t^R \\ S_t^R \\ C_t^R \end{array}\right)$		$\pi_t^R \\ 0.002 \\ (0.020) \\ 0.022 \\ (0.003)$	$ds_t \\ 0.083 \\ (0.373) \\ 0.013 \\ (0.040) \\ 5.237 \\ (0.747)$	$\begin{array}{c} L^R_t \\ -0.038 \\ (0.262) \\ -0.006 \\ (0.037) \\ -0.042 \\ (0.624) \\ 1.187 \\ (0.502) \end{array}$	$\begin{array}{c} S^R_t \\ -0.017 \\ (0.236) \\ 0.005 \\ (0.036) \\ 0.006 \\ (0.632) \\ -0.627 \\ (0.386) \\ 1.263 \\ (0.457) \end{array}$	$\begin{array}{c} C^R_t \\ 0.236 \\ (0.758) \\ 0.005 \\ (0.118) \\ -0.573 \\ (2.385) \\ \textbf{-3.493} \\ (1.360) \\ 0.619 \\ (1.035) \\ \textbf{12.864} \end{array}$	

