

Dynamics of market anomalies and measurement errors of risk-free interest rates Cho-Hoi Hui (Hong Kong Monetary Authority), Chi-Fai Lo

and Chin-To Fung (The Chinese University of Hong Kong)

For Hong Kong Monetary Authority, Federal Reserve Board and Federal Reserve Bank of Atlanta "Unconventional Monetary Policy: Lessons Learned" October 12-13, 2017, Hong Kong

The views and analysis expressed in this presentation are those of the presenter and do not necessarily represent the views of the Hong Kong Monetary Authority



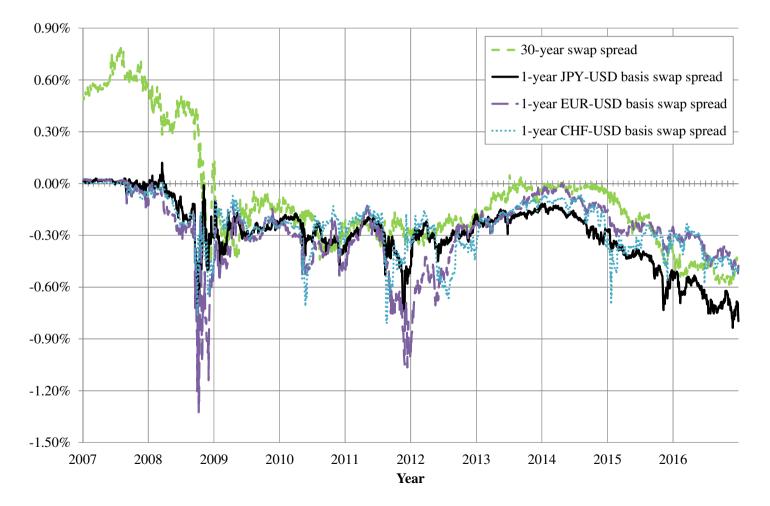
- Introduction
- Motivation of this study about two market anomalies CIP deviations and negative swap spread
- Tree-factor non-Gaussian-term structure model for the US Treasuries
- Estimations of the model
- Relationships between short-rate premium and market anomalies
- Conclusion



- The anomalies after the 2008 global financial crisis (GFC) are the widespread failure of covered interest parity (CIP); and the negative 30-year swap spread. Both phenomena are challenging for typical asset pricing models as they seem to imply a risk-free arbitrage opportunity under standard assumptions.
- Factors related to CIP deviations counterparty risk and funding liquidity risk [Baba and Packer (2009) and Hui et al. (2011) ]; strength of the USD and associated hedging demand, supply of USD funding and associated counterparty risk, banks' balance sheet structure, and asymmetric monetary policy shocks in particular normalisation in the US [Du et al. (2016), Liao (2016), Iida et al. (2016), Sushko et al. (2016), Wong et al. (2016) and Avdjiev (2016)]
- Most of these drivers have a common factor, that the USD and its interest rates play an important role in determining the CIP deviations.



30-year US dollar interest rate swap-Treasury spread and 1-year crosscurrency basis swap spreads on US dollar LIBOR of Japanese yen (JPY), euro (EUR) and Swiss franc (CHF)

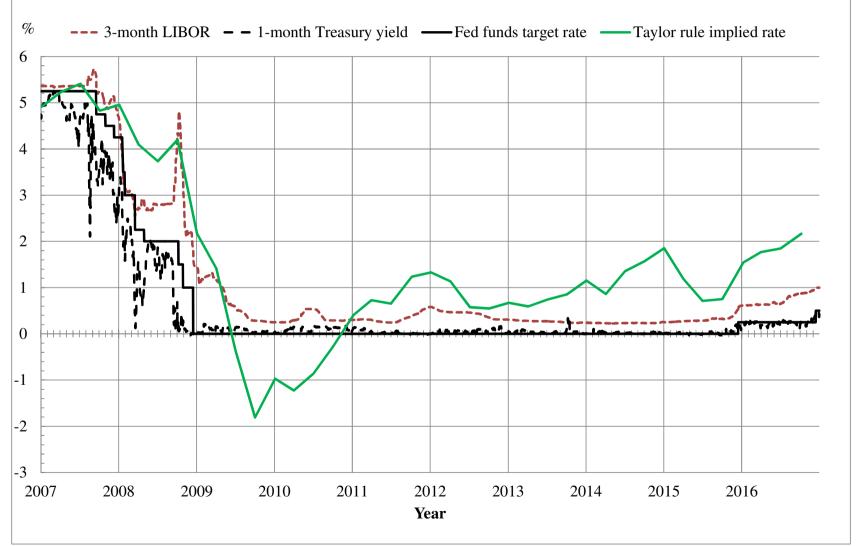




- Factors related to negative swap spreads the demand for swaps arising from duration hedging needs of underfunded pension plans; frictions for holding long-term bonds. [Klinglery and Sundaresan (2016), Jermann (2016)]
- Different from the recent studies on these two market anomalies, this paper shows that a latent factor embedded in the US Treasury yield curve displays a similar movement with the CIP deviations and 30-year swap spread over time since 2009.

## Taylor rule implied rate, 3-month US dollar LIBOR, 1month Treasury yield and Fed funds target rate





6

# Three-factor non-Gaussian term structure model of bond yields



- The first state variable in the model is the instantaneous short-term interest rate (short rate). It follows the Cox-Ingersoll-Ross (CIR) (1985) model, which is a general equilibrium model and preserves the non-negativity of interest rates.
- The second variable is a stochastic long-term mean to which the short rate reverts. This approach follows Balduzzi et al. (1998) assuming that the short rate and long-term mean are coupled stochastic processes.
- The coupled dynamics of the short rate and long-term mean can reflect observations on macro variables, such as expected inflations, consistent with fundamentals anticipated in the entire yield curve.
- The third state variable is a latent factor that captures macro information not already contained in the other two state variables. This state variable enters into the model such that the observable short-term Treasury yield is simply the sum of it and the short rate.



- The short-rate-premium factor in the term structure model likely captures the information related to demand of Treasuries due to their safety and liquidity which is not captured in the short-rate process.
- Recent studies show Treasuries carrying a convenience yield of holding them, which could be reflected in the short-term premium. Investors are willing to forgo some interest (a convenience yield) in exchange for owning a high-liquid and safe debt instrument, in particular Treasuries. [Krishnamurthy and Vissing-Jorgensen (2012), Krishnamurthy (2002), Longstaff (2004), Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014), Valchev (2016) and Del Negro et al. (2017)]
- The average (annualised) convenience yield on Treasuries ranges between 75 and 166 basis points (bps), and the estimates of the standard deviation range between 45 and 115 bps.



The instantaneous short rate r is described by the mean-reverting square-root (CIR) process:

$$dr_t = \kappa (\theta_t - r_t) dt + \sigma \sqrt{r_t} dZ_r$$

An advantage of the CIR model is that the risk-free short rate and its dynamics are determined endogenously as part of the general equilibrium. The general theory in Cox et al. (1985) shows that the nominal short-term interest rate, which can be expressed in terms of the real interest rate and the expected inflation rate.

The long-term mean  $\theta$  of the short rate in turn follows another mean-reverting square-root process:

$$d\theta_t = \alpha(\beta - \theta_t)dt + \eta\sqrt{\theta_t}dZ_{\theta}$$

In the stochastic mean model, the short rate can remain near the zero bound if the long-term mean level  $\theta$  is also low.



A factor *L* follows the stochastic process as:

 $dL_t = -\xi L_t dt + \gamma dZ_L$ 

The process is a special case of the constant elasticity of variance (CEV) model. The value L at time t proxies for macro and market information the market participants care about when trading the bonds. A similar factor is used by Piazzesi (2005), who assumes an exogenous process to capture information not contained in the other state variables that could affect the yield curve.

Given that investors value the safety and liquidity of US Treasuries, they are willing to forgo some interest in exchange for owning a high-liquid and safe debt instrument, i.e. convenience-yield component in Treasury yields. A larger convenience yield reflects higher demand for Treasuries.



The price of a zero-coupon bond is given by:

$$P_t(\tau, r, \theta, L) = E_t^Q \left[ \exp(-\int_t^T (r_t + L_t) dt) \right]$$

We assume that the risk-neutral measure has been chosen by the market in such a way that the adjusted discount rate (r + L) is the effective interest rate matching the observed bond yields.

The observed Treasury interest rate is lower than the "true" risk-free interest rate by an amount of the convenience yield. The construction of the term structure model suggests that the short rate r contains the information on the "true" riskfree interest rate. Strong demand for Treasuries will cause negative L, which pushes down the effective interest rate (r + L).

The conditional expectation in the above equation can be calculated by solving a partial differential equation (MLM by Ait-Sahalia & Kimmel, 2010) :

$$\begin{split} (r+L)P &= \frac{\partial P}{\partial t} + \frac{1}{2}\sigma^2 r \frac{\partial^2 P}{\partial r^2} + \left(\kappa\theta - (\kappa + \lambda_r \sigma)\right) \frac{\partial P}{\partial r} \\ &+ \frac{1}{2}\eta^2 \theta \frac{\partial^2 P}{\partial \theta^2} + \left(\alpha\beta - (\alpha + \lambda_\theta \eta)\right) \frac{\partial P}{\partial \theta} + \frac{1}{2}\gamma^2 \frac{\partial^2 P}{\partial L^2} - (\xi L + \lambda_L \gamma) \frac{\partial P}{\partial L} \end{split}$$

# Estimation results of term structure model



### Maximum likelihood estimates

	<b>Estimates</b>	<u>t-ratios</u>
Short rate process (r)		
mean reversion $(\kappa)$	0.6973	22.21
volatility $(\sigma)$	0.0520	26.83
risk premium $(\lambda_r)$	-8.2475	-11.53
Long-term mean process ( <i>θ</i> )		
mean reversion $(\alpha)$	0.0647	0.21
long-term mean $(\beta)$	0.0174	0.22
volatility $(\eta)$	0.0377	2.16
risk premium $(\lambda_{\theta})$	0.8010	0.10
Exogenous process (L)		
drift (ξ)	-0.0791	-4.79
volatility $(\gamma)$	0.0014	2.43
risk premium $(\lambda_L)$	-0.5826	-0.97

Note: The sample is weekly from January 1990 to December 2016.



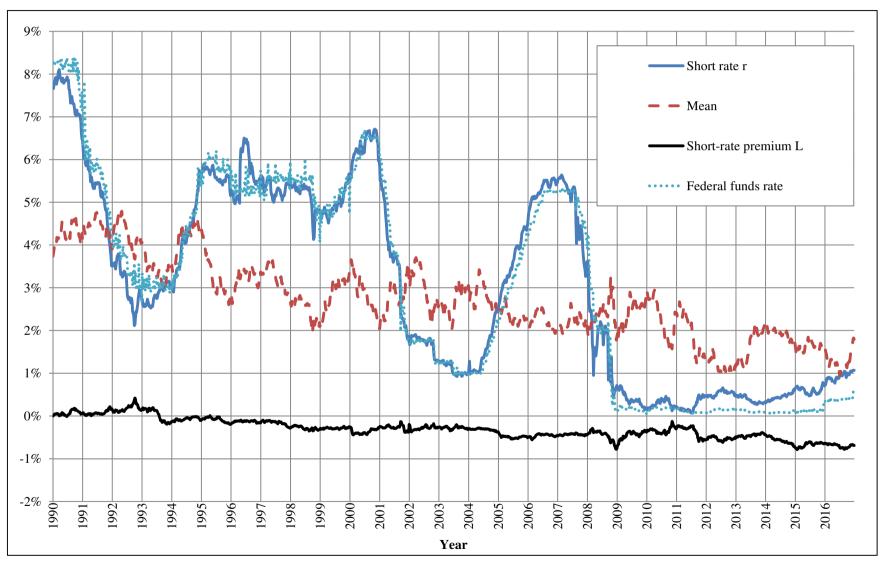
## Mean and standard deviation of absolute pricing errors (in %)

	6-month	1-yr	2-yr	3-yr	4-yr	5-yr	6-yr	7-yr	8-yr	9-yr	15-yr	20-yr
Maan												
<u>Mean</u>	0.11	0.20	0.27	0.29	0.27	0.22	0.20	0.15	0.12	0.06	0.12	0.17
Standard Deviation												
	0.08	0.13	0.19	0.21	0.20	0.17	0.14	0.10	0.07	0.04	0.07	0.11

Note: Absolute pricing errors are defined as absolute differences between the actual yields and the model-implied yields.

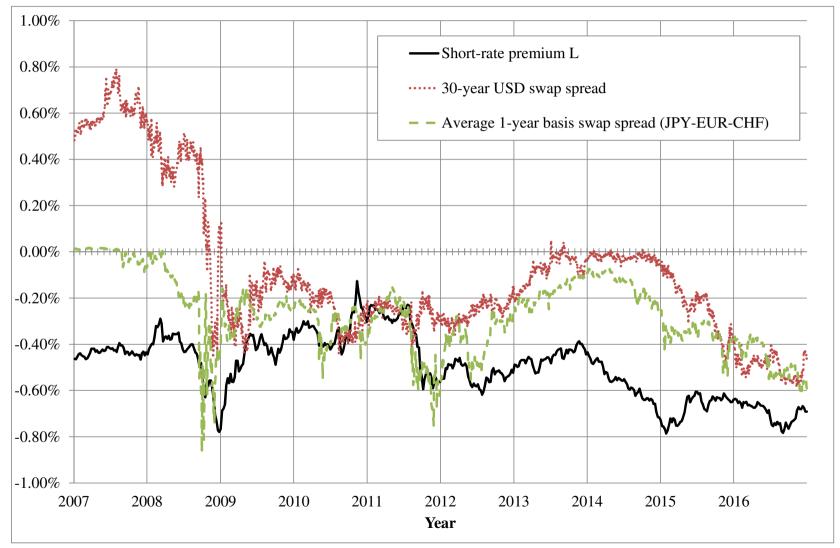
Estimated state variables





# Swap-Treasury spread, average 1-year cross-currency basis swap spreads and short-rate premium L

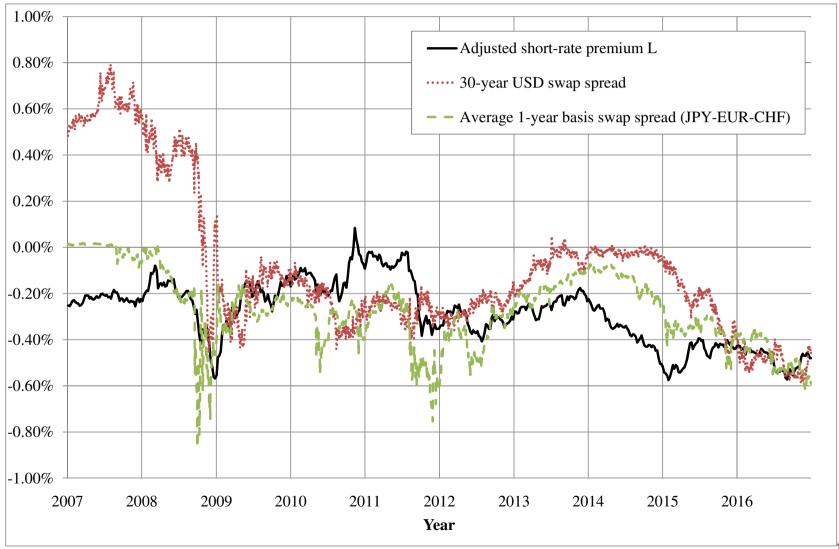




15

# Swap-Treasury spread, average 1-year cross-currency basis swap spreads and adjusted short-rate premium L





16

L



#### **Descriptive statistics**

	T 1		T 1		T 1	
-	Level	Change	Level	Change	Level	Change
Mean	-0.005	1.79E-06	-0.003	-4.12E-06	-0.002	-1.02E-05
Median	-0.005	0.000	-0.003	4.17E-07	-0.002	-5.00E-06
Maximum	-0.001	0.001	-0.001	0.002	0.001	0.002
Minimum	-0.008	-0.001	-0.007	-0.002	-0.006	-0.003
Std. Dev.	0.001	2.13E-04	0.001	3.37E-04	0.002	3.63E-04
Skewness	0.138	0.048	-0.362	0.463	-0.368	-0.671
Kurtosis	2.189	5.721	2.491	12.503	2.394	11.915
ADF test statistics	-1.679	-17.840 ***	-1.802	-27.013 ***	-2.249	-22.166 ***
Phillips-Perron test statistics	-1.995	-17.791 ***	-2.470	-27.176 **	-2.233	-22.062 ***
Correlation with $L/\Delta L$	-	-	0.394	0.254	0.189	-0.269
Observations	419	418	418	416	405	391

Average 1-year basis swap spread 30-Year USD interest rate swap

rate-Treasury spread

(USD vs JPY-EUR-CHF)

#### Notes:

1. \*\*\*, \*\* and \* indicate significance at levels of 1%, 5% and 10% respectively.

2. Both tests check the null hypothesis of unit root existence in the time series, assuming nonzero mean in the test equation.

3. The correlations for level of the variables are the correlations with L, and those for change are the correlation with  $\Delta L$ 

## Empirical results of cointegration



Tests for cointegration	Average 1-year basis swap spread (USD vs JPY-EUR-CHF)	30-Year USD interest rate swap rate- Treasury spread	
Engle-Granger single-equation test (Null hypothesis: residual has an unit root)			
ADF test statistic Phillips-Perron test statistic	-2.478 ** -2.356 **	-3.034 *** -2.984 ***	
1 mmps-1 erron test statistic	-2.550	-2.904	

Notes:

1. \*\*\*, \*\* and \* indicate significance at a level of 1%, 5% and 10% respectively.

2. The cointegration test uses the Augmented Dickey-Fuller and Phillips-Perron tests to check the null hypothesis that the residuals of the regression of L on 1year basis swap spread (USD vs JPY-EUR-CHF average) or 30-year USD interest rate swap rate-Treasury spread are non-stationary assuming zero mean in the test equation. The critical value of the test is obtained from MacKinnon (1996).

## Empirical results of cointegration



### Estimates of cointegrating vectors (i.e., the long-run part of Eq.(13))

Dependent variable:	Average 1-year basis swap spread (USD vs JPY-EUR-CHF)	30-Year USD interest rate swap rate-Treasury spread
$L(\beta)$	0.354 ***	0.199 ***
Constant	-0.001 ***	-0.001 ***

Notes:

\*\*\*, \*\* and \* indicate significance at a level of 1%, 5% and 10% respectively.

#### **Estimation results of short-run dynamics**

Dependent variable:	ΔAverage 1-year basis swap spread (USD vs JPY-EUR-CHF)	Δ30-Year USD interest rate swap rate-Treasury spread
Constant	-6.03E-06	-9.56E-07 *
Speed of adjustment	-0.026 *	-0.027 *
$\Delta L_{ ext{t-1}}$	0.019	-0.078
$\Delta L_{t-2}$	-0.074	-0.075
Dependent variable <sub>t-1</sub>	0.037	-0.296 ***
Dependent variable <sub>t-2</sub>	0.018	-0.037

Notes:

\*\*\*, \*\* and \* indicate significance at a level of 1%, 5% and 10% respectively.



### Summary statistics for explanatory variables in regression of L

Variable	Mean	Standard Deviation	Minimum	Median	Maximum	ρ
$\Delta BBB$	-0.027	0.208	-0.640	-0.011	0.797	0.176
ΔΑ	-0.023	0.200	-0.745	-0.003	0.831	0.188
∆Foreign holdings	33.152	53.391	-113.000	38.900	178.200	0.272
ΔVIX	-0.005	0.181	-0.373	-0.018	0.705	0.071
Treasury Buyback	0.001	0.005	0.000	0.000	0.025	-0.042

Note: This table reports summary statistics for the explanatory variables in the regression of  $\Delta L$ . The variables  $\Delta BBB$  and  $\Delta A$  are the monthly changes in the Bloomberg 5-year US industrial BBB and A corporate bond yield indexes respectively measured in percentage points.  $\Delta$ (Foreign Holdings) is the monthly change in the total amount of foreign holdings of U.S. Treasury bonds measured in billions of dollars.  $\Delta$ VIX is the log-difference of monthly average of VIX. Treasury Buyback is the market value in \$billions of all Treasury buybacks during the month. The data are monthly from September 2008 to December 2016. The number of observations for each time series is 100.

## Regression results of short-rate premium L



Variables	Coefficients	t-statistics	
Constant	7.61E-05*	1.677	
$\Delta L_{ ext{t-1}}$	0.273**	2.502	
$\Delta BBB_t$	2.02E-03***	3.557	
$\Delta A_t$	-2.24E-03***	-3.552	
$\Delta$ Foreign Holdings <sub>t-1</sub>	-1.74E-06**	-2.459	
$\Delta VIX_{t-1}$	-3.90E-04*	-1.676	
Treasury Buyback <sub>t-1</sub>	6.84E-03	0.919	
QE1 <sub>t</sub>	-1.97E-03***	-9.445	
QE2 <sub>t</sub>	-1.07E-03***	-5.899	
QE3 <sub>t</sub>	6.40E-05	0.617	
OT1 <sub>t</sub>	-1.09E-03***	-7.719	
OT2 <sub>t</sub>	-4.00E-04***	-6.975	
Adj. $R^2$	0.368		
No. of Observations	100		

Note: The table presents the results of estimating  $\Delta L$  on a monthly basis. \*\*\*, \*\*, and \* respectively indicate significance at the 1%, 5%, and 10% level. The robust t-statistics are based on White heteroskedasticity-consistent standard errors and covariance. Adjusted R<sup>2</sup> estimates are provided in the row labelled "Adj.  $R^{2"}$ .  $\Delta BBB$  and  $\Delta A$  are the monthly changes in the Bloomberg 5 year US industrial BBB and A corporate bond yield indexes respectively in percentage points.  $\Delta$ (Foreign Holdings) is the monthly change in the total amount of foreign holdings of U.S. Treasury bonds measured in billions of dollars.  $\Delta VIX$  is the log-difference of monthly average of VIX. Treasury Buyback is the market value in \$billions of all Treasury buybacks during the month. QE1, QE2 and QE3 are the dummy variables for the months of executing three quantitative easing programs. OT1 and OT2 are the dummy variables for the months of executing two rounds of operational twist. The sample is from September 2008 to December 2016.

## Conclusion



- Using a three-factor non-Gaussian term structure model, the short-rate premium estimated from the US Treasury yield curve tends to move in tandem with the CIP deviations and the negative swap spread over time since 2009.
- The dynamics between this premium and the two market anomalies are found to be cointegrated, suggesting a long-run equilibrium between them.
- The empirical analysis shows that the short-rate premium captures demand for Treasuries due to increased amounts of Treasuries held by foreign investors, the effects of the Fed's quantitative easing policy and operation twist, and the risk aversion in the financial market. This indicates that the short-rate premium reflects the convenience yield embedded in the Treasury yield curve.
- The above findings suggest that the anomalies manifest the "measurement error" of USD risk-free interest rates, consistent with recent studies common factor that the USD and its interest rates play an important role in determining the CIP deviations.
- Both the FX swap and interest rate swap markets could have corrected this measurement error by adding spreads on USD LIBOR, which makes reference to short-term Treasury interest rates. In other words, the FX swap and interest rate swap markets may have adjusted the USD risk-free interest rate in their corresponding instruments.



P.S. Du et al. (Aug 2017) show that in the post-GFC period, the CIP deviations for interbank rates tend to increase the US Treasury Premium because the US has lower sovereign CDS spreads than the average G10 country and swap market mispricing on average makes the swap-implied dollar yield higher than the direct dollar yield.