

Safe assets, liquidity and monetary policy*

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Abstract

This paper studies monetary policy in models where multiple assets have different liquidity properties: safe and “pseudo-safe” assets coexist. A shock worsening the liquidity properties of the pseudo-safe assets raises credit spreads and can cause a deep recession *cum* deflation. Expanding the central bank’s balance sheet fills the shortage of safe assets and counteracts the recession. Lowering the interest rate on reserves insulates market interest rates from the liquidity shock and improves risk sharing between borrowers and savers.

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1 Introduction

This paper studies monetary policy in models where multiple assets have different liquidity properties. There are some safe assets, like money, which can be perfect store of value and immediately resaleable, and other assets, labelled “pseudo-safe” assets, which are also perfect store of value but might instead have imperfect liquidity properties that can vary over time.

In fact, the recent US and European financial crises have shown sudden and dramatic changes in the quality of assets. Securities that had the reputation of safe assets with the property of being store of value and at the same time perfectly resaleable became illiquid and risky. This happened for mortgage-backed securities in the U.S. and for sovereign debt in the ongoing European crisis. A shortage of safe assets, if not immediately healed, can have effects not only on financial markets but also on the real economy. A deep recession occurred in the US followed by a slow and jobless recovery, Europe is currently experiencing a stagnation.

We look at these issues through the lens of monetary models where agents face a liquidity constraint: only some assets can be used to purchase goods, and to a different extent. In addition, the models feature a financial friction, as portfolio rebalancing takes place only after goods purchasing.

The liquidity properties of assets can change suddenly at the moment of purchasing goods and a deterioration of the quality of the pseudo-safe assets is able to bring about an adjustment in the real economy similar to that observed during the recent crisis. The overall shortage of liquidity implies a corresponding shortage of demand for goods since fewer assets remain available for goods purchasing. The mirror image of a disequilibrium in the financial market is a disequilibrium in the goods market, as often discussed in commentaries of the crisis, as Lucas (2008) and De Long (2010), bringing out ideas that go back to Mill (1829). The consequent contraction in nominal spending can depress real activity in the presence of nominal rigidities. A deep recession and a deflation can easily emerge. In asset markets, the liquidity shock raises the premium required to hold pseudo-safe assets. The funding costs for intermediaries, which borrow in the pseudo-safe assets, increase and at the same time force them to charge higher interest rates on loans. Due to rising borrowing costs, debtors need to cut on their spending, amplifying the contraction in the real economy with important distributional effects between savers and borrowers.

Monetary policy has an important role in mitigating the adverse effects on the economy, mainly along two dimensions. The central bank can heal the shortage of safe assets in the economy and prevent the contraction in nominal spending by issuing more money, which remains a perfectly safe asset in circulation. The expansion of the central bank’s balance

sheet is necessary to maintain price stability. On the other side, monetary policy can insulate the interest rates on the pseudo-safe assets from the liquidity shock, by lowering the interest rate on reserves and therefore improving risk sharing between borrowers and savers. For a large shock, the zero-lower bound can be an important constraint along this dimension. A policy in which the interest rate on reserves is lowered, while the balance sheet of the central bank is not expanded, does not prevent the contraction in nominal spending. As well, a policy in which more liquidity is injected into the system, but the policy rate is not lowered, cannot avoid the ensuing liquidity premia and the distributional costs of the liquidity shock. It is important to note that the two policy prescriptions coming out of our model do not depend on the degree of nominal rigidities. In particular nominal spending is the key variable to stabilize in the face of a liquidity shock, as frequently discussed in the recent public debate.¹

We present two simple models where pseudo-safe and safe assets coexist. In the first model, we describe the main elements of our framework through a single-agent endowment economy where the pseudo-safe assets are government bonds and the safe asset is money. The model is already rich of insights: a negative liquidity shock raises the interest rate on bonds, which can be offset by lowering the interest rate on reserves at the risk of hitting the zero-lower bound. A deflation can happen if the central bank's balance sheet is not expanded appropriately. In the second model, with heterogeneous agents, the pseudo-safe asset is an inside security, issued by intermediaries to finance lending in the economy.

Our approach to model liquidity is in line with that of Lagos (2010), where financial assets are valued for the degree to which they are useful in exchange for goods. In his model, agents are free to choose which assets to use as means of payment, between bonds and equity shares. However, he also restricts the analysis to cases in which bonds are assumed to be superior to equity shares for liquidity purposes. In Aiyagari and Gertler (1991), instead, transaction costs in trading equities are responsible for a lower degree of liquidity of the latter with respect to bonds. The finance constraint in our model, through which goods and assets are exchanged, is of a simple form in line with the works of Lucas (1982), Svensson (1985) and Townsend (1987). The way we characterize a liquidity shock, as a change in the degree of resaleability of assets, is close to Kiyotaki and Moore (2012).² In their model, entrepreneurs face a borrowing constraint to finance investment and they need to use internal resources among which money and previous holdings of equity. Equity can be used only in part to finance investment, where the fraction available is known at the time when liquidity is needed. Instead, we model the exchange of assets for goods at the level of consumption. The shortcut taken here

¹See, among others, Woodford (2012).

²Chiesa (2013) presents also a model in which liquidity holdings are an input to the investment process and assets have different degrees of pledgeability. Del Negro et al. (2012) estimate a quantitative version of Kiyotaki and Moore (2012) with nominal rigidities.

has the benefit of producing a highly tractable model of the role of liquidity, which extends standard monetary models currently used for the analysis of monetary policy. Moreover, the partial resaleability of Lagos (2010) and Kiyotaki and Moore (2012) concerns a risky asset like equity and not risk-free assets as in our model. Finally, Trani (2012) is an example of an open-economy model in which multiple assets (equities) provide collateral services with time-varying properties specific to each asset, and therefore have different liquidity premia.

In monetary analysis, several works have introduced a transaction role for bonds, although an indirect one. In Canzoneri and Diba (2005), current income can also be used for liquidity purposes in a fraction that depends on the quantity of bonds held in portfolio. In their model, bonds have indirect liquidity services since they enhance the fraction of income used to purchase goods. Woodford (1991) is an early example of a model in which current income has immediate liquidity value but bonds do not provide liquidity services. In our context, a liquidity constraint disciplines literally the exchange of assets for goods and the imperfect substitutability of pseudo-safe assets for money through a random factor. In Belongia and Ireland (2006, 2012), money and deposits are bundled together through a Dixit-Stiglitz aggregator, and can be used for liquidity purposes as in the work of Canzoneri et al. (2011) where bonds are instead imperfect substitutes for money. Canzoneri et al. (2008) consider instead a model in which bonds provide direct utility to the consumer. These latter works are not concerned with variation over time of the liquidity properties of assets.

Our analysis is complementary to a recent literature which has provided possible explanations of the macroeconomic adjustment following the recent crises. Compared to Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), we focus on different forces for understanding financial crises.³ The source of the shock in our model is to liquidity and not to the natural rate of interest. The relevant real rates for savers and borrowers do not fall under optimal policy, while in the literature the real interest rate should drop to negative values, as it is also shown in Eggertsson and Woodford (2003). In our context, zero-lower bound policies help to stabilize the market real rates and achieve a better risk-sharing of the shock. In the literature, instead, they are necessary policies to accommodate as much as possible the required fall in the real rate.

We also show that a longer stay at the zero-lower bound is not sufficient to counteract the recession: active balance-sheet policies are also required. In particular, the latter policy is a direct consequence of the key role that assets play for liquidity purposes, and of the ensuing leverage that the central bank can exploit to heal shortages of liquidity by swapping money for pseudo-safe assets.⁴ Therefore, we are able to rationalize as optimal policies the

³Justiniano, Primiceri and Tambalotti (2013) analyzes the quantitative implications of such models.

⁴Curdia and Woodford (2010, 2011), Gertler and Kyotaki (2011), Gertler and Karadi (2011) are also related models for analyzing unconventional monetary policy but in contexts in which the relevant shocks

main monetary policy actions adopted during the recent crisis such as balance-sheet and zero-lower-bound policies.

The paper is structured as follows. Section 2 presents the framework in which liquidity and pseudo-safe assets are introduced. It discusses a simple monetary model with flexible prices. In Section 3 the propagation of liquidity shocks is analyzed depending on alternative monetary policy regimes. Section 4 analyzes a monetary model with heterogenous agents (savers and borrowers) where an inside asset plays the role of a pseudo-safe security. Section 5 uses the general model to study the implications of liquidity shocks for the real economy and the role of monetary policy. Section 6 concludes.

2 A simple model with pseudo-safe assets

We model liquidity as the resaleability of an asset in exchange for consumption goods. Several assets can be brought to buy goods, but they have different liquidity properties which can be discovered only at the time of purchasing. In the goods market, the portfolio of assets cannot be rebalanced nor new assets can be traded. The following liquidity constraint applies

$$\sum_{j=1}^N \gamma_t(j)(1 + i_{t-1}(j))B_{t-1}(j) \geq P_t C_t \quad (1)$$

where N assets are available and $B_{t-1}(j)$ is the value of asset j , in units of currency, held in the agent's portfolio. At the time of purchasing goods, each security matures already a predetermined nominal interest rate, specific to the asset and given by $(1 + i_{t-1}(j))$; P_t is the nominal price index while C_t is real consumption. Securities differ for their liquidity properties which are only known when they are exchanged for goods: $\gamma_t(j)$ indicates literally the fraction of assets held from previous period that can be used to purchase goods, with $0 \leq \gamma_t(j) \leq 1$. Assets can be ordered from the worst to the best in terms of liquidity properties assuming that $\gamma_t(j)$ is a non-decreasing function of j . In this set of assets, money may have the role of the best security for liquidity purposes, meaning $\gamma_t = 1$. This happens in a fully credible fiat-money system where the liabilities of the central bank are completely resaleable and trusted.

Liquidity in this model can have a dual interpretation. On the one hand, it can simply capture the degree of “acceptance” of an asset in exchange for goods. We could think of a consumer who goes to the goods market and discovers that, among all the securities that he has carried along, only a fraction is accepted to buy goods. On the other hand, it could have a financial nature instead of being directly related to the liquidity properties of assets.

simply refer to the fraction of securities which can be fully mobilized and exchanged for goods. In line with this interpretation, it can capture the intrinsic liquidity of the asset or a sort of delay in payment. To this end, we can think of these assets as the corresponding liabilities of some other agent, not modeled, that can be liquidated only in part at the exact time in which the creditor needs to purchase goods. There is a subtle difference between the two interpretations. In the first case, liquidity is a property that the “market” (seen from the perspective of who is offering goods) attributes to the asset. This property might have to do with the trust in the security as a medium of exchange. In the second case, it is an intrinsic property of the asset, although it can vary over time. Mixed interpretations could be given since the distinction is really subtle: indeed illiquidity at the origin can also be correlated with a low degree of acceptance of the asset at destination or viceversa.

In any case, all the securities traded in this model are “risk free”, meaning that they are perfect store of value; γ_t captures just liquidity risk, and not credit risk. By this virtue, all securities are remunerated at their specific predetermined nominal interest rate. The remaining fraction $(1 - \gamma_t(j))$ – which cannot be used as liquidity – remains in the financial account becoming immediately available just after goods purchasing.

Money, the security with $\gamma_t = 1$, is the safe asset. Here safeness has a double meaning. First, it captures the property of an asset as a perfect store of value. In this model all assets share this property because each is remunerated at its specific risk-free nominal interest rate.⁵ On top of this, the safe asset is fully liquid because it can always be accepted or mobilized to purchase goods. The other assets, with $\gamma_t(j) < 1$, are imperfect substitutes as means of exchange and can be labelled “pseudo”-safe assets.

Following goods purchasing, the financial market opens and consumers reallocate their portfolio according to the following constraint

$$\sum_{j=1}^N B_t(j) = \sum_{j=1}^N (1 - \gamma_t(j))(1 + i_{t-1}(j))B_{t-1}(j) + P_t Y_t + T_t + \left[\sum_{j=1}^N \gamma_t(j)(1 + i_{t-1}(j))B_{t-1}(j) - P_t C_t \right]$$

where Y_t is exogenous output and T_t are transfers from the central bank or government. In the above constraint it is clear that the assets which are not carried in the goods market or are unspent remain in the financial account.

Given the above general framework, we start our analysis from a simple model in which there are two outside assets, money and government bonds, which can provide liquidity services. Later we consider also a model with inside assets.

⁵We could easily amend this assumption by allowing for default risk. However, the purpose of this paper is to analyze the effects of the change in the liquidity properties of assets which do not necessarily materialize in a credit event.

Consider a closed economy with a representative agent maximizing the expected discounted value of utility

$$E_{t_0} \sum_{t=t_0}^{+\infty} \beta^{t-t_0} U(C_t) \quad (2)$$

where E_t is the conditional expectation operator, β is the intertemporal discount factor with $0 < \beta < 1$; $U(\cdot)$ is the utility flow which is a function of current consumption, C , and has standard properties.

At the end of a generic period $t - 1$, the representative agent invests M_{t-1} in money and B_{t-1} in bonds. At the beginning of the next period t , money and bonds mature their nominal interest rates, given respectively by $(1 + i_{t-1}^m)$ and $(1 + i_{t-1})$, which are both predetermined. At this time, both assets can be used to purchase goods according to the following liquidity constraint

$$(1 + i_{t-1}^m)M_{t-1} + \gamma_t(1 + i_{t-1})B_{t-1} \geq P_t C_t, \quad (3)$$

where P_t is the price level. Since γ_t lies in the interval $[0, 1]$, bonds are an imperfect substitute for money for purchasing purposes. As discussed above, γ_t is a measure of the degree of saleability of bonds for goods when liquidity is needed.

After the goods market closes, the representative agent receives income and transfers from the government and, together with the unspent money and bonds, reallocates its overall wealth into new money and bonds to be carried over in the next period. The representative agent adjusts his portfolio through the following constraint

$$M_t + B_t \leq (1 - \gamma_t)(1 + i_{t-1})B_{t-1} + P_t Y_t + T_t + [(1 + i_{t-1}^m)M_{t-1} + \gamma_t(1 + i_{t-1})B_{t-1} - P_t C_t]$$

where M_t and B_t denote the holdings of money and bonds to carry in the next-period goods market. When the asset market opens the representative household receives the endowment Y_t which is also a random variable and transfers from the government, T_t . Since the endowment and the transfer are given to the agent after the goods market closes, they both have to be turned into either money or bond holdings, to be used for transactions purposes in the next period. The term in the square bracket on the second line captures the residual holdings of assets after goods purchases. It should be noted that the fraction $(1 - \gamma_t)$ of bonds, which cannot be used for transaction purposes, still remains in the financial account and is available for asset trading when the financial markets open. The above constraint simplifies to

$$P_t C_t + M_t + B_t \leq (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^m)M_{t-1} + P_t Y_t + T_t. \quad (4)$$

The representative agent maximizes the expected utility (2) under the constraints (3) and

(4), and subject to an appropriate borrowing-limit condition, by choosing consumption, C_t , asset holdings (M_t, B_t) . Given Lagrange multipliers ψ_t and λ_t attached to the constraints (3) and (4) the following first-order condition holds with respect to consumption

$$\frac{U_c(C_t)}{P_t} = \psi_t + \lambda_t \quad (5)$$

showing that the liquidity constraint creates a wedge between the marginal utility of nominal consumption and that of nominal wealth – the latter being captured by λ_t . This wedge depends on the multiplier ψ_t on the liquidity constraint. Optimality conditions with respect to money and bonds imply respectively

$$\lambda_t = \beta(1 + i_t^m)E_t(\psi_{t+1} + \lambda_{t+1}), \quad (6)$$

$$\lambda_t = \beta(1 + i_t)E_t(\gamma_{t+1}\psi_{t+1} + \lambda_{t+1}). \quad (7)$$

A unit of currency carried from period t and invested in money delivers a return $(1 + i_t^m)$ which can be used at time $t + 1$ to purchase goods or for the remaining part to contribute to next period wealth. Instead, a unit of wealth invested in bonds is remunerated at $(1 + i_t)$ but provides liquidity services only for the fraction γ_{t+1} . It should be noted that equations (6)–(7) show already that when $\gamma_{t+1} = 1$ interest rates on money and bonds are equalized because the two assets become perfect substitutes as a means of payment. This happens also when the liquidity constraint is never binding, i.e. when $\psi_t = 0$.

To see this formally, simplify the first-order conditions to

$$\frac{i_t - i_t^m}{1 + i_t}E_t \left\{ \frac{U_c(C_{t+1})}{P_{t+1}} \right\} = E_t \{ (1 - \gamma_{t+1})\psi_{t+1} \}, \quad (8)$$

$$\psi_t = \frac{U_c(C_t)}{P_t} - \beta(1 + i_t^m)E_t \left\{ \frac{U_c(C_{t+1})}{P_{t+1}} \right\}. \quad (9)$$

In general, since ψ_{t+1} and γ_{t+1} are non-negative and γ_{t+1} is bounded above by 1, money has a lower return than bonds, $i_t^m \leq i_t$, which depends on ψ_{t+1} and γ_{t+1} and their covariance. For given ψ_{t+1} , when the liquidity properties of bonds improve, the interest rate on bonds falls closer to that of money. Moreover, the premium on bonds will be high when their liquidity properties, measured by γ_{t+1} , correlate inversely with the marginal utility of liquidity, represented by ψ_{t+1} .

Money and bonds are supplied by the central bank and government, respectively. Their

integrated budget constraint can be written as

$$M_t^s + B_t^s = (1 + i_{t-1})B_{t-1}^s + (1 + i_{t-1}^m)M_{t-1}^s + T_t.$$

Equilibrium in asset markets implies

$$M_t = M_t^s,$$

$$B_t = B_t^s,$$

while in goods market

$$Y_t = C_t.$$

We solve for the equilibrium allocation of this model. The following set of equations

$$(1 + i_{t-1}^m)M_{t-1}^s + \gamma_t(1 + i_{t-1})B_{t-1}^s \geq P_t Y_t, \quad (10)$$

$$\frac{i_t - i_t^m}{1 + i_t} E_t \left\{ \frac{U_c(Y_{t+1})}{P_{t+1}} \right\} = E_t \{ (1 - \gamma_{t+1}) \psi_{t+1} \} \quad (11)$$

$$\psi_t = \frac{U_c(Y_t)}{P_t} - \beta(1 + i_t^m) E_t \left\{ \frac{U_c(Y_{t+1})}{P_{t+1}} \right\} \quad (12)$$

characterizes the equilibrium of prices, interest rates and the Lagrange multiplier ψ_t for given exogenous processes $\{Y_t, \gamma_t\}$ considering that $\psi_t \geq 0$. When $\psi_t > 0$, constraint (10) holds with equality. We further assume that there exists a technology through which the representative agent can store currency unaltered across periods so that the zero-lower bound on the nominal interest on money (or reserves) applies.⁶ The following inequalities hold $i_t \geq i_t^m \geq 0$.

The equilibrium conditions have six unknowns $\{M_t^s, B_t^s, i_t, i_t^m, P_t, \psi_t\}$ which leave room for the choice of three policy instruments. Considering an exogenous path of the supply of bonds, we are left with two dimensions along which to choose monetary policy. It should be noted that it is not necessary to specify policy in terms of money aggregates in our model. We could choose the two instruments of policy as i_t and i_t^m , for example.

⁶In our model M_t^s are the liabilities of the central bank and i_t^m is the interest rate paid by the central bank on such liabilities. We call it interest rate on money or reserves interchangeably. Note that an hypothetical corridor system is zero in our model so that the interest rate on reserves coincides at the same time with the policy rate and the interest rate on the marginal lending facility.

3 Liquidity shocks and monetary policy

We study the effects of a liquidity shock which worsens the quality of the pseudo-safe assets. At time 0 it is learnt that the liquidity properties of bonds temporarily deteriorate – meaning a fall in γ starting from period 1 – and return back to normal levels in each period with a constant probability ξ . *Ex-post*, the shock lasts T periods until period $T + 1$.⁷

There are clearly no real effects of the shock because in the simple model of the previous section prices are fully flexible. However, the way prices and interest rates react to the shock can be already meaningful to intuit what will happen in more complicated models.

The specification of monetary policy is important for the results. We consider a benchmark policy in which the monetary policymaker is completely “passive”. This policymaker keeps the interest rate on reserves unchanged and at the same time does not alter the path of money growth with respect to the previous trend. More broadly we can think of a policymaker that does not react at all to the shock either with conventional policy, through the policy rate, nor with unconventional policy, through the balance sheet.

In this context, the liquidity shock has two effects. The liquidity properties of bonds deteriorate and this is immediately reflected into a fall in their price and a rise in their yield. To hold bonds, consumers ask for a higher return to compensate for the worsening in their quality. On the other side, there is a shortage of liquidity because the pseudo-safe assets have now a lower acceptance rate in exchange for goods. The overall shortage of assets as means of payment implies a shortage of demand of consumption goods. Since prices are flexible, they fall to keep the goods market in equilibrium. These effects are shown in Figure 1 by the bold solid line. The calibration implies that before the shock the interest rate on bonds is about 5% at annual rates and the interest rate on money is at 1% while money, prices and the supply of bonds grow at 2% at annual rates. We study the effects of a full deterioration in the quality of pseudo-safe assets which brings γ from 20% to 0 for 10 quarters. This shock leads to an increase in the spread between pseudo-safe and safe assets of about 13%. The price level falls substantially with respect to previous trend through a deep deflation.⁸

We compare the benchmark policy with two other policies in which the policymaker seeks

⁷We assume that the realization of the shock is known one-period in advance, to allow monetary policy to have the possibility to stabilize it completely. This is because in our model it is the money supply of previous period that influences the current price level. In this way, a feasible policy is one in which there is complete stabilization of inflation or the price level around the target.

⁸The following quarterly calibration is used: $\beta = .99$. The initial ratio of $M/(PY)$ is set at $\bar{m} = 0.15 \cdot 4$, while that of $B/(PY)$ at $\bar{b} = .5 \cdot 4$, as implied by the US post-WWII average of the velocity of M1 and of the debt-to-GDP ratio. Such calibration implies that the steady-state share of bonds providing liquidity services consistent with the constraint (10) is about 20%, and the steady-state annualized interest rate on bonds about 5%. In the initial equilibrium, the interest rate on money is set at 1% at annual rates while the growth rates of money, prices and bonds are all 2% at annual rates.

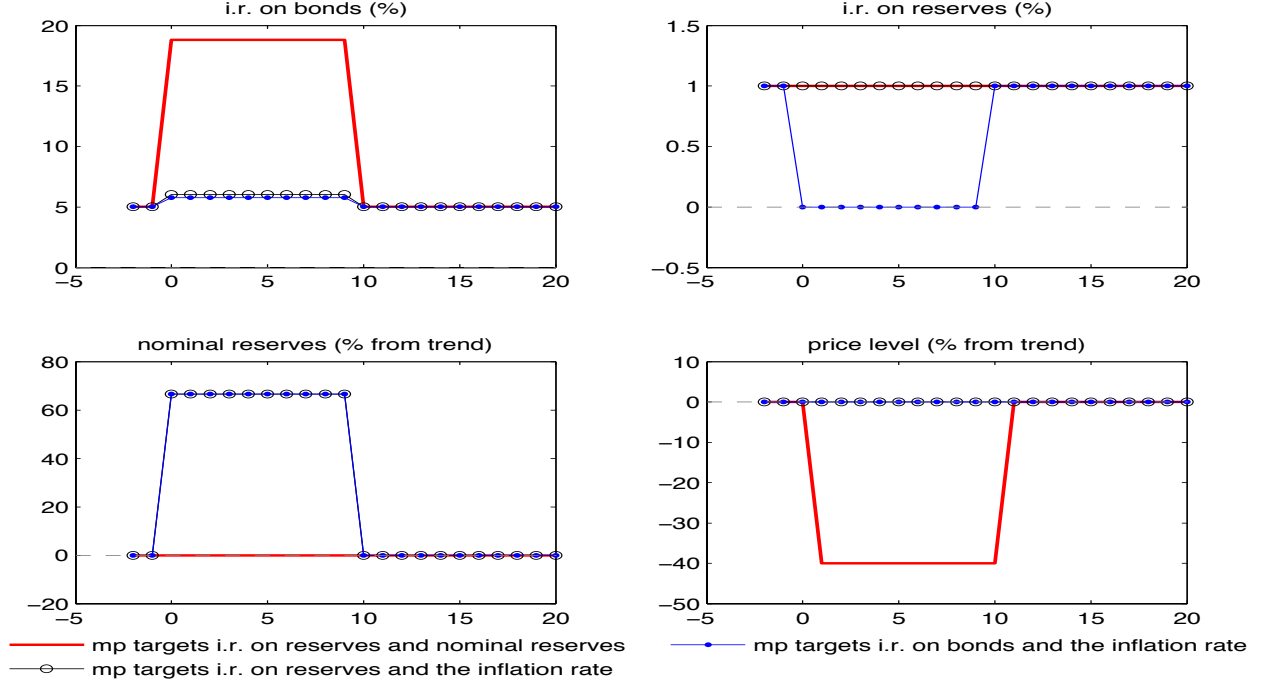


Figure 1: Response of selected variables to a 20% fall in the liquidity properties of bonds. Solid line: passive monetary policy. Circled line: monetary policy targets interest rate on reserves and the inflation rate. Dotted line: monetary policy targets interest rate on bonds and the inflation rate. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

to stabilize the inflation at the 2% target. The two policies differ because in one – the circled line in Figure 1 – the interest rate on reserves is kept unchanged at the initial level while in the other – the dotted line in Figure 1 – the policymaker tries to insulate the interest rate on bonds from the shock.

We have seen that the excess demand of liquidity and the corresponding excess supply of goods translate into a fall in the price level. To keep instead prices on their target, the excess demand of liquidity should be filled by assets with a high degree of acceptance in exchange for goods. To this end, the growth of money – the only safe asset in circulation – should increase substantially with respect to the previous target. The effectiveness is shown in Figure 1: the balance sheet is expanded and prices are stabilized. The expansion should last until the liquidity properties of bonds return back to the initial level. However, this policy does not prevent the spillovers of the liquidity shock into a higher interest rate on bonds. The expectation of a stable inflation throughout the period mutes the response of the interest rate on bonds, and the latter therefore rises less than in the case of passive policy. To completely offset this surge, the monetary policymaker can in principle lower the interest rate on reserves, up to the point in which the zero-lower bound becomes relevant. If the shock is large enough, as in Figure 1, the constraint is binding and the interest rate on

bonds still rises, although by a lower amount.⁹

At this point, it is useful to comment on the comparison between our analysis and that of Poole (1970). In Poole (1970), the driving shock is on money *demand*, and an interest-rate targeting policy fully stabilizes the economy because money supply endogenously adjusts. In our framework, instead, the driving shock is on the *supply* of liquidity, which includes money and also other assets. Given the overall demand of liquidity, a shock on the quality of pseudo-safe asset endogenously shifts the demand of money. However, in our framework, monetary policy should specify two instruments rather than one; therefore, unlike in Poole (1970), money supply can be an additional tool of monetary policy. Indeed targeting the interest rate (either on reserves or on bonds) is not sufficient to endogenously increase the money supply and fully absorb the shock.

The simple model of this section does not have welfare implications because agents get utility from consumption, which is always equal to output in equilibrium. However, two important results already emerge from the analysis. First, to prevent prices from falling with respect to the target, the shortage of liquidity should be offset by issuing more safe assets. Second, a negative liquidity shock induces an upward pressure on the interest rate on bonds. The monetary policymaker can lean against it by cutting the interest rate on reserves. The extent to which it can be successful, however, depends on whether the liquidity shock is strong enough to drive the interest rate on reserves to the zero-lower bound.

4 A model with an inside pseudo-safe asset

Building on the insights of the previous simple model, we now present a more articulated framework in which money coexists with an inside security that plays the role of the pseudo-safe asset. To the end of better characterizing the propagation mechanism of a liquidity crisis like the recent one, we model a heterogeneous-agent model where consumers are divided between savers and borrowers. Financial intermediaries channel liquidity by issuing the pseudo-safe asset (deposit) to savers in order to lend to borrowers. The real effects of a liquidity shock are analyzed in a model featuring also price rigidities.

⁹In the experiment of this section, we have focused on a monetary policy that acts directly through injection of liquidity into the system. However, since bonds and money are not perfect substitutes for liquidity purposes, the monetary policymaker can also operate by expanding money supply to buy pseudo-safe assets. In this case, the consumers' holdings of bonds, B_t , would fall during the experiment and the expansion in the central-bank's balance sheet would be even higher.

4.1 Households

Consider a closed-economy model with two types of agents: borrowers, denoted with “ b ”, and savers, with “ s ”. There is a mass χ of savers and $(1 - \chi)$ of borrowers. Utility is given by

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} [U(C_T^j) - V(L_T^j)] \quad (13)$$

for $j = b, s$ where E_t denotes the standard conditional expectation operator and β is the discount factor, with $0 < \beta < 1$. C is a consumption bundle

$$C \equiv \left[\int_0^1 C(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $C(i)$ is the consumption of a generic good i produced in the economy and θ is the intratemporal elasticity of substitution with $\theta > 1$; L^j is hours worked of quality of labor j .

At the beginning of period t the goods market opens and the following constraint limits the purchase of goods

$$(1 + i_{t-1}^m)M_{t-1}^j + \gamma_t(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j \geq P_t C_t^j, \quad (14)$$

for each $j = b, s$ where B_{t-1}^j represents the per-capita holdings of the inside security and I_t^j is an indicator function which assumes the value one only when B_{t-1}^j is positive – in which case it pays off $(1 + i_{t-1}^d)$ – and zero otherwise. When in positive holdings, the inside asset takes the form of a deposit issued by the intermediary and it is a substitute of money to a certain degree, where γ_t is the quality value that the market attaches to it for its liquidity properties. All other variables have been previously defined.

When the goods market closes, the asset market opens and agents adjust their portfolios according to

$$\begin{aligned} M_t^j + B_t^j \leq & (1 - \gamma_t)(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j + (1 + i_{t-1}^b)(1 - I_{t-1}^j)B_{t-1}^j + W_t^j L_t^j \\ & + \Psi_t^j + \Upsilon_t^j + T_t^j + [(1 + i_{t-1}^m)M_{t-1}^j + \gamma_t(1 + i_{t-1}^d)I_{t-1}^j B_{t-1}^j - P_t C_t^j] \end{aligned} \quad (15)$$

where $(1 + i_{t-1}^b)$ is the nominal interest on borrowing, i.e. when B_{t-1}^j is negative and therefore $I_{t-1}^j = 0$; W_t^j denotes the nominal wage which is specific to labor of agent $j = b, s$; Ψ_t^j are profits obtained from goods production while Υ_t^j are the profits of the intermediary sector.

Since we have assumed that agents share the same discount factor β in their preferences, we can start from an initial steady state featuring a non degenerate distribution of wealth and identify as “savers” the agents with positive holdings of the inside security, $B_t^s > 0$, and

as “borrowers” those with negative ones, $B_t^b < 0$. Furthermore, we make this steady-state distribution of wealth unique – and therefore assure convergence to it after the shock – by appropriately modelling the activity of the financial intermediaries. Finally we verify that borrowers and savers do not switch their portfolio positions during the time in which the liquidity shock hits the economy.

In models like Curdia and Woodford (2010), savers and borrowers have different preferences, which is the reason why a sector of financial intermediaries is meaningful. Instead, in our model, this role comes naturally because the inside asset has a different use for the two agents. Both borrowers and savers can transfer wealth intertemporally using the inside security, but only savers hold it in a positive amount and use it for liquidity purposes, as shown in (14). Intermediaries can raise liquidity from savers by paying the interest rate $(1 + i_t^d)$ on their deposits and use it to lend to the borrowers at a higher rate, $(1 + i_t^b)$. Positive margins of intermediation, and credit spreads, emerge endogenously in our model.

Agents choose consumption and hours worked to maximize utility (13) under (14) and (15) taking into account standard borrowing-limit constraints. First-order conditions of the two optimization problems are symmetric with respect to consumption, money and labor

$$U_c(C_t^j) = (\psi_t^j + \lambda_t^j)P_t \quad (16)$$

$$\lambda_t^j = \beta(1 + i_t^m)E_t(\psi_{t+1}^j + \lambda_{t+1}^j), \quad (17)$$

$$V_t(L_t^j) = \lambda_t^j W_t^j \quad (18)$$

for $j = b, s$, where ψ_t^j and λ_t^j are the respective Lagrange multipliers of constraints (14) and (15). The first-order condition of the savers with respect to deposit holdings implies

$$\lambda_t^s = \beta(1 + i_t^d)E_t(\gamma_{t+1}\psi_{t+1}^s + \lambda_{t+1}^s), \quad (19)$$

while that of the borrowers with respect to loans¹⁰

$$\lambda_t^b = \beta(1 + i_t^b)E_t\lambda_{t+1}^b. \quad (20)$$

We can combine more compactly the above first-order conditions to obtain the interest-rate spread between deposits and money

$$\frac{i_t^d - i_t^m}{1 + i_t^d}E_t\left\{\frac{U_c(C_{t+1}^s)}{P_{t+1}}\right\} = E_t\left\{(1 - \gamma_{t+1})\varphi_{t+1}^s\frac{U_c(C_{t+1}^s)}{P_{t+1}}\right\}, \quad (21)$$

¹⁰In writing the intertemporal first-order conditions of savers and borrowers we have already accounted for the fact that in equilibrium $B_t^s > 0$ and $B_t^b < 0$.

and between loans and money

$$\frac{i_t^b - i_t^m}{1 + i_t^b} E_t \left\{ \frac{U_c(C_{t+1}^b)}{P_{t+1}} \right\} = E_t \left\{ \varphi_{t+1}^b \frac{U_c(C_{t+1}^b)}{P_{t+1}} \right\}. \quad (22)$$

Deposit and loan rates are in general higher than the interest rate on money (or reserves) insofar as the variables φ_{t+1}^s and φ_{t+1}^b are non-zero in some contingency, where

$$\varphi_t^j = 1 - \beta(1 + i_t^m) E_t \left\{ \frac{U_c(C_{t+1}^j)}{U_c(C_t^j)} \frac{P_t}{P_{t+1}} \right\}, \quad (23)$$

and we have used the following definitions $\varphi_t^j \equiv \psi_t^j P_t / U_c(C_t^j)$ for $j = b, s$. The liquidity shock γ_t affects the interest-rate spread between deposits and money. When the liquidity properties of deposits improve, the interest rate on deposit falls closer to that on money.

Finally, we can write in a more compact way the marginal rate of substitution between leisure and consumption through the following conditions

$$\frac{V_l(L_t^j)}{U_c(C_t^j)} = (1 - \varphi_t^j) \frac{W_t^j}{P_t}, \quad (24)$$

for $j = b, s$. In this model, the liquidity constraint implies a financial friction which is captured by the variables φ_t^j . This friction creates also a wedge between the real wage and the marginal rate of substitution between leisure and consumption, as shown in (24).¹¹

4.2 Financial intermediaries

Financial intermediaries channel liquidity from savers to borrowers and have positive margins of intermediation because deposits have a liquidity value, as they can be used by savers in exchange for goods. The overall level of deposits is $D_t = \chi B_t^s$, that of loans is $A_t = -(1 - \chi) B_t^b$. The intermediaries' balance sheet in each period implies $A_t = D_t$.

In period t profits of intermediation in real terms are

$$\frac{\Upsilon_t}{P_{t-1}} = (1 + i_{t-1}^b) a_{t-1} - (1 + i_{t-1}^d) d_{t-1} - k \cdot \phi \left(\frac{(1 + i_{t-1}^d)}{(1 + \bar{i}^d)} \frac{d_{t-1}}{\bar{d}} \right)$$

which depend on the volume of lending and deposit supplied in the previous period, where $a_t = A_t / P_t$ and $d_t = D_t / P_t$. As in Belongia and Ireland (2006, 2012) and Curdia and Woodford (2010), we also assume that financial intermediaries face a cost of increasing their borrowing capacity above a certain threshold. The cost is given by the function $\phi(\cdot)$ with the

¹¹This is consistent with the cash-in-advance constraint model of Cooley and Hansen (1989).

properties $\phi(1) > 0$, $\phi'(1) = 1$ and $\phi''(1) > 0$ where $\phi'(\cdot)$ and $\phi''(\cdot)$ are respectively the first and second derivatives of $\phi(\cdot)$. The variable \bar{d} defines the steady-state level of deposits and k is an appropriate scaling factor given by $k = (1 + \bar{r}^b)\bar{\delta}\bar{d}$ where $\bar{\delta}$ is the steady-state spread between borrowing and lending rates defined by $(1 + \bar{\delta}) \equiv (1 + \bar{r}^b)/(1 + \bar{r}^d)$ which is positive because of the different liquidity properties of deposits and loans in the steady state. The function $\phi(\cdot)$ captures the costs of enlarging the deposit capacity of the intermediaries, which can be related to the managerial costs of increasing the volume of deposits to their customers, but also to the macroeconomic risk that too much borrowing of the intermediaries can create on the overall quality of their deposits. This is the reason why we assume that the cost function depends on the overall payment that intermediaries have to deliver in each period to savers. The marginal cost of raising deposits is positive and increasing with the overall expected payment. In our model, the cost function $\phi(\cdot)$ and its properties are important to determine the steady-state distribution of wealth between savers and borrowers and to assure converge to it after the shock. We assume that the costs $k \cdot \phi(\cdot)$ are paid directly to the savers as are the profits of intermediation Υ_t , which are known in period $t - 1$ and delivered in period t .¹²

In a competitive market, intermediaries set the spread between borrowing and lending rates to maximize profits

$$1 + \delta_t \equiv \frac{(1 + i_t^b)}{(1 + i_t^d)} = \left[1 + \bar{\delta}\phi' \left(\frac{(1 + i_t^d)}{(1 + \bar{r}^d)} \frac{d_t}{\bar{d}} \right) \right]. \quad (25)$$

The spread δ_t is in general increasing in the overall repayments due to depositors and is consistent with its steady-state value since $\phi'(1) = 1$. Given the formulation of the cost function, an increase in the deposit rate moves more than proportionally the loans rate and raises the spread between lending and deposit rates.

4.3 Firms

We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function $Y(i) = L(i)$ is linear in a bundle of labor which is a Cobb-Douglas index of the two types of labor: $L(i) = (L^s(i))^\chi (L^b(i))^{1-\chi}$. Given this technology, labor compensation for each type of worker is equal to total compensation $W_j L_j = WL$ where the aggregate wage index is appropriately given by $W = (W^s)^\chi (W^b)^{1-\chi}$.

¹²Quantitative results can be affected but not overturned by different assumptions on the distribution of intermediation profits, as we will discuss later.

Each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta}Y$ where aggregate output is

$$Y_t = \chi C_t^s + (1 - \chi)C_t^b. \quad (26)$$

Firms are subject to price rigidities as in Calvo's model: in each period a fraction of measure $(1 - \alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price, while the remaining fraction α of firms indexes their previously-adjusted price to the inflation-target rate $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of the profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left[\bar{\Pi}^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - (1 - \tau) \frac{W_T}{P_T} Y_T(i) \right]$$

where Λ_T is the stochastic discount factor used to evaluate profits at a generic time T , which is a linear combination of the marginal utilities of consumption of the two agents, $\Lambda_T = \beta^{T-t}[\chi U_c(C_T^s) + (1 - \chi)U_c(C_T^b)]$ and τ is an employment subsidy. The first-order condition of the optimal pricing problem implies

$$\frac{P_t^*}{P_t} = (1 - \tau) \frac{\theta}{\theta - 1} \frac{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}} \quad (27)$$

where we have set $P_t(i) = P_t^*$ since all firms adjusting their prices will fix it at the same price. Calvo's model further implies the following law of motion for the general price index

$$P_t^{1-\theta} = (1 - \alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \bar{\Pi}^{1-\theta}, \quad (28)$$

through which we can write the aggregate supply equation

$$(1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta})^{\frac{1}{1-\theta}} = (1 - \tau) \frac{\theta}{\theta - 1} \frac{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}}. \quad (29)$$

We assume that the utility flow from consumption is exponential $u(C^j) = 1 - \exp(-vC^j)$ for some positive parameter v while the disutility of working is isoelastic $v(L^j) = (L^j)^{1+\eta}/(1+\eta)$. These are convenient assumptions for aggregation purposes and to keep tractability. These features can be easily discovered by taking a weighted average of (24), for $j = s, b$, with

weights χ and $1 - \chi$ respectively obtaining¹³

$$\frac{(Y_t \Delta_t)^\eta}{v \exp(-v Y_t)} = \frac{W_t}{P_t} (1 - \varphi_t^s)^\chi (1 - \varphi_t^b)^{1-\chi}, \quad (30)$$

where aggregate output and labor are related through $Y_t \Delta_t = L_t$ and Δ_t is an index of price dispersion defined by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di,$$

which evolves as

$$\Delta_t \equiv \alpha (\Pi_t^\theta \bar{\Pi}^{-\theta}) \Delta_{t-1} + (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{\theta}{\theta-1}}. \quad (31)$$

4.4 Government budget constraint and monetary policy

To complete the characterization of the model we specify the consolidated budget constraint of government and central bank. We assume that there are no government bonds or public spending. The consolidated budget constraint simply reads as

$$M_t = (1 + i_{t-1}^m) M_{t-1} + \chi T_t^s + (1 - \chi) T_t^b + \tau W_t L_t. \quad (32)$$

It is clear that with heterogenous agents the distribution of transfers matters for the equilibrium allocation. We assume that each agent receives transfers corresponding to its holdings of money after subtracting a proportional share of the employment subsidy,

$$T_t^s = M_t^s - (1 + i_{t-1}^m) M_{t-1}^s - \tau W_t L_t, \quad (33)$$

$$T_t^b = M_t^b - (1 + i_{t-1}^m) M_{t-1}^b - \tau W_t L_t. \quad (34)$$

4.5 Equilibrium in goods and asset markets

We consider equilibria in which the constraint (14) for $j = b, s$ is binding. In this case, they imply

$$(1 + i_{t-1}^m) M_{t-1}^b = P_t C_t^b \quad (35)$$

¹³In deriving (30), the assumption of a Cobb-Douglas production technology is also critical. It should be noted that another implication of our specification of preferences and production technology is that the steady-state level of output implied by (30) does not depend on the distribution of wealth. Indeed, in a steady state in which $\Pi_t = \bar{\Pi}$ prices will be set as a mark-up over wages and moreover $\bar{\varphi}^s = \bar{\varphi}^b$ in a way that the steady-state output implied in (30) is invariant to the distribution of wealth across agents.

$$(1 + i_{t-1}^m)M_{t-1}^s + \gamma_t(1 + i_{t-1}^d)B_{t-1}^s = P_t C_t^s. \quad (36)$$

In equilibrium money supply is equal to money demand

$$M_t = \chi M_t^s + (1 - \chi)M_t^b, \quad (37)$$

while financial market equilibrium requires

$$(1 - \chi)B_t^b + \chi B_t^s = 0. \quad (38)$$

Goods market equilibrium is given by (26).

4.6 Equilibrium conditions

We collect now the equations that characterize the equilibrium of the model. On the demand side, there are equations (21) and (22), and (23) for each $j = b, s$. Lending and borrowing interest rates are connected through equation (25). The two liquidity constraints (35) and (36) can be written in real terms as

$$(1 + i_{t-1}^m) \frac{m_{t-1}^b}{\Pi_t} = C_t^b \quad (39)$$

and

$$(1 + i_{t-1}^m) \frac{m_{t-1}^s}{\Pi_t} + \gamma_t(1 + i_{t-1}^d) \frac{(1 - \chi) b_{t-1}}{\chi \Pi_t} = C_t^s \quad (40)$$

where $m_t^b \equiv M_t^b/P_t$, $m_t^s \equiv M_t^s/P_t$ while b_t denotes the real debt of the borrowers given by $b_t \equiv -B_t^b/P_t$. To obtain (40), we have also used (38).

In real terms equation (41) implies

$$\chi m_t^s + (1 - \chi) m_t^b = m_t, \quad (41)$$

where m_t denotes aggregate real money balances, defined as $m_t \equiv M_t/P_t$.

The flow budget constraint of the borrowers can be simplified, using (34), to

$$b_t = (1 + i_{t-1}^b) \frac{b_{t-1}}{\Pi_t} + C_t^b - Y_t, \quad (42)$$

where we have used the fact that the Cobb-Douglas technology implies that $W_t^b L_t^b = W_t L_t$ together with the assumption $\Psi_t = 0$.¹⁴

¹⁴If all the profits of intermediation were rebated to the borrowers, it would be easy to see that the relevant interest rate in (42) is the deposit rate, i_t^d , instead of the loan rate, i_t^b .

On the aggregate supply side, there is equation (29) together with (30) and (31) and the relationship $Y_t \Delta_t = L_t$.

The set of equations (21), (22), (23) for each $j = b, s$ together with (25), (26), (29), (30), (31), (39), (40), (41), (42), describe the equilibrium conditions of the model. There are 13 equations in the following 15 unknowns $Y_t, C_t^b, C_t^s, i_t^b, i_t^d, i_t^m, \Delta_t, W_t/P_t, P_t, b_t, \varphi_t^s, \varphi_t^b, m_t^s, m_t^b, m_t$ leaving the possibility to specify two instruments of policy.

5 Liquidity shocks and optimal monetary policy

We repeat the experiment of a shock that worsens the liquidity properties of the pseudo-safe asset. The model has now a richer transmission mechanism and there is also a propagation of the shock to the real economy, because of the redistributive effects between borrowers and savers and of nominal rigidities. We compare alternative monetary regimes with the Ramsey policy that maximizes the weighted sum of the utility of the consumers belonging to the economy:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} [\tilde{\chi}(U(C_T^s) - V(L_T^s)) + (1 - \tilde{\chi})(U(C_T^b) - V(L_T^b))], \quad (43)$$

given the equilibrium conditions of the model, in which $\tilde{\chi}$ and $(1 - \tilde{\chi})$ are the relative weights, respectively, of savers and borrowers in the objective function.

To get intuition about the underlining trade-offs, we can derive a simple quadratic loss function corresponding to just a second-order approximation of (43) under some relatively minor restrictions. In particular, we use assumptions such that the steady state resulting from the Ramsey problem coincides with the efficient steady-state allocation of consumption and labor. This efficient allocation solves the maximization of (43) under the resource constraint

$$\chi C_t^s + (1 - \chi) C_t^b = Y_t = (L_t^s)^\chi (L_t^b)^{1-\chi}.$$

In particular, as discussed in the Appendix, the first-order conditions of this problem imply

$$\frac{\tilde{\chi}}{1 - \tilde{\chi}} \frac{U_c(\bar{C}^s)}{U_c(\bar{C}^b)} = \frac{\chi}{1 - \chi} \quad (44)$$

$$\frac{V_l(\bar{L}^j)}{U_c(\bar{C}^j)} = \frac{\bar{Y}}{\bar{L}^j} \quad (45)$$

where the latter holds for each $j = b, s$. This is clearly the best allocation that can be achieved in this model, for given weights $\tilde{\chi}$ and $(1 - \tilde{\chi})$. If the Ramsey policymaker could implement this allocation in the steady state, this would correspond to the Ramsey optimal policy when

there are no stochastic disturbances. We show that the combination of policies: $\Pi_t = \bar{\Pi}$ and $i_t^m = \bar{i}^m$ where $\bar{\Pi} \geq \beta$ and $0 \leq \bar{i}^m \leq \bar{\Pi}/\beta - 1$ can indeed implement the first best, under two minor restrictions.¹⁵ The first restriction requires that the employment subsidy to firms is set at $\tau = (\bar{\mu} + \bar{\varphi})/(1 + \bar{\mu})$ where $\bar{\mu} \equiv (\theta - 1)^{-1}$ and $\bar{\varphi} = \bar{\varphi}^s = \bar{\varphi}^b$ are, respectively, the steady-state net markup and level of the financial friction. In fact a policy in which $\Pi_t = \bar{\Pi}$ implies in equation (29) that

$$\frac{\bar{W}}{\bar{P}} = \frac{1}{(1 - \tau)(1 + \bar{\mu})}$$

which can be used in the steady-state version of (24) to get

$$\frac{V_l(\bar{L}^j)}{U_c(\bar{C}^j)} = \frac{1 - \bar{\varphi}}{(1 - \tau)(1 + \bar{\mu})} \frac{\bar{Y}}{\bar{L}^j}, \quad (46)$$

for each $j = b, s$. With the chosen subsidy $\tau = (\bar{\mu} + \bar{\varphi})/(1 + \bar{\mu})$, equation (46) is equivalent to (45).

To implement (44), note that $\Pi_t = \bar{\Pi}$ and $i_t^m = \bar{i}^m$ imply that

$$\bar{C}^s = \frac{1 - \chi}{\chi} \left(\frac{1 - \beta}{\beta} \right) \bar{b} + \bar{Y} \quad (47)$$

$$\bar{C}^b = - \left(\frac{1 - \beta}{\beta} \right) \bar{b} + \bar{Y} \quad (48)$$

where we have used the steady-state version of (20), (38) and (42). Given that $\bar{b} > 0$, it follows that $\bar{C}^s > \bar{C}^b$. To be consistent with (44), we add the second restriction, that the weight $\tilde{\chi}$ is appropriately chosen to make the steady-state distribution of wealth efficient. In particular, $\tilde{\chi} > \chi$.

A corollary of these restrictions and implementation exercise is that the Ramsey policymaker can implement the first best for any choice of \bar{i}^m in the interval $0 \leq \bar{i}^m \leq \bar{\Pi}/\beta - 1$ implying, from the steady-state version of (23), that the Lagrange multiplier $\bar{\varphi}$ can fall in the interval $0 \leq \bar{\varphi} \leq 1 - \beta/\bar{\Pi}$. The Friedman's rule, which should command to completely eliminate the financial friction and set $\bar{\varphi} = 0$, is completely irrelevant for welfare in such a defined steady state. We can indeed choose a steady-state value of $\bar{\varphi}$ different from zero and get the same first-best allocation of consumption and labor and, at the same time, be consistent with the optimal choice of a Ramsey policymaker.¹⁶ A positive value of the multiplier $\bar{\varphi}$ is

¹⁵It should be recalled that we can specify two policy instruments in our model.

¹⁶Note that our framework nests standard results on the Friedman's rule. In the case in which $\bar{\mu} = 0$, $\tau = 0$ and prices are flexible, it is optimal to set $\bar{\varphi} = 0$. However, this does not necessarily implies a rate of deflation equal to the time discount factor, since \bar{i}^m is also a policy variable and therefore $\bar{\Pi} = \beta(1 + \bar{i}^m)$. Only if $\bar{i}^m = 0$, as done in the literature, $\bar{\Pi} = \beta$.

convenient to make the financial friction non-negligible in a first-order approximation of our model. To further justify this choice, it is worth noting that in heterogenous-agent stochastic models with incomplete markets the Friedman's rule is not achievable. In our model, indeed, it requires to set φ_t^s and φ_t^b simultaneously to zero at all times, but this is not feasible unless $U_c(C_{t+1}^s)/U_c(C_t^s) = U_c(C_{t+1}^b)/U_c(C_t^b)$ in all contingencies as shown in (23).¹⁷

We show in the Appendix that under these two assumptions, a second-order approximation of (43) delivers the following simple quadratic loss function

$$\frac{1}{2}E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^2 + \chi(1-\chi)\lambda_c(\hat{C}_T^s - \hat{C}_T^b)^2 + \chi(1-\chi)\lambda_l(\hat{L}_T^s - \hat{L}_T^b)^2 + \lambda_\pi(\pi_T - \bar{\pi})^2 \right] \right\}$$

where in general hat variables denote deviations from the steady state, while $\hat{C}_t^j \equiv (C_t^j - \bar{C}^j)/\bar{Y}$ for each $j = b, s$, $\pi_t = \ln P_t/P_{t-1}$ and $\bar{\pi} = \ln \bar{P}$. The positive coefficients λ_c , λ_l and λ_π are all defined in the Appendix.

The loss function contains some familiar terms to the literature. The only shock of the model is to liquidity, which is an inefficient shock, therefore deviations of output with respect to the efficient steady state are penalized appropriately. Inflation is also costly when it deviates from the trend to which price setters index prices, implying inefficient fluctuations of relative prices among goods produced according to the same technology. The other two terms in the loss function instead depend on the additional features that the heterogeneity of agents brings into the model. Since risk sharing of consumption and labor is efficient in the chosen steady state, departures from this allocation cause losses for aggregate welfare. In particular, the labor risk-sharing term can be further simplified noting that in a first-order approximation

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta}(\hat{C}_t^s - \hat{C}_t^b) - \frac{1}{(1-\bar{\varphi})(1+\eta)}(\varphi_t^s - \varphi_t^b),$$

where $\rho = v\bar{Y}$. In standard models without financial frictions, the labor risk-sharing argument is proportional to the consumption risk-sharing term. Here, instead, it is also relevant to consider the influence of the financial distortions across agents. It should be noted that in a first-order approximation of (23) we get

$$\varphi_t^j = \bar{\varphi} + (1-\bar{\varphi})E_t \left[(\pi_{t+1} - \bar{\pi}) - \hat{v}_t^m + \rho\Delta\hat{C}_{t+1}^j \right]$$

¹⁷See Woodford (1990) for a general discussion.

for $j = b, s$ and therefore we can simplify the labor risk-sharing term to

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta} E_t(\hat{C}_{t+1}^s - \hat{C}_{t+1}^b). \quad (49)$$

Because of the financial friction, labor effort at time t is producing income which is only liquid to purchase goods in the next period.¹⁸ As shown by the first-order condition (24), using (23), the consumers are optimally choosing labor given current wages and future prices taking into account their expectations of future consumption. It follows that the cross-agent difference in labor is proportional to the difference in the one-period ahead expectation of consumption. Equivalently, we can write the loss function as

$$\frac{1}{2} E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^2 + \chi(1-\chi)\lambda_c \left(\hat{C}_T^R \right)^2 + \chi(1-\chi)\tilde{\lambda}_l \left(E_T \hat{C}_{T+1}^R \right)^2 + \lambda_\pi (\pi_T - \bar{\pi})^2 \right] \right\} \quad (50)$$

for some $\tilde{\lambda}_l$, where $\hat{C}_T^R \equiv \hat{C}_T^s - \hat{C}_T^b$. We compute the optimal policy under commitment by minimizing this loss function with respect to the log-linear approximation of the equilibrium conditions.

For the numerical exercise, the model is calibrated (quarterly) as follows. We set $\beta = 0.99$ and $\bar{\pi} = 0.02/4$, to imply a steady-state annualized nominal interest rate on bonds of about 6%, while the steady-state interest rate on reserves is calibrated at $\bar{r}^m = 0.01/4$. We use the average velocity of M1 for the U.S. economy during the Great Moderation (1984-2007) to calibrate the steady-state money to GDP ratio: $M/PY = 0.125 \cdot 4$. To calibrate the ratio of households deposits to GDP, we use the average, over the same period, of M2 net of M1: $\chi B^s/PY = 0.375 \cdot 4$. The economy-wide liquidity constraint and the steady-state version of (21) imply that about one third of assets provides liquidity services (i.e. $\bar{\gamma} = 0.33$) and an annualized nominal interest rate on bank deposits of about 4.4%. We then calibrate the share of savers in the economy to 62.5% so that the equilibrium in the bond market (38) is consistent with an initial debt to income ratio of about 100%, as in Eggertsson and Krugman (2012). We follow the latter also in setting the elasticity of the credit spread to the stock of real debt to 0.049.¹⁹ A given value for the initial debt to income ratio, then, pins down a unique initial distribution of wealth: indeed, equations (47) and (48) yield the distribution of personal consumption, which in turn implies the distribution of money holdings, through equations (39) and (40). Finally, the relative risk-aversion coefficient is set to $\rho = 1$, the inverse of the Frisch-elasticity of labor supply to $\eta = 2$, the parameter α capturing the

¹⁸It should be noted that (49) is valid also when $\bar{\varphi} = 0$ and that it captures the impossibility to achieve the Friedman's rule in the stochastic equilibrium ($\varphi_t^s = \varphi_t^b$) unless $\hat{C}_{t+1}^b = \hat{C}_{t+1}^s$ at all times.

¹⁹This corresponds to the parameter ϕ defined in the Appendix.

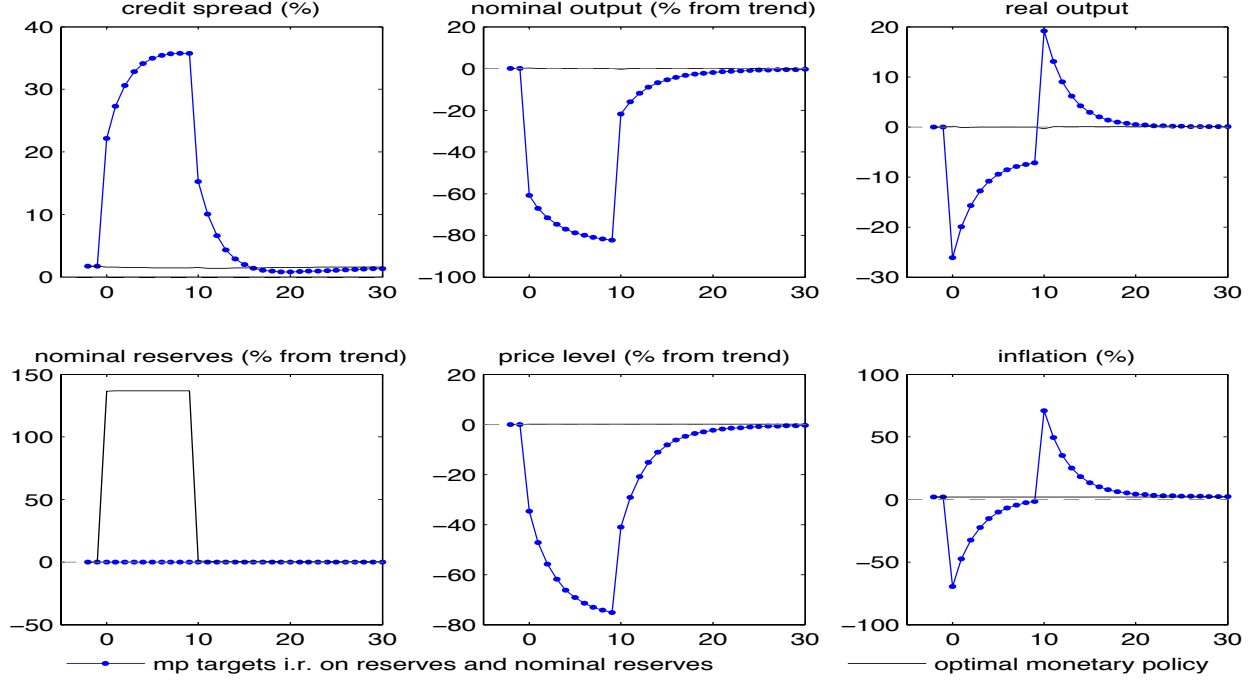


Figure 2: Responses of selected variables to a 20% fall in the liquidity properties of bonds: passive monetary regime vs optimal policy. Dotted line: monetary policy keeps both the money growth and the interest rate on reserves i_t^m constant (passive regime). Solid line: optimal monetary policy. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ($\alpha = 0.75$).

As in the previous section, we assume that at time 0 it is learnt that the liquidity properties of the pseudo-safe asset will deteriorate next period, $\gamma_t = \gamma_L < \bar{\gamma}$ for $t = 1$, and that they will revert back to their steady-state level with a fixed probability ξ in every period; *ex-post*, the shock actually lasts T periods, returning to steady state in period $T + 1$. The liquidity shock we consider is such that the share of pseudo-safe assets providing liquidity services falls 20 percentage points, from about 33% to 13%, the probability of reversal each period is $\xi = 10\%$ and the ex-post duration of the shock is $T = 10$.

In Figures 2 and 3 we compare the optimal policy (in solid line) with the passive monetary policy (dotted line) in which the interest rate on reserves and the growth rate of the nominal money supply are kept constant at the levels before the shock hits.

There are two important policy implications on what monetary policy should do when facing a liquidity shock. Inject more liquidity in the form of money, as shown in Figure 2, and lower the interest rate on reserves, as shown in Figure 3. Although it is in general hard to isolate the effects of the two channels in the general equilibrium of the model, we argue that the injection of liquidity avoids the deflation and the contraction in real activity,

while lowering the interest rate on reserves helps to achieve a better risk sharing of the shock between savers and borrowers.

The transmission of the liquidity shock can be understood in a simple way through two main mechanisms. First, the liquidity shock creates, at an aggregate level, a shortage in the supply of the assets available for the exchange for goods, because pseudo-safe assets have partially lost their qualities. To this shortage of safe assets, an excess supply of goods corresponds. Nominal spending falls, and the split between prices and real output depends on the degree of price rigidities. This is what happens under the passive policy in which real output drops dramatically, as shown in the figure, with a contraction of about 25% while prices fall by over 70% with respect to their trend, through a deep deflation. The figure illustrates clearly how important the effects on the economy can be when the monetary policymaker is completely helpless. Under optimal policy, instead, the contraction in real output is very mild as well as the response of inflation and prices which remain close to their targets. The key change in policy, that leads to a near stabilization of output and prices, is the increase in the growth rate of money, as shown in Figure 2, which goes up to a path about 140% above the previous trend. Optimal policy, therefore, requires a substantial expansion in the central bank's balance sheet and an increase in the supply of safe assets. Interestingly, the expansion should last for just the periods in which the liquidity conditions are deteriorated. Monetary policy should be ready to promptly withdraw the additional liquidity created.

The second mechanism of propagation works through asset prices. The liquidity shock requires a higher premium to hold pseudo-safe assets. This in turn increases the cost of funding for intermediaries which need to raise, more than proportionally, the interest rate on loans. Under the passive monetary policy, the spread across all securities increases as shown in Figure 3 and in particular the interest rate on loans, which reaches a range between 40% and 50% at annual rates, while the interest rate on deposits goes up to 15%. Under optimal policy, all market interest rates are instead insulated from the shock and stable around the previous steady-state levels. The important change in policy that explains this result is the reduction in the interest rate on reserves up to the point in which the zero-lower bound becomes a constraint. To intuit this result, Figure 3 displays the consumption of savers and borrowers, respectively, and their difference. In our model, as shown in the loss function (50), there is a concern for consumption risk sharing both for contemporaneous consumption and its one-period ahead expectation. Under the passive monetary policy, there is too much dispersion in consumption across savers and borrowers. Interest rates go up and nominal prices fall to imply a deflation. The increase in the real interest rate has important wealth and redistributive effects between borrowers and savers under the passive monetary regime. Borrowers are hit in a significant way by the rise in the real rate, so that they

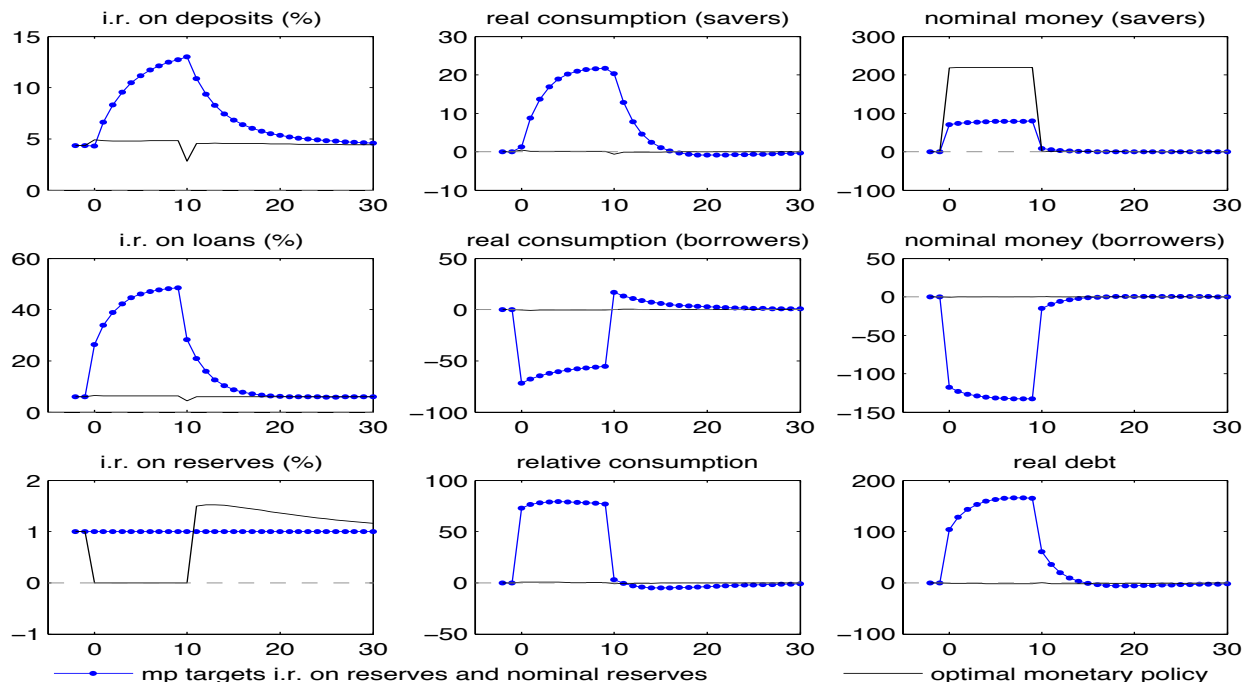


Figure 3: Responses of selected variables to a 20% fall in the liquidity properties of bonds: passive monetary regime vs optimal policy. Dotted line: monetary policy keeps both the money growth and the interest rate on reserves i_t^m constant (passive regime). Solid line: optimal monetary policy. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

have to cut on their consumption.²⁰ Their real debt even increases, mostly because of the deflation. Savers, instead, benefit in part from the liquidity shock and the ensuing higher real interest rates. They are able to increase their consumption by holding more safe assets at the expenses of borrowers. Under optimal policy, instead, the interest rate on reserves falls and this stabilizes all other market interest rates. The redistributive channel, which was strong under the passive policy, is now muted and the liquidity injection by the monetary authority prevents the deflation and allows savers to increase their money holdings without crowding out those of borrowers. The latter can maintain their previous level of consumption because the liquidity shock is perfectly absorbed by the fall in the interest rate on reserves which stabilizes the real interest rate.

Compared to the previous literature, there are two other important differences: 1) the driving shock, in the literature as in Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), is to the natural rate of interest which requires a fall in the real interest rate, while in our model the shock is to liquidity and the real rate does not move much under optimal policy; 2) in Eggertsson and Woodford (2003) the zero-lower bound on nominal

²⁰This is also the case if the profits of financial intermediary are all distributed to the borrowers. In this case, the relevant nominal interest rate is the deposit rate.

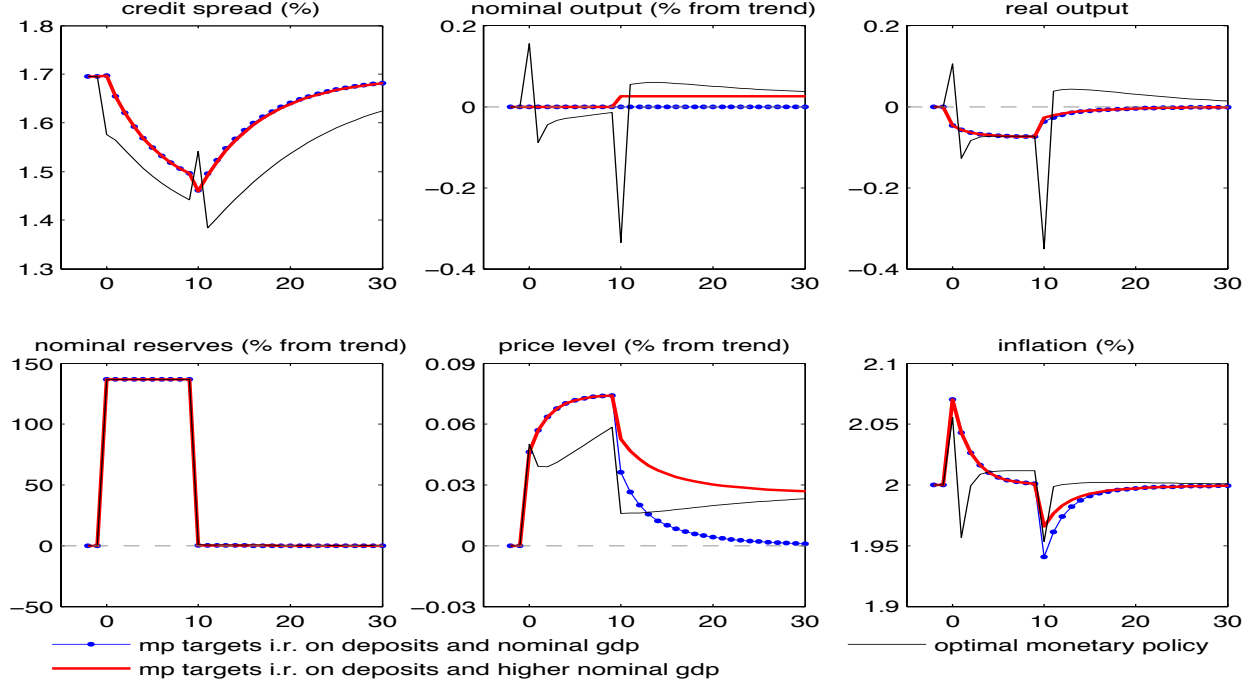


Figure 4: Responses of selected variables to a 20% fall in the liquidity properties of bonds: active monetary regime vs optimal policy. Dotted line: monetary policy implements nominal-gdp targeting and seeks stabilization of i_t^d . Bold solid line: monetary policy raises the nominal-gdp target when the shock reverts to zero. Solid line: optimal monetary policy. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

interest rate is a constraint to achieve the optimal stabilization of aggregate objectives like output and inflation, while in our model balance-sheet policies take care of the aggregate objective and the zero-lower bound is a constraint to achieve risk-sharing objectives built into the loss function.

We further investigate the features of the optimal policy and compare it with other policies that can approximate it, as shown in Figures 4 and 5. It should be noted that the scale of these figures is different from that of the previous one and enables us to appreciate more the variation of the variables of interest. We display the optimal policy in contrast with a simple policy in which nominal-GDP targeting is achieved in each period and the interest rate on reserves is lowered in order to insulate the interest rate on deposits from the liquidity shock. Under the calibration considered, this requires keeping i_t^m to zero until the time in which the liquidity properties of the pseudo-safe assets return to normal. We also add another policy regime, which slightly modifies the previous one: the central bank permanently raises its nominal-GDP target when the liquidity properties of the pseudo-safe asset return to normal. The chosen simple targeting regimes can approximate well the optimal policy along the objectives of the loss function (50): the deviations are in general small and imply negligible losses in terms of welfare, unlike the case of passive monetary policy.

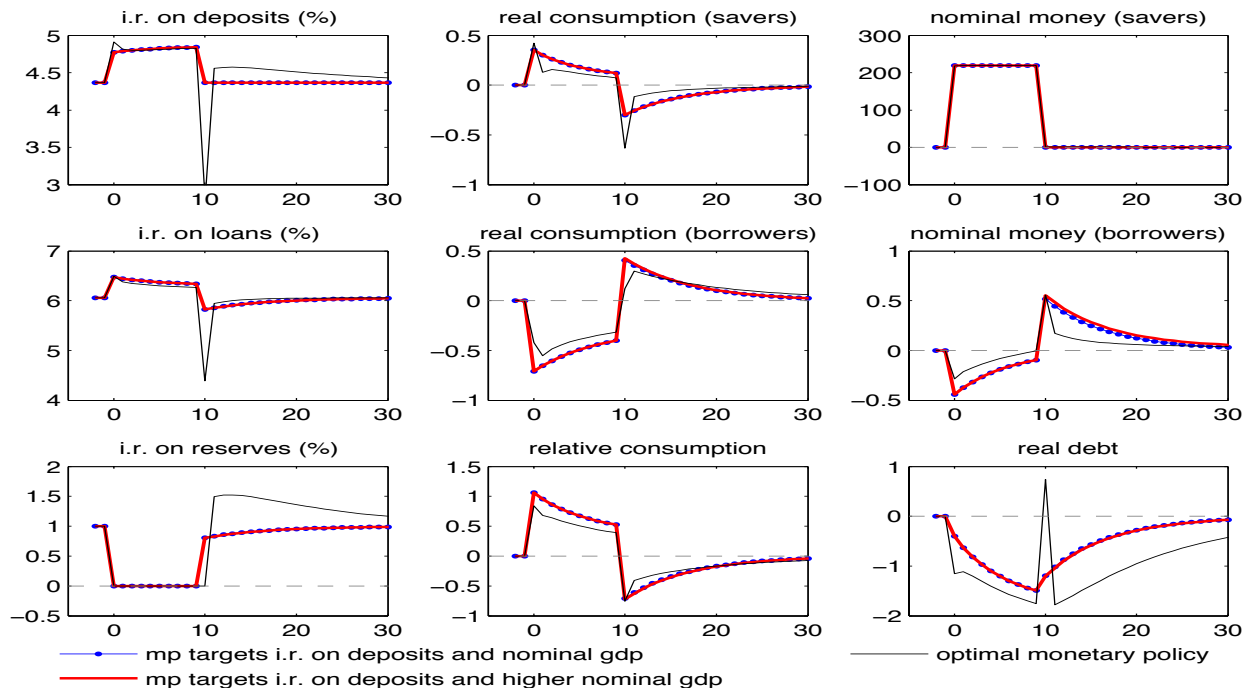


Figure 5: Responses of selected variables to a 20% fall in the liquidity properties of bonds: active monetary regime vs optimal policy. Dotted line: monetary policy implements nominal-gdp targeting and seeks stabilization of i_t^d . Bold solid line: monetary policy raises the nominal-gdp target when the shock reverts to zero. Solid line: optimal monetary policy. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

There is one interesting reason why we treat also the case of a nominal-GDP targeting with an upward revision in the path after the end of the liquidity shock: we notice, indeed, that under optimal policy both the price level and nominal GDP converge to new levels in the long run, while a policy of strict nominal-GDP targeting or inflation targeting implies a return of the price level to the initial trend in the long run.²¹

Figures 4 and 5 show, however, that there are no important differences in the real effects of the two different nominal-GDP targeting policies, and that the main difference is in the path of prices and inflation.

The simple targeting policies just described are also useful to disentangle the two transmission channels discussed above. To this end, Figure 6 contrasts the responses of some relevant variables under optimal policy with those under two alternative targeting regimes: in one the central bank targets the nominal GDP (without upward revision after the shock)

²¹This is also an important feature of optimal policy in the model of Eggertsson and Woodford (2003). However, in their context, a policy of strict inflation or nominal-GDP targeting is not feasible for the presence of the zero-lower bound. In our model, instead, an inflation targeting policy or a nominal-GDP targeting policy can be implemented in all periods if the central bank is willing to commit to it, but this necessarily requires an expansion in the balance sheet exactly for the periods in which the liquidity shock hits the economy. The zero-lower bound is not necessarily a constraint in our model.

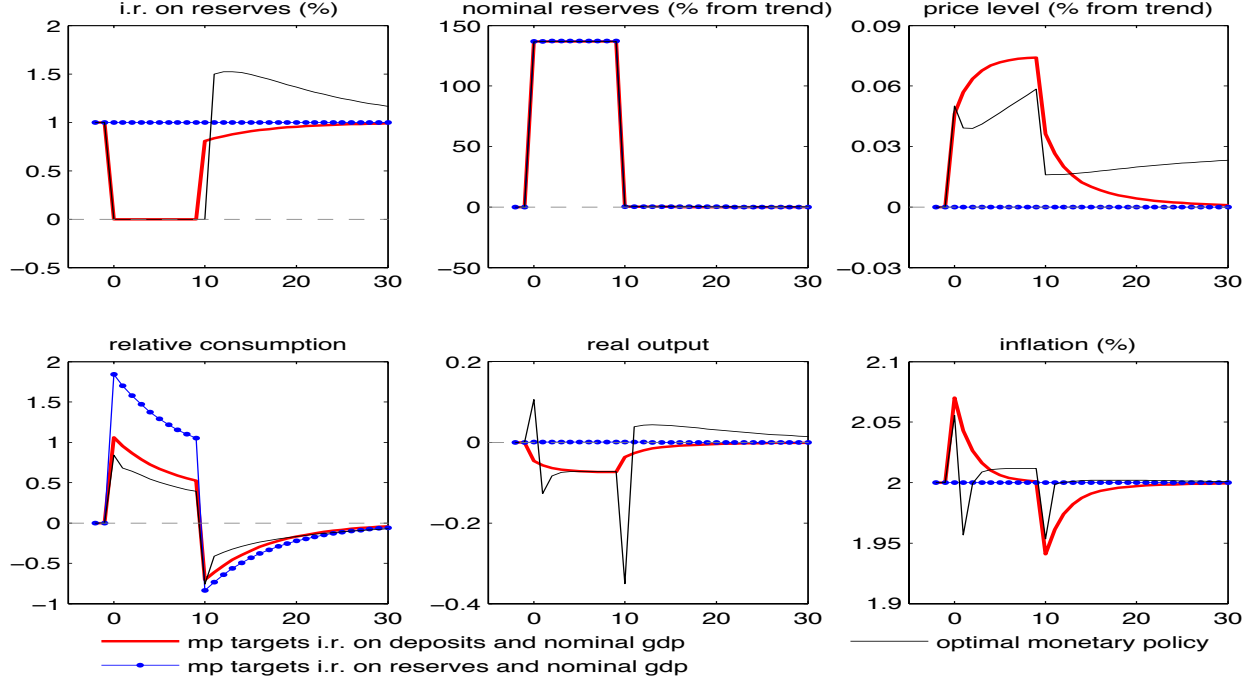


Figure 6: Responses of selected variables to a 20% fall in the liquidity properties of bonds: alternative active monetary regimes vs optimal policy. Bold solid line: monetary policy implements nominal-gdp targeting and seeks stabilization of i_t^d . Dotted line: same as before but now monetary policy keeps i_t^m constant. Solid line: optimal monetary policy. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

and keeps the interest rate on deposits constant, while in the other one it targets nominal GDP but keeps the interest rate on reserves constant. The figure clearly shows that the liquidity shock creates a trade-off between aggregate targets (output and inflation) on one side, and cross-sectional ones (consumption dispersion) on the other. The two policy regimes, indeed, imply the same degree of balance sheet expansion. However, when the central bank does not change the interest rate on reserves, full stabilization of both inflation and output can be achieved under nominal-GDP targeting, while consumption dispersion substantially increases. As we conjectured, therefore, it is the cut in the interest rate on reserves that allows the central bank to improve the risk sharing of the liquidity shock, albeit at the cost of slightly more volatile output and inflation.

Figures 7 and 8 finally explore the dimension of optimal policy that requires commitment to a permanently higher price-level path in the future. Figure 7 displays the responses of the interest rate on reserves, the detrended level of reserves and the price level to negative liquidity shocks of different sizes, and shows that the commitment to a permanently higher path of the future price level depends on the duration of the zero-lower-bound policy. When the liquidity shock is weak enough, the interest rate on deposits can be insulated from the shock without bringing the interest rate on reserves to zero, the price level rises during

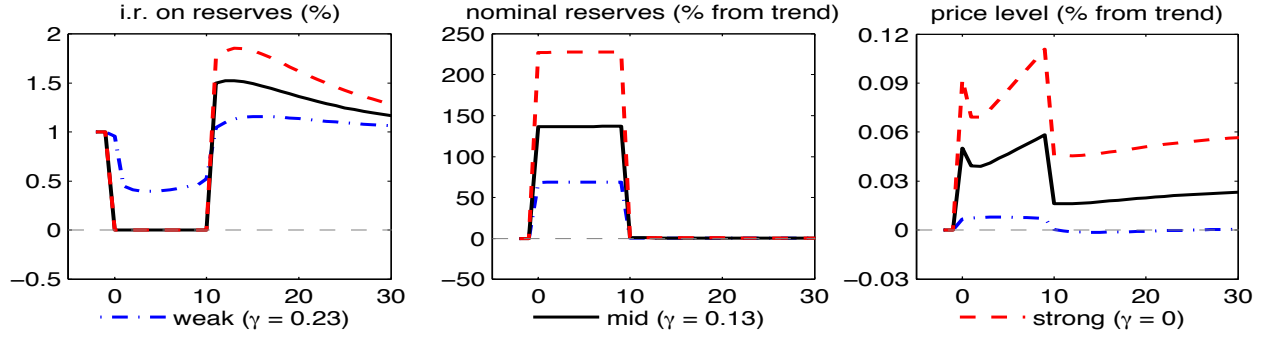


Figure 7: Responses of selected variables to negative liquidity shocks of different sizes, under the optimal monetary policy. Dash-dotted line: weak shock, $\gamma_L = 0.23$. Solid line: intermediate shock, $\gamma_L = 0.13$. Dashed line: strong shock, $\gamma_L = 0$. The probability that in each period the shock returns back to steady state is $\xi = 10\%$; the shock actually returns back after 10 quarters.

the period in which the liquidity properties of the pseudo-safe asset worsen, but it quickly converges to the previous trend thereafter. On the contrary, if the liquidity shock is strong enough to require a zero interest-rate policy, then the latter will have to last longer, the stronger the shock. The price-level path to which monetary policy has to commit will be higher, the longer the stay at the zero-lower bound.²²

Figure 8 shows that the degree of price stickiness is also important in driving this result. In particular, the figure displays the response of the same variables as in Figure 7 to a 20% deterioration of the liquidity properties of the pseudo-safe asset, for different degrees of price stickiness. Interestingly, the figure shows that under the optimal monetary policy, the economy is required to stay at the zero-lower bound longer than the duration of the shock, the more so the stickier are consumer prices. Differently from Figure 7, however, the stay at the zero-lower bound in this case is inversely related to the increase in the price-level path to which monetary policy has to commit in the future. On the one hand, indeed, more flexible consumer prices reduce the welfare costs of inflation, thereby allowing the central bank to focus more actively on the other stabilization objectives by committing strongly to increase the long-run price level. On the other hand, the stronger commitment to an increase of the future price level requires a shorter stay at the zero-lower bound, because more flexible consumer prices favor a quicker convergence to the new target path.

An additional important insight of Figure 8 is that the main policy implications of our model do not depend much on the degree of nominal rigidity. The size of the expansion in the balance sheet of the central bank is independent of the degree of price stickiness. Monetary policy should offset the shortage of safe assets by issuing more money. The need to lower the

²²If nominal interest rates could turn negative, the price level would increase more, the stronger the shock, but it would always converge back to the initial trend right after the shock has reversed to zero, regardless of the strength.

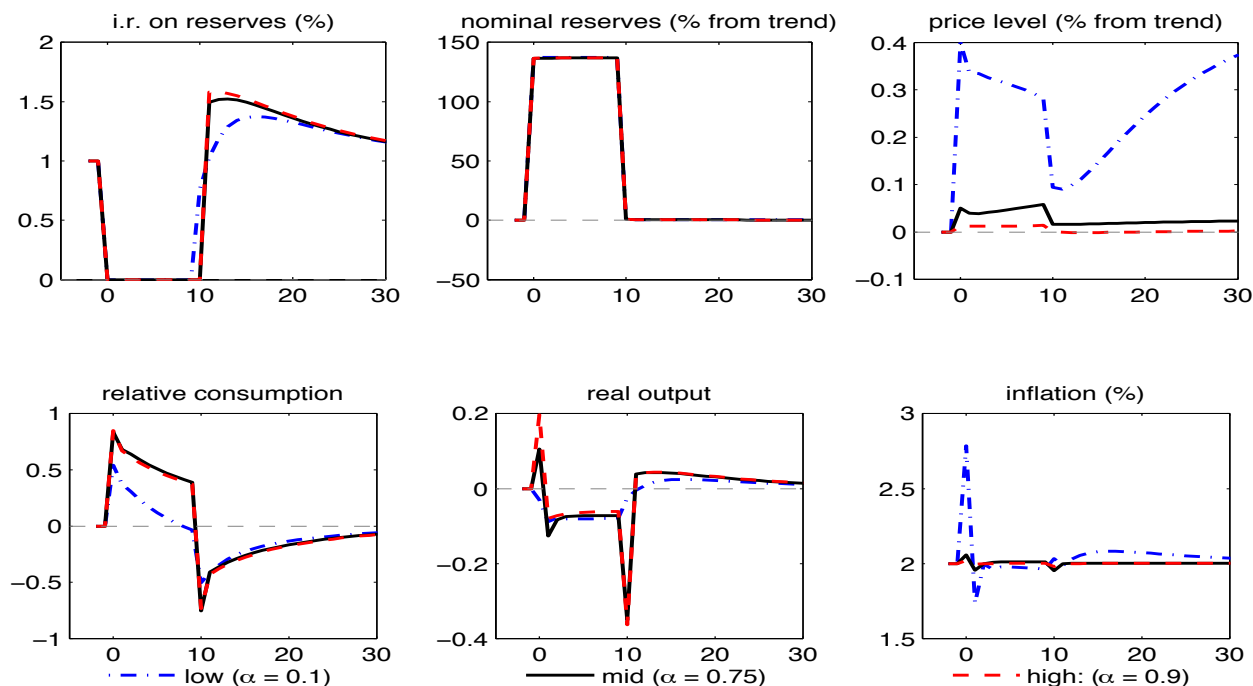


Figure 8: Responses of selected variables to a 20% fall in the liquidity properties of bonds under the optimal monetary policy: the role of price stickiness. Dash-dotted line: prices are reset every 13 weeks. Solid line: prices are reset every 4 quarters. Dashed line: prices are reset every 10 quarters. The probability that in each period the shock reverts back to steady state is $\xi = 10\%$; the shock actually reverts back after 10 quarters.

interest-rate on reserves, at least for as long as the duration of shock, is also independent of the degree of price stickiness. Indeed, even under high price flexibility (the dash-dotted line in the figure) a liquidity shock has substantial real redistributive effects between savers and borrowers, which require policy intervention.

6 Conclusions

We have presented monetary models in which the main novelty is that financial assets can have different liquidity properties. In this framework, we studied the effects on the economy of a change in these properties for some assets, which we labelled pseudo-safe assets. The overall shortage of safe assets can produce significant effects on nominal spending, and thereby on aggregate prices and real activity, in a proportion that depends on the degree of nominal rigidities. A deep recession cum deflation can emerge for a reasonable parametrization. At the same time, in a model in which the pseudo-safe asset is a deposit security through which intermediaries finance their loans, a liquidity shock raises the funding costs of intermediaries which is passed through into higher loan rates. This shock has important distributional effects between borrowers and savers, with borrowers adversely hit by the rise in the loan rates.

The role of monetary policy is critical for the propagation of the shock. Two instruments can be used to minimize the welfare consequences of the shock both at the aggregate and distributional level. The monetary policymaker should offset the shortage of safe assets by issuing more liquidity in the form of money, which remains the safe asset in circulation in the model. This can be achieved by a policy of increasing the path of nominal reserves in the vein of Quantitative Easing, to stabilize the inflation rate around the target as well as nominal and real output. Moreover, the interest rate on reserves should be reduced in order to insulate the interest rate on the pseudo-safe assets and the credit spreads. This policy improves the risk sharing of the liquidity shock between savers and borrowers and avoids a consumption recession, in particular for borrowers. For large shocks, the zero-lower bound becomes a constraint to this action.

Our work has contributed to an ongoing literature studying the cause and propagation of the financial crisis by analyzing liquidity in monetary models which have been frequently used for policy analysis before the crisis. A small departure from the standard framework is sufficient to produce an interesting transmission mechanism of a liquidity shock and capture macroeconomic behavior and policy intervention close to what economies have experienced. We see our work to be complementary to other approaches like Eggertsson and Krugman (2012) who have emphasized shocks to the natural rate in the form of deleveraging to explain the propagation mechanism of the crisis. However, in their case, a reduction in the real interest rate is important to mitigate the costs of the recession, as it is also in Eggertsson and Woodford (2003), while in our model the real rates relevant for borrowers and savers should remain stable under optimal policy. Interestingly the policy rate in our model should go to the zero-lower bound as in the literature but for different reasons, namely to improve risk sharing.

There are some limitations of our framework which can constitute ground for further work and analysis. We have abstracted from credit risk and credit events, which can be also an important channel of transmission mechanism of the recent crisis. However, the research objective of this work is to identify clearly a liquidity shock and liquidity risk as drivers of the macroeconomic adjustment. The sector of intermediaries is quite rudimentary and could be further elaborated to endogenize the creation of pseudo-safe assets. In relation to this point, the degree of acceptance of assets in exchange of goods is exogenous as in Lagos (2010), but the literature spurred from the latter work has been trying to endogenize it through differences in the information set on the quality of assets between borrowers and savers. This might be an important qualification to add to our analysis which could change some policy implications. In this vein, it could be interesting to model the exchange of assets for goods through a bargaining process instead of market equilibrium conditions. These are clearly

important avenues of future research, but the strategy in this paper is to keep the analysis simple in order to enlight the novelties of the transmission mechanism of a liquidity shock.

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Appendix A

In this appendix we present the log-linear approximation of the model of Section 2 which is used for the analysis of Section 3. The first-order approximation is taken around a deterministic steady state where we can combine equations (11) and (12) to imply

$$\frac{\bar{i} - \bar{i}^m}{1 + \bar{i}} = (1 - \bar{\gamma}) \left[1 - \frac{\beta(1 + \bar{i}^m)}{\bar{\Pi}} \right] \quad (51)$$

in which a bar denotes the steady-state value. A first-order approximation of equations (11) and (12) around the above steady state delivers

$$\hat{i}_t - \hat{i}_t^m = -\vartheta_1 E_t \hat{\gamma}_{t+1} + \vartheta_2 E_t (\hat{r}_{t+1} + (\hat{\pi}_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m) \quad (52)$$

where we have defined $\hat{i}_t \equiv \ln(1 + i_t)/(1 + \bar{i})$, $\hat{i}_t^m \equiv \ln(1 + i_t^m)/(1 + \bar{i}^m)$, $\hat{\gamma}_{t+1} = (\gamma_{t+1} - \bar{\gamma})/(1 - \bar{\gamma})$, $\pi_t = \ln P_t/P_{t-1}$ and the coefficients ϑ_1 and ϑ_2 are $\vartheta_1 \equiv (\bar{i} - \bar{i}^m)/(1 + \bar{i}^m)$ and $\vartheta_2 \equiv 1 - \bar{\gamma}(1 + \bar{i})/(1 + \bar{i}^m)$. Notice that $\vartheta_1 \geq 0$ and in particular $\vartheta_1 = 0$ when the cash-in-advance constraint is not binding, while $0 \leq \vartheta_2 \leq 1$.²³

In equation (52), \hat{r}_{t+1} captures the real interest rate that would apply in a model in which money and bonds are perfect substitutes and is defined as

$$\hat{r}_{t+1} = \rho E_{t+1} (\hat{Y}_{t+2} - \hat{Y}_{t+1})$$

where $\rho \equiv -\bar{U}_{cc}\bar{Y}/\bar{U}_c$; $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$. To complete the characterization of the equilibrium condition through a first-order approximation, we approximate the cash-in-advance constraint (10) to obtain

$$\hat{m}_t + s_b(\hat{b}_t + \vartheta_3 \hat{\gamma}_t) = 0 \quad (53)$$

where \hat{m}_t represents the log-deviations from the steady state of the ratio of money over nominal GDP defined as $m_t \equiv \tilde{M}_{t-1}^s/(P_t Y_t)$ where \bar{m} is its steady-state value; \hat{b}_t is instead the log-deviations from the steady state of the ratio of bonds over nominal GDP, defined as $b_t = \tilde{B}_{t-1}^s/(P_t Y_t)$ with steady-state value \bar{b} , while $\vartheta_3 \equiv (1 - \bar{\gamma})/\bar{\gamma}$ and $s_b \equiv \bar{\gamma}\bar{b}/\bar{m}$. Moreover

$$\hat{m}_t = \hat{m}_{t-1} + \mu_{t-1}^m - \pi_t \quad (54)$$

$$\hat{b}_t = \hat{b}_{t-1} + \mu_{t-1}^b - \pi_t \quad (55)$$

where μ_t^m and μ_t^b are the rate of growth of money supply and bond supply from time $t - 1$ to time t . Given exogenous processes $\{\mu_t^b, \hat{\gamma}_t\}$ equations (52), (53), (54) and (55) determine the path of $\{\hat{m}_t, \mu_t^m, \pi_t, \hat{b}_t, \hat{i}_t, \hat{i}_t^m\}$. Accordingly, monetary policy should specify the path of two instruments of policy.

We describe now some analytical results which are used to produce Figures 1 and 2.

Under the regime in which the monetary policymaker is passive and keeps the interest rate on reserves and the growth of money constant at the rate followed before the shock hits,

²³Indeed, equation (51) implies $\bar{\gamma}(1 + \bar{i})/(1 + \bar{i}^m) = \bar{\gamma}/[\bar{\gamma} + (1 - \bar{\gamma})\bar{\Pi}^{-1}\beta(1 + \bar{i}^m)] \in [0, 1]$, which in turn implies $\vartheta_2 \in [0, 1]$.

we have that the growth of money is given by

$$\bar{\mu}^m = (1 + s_b) \bar{\pi} - s_b \bar{\mu}^b. \quad (56)$$

while the inflation rates vary with the liquidity shock

$$\pi_t = \bar{\pi} + \frac{s_b}{1 + s_b} \vartheta_3 \Delta \hat{\gamma}_t. \quad (57)$$

and the path of the interest rate on bonds follows

$$\hat{i}_t = -\vartheta_1 \hat{\gamma}_{t+1} + \vartheta_2 \vartheta_3 \frac{s_b}{1 + s_b} E_t \Delta \hat{\gamma}_{t+2}.$$

When the policymaker set inflation rate always at the steady state level of 2% at annual rate, as shown with the solid line in Figure 1, the path of money growth follows

$$\mu_{t-1}^m = (1 + s_b) \bar{\pi} - s_b (\bar{\mu}^b + \vartheta_3 \Delta \hat{\gamma}_t). \quad (58)$$

To keep inflation on target, the growth rate of money supply rises momentarily when the liquidity properties of bonds deteriorate, and falls to return to the previous path when liquidity conditions improve. A negative liquidity shock raises the interest rate on bonds which, under inflation targeting, follows

$$\hat{i}_t = -\vartheta_1 \hat{\gamma}_{t+1}.$$

When, instead, the policymaker insulates the interest-rate on bonds from the shock, the interest rate on money follows

$$\hat{i}_t^m = \max \left(-\ln(1 + \bar{i}^m), \vartheta_1 \sum_{s=0}^{\infty} \vartheta_2^s E_t \hat{\gamma}_{t+1+s} \right)$$

which for the large shock discussed in the text can hit the zero-lower bound.

Appendix B

We solve the model of Section 4 by taking a first-order approximation around the initial steady state. The Euler equations of the savers imply

$$\hat{i}_t^d - \hat{i}_t^m = -\vartheta_1^s E_t \hat{\gamma}_{t+1} + \vartheta_2^s E_t (\hat{r}_{t+1}^s + (\pi_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m) \quad (59)$$

where we introduce the following additional notation with respect to previous sections: $\hat{i}_t^d \equiv \ln(1 + i_t^d)/(1 + \bar{i}^d)$, and the coefficients ϑ_1^s and ϑ_2^s are defined as $\vartheta_1^s \equiv (\bar{i}^d - \bar{i}^m)/(1 + \bar{i}^m)$ and $\vartheta_2^s \equiv 1 - \bar{\gamma}(1 + \bar{i}^d)/(1 + \bar{i}^m)$. The Euler equation of the borrowers read in a first-order approximation as

$$\hat{i}_t^b - \hat{i}_t^m = E_t (\hat{r}_{t+1}^b + (\pi_{t+2} - \bar{\pi}) - \hat{i}_{t+1}^m). \quad (60)$$

In both equations

$$\hat{r}_{t+1}^j = \rho E_{t+1} (\hat{C}_{t+2}^j - \hat{C}_{t+1}^j)$$

for each $j = b, s$ where $\rho \equiv v\bar{Y}$ while \bar{Y} is the steady-state output and we use the following definitions $\hat{C}_t^j \equiv (C_t^j - \bar{C}^j)/\bar{Y}$ for each $j = b, s$.

Appropriately, goods market equilibrium (26) implies in a first-order approximation that

$$\hat{Y}_t = \chi \hat{C}_t^s + (1 - \chi) \hat{C}_t^b \quad (61)$$

where now $\hat{Y}_t = (Y_t - \bar{Y})/\bar{Y}$.

Finally in a first-order approximation the spread schedule (25) implies

$$\hat{i}_t^b = (1 + \phi\tilde{b})\hat{i}_t^d + \phi\hat{b}_t \quad (62)$$

for some parameter ϕ where $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$ and $\tilde{b} \equiv \bar{b}/\bar{Y}$.

A first-order approximation of the flow budget constraint of the borrowers (42) implies that

$$\beta\hat{b}_t = \hat{b}_{t-1} + \tilde{b} \cdot \hat{i}_{t-1}^b - \tilde{b}(\pi_t - \bar{\pi}) + \beta\hat{C}_t^b - \beta\hat{Y}_t. \quad (63)$$

Euler equations (59) and (60) together with (61), (62) and (63) constitute the aggregate demand block of the model.

In a log-linear approximation, the supply block comes from approximating (29), (30) taking into account the definitions of φ_t^j for $j = b, s$. The following modified New-Keynesian Phillips curve is obtained

$$\pi_t - \bar{\pi} = \kappa(\eta + \rho)\hat{Y}_t + \kappa[\hat{r}_t + E_t(\pi_{t+1} - \bar{\pi}) - \hat{i}_t^m] + \beta E_t(\pi_{t+1} - \bar{\pi}) \quad (64)$$

where we have defined $\kappa \equiv (1 - \alpha)(1 - \alpha\beta)/\alpha$ and now

$$\hat{r}_t = \rho E_t(\hat{Y}_{t+1} - \hat{Y}_t).$$

The New-Keynesian Phillips curve is augmented by a term reflecting the variations in the monetary frictions at the aggregate level.

Finally we take a first-order approximation of the equilibrium conditions for the money market obtaining

$$\hat{m}_{t-1}^s + \hat{i}_{t-1}^m + \vartheta_3 \left(\hat{b}_{t-1} + \tilde{b} \cdot \hat{i}_{t-1}^d + \vartheta_4 \hat{\gamma}_t \right) = \frac{1 + \vartheta_3 \tilde{b}}{\tilde{c}^s} \left(\hat{C}_t^s + \tilde{c}^s(\pi_t - \bar{\pi}) \right) \quad (65)$$

$$\hat{m}_{t-1}^b + \hat{i}_{t-1}^m = \frac{1}{\tilde{c}^b} \hat{C}_t^b + (\pi_t - \bar{\pi}), \quad (66)$$

where $\vartheta_3 \equiv \bar{\gamma} \frac{1-\chi}{\chi} \frac{1+\tilde{v}^d}{1+\tilde{v}^m} (\tilde{m}^s)^{-1}$, $\tilde{m}^s = \bar{m}^s/\bar{Y}$, $\vartheta_4 \equiv \tilde{b}(1 - \bar{\gamma})/\bar{\gamma}$ and $\tilde{c}^j = \bar{C}^j/\bar{Y}$ for each j . Real money balances follow

$$\hat{m}_t \equiv \chi \hat{m}_t^s + (1 - \chi) \hat{m}_t^b \quad (67)$$

$$\hat{m}_t = \hat{m}_{t-1} + \mu_t - \pi_t \quad (68)$$

and μ_t is the nominal money-supply growth.

Equations (59), (60), (61), (62), (63) together with (64), (65), (66), (67), (68) and the definitions of \hat{r}_{t+1}^s , \hat{r}_{t+1}^b and \hat{r}_{t+1} determine the equilibrium allocation for π_t , \hat{C}_t^b , \hat{C}_t^s , \hat{Y}_t , \hat{i}_t^b , \hat{i}_t^d , \hat{i}_t^m ,

$\hat{b}_t, \hat{m}_t^s, \hat{m}_t^b, \hat{m}_t, \mu_t$, where two policy instruments should be specified.

Appendix C

In this appendix, we show the derivations of the second-order approximation of the welfare (43). The approximation is taken with respect to an efficient steady state. This efficient steady state maximizes (43) under the resource constraint (26) considering that $L = (L^s)^\chi (L^b)^{1-\chi}$.

At the efficient steady state the following conditions hold

$$\begin{aligned}\tilde{\chi} \bar{U}_c^s &= \chi \bar{\lambda}; \\ (1 - \tilde{\chi}) \bar{U}_c^b &= (1 - \chi) \bar{\lambda}; \\ \tilde{\chi} \bar{V}_l^s &= \chi \bar{\lambda} \frac{\bar{Y}}{\bar{L}^s}; \\ (1 - \tilde{\chi}) \bar{V}_l^b &= (1 - \chi) \bar{\lambda} \frac{\bar{Y}}{\bar{L}^b}\end{aligned}$$

where all upper bars denote steady-state values and $\bar{\lambda}$ is the steady-state value of the Lagrange multiplier associated with the constraint (26). Note that the above conditions imply $\bar{U}_c^s / \bar{U}_c^b = \chi(1 - \tilde{\chi}) / [(1 - \chi)\tilde{\chi}]$ so that an appropriately chosen $\tilde{\chi}$ determines the efficient distribution of wealth in a consistent way with the steady state debt position of the borrowers in the model, given by \bar{b} . For the above efficient steady-state to be consistent with the steady-state of the model we need to offset the distortions of the model appropriately. Note that at the efficient steady state

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j}$$

for each $j = b, s$. On the other side, the steady-state of the model, when inflation is at the target level, implies

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j} \frac{\bar{W}}{\bar{P}} (1 - \bar{\varphi}),$$

for each $j = b, s$ and

$$\frac{\bar{W}}{\bar{P}} = \frac{1}{(1 - \tau)(1 + \bar{\mu})}.$$

where $\bar{\mu} \equiv 1/(\theta - 1)$ while

$$\bar{\varphi} = 1 - \frac{\beta(1 + \bar{v}^m)}{\bar{\Pi}}.$$

It is clear from the above equations that we just need to set the employment subsidy at the level

$$\tau = \frac{\bar{\mu} + \bar{\varphi}}{1 + \bar{\mu}}$$

in order to make the steady-state of the decentralized allocation efficient.

Having defined the efficient steady state, we take a second-order expansion of the utility

flow around it to obtain

$$U_t = \bar{U} + \tilde{\chi} \left[\bar{U}_c^s (C_t^s - \bar{C}^s) + \frac{1}{2} \bar{U}_{cc}^s (C_t^s - \bar{C}^s)^2 \right] + (1 - \tilde{\chi}) \left[\bar{U}_c^b (C_t^b - \bar{C}^b) + \frac{1}{2} \bar{U}_{cc}^b (C_t^b - \bar{C}^b)^2 \right] + \\ - \tilde{\chi} \left[\bar{V}_l^s (L_t^s - \bar{L}^s) + \frac{1}{2} \bar{V}_{ll}^s (L_t^s - \bar{L}^s)^2 \right] - (1 - \tilde{\chi}) \left[\bar{V}_l^b (L_t^b - \bar{L}^b) + \frac{1}{2} \bar{V}_{ll}^b (L_t^b - \bar{L}^b)^2 \right] + \mathcal{O}(\|\xi\|^3)$$

where an upper-bar variable denotes the efficient steady state while $\mathcal{O}(\|\xi\|^3)$ collects terms in the expansion which are of order higher than the second. We can use the steady-state conditions to write the above equation as

$$U_t = \bar{U} + \chi \bar{\lambda} \left[(C_t^s - \bar{C}^s) + \frac{1}{2} \frac{\bar{U}_{cc}^s}{\bar{U}_c^s} (C_t^s - \bar{C}^s)^2 \right] + (1 - \chi) \bar{\lambda} \left[(C_t^b - \bar{C}^b) + \frac{1}{2} \frac{\bar{U}_{cc}^b}{\bar{U}_c^b} (C_t^b - \bar{C}^b)^2 \right] + \\ - \chi \bar{\lambda} \frac{\bar{Y}}{\bar{L}^s} \left[(L_t^s - \bar{L}^s) + \frac{1}{2} \frac{\bar{V}_{ll}^s}{\bar{V}_l^s} (L_t^s - \bar{L}^s)^2 \right] - (1 - \chi) \bar{\lambda} \frac{\bar{Y}}{\bar{L}^b} \left[(L_t^b - \bar{L}^b) + \frac{1}{2} \frac{\bar{V}_{ll}^b}{\bar{V}_l^b} (L_t^b - \bar{L}^b)^2 \right] + \mathcal{O}(\|\xi\|^3).$$

Note that for a generic variable X , we have

$$X_t = \bar{X} \left(1 + \hat{X}_t + \frac{1}{2} \hat{X}_t^2 \right) + \mathcal{O}(\|\xi\|^3)$$

where $\hat{X}_t \equiv \ln X_t / \bar{X}$ and moreover recall that

$$Y_t = \chi C_t^s + (1 - \chi) C_t^b,$$

implying that

$$\chi(C_t^s - \bar{C}^s) + (1 - \chi)(C_t^b - \bar{C}^b) = \bar{Y} \left[\hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right] + \mathcal{O}(\|\xi\|^3)$$

We can write the above approximation as

$$U_t = \bar{U} + \bar{\lambda} \bar{Y} \left[\hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right] - \frac{1}{2} \bar{\lambda} v \left[\chi(C_t^s - \bar{C}^s)^2 + (1 - \chi)(C_t^b - \bar{C}^b)^2 \right] \\ - \chi \bar{\lambda} \bar{Y} \left[\hat{L}_t^s + \frac{1}{2} (1 + \eta) (\hat{L}_t^s)^2 \right] - (1 - \chi) \bar{\lambda} \bar{Y} \left[\hat{L}_t^b + \frac{1}{2} (1 + \eta) (\hat{L}_t^b)^2 \right] + \mathcal{O}(\|\xi\|^3), \quad (69)$$

where we have also used the fact that with the preference specification used $\bar{U}_{cc}^s / \bar{U}_c^s = \bar{U}_{cc}^b / \bar{U}_c^b = -v$ and $\bar{V}_{ll}^s \bar{L}^s / \bar{V}_l^s = \bar{V}_{ll}^b \bar{L}^b / \bar{V}_l^b = \eta$.

Note that in equilibrium $L_t = \Delta_t Y_t$ where $L_t = (L^s)^\chi (L^b)^{1-\chi}$. It follows that the following condition holds exactly

$$\hat{Y}_t = \chi \hat{L}_t^s + (1 - \chi) \hat{L}_t^b + \hat{\Delta}_t.$$

Using the above equation in (69), the latter can be simplified to

$$\begin{aligned} \frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} &= \frac{1}{2}\hat{Y}_t^2 - \frac{1}{2}\rho \left[\chi(\hat{C}_t^s)^2 + (1-\chi)(\hat{C}_t^b)^2 \right] \\ &\quad - \frac{1}{2}(1+\eta) \left[\chi(\hat{L}_t^s)^2 + (1-\chi)(\hat{L}_t^b)^2 \right] - \hat{\Delta}_t + \mathcal{O}(\|\xi\|^3), \end{aligned} \quad (70)$$

where $\rho \equiv v\bar{Y}$ and we have used the definitions of \hat{C}_t^s and \hat{C}_t^b . Note that to a first-order approximation

$$\begin{aligned} \hat{C}_t^s &= \hat{Y}_t - (1-\chi)(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{C}_t^b &= \hat{Y}_t + \chi(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{L}_t^s &= \hat{Y}_t - (1-\chi)(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(\|\xi\|^2) \\ \hat{L}_t^b &= \hat{Y}_t + \chi(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(\|\xi\|^2) \end{aligned}$$

which can be used to simplify (70) to

$$\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = -\frac{1}{2}(\rho + \eta)\hat{Y}_t^2 - \frac{1}{2}\chi(1-\chi)\rho(\hat{C}_t^s - \hat{C}_t^b)^2 - \frac{1}{2}\chi(1-\chi)(1+\eta)(\hat{L}_t^s - \hat{L}_t^b)^2 - \hat{\Delta}_t + \mathcal{O}(\|\xi\|^3).$$

Note that

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\bar{\Pi}} \right)^\theta \Delta_{t-1} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}}$$

By taking a second-order approximation of $\hat{\Delta}_t$, as it is standard in the literature and integrating appropriately across time, we obtain that

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \theta \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{(\pi_t - \bar{\pi})^2}{2} + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)$$

We can then obtain a second-order approximation of the utility of the consumers as

$$W_t = -\bar{\lambda}(\eta + \rho)\bar{Y} \cdot \frac{1}{2}E_t \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} Loss_t \right\} + \text{t.i.p.} + \mathcal{O}(\|\xi\|^3)$$

where

$$Loss_t = \hat{Y}_t^2 + \chi(1-\chi)\lambda_c(\hat{C}_t^s - \hat{C}_t^b)^2 + \chi(1-\chi)\lambda_l(\hat{L}_t^s - \hat{L}_t^b)^2 + \lambda_\pi(\pi_t - \bar{\pi})^2$$

where we have defined

$$\begin{aligned} \lambda_c &\equiv \frac{\rho}{\rho + \eta} \\ \lambda_l &\equiv \frac{1 + \eta}{\rho + \eta} \end{aligned}$$

$$\lambda_\pi \equiv \frac{\theta}{\kappa}.$$

Note finally that

$$\begin{aligned}\frac{(L_t^s)^{1+\eta}}{v \exp(-vC_t^s)} &= \frac{W_t}{P_t} \Delta_t Y_t (1 - \varphi_t^s) \\ \frac{(L_t^b)^{1+\eta}}{v \exp(-vC_t^b)} &= \frac{W_t}{P_t} \Delta_t Y_t (1 - \varphi_t^b)\end{aligned}$$

which imply in a log-linear approximation that

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta}(\hat{C}_t^s - \hat{C}_t^b) - \frac{\bar{\varphi}}{(1-\bar{\varphi})(1+\eta)}(\hat{\varphi}_t^s - \hat{\varphi}_t^b)$$

Moreover from log-linear approximations of (23), we get

$$\hat{\varphi}_t^j = \frac{1-\bar{\varphi}}{\bar{\varphi}} E_t \left[(\pi_{t+1} - \bar{\pi}) - \hat{i}_t^m + \rho \Delta \hat{C}_{t+1}^j \right]$$

for $j = b, s$ and therefore

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta} E_t (\hat{C}_{t+1}^s - \hat{C}_{t+1}^b),$$

which can be also used in the loss function to replace the term $\hat{L}_t^s - \hat{L}_t^b$.