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Does Bitcoin behave as a currency?: A standard monetary model approach *

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Abstract

We derive the Bitcoin exchange rate dynamics by solving the exchange rate equation of the standard flexibleprice monetary model to investigate whether Bitcoin behaves like a currency. The dynamics is driven by an asymmetric mean-reverting fundamental shock which can be attributed to a money demand shock. A crash occurs when the exchange rate breaches a lower boundary where a smooth-pasting condition is imposed. The Bitcoin exchange rate is quasi-bounded at the boundary, and generates skewed distributions consistent with empirical observations. The crash risk increases with a weakened mean-reverting force for the exchange rate. The empirical results show the exchange rate dynamics can be calibrated according to the model, in which the mean reversion of the dynamics is positively co-integrated with the Bitcoin transaction volume indicating demand for Bitcoin; and with the risk reversals of the Australian dollar and Canadian dollar in currency option markets. The analysis based on the monetary model shows that the Bitcoin exchange rate shares some characteristics of a currency with crash risk.

Keywords: Bitcoin, money demand, currency crash, flexible-price monetary model

JEL classification: F31, G13

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1. Introduction

In the past few years, different forms of "cryptocurrencies", notably Bitcoin, have emerged as a medium of exchange and an investment asset. These crypto-currencies normally deploy the Distributed Ledger Technology and have a limited amount of issuance based on "mining" by participants. They can be transferred between participants through the internet without a central clearing agent. There are many trading platforms that allow these cryptocurrencies to be traded.¹ The transaction volumes of Bitcoin have risen rapidly from a daily volume of around 100,000 transactions in 2015 to a peak of 420,000 transactions in 2017. There has been much debate on whether Bitcoin should be considered as a currency (see discussions in Yermack (2015), and Raskin and Yermack (2016) and references therein). While some people have argued that crypto-currencies such as Bitcoin would disrupt or at least seriously challenge the traditional fiat money, there is a lot of uncertainty about how widely Bitcoin will be used as a currency and how serious authorities will regulate it. With such uncertainty, the Bitcoin exchange rate against the US dollar (XBT/USD) has experienced dramatic fluctuations with a drastic surge close to US\$20,000 in December 2017 and a subsequent crash of 60% in March 2018 (see Figure 1, Panel B). Bitcoin is therefore subject to crash risk similar to currency crash risk, which has long been a subject of interest in international finance, found in both developed and developing economies.

This paper proposes a model for the Bitcoin exchange rate dynamics based on the standard flexible-price rate monetary model to investigate whether it behaves like a currency and better understand its exchange rate fluctuations. The solution of the Bitcoin exchange rate in the equation of the model is a function of a stochastic fundamental and market participants' expectations of changes in the Bitcoin exchange rate. Given that Bitcoin's money supply grows at a steady pace, its exchange rate fluctuations are mainly driven by its demand, i.e., its

¹ On other features of Bitcoin, see Nakamoto (2008), Fernandes-Villaverde and Sanches (2016), Chiu and Koeppl (2017) and Saleh (2017).

expected adoption. The approach of the proposed model shares some features of the model developed by Jermann (2018) who studies the driver of Bitcoin's exchange rate fluctuations in a framework following Cagan (1956)'s model of hyperinflation.² This is in the line with linear rational expectations models of exchange rates (see Engel and West (2005) and references therein). Jermann argues that the exchange rate is driven by stochastic adoption and payments technology, as well as endogenous expectations of future changes in the price of Bitcoin, which represent money demand shock for Bitcoin. Given that Bitcoin adoption has grown and changes in the payments technology affect the transaction velocity and Bitcoin's exchange rate, the analysis by Jermann suggests shocks to transaction volumes play a dominant role in explaining Bitcoin exchange rate fluctuations.³

The money demand for Bitcoin is incorporated into the fundamental dynamics in the Bitcoin exchange rate equation by assuming that there is a restoring force for the fundamental which moves towards a mean level proportional to the deviation from the mean. The driving force behind the mean-reverting property can represent the adjustment of the demand for Bitcoin (its adoption), an error-correction or speculative actions taken by market participants to pull the Bitcoin exchange rate back to its long-run equilibrium whenever it drifts too far from the equilibrium. As market participants would take actions more intensively when the Bitcoin exchange rate falls than rises sharply to avoid substantial loss in their Bitcoin investments, the corresponding mean-reverting fundamental shock is likely to be asymmetric with stronger force pushing up the exchange rate away from low levels. Based on rigorous analysis with extensive robustness checks, Gandal et al. (2018) demonstrate that the suspicious trading activity likely caused the unprecedented spike in the Bitcoin exchange rate

² On Bitcoin pricing, see Athey et al (2016), Bolt and van Oordt (2018), and Prat and Walter (2018).

³ Further changes in Bitcoin's payment technology are anticipated (for instance, initiative such as the Lightning Network, or attempts to increase the blockchain size) and they may influence adoption of Bitcoin. The Lightning Network allows more small volume payments through cryptocurrency and new players may come into the space to provide payment services. Blockstream's Liquid sidechain for Bitcoin went live on 10 October 2018 and allows increased transaction speed and size between crypto exchanges, brokers, etc (https://www.coindesk.com/liquid-goes-live-blockstreams-first-bitcoin-sidechain-has-finally-arrived).

in late 2013, when the exchange rate jumped from around US\$150 to more than US\$1,000 in two months (see Figure 1, Panel A), suggesting that in "thin" markets of cryptocurrencies exchange rate manipulation is quite feasible. Those market manipulators could be capable of defending the exchange rate above certain low levels when the demand for Bitcoin falls substantially.

Bitcoin is expected to share another characteristic of a currency - crash risk - as demonstrated by the crash in March 2018. Earlier studies on currency crashes focused more on developing economies where currency crashes occurred due to those economies' authorities not being able to defend the devaluation of their currencies triggered by countryspecific macro-economic variables, such as current account deficit, inadequate foreign exchange reserves and budget deficit.⁴ Recent studies find that carry trades in major currencies of developed economies are also subject to crash risk, such as the large crash of carry trade in 2008 when the global financial crisis emerged.⁵ Several studies, including Burnside et al. (2011), Lustig et al. (2011), Lettau et al. (2014) and Dobrynskaya (2014), explain the linkage between high returns in carry trades and currency crashes. The excess returns earned by currency carry trades may represent compensation for the crash risk in currencies with relatively higher interest rates. Brunnermeier et al. (2009) show that the price of currency crash risk is reflected by currency option prices which forms the risk reversal. The risk reversal measures the implied volatility difference between an out-of-the-money call on the currency and an out-of-the-money put at the same (absolute) delta. The 10%- and 25%-deltas are commonly used for such measure. Farhi et al. (2015) propose a disaster-based structural model which assumes that investors price in a currency crash-risk premium into the

⁴ See Eichergreen et al. (1996), Frankel and Rose (1996), Kaminsky et al. (1998), and Kumar et al. (2003).

⁵ In a carry trade, an investor sells a currency with a relatively low interest rate and uses the funds to buy a different currency yielding a higher interest rate. This strategy attempts to capture the difference between the rates of the two currencies provided that their exchange rate is stable.

value of a currency's exchange rate.⁶ Farhi and Gabaix (2016) develop a model that makes predictions regarding the link between exchange rates and signs of crash risk in currency options. Jurek (2014) shows that premiums on currency crashes in option prices can be used to explain at most one-third of the portion of carry trade returns. Chernov et al. (2018) demonstrate out-of-the-money options price in jump risk related to currency crashes. Husted et al. (2017) show that the cost of protection against currency crash risk reflects an increase in uncertainty in financial markets.

The risk reversal reflects asymmetric expectations on the directions of exchange rate movement and market participants' caution about the exchange rate falling below a certain level. Based on the feature of currency crashes represented by risk reversals which measure sharp falls of exchange rates below certain boundaries as the strike prices of out-of-themoney options and following the ensuing analysis in Jurek (2014) which studies compensation for exposure to the risk of large currency devaluations, we define a crash as the Bitcoin exchange rate shock that exceeds a pre-specified threshold. A smooth-pasting condition is imposed at a lower boundary for the Bitcoin exchange rate equation, suggesting an optimal boundary condition for the process with no foreseeable jump and no arbitrage condition at the boundary. The smooth-pasting condition ensures that a Bitcoin crash is rare and only occurs when the exchange rate dynamics causes the condition to break down. We derive a solution from the Bitcoin exchange rate equation in which the boundary is a moving average of the current and past exchange rates over a time horizon. This suggests some market participants take actions to keep within an assigned band that is not the current level of the Bitcoin exchange rate.

By normalising the Bitcoin exchange rate with a moving boundary, the relationship between the Bitcoin exchange rate and the fundamental in the model depends upon the past

⁶ The importance of downside risk can be related to the rare disasters model proposed by Barro (2006).

history of the exchange rate. The proposed model shares a key feature of the soft exchange rate target zone model for the European Exchange Rate Mechanism proposed by Bartolini and Pratib (1999), which shifts the reference for intervention from the level of the exchange rate at each instant to the behaviour of the exchange rate over a time interval, by featuring the central bank to keep only a moving average of past exchange rates within a range. The main implication is that it allows exchange rates to fluctuate within a wider range over short time horizons. According to such feature, Bitcoin market participants have more time to observe the exchange rate movements and market reactions such that they can postpone their decisions to defend until the negative Bitcoin demand shocks have worn out. When the exchange rate breaches the boundary under a shock, this leads to a discrete drop in the exchange rate with a magnitude that reflects the extent of uncertainty.

By using the asymmetric mean-reverting fundamental shock representing a money demand shock, the solution of the Bitcoin exchange rate equation shows the log-normalised Bitcoin exchange rate follows a mean-reverting square-root process which has a closed-form probability density function. The analysis illustrates that the Bitcoin exchange rate is quasibounded at the lower boundary. When it breaches the boundary (i.e., the smooth-pasting condition does not hold), provided the probability leakage condition is met, a crash occurs. The crash risk increases when the mean-reverting force for the Bitcoin exchange rate weakens. The probability density function is able to generate skewed exchange rate distributions consistent with empirical observations. The Bitcoin exchange rate dynamics can be calibrated according to the model using market Bitcoin exchange rate data.

The model suggests that a negative demand shock on Bitcoin increases its crash risk, which is reflected by a weakened mean-reverting force in its exchange rate dynamics. To test the validity of incorporating money demand shocks and crash risk into the model empirically, a co-integration analysis is used to test any positive relationship between the mean reversion in the Bitcoin exchange rate dynamic derived from the model and the Bitcoin transaction volume which indicates demand for Bitcoin. The empirical results show the mean reversion is positively co-integrated with the Bitcoin transaction volume, suggesting that demand for Bitcoin is adequately captured by the proposed fundamental dynamics. The analysis is also applied to test the relationship between the mean reversion and the risk reversals in the currency option market given that Bitcoin is expected to share a characteristic of currencies with crash risk. The risk reversals of the Australian dollar (AUD) and Canadian dollar (CAD) are shown to have a positive relationship with the mean reversion, supporting the incorporation of crash risk and the use of the asymmetric mean-reverting fundamental shock in the model.

The paper is organised as follows. We develop the Bitcoin exchange rate model associated with crash risk in the following section. The corresponding exchange rate dynamics and probability density function are derived and discussed. The calibrations of the Bitcoin exchange rate dynamics and its probability leakage condition are presented in section 3. The exchange rate dynamics' relationship with the Bitcoin transaction volume and currency risk reversals in currency option markets are studied by a co-integration analysis in section 4. The final section of the paper concludes.

2. Bitcoin exchange rate solution with crash risk

2.1 Bitcoin exchange rate model and smooth-pasting boundary condition

To incorporate crash risk into the model, we consider that Bitcoin may face a large fall in its value. To qualify how big a change in the exchange rate *S* (USD per Bitcoin), we define a lower boundary as a tolerance limit for a distribution of the exchange rate's statistics. Without assuming any distribution of the exchange rate, the lower boundary S_L is taken to be

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the number (Δ) of standard deviations (Σ) from its mean \overline{S} : $S_L = \overline{S} - \Delta \Sigma$.⁷ The idea is similar to the analysis of currency crashes in Jurek (2014) which sets a threshold of a crash according to the strikes of the 25% delta and 10% delta options corresponding to 0.70 and 1.4 standard deviations, respectively, away from the exchange rates.⁸ The choice of the level of the lower boundary (provided that it is adequately low) does not affect the process of the exchange rate dynamics. It is not necessary for the exchange rate breaching the lower boundary to capture its dynamics, as shown by the calibration using market data in the next section.

Historical exchange rates can be used as a guide to set a trading band confining the exchange rate. The historical trend of the exchange rate can be measured by a moving average $S_A(t)$ of the current and past exchange rate. For Bitcoin suffering from downwards exchange rate pressure subject to crash risk, the moving average can be scaled by a parameter η_L , with $0 < \eta_L < 1$, such that $\eta_L S_A(t)$ forms a lower boundary for the exchange rate movement. If the Bitcoin exchange rate is assumed to be normally distributed, $\eta_L S_A(t)$ corresponds to the number of standard deviations from its moving average. The parameter η_L tells how far market participants tolerate a fall in the Bitcoin to be in terms of a fraction of the moving average value $S_A(t)$. A smaller η_L suggests the market expects wider fluctuations over short horizons. Such specification of a lower boundary assumes market participants care about the behaviour of the Bitcoin exchange rate over a time interval, rather than just its current level. The particular way in which past Bitcoin exchange rate is brought into play

⁷ If a normal distribution is assumed and Δ is set equal to 1.5 and 2, the cumulative normal probabilities when the exchange rate falls below the boundaries are 0.0668 and 0.0227 respectively. It is noted that even when the exchange rate is not normally distributed, 1.5- and 2-standard deviations still cover a large area under the distribution of the exchange rate in a given time horizon, suggesting that falling to the lower boundary is a crash.

⁸ The idea of the definition of a crash is also similar to value-at-risk (VaR), which is a statistical measure of the riskiness of financial entities or portfolios of assets. VaR is defined as the maximum expected loss at a predefined confidence level (say 95%) over a given time horizon.

does not affect the derivation of the Bitcoin exchange rate solution and the qualitative results of our analysis. Similarly, an upper boundary can be set by using a scaling parameter η_U , with $\eta_U > 1$. With no loss of generality, the normalised log Bitcoin exchange rate *s* is defined by:

$$s = \ln \left[\frac{\eta_U S_{At} - S_t}{(\eta_U - \eta_L) S_{At}} \right],\tag{1}$$

where η_L and η_U are adjustable parameters for the lower and upper boundaries of a band respectively.

We use the standard flexible-price monetary model for the Bitcoin exchange rate.⁹ The log exchange rate *s* at time *t* follows the following equation:

$$s(t) = m + \nu + \alpha \frac{E[ds(t)]}{dt},$$
(2)

where *m* is the logarithm of the constant money supply, *v* is a monetary demand shock term, α is the absolute value of semi-elasticity of the exchange rate with respect to its expected rate of change, and *E* the expectation operator. The last term captures the expected exchange rate change. The "fundamental" (*v*) is the source of uncertainty and is assumed to follow a stochastic process with a drift μ_v which is a function of *v* and instantaneous standard deviation σ_v :

$$d\nu = \mu_{\nu}dt + \sigma_{\nu}dZ,\tag{2}$$

where dZ is a Wiener process with E[dZ] = 0 and $E[dZ^2] = dt$. The drift μ_v represents the behaviour of market participants in relation to money demand, and σ_v is the random shock. We apply Ito's lemma to Eqs.(1) and (2), and have

$$\frac{E[ds(t)]}{dt} = \mu_v \frac{ds}{dv} + \frac{1}{2}\sigma_v^2 \frac{d^2s}{dv^2}.$$
(3)

⁹ The model is based on the existence of a money demand function, the purchasing power parity and the uncovered interest rate parity. These theories form a flexible-exchange rate model of exchange rates.

Then substituting Eq.(3) into Eq.(1) yields

$$\frac{1}{2}\alpha\sigma_v^2\frac{d^2s}{dv^2} + \alpha\mu_v\frac{ds}{dv} - s = -v - m \quad , \tag{4}$$

which is a second-order linear ordinary differential equation.

To solve Eq.(4) with a crash when the exchange rate *s* breaches the lower boundary at s = 0 ($S_t = \eta_L S_{At}$), we specify the following boundary conditions at the fundamental of v = 0:

$$s(0) = 0, \tag{5}$$

$$\left. \frac{ds(v)}{dv} \right|_{v=0} = 0, \tag{6}$$

where the former condition ensures a proper normalisation of the Bitcoin exchange rate and the latter is the smooth-pasting boundary condition at v = 0, suggesting an optimal boundary condition for the process with no foreseeable jump in the exchange rate and no arbitrage condition. Krugman and Rotemberg (1990) show that the smooth-pasting condition ensures that the exchange rate does not cross the boundary. If the condition does not hold, the exchange rate could jump across the boundary, indicating a crash, which is a rare event.

Lo et al. (2019) provide a rigorous derivation of the fundamental of v which is uniquely determined under the boundary conditions in Eqs.(5) and (6). They show that the fundamental of v follows an asymmetric mean-reverting process with the following specification:

$$d\nu = \left(\frac{A_{-1}}{\nu} + A_1\nu\right)dt + \sigma_\nu dZ , \qquad (7)$$

where $A_1 < 0$, $A_{-1} > 0$, $-\infty < \nu \le 0$.¹⁰ Using this asymmetric mean-reverting fundamental dynamics, Lo et at. (2015) and Hui et al. (2016) find the associated exchange rate dynamics

¹⁰ Lo et al. (2019) derive the asymmetric mean-reverting fundamental dynamics proposed by Lo et at. (2015) et al. et al. (2016) for target-zone exchange rates, and has also shown that the proposed fundamental dynamics is indeed the unique choice and is described by the Rayleigh process. The generalized Rayleigh process is a diffusion process with mean reversion, of which some stochastic processes such as the Ornstein-Uhlenbeck

and interest rate differentials derived from the flexible-price monetary model can describe the market data for the Hong Kong dollar against USD in a target zone and the Swiss franc against the euro during the target zone regime of September 2011 – January 2015 respectively. The asymmetric mean-reverting fundamental dynamics are similar to asymmetric country-specific and global shocks in the context of contributions to violations of uncovered interest rate parity (Backus et al. (2001)) and exchange rate option (Bakshi et al. (2008); Jurek and Xu (2014)). It is also consistent with risk reversals (i.e., currency crashes), which are inherently asymmetric, given that crashes are one-sided events.

To understand and visualise the asymmetric mean-reverting fundamental shock, Lo et al. (2019) obtain a "potential well" U(v) by integrating the drift term in Eq.(7), in a negative form, with respect to v:

$$U(\nu) = -\int \left(\frac{A_{-1}}{\nu} + A_1\nu\right) d\nu = -A_{-1}\ln|\nu| - \frac{1}{2}A_1\nu^2, \qquad (8)$$

in which the fundamental variable v is similar to a ball moving in a well, as shown in Figure 1 in Lo et al. (2019) by plotting Eq.(8) with different values of A_{-1} and A_1 . The restoring force (an increase in exchange rate) given by the first term in the mean-reverting drift with v close to zero is stronger than the force (a decrease in exchange rate) provided by the second term. Therefore, the mean-reverting force in Eq.(7) is not symmetric. This is consistent with the intuition that when the demand for Bitcoin is extremely weak such that the Bitcoin exchange rate falls significantly, market participants who hold significant amounts of Bitcoin have an incentive to defend the exchange rate above the lower boundary. Those market participants could be market manipulators as found in Gandal et al. (2018) that the suspicious trading activity likely caused the unprecedented spike in the Bitcoin exchange rate in late 2013, suggesting that Bitcoin exchange rate manipulation is quite feasible in such "thin"

process are special cases. It has been considered in the context of the path-dependent option pricing models used in economics and stochastic finance studies (see Davidov and Linetsky (2001)).

market. Regarding the shape of the potential well, decreasing the magnitude of the parameters A_{-1} or A_1 reduces the mean-reverting force for the fundamental and gives a very flat potential well, such that the fundamental variable simply moves randomly above the lower boundary, i.e., increasing the crash risk of v breaching the origin. This shows that the strength of the mean reversion in the fundamental dynamics determines the crash risk of the Bitcoin exchange rate. The quasi-bounded process for the Bitcoin exchange rate will be derived from Eq.(4) according to the asymmetric mean-reverting fundamental shock and discussed in the following subsection.

2.2 Bitcoin exchange rate solution and probability density function

By the power series method we are able to obtain the desired solution of the ordinary differential equation in Eq.(4) with the boundary conditions specified by Eq.(5) and Eq.(6) in the form:

$$s(\nu) = \nu^2 \sum_{n=0}^{\infty} B_n \nu^n \tag{9}$$

which vanishes at v = 0. Lo et al. (2019) derive the coefficients in Eq.(9) as

$$B_0 = -\frac{m}{\alpha(\sigma_v^2 + 2A_{-1})}, \qquad B_1 = -\frac{1}{3\alpha(A_{-1} + \sigma_v^2)}$$
$$B_{n+2} = \frac{2[1 - \alpha(n+2)A_1]}{\alpha(n+4)[2A_{-1} + (n+3)\sigma_v^2]}B_n \qquad \text{for } n = 0, 1, 2, \dots$$

and show that:

$$B_0 = \frac{1}{2} \frac{d^2 s}{d\nu^2} \Big|_{\nu=0} = -\frac{m}{\alpha(\sigma_{\nu}^2 + 2A_{-1})} < 0 \quad , \tag{10}$$

suggesting *s* attains its maximum at v = 0. The second-order linear ordinary differential equation of Eq.(4) uniquely determines the second-order derivative of *s* with respect to *v* at v = 0 by itself.

Lo et al. (2019) show the series solution is a convergent series for all v by means of the ratio test. Motivated by the rapid convergence of the series solution shown in those studies, we propose to approximate the exact solution by an optimal approximate solution of the form:

$$s(\nu) = B_0 \nu^2 = -\frac{m}{\alpha(\sigma_{\nu}^2 + 2A_{-1})} \nu^2$$
(11)

Figure 2 plots the relationship between the Bitcoin exchange rate and the fundamental expressed in Eq.(11). It shows that changes in the exchange rate flatten with changes in the fundamental at the two boundaries. This means the exchange rate could marginally move away from the boundaries even though the fundamental changes materially. When the exchange rate moves towards its lower boundary due to a negative demand shock in the fundamental, there is a counteracting tendency of a mean reversion back to the equilibrium level which acts as a stabilising force as shown in Eq.(7) to limit further fall in the exchange rate. Based on the model, as changes in the Bitcoin fundamental, the Bitcoin exchange rate could move from C to C" or C', where the paths depend on the coefficient B_0 in Eq.(11), which represents the state of the Bitcoin market, including the Bitcoin supply (*m*), parameters (A_{-1}) of the asymmetric fundamental shock, and sensitivity (α) of the exchange rate to its expected rate of change. A larger B_0 suggests that the Bitcoin exchange rate is more sensitive to changes in the fundamental (money demand). This happened when the Bitcoin supply *m* is relatively ample and/or both A_{-1} and α are relatively small. As the Bitcoin supply is quite steady, this suggests when the restoring force in the fundamental dynamics is weak and/or the exchange rate is less sensitive to the expected exchange rate, the crash risk of Bitcoin is higher.

To demonstrate the Bitcoin exchange rate dynamics, it is convenient to concentrate on the magnitude of s and introduce the new variable $x \equiv -s$ so $0 \le x < \infty$ with x = 0 corresponding to the lower boundary. By applying Ito's lemma to Eq.(7) with Eq.(11), x is shown to follow a mean-reverting square-root (MRSR) process:

$$dx = \kappa (\theta - x)dt + \sigma_x \sqrt{x}dZ , \qquad (12)$$

where

$$\kappa = 2|A_1|, \qquad \theta = \left|\frac{B_0}{A_1}\right| \left(A_{-1} + \frac{1}{2}\sigma_{\nu}^2\right) \tag{13}$$

$$\sigma_x = 2\sigma_v \sqrt{|B_0|} \qquad . \tag{14}$$

In Eq.(12), κ determines the speed of the mean-reverting drift towards the long-term mean θ which is a time-varying equilibrium level and determined through actions by market participants to drive the exchange rate towards its mean level. When the exchange rate is close to zero (i.e., the lower boundary), the standard deviation $\sigma_x \sqrt{x}$ also becomes very small. The corresponding exchange rate dynamics become dominated by the mean-reverting drift, which pushes the exchange rate towards the mean and away from the boundary and reduces crash risk. The properties of the MRSR process is also shown by the well-known Cox–Ingersoll–Ross (CIR) model (1985) for interest rate term structures.

Following Feller's classification of boundary points, it can be inferred that there is a non-attractive natural boundary at infinity and the one at the origin is a boundary of no probability leakage for $(\sigma_x^2/4\kappa\theta) < 1$ in Eq.(12), and it is not otherwise.¹¹ The no-leakage condition ensures the exchange rate will not breach the origin (the lower boundary) and there is no Bitcoin crash; otherwise, the exchange rate may pass through the boundary, i.e., the exchange rate is quasi-bounded at the origin.^{12,13} The boundary condition at the origin under

¹¹ For boundary condition definitions, see Karlin and Taylor (1981).

¹² Lo et al. (2015) et al. et al. (2016) show that the quasi-bounded process can describe the exchange rate dynamics and interest rate differentials of the Hong Kong dollar against the US dollar in a target zone and the Swiss franc against the euro during the target zone regime of September 2011 to January 2015 respectively. Regarding the Swiss franc exchange rate, the condition for breaching the strong-side limit was met in November 2014 using only information until that point, i.e., about two months before abandoning the limit. This demonstrates that the asymmetric mean-reverting fundamental dynamics incorporates the features of intervention and realignment.

the MRSR process is studied in CIR (1985) and Longstaff (1989, 1992). If the no-leakage condition does not hold at the boundary, the smooth-pasting condition of Eq.(6) may break down in the model and a Bitcoin crash could occur.

The probability density function (PDF) of *x* under the MRSR process is given by:

$$G(x,t;x',t') = \frac{2}{\sigma_x^2 C_1(t-t')} \left(\frac{x}{x'}\right)^{\omega/2} \exp\left[-\frac{\omega+2}{2} C_2(t-t')\right] \times \exp\left\{-\frac{2x'+2x \exp\left[-C_2(t-t')\right]}{\sigma_x^2 C_1(t-t')}\right\} \times , \qquad (15)$$
$$I_{\omega}\left\{\frac{4x^{1/2} x'^{1/2} \exp\left[-C_2(t-t')/2\right]}{\sigma_x^2 C_1(t-t')}\right\}$$

where $\omega = 2\kappa\theta/\sigma_x^2 - 1$, $C_1(\tau) = [\exp(\kappa\tau) - 1]/\kappa$, $C_2(\tau) = -\kappa\tau$, I_{ω} is the modified Bessel function of the first kind of order ω . The associated asymptotic PDF will eventually approach the steady-state exchange rate distribution, which is:

$$K(x,t \to \infty; x',t') = \frac{2x^{\omega+1/2}}{\Gamma(\omega+1)} \left(\frac{2\kappa}{\sigma_x^2}\right)^{\omega+1} \exp\left(-\frac{2\kappa}{\sigma_x^2}x\right),$$
(16)

where Γ is the gamma function. Given the PDF in Eq.(15), the parameters of the MRSR process for the exchange rate dynamics are calibrated in section 3 using market exchange rate data.

Figure 3 shows the steady-state exchange rate distributions in *S* based on Eq.(16) with two values of the long-term mean θ of 0.3, 1.0 and 1.5: the smaller θ is closer to the lower boundary. We use the model parameters for $\sigma_x = 0.05$, 0.13 and 0.2, and $\kappa = 0.05$ and 0.25, which are consistent with the estimations in section 3, with the lower boundary $\eta_L S_{At} = S_L =$ 3793 and upper boundary $\eta_U S_{At} = S_U = 17360$. The distributions with $\theta = 1.0$ and 1.5 have their peaks at the right, showing the PDF decays slower than a Gaussian distribution at

¹³ Such a property is similar to the bounded exchange rate dynamics in Ingersoll (1996) and Larsen and Sørensen (2007). In their models the exchange rate is completely bounded under all circumstances.

the left, suggesting the fat-tails effect with the probability of outlier negative returns. On the other hand, the distributions with $\theta = 0.3$ have their peaks at the left and the probability of outlier positive returns. The different skewness of the distributions is consistent with the empirical observations of Bitcoin exchange rate returns and the both left- and right-skewed distributions found in Nadarajah and Chu (2017), Liu and Tsyvinski (2018), and Chevapatrakul and Mascia (2018) depending on the sample periods of their studies.

All panels in Figure 3 show fatter left tails of the exchange rate distributions with the mean θ further away from the lower boundary, demonstrating that the probability of outlier negative returns becomes more significant for the Bitcoin exchange rate expected to increase in the near term. Comparison among Panel A, B and C, where σ_x increases from 0.05 to 0.2, shows the left tails of the distributions become much fatter and hump shaped, and their left-skewness is sensitive to an increase in the exchange rate volatility. The higher exchange rate volatility increases the likelihood of a crash, which is reflected by the fat left tails. By keeping $\sigma_x = 0.2$ and increasing κ from 0.15 to 0.25 in Panel D, the exchange rate distributions revert to the shapes similar to those in Panel A with less fat left tails, showing an increase in the mean reversion in the exchange rate dynamics reduces crash risk. The changes in the distributions in Figure 3 with different exchange rate parameters demonstrate that the leakage condition of the MRSR process of the exchange rate dynamics derived from the model is consistent with the left-skewed distributions for exchange rates with crash risk.

Similarly, Panels A and B show the distributions with $\theta = 0.3$ have flatter right tails when σ_x increases from 0.05 to 0.13, suggesting that the probability of outlier positive returns becomes more significant. By keeping $\sigma_x = 0.2$ and increasing κ from 0.15 to 0.25 in Panel D, the exchange rate distribution with $\theta = 0.3$ does not change much from that in Panel C, demonstrating that a stronger mean reversion is required to change the shape of the distributions when the mean θ is close to the lower boundary.

3. Calibration of Bitcoin exchange rate dynamics

In this section, we examine whether the Bitcoin exchange rate dynamics can be characterised by the proposed exchange rate model. By using the log-likelihood function based on the PDF of Eq.(15), we can calibrate the model parameters of the process specified in Eq.(12). Regarding the sample period, the calibration is conducted by applying the maximum likelihood estimation (MLE) to daily Bitcoin exchange rate data from 19 July 2010 to 9 November 2018. Figure 1 shows the Bitcoin exchange rates in *S* and the associated moving lower and upper boundaries with the parameters of $\eta_L = 0.59$ and $\eta_U = 2.7$ on the 1-month moving average, and the transformed exchange rate in *x* of the time series.¹⁴ There was a sharp fall in early 2018 which caused the exchange rate to breach the lower boundary.

Based on the 2-year rolling window estimations results, Figure 4 reported statistically significant estimates of the drift term κ (Panel A) with the respective *z*-statistic maintaining above the value of 1.96 (i.e., at the 5% significance level). κ decreased from 0.2 to about 0.05 in 2015, indicating a weakened force in restoring the Bitcoin exchange rate towards its long-term mean after the sharp rise (from US\$50 to US\$1000) and then the fall (back to US\$200) during 2013 – 2015 as shown in Figure 1 (Panel A). The drift dropped further in early 2018 when there was a shape fall in the exchange rate after a drastic surge in late 2017. As the Bitcoin exchange rate dropped towards its lower boundary during this period as shown in Figure 1 (Panel B), the mean-reverting force weakened with the Bitcoin crash. It is noted that the estimation of κ became insignificant in a very short period of time after the crash with κ

¹⁴ The lower and upper boundaries correspond to about 1.69 and 7 standard deviations respectively.

not significantly different from zero. Subsequently, the estimation rebounded to the 0.04 level with the *z*-statistic higher than 1.96.

Panel B of Figure 4 shows a steady estimated mean θ with the values ranging between 0.2 and 0.35 and the corresponding *z*-statistic staying above the 1.96 level. Similar to the changes in κ , the estimation of θ became insignificant in a very short period of time after the crash in early 2018. Regarding the statistical significance, the mean reversion of the Bitcoin exchange rate dynamics, which can be expressed by κ and θ in the model, exhibited similar patterns.

The volatility σ_x , which is displayed in Panel C of Figure 4, is estimated to take the value between 0.05 and 0.21. The corresponding *z*-statistic is much higher than 1.96, indicating that the estimated σ_x is highly significant. The results suggest that the estimation of the square-root-process part of the quasi-bounded dynamics is robust. The volatility decreased in 2015 after the exchange rate dropped to US\$200 from US\$800 in 2014.

As the probability leakage condition of $(\sigma_x^2/4\kappa\theta)$ can portray the crash risk of the Bitcoin exchange rate at the lower boundary, Panel D of Figure 4 displays this measure to identify periods with the leakage condition greater than 1. The measure was about 0.2 during 2013 – 2015 when the Bitcoin exchange rate fell from about \$1,000 to \$300, suggesting that the crash risk was not immaterial while the Bitcoin exchange rate was bounded above the lower boundary. After the measure stayed at the level of 0.1, it rose sharply and breached 1.0 in February 2018 with the existence of the leakage condition when the exchange rate fell sharply after a drastic surge close to \$20,000 in December 2017. The fall in the exchange rate reflected that the demand for Bitcoin was expected to drop sharply as some economists, renowned investors, and finance professionals warned that rapidly increasing cryptocurrency

exchange rates could cause the "bubble" to burst.¹⁵ At the same time, the Chicago Board Options Exchange and Chicago Mercantile Exchange launched Bitcoin futures which allowed investors to increase their exposure in shorting Bitcoin. In addition, there was regulatory uncertainty about global coordination on how to regulate the cryptocurrencies. Officials were expected to debate the rise of Bitcoin at the G20 summit in Argentina in March 2018. Subsequently, different jurisdictions continue to focus on taking enforcement action on cryptocurrencies. For example, the North American Securities Administrators Association reported more than 200 active investigations by subnational agencies into initial coin offerings and other crypto-related investment products.¹⁶ The US Securities and Exchange Commission has cracked down on decentralised/ unregistered exchanges.¹⁷ Japan's Financial Services Agency has tightened registration screening and monitoring of crypto-asset trading platforms.¹⁸ After the crash of the Bitcoin exchange rate, the measure returned to the level of 0.15.

In summary, we found empirical evidence that the MRSR process adequately describes the Bitcoin exchange rate dynamics by incorporating a feature of lower boundary representing the level for a crash. By using the MLE estimation and the 2-year rolling window, the Bitcoin exchange rate dynamics is calibrated according to the model, and the estimated parameters are found to be time-varying. The mean-reverting force, which is represented by the parameters κ and θ , is estimated to be present during the estimation period. The diminishing mean-reverting force in Bitcoin exchange rate and the existence of the leakage condition reflect that crash risk built up at the lower boundary during the Bitcoin crash in February 2018.

¹⁵ See for example "Bitcoin's rollercoaster ride after hitting \$17,000" at https://www.bbc.com/news/business-42275564.

¹⁶ http://www.nasaa.org/46226/nasaa-marks-cryptocurrency-anniversary-with-a-word-of-caution/

¹⁷ https://www.sec.gov/news/press-release/2018-258.

¹⁸ https://www.japantimes.co.jp/news/2018/09/02/business/financial-markets/japan-fsa-tightens-screening-cryptocurrency-exchanges

4. Dynamic relationships of Bitcoin exchange rate dynamics with Bitcoin transactions and currency risk reversals

To test the validity of incorporating money demand shocks and crash risk into the model, a co-integration analysis is used to test the relationships between the mean reversion in the Bitcoin exchange rate dynamics derived from the model and the Bitcoin transaction volume which indicates demand for Bitcoin; and the risk reversals of AUD and CAD.

4.1 Relationships between mean reversion in Bitcoin exchange rate and transactions

The Bitcoin exchange rate and its transaction volume shown in the upper panel in Figure 5 exhibit that declines in the Bitcoin transaction volume often occurred along with depreciations of Bitcoin. According to Jermann (2018)'s argument, significant technological obstacles or regulatory measures which affect adoption of Bitcoin will trigger a demand shock on Bitcoin. The weakened demand will be reflected by lower transaction volumes and cause a sharp decline in the Bitcoin exchange rate that increases the Bitcoin crash risk. The Bitcoin exchange rate model expects a weakened mean-reverting force, i.e. lower κ and θ , in the exchange rate dynamics under such demand shock.

If there exists a long-run equilibrium relationship between the model parameters and Bitcoin transaction volume, their short-run dynamics can be studied through the following dynamical error-correction representation:

$$\Delta y_t = a_{10} + \alpha_y (y_{t-1} - \beta_1 X_{t-1}) + \sum_k b_{1k} \Delta y_{t-k} + \sum_k c_{1k} \Delta X_{t-k} + \varepsilon_{yt} , \qquad (17)$$

where y_t is either κ or θ at time t, and α_y is less than zero. X_{t-1} is logarithm of Bitcoin transaction volume ln(txn_vol) at time t-1. Under this representation, the model parameters (as represented by y_t) will respond to stochastic shocks (represented by ε_{yt}) and also the long-run equilibrium deviation in previous period (i.e., $y_{t-1} - \beta_1 X_{t-1}$). The estimated speed of adjustment (i.e. α_y) should be negative and nonzero for the co-integration relationship to be validly specified by the error-correction. In terms of absolute magnitude, a larger estimated value of α_y reflects a higher sensitivity of y_t to the long-run equilibrium deviation in the previous period.

The estimation is conducted at a weekly frequency starting from 13 November 2015 to 9 November 2018 when the transaction volume was sufficient and well above 100,000.¹⁹ The week-end data for the model parameters κ and θ based on the calibration results in section 3 are used. Table 1 reports the summary statistics, correlation coefficient and the respective Augmented Dickey–Fuller (ADF) test results for the variables both in levels and first differences. The ADF test results reflect that the existence of unit root for κ , θ and logarithm value of the Bitcoin transaction volume in level form cannot be rejected at the 10% significance level. Nonetheless, the respective ADF tests for the first differenced variables indicate no presence of unit root at the 1% level. Thus, the above results suggest that these three variables are all co-integrated of same order I (i.e., I(1)).

We adopt the single-equation test proposed by Engle and Granger (1987) to test the co-integration relationship between κ , θ and the Bitcoin transaction volume. This Engle-Granger single-equation test essentially examines whether the residuals of the linear combinations among i.) κ and the Bitcoin transaction volume; ii.) θ and the Bitcoin transaction volume are stationary. Table 2 reports the co-integration tests between the transaction volume and κ , θ , with the ordinary least squared regression residuals being tested by both the ADF and Phillips-Perron tests.²⁰ Overall, the results favour the alternative

¹⁹ The transaction volume data are from https://blockchain.info/.

²⁰ The critical values of the tests are based on MacKinnon (1996) and the lag length is determined by the Akaike information criterion. In addition, taking into account of the possibility of a regime shift in the co-integration model, we also test the null hypothesis of no co-integration relationship with the residual-based tests derived in

hypothesis of the presence of at least one co-integrating vector among Bitcoin transaction volume and the model parameters (κ and θ) respectively, given the statistical significances for κ and θ at the 5% or 1% level.

Table 3 shows that the co-integrating vectors (expressed by β) between the Bitcoin transaction volume and κ , θ_x are estimated to be 0.0191 and 0.0445 at the 1% or 10% level respectively. The estimated positive coefficients indicate that a higher Bitcoin transaction volume would increase κ and θ , holding other things constant. Intuitively, the positive relationship suggests that when the transaction volume declines, the crash risk of Bitcoin increases, which is reflected from the weakened restoring force of the Bitcoin exchange rate dynamics.

As reported in Table 4, the estimates of the speed of adjustment (that is, α_y) for κ and θ are -0.1729 and -0.0989 respectively, which are negative but greater than -1, reflecting that the model parameters will subsequently adjust to restore the long-run equilibrium. This demonstrates a valid error correction specification and the presence of a self-restoring force, which will close the spread of the link between the mean reversion parameters (κ and θ) and the Bitcoin transaction volume.

4.2 Relationships between mean reversion in Bitcoin exchange rate and currency risk reversals

A co-integration analysis is used to examine whether crash risk of Bitcoin is adequately incorporated into the model such that its exchange rate dynamics, i.e. the mean reversion, is related to crash risk anticipated in the market. Given that there is no liquid Bitcoin option market and Bitcoin is expected to share a characteristic of currencies with

Gregory and Hansen (1996) for the Engle-Granger regressions in Tables 2 and 5. The results from the Gregory-Hansen test also suggest the null hypothesis for no co-integration relationship among these interest variables is rejected at 5% or 10% significance level. The results are available upon request.

crash risk in which the corresponding prices are reflected from risk reversals, we use the risk reversals of AUD against the Japanese yen (JPY) and CAD against USD in the analysis to study their relationship with the mean reversion in the Bitcoin exchange rate dynamics. The AUD/JPY and CAD/USD are currency pairs commonly used for carry trades being studied by Brunnermeier et al. (2009) showing that carry trades are subject to crash risk. In carry trades, exchange rate movements between high interest rate and low interest rate currencies are negatively (left) skewed. The price of currency crash risk is reflected by the price of the risk-reversal. Lustig et al. (2011) find the effects of broadly defined "global risk aversion" on the profitability of carry trades. Jurek (2014) derives a measure of crash risk from currency options and finds that exposure to a currency crash can be used to explain at most one-third of the portion of carry trade returns. Several studies, including Burnside et al. (2011), Lettau et al. (2014) and Dobrynskaya (2014), explain high returns to carry trades and investigate downside factors related to currency crashes.

Given that currency crash risk is reflected by currency option risk reversals, a more negative risk reversal of a currency suggests that exchange rates of hedging against downside risk (crash risk) of the currency are higher than its up-side risk. In line with the Bitcoin exchange rate dynamics, the exchange rate crash risk heightens when the mean-reverting force for the exchange rate diminishes, indicating rising probability leakage for the rate across the lower boundary. Therefore, we postulate that the currency risk reversals are positively correlated with the mean-version of the Bitcoin exchange rate. The middle and lower panels in Figure 5 show the Bitcoin exchange rate and the 1-month 25-delta risk reversals of AUD against JPY as well as CAD against USD respectively. It is particularly noticeable that AUD/JPY risk reversal displays strong positive relationships with the Bitcoin exchange rate when there are substantial declines in the Bitcoin exchange rate during the Bitcoin crash in early 2018.

We apply a similar methodology as in section 4.1 to examine whether there is a longrun relationship between the model parameters (κ and θ) and the two currency risk reversals. Their short-run representations in a dynamical error-correction form are essentially the same as in Eq.(17), with the currency risk reversals of AUD (AUD_rr) and CAD (CAD_rr) respectively at time *t*-1 denoted by X_{t-1} and the estimation sample using weekly data starting from 13 November 2015 to 9 November 2018.²¹ The summary statistics, correlation coefficients and the ADF test statistics for the time series of the four variables (κ , θ , AUD_rr and CAD_rr) in levels and changes are also reported in Table 1. The ADF test fails to reject at the 10% level the presence of a unit root for these variables in level, but rejects the same hypothesis for the first difference at 1% level or less, suggesting that the two currency risk reversals and model parameters (κ and θ) are I(1).

Table 5 reports the co-integration results for the AUD and CAD risk reversals with the model parameters in the two panels respectively, of which the Engle-Granger singleequation test is applied again. The results of the ADF and Philips-Perron tests reject the residuals from the regressions of two risk reversals with κ and θ to have a unit root at the 5% and below, except for the ADF test for CAD_rr and θ at the 10% level. Therefore, we favour the alternative hypothesis that there is at least one co-integrating vector among each of the two risk reversals with the model parameters κ and θ .

The two panels in Table 6 report the estimated co-integrating vectors between the AUD (and CAD) risk reversal and the mean reversion parameters κ and θ . The positive coefficients β for κ and θ are 0.0057 (0.0177) and 0.0125 (0.0333) respectively at the 5% or 10% significance level, suggesting that a more negative AUD (CAD) risk reversal is correlated to decreases in κ and θ . Given that the mean reversion in the Bitcoin exchange rate dynamics weakens when the Bitcoin crash risk increases, the positive relationship illustrates

²¹ The risk reversal data are from Bloomberg.

that market participants expect the AUD/JPY and CAD/USD exchange rates exhibit certain degree of co-movement with the Bitcoin exchange rate with material crash risk. When there is risk aversion against Bitcoin, such expectation may generate strong demand for put options on AUD and CAD to hedge against potential loss of long AUD and CAD (carry-trade) positions funded by JPY and USD respectively, and cause more negative AUD and CAD risk reversals. Table 7 shows the estimates of α_y for κ and θ are negative but greater than -1, reflecting that there is a restoring force to subsequently adjust κ and θ towards their long-run equilibria.

The results of the co-integration analysis show the mean reversion is positively cointegrated with the Bitcoin transaction volume, suggesting that demand for Bitcoin is adequately captured by the proposed fundamental dynamics. The AUD and CAD risk reversals are also shown to have a positive relationship with the mean reversion, supporting the incorporation of crash risk and the use of the asymmetric mean-reverting fundamental shock in the model.

5. Conclusion

We derive the Bitcoin exchange rate dynamics by solving the exchange rate equation of the standard flexible-price monetary model to investigate whether Bitcoin behaves like a currency. A Bitcoin crash occurs when its exchange rate breaches a moving lower boundary where a smooth-pasting boundary condition is imposed for the equation. The boundary condition ensures a crash is rare and assumes market participants, including manipulators who hold substantial amounts of Bitcoin for investment, will defend the Bitcoin exchange rate which falls close to the boundary.

The fundamental dynamics in the exchange rate equation is driven by an asymmetric mean-reverting fundamental shock which can be attributed to a money demand shock. The

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solution of the equation shows the Bitcoin exchange rate follows a mean-reverting squareroot process, which is quasi-bounded at the lower boundary and can breach the boundary with a weakened mean reversion. The exchange rate solution generates both left- and rightskewed exchange rate distributions consistent with empirical observations. The empirical results using market data suggest that the model can describe the Bitcoin exchange rate dynamics. While the exchange rate was bounded above the boundary during most of the time, the condition for breaching the boundary was met in early 2018 when the rate fell sharply. The crash reflected that the demand for Bitcoin was expected to fall sharply after rapidly increased cryptocurrency exchange rates and uncertainty about global coordination on how to regulate the cryptocurrencies.

The co-integration tests show that the mean reversion in the Bitcoin exchange rate dynamic are positively co-integrated with the Bitcoin transaction volume which represents money demand; and with the risk reversals of AUD and CAD. The results suggest that money demand for Bitcoin is adequately captured by the proposed fundamental dynamics in the standard flexible-price monetary model, and support the incorporation of crash risk and the use of the asymmetric mean-reverting fundamental shock in the model. The analysis shows that the Bitcoin exchange rate shares some characteristics of a currency with crash risk.

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References

Athey, S., Parashkevov, I., Sarukkai, V., Xia, J., 2016. Bitcoin pricing, adoption, and usage: Theory and evidence. Working Paper, Stanford University.

Backus, D.K., Foresi, S., Telmer, C., 2001. Affine term structure models and the forward premium anomaly. *Journal of Finance* 56, 279-304.

Bakshi, G., Carr, P., Wu, L., 2008. Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies. *Journal of Financial Economics* 87, 132-156.

Bartolini, L., Prati, A., 1999. Soft exchange rate bands and speculative attacks: theory, and evidence from the ERM since August 1993. *Journal of International Economics* 49(1), 1-29.

Bolt, W., van Oordt, M.R.C., 2019. On the value of virtual currencies. *Journal of Money, Credit and Banking*. doi.org/10.1111/jmcb.12619.

Brunnermeier, M.K., Nagel, S., Pedersen, L.H., 2009. Carry trades and currency crashes. *NBER Macroeconomics Annual* 23, 313–347.

Burnside, C., Eichenbaum, M., Kleshchelski, I., Rebelo, S., 2011. Do peso problems explain the returns to the carry trade? *Review of Financial Studies* 24 (3), 853-891.

Cagan, P., 1956. The monetary dynamics of hyperinflation, In: Friedman, M., Ed., Studies in the Quantity Theory of Money, The University of Chicago Press, Chicago, 25-117.

Chernov, M., Graveline, J., Zviadadze, I., 2018. Crask risk in currency returns. *Journal of Financial and Quantitative Analysis* 53 (1), 137-170.

Chevapatrakul, T., Mascia, D.V., 2018. Detecting overreaction in the Bitcoin market: A quantile autoregression approach. *Finance Research Letters*, Available online 6 November 2018.

Chiu, J., Koeppl, T.V., 2017. The economics of cryptocurrencies – Bitcoin and beyond. Working Paper, Bank of Canada. Cox, J.C., Ingersoll J.E., Ross S.A., 1985. A theory of the term structure of interest rates. *Econometrica* 53, 385-407.

Davidov, D., Linetsky, V. 2001. Pricing and hedging path-dependent options under CEV processes. *Management Sciences* 47, 949-965.

Dobrynskaya, V., 2014. Downside market risk of carry trades. *Review of Finance* 18 (5), 1885-1913.

Eichengreen, B., Rose, A. Wyplosz, C., 1996. Contagious currency crises: First test. *Scandinavian Journal of Economics* 98 (4), 463-484.

Engel C., West K., 2005. Exchange rates and fundamentals. *Journal of Political Economy* 113 (3), 485-517.

Engle, R.F., Granger, C.W.J., 1987. Co-integration and error correction: Representation, estimation and testing. *Econometrica* 55, 251-276.

Farhi, E., Fraiberger, S.P., Gabaix, X., Ranciere, R., Verdelhan, A., 2015. Crash risk in currency markets. Working Paper, New York University.

Farhi, E., Gabaix, X., 2016. Rare disasters and exchange rates. *Quarterly Journal of Economics* 131 (1), 1-51.

Fernandes-Villaverde, J., Sanches, D., 2016. Can currency competition work? NBER Working Paper No. 22157.

Frankel, J., Rose, A., 1996. Currency crashes in emerging markets: An empirical treatment, *Journal of International Economics* 41, 351-366.

Gandal, N., Hamrick, J.T., Mooreb, T., Oberman, T., 2018. Price manipulation in the Bitcoin ecosystem. *Journal of Monetary Economics* 95, 86-96.

Gregory, A., Hansen, B., 1996. Residual-based tests for cointegration in models with regime shift. *Journal of Econometrics* 70, 99-126.

Hui, C.H., Lo, C.F., Fong, T., 2016. Swiss franc's one-sided target zone during 2011-2015. *International Review of Economics and Finance* 44, 54-67.

Husted, L., Rogers, J., Sun, B., 2018. Uncertainty, currency excess returns, and risk reversals. *Journal of International Money and Finance* 88(C), 228-241.

Ingersoll, J.E., Jr., 1997. Valuing foreign exchange rate derivatives with a bounded exchange process. *Review of Derivatives Research* 1, 159-81.

Jermann, U., 2018. Bitcoin and Cagan's model of hyperinflation. Working Paper, University of Pennsylvania.

Jurek, J.W., 2014. Crash-neutral currency carry trades. *Journal of Financial Economics* 113, 325-347.

Jurek, J.W., Xu, Z., 2014. Option-implied currency risk premia. Working Paper, University of Pennsylvania.

Kaminsky, G.L., Lizondo, S., Reinhart, C.M., 1998. Leading indicators of currency crises. *International Monetary Fund Staff Papers* 45 (1), 1-48.

Karlin, S., Taylor, H.M., 1981. A Second Course in Stochastic Processes. Academic Press, New York.

Krugman, P., Rotemberg, J., 1990. Target zones with limited reserves. NBER Working Paper No. 3418.

Kumar, M., Moorthy, U., Perraudin, W., 2003. Predicting emerging market currency crashes. *Journal of Empirical Finance* 10 (4), 427-454.

Larsen, K.S., Sørensen, M., 2007. A diffusion model for exchange rates in a target zone. *Mathematical Finance* 17, 285-306.

Lettau, M., Maggiori, M., Weber, M., 2014. Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics* 114, 197-225.

Liu, Y., Tsyvinski, A., 2018. Risks and returns of cryptocurrency. Working Paper, Yale University.

Lo, C.F., Hui, C.H., Fong, T., Chu, S.W., 2015. A quasi-bounded target zone model – Theory and application to Hong Kong dollar. *International Review of Economics and Finance* 37, 1-17.

Lo, C.H., Hui, C.H., Liu, C.H., 2019. Unique fundamental dynamics of target-zone exchange rates. Manuscript, The Chinese University of Hong Kong. Available at SSRN: http://dx.doi.org/10.2139/ssrn.3394637.

Longstaff, F., 1989. A non-linear general equilibrium model of the term structure of interest rates. *Journal of Financial Economics* 23, 195-224.

Longstaff, F., 1992. Multiple equilibria and term structure models. *Journal of Financial Economics* 32, 333-344.

Lustig, H., Roussanov, N., Verdelhan, A., 2011. Common risk factors in currency markets. *Review of Financial Studies* 24, 3731-3777.

MacKinnon, J.G., 1996. Numerical distribution functions for unit root and cointegration test. *Journal of Applied Econometrics* 11, 601-618.

Nakamoto, S., 2008. Bitcoin: A peer-to-peer electronic cash system. Available from http://pdos.csail.mit.edu/6.824/papers/bitcoin.pdf.

Nadarajah S., Chu, J., 2017. On the inefficiency of Bitcoin. Economics Letters 150, 6-9.

Prat, J., Walter, B., 2018. An equilibrium model of the market for Bitcoin mining. CESifo Working Paper Series 6865.

Raskin, M., Yermack, D., 2016. Digital currencies, decentralized ledgers, and the future of central banking. NBER Working Paper No. 22238.

Saleh, F., 2017. Blockchain without waste: Proof-of-stake. Working Paper, New York University.

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Yermack, D., 2015. Is Bitcoin a real currency? An economic appraisal. Handbook of digital currency, 31-43.



Figure 1: Bitcoin exchange rate in S-scale and x-scale, and upper and lower boundaries in S with $\eta_L = 0.59$ and $\eta_U = 2.7$ on 1-month moving average.



Figure 2: Relationship between Bitcoin exchange rate (*S*) and fundamental (ν) based on Eq.(11) with $B_0 = 1$ and 1.5.



Figure 3: Bitcoin exchange rate distributions with different values of model parameters σ_x , κ and θ under the normalisation on 1/11/2018.

Figure 4: Estimated κ (Panel A), θ (Panel B), σ_x (Panel C), corresponding *z*-statistic, and leakage ratio ($\sigma_x^2/4\kappa\theta_x$) with 1-month moving average using 2-year rolling window.



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Figure 5: Logarithm of Bitcoin exchange rate and transaction volume, AUD/JPY risk reversal (AUD_rr) and CAD/USD risk reversal (CAD_rr).



Level	К	θ_x	AUD_rr	CAD_rr	ln_txn
Mean	0.0560	0.2432	-2.8278	-0.6777	12.3718
Median	0.0547	0.2486	-2.6300	-0.6200	12.3446
Maximum	0.0845	0.2876	-1.3650	-0.2150	12.8751
Minimum	0.0341	0.2063	-5.1300	-1.4450	11.9392
Std. Dev.	0.0142	0.0197	0.8695	0.2973	0.1913
Skewness	-0.0273	-0.1119	-0.4116	-0.6062	0.2161
Kurtosis	1.7614	1.8843	2.2820	2.4735	2.7490
Observations	157	157	157	157	157
ADF test statistics	-2.086	-1.575	-2.772	-1.278	-2.127
Correlation with κ			-0.0580	-0.0027	0.4509
Correlation with θ_x			0.8122	0.6881	0.3810
Change	К	θ_{x}	AUD_rr	CAD_rr	ln_txn
Mean	-0.0003	0.0003	0.0078	0.0029	0.0032
Median	0.0000	0.0003	0.0475	0.0100	0.0067
Maximum	0.0265	0.0159	0.9800	0.2800	0.2241
Minimum	-0.0155	-0.0124	-1.4975	-0.3650	-0.1978
Std. Dev.	0.0045	0.0036	0.3935	0.0996	0.0579
Skewness	0.9343	0.1378	-0.5270	-0.6151	-0.1727
Kurtosis	13.1476	5.9495	4.3044	5.2799	5.0815
ADF test statistics	-13.249 ***	-8.482 ***	-9.368 ***	-9.417 ***	-5.299 ***
Correlation with $\Delta \kappa$			0.1323	0.1334	-0.0544
Correlation with $\Delta \theta_x$			-0.0515	-0.0120	0.1998

Table 1: Descriptive statistics of Bitcoin transaction volume, AUD and CAD risk reversals, κ and θ .

1. AUD_rr (CAD_rr) is the 1-month 25-delta risk reversal of AUD (CAD) against JPY (USD) whereas (ln_txn) is the natural logarithm of two-week moving average of the Bitcoin transaction volume. The correlations for level of the variables are the correlations with κ and θ , and those for change are the correlation with $\Delta \kappa$ and $\Delta \theta$.

2. The ADF test checks the null hypothesis of unit root existence in the time series, assuming nonzero mean in the test equation, with lag length determined by Akaike information criterion up to maximum length of 4. *** indicates significance at levels of 1% respectively.

Table 2: Tests for co-integration of Bitcoin transaction volume (ln_txn), κ and θ .

Engle-Granger single-equation test²

(Null hypothesis: residual has an unit root)

On ln_txn	ADF test statistic	Phillips-Perron test statistic	
Equation:			
$\Box \kappa$	-3.000 ***	-2.904 **	
$\Box \theta_x$	-3.828 ***	-3.810 ***	

Notes:

1. ***, ** and * indicate significance at the 1% ,5% and 10% level respectively.

2. The Engle-Granger single-equation test (ADF and Phillips-Perron tests) examines the null hypothesis that the residuals of the regressions of κ on ln_txn, and θ on ln_txn plus one dummy specified as in Table 4 respectively, given that κ , θ and ln_txn are non-stationary. The test assumes the existence of zero mean of the residuals in the test equation. The critical value of the test is based on MacKinnon (1996).

3. Alternatively, Gregory and Hansen (1996) derived residual-based tests for testing cointegration with regime shifts. We test the residual from the regression of θ on ln_txn alone based on the Gregory-Hansen cointegration test for the type of regressions with a level shift. The results based on ADF test statistics (Philip test statistic based on Zt) also indicate that the null hypothesis for no cointegration between the variables is rejected at 10% (5%) significance level (with the lag length determined by Akaike information criterion up to maximum length of 4). The identifications of break date vary across the types of shift models chosen for the test, our choice of starting date for the dummy specified as in Table 4 is quite close to the date identified by the Gregory-Hansen test based on a regime-shift-type model. Table 3: Estimates of long-run coefficient (β) for Bitcoin transaction volume (ln_txn), κ and θ .

Dependent variable:	K _t	θ_{xt}
ln_txn _t	0.0191 *	0.0445 ***

Notes: *** and * indicate significance at a level of 1% and 10% respectively. The coefficients are estimated by using the Engle-Granger single-equation and the coefficients of the short-run dynamic are in Table 4.

Table 4: Estimation results of the short-run dynamics for Bitcoin transaction volume (ln_txn), κ and θ .

Dependent variable:	$\Delta \kappa_{ m t}$	$\Delta \theta_{xt}$
Constant	-0.0303	-0.0314 *
Speed of adjustment	-0.1729 ***	-0.0989 ***
$\Delta ln_t x n_{t-1}$	-0.0075	-0.0041
$\Delta ln_t x n_{t-3}$	0.00614	-0.0113 **
$\Delta \kappa_{t-1}$	0.0309	
$\Delta heta_{xt-4}$		-0.1486 **
dummy_bitcoin_crash	-0.0042 ***	
dummy_bitcoin_optimism		0.0026 **

Notes: ***, ** and * indicate significance at a level of 1%, 5% and 10% respectively. For the short-run equation of $(\ln_t xn)$ and κ , a dummy variable for the period after the Bitcoin crash (taking a value of 1 since February 2018) is added for controlling a lower level of κ when the Bitcoin exchange rate declined from peak value. The dummy is statistically significant at the 1% level. For the short-run equation of $\ln_t xn$ and θ , we add a dummy variable for the period for the Bitcoin optimism during 1 June 2017 to 9 November 2018 for controlling the fast growth in θ due to market optimism over Bitcoin adoption. The dummy is again statistically significant at the 5% level. The coefficients are estimated by using the Engle-Granger single-equation and the long-run coefficients are in Table 3.

Table 5: Tests for co-integration of AUD and CAD risk reversals, κ and θ .

Engle-Granger single-equation test²

(Null hypothesis: residual has an unit root)

On AUD_rr	ADF test statistic	Phillips-Perron test statistic
Equation:		
$\Box \kappa$	-3.866 ***	-4.533 ***
$\Box \theta_x$	-5.453 ***	-5.232 ***
On CAD_rr	ADF test statistic	Phillips-Perron test statistic
On CAD_rr <i>Equation:</i>	ADF test statistic	Phillips-Perron test statistic
On CAD_rr <i>Equation:</i>]к	ADF test statistic	Phillips-Perron test statistic

Notes:

1. ***, ** and * indicate significance at the 1% ,5% and 10% level respectively.

2. The Engle-Granger single-equation test (ADF and Phillips-Perron tests) examines the null hypothesis that the residuals of the regressions of κ on AUD_rr (CAD_rr) plus a dummy for period after the Bitcoin crash, and θ on AUD_rr (CAD_rr) respectively, given that κ , θ and AUD_rr (CAD_rr) are non-stationary. The test assumes the existence of zero mean of the residuals in the test equation. The critical value of the test is based on MacKinnon (1996).

3. Alternatively, Gregory and Hansen (1996) derived residual-based tests for testing cointegration with regime shifts. We test the residual from the regression of θ on ln_txn alone based on the Gregory-Hansen cointegration test for the type of regressions with a level shift. The results based on ADF test statistics (Philip test statistic based on Zt) also indicate that the null hypothesis for no cointegration between the variables is rejected at 5% significance level (with the lag length determined by Akaike information criterion up to maximum length of 4). The break date identified by the Gregory-Hansen test is very close to the starting date for the dummy specified for the Bitcoin crash, regardless of the type of shift models considered.

Dependent variable:	Kt	θ_{xt}
AUD_rr _t	0.0057 **	0.0125 *
CAD_rr _t	0.0177 ***	0.0333 *

Table 6: Estimates of long-run coefficient (β) for AUD and CAD risk reversals, κ and θ .

Notes: ***, ** and * indicate significance at a level of 1%, 5% and 10% respectively. The coefficients are estimated by using the Engle-Granger single-equation and the coefficients of the short-run dynamic are in Table 7.

Table 7: Estimation results of the short-run dynamics for AUD and CAD risk reversals, κ and θ .

	(With AUD_rr)	(With CAD_rr)	(With AUD_rr)	(With CAD_rr)
Dependent variable:	$\Delta \kappa_{ m t}$	$\Delta \kappa_{\rm t}$	$\Delta \theta_{xt}$	$\Delta heta_{x ext{t}}$
Constant	0.0164 ***	0.0164 ***	0.0167 **	0.0144 **
Speed of adjustment	-0.2067 ***	-0.2162 ***	-0.0584 **	-0.0523 **
ΔAUD_rr_{t-1}	-0.0007		-0.0011	
$\Delta CAD_{rr_{t-1}}$		-0.0030		-0.0021
$\Delta \kappa_{t-1}$	0.0221	0.0354		
$\Delta heta_{xt-4}$			-0.1771 ***	-0.1700 ***
dummy_bitcoin_crash	-0.0066 ***	-0.0070 ***		
dummy_currency_turmoil			-0.0021 *	-0.0023 *

Notes: ***, ** and * indicate significance at a level of 1%, 5% and 10% respectively. For the short-run equation for the risk reversals and κ , a dummy variable for the period after the Bitcoin crash (taking a value of 1 since February 2018) is added separately in each equation for controlling a low level of κ when the Bitcoin exchange rate declined from the peak. It is statistically significant at the 1% level. For the short-run equation for the risk reversals and θ , a dummy variable for the period from 1 November 2015 to 15 February 2016 for controlling the very negative risk reversals due to the short-lived turmoil in the exchange rate markets of some commodity currencies and emerging economies. The coefficients are estimated by using the Engle-Granger single-equation and the long-run coefficients are in Table 6.