Capital Controls and Currency Wars

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Abstract

Policy measures that affect international capital flows have led to considerable controversy in international policy circles. This paper analyzes the welfare effects of such measures in a general equilibrium model of the world economy. Capital controls or reserve accumulation in one country leads to significant international spillover effects via lower world interest rates and greater flows to other countries. If controls are designed to offset domestic externalities, the resulting equilibrium is nonetheless Pareto efficient, i.e. a global planner would impose the same measure and there is no role for global coordination. We illustrate this for several examples of externalities, including financial stability externalities, and capital controls that act as second-best devices to correct a domestic distortion. On the other hand, if policymakers face an imperfect set of instruments, e.g. targeting problems or costly enforcement, then multilateral coordination is desirable in order to mitigate the inefficiencies arising from such imperfections.

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1 Introduction

Policy measures that affect international capital flows have led to considerable controversy in international policy circles in recent years (see e.g. Ostry et al, 2012). This paper determines the welfare effects of such measures in a general equilibrium model of the world economy and analyzes under what conditions global coordination of capital account policies is indicated.

The paper starts out with a benchmark model of a global economy in which individual countries engage in intertemporal trade with each other, i.e. they borrow and lend. We characterize equilibrium and establish conditions for the efficiency of the decentralized market equilibrium. Next, we extend the benchmark model to include non-tradable goods and a real exchange rate. If a country imposes capital controls in the form of taxes on borrowing from abroad, it reduces borrowing and consumption and depreciates its real exchange rate. At the same time, it pushes down the world interest rate, which induces other countries to borrow and spend more. The decline in the world interest rate increases the welfare of all net borrowers and hurts all net lenders in the world economy.

When capital accounts are open for the borrowing/lending transactions of private agents, we derive a Ricardian equivalence result for reserve accumulation, i.e. it will be undone. When capital accounts are closed for private transactions, reserve accumulation determines the level of international borrowing and lending. There is an isomorphism between capital controls and reserve accumulation – any level of capital controls can be replicated by a corresponding level of reserve accumulation. We extend the model by introducing uncertainty and show that our results continue to hold both for the case of complete markets and risk-free bonds only.

The equilibrium in our benchmark model is Pareto efficient if all countries impose a uniform level of capital controls, e.g. the same tax on flows in all inflow countries and subsidy in all outflow countries. However, different levels of this uniform capital control correspond to different world interest rates and therefore lead to redistributions between borrowers and lenders. In a two-country framework, a global planner can replicate any transfers between the countries by setting a corresponding level of uniform capital controls.

The second part of the paper analyzes several potential motives for imposing capital controls: terms-of-trade manipulation, and domestic externalities from borrowing/lending. To analyze terms-of-trade manipulation, we investigate the incentives of a national planner in a large country to exert market power over the country's intertemporal terms of trade, i.e. the world interest rate. We show that such a planner has incentives to reduce the quantity transacted, to mitigate real exchange rate fluctuations, and to distort capital flows towards less state contingent forms than in the decentralized equilibrium. As is common in monopolistic settings, such intervention reduces the global gains from intertemporal trade and is Pareto-inefficient. Since the Nash equilibrium between national planners in large countries is characterized by non-zero capital account intervention, it is desirable to come to a global agreement that capital controls aimed at manipulating the world interest rate will not be used.

This contrasts with the situation of national planners who face externalities within their domestic economies. If national planners impose capital controls to offset such domestic externalities, global welfare is unambiguously increased. We show that the uncoordinated global equilibrium is Pareto efficient as long as national planners behave competitively in the world market and impose capital controls that offset domestic externalities while ignoring their general equilibrium effects. This holds true even though such capital controls in general equilibrium affect the world interest rate. Conceptually, we can view the national planners in different countries that internalize externalities but do not exert market power as competitive agents to which the welfare theorems apply. Changes in the world interest rate that stem from capital controls therefore constitute pecuniary externalities that cancel out. Furthermore, we find that a seeming "arms race" of escalating capital controls does not necessarily indicate inefficiency but may be the tatonnement process through which multiple countries optimally adjust their capital controls when financial fragility increases.

The lesson for international policy coordination is that it is important to distinguish between 'distortive' capital controls that are designed to manipulate a country's terms of trade and 'corrective' capital controls that are imposed to offset domestic externalities. The former are always undesirable, whereas the latter are generally desirable.

An additional motive for coordinating capital controls arises when policymakers face restrictions on the set of available policy instruments. For example, if capital controls not only correct distorted incentives to borrow/lend but also impose an additional cost arising from costly implementation or corruption, then there is scope for global coordination of capital account policies: a global planner recognizes that adjusting all capital controls worldwide by the same factor may reduce the distortions created by capital controls but would leave the marginal incentives of all actors in the world economy unaffected.

Next, we study a particular example of externalities: we focus on prudential capital controls that are designed to internalize pecuniary externalities that arise from domestic financial instability (see Korinek, 2010, 2011). We show that although such capital controls reduce borrowing in good times, they relax financial constraints in bad times, which allows for more borrowing and lending to take place and leads to larger gains from trade. These can be shared among both borrowing and lending countries so as to make everybody better off.

Literature There is a growing recent literature that finds that capital controls may improve welfare from the perspective of a single country if they are designed to correct domestic externalities. An important example are prudential capital controls that reduce the risk of financial crises, as analyzed in the small open economy literature by Korinek (2007, 2010, 2011b), Ostry et al. (2010, 2011) and Bianchi (2011). This paper provides a normative analysis of the resulting general equilibrium effects and discusses whether global coordination of such policies is desirable.[1] We find that in a benchmark case in which national regulators can optimally control domestic externalities, coordination is not indicated. By contrast, Bengui (2011) studies the role for coordination between national regulators in a multi-country framework of banking regulation. He shows that liquidity in the global interbank market is a global public good. In the presence of such global externalities, there exists a case for global coordination of liquidity requirements.

Earlier work by MacDougall (1960), Kemp (1962), Hamada (1966), Jones (1967) and Obstfeld and Rogoff (1996) investigated how a national planner of a large country in the world economy may impose capital controls to exert monopoly/monopsony power over intertemporal prices. As in optimal tariff theory, such policies are beggar-thy-neighbor, i.e. they improve national welfare at the expense of reducing overall global welfare. In a recent contribution to this literature, Costinot et al. (2011) analyze the optimal time path of monopolistic capital controls under commitment and show how they can be used to distort relative prices even in contemporaneous goods markets. Our paper contrasts the global welfare effects of distortive (monopolistic) capital controls with corrective capital controls that are designed to offset domestic externalities, as was invoked by a rising number of countries that have imposed such controls in recent years. Jeanne et al. (2012), Gallagher et al. (2012) and Ostry et al. (2012) discuss the desirability and the multilateral implications of capital controls from a policy perspective.

Persson and Tabellini (1995) show that coordination of national fiscal and/or monetary policies is desirable if countries have incentives to employ such policies to exert monopoly power over international prices. Korinek (2011a) analyzes the positive implications of prudential capital controls in a multi-country setting.

The link between reserve accumulation and real exchange rate valuation is also investigated in Rodrik (2008) and Korinek and Serven (2010). Ghosh and Kim (2009) and Jeanne (2012) show how a combination of capital controls and tax measures can be used to undervalue a country's real exchange rate. These papers look at the exchange rate effects of various capital account policies in a small open economy, whereas we focus explicitly on global general equilibrium effects.

Magud et al. (2011) provide a survey of the empirical literature on the effects of capital controls on the country imposing the controls. Forbes et al. (2011) and Lambert et al. (2011) investigate the spillover effects of capital controls empirically. They find evidence that when Brazil imposed capital controls, there was diversion of capital flows to other countries that were expected to maintain free capital flows.[2]

[1] Ostry et al. (2012) discusses the multilateral aspects of policies to manage the capital account from a policy perspective.

[2] Forbes et al. (2011) also document negative spillover effects on countries that were likely to follow the example of Brazil to impose controls.

	$\tau^i_{t+1} > 0$	$\tau^i_{t+1} < 0$
$b^i_{t+1} > 0$ (lender)	outflow subsidy	outflow tax
$b^i_{t+1} < 0$ (borrower)	inflow tax	inflow subsidy

Table 1: Interpretation of capital control τ^i_{t+1}

To the extent that the capital controls imposed by Brazil were imposed to correct a domestic distortion, our analysis suggests that this was a Pareto-efficient equilibrium response and did not introduce distortions in the global allocation of capital.

2 Benchmark Intertemporal Model

We describe a world economy with $N \geq 2$ countries indexed by $i = 1, ...N$ and a single homogenous tradable consumption good. Time is indexed by $t = 0, ...$ We assume that the mass of each country i in the world economy is m^i, where $\Sigma_{i=1}^N m^i = 1$.

2.1 Country Setup

Each country is inhabited by a unit mass of identical consumers who value the consumption c^i_t of a tradable good according to the utility function

$$U = \sum \beta^t u\left(c^i_t\right)$$

where $u\left(\cdot\right)$ is a standard neoclassical period utility function and $\beta < 1$ is a time discount factor. Consumers start each period t with an endowment of y^i_t of tradable goods and financial net worth b^i_t. They choose how much to consume and how much to save by purchasing b^i_{t+1} zero coupon bonds at a price $\left(1 - \tau^i_{t+1}\right)/R_{t+1}$ that pay off one unit of tradable goods in period $t+1$, where R_{t+1} represents the gross world interest rate between periods t and $t+1$.

The variable τ^i_{t+1} is a proportional subsidy to bond purchases b^i_{t+1}/R_{t+1} if $b^i_{t+1} > 0$, or a proportional tax on bond sales if $b^i_{t+1} < 0$. We assume that the revenue is raised/rebated as a lump-sum transfer $T^i_t = -\tau^i_{t+1} b^i_{t+1}/R_{t+1}$. If the country is a net saver so $b^i_{t+1} > 0$, then τ^i_{t+1} constitues a subsidy on saving or, equivalently, a subsidy on capital outflows. Since capital outflows go hand in hand with positive net exports, we can also think of it as an export subsidy. If the country is a net borrower so $b^i_{t+1} < 0$, then it can be interpreted as a tax on foreign borrowing, or on capital inflows. Since capital inflows imply positive net imports, the tax can be thought of as an import tariff. See table 1. To ensure that bond demand is bounded, we impose the assumption that $\tau^i_{t+1} < 1 \forall i, t$. In the following, we will loosely refer to τ^i_{t+1} as the "capital control" in period t.

Since there is a single representative consumer in the economy, we can interpret borrowing/savings decisions in terms of international capital flows and the current account. The term b_t^i by itself is the gross return on savings that the consumer receives at the beginning of period t. The fraction b_t^i/R_t captures how much the economy saved in period $t-1$ in order to receive b_t^i units of goods in period t. Therefore the interest earnings in period t are $b_t(1-1/R_t)$. The trade balance of the country in period t has to equal the difference between new savings and the value of bond holdings at the beginning of the period , $b_{t+1}^i/R_{t+1}-b_t^i$. And finally, the current account balance in period t is the sum of interest earnings and the trade balance, $b_{t+1}^i/R_{t+1}-b_t^i/R_t$, and corresponds to the change in the net asset position of the country between period $t-1$ and t.

The budget constraint of the representative consumer in period t is

$$c_t^i + \frac{\left(1-\tau_{t+1}^i\right) b_{t+1}^i}{R_{t+1}} = y_t^i + b_t^i + T_t^i \tag{1}$$

We write the utility of the consumer in recursive form as

$$V^i\left(b_t^i\right) = \max_{c_t^i, b_{t+1}^i} u\left(c_t^i\right) + \beta V^i\left(b_{t+1}^i\right) \tag{2}$$

The consumer takes T_t^i, R_{t+1} and τ_{t+1}^i as given and maximizes utility (2) subject to the budget constraint (1). This leads to the Euler equation

$$\left(1-\tau_{t+1}^i\right) u'\left(c_t^i\right) = \beta R_{t+1} V^{i\prime}\left(b_{t+1}^i\right) \tag{3}$$

where by the envelope theorem $V^{i\prime}\left(b_{t+1}^i\right) = u'\left(c_{t+1}^i\right)$ is a strictly decreasing function. Taking the continuation utility $V^{i\prime}(\cdot)$ as given, the Euler equation implies a bond demand function $b_{t+1}^i\left(R_{t+1};\tau_{t+1}^i\right)$ that is strictly increasing in the capital control τ_{t+1}^i. To better characterize this function, we impose the following common assumption:

Assumption 1 (Elasticity of Intertemporal Substitution) *The savings/consumption ratio of country i is greater than the negative of the elasticity of intertemporal substitution,*

$$\frac{b_{t+1}^i/R_{t+1}}{c_t^i} > -\sigma\left(c_t^i\right)$$

This assumption implies that the substitution effect of a change in the world interest rate outweighs the income effect so that higher interest rates induce the country to save more. Therefore the bond demand $b_{t+1}^i\left(R_{t+1};\tau_{t+1}^i\right)$ is strictly increasing in R_{t+1} and can be inverted into a strictly increasing inverse demand function $R_{t+1}^i\left(b_{t+1}^i;\tau_{t+1}^i\right)$. For a given elasticity σ, the assumption is satisfied if the country is either a lender or a borrower that is not too indebted.

2.2 Partial Equilibrium Effects

In small open economies, we take the world interest rate R_{t+1} as given and analyze the effects of changes in capital controls and in the world interest rate on the economy. (These results can alternatively be interpreted as partial equilibrium effects in a large economy.)

Lemma 1 (Partial Equilibrium Effects of Capital Controls) *Ceteris paribus, an increase in the capital control τ_{t+1}^i in a given period t:*

1. *increases saving $\partial \left(b_{t+1}^i / R_{t+1}\right) / \tau_{t+1}^i > 0$ and bond holdings $\partial b_{t+1}^i / \partial \tau_{t+1}^i > 0$ or, conversely, reduces borrowing,*
2. *reduces consumption c_t^i, i.e. $\partial c_t^i / \partial \tau_{t+1}^i < 0$,*
3. *reduces welfare the further it moves the consumer away from free capital flows with $\tau_{t+1}^i = 0$.*

Proof. Points 1. and 2. follow from implicitly differentiating the Euler equation of the consumer to express $\partial b_{t+1}^i / \partial \tau_{t+1}^i$, dividing by R_{t+1} and applying the period t budget constraint.

In our benchmark model (in the absence of frictions), point 3. follows since capital controls distort the consumer's optimality condition. ■

Lemma 2 (Effects of Interest Rate) *Ceteris paribus, an increase in the world interest rate between two time periods:*

1. *increases bond holdings $\partial b_{t+1}^i / \partial R_{t+1} > 0$,*
2. *reduces consumption $\partial c_t^i / \partial R_{t+1} < 0$ while increasing net saving $\partial \left(b_{t+1}^i / R_{t+1}\right) / \partial R_{t+1} > 0$ if the country's elasticity of saving satisfies $\eta_{bR}^i > 1$ and vice versa otherwise.*
3. *increases welfare in lending countries $b_t^i > 0$ and reduces welfare in borrowing countries $b_t^i < 0$.*

Observe that the condition on the elasticity of saving in point 2 can also be written as $b_{t+1}^i / R_{t+1} < \partial b_{t+1}^i / \partial R_{t+1}$, i.e. that saving is small compared to the slope of the demand curve. This is satisfied if the country is either a net borrower or a modest saver.

Proof.

1. The first result follows from implicitly differentiating the consumer's Euler equation (see appendix A.1) and combining $\partial b_{t+1}^i / \partial R_{t+1} = -\frac{\partial F / \partial R}{\partial F / \partial b^i}$ with assumption 1.

2. Apply the implicit function theorem to the consumer's Euler equation to find

$$\frac{\partial b^i_{t+1}/R_{t+1}}{\partial R_{t+1}} = \frac{\partial b^i_{t+1}/\partial R_{t+1}}{R_{t+1}} - \frac{b^i_{t+1}}{(R_{t+1})^2} = \frac{b^i_{t+1}}{(R_{t+1})^2}\left(\eta^i_{bR} - 1\right)$$

   The first term in the expression in the middle captures the substitution effect – a higher bond price makes it less desirable to save. This effect has a positive sign given the assumptions of our setup. The second term captures the income effect. The overall sign depends on the elasiticty η^i_{bR}, i.e. on the sign and magnitude of the country's bond holdings: if the country is a net saver, then a higher world interest rate reduces the amount that needs to be saved in order to carry b_{t+1} bonds into the following period. If the country is a net borrower, a higher interest rate reduces what is obtained today in exchange for a promise to repay b_{t+1} tomorrow. Combine this with the period t budget constraint of the consumer to obtain the results on consumption.

3. Take the derivative of the consumer's utility function

$$\frac{dU^i}{dR_{t+1}} = \frac{b^i_{t+1}}{R^2_{t+1}} \cdot \beta^t u'\left(c^i\right) \tag{4}$$

■

It is straightforward that saving, future bond holdings and consumption increase in the economy's endowment y^i_t in a given period.

Numerical Illustration We illustrate the effects of changes in capital controls and in world interest rates numerically. Let us assume a small open economy i with constant intertemporal elasticity of substitution $\sigma = 1/2$ so that $u(c) = c^{1-1/\sigma}/(1-1/\sigma)$. Assume a steady state with $c^i = y^i = const$, $\beta R = 1$, $b^i = 0$ and $\tau = 0$. Then evaluating the two partial equilibrium derivatives of saving relative to steady-state output b^i/y^i yields

$$\begin{aligned} b^i_\tau/y^i &= \frac{\partial b^i/y^i}{\partial \tau^i} = \frac{\sigma}{1+\beta} \\ b^i_R/y^i &= \frac{\partial b^i/y^i}{\partial R} = \frac{\beta\sigma}{1+\beta} \end{aligned}$$

In short, an increase in the capital control or an increase in the world interest rate both increase the net savings of the country by approximately half of the intertemporal elasticity of substitution, i.e. by a quarter percentage point. (The small difference between the two expressions – the second one is pre-multiplied by β – stems from the fact that we assumed that interest is compounded in period $t+1$ whereas the capital control is imposed in period t.)

For the standard value of the elasticity of substitution $\sigma = 1/2$, an increase in capital controls or in the interest rate both result in an increase in domestic savings by approximately a quarter percent of GDP. (We note that there is some disagreement among economists about the correct value of the intertemporal elasticity of substitution. See e.g. Bansal and Yaron (2004) for a discussion. The formulas we derived above deliver transparent results for arbitrary values of the intertemporal elasticity of substitution.)

2.3 General Equilibrium

Let us define the global excess demand for bonds in period t as a function of the world interest rate R_{t+1} and the vector $\tau_{t+1} = \left\{\tau_{t+1}^i\right\}_{i=1}^N$ of capital controls as

$$B_{t+1}\left(R_{t+1}; \tau_{t+1}\right) = \sum_{i=1}^{N} m^i b_{t+1}^i \left(R_{t+1}; \tau_{t+1}^i\right)$$

which is – by our earlier assumptions – strictly increasing.[3]

Definition 1 (Decentralized Equilibrium) *For a given series of vectors $\{\tau_{t+1}\}$, the decentralized equilibrium of the world economy is given by a series of world interest rates $\{R_{t+1}\}$ that solves the market clearing condition*

$$B_{t+1}\left(R_{t+1}; \tau_{t+1}\right) = 0 \tag{5}$$

together with series of individual country bond positions $\left\{b_{t+1}^i\right\}_{i=1}^N$ that satisfy $b_{t+1}^i = b_{t+1}^i\left(R_{t+1}; \tau_{t+1}^i\right)$.

Lemma 3 (General Equilibrium Effects of Capital Controls) *In general equilibrium, an exogenous increase in the capital control τ_{t+1}^i in country i with $m^i > 0$ reduces the world interest rate*

$$\frac{dR_{t+1}}{d\tau_{t+1}^i} = -\frac{b_\tau^i}{B_R} < 0 \tag{6}$$

Proof. The equation follows from applying the implicit function theorem to the global market clearing condition (5). The inequality holds because $B_R = \partial B_{t+1}/\partial R_{t+1} > 0$ and $\partial B_{t+1}/\partial \tau_{t+1}^i = \partial b_{t+1}^i/\partial \tau_{t+1}^i = b_\tau^i > 0$. ■

Intuitively, capital controls raise the desired bond holdings of country i for a given world interest rate. This shifts the global excess demand for bonds $B_{t+1}\left(R_{t+1}; \tau_{t+1}\right)$ upwards. For the global bond market to clear, a decline in the world interest rate is required.

[3] This is our analogon of the Marshall-Lerner condition that an increase in the world interest rate increases the global excess demand for bonds.

As a result, all the effects of changes in world interest rates that we listed in lemma 2 are triggered: all other countries j reduce their bond holdings and (if $\eta^j_{Rb} > 1$) increase consumption. All net borrowers benefit from the lower world interest rate; all net lenders are hurt.

Numerical Illustration Generalizing our numerical illustration to general equilibrium, let us assume a steady state in which world output and consumption are given by $C = Y$. Then the global demand for bonds as a fraction of global GDP B/Y satisfies

$$B_R/Y = \frac{\partial B/Y}{\partial R} = \frac{\beta\sigma}{1+\beta}$$

We combine this with the steady-state expression $b^i_\tau/y^i = \frac{\sigma}{1+\beta}$ in equation (6) to find the effect of capital controls in a country i of relative size $m^i = y^i/Y$ on the world interest rate is

$$\frac{dR_{t+1}/R}{d\tau^i_{t+1}} = -\frac{b^i_\tau/R}{B_R} = -m^i$$

In short, if a country that has a relative share m^i of world GDP imposes a 1% capital control, the world interest rate will decline by m^i %. Observe that this expression is independent of the intertemporal elasticity of substitution.

Combining this with our earlier findings for the partial equilibrium effect b^i_R, this change in the world interest rate mitigates the effects of the capital control on savings in the country imposing them, delivering a total effect of

$$\frac{db^i/y^i}{d\tau^i} = b^i_\tau/y^i + b^i_R/y^i \cdot \frac{dR_{t+1}}{d\tau^i_{t+1}} = \frac{\sigma\left(1-m^i\right)}{1+\beta}.$$

In short, for large countries the general equilibrium effect of imposing capital controls is diminished by $(1 - m^i)$ because the world interest rate adjusts.

In Table 2, we illustrate the estimated effects of capital controls on the world interest rate and on global bond flows of a number of countries that were viewed to be important players in international financial markets in recent years.

2.4 Welfare Analysis

We next turn our attention to analyzing under what capital control policies $\{\tau_{t+1}\}$ the decentralized equilibrium is Pareto efficient. Since the first welfare theorem applies in the described economy, it is straightforward that the laissez faire equilibrium with $\tau^i_{t+1} = 0\ \forall i, t$ is Pareto efficient. However, as we show in the following, there are further configurations of capital controls that implement a Pareto efficient equilibrium and that have different redistributive implications.

Proposition 1 (Efficiency of Decentralized Equilibrium) *The decentralized equilibrium in the described economy with capital controls is Pareto efficient if and only*

Country	GDP^i	$\$\Delta b^i/R$	$\Delta R/R$
World	$62,634bn	···	–1%
United States	$14,447bn	$28.4bn	–0.231%
China	$5,739bn	$13.3bn	–0.092%
Japan	$5,459bn	$12.7bn	–0.087%
Brazil	$2,089bn	$5.2bn	–0.033%
India	$1,722bn	$4.3bn	–0.027%
South Korea	$1,014bn	$2.5bn	–0.016%
Indonesia	$707bn	$1.8bn	–0.011%
Argentina	$370bn	$0.9bn	–0.006%

Table 2: Effects of 1% domestic capital control on saving and world interest rate (Source: IMF IFS and author's calculations)

if capital controls are uniform across countries in every time period, i.e. $\tau^i_{t+1} = \bar{\tau}_{t+1}\ \forall i, t$. *One example is the laissez-faire equilibrium with* $\tau^i_{t+1} = 0\ \forall i, t$.

Proof. The allocation of a decentralized equilibrium is Pareto-efficient if and only if it is the solution of a global planning problem for some vector of welfare weights $\{\phi^i > 0\}$

$$\max \sum_{i,t} m^i \phi^i \beta^t u\left(c^i_t\right) \quad \text{s.t.} \quad \sum_i m^i (c^i_t - y^i_t) = 0 \forall t \tag{7}$$

Assigning shadow prices λ_t to the period t resource constraints, the planner's optimality conditions are

$$FOC(c^i_t): \quad \phi^i \beta^t u'\left(c^i_t\right) = \lambda_t\ \forall i, t$$

Consider the allocation of the decentralized equilibrium in definition 1 for a series of capital controls $\{\bar{\tau}_{t+1}\}$ that are uniform across countries at any given time. If we set $\phi^i = 1/u'\left(c^i_0\right)$, $\lambda_0 = 1$ and $\lambda_{t+1} = \left(1 - \bar{\tau}_{t+1}\right)/R_{t+1}\lambda_t \forall t$, then it can be seen that the allocations of the decentralized equilibrium satisfy the planner's optimality conditions above for all i, t. Conversely, it can be seen that it is not possible to simultaneously fulfill all of the planner's optimality conditions if the capital controls are not set at a uniform level. ■

Intuitively, as long as a given level of capital controls $\bar{\tau}_{t+1}$ is uniform across countries in a given time period, the intertemporal marginal rates of substitution of all agents are equated at $\frac{1-\bar{\tau}_{t+1}}{R_{t+1}}$ and the equilibrium is efficient. A synchronized increase in $\bar{\tau}_{t+1}$ raises the world interest rate by an equivalent amount $dR_{t+1}/d\bar{\tau}_{t+1} = -B_{\bar{\tau}}/B_R > 0$ to ensure that the global bond market clears. The change in the world interest rate creates positive income effects for all borrowers and negative income effects for savers. (If we held the income of countries constant, we could show analytically that $dR_{t+1}/d\bar{\tau}_{t+1} = 1$. In the presence of income effects, heterogeneities in the response of borrowers and lenders may cause small deviations from this benchmark.)

A global planner can employ the income effects of capital controls to generate transfers between countries. We can characterize this in more detail if there are only two countries (or two types of countries) in the world economy. These transfers are relevant for two reasons: First, the income effects of capital controls may have important political economy implications. Secondly, they may shed light on how to achieve Pareto improvements in the sections below when we study the global coordination of capital account policies if a global planner has access to transfers.

Corollary 1 (Transfer Effect of Capital Controls) *Assume $N = 2$ countries and consider a transfer between the two at time t but zero capital controls. In the resulting post-transfer equilibrium, denote by i the country that is saving, $b^i_{t+1} > 0$, by j the country that is borrowing, by $\tilde{T}_t$ the transfer given by country i and by $\tilde{R}_{t+1}$ the prevailing interest rate. If $\tilde{T}_t > -b^i_{t+1}/\tilde{R}_{t+1}$, then a global planner can replicate the transfer by imposing a uniform level of capital controls*

$$\bar{\tau}_{t+1} = \frac{\tilde{T}_t}{\tilde{T}_t + b^i_{t+1}/\tilde{R}_{t+1}}. \tag{8}$$

Proof. In the equilibrium after the transfer $\tilde{T}_t$ has been made but with zero capital controls, the Euler equation of country i is

$$u'\left(y^i_t + b^i_t - \tilde{T}_t - b^i_{t+1}/\tilde{R}_{t+1}\right) = \beta\tilde{R}_{t+1}V^{i\prime}\left(b^i_{t+1}\right)$$

and similarly but with opposite sign on the transfer and borrowing for country j. Next consider an equilibrium with a uniform level of capital controls $\bar{\tau}_{t+1}$ and an interest rate $\bar{R}_{t+1}$ across the two countries and observe the Euler equation for country i,

$$u'\left(y^i_t + b^i_t - b^i_{t+1}/\bar{R}_{t+1}\right) = \beta\frac{\bar{R}_{t+1}}{1-\bar{\tau}_{t+1}}V^{i\prime}\left(b^i_{t+1}\right)$$

and similarly for country j.

For given y^i_t, b^i_t and b^i_{t+1}, it can be seen that the two equations coincide as long as

$$\tilde{T}_t + \frac{b^i_{t+1}}{\tilde{R}_{t+1}} = \frac{b^i_{t+1}}{\bar{R}_{t+1}} \quad \text{and} \quad \tilde{R}_{t+1} = \frac{\bar{R}_{t+1}}{1-\bar{\tau}_{t+1}}$$

For a given transfer $\tilde{T}_t$, this is a system in two equations and two variables, $\bar{R}_{t+1}$ and $\bar{\tau}_{t+1}$. Observe that the same two conditions can be derived for country j since both the transfer and the bond positions enter with opposite sign and are scaled by m^j/m^i. Eliminating $\bar{R}_{t+1}$ from the two equations yields the equivalent uniform capital control in equation (8). To ensure that the resulting capital control satisfies the restriction $\bar{\tau}_{t+1} < 1$ in both countries, it is necessary that the transfer satisfies $\tilde{T}_t > -b^i_{t+1}/\tilde{R}_{t+1}$. (If $b^i_{t+1} = 0$, then capital controls cannot replicate the effects of a transfer.)

Going in the opposite direction, any equilibrium with uniform capital controls $\bar{\tau}_{t+1}$ can be replicated by an equilibrium with zero capital controls and a transfer

$$\tilde{T}_t = \bar{\tau}_{t+1} \frac{b^i_{t+1}}{\bar{R}_{t+1}}$$

■

The intuition for the lower bound $\tilde{T}_t > -b^i_{t+1}/\tilde{R}_{t+1}$ is that the planner has to impose the capital control tax on saving $b^i_{t+1}/\tilde{R}_{t+1}$. If the base for this tax is small, then only a small negative transfer can be emulated. There is no upper bound on the transfer from the lending to the borrowing country, except that the lending country still needs to be lending in the post-transfer equilibrium. (For large transfers away from a given country i, b^i_{t+1} would turn negative.)

From the perspective of the global planner, performing a transfer $\tilde{T}_t$ is equivalent to solving the global optimization problem with different relative welfare weights ϕ^i/ϕ^j. The proposition thus also implies that a global planner can implement Pareto-efficient equilibrium with different welfare weights by varying the global level of capital controls.

2.5 Extension 1: Real Exchange Rate

Policymakers who are concerned about capital flows often refer to the effects on the real exchange rate as a reason for intervention. To capture such effects, we extend our benchmark model to include a non-traded good in each country, which allows us to introduce a real exchange rate.

We distinguish variables that refer to traded versus non-traded goods by the subindices T and N. We maintain our assumption that there is a single homogenous traded good which is the numeraire good, and we denote the relative price of non-traded goods in country i by p^i_N, which constitutes a measure of the real exchange rate.[4] Observe that we index p^i_N by country i since the prices of non-traded goods in different countries are different.

The recursive utility of a representative consumer in country i is

$$V^i\left(b^i_t\right) = u\left(c^i_{T,t}, c^i_{N,t}\right) + \beta V^i\left(b^i_{t+1}\right)$$

[4] The official definition of the real exchange rate is the price of a consumption basket of domestic goods expressed in terms of a consumption basked of foreign goods. Ceteris paribus, a rise in the relative price of non-tradables increases the price of a consumption basket of domestic goods, implying a strictly monotonic relationship between the official real exchange rate and our measure p^i_N.

We denote the partial derivatives of the utility function as $u_{T,t} = \partial u / \partial c^i_{T,t}$ and similar for $u_{N,t}$, $u_{NT,t}$ etc. We impose the assumptions $u_T > 0 > u_{TT}$, $u_N > 0 > u_{NN}$ and $u_{NT} u_T - u_N u_{TT} > 0$, i.e. the two goods are complements or at most mild substitutes in the utility function of domestic agents.[5]

The period 0 consumer budget constraint augmented by non-traded goods is

$$c^i_{T,t} + p^i_{N,t} c^i_{N,t} + \left(1 - \tau^i_{t+1}\right) b^i_{t+1} / R_{t+1} = y^i_{T,t} + p^i_{N,t} y^i_{N,t} + b_t + T_t \tag{9}$$

The consumer's optimality conditions are

$$\left(1 - \tau^i_{t+1}\right) u_T \left(c^i_{T,t}, c^i_{N,t}\right) = \beta R_{t+1} V^{i\prime} \left(b^i_{t+1}\right) \tag{10}$$

$$p^i_{N,t} = \frac{u_N \left(c^i_{T,t}, c^i_{N,t}\right)}{u_T \left(c^i_{T,t}, c^i_{N,t}\right)} \tag{11}$$

The first optimality condition is analogous to the Euler equation (3) if we replace u' with u_T and defines an increasing bond demand function $b^i \left(R_{t+1}\right)$ under assumption 1. The second optimality condition states that the real exchange rate is the marginal rate of substitution between traded and non-traded goods. Market clearing for non-traded goods requires that $c^i_{N,t} = y^i_{N,t}$.

We have set up the structure of the real exchange rate model such that the results from our benchmark model carry over. To put it formally:

Corollary 2 (Real Exchange Rate Model) *For given levels of non-traded output* $\left\{y^i_{N,t}\right\}^{N,\infty}_{i,t=1}$*, the real exchange rate model is isomorphic to our benchmark model, with the real exchange rate being a strictly increasing function of tradable consumption* $p^i_{N,t} = p^i_N(c^i_{T,t})$*. Therefore we find:*

1. *An increase in the capital control* τ^i *depreciates the exchange rate* $\partial p^i_{N,t} / \partial \tau^i_{t+1} < 0$ *in country* i*.*

2. *An exogenous increase in the world interest rate depreciates the real exchange rate* $\partial p^i_{N,t} / \partial R_{t+1} < 0$ *for borrowers or mild savers* ($b^i_{t+1}/R_{t+1} < b^i_R$)*, and appreciates the real exchange rate* $\partial p^i_{N,t} / \partial R_{t+1} > 0$ *for high savers* ($b^i_{t+1}/R_{t+1} > b^i_R$)*.*

3. *An increase in the capital control* τ^i_{t+1} *in country* i *appreciates the exchange rates* $\partial p^j_{N,t} / \partial \tau^i_{t+1} > 0$ *of all other countries* $j \neq i$ *that are borrowers or modest savers* ($b^j_{t+1}/R_{t+1} < b^j_R$) *and vice versa for large savers* ($b^j_{t+1}/R_{t+1} > b^j_R$)*.*

[5] Empirically, Mendoza (1995) and Stockman and Tesar (1995) find that traded and non-traded goods are clear complements.

Proof. To show the isomorphism, we define the utility function $u\left(c^i_{T,t}\right) = u\left(c^i_{T,t}, y^i_{N,t}\right)$ for each country by substituting the market-clearing condition for non-traded goods $c^i_{N,t} = y^i_{N,t}$. This utility function satisfies the restrictions required by the benchmark model. Non-traded consumption and endowment cancel from the budget constraint (9). The remaining problem is identical to our benchmark setup and leads to idential optimality conditions.

After substituting the market-clearing condition in (11), we observe that tradable consumption is the only endogenous variable driving the real exchange rate. The derivative

$$\frac{dp^i_{N,t}}{dc^i_{T,t}} = \frac{u_{NT}u_T - u_N u_{TT}}{\left(u_T\right)^2} > 0$$

is positive by our earlier assumptions on the utility function. The proofs of the remaining points follow from lemmas 1 and 2. ■

The intuition is that higher availability of traded goods implies that non-traded goods, which are in fixed supply in the economy, become relatively more valuable. Capital controls shift tradable consumption from one country to another and therefore affect the real exchange rate.

Remark Introducing an additional tax instrument $\tau_{N,t+1}$ on non-tradable consumption that is rebated lump-sum does not affect real allocations. Such a tax scales down the real exchange rate $p_{N,t}$ by a factor $1+\tau_{N,t+1}$, but market clearing implies that non-tradable consumption is unchanged. Therefore the tax does not affect the marginal utility of tradable consumption and the intertemporal Euler equation of consumers.[6]

2.6 Extension 2: Reserve Accumulation

We extend our framework to study reserve accumulation.[7] Assume a planner in country i levies a lump-sum tax in the amount of a^i_{t+1}/R_{t+1} on domestic agents in period 0 and uses it to purchase a quantity a^i_{t+1} of bonds in the global market, which she rebates to consumers in period 1. We may think of these bond holdings as reserves. This changes the period 0 budget constraint to

$$c^i_{T,t} + p^i_{N,t}c^i_{N,t} + a^i_{t+1}/R_{t+1} + \left(1 - \tau^i_{t+1}\right) b^i_{t+1}/R_{t+1} = y^i_{T,t} + p^i_{N,t}y^i_{N,t} + T^i_t \qquad (12)$$

It also modifies the continuation utility of domestic agents to $V^i(a^i_{t+1}+b^i_{t+1})$. Suppose first that domestic agents continue to have free access to international bond markets, i.e. their choice of b^i_{t+1} is unrestricted.

[6] This observation would change if we endogenize the supply of non-tradable goods: a tax on non-tradable goods reduces their relative supply, which decreases the marginal utility of tradable goods u_T and induces consumers to save more in international capital markets.

[7] This extension can be introduced either in our benchmark model or in our model with a real exchange rate. We perform our analysis in the latter model since this naturally allows us to discuss the implications of reserve accumulation for the real exchange rate.

Proposition 2 (Reserve Accumulation) *(i) Under open capital accounts, domestic agents will undo any reserve accumulation* a_{t+1}^i *by adjusting their private bond holdings by* $\Delta b_{t+1}^i = -a_{t+1}^i$.

(ii) Under closed capital accounts, reserve accumulation cannot be undone. It reduces domestic consumption $\partial c_t^i / \partial a_{t+1}^i < 0$ *and depreciates the real exchange rate* $\partial p_{N,t}^i / \partial a_{t+1}^i < 0$ *of country i. If the mass of the country is positive, it also reduces the world interest rate* $\partial R_{t+1} / \partial a_{t+1}^i < 0$.

(iii) There is a one-to-one correspondence between a given level of capital controls τ_{t+1}^i *under open capital accounts and a given amount of reserve accumulation* a_{t+1}^i *under closed capital accounts.*

Proof. To show part (i), observe that if private bond holdings b_{t+1}^i satisfied the consumer's optimality condition in the absence of reserve accumulation, then private bond holdings $b_{t+1}^i - a_{t+1}^i$ satisfy the optimality condition given reserve accumulation a_{t+1}^i.

If consumers have unconstrained access to capital markets, then reserve accumulation is ineffective, even if the planner has imposed price controls τ_{t+1}^i on international capital flows. What matters for the real allocations of the consumer is solely the level of capital controls τ_{t+1}^i, not the level of reserves a_{t+1}^i. This is a form of Ricardian equivalence – a representative consumer internalizes that government bond holdings are equivalent to private bond holdings.[8]

Under closed capital accounts in part (ii), private agents are restricted to a zero international bond position $b_{t+1}^i = 0$ and international capital flows are solely determined by reserve accumulation. Reserve accumulation/decumulation constitutes forces saving/dissaving. The effects of reserve accumulation therefore mirror the effects of private capital flows in proposition 3.

To show point (iii), we observe that a capital control τ_{t+1}^i under open capital accounts leads private consumers to accumulate $b^i\left(R_{t+1}; \tau_{t+1}^i\right)$ bonds and is therefore equivalent to reserve accumulation $a_{t+1}^i = b^i\left(R_{t+1}; \tau_{t+1}^i\right)$ under closed capital accounts. Since the function $b^i\left(R_{t+1}; \tau_{t+1}^i\right)$ is strictly decreasing in τ_{t+1}^i and its range is $\Re$, any level of reserve accumulation can be replicated by a capital control τ_{t+1}^i. ■

Numerical Illustration We continue our numerical illustration to investigate the isomorphism between reserve accumulation and capital controls. Under the assumption of a small open economy that is in steady state and in which reserve accumulation is not undone by private agents, the increase in capital controls that is equivalent to a certain increase in reserve accumulation as a fraction of GDP a^i/y^i is

$$\frac{\partial \tau^i}{\partial a^i / y^i} = \frac{1+\beta}{\sigma}$$

[8] The result is therefore subject to the same limitations as Ricardian equivalence. In particular, it critically relies on the assumption that consumers can access bond markets at the same conditions as governments.

For the standard value of the intertemporal elasticity of substitution $\sigma = 1/2$, this term is approximately $\partial\tau^i/\partial(a^i/y^i) \approx 4$. In short, accumulating an extra percent of GDP in reserves is equivalent to imposing a 4% capital control or, vice versa, a 1% capital control is equivalent to accumulating a quarter percent of GDP in reserves.

For more detailed numerical results, we refer back to Table 2 on page 11. In the Table, we illustrated that a 1% capital control improves the current account by $\Delta b^i/R$. But, given the isomorphism, we can read the table in both directions. If China, for example, accumulates an extra $13bn in foreign reserves, this is equivalent to a 1% capital control. Similarly, if Brazil accumulates an extra $5bn in foreign reserves, it is equivalent to a 1% capital control (under the assumption that the transaction is not undone by private agents.)

2.7 Extension 3: Uncertainty

Our benchmark model can easily be extended to incorporate uncertainty. Assume that a state of nature $\omega \in \Omega$ is realized at the beginning of period t and that the probability of state ω is denoted by π^ω. The continuation utility of the consumer in state ω is $V^{i,\omega}(b^{i,\omega})$ is strictly increasing, continuously differentiable and strictly concave and follows the axioms of expected utility. The total expected utility of the consumer is then

$$V^i\left(b_t^i\right) = u\left(c_t^i\right) + \beta E\left[V^{i,\omega}\left(b_{t+1}^{i,\omega}\right)\right]$$

2.7.1 Bond-only Economy

Assume first that the world economy only trades a risk-free bond, as in our benchmark model. Then it is easy to see that the economy is isomorphic to our benchmark economy:

Lemma 4 (Isomorphy of Stochastic Bond-Only Model) *The stochastic economy with a single bond is isomorphic to our benchmark model.*

Proof. We define $\tilde{V}(b^i) = E\left[V^{i,\omega}(b^i)\right]$. Since $\tilde{V}(b^i)$ is strictly increasing, continuously differentiable and strictly concave it satisfies all the conditions of the continuation utility in our benchmark model and the results from that section continue to apply. ■

This implies that in an incomplete markets economy in which only a bond is traded, all our earlier results continue to apply.

2.7.2 Complete Markets

Assume next that there is a complete set of securities contingent on the state of nature ω, and denote a representative consumer i's holdings of securities contingent on state

ω by $b^{i,\omega}$. The required return of a security that pays off one unit in state ω is R^{ω}_{t+1}. We denote the inverse of this required return as the state price $q^{\omega}_{t+1} = 1/R^{\omega}_{t+1}$, i.e the price of a security that pays off one unit in state ω of period 1. Furthermore, assume that a planner imposes a capital control $\tau^{i,\omega}_{t+1}$ on each of the securities and rebates the net revenue as a lump sum $T^i_t = -\sum_{\omega\in\Omega} \tau^{i,\omega}_{t+1} b^{i,\omega}_{t+1}$.

A representative consumer maximizes his expected utility subject to the budget constraint

$$c^i_t + \sum_{\omega\in\Omega} \left(1 - \tau^{i,\omega}_{t+1}\right) q^{\omega}_{t+1} b^{i,\omega}_{t+1} = y^i_t + b^i_t + T^i_t$$

By imposing a version of assumption 1 that is adjusted for uncertainty, we obtain the same comparative static effects of changes in the prices of state-contingent securities q^{ω}_{t+1} and of capital controls τ^{ω}_{t+1} as in lemmas 2 and 1.

We define a global excess demand function for state-contingent securities $B^{\omega}\left(q^{\omega}_{t+1};\tau^{\omega}_{t+1}\right) = \sum_{i=1}^{N} b^{i,\omega}\left(q^{\omega}_{t+1};\tau^{i,\omega}_{t+1}\right)$ and impose market clearing $B^{\omega}_{t+1} = 0$ for each state ω. The logic of proposition 3 implies that a capital control $\tau^{i,\omega}_{t+1}$ in country i, state ω pushes up the price q^{ω}_{t+1} of payoffs in that state, i.e. $\partial q^{\omega}_{t+1}/\partial \tau^{i,\omega}_{t+1} > 0$ and induces other countries to save less contingent on that state, i.e. reduce $b^{j,\omega}_{t+1}$.

3 Distortive Capital Controls

3.1 Setup

Suppose that there is a domestic planner in each country i with positive mass $m^i > 0$ that maximizes the utility of the representative consumer U^i and internalizes that she has market power over the world interest rate R_{t+1}, which affects consumer welfare as we observed in lemma 2. (For simplicity we omit time subscripts.)

Global market clearing requires that world-wide savings add up to zero,

$$m^i b^i_{t+1} + B^{-i}_{t+1} = 0 \qquad \text{where} \qquad B^{-i}_{t+1} = \sum_{j\neq i} m^j b^j_{t+1}$$

B^{-i}_{t+1} denotes the rest-of-the-world bond holdings excluding country i. We can express these as a function $B^{-i}_{t+1}\left(R_{t+1};\tau^{-i}_{t+1}\right)$ that is strictly increasing in R_{t+1} and increasing in each element of the rest-of-the world's capital controls $\tau^{-i}_{t+1} = \left\{\tau^{j}_{t+1}\right\}_{j\neq i}$. We invert this function to obtain the inverse rest-of-the-world bond demand $R^{-i}\left(B^{-i}_{t+1};\tau^{-i}_{t+1}\right)$, which is strictly increasing in B^{-i}_{t+1} and declining in each element of τ^{-i}_{t+1}.

A domestic planner in country i recognizes that market clearing requires $B^{-i}_{t+1} = -m^i b^i_{t+1}$ and that the world interest rate therefore satisfies $R_{t+1} = R^{-i}\left(-m^i b^i_{t+1};\tau^{-i}_{t+1}\right)$. We formulate the recursive optimization problem of planner who takes the vector of policies imposed by other countries as given,[9]

$$\max_{b^i} u\left(y_t^i + b_t^i - \frac{b_{t+1}^i}{R^{-i}(-m^i b_{t+1}^i; \tau_{t+1}^{-i})}\right) + \beta V^i\left(b_{t+1}^i\right)$$

leading to the generalized Euler equation

$$u'\left(c_t^i\right)\left[1 - \frac{\partial R_{t+1}}{\partial B_{t+1}^{-i}} \cdot \frac{m^i b_{t+1}^i}{R_{t+1}}\right] = \beta R_{t+1} V^{i\prime}\left(b_{t+1}^i\right) \tag{13}$$

where the term $\eta_{RB^{-i}} = \partial R_{t+1}/\partial B_{t+1}^{-i} \cdot m b_{t+1}^i / R_{t+1}$ captures the inverse elasticity of global savings. This term distinguishes the Euler equation of a monopolistic planner from the Euler equation of decentralized agents (3). It reflects that increasing domestic saving b_{t+1}^i pushes down the world interest rate $\partial R_{t+1}/\partial b_{t+1}^i = -\partial R_{t+1}/\partial B_{t+1}^{-i} < 0$. If the country is a net saver, then $\eta_{RB^{-i}} < 0$ and the planner finds it desirable to push up the world interest rate by taxing capital outflows and reduing saving; if the country is a net borrower, then $\eta_{RB^{-i}} > 0$ and the planner finds it optimal to tax capital inflows.

Naturally the effect is scaled by the weight m^i of the country in the world economy. The planner distorts the domestic saving decision to the point where the marginal benefit of manipulating the world interest rate – the interest rate impact times the amount saved valued at the country's marginal utility $\partial R_{t+1}^{-i}/\partial B_{t+1}^{-i} \cdot b_{t+1}^i / R_{t+1} \cdot u'\left(c_t^i\right)$ – equals the marginal cost of having an unsmooth consumption profile $\beta R_{t+1} V^{i\prime}\left(b_{t+1}^i\right) - u'\left(c_t^i\right)$.

Proposition 3 (Market Power and Capital Controls) *A domestic planner who internalizes the country's market power over the world interest rate imposes a monopolistic capital control*

$$\tau_{t+1}^{i,M} = m^i \eta_{RB^{-i}} = \frac{\partial R_{t+1}}{\partial B_{t+1}^{-i}} \frac{m^i b_{t+1}^i}{R_{t+1}} \tag{14}$$

Proof. The tax $\tau_{t+1}^{i,M}$ ensures that the private optimality condition of consumers (3) replicates the planner's generalized Euler equation (13). ■

This leads us to the following observations about the optimal tax rate:

1. The optimal tax carries the opposite sign as the country's bond position b_{t+1}^i. If the country is a net saver, the planner taxes saving $\tau_{t+1}^i < 0$. If the country is a net borrower, the planner taxes inflows $\tau_{t+1}^i > 0$.

[9] The described setup solves for the optimal level of distortive capital controls in a time-consistent setting. For an analysis of optimal distortive capital controls under commitment see Costinot et al. (2011).

2. Ceteris paribus, the absolute value of the optimal tax $|\tau_{t+1}^i|$ is linear in the mass of the country m^i and in the absolute size of the country's bond position $|b_{t+1}^i|$. If the country is small compared to the world economy $m^i = 0$ or is a zero saver $b_{t+1}^i = 0$, then the optimal tax rate is $\tau_{t+1}^i = 0$.

3. The absolute magnitude of the optimal tax is higher the greater the elasticity of the world interest rate with respect to global savings $\eta_{RB^{-i}}$.

4. For a given elasticity $\eta_{RB^{-i}}$, the optimal tax rate is a decreasing function of initial output, $\partial\tau_{t+1}^i/\partial y_t^i > 0$ because output increases saving. Countries that are comparatively rich in period 0 tax saving; countries that are comparatively poor tax borrowing.

5. For a given country, the optimal tax rate reduces the magnitude of capital flows but does not change their direction.

6. The optimal policy can equivalently be implemented via quantity restrictions. If a country is a net saver, the tax is equivalent to a quota or ceiling on capital outflows $\bar{b}_{t+1}^i = b^i\left(R_{t+1}; \tau_{t+1}^i\right)$ that restricts $b_{t+1}^i \leq \bar{b}_{t+1}$. If the country is a net borrower so $b_{t+1}^i < 0$, the tax is equivalent to a quota or ceiling on capital inflows $\underline{b}_{t+1}^i = b^i\left(R_{t+1}; \tau_{t+1}^i\right)$ that restricts inflows to $b_{t+1}^i \geq \underline{b}_{t+1}^i$.

7. Finally, optimal distortive capital controls are isomorphic to reduced reserve accumulation under closed capital accounts.[10]

Numerical Illustration In the following we determine the optimal monopolistic level of capital controls for a variety of countries numerically based on equation (14). From our earlier analysis, we observe that the steady-state response of the world interest rate to additional saving is $\partial R/\partial B^{-i} = \frac{1+\beta}{\sigma\beta\bar{Y}^{-i}}$. If we identify $m^i b^i/R$ in the data by the current account balance CA^i of different countries, then we can express the optimal monopolistic capital controls of a given country i by

$$\tau_{t+1}^{i,M} = \frac{1+\beta}{\sigma\beta} \cdot \frac{CA^i}{\bar{Y}^{-i}}$$

For our earlier value of the intertemporal elasticity of substitution, the first term in this expression is approximately 4. In short, the optimal capital control of a monopolistic country is four times its current account relative to the GDP of the rest of the world.

We report the resulting calculations for a number of countries in Table 3 below. It can be seen that countries for which the current account balance represents a significant fraction of rest-of-the-world GDP have a strong motive for imposing monopolistic

[10] For example, when a country that accumulates reserves is concerned that it is not earning "sufficient" interest on its reserves because its accumulation is pushing down the world interest rate, this is non-competitive behavior and is equivalent to distortive capital controls.

Country	GDP^i	CA^i	CA^i/Y^{-i}	$\tau^{i,M}$
World	\$62,634bn	...	...	...
United States	\$14,447bn	\$-474bn	-0.98%	4.02%
China	\$5,739bn	\$281bn	0.49%	-2.02%
Japan	\$5,459bn	\$123bn	0.22%	-0.88%
Brazil	\$2,089bn	\$-63bn	-0.1%	0.42%
India	\$1,722bn	\$-63bn	-0.1%	0.42%
South Korea	\$1,014bn	\$30bn	0.05%	-0.2%
Indonesia	\$707bn	\$6bn	0.01%	-0.04%
Argentina	\$370bn	\$0bn	0%	0%
Colombia	\$288bn	\$-7bn	-0.01%	0.05%
Malaysia	\$238bn	\$33bn	0.05%	-0.22%

Table 3: Optimal monopolistic capital controls (Source: IMF IFS and author's calculations)

capital controls. The United States, for example, would optimally impose a 4% tax on capital inflows so as to exert monopoly power over the availability of global savings instruments and benefit from a lower world interest rate. By contrast, China would optimally impose a 2% on capital outflows (or subsidy on capital inflows) so as exert monopoly power over its supply of worldwide savings and raise the interest rate. Countries that make up a small share of the world capital market have less market power and choose accordingly smaller capital controls since they cannot affect world interest rates.

The table highlights that it is in many cases difficult to reconcile the capital controls observed in the real world with the monopolistic motive for imposing capital controls. We will therefore investigate a number of more promising motives in future sections.

3.2 Welfare Analysis of Exerting Market Power

In this subsection we analyze the welfare effects if one country imposes capital controls to exert market power. We first discuss the spillover effects on other countries; then we investigate the Pareto efficiency of the resulting global equilibrium.

Corollary 3 (Spillover Effects of Exerting Market Power) *An increase in the capital control τ^i_{t+1} has positive welfare effects for borrowing countries $b^j_{t+1} < 0$ and negative welfare effects for lending countries $b^j_{t+1} > 0$.*

Proof. We observed in proposition 3 that the effects of raising the capital control τ^i on the world interest rate is $dR/d\tau^i = -m^i b^i_\tau / B_R < 0$. The interest rate in turn

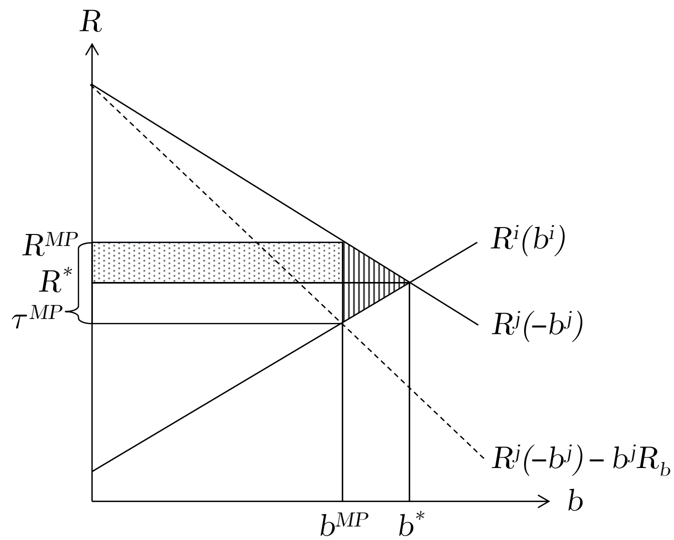

Figure 1: Optimal capital control imposed by a domestic planner to exert market power

affects the welfare of another country j as follows

$$\frac{dV_t^j}{dR_{t+1}} = \beta^t u'\left(c_{t+1}^j\right) \cdot \frac{b_{t+1}^j}{R_{t+1}^2} \gtrless 0$$

If the country is a net saver ($b_{t+1}^j > 0$), it is hurt by a low interest rate and by capital controls in country i. If the country is a net borrower ($b_{t+1}^j < 0$), it benefits from a low interest rate and from capital controls in country i. In short, it is in the interest of all lending countries to improve their intertemporal terms-of-trade and push up the world interest rate by reducing the supply of bonds on world capital markets. It is in the interest of all borrowing countries to lower the world interest rate and reduce the demand for bonds on world capital markets. ■

Figure 1 illustrates our results in a framework of two countries i and j of equal mass for a given time period. The line $R^i(b^i)$ represents the (inverse) supply of bonds, the two lines $R^j(-b^j)$ and $R^j(-b^j) - b^j R_b$ represent the demand for bonds as well as the marginal revenue curve for country i, i.e. taking into account the decline in the interest rate from supplying additional bonds. The decentralized equilibrium is characterized by an interest rate R^* and bond positions $b^i = b^* = -b^j$. A monopolistic planner in country i would reduce the quantity of bonds supplied to j such that her marginal valuation $R^i(b^i)$ equals the marginal revenue derived from country j. This monopolistic equilibrium is indicated by the quantity of bonds sold b^{MP} and interest rate R^{MP}. The described policy shifts the surplus between R^{MP} and R^*, marked by the dotted area in the figure, from country j to country i. It also introduces a deadweight loss indicated by the triangular vertically-shaded area. Because of this deadweight loss, monopolistic capital controls constitute a classic beggar-thy-neighbor policy and are always inefficient: they introduce a distortion into the Euler equation of domestic agents, which reduces global welfare, in order to shift welfare from foreigners to domestic agents – the policy represents a "negative-sum" game overall.

Proposition 4 (Inefficiency of Exerting Market Power) *An equilibrium in which domestic planners impose capital controls to exert market power is Pareto-inefficient.*

Proof. The result is a straightforward application of the first welfare theorem that we captured in proposition 1. ∎

If one or more countries impose capital controls to exert market power, then capital controls are not equal across countries (lenders impose outflow controls; borrowers impose inflow controls). The intertemporal marginal rates of substitution of different agents differ, and the necessary conditions for Pareto efficiency of proposition 1 are violated.

Market Power and the Real Exchange Rate The real exchange rate model allows us to study the effect of monopolistic capital controls on the real exchange rate as well as the scope for monopolistic real exchange rate intervention.

If a country experiences capital outflows and is a net saver in a given period ($b^i_{t+1} > 0$), then its exchange rate is depreciated compared to the autarky level. A monopolistic planner would tax saving abroad ($\tau^i_{t+1} < 0$), which would reduce capital outflows and push up the world interest rate. In the domestic economy, this policy appreciates the real exchange rate. Alternatively, in a country with closed capital accounts, the planner would reduce a^i_{t+1}, i.e. reduce reserve accumulation to keep the world interest rate elevated.

If a country is a net borrower ($b^i_{t+1} < 0$), the opposite lessons apply. The country's real exchange rate is appreciated compared to the autarky level. A monopolistic planner would tax capital inflows to push down the world interest rate. In doing so, she would also put downward pressure on the domestic real exchange rate. Alternatively, with closed capital accounts, the planner would increase a^i_{t+1}, i.e. increase reserves or borrow less from abroad, to push down the world interest rate.

In short, a monopolistic planner would reduce deviations of the real exchange rate from its steady state.

Market Power and Uncertainty Our findings on market power carry over to the model with uncertainty that we outlined in section 2.7. In particular, a monopolistic domestic planner finds it optimal to impose state-contingent capital controls of

$$\tau^{i,\omega}_{t+1} = m^i \eta_{RB^{\omega,-i}}$$

In a world economy with idiosyncratic country shocks, optimal risk-sharing requires that each country purchases insurance contingent on states in which it is relatively worse off: a country sells more securities contingent on states in which it is relatively better off than on states in which it is worse off compared to the rest of the world.

For example, if the country is a net lender and is relatively better off in state ω than in state ψ, then optimal risk-sharing implies $0 < b^{i,\omega} < b^{i,\psi}$ which insures consumers against state ψ. Under the usual regularity conditions for $\partial R^{\omega}/\partial B^{\omega,-i}$, a monopolistic domestic planner sets $0 > \tau^{i,\omega} > \tau^{i,\psi}$, i.e. the planner taxes carrying resources (insurance) into state ψ more than carrying resources into state ω. This diminishes international risk-sharing. Practically speaking, lending countries will lend too much in hard claims and too little in contingent forms of finance such as FDI.

By the same token, countries that are net borrowers and want to exert monopoly power will borrow too much in foreign currency and too little in terms of FDI, again diminishing international risk-sharing.

4 Corrective Capital Controls

4.1 Setup

We next focus on capital controls that are designed to correct externalities in the country in which they are imposed. Such capital controls may be imposed for example for prudential reasons, to internalize learning-by-exporting effects or because of aggregate demand externalities.

To side-step the issue of monopoly power, which we analyzed in the previous section, we assume a setup of $N \geq 1$ regions in the world economy that each consist of a mass m^i of identical atomistic countries, and each country in turn consists of a unit mass of representative consumers. To simplify notation, we will denote by "country i" a representative country in the region $i \in \{1, ...N\}$.

The representative agent in a representative country i maximizes utility $V^i\left(b_t^i\right) = u\left(c_t^i\right) + \beta V^i\left(b_{t+1}^i; \bar{b}_{t+1}^i\right)$ as described in section 2, but with a second argument $\bar{b}_{t+1}^i = \int_0^1 b_{t+1}^{i,s} ds = b_{t+1}^i$ in the continuation utility function $V(\cdot)$, where $s \in [0,1]$ represents the unit mass of consumers in the represenative country and $\bar{b}_{t+1}^i$ reflects their aggregate bond holdings. We assume that if we set $\bar{b}_{t+1} = b_{t+1}^i$, the function $V^i\left(b_{t+1}^i; b_{t+1}^i\right)$ is strictly increasing, continuously differentiable and strictly concave in b_{t+1}^i, i.e. that $\partial V^i\left(b_{t+1}^i; b_{t+1}^i\right)/\partial b_{t+1}^i = V_1^i + V_2^i > 0$ etc. Furthermore, we assume that assumption 1 continues to hold. We will discuss several alternative specifications for $V^i(\cdot)$ below.

An individual consumer takes the aggregate bond holdings of the economy as given and, if we use the notation $V^{i\prime}\left(b_{t+1}^i\right) = V_1^i\left(b_{t+1}^i; b_{t+1}^i\right)$, arrives at the same standard Euler equation (3) as above.

By contrast, the domestic planner in country i internalizes that $b_{t+1}^i = \bar{b}_{t+1}^i$ when making her optimal decisions. When she maximizes the welfare of her consumers, she arrives at the Euler equation

$$u'\left(c_t^i\right) = \beta R_{t+1}\left[V^{i\prime}\left(b_{t+1}^i\right) + \xi_{t+1}^i\left(b_{t+1}^i\right)\right] \tag{15}$$

where we use the short-hand notation $\xi^i_{t+1}\left(b^i_{t+1}\right) = V^i_2\left(b^i_{t+1};\bar{b}^i_{t+1}\right) = \partial V^i(\cdot)/\partial\bar{b}^i_{t+1}$ for the marginal externalities associated with economy-wide saving or borrowing. We obtain the demand and inverse demand functions of the planner for country i, which we denote by $\tilde{b}^i(R_{t+1})$ and $\tilde{R}^i(b_{t+1})$, by following the steps outlined in appendix A.1.

Proposition 5 (Unilaterally Correcting Externalities) *The domestic planner in country i implements the optimal bond allocation $\tilde{b}^i$ for country i by unilaterally imposing a capital control $\tilde{\tau}^i_{t+1}$ that satisfies*

$$\tilde{\tau}^i_{t+1} = \frac{\beta R_{t+1}\xi^i_{t+1}\left(\tilde{b}^i_{t+1}\right)}{u'\left(c^i_t\right)} \tag{16}$$

Proof. Substituting the tax $\tilde{\tau}^i_{t+1}$ in the Euler equation of private agents (3) replicates the planner's optimality condition (15). ■

The optimal tax offsets the externality that private agents fail to internalize and therefore induces them to choose the socially efficient level of saving from a domestic perspective. Let us next turn to the worldwide general equilibrium effects of such a policy.

4.2 Welfare Analysis of Correcting Domestic Externalities

In this subsection, we analyze the global efficiency of capital controls that are imposed to correct domestic externalities. First, we investigate the global spillover effects of capital controls that are imposed unilaterally to correct domestic externalities and show that they generally make some countries better off and others worse off. Next we show that such capital controls nonetheless lead to a Pareto efficient outcome. Finally, we investigate the scope for global capital control policies that lead to Pareto improvements, i.e. that make all countries better off. We show that such 'Pareto-improving' capital controls are possible if a global planner can coordinate the capital control policies of all individual countries.

The capital controls described in proposition 5 are optimal from a domestic perspective, but they have nonetheless global spillover effects, as we described in proposition 3 and lemma 2. If country i imposes w.l.o.g. its unilaterally optimal capital control $\tilde{\tau}^i_{t+1} > 0$, then its demand for bonds declines, the world interest rate falls $dR_{t+1}/d\tau^i_{t+1} < 0$, and capital is diverted to other countries.

Figure 2 illustrates our finding graphically in a two-country world: $R^j(b^j)$ depicts the (inverse) supply of bonds of country j, $R^i(-b^i)$ represents the inverse demand for borrowing in country i. The intersection of the two, marked by R^{DE} and b^{DE}, determines the decentralized equilibrium of the economy. However, suppose that there is a negative externality ξ^i associated to borrowing by country i, for example because of the financial instability caused by debt. Then a domestic planner would internalize that the social benefit of debt $R^{i,DP}(-b^i)$ is less than the private benefit

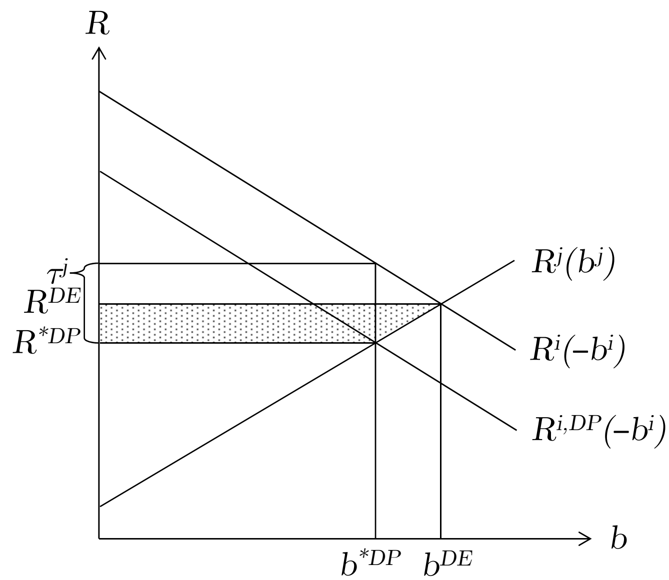

Figure 2: Optimal capital control to internalize domestic externalities

and would impose a tax τ^i on borrowing to induce private agents to account for the difference. The resulting equilibrium exhibits less borrowing/lending b^{*DP} and a lower world interest rate R^{*DP}. Note that country j looses the surplus that is marked by the shaded area in the figure.

Although there are spillover effects on other countries when a domestic planner imposes her unilaterally optimal capital controls, the outcome is Pareto efficient, i.e. in the resulting equilibrium, no country can be made better off without hurting another country:

Proposition 6 (Efficiency of Unilaterally Correcting Externalities) *The global equilibrium in which each domestic planner i corrects domestic externalities by using her unilaterally optimal capital control $\tilde{\tau}^i_{t+1}$ is Pareto efficient.*

Conversely, if there are $N > 2$ countries and there are domestic externalities $\xi^i \neq 0$ that differ across countries, then the free market equilibrium without policy intervention is inefficient as private agents do not internalize the externalities of global capital flows.

Proof. The described allocation is Pareto efficient if it maximizes global welfare for some vector of country welfare weights $\left\{\phi^i > 0\right\}_{i=1}^N$

$$\max \sum_{i,t} m^i \phi^i \beta^t \left[u\left(c^i_t\right)\right] \quad \text{s.t.} \quad \sum_i m^i \left(c^i - y^i\right) = 0 \forall t$$

subject to the global bond market clearing condition $\sum_i m^i b^i = 0$,

$$\max_{\{b^i, R\}} \sum_i \phi^i \left[u\left(y^i - b^i/R\right) + \beta V^i\left(b^i, b^i\right)\right] + \nu \sum_i m^i b^i$$

Using our notation for ξ^i from above, the first-order condition on b^i yields

$$\frac{\phi^i}{m^i}\left\{u'\left(c^i\right)-\beta R\left[V^{i\prime}\left(b^i\right)+\xi^i\left(b^i\right)\right]\right\}=\nu \quad \forall i \tag{17}$$

If $\tau^i=\beta R\xi^i\left(\tilde{b}^i\right)/u'\left(c^i\right)\forall i$ in the private Euler equation of decentralized agents, then $\nu=0$ and the condition is satisfied for all countries. The first-order condition on R yields

$$\sum_i \phi^i u'\left(c^i\right)b^i=0$$

If we set the country welfare weights $\phi^i=m^i/u'\left(c^i\right)$ then this condition coincides with the market clearing condition and is also satisfied. Therefore the allocation constitutes a Pareto optimum.

Conversely, if $\tau^i=0$ but $\xi^i\neq 0$, then the optimality condition (17) cannot generally be satisfied for all countries simultaneously. The resulting allocation is not Pareto efficient. ■

At first blush, there may seem to be a conflict between lemma **??** and proposition 6: some have interpreted the existence of spillover effects from capital controls as evidence that there is an inefficiency. However, the main insight of proposition 6 is that these spillover effects on the world interest rate R constitute efficient pecuniary externalities that reflect the response of the market to the new balance of demand and supply for bonds. Such pecuniary externalities are redistributions between borrowers and lenders and do not necessarily lead to Pareto inefficiencies, since the benchmark of Pareto efficiency is indifferent about the distribution of resources. In standard consumer theory, pecuniary externalities correspond to income and wealth effects and are usually disregarded because they can be undone by lump-sum transfers, as we show in the following proposition:

Proposition 7 (Pareto-Improving Capital Controls, With Transfers) *Suppose a world economy in a free market equilibrium without capital controls. If a global planner identifies domestic externalities $\left\{\xi^i\left(b^i\right)\right\}_{i=1}^N$ with at least one $\xi^i\left(b^i\right)\neq 0$, he can achieve a global Pareto improvement by setting capital controls in all countries such that $\tau^i=\xi^i\left(b^i\right)$ and engaging in compensatory transfers $T^i\lesseqgtr 0$ that satisfy $\sum_i T^i=0$.*

Proof. Denote the saving/consumption allocation in the free market equilibrium and the associated world interest rate by $\left\{\left(b^{DE,i},c^{DE,i}\right)\right\}_{i=1}^N$ and R^{DE}. Denote the allocations and world interest rate in the planner's new equilibrium by $\left\{\left(b^{P,i},c^{P,i}\right)\right\}_{i=1}^N$ and R^P, and let him set the transfers such that $T^i=c^{DE,i}-c^{P,i}+\frac{b^{DE,i}-b^{P,i}}{R^P}$. Observe that $\sum T^i=0$ since both sets of allocations (DE and P) satisfy market clearing. Furthermore, observe that given these transfers, each country i can still afford the allocation that prevailed in the free market equilibrium. If $\xi^i\left(b^i\right)\neq 0$, then the allocation chosen for the country by the global planner differs from the allocation in the free market equilibrium since the Euler equations (3) and (15) differ. Given that the old allocation was still feasible, revealed preference implies that country i must be better off under the new allocation. ■

In an international context, compensatory transfers may be even less feasible than within a domestic economy. However, as we will show in the following, if a global planner can globally coordinate capital control policies, he can correct the domestic externalities in all economies while holding the world interest rate constant so that no income and wealth effects arise. As a result, the global planner's capital control policy constitutes a global Pareto improvement at a first-order approximation.[11]

We first demonstrate in a lemma how a global planner can manipulate the world interest rate by simultaneously adjusting the worldwide level of capital controls; then we show how this mechanism can be used to offset the pecuniary externalities that arise from capital controls that correct domestic externalities in a given country. Using this mechanism, a global planner can ensure that capital controls imposed to correct domestic externalities achieve a global Pareto improvement at a first-order approximation.

Lemma 5 *Suppose the world economy is in an equilibrium with a level of individual country bond holdings* $\{b^j\}$*, capital controls* $\{\tau^j\}$ *and a world interest rate* R*. A global planner can increase the world interest rate by* dR *while keeping* $\{b^j\}$ *constant for all countries by moving the capital control in each country* $j = 1...N$ *by*

$$\frac{d\tau^j}{dR} = -\frac{b^j_R}{b^j_\tau} \tag{18}$$

Proof. Taking the total differential of the bond demand function of a given country j we find

$$db^j = b^j_R dR + b^j_\tau d\tau^j$$

Setting $db^j = 0$, we find that the required change in the capital control for a given dR is given by equation (18) for every country j. ■

Proposition 8 (Pareto-Improving Capital Controls, No Transfers) *A global planner can correct for an exogenous marginal increase in the domestic externality*

[11] Holding the world interest rate constant while correcting for an infinitesimal change in externalities $d\xi^i$ implies that welfare in all other countries $j \neq i$ remains constant at a first-order approximation. However, the capital controls in country i also affect the amounts borrowed and lent b^j in other countries j, which has welfare effects that are second-order (i.e. negligible for infinitesimal changes but growing in the square of the deviation). W.l.o.g., for $\tau > 0$, these second-order welfare effects are positive for borrowers and negative for lenders. For non-infinitesimal changes in the externality $\Delta\xi^i$, a global planner could undo these second-order effects via further adjustments in the world interest rate R under certain cirumstances (esp. a uniform demand elasticity b^j_R across all countries).

$d\xi^i > 0$ in country i while keeping the world interest rate R constant to avoid income and wealth effects by adjusting

$$\frac{d\tau^j}{d\xi^i} = -\frac{\beta R}{u'(c^i)} \cdot \frac{m^i b^i_\tau}{B_R} \cdot \frac{b^j_R}{b^j_\tau}$$
$$\frac{d\tau^i}{d\xi^i} = \frac{\beta R}{u'(c^i)} \cdot \left(1 - \frac{m^i b^i_R}{B_R}\right)$$

Proof. We found in proposition 5 that the optimal unilateral response (uni) for a planner in country i is $d\tau^i/d\xi^i\big|_{uni} = \beta R/u'(c^i)$. By lemma **??**, the capital control moves the world interest rate by $dR/d\tau^i = -\beta R/u'(c^i) \cdot m^i b^i_\tau / B_R$. Finally, according lemma 5, the move in the interest rate can be undone if the capital controls of all countries $j = 1...N$ are simultaneously adjusted by $-d\tau^j/dR \cdot dR/d\tau^i$. The first equation of the proposition can be obtained by multiplying out the three derivatives $-d\tau^j/dR \cdot dR/d\tau^i \cdot d\tau^i/d\xi^i\big|_{uni}$. The second equation can be obtained by adding the initial unilateral response of the capital control $d\tau^i/d\xi^i\big|_{uni}$ plus the adjustment given in the first equation with $j = i$. As a result, the increase in the externality $d\xi^i$ is corrected but the world interest rate is unchanged. ■

Returning to our illustration in figure 2 where we corrected for a negative externality to borrowing in country i, a global planner would impose a tax on capital outflows in country j in the amount of $\left(R^{DE} - R^{*DP}\right)$ and a tax on capital inflows for the remaining part of τ^j in country j. As a result, the interest rate would be unchanged at R^{DE} and the welfare loss by country i, as indicated by the shaded area in the figure, would be largely avoided.[12]

Arms Race of Capital Controls Optimally imposed capital controls may lead to dynamics that look like an arms race, but this does not necessarily indicate inefficiency: Suppose that there are two regions i and j that each have capital controls in place in order to offset a domestic externality $\xi^i\left(\bar{b}^i\right)$, which is increases in $\bar{b}^i$, i.e. $\xi^{i\prime} > 0$. Assume that region i experiences an exogenous increase in ξ^i that makes it optimal to increase the capital control τ^i. As a result, the supply of capital to region j increases, $\bar{b}^j$ rises, and it is optimal for region j to raise their capital controls. However, based on the response of country j, country i may find it optimal to increase its capital control τ^i yet further.

The resulting dynamics may give the appearance of an arms race but are not, in themselves, inefficient. In fact, as long as the conditions of proposition 6 are satisfied, this is the mechanism of tatonnement through which the globally efficient equilibrium is restored. Under the conditions of the proposition, each successive round of increases

[12] Country i would still loose the triangular part of the shaded area between b^{*DP} and b^{DE}, but this area is second order. For large interventions, country i could be compensated for this by raising the interest rate on the remainder of his bond holdings.

in capital controls will be smaller and the capital controls in the two regions τ^i and τ^j will converge towards the efficient level.

5 Imperfect Capital Controls

This section analyzes capital controls that are imperfect policy tools and investigates under what circumstances such imperfections lead to a case for global coordination of capital control policies. In the previous section, we emphasized that the international spillover effects of perfectly targeted capital controls constitute pecuniary externalities that are mediated through a well-functioning market and therefore lead to Pareto-efficient outcomes, as long as domestic policymakers act competitively and impose such controls to internalize domestic externalities. This result follows from the first welfare theorem if we view the domestic policymakers in each country as competitive agents who optimize domestic welfare. By implication, we found that there is no need for global coordination to achieve Pareto-efficient outcomes. Our result relies on the assumption that domestic policymakers have the instruments to perfectly and costlessly control the amount of capital flows to the country.

In practice capital controls sometimes differ from the perfect policy instruments that we have depicted in our earlier analysis in that they create ancillary distortions. In the following two subsections, we analyze two types of such distortions in more detail: implementation costs of capital controls and imperfect targeting of capital controls. We formalize both examples and analyze whether a global planner could achieve a Pareto improvement by coordinating the capital control policies of different countries in the presence of such ancillary distortions.

5.1 Costly Capital Controls

The simplest specification of such a setup is to assume that capital controls impose a resource cost $C^i(\tau)$ on the economy that represents enforcement costs or distortions arising from attempts at circumvention. Assume that the function $C^i(\cdot)$ is twice continuously differentiable and satisfies $C(0) = C'(0) = 0$ and $C''(\tau) > 0 \forall \tau$.[13]

The optimization problem of a national policymaker, where we use the summary notation $W^i(b^i) = V^i(b^i; b^i)$, is then

$$\max_{b^i, c^i, \tau^i} u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i \left[c^i - y^i + \frac{b^i}{R} + C^i\left(\tau^i\right)\right] - \mu^i \left[\left(1 - \tau^i\right) u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right]$$

[13] Analogous results can be derived if the cost of capital controls is linear in bond holdings, e.g. $c(\tau, b) = C(\tau) \cdot |b|$, which may better capture the costs associated with attempts at circumvention.

The first-order conditions are

$$\begin{aligned}
FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i/R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) \\
FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i\left(1-\tau^i\right)u''\left(c^i\right) \\
FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right)
\end{aligned}$$

and can be combined to the optimality condition

$$u'\left(c^i\right) = \beta R W'\left(b\right) \frac{1+\frac{\beta R V' u''}{(u')^2}C^{i\prime}}{1-\frac{\beta R V''}{u'}C^{i\prime}} \tag{19}$$

We find:

Proposition 9 (Costly Capital Controls) *If capital controls impose a resource cost $C^i\left(\tau^i\right)$ as defined above and if $\xi^i \neq 0$, then a national planner imposes an optimal level of capital controls of the same sign as ξ^i but of smaller absolute magnitude, i.e. τ^i satisfies $0 < \left|\tau^i\right| < \left|\xi^i\right|$.*

Proof. The planner implements the optimality condition (19) by setting the capital control in the decentralized optimality condition (3) to

$$\tau^i = \frac{\beta R \xi^i}{u'\left(c^i\right)} + \beta R C^{i\prime} \cdot \frac{V' u'' + u' V''}{(u')^2 + \beta R V' u'' C'}$$

The first additive term corresponds to the optimal costless capital controls $\tilde{\tau}^i$. If this term is positive because the country experiences a negative externality $\xi^i > 0$ from capital inflows, then $C^{i\prime} > 0$ and the second additive term is negative, which mitigates the optimal magnitude of the capital control to $\tau^i < \tilde{\tau}^i$. (This holds as long as the denominator is positive, i.e. $(u')^2 + \beta R V' u'' C' > 0$, which is satisfied as long as the marginal cost of the capital control C' is not too large.) For $\xi^i > 0$, the second term never flips the sign of the control τ^i to make it negative. If it did, then $C^{i\prime}$ would switch sign as well and the second term would become positive, leading to a contradiction. The argument for $\xi^i < \tau^i < 0$ follows along the same lines. ■

5.2 Global Coordination of Costly Capital Controls

We next determine under what conditions the equilibrium in which each national planner imposes capital controls according to equation (19) is globally Pareto efficient. In other words, if national planners follow the described rule, can a global planner achieve a Pareto improvement on the resulting equilibrium? It turns out that the answer depends critically on the set of instruments available to the planner.

5.2.1 Global Coordination with Transfers

First, we analyze a global planner who maximizes global welfare in the described environment who has access to lump-sum transfers between countries. This implies that he is not bound by the period 1 budget constraints of individual countries and can undo the redistributions that stem from changes in the world interest rate.

Formally, a global planner maximizes the sum of the surplus of all nations for some set of welfare weights $\{\phi^i\}$. He internalizes that the world interest rate R is a choice variable and that the optimality conditions of individual agents (with shadow price μ^i) as well as global market clearing must hold, i.e. $\Sigma_i m^i b^i = 0$ (with shadow price ν). In addition, we include a transfer T^i in our optimization problem, which needs to satisfy global market clearing $\Sigma_i m^i T^i = 0$ (with shadow price γ). The associated Lagrangian is

$$\mathcal{L} = \sum_i \phi^i \left\{ u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i \left[c^i - y^i + b^i/R + C^i\left(\tau^i\right) - T^i\right] - \right.$$
$$\left. -\mu^i \left[\left(1-\tau^i\right) u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right] \right\} - \nu \sum_i m^i b^i - \gamma \sum_i m^i T^i$$

The first-order conditions of the global planner are

$$\begin{aligned}
FOC\left(b^i\right) &: \beta W^{i\prime}\left(b^i\right) = \lambda^i/R - \mu^i \beta R V^{i\prime\prime}\left(b^i\right) + m^i \nu/\phi^i \\
FOC\left(c^i\right) &: u'\left(c^i\right) = \lambda^i + \mu^i\left(1-\tau^i\right) u''\left(c^i\right) \\
FOC\left(\tau^i\right) &: \lambda^i C^{i\prime}\left(\tau^i\right) = \mu^i u'\left(c^i\right) \\
FOC\left(R\right) &: \sum_i \phi^i \left\{ \frac{\lambda^i b^i}{R} + \mu^i\left(1-\tau^i\right) u'\left(c^i\right) \right\} = 0 \\
FOC\left(T^i\right) &: \phi^i \lambda^i = \gamma m^i
\end{aligned}$$

The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights $\{\phi^i\}$ such that the allocations of national planners satisfy the maximization problem of the global planner. If we substitute the allocations from the Nash equilibrium, we find that the second and third optimality conditions are unchanged compared to the national planner's equilibrium and can be solved for λ^i and μ^i that are identical to the shadow prices in the Nash equilibrium between national planners. Substituting these in the optimality condition $FOC\left(b^i\right)$, we find that this condition is satisfied for all countries if we set $\nu = 0$. The fifth optimality condition is satisfied if we set $\phi^i = \gamma m^i/\lambda^i \forall i$. The Nash equilibrium among planners is therefore efficient if the described variables also satisfy the fourth optimality condition $FOC\left(R\right)$.

Proposition 10 (Coordination of Costly Controls with Transfers) *If capital controls to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a global planner* *who can engage in transfers if and only if the resulting allocation satisfies*

$$\sum_i m^i \left(1-\tau^i\right) C'\left(\tau^i\right) = 0 \tag{20}$$

Proof. The optimality condition (20) can be obtained by substituting $FOC\left(T^i\right)$ into the condition $FOC\left(R\right)$ and accounting for market clearing $\Sigma_i m^i b^i = 0$ as well as for $FOC\left(\tau^i\right)$. ∎

In a Pareto-optimal allocation, the weighted average marginal distortion imposed by capital controls must be zero. If there are no externalities, this can be achieved by having zero controls in all countries. Otherwise, the planner combines controls in capital inflow and outflow countries in a way that their weighted average marginal distortion is zero.

The planner's country weights ϕ^i do not show up in condition (20) since the condition is purely about efficiency, i.e. about minimizing the overall resource cost of imposing capital controls. Since the planner has access to lump-sum transfers, she can undo any redistributions created by movements in the interest rate according to her welfare weights.

We illustrate our findings in the following examples:

Example 1: Single country/symmetric countries Assume a world economy that consists of $k \geq 1$ identical countries that impose costly capital controls $0 < \tau^i < \xi$ to offset domestic externalities. In doing so they incur a resource cost $C^i\left(\tau^i\right) > 0$. However, since they are identical, their net bond positions is $b^i = 0$ in equilibrium. There is a clear scope for reducing capital controls to zero and avoiding the resource cost, making all countries better off. Observe that the reduction in capital controls leads to a parallel increase in the world interest rate R.

Analytically, since all countries are symmetric, the only non-degenerate solution to equation (20) is $C^{i\prime}\left(\tau^i\right) = 0 \forall i$. A global planner would reduce the controls in all countries to zero.

Example 2: Two countries with asymmetric externality Assume two countries that are identical, except that one of them experiences a negative externality from selling bonds $\xi > 0$. In the Nash equilibrium of national planners, country i imposes a capital control $0 < \tau^i < \xi$ (the inequality holds because capital controls are costly) and country j doesn't. Country i therefore experiences capital outflows and incurs a resource cost $C\left(\tau^i\right) > 0$, whereas country j receives capital inflows. The decentralized equilibrium is inefficient and the optimality condition (20) is not satisfied.

The global planner would lower the capital control $\tau^i > 0$ on inflows in country i and impose a control on outflows $\tau^j < 0$ in country j to minimize the total resource cost $C\left(\cdot\right)$ of controls. This would increase the world interest rate, but the planner can undo the resulting redistribution to make sure that a Pareto-improvement takes place.

Example 3: Two countries, restricted instruments Let us add to the previous example a restriction that country j cannot use its capital control instrument so $\tau^j \equiv 0$. The Nash equilibrium of national planners is unaffected since country j already found it optimal to impose a zero capital control.

Analytically, we may capture the restriction $\tau^j \equiv 0$ by assuming that it is arbitrarily costly for country j to deviate from zero capital controls, e.g. $C^j(\tau^j) = \alpha(\tau^j)^2$ with $\alpha \to \infty$. In the limit, the optimality condition (20) is satisfied for the allocation in the Nash equilibrium among national planners. The global planner balances the marginal cost of changing the capital control in countries i and j which requires $m^i(1-\tau^i)C^{i\prime}(\tau^i) = -m^j(1-\tau^j)C^{j\prime}(\tau^j)$ and which holds for $\tau^j \to 0$. The intuition for the result is that there is no scope for sharing the burden of regulation with country j if it is infinitely expensive for country j to impose even minimal capital controls.

Example 4: More countries Observe that all our examples on coordination continue to hold for the case of more than two countries. In particular, equation (20) weighs each country by its mass m^i in the world economy and adds up the marginal distortion it experiences. If we replace one country of mass m^i by k identical countries of mass m^i/k, then the national planner in each of these countries will find it optimal to choose precisely the same allocation as the one in the large country of mass m^i. Moreover, the global planner treats the sum of the k small countries in the same fashion as the one large country. This is because we assumed that the planners in the current section do not exert market power. (The case of $k > 1$ in example 1 is an application of this finding.)

In summary, our examples illustrate that there may be a rationale for global coordination of capital controls if such controls impose deadweight costs that can be reduced by sharing the burden of controlling capital flows and if a global planner can engage in compensatory transfers. The goal of coordination is to minimize the aggregate deadweight loss from capital controls by distributing the burden of imposing controls between borrowing and lending countries.

5.2.2 Global Coordination without Transfers

If the global planner cannot engage in transfers between the countries involved, then the conditions under which a global Pareto improvement can be achieved are highly restrictive.

A global planner maximizes the sum of the surplus of all nations for some set of welfare weights $\{\phi^i\}$. He internalizes that the world interest rate R is a choice variable and that global market clearing must hold, i.e. $\Sigma_i m^i b^i = 0$ (with shadow price ν):

$$\mathcal{L} = \sum_i \phi^i \left\{ u\left(c^i\right) + \beta W^i\left(b^i\right) - \lambda^i \left[c^i - y^i + b^i/R + C^i\left(\tau^i\right)\right] - \right.$$
$$\left. -\mu^i \left[\left(1-\tau^i\right) u'\left(c^i\right) - \beta R V^{i\prime}\left(b^i\right)\right] \right\} - \nu \sum_i m^i b^i$$

The first-order conditions of the global planner are the same as the first four conditions above. The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights $\left\{\phi^i\right\}$ such that the allocations of national planners satisfy the maximization problem of the global planner.

If we substitute the allocations from the Nash equilibrium, we find as before that the first three optimality condition are satisfied for all i if we set $\nu = 0$. The fourth optimality condition captures the effects of varying the world interest rate, which has both a redistributive effect that depends on the sign of b^i/R and an effect on the tightness of the implementability constraint μ^i of each country. We can reformulate this optimality condition as

$$\sum_i \phi^i \lambda^i \left\{ \frac{b^i}{R} + \left(1-\tau^i\right) C^{i\prime}\left(\tau^i\right) \right\} = 0 \tag{21}$$

Proposition 11 (Coordination of Costly Controls, No Transfers) *If capital controls to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a constrained global planner who cannot engage in transfers as long as either (i) there is no trade or (ii) there is at least one borrower and one lender for whom the marginal distortions imposed by costly capital controls are smaller than the country's bond positions, i.e.* $\left(1-\tau^i\right) C^{i\prime}\left(\tau^i\right) < -b^i/R$ *for borrowers and* $-\left(1-\tau^i\right) C^{i\prime}\left(\tau^i\right) < b^i/R$ *for lenders.*

Proof. The optimality condition (21) has a non-trivial solution with non-degenerate welfare weights $\phi^i > 0 \forall i$ if and only if the term $\left\{\frac{b^i}{R} + \left(1-\tau^i\right) C^{i\prime}\left(\tau^i\right)\right\}$ is either zero for all countries or is positive for some and negative for other countries. ■

These conditions are typically satisfied since some countries are lenders $b^i > 0$, others are borrowers $b^i < 0$, and since the capital control τ^i and the marginal distortion $C^{i\prime}$ are small in absolute value.

Inefficiency arises if the capital controls of national planners impose significant marginal costs $C^{i\prime}$ compared to the amount of borrowing/lending b^i/R that agents engage in. We provide two examples in which this may be the case:

Example 5: Single country/symmetric countries Returning to our earlier example of $k \geq 1$ identical countries with costly capital controls $0 < \tau^i < \xi$, we observe that their net bond positions are $b^i = 0$ in equilibrium. Therefore a coordinated reduction in capital controls and the associated increase in the world interest rate do not have redistributive effects. A planner can achieve a Pareto improvement without engaging in transfers by reducing capital controls to zero.

Analytically, we observe that in the Nash equilibrium between national planners, $b^i = 0$ and $C^{i\prime}(\tau^i) > 0 \forall i$. Therefore the only solution to the optimality condition (21) is the degenerate solution $\phi^i = 0 \forall i$. The allocation therefore cannot be the outcome of the constrained planner's optimization and is constrained inefficient.

Example 6: Highly distortive capital controls Assume a world economy that consists of a borrowing country i with $b^i < 0$ and a lending country j of the same size with $b^j = -b^i > 0$. The national planner in the borrowing country is subject to an externality ξ^i and corrects it using a capital control τ^i that is so highly distortive that $(1 - \tau^i) C^{i\prime}(\tau^i) > -b^i/R$. By contrast, the lender does not suffer from externalities and sets $\tau^j = 0$. In the described situation, a global planner recognizes that both countries would be better off if she reduces the capital controls in both countries in parallel. ($\tau^j < 0$ for the lending country j amounts to an export tax on capital.) This policy reduces the marginal distortion $C^{i\prime}(\tau^i)$ in country i while introducing a small distortion in country j. (Recall that C^i is convex.) However, in general equilibrium the parallel reduction in capital controls pushes up the world interest rate R, which benefits country j. In country i, the cost of the increase in the interest rate is offset by the reduced distortion since $(1 - \tau^i) C^{i\prime}(\tau^i) > -b^i/R$. Therefore both countries are better off.

Analytically, all the terms in the curly brackets of condition (21) are positive so that the only solution is degenerate, $\phi^i = 0 \forall i$. The allocation therefore cannot be the outcome of the constrained planner's optimization and is constrained inefficient.

Example 7: Modestly distortive capital controls We continue to assume that country j is a lender with zero capital controls and country i is a borrower that imposes a capital control $\tau^i > 0$, but that the distortion arising from the capital control is more modest, i.e. $(1 - \tau^i) C^{i\prime}(\tau^i) < -b^i/R$. This is plausible if we believe that the marginal cost of capital controls is less than the stock of foreign capital that a country is borrowing. Then the term in curly brackets in optimality condition (21) is negative for the borrower and positive for the lender. It is clear that we can find welfare weights ϕ^i and ϕ^j such that the optimality condition is satisfied and we can conclude that the capital control imposed by the national planner in country i is constrained Pareto efficient.

Observe that a critical element of proposition 11 is that it is sufficient to achieve constrained Pareto efficiency if there is a single borrowing and a single lending country in the world economy without large externalities or without large distortions from capital controls. As long as this is the case, there will be a loser for any policy that shifts the world interest rate, and it is impossible for a global planner to achieve a Pareto improvement.

5.3 Imperfectly Targeted Capital Controls

We now introduce the possibility that a planner cannot perfectly target different forms of capital flows and study the implications for the desirability of international coordination in the setting of capital controls.

For simplicity, we use our earlier state-contingent setup and assume that there are two states of nature $\omega = L, H$ at $t = 1$ with probabilities π^ω and two securities b^ω that are contingent on these two states, but the planner in each country i has only one capital control instrument τ^i that equally applies to both. One possible interpretation of the two securities is that security L represents a payoff in a low state of nature in which additional insurance mandated by the planner is desirable and H represents a payoff in a high state in which no insurance is necessary.

5.3.1 Single Country Problem

$$\max_{b^{i,\omega},c^i,\tau^i} u\left(c^i\right)+\beta E\left[W^\omega\left(b^{i,\omega}\right)\right]-\lambda^i\left[c^i+\Sigma_\omega q^\omega b^{i,\omega}-y^i\right]-\sum_\omega \mu^{i,\omega}\left[1-\tau^i-\frac{\beta\pi^\omega V^{\omega\prime}\left(b^{i,\omega}\right)}{q^\omega u'\left(c^i\right)}\right] \tag{22}$$

The first-order conditions are

$$\begin{aligned}
FOC\left(c^i\right) &: \lambda^i = u'\left(c^i\right)-\left(1-\tau^i\right)\frac{u''\left(c^i\right)}{u'\left(c^i\right)}\sum_\omega \mu^{i,\omega} \\
FOC\left(b^{i,\omega}\right) &: q^\omega\lambda^i = \pi^\omega\beta W^{\omega\prime}\left(b^{i,\omega}\right)+\mu^{i,\omega}\left(1-\tau^i\right)\frac{V^{\omega\prime\prime}\left(b^{i,\omega}\right)}{V^{\omega\prime}\left(b^{i,\omega}\right)} \\
FOC\left(\tau^i\right) &: \sum_\omega \mu^{i,\omega}=0
\end{aligned}$$

The first condition captures that the marginal utility of wealth is equal to the marginal utility of consumption plus the benefit of relaxing the planner's implementability constraints that stems from consumption. Combining this condition with the third condition yields $\lambda^i = u'(c^i)$. The second condition is the Euler equation for state ω, by which the planner equates the marginal cost of saving in state ω (lhs) to the marginal social benefit (first term on rhs) plus the effects on the implementability constraint in state ω. It can be reformulated to express the shadow value

$$\mu^{i,\omega}=\frac{q^\omega u'(c^i)-\pi^\omega\beta W^{\omega\prime}\left(b^{i,\omega}\right)}{(1-\tau^i)\,V^{\omega\prime\prime}\left(b^{i,\omega}\right)/V^{\omega\prime}\left(b^{i,\omega}\right)}$$

If private agents save too much in that state compared to what is optimal in the first-best, $q^\omega u'(c^i) > \pi^\omega \beta W^{\omega\prime}$, then the shadow price $\mu^{i,\omega}$ is negative, indicating that it is desirable to increase the capital control from the perspective of this state; if they save less than optimal, then the shadow price in that state is positive. The third optimality condition states the planner sets the capital control τ^i such that saving is on average at the right level, as indicated by these $\mu^{i,\omega}$'s.

As in the case of costly capital controls, there are two variants of the global planning problem that we can solve. The first variant corresponds to the traditional test for Pareto efficiency, in which a planner is only concerned about the efficiency implications of her actions not the redistributive effects. In this setup, we allow the planner to have access to lump-sum transfers and ask if we can find weights ϕ^i such that the global planner's solution replicates the allocations in the decentralized equilibrium. If such weights can be found, then we call the decentralized equilibrium Pareto efficient.

5.3.2 Global Coordination with Transfers

[tk]

5.3.3 Global Coordination without Transfers

In the second variant, we assume that the planner does not have access to compensatory transfers. This may better reflect the reality of our global system of governance, in which countries rarely compensate each other for the international effects of their policy actions.

We setup a global planning problem without compensatory transfers to determine if there is scope for international cooperation in the setting of imperfectly targeted capital controls. We assume that the global planner places weight ϕ^i on the objective of each country i as described in problem (22) and includes the constraints on global market clearing in each state-contingent security (with multiplier ν^ω),

$$\sum_i m^i b^{i,\omega} = 0 \forall \omega$$

If the planner has access to precisely the same set of instruments as decentralized agents but internalizes the endogeneity of the prices q^ω, then her objective function is

$$\max_{b^{i,\omega}, c^i, \tau^i, q^\omega} \sum_i \phi^i \Bigg\{ u\left(c^i\right) + \beta E\left[W^\omega\left(b^{i,\omega}\right)\right] - \lambda^i \left[c^i + \Sigma_\omega q^\omega b^{i,\omega} - y^i\right] - \\ - \sum_\omega \mu^{i,\omega} \left[1 - \tau^i - \frac{\beta \pi^\omega V^{\omega\prime}\left(b^{i,\omega}\right)}{q^\omega u'\left(c^i\right)}\right] \Bigg\} + \sum_i m^i \sum_\omega \nu^\omega b^{i,\omega}$$

The global planner's optimality conditions are

$$
\begin{aligned}
FOC\left(c^i\right) &: \lambda^i = u'\left(c^i\right) - \left(1-\tau^i\right)\frac{u''\left(c^i\right)}{u'\left(c^i\right)}\sum_\omega \mu^{i,\omega} \\
FOC\left(b^{i,\omega}\right) &: q^\omega \lambda^i = \pi^\omega \beta W^{\omega\prime}\left(b^{i,\omega}\right) + \mu^{i,\omega}\left(1-\tau^i\right)\frac{V^{\omega\prime\prime}\left(b^{i,\omega}\right)}{V^{\omega\prime}\left(b^{i,\omega}\right)} + m^i\nu^\omega/\phi^i \quad \forall\omega \\
FOC\left(\tau^i\right) &: \sum_\omega \mu^{i,\omega} = 0 \\
FOC\left(q^\omega\right) &: \sum_i \phi^i\left[\lambda^i b^{i,\omega} + \mu^{i,\omega}\frac{1-\tau^i}{q^\omega}\right] = 0 \quad \forall\omega
\end{aligned}
$$

We find the following result:

Proposition 12 *The decentralized equilibrium in our problem with imperfect targeting is Pareto efficient if*

$$xyz$$

Proof. The decentralized equilibrium is Pareto efficient if we can find a set of $\left\{\phi^i\right\}$ such that the allocations of the decentralized equilibrium satisfy the optimality conditions of the global planner.

[to be completed]

Combining the first and the third condition we find $\lambda^i = u'\left(c^i\right)$, as we did in the decentralized equilibrium. This allows us to express the new (fourth) optimality condition as

$$\sum_i \phi^i\left[q^\omega b^{i,\omega}u'\left(c^i\right) + \mu^{i,\omega}\left(1-\tau^i\right)\right] = 0$$

We sum this equation over all states ω, and using the third optimality condition we obtain

$$\sum_i \phi^i u'\left(c^i\right)\cdot\sum_\omega q^\omega b^{i,\omega} = 0$$

We need to set $\phi^i = m^i/u'\left(c^i\right)$ for this condition to be satisfied by global market clearing condition.

We substitute these welfare weights back into the optimality condition $FOC\left(q^\omega\right)$ and find that the first term drops out by market clearing. The equilibrium is Pareto-efficient if

$$\sum_i \frac{m^i\mu^{i,\omega}\left(1-\tau^i\right)}{u'\left(c^i\right)} = 0 \quad \forall\omega$$

■

6 Externalities from Financial Constraints

In our final section we study the global effects of prudential capital controls that are imposed to combat financial instability, as in Korinek (2010). We describe a world economy in infinite discrete time $t = 0, ...$ with $I \geq 2$ countries indexed by $i = 1, ...I$ of mass m^i each such that $\Sigma_i^I m^i = 1$. Within each country, there is a unit mass of identical consumers with utility function

$$U^i = \sum_{t=0}^{\infty} \beta^t u\left(c_{T,t}^i, c_{N,t}^i\right) \tag{23}$$

where $\beta < 1$ is a time discount factor and $u(c_T, c_N)$ is the consumer's period utility over traded and non-traded goods consumption. We denote the partial derivatives of this function as $u_T = \partial u(c_T, c_N)/\partial c_T$ and similar for u_N, u_{NT} etc. and impose the assumptions $u_T > 0 > u_{TT}$, $u_N > 0 > u_{NN}$ and $u_{NT}u_T - u_N u_{TT} > 0$, i.e. the two goods are complements or at most mild substitutes.[14]

Consumers enter period t with bond holdings b_t^i, obtain a pair of endowments $\left(y_{T,t}^i, y_{N,t}^i\right)$ and choose how much to consume and carry into the following period, leading to a budget constraint

$$c_{T,t}^i + p_t^i c_{N,t}^i + \frac{\left(1 - \tau_{t+1}^i\right) b_{t+1}^i}{R_{t+1}} = y_{T,t}^i + p_t^i y_{N,t}^i + b_t^i - T_t^i \tag{24}$$

We denote by p_t^i the relative price of the non-traded good in country i in terms of the traded (numeraire) good, which constitutes a measure of the real exchange rate.[15] Observe that we index p_t^i by country i since the relative prices of the non-traded goods of different countries are in general different.

The policy instrument τ_{t+1}^i is a subsidy to bond purchases (capital outflows) or, if b_{t+1}^i is negative, a tax on bond sales (capital inflows). Given the current account identity, we can equivalently interpret τ_{t+1}^i as a net export subsidy or, if b_{t+1}^i is negative, a net import tariff. In the following, we will loosely refer to τ_t^i as a "capital control." The government revenue involved is raised/rebated as a lump sum T_t^i so as to make the instrument wealth-neutral. R_{t+1} represents the common world interest rate on risk-free bonds.

Consumers in each country are subject to a commitment problem that limits how much they can borrow from international lenders.[16] Specifically, we follow Korinek

[14] Empirically, Mendoza (1995) and Stockman and Tesar (1995) find that traded and non-traded goods are clear complements.

[15] The official definition of the real exchange rate is the price of a consumption basket of domestic goods expressed in terms of a consumption basket of foreign goods. Ceteris paribus, a rise in the relative price of non-tradables increases the price of a consumption basket of domestic goods, implying a strictly monotonic relationship between the official real exchange rate and our measure p_t^i.

[16] Since all agents within a given economy are identical, there is no domestic bond market.

(2010) in assuming that consumers may threaten to abscond and renegotiate after taking on their new debts b^i_{t+1}/R_{t+1}. If they do so, international lenders can take them to court and seize at most a fraction ϕ of their income, which they convert into traded goods at the prevailing market price p^i_t. To avoid absconding, lenders impose a borrowing constraint

$$\frac{b^i_{t+1}}{R_{t+1}} \geq \frac{\bar{b}^i_{t+1}}{R_{t+1}} = \phi \left(y^i_{T,t} + p^i_t y^i_{N,t} \right) \tag{25}$$

6.1 Optimality Conditions

In the decentralized equilibrium of the economy, the representative consumer chooses $\left(c^i_{T,t}, c^i_{N,t}, b^i_{t+1}\right)$ every period so as to maximize the expectation of her utility (2). Using the short-hand notation $u^i_{T,t} = u_T\left(c^i_{T,t}, c^i_{N,t}\right)$ for the marginal utility of tradable consumption and λ^i_{t+1} for the shadow price on the borrowing constraint in period t, the consumer's Euler equation is

$$\left(1 - \tau^i_{t+1}\right) u^i_{T,t} = \beta R_{t+1} \cdot u_{T,t+1} + \lambda^i_{t+1} \tag{26}$$

This equation defines a supply of bonds function $b^i_{t+1}\left(R_{t+1}; \tau^i_{t+1}\right)$ that is strictly increasing in the interest rate R_{t+1} and in the capital control τ^i_{t+1} under assumption 1.

The consumer's first order condition on non-tradable consumption implies

$$p^i_t = \frac{u^i_{N,t}}{u^i_{T,t}} \tag{27}$$

The real exchange rate equals the marginal rate of substitution between traded and non-traded goods, which is strictly increasing in $c^i_{T,t}$ by our assumptions on the period utility function. Observe that market clearing for non-traded goods requires that $c^i_{N,t} = y^i_{N,t}$ every period so that $c^i_{T,t}$ is the only endogenous variable affecting the real exchange rate.

6.2 Global Equilibrium

Let us define the global excess supply of bonds as a function of the world interest rate R_{t+1} and the vector $\tau_{t+1} = \left\{\tau^i_{t+1}\right\}^I_{i=1}$ of capital controls as

$$B_{t+1}\left(R_{t+1}; \tau_{t+1}\right) = \sum_{i=1}^{I} m^i b^i_{t+1}\left(R_{t+1}; \tau^i_{t+1}\right)$$

which is – by our earlier assumptions – strictly increasing.[17]

[17] This condition constitutes the analogon of the Marshall-Lerner condition in our framework.

Definition 2 *For a given series of vectors* $\{\tau_t\}$*, the decentralized equilibrium of the world economy is given by a series of interest rates* $\{R_t\}$ *that solve the bond market clearing condition*

$$B_{t+1}\left(R_{t+1};\tau_{t+1}\right)=0 \quad \forall \quad t$$

together with individual country bond positions $\left\{b^i_{t+1}\right\}_{i=1}^{I}$ *that satisfy* $b^i_{t+1}=b^i_{t+1}\left(R_{t+1};\tau^i_{t+1}\right)$.

6.3 Real Exchange Rate and Market Power

Let us now replicates our analysis of monopolistic concerns as a rationale for capital controls for the described setup. For simplicity, we assume without loss of generality that financial constraints are loose and can be ignored throughout this section.

Suppose that a domestic planner in country i chooses τ^i_{t+1} in period t so as to maximize the welfare of her representative consumer, while taking the vector of capital controls imposed by other countries $\tau^{-i}_{t+1}=\left\{\tau^j_{t+1}\right\}_{j\neq i}$ as given. If $m^i>0$, the planner internalizes that she has market power over the world interest rate. Global market clearing requires that world-wide savings add up to zero,

$$m^i b^i_{t+1}+B^{-i}_{t+1}=0 \qquad \text{where} \qquad B^{-i}_{t+1}=\sum_{j\neq i} m^j b^j_{t+1}$$

B^{-i}_{t+1} denotes the bond holdings of the rest-of-the-world excluding country i. We can express these as a function $B^{-i}_{t+1}\left(R_{t+1};\tau^{-i}_{t+1}\right)$ that is strictly increasing in R_{t+1}. We invert this function to obtain the inverse rest-of-the-world bond supply $R^{-i}_{t+1}\left(B^{-i}_{t+1};\tau^{-i}_{t+1}\right)$, which is also strictly increasing in B^{-i}_{t+1} and in each element of τ^{-i}_{t+1}.

Global bond market clearing requires $B^{-i}_{t+1}=-m^i b^i_{t+1}$. The planner therefore recognizes that $R_{t+1}=R^{-i}_{t+1}\left(-b^i_{t+1};\tau^{-i}_{t+1}\right)$. In recursive formulation, we denote the resulting optimization problem as

$$V^i\left(b^i_t\right)=\max_{b^i_{t+1}} u\left(y^i_{T,t}+b^i_t-b^i_{t+1}/R^{-i}_{t+1}(-m^i b^i_{t+1};\tau^{-i}_{t+1}),y^i_{N,t}\right)+\beta V^i\left(b^i_{t+1}\right)$$

leading to the generalized Euler equation

$$u^i_{T,t}\cdot\left(1-m^i\eta_{RB^{-i}}\right)=\beta R_{t+1}u^i_{T,t+1} \tag{28}$$

As captured by the term with the elasticity of the interest rate $\eta_{RB^{-i}}=\frac{\partial R_{t+1}}{\partial B^{-i}_{t+1}}\cdot\frac{B^{-i}_{t+1}}{R_{t+1}}$, the planner internalizes that increasing domestic saving b^i_{t+1} requires a reduction in rest-of-the-world saving B^{-i}_{t+1}, which necessitates a fall in the world interest rate. The planner distorts the domestic saving decision to the point where the marginal benefit of manipulating the world interest rate equals the marginal cost of having an unsmooth consumption profile.

Proposition 13 (Market Power and Capital Controls) *A time-consistent domestic planner who optimally exerts market power over the country's terms-of-trade effects in period t imposes a capital control*

$$\tau_{t+1}^{i} = m^{i} \eta_{RB^{-i}} \tag{29}$$

This reduces the country's transactions in absolute value so that the bond position $b_{t+1}^{M,i}$ under market power satisfies $\left|b_{t+1}^{M,i}\right| < \left|b_{t+1}^{i}\right|$.

Proof. The optimal tax τ_{t+1}^{i} ensures that the private optimality condition of consumers (3) replicates the planner's generalized Euler equation (13).

Note that the elasticity of the world interest rate $\eta_{RB^{-i}}$ – and therefore the optimal capital control – is of the same sign as B_{t+1}^{-i} and therefore of the opposite sign as b_{t+1}^{i}. If the country is a net saver in the decentralized equilibrium ($b_{t+1}^{i} > 0$), the planner taxes capital outflows $\tau_{t+1}^{i} < 0$ to reduce saving to $b_{t+1}^{M,i} < b_{t+1}^{i}$ and push up the world interest rate. If the country is a net borrower ($b_{t+1}^{i} < 0$), the planner taxes inflows $\tau_{t+1}^{i} > 0$ to reduce borrowing to $0 > b_{t+1}^{M,i} > b_{t+1}^{i}$ and push down the world interest rate. ■

Corollary 4 (Spillover Effects of Capital Controls) *An increase in the capital control τ_{t+1}^{i} has positive effects on all countries borrowing and negative effects on all countries lending in period t.*

Proof. Observe first that if country i raises its capital control by a marginal unit, this reduces the world interest rate,

$$\frac{dR_{t+1}}{d\tau_{t+1}^{i}} = -\frac{\partial R_{t+1}}{\partial B_{t+1}^{-i}} \cdot m^{i} \frac{\partial b_{t+1}^{i}}{\partial \tau_{t+1}^{i}} < 0 \tag{30}$$

The interest rate in turn affects the welfare of another country j as follows

$$\frac{dW_{t}^{j}}{dR_{t+1}} = u_{T,t}^{j} \cdot \frac{b_{t+1}^{j}}{R_{t+1}^{2}} \gtrless 0 \tag{31}$$

If the country is a net saver ($b_{t+1}^{j} > 0$), it is hurt by a low interest rate and by capital controls in country i. If the country is a net borrower ($b_{t+1}^{j} < 0$), it benefits from a low interest rate and from capital controls in country i. ■

In our benchmark model, the first welfare theorem implies that capital controls that are imposed to exert market power reduce welfare. Specifically, they introduce a distortion into the Euler equation of the country that imposes the controls, which imposes a welfare cost on that country, in order to manipulate the world interest rate and shift welfare from foreigners to domestic agents.

6.4 Welfare Effects of Financial Constraints

This section analyzes the welfare effects of financial constraints in our setup. We focus on a country i in which the constraint (25) is binding so that $b^i_{t+1} = \bar{b}^i_{t+1} < 0$ and study the effects of exogenous changes in the level of $\bar{b}^i_{t+1}$ on the world interest rate and on welfare. Since b^i_{t+1} denotes saving, an increase in $\bar{b}^i_{t+1}$ corresponds to a tighter borrowing limit. A tighter borrowing limit has a similar effect on the world interest rate as an increase in capital controls,

$$\frac{dR_{t+1}}{d\bar{b}^i_{t+1}} = -m^i \frac{\partial R_{t+1}}{\partial B^{-i}_{t+1}} < 0$$

The welfare effects of the tighter borrowing limit on welfare in country i are made up by the interest rate effect, which is always positive since the constrained country is a borrower, plus the welfare costs of not being able to borrow as much as they want to, as captured by the wedge λ^i_{t+1} in the Euler equation, which is always negative,

$$\frac{dW^i_t}{d\bar{b}^i_{t+1}} = u^i_{T,t} m^i \eta_{RB^{-i}} - \frac{\lambda^i_{t+1}}{R_{t+1}} \gtrless 0 \tag{32}$$

If the borrowing constraint is laxer than the optimal level of borrowing under market power ($\bar{b}^i_{t+1} < b^{M,i}_{t+1}$), then the first term is larger than the second term, and the constrained country is better off in welfare terms. This is because the tightening of the constraint moves the country closer to the "monopoly" solution. On the other hand, if the borrowing constraint is tighter than the optimal level of borrowing under market power ($\bar{b}^i_{t+1} > b^{M,i}_{t+1}$), then the welfare cost of the constraint λ^i_t is greater than the interest rate effect, and the country is worse off.

The spillover effects of a change in the world interest rate on other countries are identical to those detailed in equation (31) in the section on market power – a tightening constraint in country i improves the welfare of other borrowing countries (who compete for funds) and reduces the welfare of lending countries (who experience a decline in the effective demand for their lending).

6.5 Global Planner Restoring First-Best Allocation

Next we study the policy responses that a global planner can take in the face of binding constraints. We show that under certain circumstances, a global planner can restore the first-best equilibrium in our setup.

Proposition 14 (Restoring the First-Best) *In a world with two countries i, j that are subject to the financial constraint (25), a global planner who can determine the capital controls τ^i_t, τ^j_t of both countries can implement the first-best equilibrium.*

The result arises because the planner can take advantage of an indeterminacy in the setting of capital controls and the world interest rate across countries. We can rewrite the Euler equation (3) of decentralized agents as

$$u_{T,t} = \beta \frac{R_{t+1}}{1-\tau_{t+1}^i} \cdot u_{T,t+1} + \lambda_{t+1}^i \qquad \forall i, t$$

What matters for the decisions of decentralized agents is the fraction $\frac{R_{t+1}}{1-\tau_{t+1}^i}$ not the level of the interest rate and the capital controls themselves. Any given real allocation that arises under a set of interest rates and capital controls $\{R_t, 1-\tau_t\}$ can be replicated by an alternative set $\{xR_t, x(1-\tau_t)\}$ for an arbitrary scaling factor $x > 0$. This provides the planner with a degree of freedom to manipulate the level of the variable b_{t+1}^i that is subject to the financial constraint.

Proof. Denote variables in the first-best equilibrium with no capital controls by $\{c_t^{*i}\}$, $\{b_t^{*i}\}$ and $\{R_t^*\}$ and focus on an arbitrary period t in which the first-best level of new borrowing b_{t+1}^{*i}/R_{t+1}^* in country i is by Δ below what the financial constraint permits. The planner implements the first-best allocation by reducing both the period t repayment and new borrowing by Δ, i.e. by setting $b_t^i = b_t^{*i} + \Delta$ and $b_{t+1}^i/R_{t+1} = b_{t+1}^{*i}/R_{t+1}^* + \Delta$, which leaves period t consumption unchanged. (Recall that borrowing is captured by $b_{t+1}^i < 0$ in our framework.) At the same time, the planner uses his control over the interest rates R_t and R_{t+1} to keep borrowing in the previous period $b_t^i/R_t = b_t^{*i}/R_t^*$ and the repayment next period $b_{t+1}^i = b_{t+1}^{*i}$ constant at the first-best levels, which guarantees that consumption for both countries in all time periods is unchanged. Substituting the latter two equations into the former two, we find

$$R_t = R_t^* \cdot \underbrace{\frac{b_t^{*i} + \Delta}{b_t^{*i}}}_{<1} \qquad \text{and} \qquad R_{t+1} = R_{t+1}^* \cdot \underbrace{\frac{b_{t+1}^{*i}/R_{t+1}^*}{b_{t+1}^{*i}/R_{t+1}^* + \Delta}}_{>1}$$

In other words, the planner reduces the world interest rate for repayments and increases it proportionately for new borrowing in period t. To achieve this, she imposes capital controls

$$\tau_t = -\frac{\Delta}{b_t^*} > 0 \quad \text{and} \quad \tau_{t+1} = \frac{\Delta}{b_{t+1}^{*i}/R_{t+1}^* + \Delta} < 0$$

By engaging in this manipulation in a given period t, the planner can circumvent any level of the borrowing constraint that satisfies $\bar{b}_{t+1}^i < 0$. The intervention can be repeated for arbitrarily many periods. ■

Intuitively, we can interpret this form of global cooperation as follows: in period $t-1$, both countries agree to impose capital controls $\tau_t > 0$ (i.e. controls on inflows in the borrowing country and subsidies to outflows in the lending country) to push down the world interest rate and "help" country i, which would otherwise be constrained in the following period, to reduce its need for borrowing b_t^i (or, more precisely, to issue a smaller face value of debt b_t^i for a given amount of funds borrowed b_t^i/R_t). In period t, both countries agree to impose capital controls in the opposite direction (i.e. subsidies on inflows in the borrowing country and taxes on outflows in the lending country) to push up the world interest rate and make up for the loss in interest payments that the lending country would otherwise have suffered.

Multiple countries The result relies on the planner's ability to keep consumption in all countries unchanged at the first-best level while proportionately scaling down the repayment and new borrowing of the country that is constrained in period t. In the case of multiple countries, this can can only be done if the ratio of their repayment and new borrowing is the same, i.e. if the following condition is met:

$$b_t^i/b_{t+1}^i = b_t^j/b_{t+1}^j \forall i, j \tag{33}$$

In our two-country example this condition is naturally fulfilled since $b_t^i = -b_t^j \ \forall \ t$. However, if there are more than two countries and they are subject to idiosyncratic shocks, this condition may no longer be satisfied.

Robustness to Alternative Specifications of Constraint Observe that proposition 14 does not rely on the specific form of the financial constraint. In particular, the same reasoning could be applied if the interest rate was omitted in the denominator of the constraint (25). What matters is the the planner can change the amount borrowed b_t/R_t and the amount repaid b_t independently because she can freely choose the level of the interest rate R_t.

Generalization to Risk In a multi-state world with two countries in which contingent securities b_{t+1}^ω for each state of nature $\omega \in \Omega$ exist, a planner can still implement the first-best equilibrium by following the recipe of proposition 14. She would focus on reducing the payoffs of contingent liabilities of the borrowing country i that pay out in those states of nature when the constraint is binding (e.g. "hard" dollar debt) by imposing capital controls $\tau_{t-1}^\omega > 0$ on such securities (i.e. controls on inflows in the recipient country and subsidies to outflows in the source country). This reduces the need for new financing in country i if one of those states of nature materializes. In that event, the planner would subsequently impose capital controls in the opposite direction on *all* securities (i.e. subsidies on inflows in the recipient country and on outflows in source country) to push up the world interest rate and compensate the source country for the lower returns in the prior period. If a different state of nature materializes in which there is no risk of binding constraints, the planner would take no further action in period t. Again, the resulting real allocations replicate the first-best.

The first-best interventions that we described in this section are ultimately a device to circumvent a country's financial constraint in a given period by funneling more resources to the country in period t and receiving a larger repayment in period $t+1$ than in the decentralized equilibrium with binding constraints. In the given setup, the planner achieves these transfers through manipulations in the world interest rate. These may be difficult to implement in practice as they require a significant amount of international coordination and commitment. Furthermore, they can only restore the first-best if condition 33 is met, which is unlikely in practice. However, similar effects could be obtained through "crisis lending" – if lenders create an institution with a superior enforcement technology that can provide a constrained country with additional finance beyond what the financial constraint (25) permits.

6.6 Prudential Capital Controls

Given the practical difficulty of using tax-cum-subsidy schemes to restore the first-best equilibrium, most of the discussion on interventions in international capital markets has focused on what we may call prudential capital controls, i.e. policy interventions that reduce capital inflows into countries during booms as a second-best devide to mitigate outflows during busts when financial constraints become binding (see e.g. Korinek, 2011). This section focuses on the general equilibrium effects of this type of capital controls.

We focus on a country i that is a borrower in periods $t-1$ and t in which the constraint (25) is binding loose in period $t-1$ (corresponding to good times) and binding in period t so that $b^i_{t+1} = \bar{b}^i_{t+1} < 0$ (corresponding to bad times). We study the effects of a "prudential" intervention that consists of reducing borrowing in period $t-1$ as a second-best device to mitigate the severity of the binding constraint in that period,

$$\begin{aligned} \frac{\partial \bar{b}^i_{t+1}}{\partial b^i_t} &= -\phi R_{t+1} p'\left(c^i_{T,t}\right) - m^i \eta_{RB^{-i}_{t+1}} \cdot \frac{\partial \bar{b}^i_{t+1}}{\partial b^i_t} = \\ &= -\frac{\phi R_{t+1} p'\left(c^i_{T,t}\right)}{1 + m^i \eta_{RB^{-i}_{t+1}}} < 0 \end{aligned}$$

The first line of this expression captures that higher net worth b^i_t increases tradable consumption $c^i_{T,t}$ and the real exchange rate by $p'\left(c^i_{T,t}\right)$, which relaxes the constraint (25) on new borrowing $\bar{b}^i_{t+1}/R_{t+1}$. However, higher loan demand pushes up the world interest rate R_{t+1} in the denominator of this expression, which mitigates the effect on $\bar{b}^i_{t+1}$ itself.

We summarize the welfare effects of a prudential intervention by adding up the effects of three welfare-relevant pecuniary externalities, on the two interest rates R_t

and R_{t+1} as well as on the exchange rate p_t^i of the country. This yields

$$\begin{aligned}\frac{dW_{t-1}^i}{db_t^i} &= -u_{T,t-1}^i \cdot \overbrace{\frac{b_t^i}{R_t^2} \cdot \frac{\partial R_t}{\partial b_t^i}}^{-m^i \eta_{RB_t^{-i}}<0} - \overbrace{\frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i}}^{<0} \cdot \beta \frac{dW_t^i}{d\bar{b}_{t+1}^i} = \\ &= u_{T,t-1}^i \cdot m^i \eta_{RB_t^{-i}} - \frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i} \cdot \frac{\beta}{R_{t+1}} \left(\lambda_{t+1}^i - u_{T,t}^i \cdot \eta_{RB_{t+1}^{-i}} \right) \end{aligned} \tag{34}$$

The first term captures that country i benefits from a reduction in the borrowing rate R_t – similar to a country that reduces borrowing for monopolistic reasons. The term $\frac{\partial \bar{b}_{t+1}^i}{b_t^i} < 0$ captures how much a precautionary reduction in borrowing b_t^i relaxes the binding constraint on b_{t+1}^i, which entails the two effects expressed above in equation (32), i.e. the direct positive effect of being less constrained and an indirect negative effect that stems from the increase in the world interest rate from the higher effective world demand for bonds.

The spillover effects on welfare in other (unconstrained) countries $j \neq i$ reflect the sum of the redistributions stemming from changes in the world interest rates,

$$\frac{dW_{t-1}^j}{db_1^i} = u_{T,t-1}^j \cdot \frac{b_t^j}{R_t^2} \frac{\partial R_t}{\partial b_t^i} + \frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i} \cdot \beta u_{T,t}^j \cdot \frac{b_{t+1}^j}{R_{t+1}^2} \frac{\partial R_{t+1}}{\partial b_{t+1}^i}$$

If the country is a net lender, it suffers from the lower demand for borrowing in period $t-1$, but benefits from higher demand for borrowing in period t, and vice versa for countries that are net borrowers. The relative magnitude of the two effects depends, aside from the interest rate elasticities, on the extent of financial amplification $\frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i}$.

Proposition 15 (Pareto-Improving Capital Controls) *In a two country world in which country i is unconstrained in period $t-1$ but experiences binding constraints in period t, prudential capital controls imposed in country i are Pareto-improving if and only if*

$$\underbrace{-\frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i} \cdot \frac{\beta R_t \cdot \lambda_{t+1}^i}{R_{t+1} \cdot u_{T,t-1}^j}}_{\text{efficiency gain to borrower}} \geq \underbrace{-\eta_{RB_t^{-i}} - \frac{\partial \bar{b}_{t+1}^i}{\partial b_t^i} \cdot \frac{\eta_{RB_{t+1}^{-i}}}{R_{t+1}}}_{\text{redistribution to lender}} \geq 0 \tag{35}$$

with at least one strict inequality.

Proof. The first inequality guarantees that the country i imposing the controls experiences a welfare gain, i.e. that the benefit of relaxing the constraint as captured by λ_{t+1}^i surpasses redistributive effects from changes in the world interest rate. The second inequality ensures that these redistributive effects are in favor of the lending country. ■

The relative strength of the two effects depends critically on two magnitudes, λ_{t+1}^{i} and $\frac{\partial \bar{b}_{t+1}^{i}}{\partial b_{t}^{i}}$. The first part of the condition is satisfied if the efficiency gain to the borrower is sufficiently large, i.e. if the capital controls mitigate severely binding constraints. The second part of the condition is satisfied if the gain in interest earnings in period $t+1$ outweighs the loss in period t. This is the case if $\left|\frac{\partial \bar{b}_{t+1}^{i}}{\partial b_{t}^{i}}\right|$ is sufficiently large, i.e. if amplification effects in country i are large. If $\eta_{RB_{t}^{-i}} \approx \eta_{RB_{t+1}^{-i}}$, the second part of the condition is satisfied whenever $\left|\frac{\partial \bar{b}_{t+1}^{i}}{\partial b_{t}^{i}}\right| > R_{t+1}$.

General Sharing of Efficiency Gains A global planner who coordinates the capital control measures of all countries in the world economy can ensure that the efficiency gains, as captured by the term on the left-hand side of condition (35), are spread between a borrowing country i and its lender $j \neq i$ in a way that both parties are better off even if the inequalities in condition (35) are violated. Specifically, the term on the left-hand side (i.e. the efficiency effects of imposing capital controls in a contrained country) are always positive. A global planner can redistribute between borrower and lender by rescaling the capital control measures of both countries $(1-\tau^{i})$ and $(1-\tau^{j})$ in periods t and $t+1$ by a given factor without affecting the levels of borrowing b_{t}^{i} and b_{t+1}^{i} and therefore without reducing exchange efficiency.

A particular example of a Pareto-improvement, which ensures that the borrowing country is strictly better off and all other countries are indifferent at a first-order approximation, is to set the level of capital controls in all countries such that the world interest rate is unchanged. In period $t-1$, this would require increasing the interest rate R_t compared to the levels that prevail under a unilateral policy of imposing prudential capital controls.

Following equation (30), if the optimal unilateral capital control in country i is τ_{t}^{i*}, then a first-order approximation implies that this would change the world interest rate by $\Delta R_t = \tau_{t}^{i*} \cdot \frac{dR_t}{d\tau^*} < 0$. To counteract this decline, the global planner has to rescale the world interest rate up by a factor $1 + \frac{\Delta R_t}{R_t}$, i.e. reduce the control in country i at a first-order approximation to $\tau_{t}^{i} = \tau_{t}^{i*} - \frac{\Delta R_t}{R_t}$ and in all other countries to $\tau_{t}^{j} = -\frac{\Delta R_t}{R_t}$. This corresponds to an outflow tax in lending countries and an inflow subsidy in other borrowing countries. (The outflow tax is comparable to the voluntary export restrictions (VERs) that are sometimes used in trade policy.)

Similarly, in period t, the policy involves reducing the world interest rate R_{t+1} to the pre-intervention level, which requires, at a first-order approximation, an increase in capital controls around the world by

$$\Delta\tau_{t+1}^{j} = \frac{\Delta R_{t+1}}{R_{t+1}} = -\eta_{RB_{t+1}^{-i}} \cdot \frac{\partial \bar{b}_{t+1}^{i}}{\partial b_{t}^{i}} \frac{\partial b_{t}^{i}}{\tau_{t}^{i}} \frac{\tau_{t}^{*i}}{R_{t+1}} > 0$$

This corresponds to an inflow tax in all borrowing countries (including country i) and an outflow subsidy in all lending countries.

Spillover Effects in Multiple Country Framework In the absence of such a global planner, it is generally more difficult to obtain a Pareto improvement from imposing prudential capital controls in a framework with multiple countries. Although the country imposing the controls is better off whenever λ_{t+1}^i is sufficiently large, corresponding to the first inequality in condition (35), the welfare effects on borrowing and lending countries are of opposite signs. If lenders are better off from imposing capital controls in a vulnerable borrowing country, corresponding to the second inequality in condition (35), then other borrowers will necessarily be worse off, and vice versa. However, if all borrowing countries are vulnerable to binding financial constraints, then everybody in the world economy may be better off if they simultaneously impose capital controls. (Proposition 15 can be viewed as an extreme example of this in which there is a mass of identical borrowers and a mass of identical lenders.)

7 Conclusions

This paper has studied the effects of capital controls in a general equilibrium model of the world economy and has delineated under what conditions such controls may be desirable from a global welfare perspective. In our positive analysis, we found that capital controls in one country push down world interest rate and induce other countries to borrow and spend more. We then analyzed three motives for imposing capital controls. If national planners impose capital controls to exert market power and manipulate a country's terms of trade, then they have beggar-thy-neighbor effects and reduce global welfare.

On the other hand, if capital controls are imposed to combat national technological externalities, then controls are Pareto efficient from a global welfare perspective. As long as national policymakers can impose such controls optimally, there is no need for global coordination of such controls as the Nash equilibrium between national planners is socially efficient. Under fairly mild conditions, capital controls that combat national externalities can make everybody in the world economy better off.

If we deviate from the assumption that national policymakers can optimally address externalities, for example, if imposing capital controls has distortionary side-effects or if they cannot perfectly target different types of capital flows, then global policy coordination is desirable. The goal of such coordination is to minimize the aggregate distortions created from capital controls.

Finally, if prudential capital controls are imposed that are designed to mitigate the risk of systemic crises after a surge in capital inflows, we have shown that our insights on technological externalities carry through. In particular, capital controls are Pareto efficient from a global perspective. Under certain circumstances, they may even lead to a global Pareto improvement since they reduce financial instability and create the potential for larger gains from trade in the future.

There are a number of issues that remain for future research. First, we have not considered New Keynesian arguments that capital controls may be useful in aggregate demand management. This opens up an additional set of issues that may be important in short-term macroeconomic management. See e.g. Jeanne (2009) and Farhi and Werning (2012). Secondly, we have assumed throughout that global capital markets function efficiently. Accounting for potential imperfections in global capital markets may lead to additional implications for the desirability of global coordination of capital controls.

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A Mathematical Appendix

A.1 Decentralized demand for bonds

The consumer's Euler equation, after substituting the government budget constraint, defines an implicit function

$$F = \left(1-\tau^i\right) u'\left(y^i - b^i/R\right) - \beta R V^{i\prime}\left(b^i\right)$$

$$\begin{aligned} \text{which satisfies } \frac{\partial F}{\partial b^i} &= -\left(1-\tau^i\right) u''\left(c^i\right)/R - \beta R V^{i\prime\prime}\left(b^i\right) > 0 \\ \frac{\partial F}{\partial R} &= \left(1-\tau^i\right) u''\left(c^i\right) \cdot b^i/R^2 - \beta V^{i\prime}\left(b^i\right) \gtrless 0 \\ \frac{\partial F}{\partial \tau^i} &= -u'\left(c^i\right) < 0 \end{aligned}$$

The first partial derivative is always positive, allowing us to implicitly define a demand function $b^i(R)$.

The second partial derivative is negative as long as saving b^i is sufficiently high. Specifically, we write the condition as

$$\left(1-\tau^i\right) u''\left(c^i\right) \cdot b^i/R^2 - \beta R V^{i\prime}\left(b^i\right)/R < 0$$

We employ the Euler equation to substitute for $V^{i\prime}$ and rearrange to

$$\begin{aligned} b^i/R &> \frac{u'\left(c^i\right)}{u''\left(c^i\right)} \\ \text{or } \frac{b^i/R}{c^i} &> \frac{u'\left(c^i\right)}{c^i u''\left(c^i\right)} = -\sigma\left(c^i\right) \end{aligned}$$

i.e. the savings/consumption ratio is greater than the negative of the elasticity of intertemporal substitution $\sigma(c^i)$, as we stated in assumption 1. If this inequality is satisfied then the demand function $b^i(R)$ is strictly decreasing, which allows us to invert it into a strictly decreasing inverse demand function $R(b^i)$.

The third partial derivative is always negative – this is because we assumed that the revenue from capital controls is rebated so that there are only substitution effects and no income effects from capital controls.