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Exchange rate dynamics under a currency board when policy rates are zero

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Abstract

In a target-zone exchange rate system, both fundamentals and exchange rate expectations, reflected in interest rate differentials between the domestic and anchor currency, determine the exchange rate. However, the scope to capture exchange rate expectations is limited when policy rates are close to their zero lower bound, especially in a narrow-band target zone or currency board. Cook and Yetman (2014) introduce a new mechanism, based on a central bank's balance sheet, which works to bring about equilibrium in currency markets even when interest rates are zero. To investigate how interest rate differentials and balance sheets (monetary base) affect exchange rate dynamics, this paper uses a target-zone model with asymmetric mean-reverting fundamental dynamics to test the data for the Hong Kong dollar (HKD) pegged with the US dollar (USD) under a zero-interest rate environment. The empirical results suggest that the restoring force and long-term mean of the exchange rate dynamics are cointegrated with the monetary base, as well as HKD-USD interest rate differentials. Appreciation (depreciation) expectations of the HKD reflected in the dynamics are positively (negatively) related to capital inflows (interest rate differentials).

Keywords: Target zone model, currency board, Hong Kong dollar

JEL classification: F31, G13

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1. Introduction

Essential to a currency board system is a rule that requires any change in the monetary base to be brought about only by a corresponding change in foreign reserves in an anchor currency at a fixed exchange rate. The monetary base and foreign reserves should respectively be on the liability and asset side of the balance sheet of the monetary authority which operates the Hong Kong currency board. The settlement of transactions arising from the convertibility undertaking between the domestic currency and anchor currency changes the monetary base so that the automatic adjustment mechanism under a currency board system can function to ensure exchange rate stability. A currency board is thus a highly credible monetary policy framework provided that the domestic currency is backed by liquid, risk-free assets denominated in the anchor currency.

In addition to exchange rate adjustment for deviations between the market and fixed exchange rates, a more effective adjustment mechanism of a currency board system works through interest rates. Capital inflows (outflows) will lead to corresponding decreases (increases) in domestic interbank interest rates. Provided that the currency board is credible, the resultant interest rate differentials create interest rate arbitrage opportunities therefore generating offsetting capital flows. Investors sell (buy) domestic currency in exchange for the anchor currency. This process continues until the domestic and anchor interest rates are equalized. Meanwhile, the market exchange rate remains close to the fixed exchange rate level or within the narrow target zone.

However, this mechanism may not work when the policy interest rate of the anchor currency is close to the zero lower bound and the domestic currency is expected to appreciate. While capital flows into the domestic currency, there is no depressing effect on the domestic interest rate, which is constrained by the zero lower bound. ¹ In other words, the effect of appreciation expectations on interest rate differentials is limited when interest rates are close to the zero lower bound. As a result, the inflows causes foreign reserves (monetary base) to continue to increase amid the appreciation expectation. In view of the limitations of the conventional mechanism of currency-board operations,

¹ Although nominal interest rates can be negative when central banks adopt negative interest rate policy, such policy has not been in place for Hong Kong and the US. Hence, the zero lower bound of interest rates remains a constraint for the Hong Kong dollar.

Cook and Yetman (2014) introduce a new mechanism which works through the central bank balance sheet which helps to bring about equilibrium in currency markets even when interest rates are zero. They develop a simple model to illustrate such a mechanism using data for Hong Kong. To investigate how such a mechanism affects exchange rate dynamics, this paper uses a target-zone model with asymmetric mean-reverting fundamental dynamics as proposed by Lo et al. (2015) and Hui et al. (2016). This is applied to data for the Hong Kong dollar (HKD), which is pegged with the US dollar (USD), under a zero-interest rate environment. The empirical results suggest that a target-zone model can describe exchange rate dynamics, where the drifting force and long-term mean are cointegrated with the monetary base and HKD-USD interest rate differentials.

Hong Kong's Linked Exchange Rate System (LERS) has been in operation since 1983, whereby the HKD is fixed at a rate of 7.8 per USD. The LERS is in essence a Currency Board system, which requires both the stock and the flow of the Monetary Base to be fully backed by foreign reserves. Any change in the size of the Monetary Base has to be fully matched by a corresponding change in foreign reserves. Although in theory an exchange rate commitment involving currency only can be expected to lead to convergence between the exchange rate in the interbank market and the fixed rate for currency through arbitrage, this has not always been the case. ² Thus during the Asia financial crisis in September 1998, a wider exchange rate commitment for reserve balances of banks was introduced at a rate of 7.75 HKD/USD. The Hong Kong Monetary Authority (HKMA) stood ready to purchase unlimited amounts of HKDs using USDs to prevent the currency weakening beyond that rate. ³ This commitment was gradually moved from 7.75 to 7.8 between April 1999 and July 2000.

Starting in the fall of 2003 the Chinese renminibic came under pressure to appreciate. The HKD also came under pressure, perhaps due to expectations in the market that the Hong Kong authorities would follow any move by their counterparts on the Mainland to allow the currency to appreciate with respect to the USD. The combination of very low interest rates, which risked creating overheating in

² Arbitrage is a mechanism through which economic agents seek to profit from the deviation between the official exchange rate applicable to the issue and redemption of banknotes and the market exchange rate. For example, when the market exchange rate (say 7.76 for HKD) is stronger than the official exchange rate, banks can buy foreign currency (i.e., USD) in the foreign exchange market, surrender it to the monetary authority in exchange for domestic currency at the fixed exchange rate (i.e., 7.8), and thereby make a profit from the differential between the two rates.

³ The HKMA was established on 1 April 1993, by merging the Office of the Exchange Fund (which was responsible for the LERS) with the Office of the Commissioner of Banking. Its main functions and responsibilities are governed by the Exchange Fund Ordinance and the Banking Ordinance and it reports to the Financial Secretary. One of the functions regarding the exchange rate of the HKD is to maintain currency stability, within the framework of the LERS, through sound management of the exchange fund, monetary policy operations, and other means deemed necessary. See Genberg and Hui (2011) about the development of the LERS.

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the economy in general and in asset markets in particular, and increased uncertainty about the exchange rate level, called into question the credibility of the LERS. In response, the HKMA introduced "Three Refinements" to the LERS on 18 May 2005. These were (i) the introduction of a strong-side Convertibility Undertaking at 7.75 HKD/USD, (ii) moving the weak-side Convertibility Undertaking to 7.85 (in small steps over a five week period) thus creating a symmetric convertibility zone around 7.8, i.e., a target zone, and (iii) giving the HKMA the possibility to intervene inside the convertibility zone. The first two refinements brought the HKD into a narrow-band target zone. Genberg and Hui (2011) find that the LERS has become more credible over time, and their sample period includes the introduction of the convertibility zone. Their assessment is based on extracting information about exchange rate expectations from market prices, in part by investigating exchange rate and interest rate volatility, as well as studying the dynamics of the exchange rate itself.

The target-zone model used for the empirical tests in this paper is the one proposed by Lo et al. (2015) and Hui et al. (2016), in which the fundamentals follows an asymmetric mean-reverting process. The model uses the basic log linear model of the exchange rate on which most of the target-zone literature is based for a small open economy. The log exchange rate is equal to a 'fundamental' term plus a term proportional to the expected change in the log exchange rate. Therefore, the model is in line with those in the target-zone literature with a stochastic differential equation that links the exchange rate between two currencies to underlying fundamentals. The asymmetric mean-reverting fundamental dynamics with a strong force pushing the exchange rate away from the weak-side limit is consistent with the idea of more intensive central bank intervention, and stability speculation by market participants, when the currency depreciates toward the weak-side limit in order to maintain the credibility of the target-zone regime. ⁴

The force on the exchange rate depends on the values of the model parameters on the fundamentals (i.e., the exchange rate dynamics). When both the interest rate of the anchor and domestic currencies are close to their zero lower bound, with the monetary authority operating intervention at the strong-side limit (buying the anchor currency and selling the domestic currency in the market), the model parameters are expected to be adjusted according to changes in the monetary base due to exchange

⁴ The literature on target-zone models is vast and previous studies generally agree that the empirical performance of the Krugman model is not satisfactory. For instance, Bertola and Cabellero (1992) provide data and a model which allows for intra-marginal intervention for situations where the target zone is not credible.

rate expectations as suggested in Cook and Yetman (2014). If the anchor currency's interest rate increases from the zero lower bound such that the scope for exchange rate expectations is relaxed and the interest rate differential widens accordingly, the restoring force and long-term mean of the exchange rate will change with interest rate differentials. The theoretical implications of the model, and how they are associated with macro-financial factors, will be tested empirically by estimating the exchange rate dynamics. These are derived from the asymmetric mean-reverting fundamental dynamics using the HKD exchange rate data covering the period of the zero-interest rate environment after the 2008 global financial crisis.⁵

We discuss the asymmetric mean-reverting fundamental dynamics and its associated exchange rate dynamics in the following section. The probability density function of the exchange rate dynamics is illustrated. A brief discussion on the HKD exchange rate since the adoption of the strong-side and weak-side Convertibility Undertakings in May 2005 is presented in Section 3. Empirical estimates for the HKD based on the probability density function after the introduction of a convertibility zone, and the relationship between the model parameters and macro-financial factors related to capital flows into the HKD in the zero-interest rate environment are discussed in Section 4. The final section of the paper concludes.

2. Asymmetric mean-reverting fundamental dynamics

2.1 Characteristics of fundamental

To establish the relationship between the exchange rate and fundamentals in a target-zone model, we use a basic log linear model of the exchange rate on which most of the target-zone literature is based for a small open economy. The exchange rate is equal to fundamentals f(t) plus a term proportional to the expected change in the log exchange rate:

$$s(t) = m + v(t) + \alpha \frac{\mathrm{E}[ds(t)]}{dt}.$$
(1)

⁵ Lo et al. (2015) and Hui et al. (2016) show that the mean-reverting force is inversely related to central banks' foreign reserves for both the HKD and Swiss franc respectively.

where α is the absolute value of the semi-elasticity of the exchange rate with respect to its expected rate of change, m is the logarithm of the constant relative money supply of the domestic currency, v(t) follows a stochastic process and is the logarithm of a general purpose term encompassing changes in real output, money demand, and other factors in the economy other than the money supply and expected currency depreciation or appreciation, and E is the expectation operator. The monetary authority is prepared to change m by increasing (decreasing) the money supply to prevent s from breaching the band in the case of capital inflows (outflows). But as long as s lies within the band, the money supply is unchanged.

The fundamentals are assumed to follow a stochastic process with drift μ_{ν} and instantaneous standard deviation σ_{ν} :

$$df = dv = \mu_v dt + \sigma_v dZ , \qquad (2)$$

where dZ is a Wiener process with E[dZ] = 0 and $E[dZ^2] = dt$. A zero drift for the fundamentals v used in the basic Krugman model and defined in Eq.(2) suggests central banks only conduct marginal intervention in a target zone. However, extensions of the basic model have been developed to capture features of intra-marginal interventions. Froot and Obstfeld (1991) and Delgado and Dumas (1992) incorporate a simple way to model such interventions with imperfect credibility by specifying that the fundamentals follow a constant trend, and a drift term towards central parity which is proportional to the deviation from central parity, i.e., mean reversion. The constant trend suggests that the domestic currency is inherently weak (strong) with a negative (positive) drift if there is no intervention. The driving force behind the mean-reverting property is central bank intervention within the band (intra-marginal intervention), reflecting a policy of "leaning against the wind". "Stability speculation" by market participants also produces forces to pull the exchange rate back to its long-run equilibrium whenever it drifts too far from it. The mean-reverting fundamentals can be represented by an error-correction term reflecting action by the authorities in the foreign exchange market and market forces respectively. During the Exchange Rate Mechanism (ERM) crisis, speculation in the foreign exchange market and official interventions were more intensive on the weak side than the strong side of the band, with 85% of all European Monetary System interventions done intra-marginally. The corresponding mean-reverting fundamental shocks, due to interventions are likely to be asymmetric with stronger action on the weak side. These observations of central banks' interventions and market forces allow us to model a generalized fundamental shock as used in Lo et al. (2015) for the HKD, given by:

$$dv = \mu_{v}dt + \sigma_{v}dZ$$

= $\frac{1}{2}\left(-\kappa v - \beta + \frac{4\kappa\theta - \sigma^{2}}{4v}\right)dt + \frac{\sigma}{2}dZ$ (3)

for β , θ and $\sigma \ge 0$, and $\sigma_{\nu} = \sigma/2$. Eq.(3) generates a class of parametric drifts, including a zero drift ($\kappa = 0$, $\kappa \theta = \sigma^2/4$ and $\beta = 0$), symmetric mean-reverting drift ($\kappa \theta = \sigma^2/4$) and asymmetric mean-reverting drift ($\beta = 0$). As illustrated in the next subsection, μ_{ν} and σ_{ν} are specified such that via Ito's lemma the solution given by Eq.(6) to Eq.(1) can give rise to a mean-reverting square-root (MRSR) process for the log-exchange rate. Lo et al. (2015) and Hui et al. (2016) find empirical evidence that the asymmetric mean-reverting fundamental dynamics can describe the exchange rate dynamics and interest rate differentials of the HKD in a target zone during 2005-2013, and also the Swiss franc against the euro during the one-sided target zone regime in place between September 2011 and January 2015.

Regarding the asymmetric mean-reverting shock, when v becomes small and approaches its origin (the weak-side limit at v = 0), the last term of the drift in Eq.(3) will push v to a higher value given that the term $(4\kappa\theta - \sigma^2) > 0$. The corresponding domestic currency will appreciate and the exchange rate will move away from its weak-side limit (the origin). Conversely, when v moves close to the strong-side limit, the first term of the drift will push v back to the origin and the domestic currency will depreciate accordingly. The mean-reverting force is not symmetric at the mean level θ . The upward force (domestic currency appreciation) given by the last term in the drift with v close to zero (the weak-side limit) is stronger than the downward force (domestic currency depreciation) provided by the first term. Such an asymmetric mean-reverting property with a strong force pushing the exchange rate

away from its weak-side limit is consistent with the ideas of more intensive central bank intervention, and stability speculation by market participants, when the currency depreciates toward its weak-side limit in order to maintain the credibility of the target-zone regime.

The force on the exchange rate depends on the values of the parameters κ and θ . When both the interest rates of the anchor and domestic currencies are close to the zero lower bound, the scope for exchange rate expectations is limited. Under the mechanism proposed by Cook and Yetman (2014) based on central bank balance sheets, an expectation of exchange rate appreciation will cause foreign reserves to swell reflecting the increase in the monetary base. This lowers the likelihood of the target zone being abandoned. With the monetary authority operating intervention at the strong-side limit (buying the anchor currency and selling the domestic currency in the market), the model parameters are expected to be adjusted according to the mechanism that brings about equilibrium in currency markets. In the mean-reverting fundamental dynamics, interventions will increase κ ; indicating an increase in the restoring force towards the exchange rate and its long-term mean which is related to θ as shown in Eq.(10) below. When the exchange rate and its long-term mean move towards its strong-side limit as a result of capital inflows, the tendency to mean-reversion can act as a stabilizing force limiting the upward movement of the exchange rate.

To further understand the asymmetric mean-reverting fundamental dynamics, we obtain a "potential well" U(v) by integrating the drift term in Eq.(3), in negative form, with respective to v:

$$U(\nu) = -\int \frac{1}{2} \left(-\kappa \nu + \frac{4\kappa\theta - \sigma^2}{4\nu} \right)$$

= $-\left(\kappa\theta - \frac{\sigma^2}{4}\right) \ln\nu + \frac{\kappa\nu^2}{4}$, (4)

in which the fundamental variable v is like a ball moving in a well as shown in Figure 1 by plotting Eq.(4) with different values of κ and θ . Decreasing κ will give an extremely flat potential well covering the whole v, such that the Brownian force in the stochastic term will dominate the motion of the fundamental variable. The fundamental and thus the exchange rate can then move more randomly with a weaker restoring force within the target zone. On the other hand, decreasing θ will allow the

fundamentals to approach the origin more easily, given the trough of the "well" moving towards the origin. As the force due to κ remains unchanged, the Brownian force is too weak to enable the fundamentals to move freely towards the strong side. In the subsection 2.2 below, the interest rate differential between the domestic and anchor currencies (i.e., the exchange rate expectation) is shown to be related to the parameter θ , according to the model.

In Section 4, the above theoretical implications and how they are associated with macro-financial factors will be tested empirically by estimating the exchange rate dynamics derived from the asymmetric mean-reverting fundamental dynamics using the HKD exchange rate data.

2.2 Exchange rate dynamics and probability density function

As shown in Lo et al. (2015) and Hui et al. (2016), the desired solution of Eq.(1) is given by:⁶

$$s(v) = v^2 \sum_{n=0}^{\infty} A_n v^n \tag{5}$$

where

$$A_{0} = -\frac{m}{\alpha\kappa\theta} < 0$$

$$A_{1} = -\frac{1}{\alpha\kappa\theta} \left\{ \frac{2}{3} \left(1 + \frac{\sigma^{2}}{4\kappa\theta} \right)^{-1} \right\}$$

$$A_{n+2} = \frac{1}{\alpha\kappa\theta} \left\{ \frac{2 + (n+2)\alpha\kappa}{n+4} \left[1 + \frac{(n+2)\sigma^{2}}{4\kappa\theta} \right]^{-1} \right\} A_{n}.$$

⁶ See the Appendix of Lo et al. (2015) for details.

For n = 0,1,2,... The series solution is shown to be a convergent series for all v by means of the ratio test as $\lim_{n\to\infty} |A_{n+2} / A_n| \to 0$ and numerically in Lo et al. (2015) and Hui et al. (2016). As a result, the leading term of the expression:

$$s(v) = A_0 v^2$$

= $-mv^2 / (\alpha \kappa \theta)$ (6)

is a good approximation of the exact relationship between s and v, with the prescribed boundary condition:

$$\left. \frac{ds(v)}{dv} \right|_{v=0} = 0.$$
⁽⁷⁾

Eq.(7) is a smooth-pasting boundary condition at the weak-side limit implying an optimal boundary condition for the process, where interventions involve buying the domestic currency from the market. At the boundary, there is no foreseeable jump in the exchange rate, i.e., no arbitrage condition. Krugman and Rotemberg (1990) show that the smooth-pasting condition at the strong-side boundary ensures that the exchange rate does not cross the bound of the target zone.

We consider the exchange rate S defined as a domestic currency value (i.e., HKD) of a unit of a foreign currency (i.e., USD), and let $S_S = 7.75$, $S_W = 7.85$ and $S_{cp} = 7.8$ be the strong-side limit, weak-side limit and central parity respectively. With no loss of generality, the normalized dimensionless log exchange rate x is defined by:

$$x \equiv -s = -\ln\left(\frac{S - S_s}{S_w - S_s}\right) \tag{8}$$

where $x = \infty$ corresponds to the strong-side limit which is thus inaccessible, and x = 0 corresponds to the weak-side limit.

By applying Ito's lemma to Eq.(3) with the asymmetric mean-reverting fundamental dynamics and Eq.(6), the dynamics of x are shown to follow a MRSR:

$$dx = \kappa (\theta_x - x)dt + \sigma_x x^{1/2} dZ, \qquad (9)$$

where

$$\theta_{\rm r} = -A_0 \theta > 0 \,, \tag{10}$$

$$\sigma_x = \sigma \sqrt{-A_0} . \tag{11}$$

 $\sigma_x^2 x$ is the variance that depends upon the level of x, and κ determines the speed of the meanreverting drift towards the long-term mean θ_x . The empirical results in section 4 below demonstrate that the dynamics specified in Eq.(9) can adequately describe the movements of the HKD. Given the linear relationship between the model parameters of the exchange rate x and the fundamentals v in Eqs.(10) and (11), the relationship between macro-financial factors and model parameters of the fundamental dynamics can be tested directly using the exchange rate data.

The maximum likelihood estimation (MLE) is used to estimate the model parameters of the process specified in Eq.(9) based on a log-likelihood function which is constructed by the probability density function (p.d.f.) of Eq.(5) in Lo et al. (2015) of the process:

$$G(x,t;x',t') = \frac{4}{\sigma_x^2 C_1(t-t')} \left(\frac{x}{x'}\right)^{\omega/2} \exp\left[-\frac{\omega+2}{2}C_2(t-t')\right] \times \\ \exp\left\{-\frac{2x'+2x \exp\left[-C_2(t-t')\right]}{\sigma_x^2 C_1(t-t')}\right\} \times , \qquad (12)$$

$$I_{\omega}\left\{\frac{4x^{1/2} x'^{1/2} \exp\left[-C_2(t-t')/2\right]}{\sigma_x^2 C_1(t-t')}\right\}$$

where $\omega = 2\kappa \theta_x / \sigma_x^2 - 1$, $C_1(\tau) = [\exp(\kappa \tau) - 1] / \kappa$, $C_2(\tau) = -\kappa \tau$, and I_{ω} is the modified Bessel function of the first kind of order ω .

While the exchange rate dynamics keeps a smooth-pasting boundary condition Eq.(7) at the weakside limit of the target zone, the exchange rate can breach the limit (i.e., realignment or abandoning the target zone) and therefore evoke a breakdown of the smooth-pasting boundary condition under the leakage condition of $(4\kappa\theta_x - \sigma_x^2 < 0)$ of the relationship between the parameters of the drift term and stochastic part of the process.⁷ The exchange rate is thus quasi-bounded in the target zone. The property of the quasi-bounded dynamics is discussed in detail in Hui et al. (2016).^{8,9}

The asymmetric mean-reverting fundamental dynamics has implications for the relation between the exchange rate and interest rate differential. By following Svensson (1991) and assuming that uncovered interest rate parity (UIP) continually holds in the transformed exchange rate x(t), Lo et al. (2015) show:

$$\delta(f,t) = \frac{x_0 - x_0 \exp(-\kappa t) - \theta_x [1 - \exp(-\kappa t)]}{t},$$

$$= (x_0 - \theta_x) \left\{ \frac{1 - \exp(-\kappa t)}{t} \right\},$$
(13)

where $\delta(f,t) = r(t) - r^*(t)$ is the interest rate differential in which r(t) and $r^*(t)$ are the domestic (HKD) and foreign (USD) interest rates of the exchange rate x(t) respectively.

Eq.(13) suggests that when the spot exchange rate is momentarily weaker (stronger) relative to the mean level θ_x , the interest rate differential $\delta(f,t)$ is negative (positive), i.e., the domestic (HKD) interest rate is lower (higher) than its foreign (USD) counterpart. It is because market participants expect foreign exchange gains (losses) from the future exchange rate appreciation (depreciation) under UIP.¹⁰ This property is consistent with how the target zone of the HKD operates when the USD interest rate is at the zero lower bound. If a strong HKD is expected, θ_x will be close to the strong-side boundary and scope for exchange rate expectations is limited as the HKD interest rates is also close

⁷ The boundary behaviour is determined by the values of the parameters of the process. See Karlin and Taylor (1981, ch. 15).

⁸ Hui et al. (2016) find empirical evidence that the quasi-bounded process can describe the exchange rate dynamics and interest rate differential of the Swiss franc against the euro during the target zone regime of September 2011 to January 2015. While the exchange rate was bounded below the strong-side limit during most of the time, as indicated by its dynamics, the condition for breaching the limit was met in November 2014 using only information until that point, i.e., about two months before abandoning the limit. The asymmetric mean-reverting fundamental therefore incorporates both the characteristics of intervention and realignment.

⁹ Such a property is similar to the bounded exchange rate dynamics in Ingersoll (1996) and Larsen and Sørensen (2007) in which the variance of the exchange rate vanishes at both the weak-side and strong-side limits in a two-sided target zone. In their models the exchange rate is completely bounded under all circumstances determined by the model parameters. However, the exchange rate following the quasi-bounded process can breach the limit under particular conditions.

¹⁰ This feature is different from that in the original Krugman target zone in which the interest rate differential is positive (negative) in the regime where the exchange rate is stronger (weaker) than the central parity, as the exchange rate in the Krugman model is stationary around the central parity.

to the zero lower bound. The monetary authority thus needs to sell HKD in the market to prevent the exchange rate breaching the boundary, i.e., increases in the monetary base and foreign reserves. Once the USD interest rate increases, a negative interest rate differential will emerge with the spot exchange rate weaker than θ_x given that the strong HKD remains to be expected. The actual market interest rate differential in respond to this mechanism is illustrated in the following section.

3. HKD exchange rate dynamics under zero interest rates

Figures 2 and 3 show the HKD exchange rate in S and the transformed exchange rate in x respectively from 18 May 2005 to 30 June 2017. After trading within a relatively wide range between May 2005 and August 2008, the HKD exchange rate strengthened towards the strong-side Convertibility Undertaking of 7.75 after September 2008 due to capital inflows, with the strong-side Convertibility Undertaking triggered repeatedly thereby prompting the HKMA to passively inject liquidity into the banking system.¹¹ With the capital inflows, the exchange rate stayed close to the 7.75 for a considerable period of time as shown in Figure 2.¹² Subsequently, the HKD exchange rate weakened but stayed on the strong side for most of the time.

The suspension of trading after the drop of the Chinese stock market on January 2016 led to a sharp correction in global asset prices, including the HKD exchange rate which weakened sharply by nearly 600 pips in a week. With the subsequent stabilization of the external environment, the HKD exchange rate quickly rebounded and stayed in the strong-side convertibility zone for the rest of 2016. As the US Federal Reserve repeatedly raised its policy rates in March and June of 2017, the widening of the HKD-USD interest rate differential incentivised more arbitrage activities and led to a notable

¹¹ The capital inflows into the HKD after the global financial crisis intensified in September 2008 is discussed in "Latest Analysis of Fund Inflows Into the Hong Kong Dollar" by Norman Chan at <u>http://www.hkma.gov.hk/eng/key-information/insight/20100201.shtml</u>

¹² The Monetary Base (a part of the monetary liabilities of a central bank) is defined, at the minimum, as the sum of the currency in circulation (banknotes and coins) and the balance of the banking system held with the central bank (the reserve balance or the clearing balance). In Hong Kong, the Monetary Base comprises Certificates of Indebtedness (for backing the banknotes issued by the note-issuing banks), government-issued currency in circulation, the balance of the clearing accounts of banks kept with the HKMA (the Aggregate Balance), and Exchange Fund Bills and Notes.

weakening of the HKD exchange rate. At the end of June 2017, the HKD exchange rate closed at 7.8053.

4. Empirical test of mean-reverting square-root (MRSR) process for HKD

4.1 Estimations of model parameters

In this section, we investigate whether a MRSR process for exchange rate dynamics within a target zone can describe movements in the HKD. The maximum likelihood estimation (MLE) is used to estimate the model parameters of the process specified in Eq.(9) based on a log-likelihood function which is constructed by the analytical p.d.f. of Eq.(12). The MLE on the daily exchange rate uses time series data from 18 May 2005 to 30 June 2017 for estimation. The estimation uses a rolling three-year window with the initial window covering the period between May 2005 and May 2008.

The estimated σ_x shown in Figure 4 ranges between 0.11 and 0.23, which is relatively steady. The corresponding *z*-statistic is much higher than 1.96 (i.e., at 5% significance level) indicating that the estimation of σ_x is highly significant. The results suggest that estimation of the square-root-process part of the MRSR dynamics is robust, and the changes of σ_x over time are within a relatively narrow range.

Figure 5 shows that the estimates of the drift term κ are more significant in terms of the z-statistic (higher than the 5% significance level) when κ is higher than 0.01. In other words, the drift becomes weaker and less significant when κ is not significantly different from zero. κ fluctuated in a narrow range in the early part of the sample but edged up significantly in 2014. Th1e increase in κ occurred at a time when there were continued capital inflows to the HKD. κ continued to surge until January 2016 when it dropped notably due to an abrupt weakening of the exchange rate amid the suspension of trading in the Chinese stock market. Consistent with the dynamics of the exchange rate, κ quickly

stabilised at 0.04 for the rest of 2016. In March 2017, the exchange rate weakened and κ declined again due to more interest rate arbitrage activities induced by a widening HKD-USD interest rate differential.¹³ In section 4.2 below we study how the estimated model parameters are related to macro-financial factors such as the monetary base and interest rate differentials.

Figure 6 shows that the estimated long-term mean θ_x is significant and ranges from x = 1.31 (7.7769 in S-scale) to x = 3.94 (7.7520 in S-scale) which falls within the strong-side convertibility zone. It is noteworthy that θ_x tends to strengthen further when there are significant capital inflows into the HKD. Meanwhile, the dynamics of θ_x share a similar picture to κ and both show a notable decline from March 2017 onwards.

In summary, the estimation results based on the MLE shown in Figures 4-6 provide evidence that the MRSR process adequately fits the data on the HKD exchange rate in the convertibility zone. Although there has been no pressure on the weak-side limit during the estimation period, the dynamics of the exchange rate may still be informative about the credibility of the target zone if the same dynamics occur on the weak side of the convertibility zone. To check whether the weak-side limit is bounded given the estimated parameters, the value of $\sigma_x^2/4\kappa\theta_x$ is presented in Figure 7. As the condition of $\sigma_x^2/4\kappa\theta_x < 1$ indicates no probability leakage, the results show that, the exchange rate is bounded at the weak-side limit.

According to Eq. (13), the model predicts that when the spot exchange rate is weaker than the longterm mean, the interest rate differential should be negative. To see this empirically, we plot the difference between the spot and estimated long-term mean exchange rate, and the HKD-USD interest rate differential as measured by the 3-month HIBOR and LIBOR spread in Figure 8. For illustrative purposes, we recast θ_x in *S* space and plot its difference with the HKD spot exchange rate (*S*). It can be seen that negative interest rate differentials typically occur when $\theta_s - S$ is negative (i.e., the spot exchange rate is weaker than the long-term mean). Hence, this provides further evidence of consistency between the observed dynamics and the MRSR process of the HKD exchange rate.

¹³ The weakening of exchange rate and decline in *k*⁻ induced by interest rate arbitrage are possible as the interest rate of the anchor currency (USD) is no longer constrained at the zero lower bound.

4.2. Relationship between model parameters and macro-financial factors

In Cook and Yetman's proposed mechanism, much larger changes in foreign reserves are required to equilibrate currency markets when interest rates are zero, as an expectation of exchange rate appreciation will cause foreign reserves to swell. Therefore, the estimated model parameters κ and θ_x in the HKD exchange rate dynamics are expected to be related to changes in the monetary based which is fully backed by foreign reserves and the HKD-USD interest rate differential. We examine their inter relationship using the cointegration method. We measure capital inflows using the logarithm of the monetary base (*MB*) as shown in Figure 9. ¹⁴ The interest rate differential (*ID*) is measured by the three-month HIBOR and LIBOR spread.

The empirical relationship between capital flows and the model parameters is motivated by the experience of the Swiss francs. To overcome the over-valuation of the Swiss francs, the Swiss National Bank (SNB) introduced an exchange rate peg in 2011 at a fixed rate of 1.2 Swiss francs against 1 Euro. Partly due to its feature as a "safe heaven" asset, investors continue to demand Swiss francs after the introduction of the peg. To maintain the credibility of the exchange rate limit when there were capital inflows into the Swiss franc, SNB sold Swiss francs in the foreign exchange market to keep the exchange rate below the strong-side limit. These interventions not only increase the banks' foreign reserves, but also affect the exchange rate dynamics. In particular, Hui et al. (2016) show that the drift term κ is an increasing function of the SNB's reserves. We extend the analysis in Hui et al. (2016) by postulating that there is a long-run equilibrium relationship between the model parameters, capital flows and interest rate differential. The short-run dynamics represented as a dynamical error-correction model are given by:

$$\Delta y_{t} = a_{10} + \alpha_{y} (y_{t-1} - \beta_{1} X_{t-1}) + \sum_{k} b_{1k} \Delta y_{t-k} + \sum_{k} c_{1k} \Delta X_{t-k} + \varepsilon_{yt}$$
(14)

¹⁴ In unreported regressions, we have replaced the proxy of capital flows from monetary base to domestic property prices and all empirical results remain valid. The two proxies of capital flows enter the error-correction model separately because they are highly collinear. Details are available upon request.

where y_t is either κ or θ_x at time t, and α_y is greater than zero. X_{t-1} is a linear combination of the monetary base and interest rate differentials at time $t \cdot 1$ (i.e., $X_{t-1} = \gamma_1 MB_{t-1} + \gamma_2 ID_{t-1}$). As specified, the variable will change in response to stochastic shocks (represented by ε_{yt}) and to the previous period's gap from the long-run equilibrium (i.e., $y_{t-1} - \beta_1 X_{t-1}$). The parameter α_y is the speed of adjustment. In absolute terms, the larger is α_y the greater the response of y_t to the previous period's gap from the long-run equilibrium. If α_y is equal to zero, the long-run equilibrium relationship does not appear and the model is not an error-correction one or cointegrated. Thus, for a meaningful cointegration and error-correction model, the speed of adjustment α_y must be non-zero.

The estimation is conducted at monthly frequency using data between January 2009 and June 2017 and month-end figures for the model parameters κ and θ_x are used. Table 1 provides the Augmented Dickey-Fuller (ADF) test results for κ and θ_x in levels and changes. It fails to reject at the 10% level the presence of a unit root for κ and θ_x in levels. However, the test for the first differences is significant at the 5% level or less. Thus, their levels are non-stationary while the changes are stationary. This suggests that the variables considered are all I(1) (i.e., integrated of the same order 1), which satisfies the requirement for the variables to be cointegrated. Table 2 reports the Engle-Granger (1987)'s and Johansen (1991)'s cointegration tests. As can be seen, both conintegration tests in each row of the regressions considered are all significant at conventional statistical significance levels. Thus, we reject the null hypothesis that the model parameters and macro-financial factors are not cointegrated (i.e., there exists at least one cointegrating vector).

Table 3 reports the estimated cointegrating vectors. We find that κ is positively related to the monetary base, indicating that a stronger capital inflow to the HKD would increase the restoring force towards the long term mean. Intuitively, it means that when capital inflows push the HKD exchange rates towards its strong-side limit, there is a stronger tendency of mean reversion which acts as a stabilizing force to limit further appreciation of the domestic currency. Regarding the role of the interest rate differential, Cook and Yetman (2014) argue that the HKD-USD interest rate differential should be negatively related to the expected appreciation of the domestic currency. A strong

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appreciation expectation would mimic the effect of capital inflows and hence we expect that a widening negative interest rate differential would increase restoring force through κ so as to pull the exchange rate back to its long-term mean. Our empirical analysis in which κ is negatively related to the HKD-USD interest rate differential supports this argument. In addition, Eq.(13) suggests that an increase in κ widens the interest rate differential. The empirical result is thus consistent with Eq.(13) derived from the target zone model.

The second column of Table 3 provides the results for θ_x . The coefficient on the monetary base is positive, suggesting that more capital inflows induce an appreciation of the long-term mean of the exchange rate. The result supports the mechanism proposed by Cook and Yetman (2014) in which an expectation of exchange rate appreciation causes foreign reserves (monetary base) to swell. Similar to the analysis of κ above, a widening negative interest rate differential can be interpreted as capturing a stronger appreciation expectation of the HKD, which induces a stronger long-term mean of the HKD exchange rate. Eq.(13) also suggests that an increase in θ_x widens the negative interest rate differential. The negative relationship found in the empirical results in Table 3 is consistent with this analysis. While a widening negative HKD-USD interest rate differential would induce interest rate arbitrage activities, that is selling the HKD (lower yield currency) and buying the USD (higher yield currency) and thus inevitably weaken the HKD exchange rate, the strong HKD expectation remains a dominating factor in the estimation period with the long-term mean of the exchange rate are negatively related to the interest rate differential.

Finally, Table 4 reports the estimates of the short-run dynamics (i.e., Eq. (14)). In all regressions considered, the speed of adjustment parameters are significant and smaller than 1 in absolute value. This suggests that the error correction specification is valid and there is a self-restoring force to close the gap of the link between model parameters and macro-financial factors.

5. Conclusion

Exchange rate adjustment for deviations between the market and fixed exchange rates under a currency board through interest rates may not work when the interest rate of the anchor currency is close to the zero lower bound and the domestic currency is expected to appreciate. Cook and Yetman (2014) suggest that expansion of the central bank balance sheet can help the exchange rate move toward equilibrium in currency markets even when interest rates are zero. To investigate how the interest rate differential and central bank's balance sheet (monetary base) affect exchange rate dynamics, this paper uses a target-zone model with an asymmetric mean-reverting fundamental dynamics as proposed by Lo et al. (2015) and Hui et al. (2016). It tests the model on data for the HKD pegged to the USD under a zero-interest rate environment.

The empirical results suggest that the mean-reverting square-root process derived from the targetzone model can describe the exchange rate dynamics, where the restoring force and long-term mean are cointegrated with capital flows (the monetary base) as well as the HKD-USD interest rate differential. When both the interest rates of the anchor and domestic currencies are close to the zero lower bound, the model parameters associated with the restoring force and long-term mean of the exchange rate, reflecting the appreciation expectation for the HKD, are positively related to capital inflows as suggested in Cook and Yetman (2014). The empirical results also show that a widening of the negative interest rate differential is related to increases in the restoring force and long-term mean of the exchange rate (i.e., appreciation expectation of the HKD) that is consistent with the mechanism proposed by Cook and Yetman (2014). In addition, the interest rate differential derived from the target-zone model is consistent with market data and our empirical results. While a widening negative HKD-USD interest rate differential would induce interest rate arbitrage activities (selling the HKD and buying the USD), and inevitably weaken the HKD exchange rate, the strong HKD expectation remains a dominating factor over the estimation period with the long-term mean exchange rate stronger than the spot rate.

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Figure 1: Eq.(4) of U(v) on asymmetric mean-reverting fundamental dynamics with different model parameters (kappa) and (theta), where $\sigma = 0.25$

Figure 2: HKD-USD exchange rate in the convertibility zone with strong-side limit at and weak-side limit at S = 7.85.





Figure 2: HKD-USD exchange rate in the convertibility zone with strong-side limit at $x = \infty$ and weak-side limit at S = 7.85

Figure 4: Estimated σ_x and corresponding z-statistic using three-year rolling window





Figure 5: Estimated κ and corresponding z-statistic using three-year rolling window.

4.5 16 4 14 3.5 θ_x (LHS) 12 3 10 z-statistic 2.5 8 θ_x 2 6 1.5 4 1 z-statistic (RHS) 2 0.5 0 0 Nov-08 Nov-16 May-08 Nov-11 May-12 Nov-12 May-13 Nov-13 May-14 May-15 Nov-15 May-16 May-17 May-09 Nov-09 May-10 Nov-10 May-11 Nov-14

Figure 6: Estimated σ_x and corresponding z-statistic using three-year rolling window





Figure 8: Estimated values of the gap between the long term equilibrium and the spot HKD exchange rate ($\theta_s - S$) and the 3-month (3M) HIBOR-LIBOR interest rate differential



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Figure 9: Monetary Base of HKD (before discount window activity)

	ADF test statistic			
Null hypothesis: The variable has a unit root.				
	Level	<u>Change</u>		
κ	-1.786	-2.821*		
θ_{r}	-1.678	-4.145 ***		
ŴВ	-1.257	-3.699 ***		
ID	-2.667	-7.552***		

Table 1: Unit r	oot tests for va	ariables in Eq. (14)
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Notes:

1.*** and * indicate significance at the 1% and 10% level respectively.

2.All the tests include nonzero mean in the test equation, with the exception of *ID*, which also include a trend

	Engle-Granger single-equation test ²		Johansen sy	vstem test ³	
	ADF test statistic	Trace test	statistics	Maxim eigenvalu statist	um ue test ics
Null hypothesis:		r = 0	<i>r</i> = 1	r = 0	<i>r</i> = 1
κ , <i>MB</i> and <i>ID</i>	-3.251 ***	53.96***	11.57	42.39 ***	6.79
θ_x , <i>MB</i> and <i>ID</i>	-2.579 **	30.19***	9.33	20.85 **	6.02

Table 2: Tests for cointegration

Notes:

vectors.

1.***, ** and * indicate significance at the 1%, 5% and 10% level respectively.

2. The Engle-Granger single-equation test examines the null hypothesis that the residuals of the regressions of the model parameters κ and θx on macro-financial factors *MB*, *HP* and *ID* are non-stationary. The test assumes zeromean of the residuals in the test equation. The critical value of the test is based on MacKinnon (1996). 3. The Johansen system test performs a sequential testing on null hypotheses of the number of cointegration vectors (*r*) in the system formed by the model parameters κ and θ_x on macro-financial factors *MB*, *HP* and *ID*. There are two types of the Johansen test, either with the uses of trace or the maximum eigenvalue, where the alternative hypotheses are different (Trace: $r = r^* < k$ vs. r = k; Maximum eigenvalue: $r = r^*$ vs. $r = r^* + 1$). Nevertheless, in both cases the first non-rejection of the null hypothesis will be used as the estimate of the number of cointegration

Table 3: Estimates of cointegrating vectors

	κ	$ heta_x$
Constant	-0.295 ***	-10.92 ***
МВ	0.044 ***	1.932 ***
ID	-0.018 ***	-0.622***

Notes:

*** and * indicate significance at the 1% and 10% level respectively.

Table 4: Estimation results of short-run dynamics			
Dependent variables	κ	$ heta_{\chi}$	
Constant	-0.00004	0.022	
Speed of adjustment	-0.048**	-0.059 **	
$\Delta \kappa_{t-1}$	0.116		
$\Delta \theta_{x,t-1}$		0.099	
ΔID_{t-1}	-0.002	0.046	
ΔMB_{t-1}	0.013	-0.639	

Notes:

** and * indicate significance at the 5% and 10% level respectively.