VaR and Stress Test: The Impact of Fat-Tail Risk and Systemic Risk on Capital Requirements of Financial Institutions

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Outline

- Objectives
- Historical Crises
- The Model
- Extreme Market Losses (EML) and Tail Risk Tolerance (TRT)
- Methodology, Data and Hypotheses
- Empirical Results
- Conclusion

Objectives

- "*Stress tests complement standard capital ratios by adding a more forward-looking perspective and by being more oriented toward protection against so called tail risks; by design, stress tests help ensure that banks will have enough capital to keep lending even under highly adverse circumstances...*" Bernanke (2013)
- Use "tail risks" employed by Bernanke (2013) and a unified approach recommended by Berkowitz (1999) and Kupiec (2000) to eliminate the weaknesses of the stress tests and VaR

Objectives

- Proposed a statistically parsimonious model that allows stable Paretian distributions to capture the tail risk of the VaR as well as the “fat-tail” risk embedded in the “severe” stress scenarios
- Proposed a new measure “Probability of EML” that is the actual likelihood of EML in the future
- Proposed a new measure “Tail Risk Tolerance (TRT)”, it accesses the Probability of EML that the bank is able to bear without getting into bankruptcy

Historical Crises

Table 1: The annual EML of S&P 500 (losses more than 20%) within 70 years (after World War II)

Years	Losses of S&P 500	Events
1974	26%	The collapse of the Bretton Woods system over the previous two years
2002	22%	Internet Bubble Burst
2008	37%	Subprime Mortgage Crisis

*The annual return in 1981, 1 year after the beginning of the saving and loan crisis, is −5%.

*The annual return in 2001, the 911 attacks, is −12%.

Historical Crises

Table 2: The impact of two major financial crises in U.S.

Crises	Start	End	Bailout (Billion)	Output Loss/1	Fiscal Cost/2
Saving and Loan Crisis/3	1980	1989	153	0.0%	3.7%
Subprime Mortgage Crisis/4	2007	2009	9,700	31.0%	4.5%

/1: In percent of GDP. Output losses are computed as the cumulative sum of the differences between actual and trend real GDP over the period [T, T+3], expressed as a percentage of trend real GDP, with T the starting year of the crisis.

/2: In percent of GDP. Fiscal costs are defined as the component of gross fiscal outlays related to the restructuring of the financial sector. They include fiscal costs associated with bank recapitalizations but exclude asset purchases and direct liquidity assistance from the treasury.

/1 & /2 are provided by Laeven and Valencia (2012).

/3 The detail of saving and loaning crisis can be obtained in Federal Deposit Insurance Corporation (FDIC) website. According to the "FINANCIAL AUDIT: Resolution Trust Corporation's 1995 and 1994 Financial Statements" in July 1996, the direct cost used to solve saving and loaning crisis is about $152.6 billion. The direct cost comprises the Federal Savings and Loan Insurance Corporation cost and Resolution Trust Corporation resolution cost.

/4 The subprime mortgage crisis starts in mid-2007 and the U.S. congress passes the American Recovery and Reinvestment Act of 2009 (ARRA) in early 2009. According to Bloomberg news at February 9, 2009, the government bailout is around 9.7 trillion U.S. dollars.

Figure 1: The density functions of normal distribution (the line with triangles), stable distribution (the line with circles) and empirical Distribution (histogram), the data is bond returns in 2006

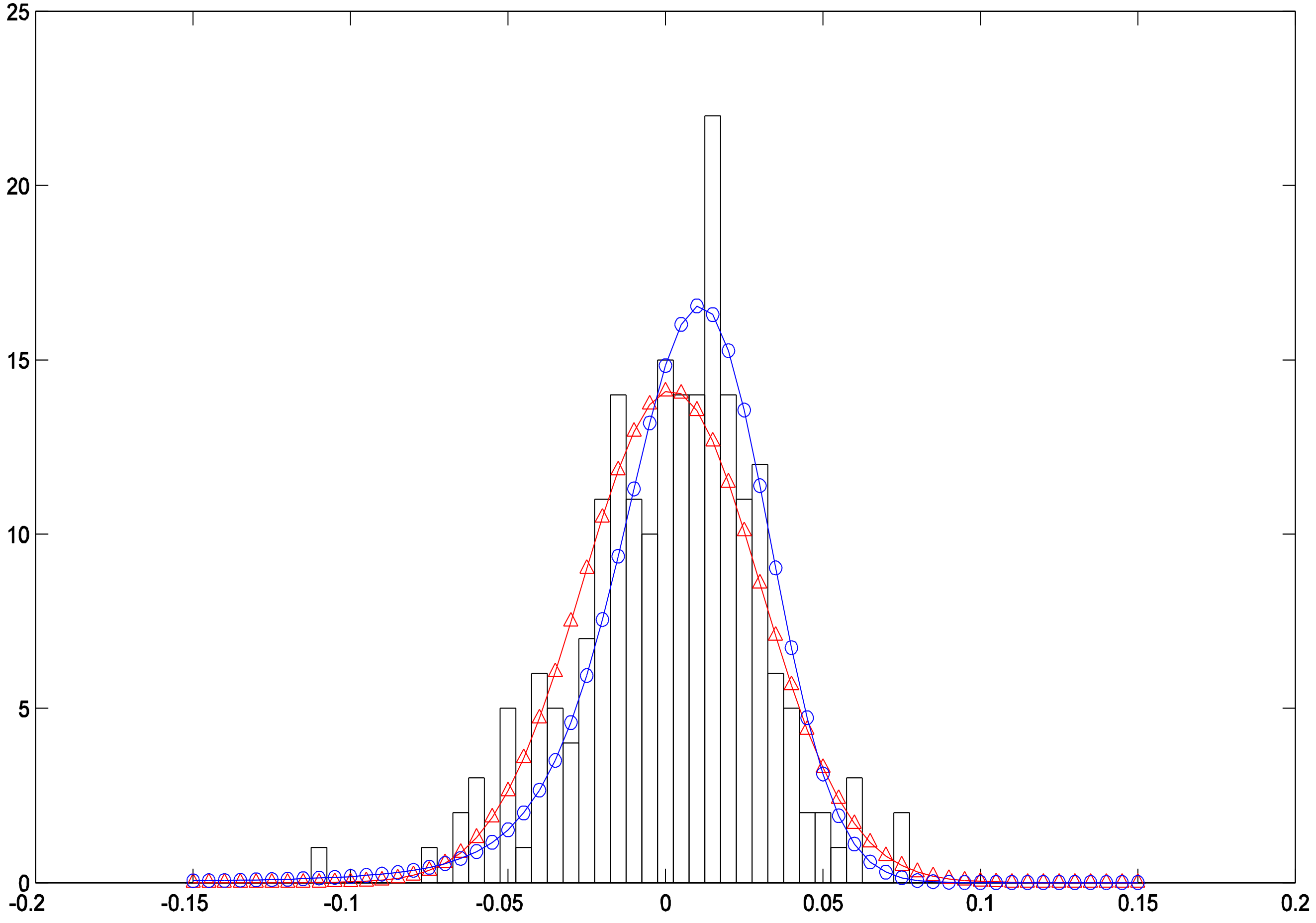
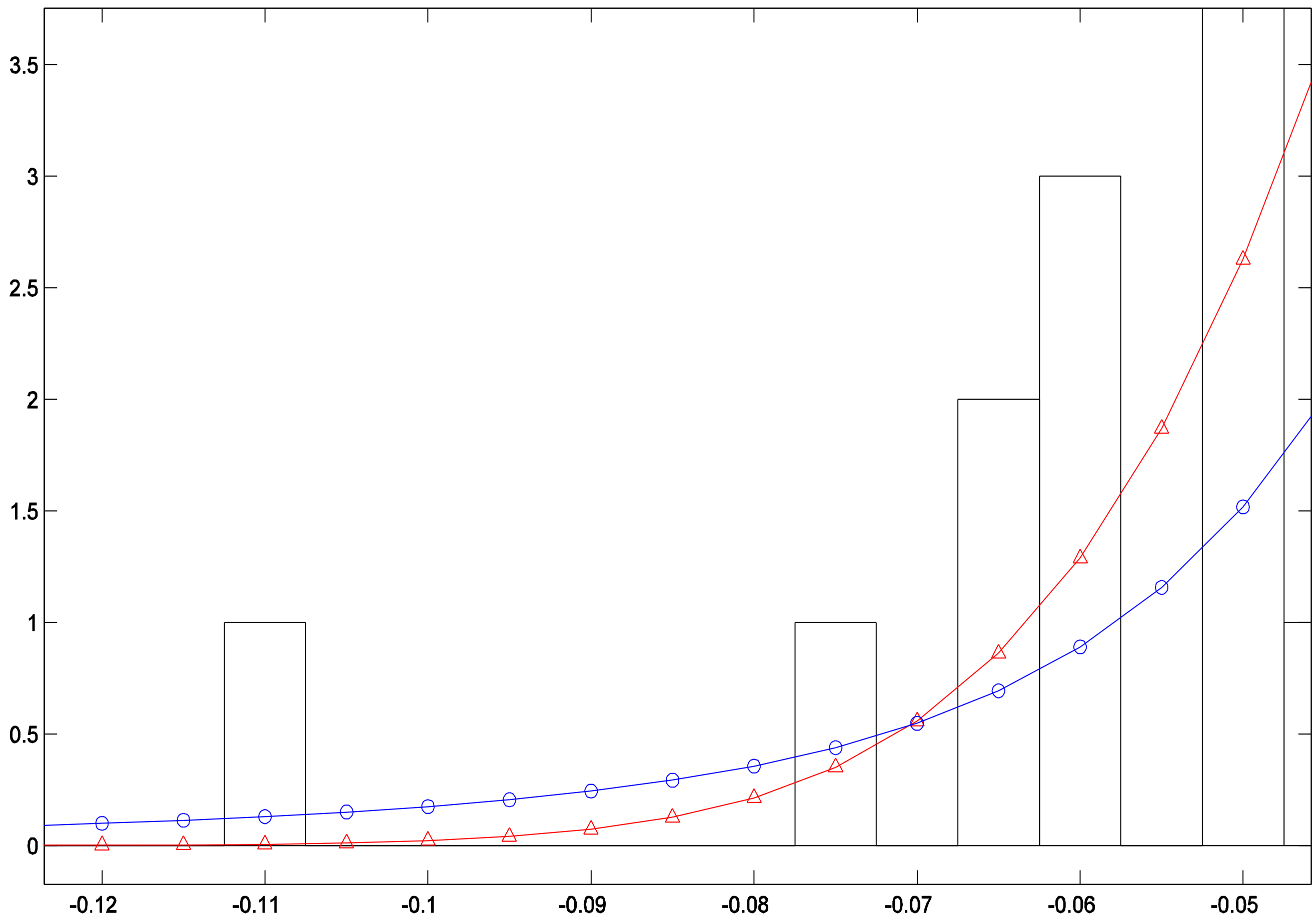


Figure 1 (Continued)



The Model

- Berkowitz (1999, 2000) recommend to partition the states of nature into normal and stress stages.
  - $R_t = P(H_t(f))$

    where R_t is the portfolio returns of commercial banks, $P(\cdot)$ is the pricing model with factors H_t and a simulated distribution $f(\cdot)$
  - $R_{s,t} = P(H_t(f_s))$

    where the subscript s represents stressful economic conditions

The Model

- Basak and Shapiro (1998)
  - Maximize: $\mathbb{E}_g[U(R_{t+1})]$
  - Subject to: $\Pr(R_{t+1} \geq \bar{R}) \geq 1 - p$
- Berkowitz (2000) recommends to apply the optimal combination of normal and stress forecasts: $h\big(g(R_t), g_s(R_t)\big)$
  - Maximize: $\mathbb{E}_h[U(R_{t+1})]$
  - Subject to: $\Pr(R_{t+1} \geq \bar{R}) \geq 1 - p$
- Meta-distribution [Berkowitz (1999)]
  - $x \sim \begin{cases} f(\cdot) & \text{with probability } 1-\alpha \\ f_s(\cdot) & \text{with probability } \alpha \end{cases}$

The Model

- For normal distributions, risks are determined by the scale parameter: σ (one factor)
- For stable Paretian distributions, risks are determined by three factors:
  (i) Characteristic exponent (α) captures the extra-ordinary risk.
  (ii) Skewness (β) depicts the asymmetric movements of returns.
  (iii) Scale (c) values the ordinary risk.

The Model

- Stress tests try to compensate the extra losses that is underestimated by VaR
- Berkowitz (1999) proposes meta distributions to give a better estimate of the density function under normal and stress scenarios with a subjective probability α
- Stable Paretian distributions provide a unified distribution with the tail fatter than normal without any subjective parameter

The Model

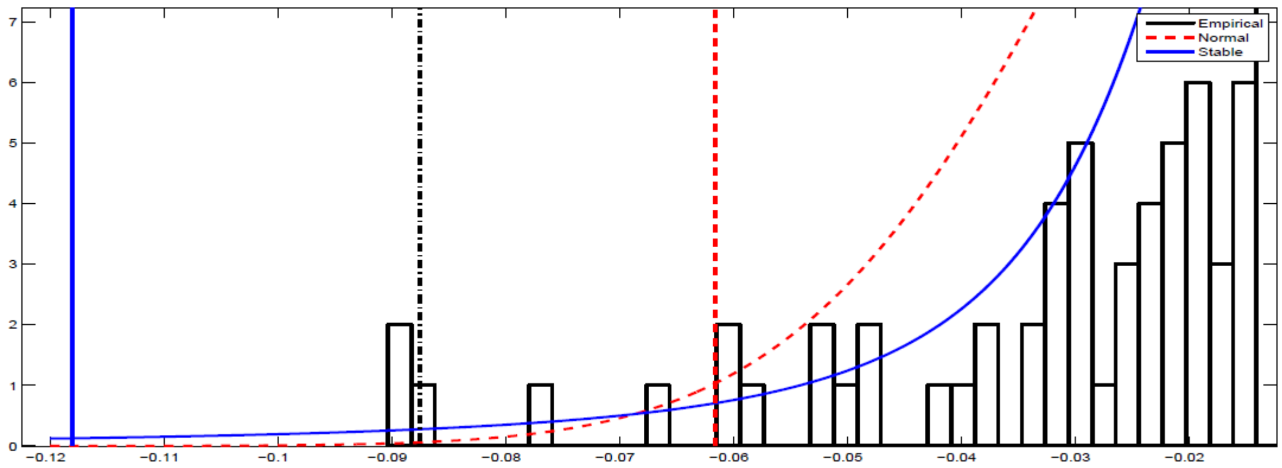


Figure 1: The figure shows the density functions and VaR estimates of the normal distribution and the stable distribution where S&P500 daily returns from 2008 to 2009 are used. (1) Histogram–empirical returns; (2) Red dash line–The normal distribution ($\mu = -0.16\%, \sigma = 2.58\%$); (3) Blue solid line–the stable Paretian distribution ($\alpha = 1.42, \beta = -0.0964, c = 1.18\%, \delta = -0.24\%$). Three vertical lines from left to right are the VaR(1%) under the stable Paretian distribution, empirical data (1% quantile) and the normal distribution respectively.

The Model

- Stable-GARCH(1,1) and Stable-GJR(1,1)
- $R_{i,t} = \delta_i + \varepsilon_{i,t}$,
- GARCH(1,1): $c_{i,t}^{\eta} = \mu_i + \gamma_{i1}\left|\varepsilon_{i,t-1}\right|^{\eta} + \phi_{i1} c_{i,t-1}^{\eta}$
- GJR(1,1):
  $c_{i,t}^{\eta} = \hat{\mu}_i + \left(\hat{\gamma}_{i1} + \hat{\psi}_{i1} I^{-}\left(\varepsilon_{i,t-1}\right)\right)\left|\varepsilon_{i,t-1}\right|^{\eta} + \phi_{i1} c_{i,t-1}^{\eta}$
  where $I^{-}(x) = 1, \text{if } x < 0$ otherwise is 0
- For the stable Paretian distribution $\eta = \alpha$
- For the normal distribution $\eta = 2$

The Model

- Develop from Bawa and Lindenberg (1977) and Ang et al. (2006): asymmetric systematic risk CAPM; Schwer and Seguin (1990): dynamic CAPM
- $R_{i,t} = \delta_i + \left(a_i + b_i I^{-}\left(\tilde{R}_{m,t}\right)\right)\frac{c_{i,t}}{c_{m,t}}\left(R_{m,t} - \delta_m\right) + c_{i,t}\tilde{\xi}_{i,t}$

  where $\tilde{R}_{m,t}$ is the standardized market returns

The Model

- stable: Residuals of R_m (ε_m) are stably distributed
- sstable: Residuals of R_m (ε_m) are symmetric stably distributed
- nstable: Residuals of R_m (ε_m) are stably distributed with maximum negative skewness
- normal: Residuals of R_m (ε_m) are normally distributed
- GED: Residuals of R_m (ε_m) are generalized error distributed
- student-t: Residuals of R_m (ε_m) are student-t distributed

Systemic Risk

- Acharya et al. (2010, 2012) define "Systemic Risk" as the amount of capital that a financial institution in need to raise in order to function normally if we have another financial crisis
- $SRISK_{i,t} = \mathbb{E}_{t-1}(\text{Capital Shortfall}_t | \text{Crisis})$
- $SRISK_{\theta,i,t} = kd_{i,t} - (1-k)\left(1 - LRMES_{\theta,i,t}\right)e_{i,t}$

where k is capital adequacy ratio, $d_{i,t}$ and $e_{i,t}$ are debts and equity at time t.

Systemic Risk

- $LRMES_{\theta,i,t}$ is the marginal expected shortfall in a specified time period
- $LRMES_{\theta,i,t} = 1 - \exp\left(-\sum_{j=1}^{n} MES_{\theta,i,t_j}\right)$
$$\approx 1 - \exp\left(-\rho_t (t_N - t) MES_{\theta,i,t}\right) = \widehat{LRMES}_{\theta,i,t}$$

where t_j is the day that the loss of market returns is higher than $\mathrm{VaR}(R_m)$, $t_N - t$ is the specified time horizon, $\rho_t = n/(t_N - t)$. In Acharya et al. (2010, 2012), $\rho_t(t_N - t) = 18$

- $MES_{\theta,i,t}$ is the expected one day loss:
- $MES_{\theta,i,t} = \mathbb{E}_{t-1}\left[-R_{i,t} \middle| R_{m,t} \le -\mathrm{VaR}\left(\theta, R_{m,t}\right)\right]$

Extreme Market Losses (EMLs)

- Daily market returns less than $-\mathrm{VaR}(1\%\ )$ are denoted as EMLs.
- $t_N - t = 252$ trading days in 1 year
- The probability of EMLs (ρ_t) is

$$\rho_t = \frac{\#\ \mathrm{of\ market\ returns\ between}\ t\ \mathrm{and}\ t_N < -\mathrm{VaR}(1\%)}{252}$$

Tail Risk Tolerance (TRT)

- TRT of the bank *i* ($\rho_{i,t}$) measures the level of ρ_t that the bank is able to remain solvent in the future
- $\rho_{i,t} = \dfrac{\ln\left(\frac{1-k}{k \cdot LVG_{i,t}}\right)}{252 \cdot MES_{\theta,i,t}}$
- $\dfrac{\partial \rho_{i,t}}{\partial e_{i,t}} = \dfrac{1}{252 \cdot e_{i,t} \cdot MES_{\theta,i,t}}$
- $\dfrac{\partial \rho_{i,t}}{\partial d_{i,t}} = -\dfrac{1}{252 \cdot d_{i,t} \cdot MES_{\theta,i,t}}$

where $LVG_{i,t}$ is the leverage of the bank *i* at time *t*

Aggregate TRT

- Using the aggregate leverage ($LVG_{a,t}$) and weighted average of MES ($MES_{\theta,a,t}$)
- $\rho_{a,t} = \frac{\ln\left(\frac{1-k}{k \cdot LVG_{a,t}}\right)}{252 \cdot MES_{\theta,a,t}}$

Methodology

- Fourier-cosine expansion of the probability density function $(f(x))$ [Fang and Oosterlee (2008)],
- $f(x) \approx \sum_{n=0}^{N} \left(1 - \frac{1}{2} I_0(n)\right) A_n \cos\left(\frac{n\pi(x-u)}{v-u}\right)$,
- $A_n = \frac{2}{v-u} \mathrm{Re}\left\{\Phi\left(\frac{n\pi}{v-u}\right) \cdot \exp\left(-\mathrm{i}\,\frac{n\pi u}{v-u}\right)\right\}$

where $I_0(n) = 1, \text{if } n = 0$, otherwise is 0, $\mathrm{Re}(x)$ is the real part of x, Φ is the characteristic function, $[u, v]$ is the truncated range of Fourier expansion

Methodology

- VaR under stable Paretian distributions can be calculated after pdf is expressed by FC expansion
- $MES_{\theta,i,t} = -\delta_i - \frac{(a_i+b_i)c_{i,t}}{F(\tau)}\left[\int_{-\infty}^{\kappa} x \, \mathrm{d}F(x) + \int_{\kappa}^{\tau} x \, f(x)\mathrm{d}x\right]$

where $\tau = -\left(\mathrm{VaR}\left(\theta, R_{m,t}\right) + \delta_m\right)/c_{m,t}$, κ is the cutoff value that the pdf can be approximated by power law

Methodology

- For stable Paretian distributions
- $\int_{-\infty}^{\kappa} x \, \mathrm{d}F(x) = \frac{\alpha}{1-\alpha} c_{m,t}^{\alpha} D_{\alpha}(1-\beta)\kappa^{1-\alpha}$
- $\int_{\kappa}^{\tau} x f(x) \mathrm{d}x \approx \frac{\tau^2-\kappa^2}{2(v-u)} + \sum_{n=1}^{N} A_n \frac{v-u}{n\pi} \left\{ \begin{array}{c} \tau \sin\left(n\pi \frac{\tau-u}{v-u}\right) - \kappa \sin\left(n\pi \frac{\kappa-u}{v-u}\right) \\ + \frac{v-u}{n\pi}\left[\cos\left(n\pi \frac{\tau-u}{v-u}\right) - \cos\left(n\pi \frac{\kappa-u}{v-u}\right)\right] \end{array} \right\},$
- MES can be calculated, so that we can obtain $\rho_{i,t}$

Methodology

- Proposition 1. For the GARCH(p,q) and GJR(p,q) models, given all the information at time t-1, then

$$\mathbb{E}_{t-1}\left[c_{i,t+k}^{\alpha}\right] \leq c_{i,\max}^{\alpha}$$

for all $k \geq 1$, where $c_{i,\max} = \max\left\{c_{i,t}, c_{i,t-1}, \dots, c_{i,t-r}\right\}$, $r = \max\{p, q\}$. For the stable distributional assumption, we define $\mathbb{E}_{t-1}\left[\left|\varepsilon_{i,t+j}\right|^{\alpha}\right] = \mathbb{E}\left[c_{i,t+j}^{\alpha}\right]$, for all $j \geq 0$, and $\mathbb{E}\left[\left|\varepsilon_{i,t}\right|^{\alpha}\right] = \mathbb{E}\left[c_{i,t}^{\alpha}\right] = \mathbb{E}[c_{i}^{\alpha}]$

Methodology

- Proposition 2. The estimated LRMES ($\widehat{LRMES}_{\theta,i,t}$) is the upper bound of LRMES at time t, if the market returns are following the GARCH(1,1) or GJR(1,1) and Proposition 1 holds, so that as the estimated systemic risk
- $LRMES_{\theta,i,t} \leq \widehat{LRMES}_{\theta,i,t}$
- $SRISK_{\theta,i,t} \leq \widehat{SRISK}_{\theta,i,t} = kd_{i,t} - (1-k)\left(1 - \widehat{LRMES}_{\theta,i,t}\right)e_{i,t}$

Empirical Results

- Market returns, stock returns, market equity and liabilities of banks
- Market index: S&P500 index
- Stocks: 51 financial institutions (market equity greater than 1 million, without filing for bankruptcy before 30th June, 2008, without heteroskedasticity after standardized by GARCH(1,1))
- Daily data between 30th June 1987 to 31st December 2014
- Data between 30th June 1987 and 30th June 2007 is used to estimate the parameters of the models
- Market returns and stock returns are from the CRSP
- Quarterly liabilities of banks are from the Compustat

Empirical Results

- Test for heteroskedasticity
  - Ljung-Box Q-test
  - Engle’s ARCH test
- Test the fitness of the Models
  - Maximum likelihood function value
  - Akaike information criterions (AICC)
  - Bayesian information criterions (BIC)
  - Kolmogorov-Smirnov test (KS p-value)
- Fat-tail tests
  - Goodness of fit test
  - Extreme value distributions
  - Structural change tests [Quintos et al.(2001)]

Table 4: Maximum Likelihood estimates of the GARCH (upper panel) and GJR (lower panel) models with different distributional assumptions.

GARCH	δ_m	μ	γ	ψ	ϕ	α or ν	β	ML	AICC	BIC	KS p-value
stable	0.053**	0.003**	0.025**	–	0.941**	$1.885^{\dagger}$	-0.282*	-6436	12871	12922	0.117
std	(0.011)	(0.001)	(0.003)	(–)	(0.007)	(0.020)	(0.117)				
normal	0.000	0.012	0.077*	–	0.914**	2.000	0.000	-6627	13254	13305	< 0.01
std	(0.011)	(0.007)	(0.030)	(–)	(0.032)	(–)	(–)				
GED	0.000	0.006**	0.057**	–	0.938**	$1.302^{\dagger}$	–	-6467	12934	12985	< 0.01
std	(0.000)	(0.002)	(0.012)	(–)	(0.012)	(0.055)	(–)				
student t	0.000	0.005**	0.050**	–	0.946**	6.279**	–	-6446	12892	12943	< 0.01
std	(0.010)	(0.002)	(0.008)	(–)	(0.009)	(0.637)	(–)				
GJR											
stable	0.035**	0.005**	0.010**	0.041**	0.925**	$1.896^{\dagger}$	-0.402**	-6396	12793	12844	0.117
std	(0.011)	(0.001)	(0.003)	(0.006)	(0.008)	(0.020)	(0.140)				
normal	0.000	0.017**	0.009	0.126**	0.915**	2.000	0.000	-6553	13106	13157	< 0.01
std	(0.014)	(0.003)	(0.005)	(0.013)	(0.008)	(–)	(–)				
GED	0.046	0.011**	0.010	0.103**	0.927**	$1.336^{\dagger}$	–	-6413	12827	12878	0.191
std	(0.009)	(0.002)	(0.007)	(0.014)	(0.009)	(0.035)	(–)				
student t	0.000	0.011**	0.009	0.106**	0.930**	6.887**	–	-6406	12811	12862	< 0.01
std	(0.015)	(0.002)	(0.006)	(0.016)	(0.008)	(0.631)	(–)				

In the stable Paretian distribution, the parameter α is the characteristic exponent, for the normal distribution, $\alpha = 2$. For the GED and student-t distributions, ν is the shape parameters. The AICC is the Akaike information criterion with correction on a finite sample sizes, the BIC is the Bayesian information criterion that gives a higher penalty on the number of variables. The p-value of the KS test is calculated by using standardized returns from 1st July 2007 to 31st December 2013. *, ** denote 95% and 99% significance levels, respectively. † is 1% significant less than 2. The standard deviation is obtained by using the 2-sided finite difference Hessian matrix.

Empirical Results

- Extreme value distributions

Table 5: Estimates of Extreme Value Distributions for different crisis periods.

	n	shape	scale	location	skewness	kurtosis	min	max	median
S&L	98	0.76 (1.34**)	0.31 (0.04**)	2.42 (0.31**)	-2.79	59.47	-20.47	9.10	0.06
Internet Bubble	49	0.65 (1.14**)	0.36 (0.44**)	2.80 (0.41**)	-0.22	5.83	-6.87	5.12	0.07
Subprime Mortgage	44	0.68 (1.43**)	0.74 (0.97**)	4.44 (0.09**)	0.06	9.31	-9.04	11.58	0.09
1987 to 2014	320	0.69 (1.38**)	0.48 (0.62**)	2.77 (0.58**)	-0.65	22.37	-20.47	11.58	0.04

n is number of observations that the absolute returns are greater than 2 standard deviations from mean. The standard errors are given in the parentheses. ** denoted as 99% significantly greater than 0. If shape parameters are greater than 0, the distributions should be Fréchet. The larger shape parameter, the fatter the distribution.

Empirical Results

- Structural change tests

Table 6: The tail indices of three financial crises in two sub-periods

Crises	Pre-break	Post-break	P_1
Saving and Loan Crisis	2.46 (2.02–2.86)	1.56 (1.28–2.73)	11.79**
Internet Bubble Crisis	1.60 (1.34–1.85)	1.91 (1.65–1.85)	5.48*
Subprime Mortgage Crisis	2.06 (1.70–2.42)	1.51 (1.23–2.34)	8.26**

The 95% confident interval of the tail indices is given in the parentheses. * and ** denote as 95% and 99% significance levels respectively.

Empirical Results

- TRT $\left(\rho_{i,t}\right)$ and the probability of EML (ρ_t)
- Table 7: TRT of 10 largest banks on 30^{th} June 2007
- Table 8: TRT of 51 banks.
  - “E”—The numbers of “Estimated Unhealthy”
  - “C”—The numbers of correct estimates
- We have 23 banks out of 51 banks in our sample are unhealthy
  - Overestimate— TRT is overestimated, the bank is unhealthy but it is estimated as healthy
  - Underestimate—TRT is underestimated, the bank is healthy but it is estimated as unhealthy
- Two measures of these errors
  - “Equal”—The average of Overestimate and Underestimate
  - “Over”—The weight of Overestimate is 1 and the weight of Underestimate is 0.5

Table 7: The tail risk tolerance $\rho_{i,t}$(s) of ten big banks.

$\rho_{i,t}$ (%)	VaR(1%)	ρ_t	C	BAC	JPM	WFC	WB	USB	WM	LEHMQ	STI	BK
GARCH(1,1)												
stable	1.66	11.11	4.67	9.51	4.01	16.83	9.03	19.38	6.17	-2.03	13.39	14.88
sstable	1.59	12.70	5.13	10.44	4.41	18.47	9.91	21.27	6.78	-2.22	14.68	16.34
nstable	1.87	8.33	3.90	7.94	3.35	14.06	7.54	16.18	5.15	-1.70	11.19	12.42
norm	1.53	13.10	7.11	13.81	6.49	25.04	13.77	31.69	9.22	-3.06	19.19	23.17
GED	1.69	11.11	6.42	12.16	5.84	22.61	12.18	28.16	8.10	-2.74	17.10	19.07
student-t	2.02	7.94	5.13	9.63	4.62	17.91	9.47	21.68	6.49	-2.18	13.66	15.88
GJR(1,1)												
stable	1.68	11.11	4.77	9.72	4.00	16.98	9.24	19.04	6.12	-1.97	13.45	14.48
sstable	1.58	12.70	5.40	10.99	4.52	19.19	10.44	21.52	6.92	-2.22	15.19	16.37
nstable	1.83	9.13	4.15	8.45	3.47	14.76	8.03	16.55	5.32	-1.71	11.71	12.59
norm	1.56	13.10	7.34	13.61	6.37	25.12	13.28	32.58	9.16	-3.01	18.37	22.02
GED	1.64	11.11	6.85	12.67	5.96	23.60	12.29	29.11	8.32	-2.79	17.61	19.99
student-t	1.97	8.33	5.46	10.03	4.77	18.59	9.70	21.74	6.70	-2.22	13.91	15.26
Healthy	-	-	N	N	N	Y	N	Y	N	N	Y	Y

The TRT is estimated at 30th June 2007, VaR(1%) is 1% Value-at-Risk in different models. ρ_t is the empirical probability of extreme market losses between June 2007 and June 2008. The row "Healthy" means the average capital ratio between June 2008 and September 2008 is greater than 8%. If the tail risk tolerance is less than "ρ_t", the bank is denoted as "Unhealthy" in the estimate.

Empirical Results

Table 8: The "Estimated unhealthy" banks under the GARCH(1,1) and GJR(1,1) models.

	GARCH(1,1)				GJR(1,1)			
Models	E	C	Equal	Over	E	C	Equal	Over
stable	26	18	6.5	6.0	27	18	7.0	6.3
sstable	27	18	7.0	6.3	27	18	7.0	6.3
nstable	22	16	6.5	6.7	25	18	6.0	5.7
norm	14	10	8.5	10.0	14	10	8.5	10.0
GED	12	9	8.5	10.3	11	9	8.0	10.0
student-t	11	9	8.0	10.0	11	9	8.0	10.0

The column "E" (estimate) is the number of unhealthy banks which is estimated by using the tail risk tolerance at 30th June 2007. The column "C" (correct) is the number of correct estimates. "Equal" is the equal weighted measure, the average of the number of overestimated banks and the number of underestimated banks. "Over" is the weighted-average of the number of overestimated banks and the number of underestimated banks. The weights of overestimated banks and underestimated banks are 1 and 0.5 respectively. There are 23 unhealthy banks. "Unhealthy" banks mean their average capital ratio between 30th June 2008 and 31st September is less than 8%.

Empirical Results

- Aggregate TRT

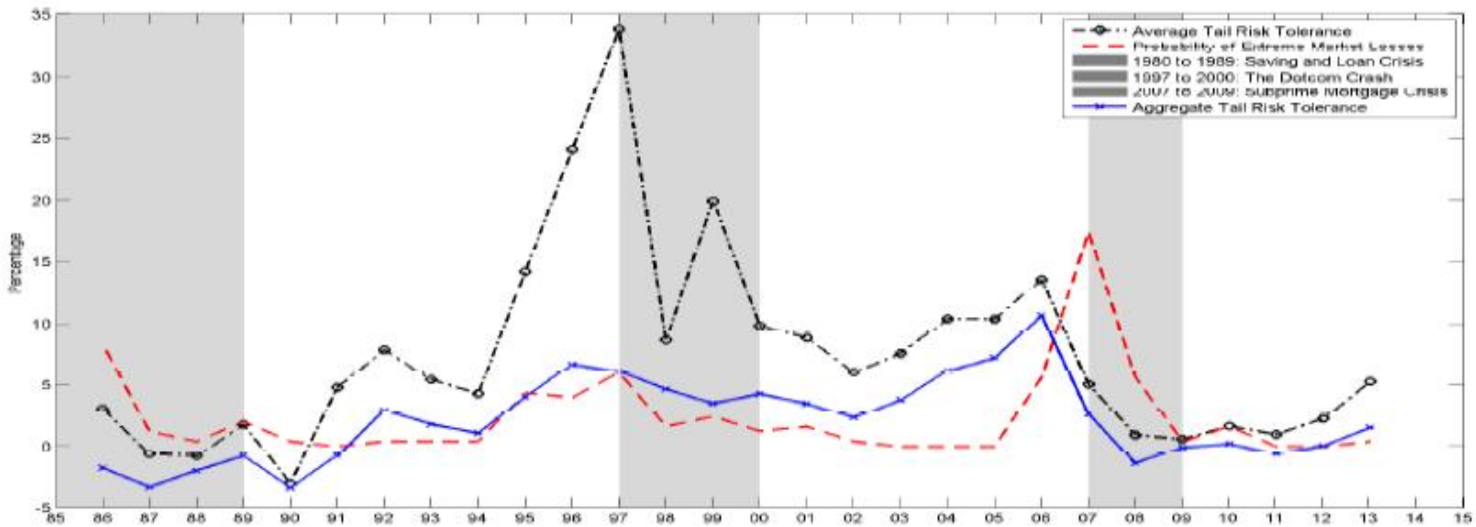


Figure 4: The aggregate tail risk tolerance vs probability of extreme market losses under the stable-GARCH model.

Empirical Results

- TRT of CITIGROUP INC

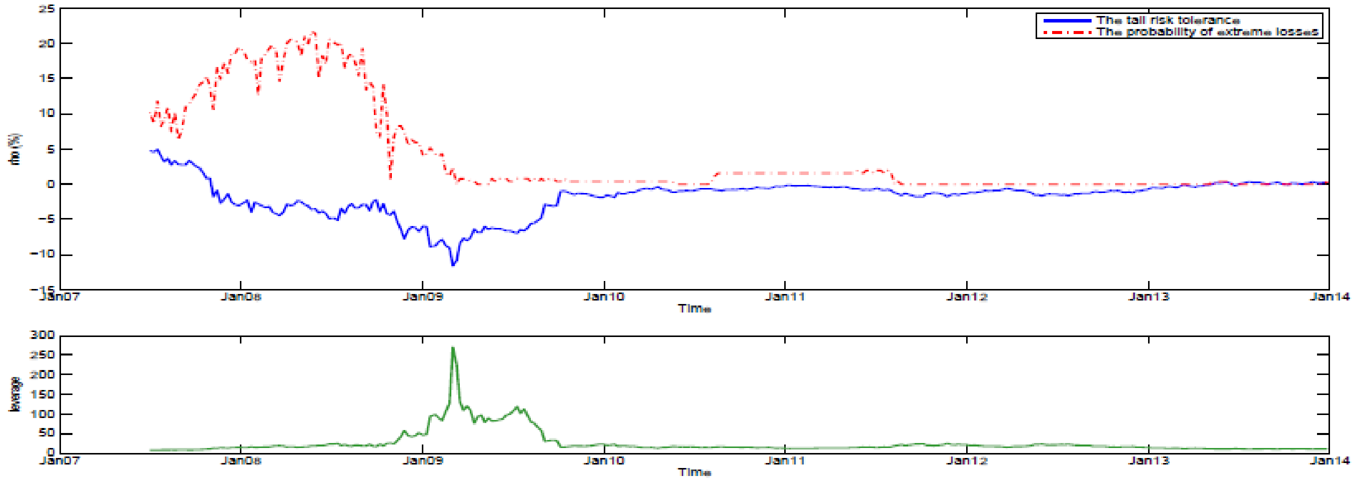


Figure 5: The upper panel: The comparison of the tail risk tolerance ($\rho_{i,t}$) and the probability of extreme market losses of CITIGROUP INC under the stable-GARCH(1,1) model, where $k = 8\%$, and $t_N - t = 252$. The lower panel: The leverage of the bank.

Empirical Results

- TRT of WELLS FARGO & CO NEW

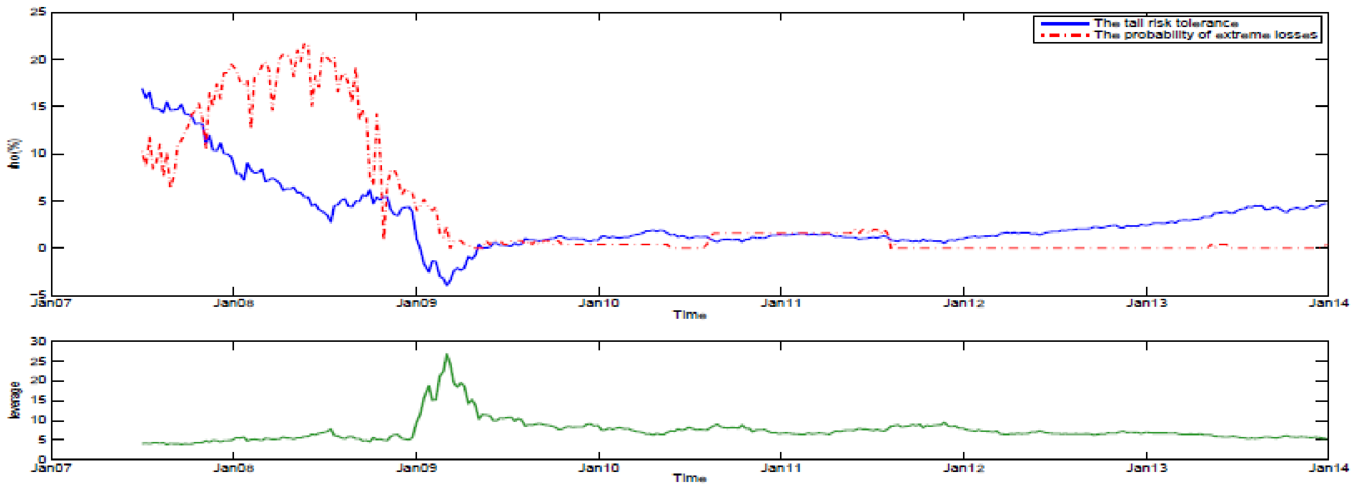


Figure 6: The upper panel: The comparison of the tail risk tolerance ($\rho_{i,t}$) and the probability of extreme market losses of WELLS FARGO & CO NEW under the stable-GARCH(1,1) model, where $k = 8\%$, and $t_N - t = 252$. The lower panel: The leverage of the bank.

Empirical Results

- TRT of BANK OF AMERICA CORP with different distributional assumptions

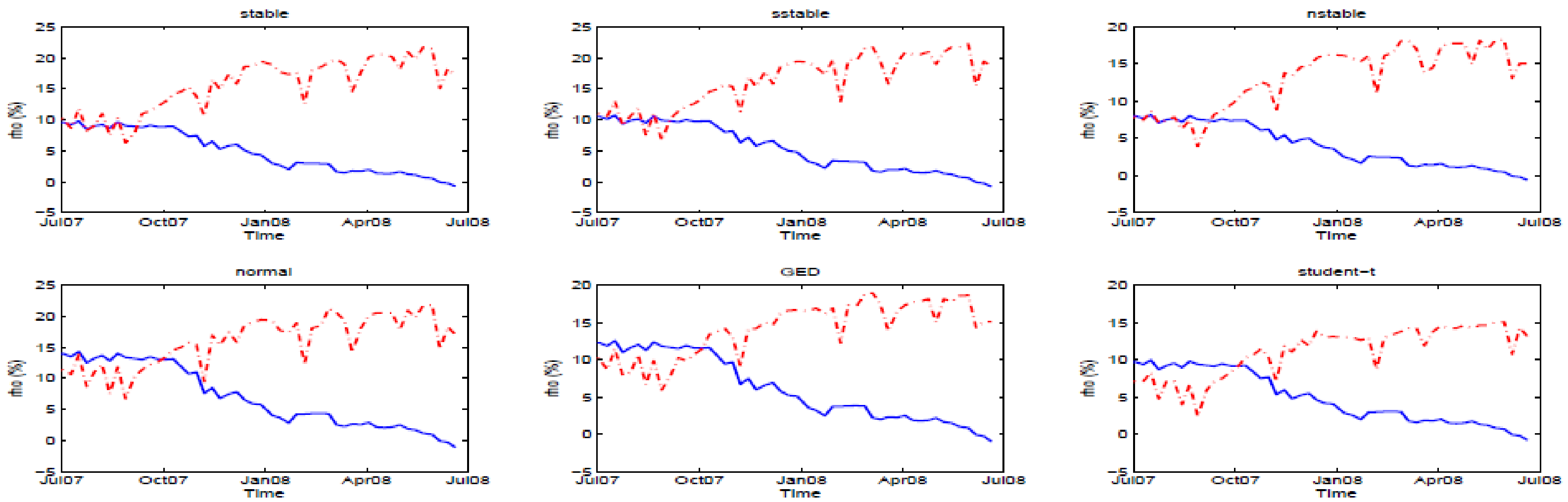


Figure 7: The comparison of the tail risk tolerance ($\rho_{i,t}$) and the probability of extreme market losses of BANK OF AMERICA CORP under the GARCH(1,1) model with different distributional assumptions, where $k = 8\%$, and $t_N - t = 252$. The blue solid line is TRT, the red dash line is empirical probability of EML.

Conclusion

- Heteroskedasticity of market returns and stock returns is observed
- Standardized returns are still fat-tail
- Shape parameters changed over time (before and after crises)
- TRT under stable distributions is recommended → It reveals the endurance of the bank to the fat-tail risks in the future
- Aggregate TRT shows that the banking system is getting trouble during the S&L crisis and Subprime mortgage crisis
- TRT of most largest banks on 30th June 2007 is not sufficient to overcome the subprime mortgage crisis during September 2008
- Government actions e.g. QE, lower-interest-rate policy, reduce the likelihood of EML after 2009. However, aggregate TRT shows that the banking system is still fragile.