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Debt Deleveraging and the Zero Bound: Potentially Perverse Effects of Real Exchange Rate Movements*

Paul Luk

Oxford University Hong Kong Institute for Monetary Research

and

David Vines

Oxford University Centre for Applied Macroeconomic Analysis Australian National University Centre for Economic Policy Research

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Abstract

We present a microfounded two-country model of global imbalances and debt deleveraging. A sustained rise in saving in one country can lead to a worldwide fall in interest rates and an accumulation of debt in the other country. When a subsequent deleveraging shock occurs, interest rates are forced down further. In the presence of a zero bound to interest rates, the deleveraging country may face a combination of a large fall in output, deflation, a rise in real interest rates and real exchange rate appreciation. Such exchange rate appreciation will intensify the loss in output, magnify the deflation and further tighten the deleveraging constraint.

Keywords: Global Imbalances, Debt Deleveraging, Liquidity Trap, Real Exchange Rate JEL Classification: E5, F3

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E-mail addresses: pskluk@hkma.gov.hk (Paul Luk), david.vines@economics.ox.ac.uk (David Vines)

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1. Introduction and Summary

This paper argues that exchange rate movements played a part in exacerbating the early stages of the global financial crisis. We present a two-country dynamic DSGE model to study the interconnections between global imbalances, deleveraging and a zero bound, and examine resulting exchange rate behaviour. We suggest that the crisis can be understood in the following way. Initially deleveraging by borrowers in the US caused a collapse in output and interest rates fell to the zero bound. The result was deflation, leading to a *rise* in real interest rates, and an *appreciation* of the US dollar. In a circular process, such an appreciation intensified the collapse in output and magnified the extent of deflation, thereby further raising real interest rates, and exacerbating the appreciation of the dollar.

This argument brings two ideas together in a formal model, for the first time. Ben Bernanke has argued that it is impossible to understand the global financial crisis (GFC) without reference to the global imbalances in trade and capital flows that began in the latter half of the 1990s as a result of the global 'savings glut'. But the extent of the savings glut is treated as exogenous. (Bernanke, 2005; see also Obstfeld and Rogoff, 2007, henceforth OR, and Obstfelt and Rogoff, 2009.) By contrast, Paul Krugman has argued that any analysis of the GFC requires an understanding of how deleveraging caused interest rates to fall to the zero bound and led to a collapse in aggregate demand. (Krugman, 2012; Eggertsson and Krugman, 2012). But Eggertsson and Krugman (henceforth EK) make no explicit reference to global issues. The model shows how these issues may be thought about together at the same time.

The two countries in the model correspond to the US and China; we examine two (extended) time periods. During the first period - that of the Great Moderation - there is a savings glut in China and US debt gradually increases. The second period starts when deleveraging begins; during this period the US external debts are gradually repaid. If deleveraging is strong enough, the zero bound may bind. Our model permits a formal account of the interconnections between the deleveraging process, the resulting interest rate changes and the emerging exchange rate movements.

Our analysis proceeds as follows. We represent the global savings glut of Bernanke, and OR, in the manner suggested by EK. There are two types of agents, savers - in the US - and borrowers in China. Savers are more patient than borrowers: we suppose that there is a rise in the subjective discount factor in China relative to that in the US, a rise which is persistent but which gradually disappears. Global interest rates fall to ensure that resources remain fully employed, and fall by more in China than in the US, since that is where the savings shock happens. In the absence of deleveraging, the Chinese real exchange rate initially depreciates and then gradually appreciates, in a way consistent with relatively lower Chinese real interest rates. As a result, the ultimate outcome is one with higher US debt.

We assume that deleveraging is imposed when US debt reaches a certain level. The consumption of

savers in China continues to follow an Euler equation, but that of borrowers in the US is constrained by their debt limit; this is assumed to gradually return to its initial level, at an exogenous rate, as in Benigno and Romei, 2012). Interest rates need to fall in order for resources to remain fully employed, when there is no zero bound, we assume that this is achieved and that prices remain stable in both countries. Interest rates need to fall by more in the US than in China, since that is where the deleveraging shock happens. The US currency initially depreciates to enable US debt to be repaid; such a depreciation assists lower US interest rates in keeping resources fully employed there. Lower interest rates in China ensure that resources there also remain fully employed, even although the Chinese currency has appreciated. Over time the US real exchange rate gradually appreciates, in a way which is consistent with the relatively lower interest rates there.¹

A zero bound is encountered in the US if the deleveraging shock is large enough.² This zero bound in the US causes a fall in output and deflation there. But as a result of such deflation the gap between real interest rates in China and the US narrows; real interest rates can become higher in the US. This means that the real exchange rate of the US appreciates, magnifying the negative effects of a zero bound on US output and the extent of the deflation. This makes the deleveraging constraint even more binding, leading to lower consumption, more deflation and an even larger fall in output. Of course, when the zero bound ceases to bind the outcome must revert to one in which the collapse in US output has disappeared, real interest rates in the US are below those in China, the US real exchange rate is depreciated, the real exchange rate is gradually appreciating as the deleveraging is unwound. But the initial appreciation of the dollar, caused by the temporarily higher real interest rate in the US, can magnify the initial negative effects on output and deflation.

Blanchard and Milesi-Ferretti (2011) suggest that, in the presence of a zero bound, the Chinese exchange rate might have depreciated as a result of a deliberate policy to maintain full employment of resources there. The analysis here provides an explanation of why something similar might have been produced as a market outcome rather than as a policy response, as a result of the relative rise in the US real interest rate caused by deflation. It provides such an explanation without appealing to the popular 'flight to safety' idea.

Our analysis is consistent with what was observed in the US immediately after the Lehman crisis of 2008. Subsequently, although a zero bound has remained in advanced countries since the crisis, the process of deflation -- which is central to our story -- came quickly to an end. It is possible that the mechanism which we identify might have worked more strongly and for a longer period of time had not other polices -- for example quantitative easing -- come into play. Something similar appears to have been at work following the crisis in Japan at the end of the 1980s.

¹ Something similar happens in Blanchard and Milesi-Ferretti (2011). That model is static and abstracts from differences in interest rates between the two regions; here we allow for such a difference, which emerges as part of the dynamic process of adjustment.

² For reasons already described, the fall in interest rates that would be required -- if there were no zero bound -- to keep resources fully employed is larger in the US than it is in China. As a consequence, we suppose that a zero bound is encountered only in the US -- an assumption which mirrors reality.

For the purposes of our analysis, we assume that consumers have Greenwood-Hercowitz-Huffman (GHH) preferences. This ensures that output and consumption move in the same direction, as required by the data. In a closed economy, this would happen even with additively separable preferences: a fall in consumption causes a fall in the real wage, a reduction in labour supply, and so a fall in output. But in an open economy, the real exchange-rate depreciation which results from the fall in the real wage will, with reasonable parameters, cause such a large increase in foreign demand for domestic goods that output will rise. GHH preferences avoid this problem.

We also need to solve a non-linear model. Our system has one endogenous state variable, foreign debt *D* and there is a unit root process in our system; the level of debt at any point in time depends on the time-path of the system after the shock has been applied.³ As a result of this unit root we do not have a unique steady state and so cannot log-linearise around it.⁴ Nevertheless, once exogenous shocks dissipate, there are no dynamics in our system. That makes it straightforward to use a shooting algorithm to solve the model. In fact, our model is the first open-economy macro model that solves a zero bound problem using such a global solution method. The working of this algorithm is explained in a technical appendix.

1.1 Related Literature

There are many other papers studying global imbalances, including that by Blanchard and Milesi-Ferretti (2011). Caballero, Farhi and Gourinchas (2008) have developed a two-country model to match low real interest rates, a current account deficit in the US and a rise in the fraction of US assets in global portfolios. Their model explains global imbalances by an inability of the financial systems of the emerging economies to supply assets to absorb the savings available. In their model, global imbalances are caused by the inadequacy of asset markets in emerging economies in the wake of the Asian financial crisis, and by fast growth of China. Here we abstract from the details of the asset market and model the aftermath of the Asian financial crisis as an exogenous rise in Chinese consumers' preference to save. This is a common approach, shared by Artige and Cavenaile (2011) and Choi, Mark and Sul (2008).

There are a number of other models of deleveraging and the zero interest rate lower bound in addition to EK, including Eggertsson and Woodford (2003) and Christiano, Eichenbaum and Rebelo (2010), Guerrieri and Lorenzoni (2012) and Philippon and Midrigan (2011).

There are many *descriptive* accounts of the connection between global imbalances and the global financial crisis, for example Bean (2009), Krugman (2009) and Truman (2009), as well as Obstfeld and Rogoff (2009). But there is no discussion of global effects on the outcomes of deleveraging using

³ The reason is well known. In the steady state, both the home and foreign consumption Euler equations solve for the world real interest rate. We are left with n - 2 equations to solve for n - 1 steady-state values.

⁴ It is not appropriate to use one of the methods in Schmitt-Grohe and Uribe (2003) since their purpose is to rule out the unit root in the evolution of debt which lies at the heart of our analysis.

a formal model. Benigno and Romei (2012) build a two-country version of EK similar to ours, but they focus on the implications to welfare of fixed exchange rate regimes and monetary unions. Fornaro (2012) studies a heterogeneous multi-country monetary union model with aggregate uncertainty, analysing the way in which deleveraging in a liquidity trap gives rise to a union-wide recession. Our work differs from these two papers in considering adjustments in a flexible-exchange-rate setting whereas they study adjustment without this feature.

1.2 The Plan of this Paper

The rest of the paper presents the argument summarised above. It is structured as follows. Section 2 sets out and calibrates the two-country model. In Section 3 we study what happens when the world is hit by a 'savings-glut' shock in China, followed by a deleveraging shock in the US. In section 4 we formally study deleveraging in the presence of a zero bound to interest rates by extending the model to include nominal rigidities. Section 5 concludes. Three appendices describe parameterisation, the system in the presence of nominal rigidities, and the solution algorithm used to solve the model.

2. The Model

In this section we describe the benchmark model in which we assume flexible prices and describe outcomes for real variables. The model closely follows Obstfeld and Rogoff (2005, 2007) and Benigno (2009). Homogenous consumers in each of the two countries supply labour, consume and save. Consumers exhibit consumption home bias, so that a change in the relative wealth of consumers in the two countries affects the relative demand for goods and so the real exchange rate. Goods are produced in each of the two countries by perfectly competitive firms which turn labour into goods. For simplicity we assume there is no investment by firms in capital. There is an international debt market. In this section, we build the benchmark model without nominal rigidities.

2.1 The Allocation of Consumption between Home and Foreign Goods

Home consumers (in the US) and foreign consumers (in China) consume composite goods C_t and C_t^* , where the composite is defined using a CES aggregator with home bias:

$$C_{t} = \left(\alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$
(1)

$$C_t^* = \left((1-\alpha)^{\frac{1}{\eta}} (C_{Ht}^*)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{Ft}^*)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},\tag{2}$$

where C_H and C_F are the home consumption of home and foreign goods, and C_H^* and C_F^* are the foreign counterparts. Home bias implies that $\alpha > 1/2$. We let P_{Ht} and P_{Ft} denote the price of home and foreign goods in the home country. Similarly P_{Ht}^* and P_{Ft}^* denote the price of home and foreign goods in the foreign country. Optimisation by consumers produces the demand functions:

$$C_{Ht} = \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \qquad C_{Ft} = (1-\alpha) \left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t,$$

$$C_{Ht}^* = (1-\alpha) \left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*, \qquad C_{Ft}^* = \alpha \left(\frac{P_{Ft}^*}{P_t^*}\right) C_t^*,$$

where the home and foreign aggregate price levels are:

$$P_t \equiv \left(\alpha P_{Ht}^{1-\eta} + (1-\alpha) P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}},\tag{3}$$

$$P_t^* \equiv \left((1 - \alpha) (P_{Ht}^*)^{1 - \eta} + \alpha (P_{Ft}^*)^{1 - \eta} \right)^{\frac{1}{1 - \eta}}.$$
(4)

We assume that the law of one price holds, *i.e.* after conversion at the ruling nominal exchange rate, e_t , each good sells at the same price in each country:

$$e_t = \frac{P_{Ht}^*}{P_{Ht}} = \frac{P_{Ft}^*}{P_{Ft}}.$$
 (5)

A rise in e_t is a depreciation of the foreign currency.

We let S_t denote the terms of trade (from the foreign country's perspective):

$$S_t \equiv \frac{P_{Ht}^*}{P_{Ft}^*} = \frac{P_{Ht}}{P_{Ft}}.$$
 (6)

A rise in S is a strengthening of the terms of trade in the home country.

2.2 Intertemporal Choice

Consumers in each of the two countries are homogeneous. We assume home consumers have the following utility:

$$U_{Ht} = E_t \sum_{s=0}^{\infty} (\prod_{\nu=1}^{s} \beta_{Ht+\nu}) u(C_{t+s}, L_{t+s}),$$
(7)

where the period utility depends on consumption *C* and labour *L*. We allow the discount factor $\beta_{H,t}$ to be variable across time.

Consumers are assumed to have Greenwood-Hercowitz-Huffman (1988) (henceforth GHH) preferences. It is well-known that this utility specification can generate a labour supply schedule that only depends on the real wage. Moreover, Correia, Neves and Rebelo (1995) show that GHH preferences are better suited to match the second moments of open economies. Raffo (2008) also shows that GHH preferences can improve the empirical performance of two-country models by generating sufficient volatility in consumption. We will explain in detail the choice of the preference and its implications in the discussion below. Specifically, the preferences used are the following:

$$u(C_t, L_t) = \ln\left(C_t - \chi \frac{L_t^{1+\varphi}}{1+\varphi}\right).$$

Consumers maximise their utility subject to the following budget constraint:

$$P_t C_t = W_t L_t + \frac{D_{t+1}}{1+i_t} - D_t.$$
(8)

In each period, home consumers earn wage income, repay a (one-period) home nominal debt D_t from the last period, obtain new borrowing amount of $D_{t+1}/(1+i_t)$ and purchase consumption goods. We assume that nominal debt is the only financial asset in the system for the tractability of the model.⁵ A little manipulation of the budget constraint yields:

$$C_t = \frac{W_t}{P_t} L_t + \frac{D_{t+1}}{P_{t+1}} \frac{1}{(1+r_t)} - \frac{D_t}{P_t},$$
(9)

where we define $(1 + r_t) \equiv P_t (1 + i_t) / P_{t+1}$ as the real interest rate.

Consumers maximise utility subject to the budget constraint and the standard transversality condition associated with debt. This yields the following first order conditions:

$$\frac{1}{\left(C_t - \chi \frac{L_t^{1+\varphi}}{1+\varphi}\right)} = E_t \left(\beta_{Ht+1}(1+i_t) \frac{P_t}{P_{t+1}} \frac{1}{\left(C_{t+1} - \chi \frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)}\right),\tag{10}$$

⁵ Our analysis of global imbalances and the global financial crisis abstracts from an elaborate financial market with multiple financial assets earning different returns. Despite the popularity of the topic, few models consider this. Exceptions include Blanchard, Giavazzi and Sa (2005) and Gourinchas, Rey and Govillot (2010), where the former model relies on imperfect substitutability across assets and the latter studies endogenous portfolio choices in response to shocks.

$$\chi L_t^{\varphi} = \frac{W_t}{P_t}.$$
(11)

The first equation is the consumption Euler equation. The second equation is the intratemporal tradeoff between consumption and leisure. Notice that this labour supply curve has no wealth effect, as a result of the assumption of GHH preferences.

Foreign consumers have a symmetric structure to home consumers. Their utility is:

$$U_{Ft} = E_t \sum_{s=0}^{\infty} \left(\prod_{\nu=1}^{s} \beta_{Ft+\nu} \right) \ln \left(C_{t+s}^* - \chi \frac{(L_{t+s}^*)^{1+\varphi}}{1+\varphi} \right).$$
(12)

The budget constraint for foreign consumers is:

$$P_t^* C_t^* = W_t^* L_t^* + \frac{e_t D_{t+1}^*}{1+i_t} - e_t D_t^* + \frac{B_{t+1}^*}{1+i_t^*} - B_t^*.$$
(13)

where D_t^* is the nominal home debt acquired by foreign consumers and e_t is the nominal exchange rate, defined in Equation (5). We introduce a foreign nominal debt B_t^* which pays the foreign nominal interest rate i_t^* . Only foreign consumers hold foreign debt. (The Chinese debt market is isolated from the rest of the world due to capital account restrictions). For the world as a whole, debts have zero net supply, which means that:

$$D_t + D_t^* = 0, \qquad B_t^* = 0.$$
 (14)

We substitute the debt market clearing condition into the foreign budget constraint to obtain:

$$C_t^* = \frac{W_t^*}{P_t^*} L_t^* - \frac{Q_t}{1 + r_t} \frac{D_{t+1}}{P_{t+1}} + Q_t \frac{D_t}{P_t},$$
(15)

where Q_t is the real exchange rate, defined as $Q_t \equiv e_t P_t / P_t^*$.

The first order conditions for the foreign consumers' utility maximisation problem are the consumption Euler equation, the labour supply curve and the uncovered interest parity (UIP) as follows:

$$\frac{1}{\left(C_t^* - \chi \frac{(L_t^*)^{1+\varphi}}{1+\varphi}\right)} = E_t \left[\beta_{Ft+1}(1+r_t^*) \frac{1}{\left(C_{t+1}^* - \chi \frac{(L_{t+1}^*)^{1+\varphi}}{1+\varphi}\right)}\right],\tag{16}$$

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$$\chi(L_t^*)^{\varphi} = \frac{W_t^*}{P_t^{*}},$$
(17)

$$0 = E_t \left[\beta_{Ft+1} \frac{\left(C_t^* - \chi \frac{(L_t^*)^{1+\varphi}}{1+\varphi} \right)}{\left(C_{t+1}^* - \chi \frac{(L_{t+1}^*)^{1+\varphi}}{1+\varphi} \right)} \left((1+r_t) \frac{Q_{t+1}}{Q_t} - (1+r_t^*) \right) \right].$$
(18)

where $(1 + r_t^*) \equiv P_t^*(1 + i_t^*)/P_{t+1}^*$ is the real interest rate in the foreign country.

2.3 Production

Home goods are produced in home firms. In this benchmark model, we assume that firms are perfectly competitive and there is no nominal rigidities. Firms produce with a simple linear technology:

$$Y_t = L_t. (19)$$

Profit maximisation means that price equals marginal cost ($W_t = P_{Ht}$) and the profit of the industry is zero. Production in the foreign economy is assumed to have an analogous structure.

2.4 Goods Market Clearing

Home output equals consumption of home goods by home and foreign consumers:

$$Y_t = \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + (1-\alpha) \left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*.$$

An analogous goods market clearing condition holds in the foreign economy:

$$Y_t^* = (1-\alpha) \left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*.$$

This completes the description of the model.

2.5 Interest Rate Determination

We suppose that the two countries use monetary policy to ensure that the real interest rate ensures full employment of resources and zero inflation. The model which is used in the following section is thus a real model. Nominal rigidities are introduced in the section on the zero bound.

2.6 Model Summary

We summarise the model as follows.

The home and foreign budget constraints are:

$$C_t = \Theta_t^{-\frac{1}{1-\eta}} Y_t + \frac{D_{t+1}}{P_{t+1}} \frac{1}{(1+r_t)} - \frac{D_t}{P_t},$$
(20)

$$C_t^* = \Xi_t^{-\frac{1}{1-\eta}} Y_t^* - \frac{Q_t}{1+r_t} \frac{D_{t+1}}{P_{t+1}} + Q_t \frac{D_t}{P_t}.$$
(21)

The home and foreign consumption Euler equations are:

$$\frac{1}{\left(C_t - \chi \frac{Y_t^{1+\varphi}}{1+\varphi}\right)} = E_t \left[\beta_{Ht+1}(1+r_t) \frac{1}{\left(C_{t+1} - \chi \frac{Y_{t+1}^{1+\varphi}}{1+\varphi}\right)}\right],$$
(22)

$$\frac{1}{\left(C_t^* - \chi \frac{(Y_t^*)^{1+\varphi}}{1+\varphi}\right)} = E_t \left[\beta_{Ft+1}(1+r_t^*) \frac{1}{\left(C_{t+1}^* - \chi \frac{(Y_{t+1}^*)^{1+\varphi}}{1+\varphi}\right)}\right].$$
(23)

The uncovered interest parity is:

$$0 = E_t \left[\beta_{Ft+1} \frac{\left(C_t^* - \chi \frac{(L_t^*)^{1+\varphi}}{1+\varphi} \right)}{\left(C_{t+1}^* - \chi \frac{(L_{t+1}^*)^{1+\varphi}}{1+\varphi} \right)} \left((1+r_t) \frac{Q_{t+1}}{Q_t} - (1+r_t^*) \right) \right].$$
(24)

The home and foreign intratemporal tradeoffs are:

$$\Theta_t^{-\frac{1}{1-\eta}} = \chi Y_t^{\varphi},\tag{25}$$

$$\Xi_t^{-\frac{1}{1-\eta}} = \chi(Y_t^*)^{\varphi}.$$
 (26)

The home and foreign goods-market clearing conditions are:

$$Y_{t} = \alpha \Theta_{t}^{\frac{\eta}{1-\eta}} C_{t} + (1-\alpha) S_{t}^{-\eta} \Xi_{t}^{\frac{\eta}{1-\eta}} C_{t}^{*},$$
(27)

$$Y_t^* = (1 - \alpha) S_t^{\eta} \Theta_t^{\frac{\eta}{1 - \eta}} C_t + \alpha \Xi_t^{\frac{\eta}{1 - \eta}} C_t^*,$$
(28)

where Θ_t , Ξ_t , Q_t are functions of the terms of trade S_t :

$$\Theta_t \equiv \alpha + (1 - \alpha) S_t^{-(1 - \eta)},\tag{29}$$

$$\Xi_t \equiv \alpha + (1 - \alpha) S_t^{1 - \eta},\tag{30}$$

$$Q_t \equiv Q_t \equiv \frac{e_t P_t}{P_t^*} = S_t \left(\frac{\Theta_t}{\Xi_t}\right)^{\frac{1}{1-\eta}}.$$
(31)

The system contains 9 equations (Equations (20) - (28)) and solves for 8 variables:

$$\{C_t, C_t^*, Y_t, Y_t^*, S_t, r_t, r_t^*, D_t/P_t\}$$

given the initial level of real debt, D_0/P_0 and the shock process which will be described below. By Walras' law, one of the equations is redundant. Real debt is the only state variable in the system.

2.7 The Steady State

We suppose that the model is initially at its steady state before being hit by any shocks, which we now characterise. Throughout this paper, we assume that the home economy has a constant discount factor ($\beta_{Ht} = \beta_H$ for all *t*). In the steady state, consumers in both countries choose consumption according to their consumption Euler equations. This means that

$$\beta_H = \frac{1}{(1+\bar{r})} = \frac{1}{(1+\bar{r}^*)} = \bar{\beta}_F,$$
(32)

where $\bar{\beta}_F$ denotes the steady state for the foreign discount factor and \bar{r} and \bar{r}^* denote the steady state for home and foreign real interest rates. This equation implies that the discount factor in the foreign economy cannot permanently deviate from the discount factor in the home country.⁶

There is a related problem concerning debt. As the steady-state version of both the home and foreign consumption Euler equations solve for the steady-state real interest rate, there is an additional degree of freedom for the other variables in the system. Hence, this model belongs to a class of open

⁶ See, for instance, Mendoza (1991), Mendoza and Uribe (2000) and Neumeyer and Perri (2001).

economy models that features a steady state that depends on initial conditions and equilibrium dynamics that possess a random walk component (Schmitt-Grohe and Uribe, 2003). After a temporary shock hits the system, the system will not, in general, return to the pre-shock steady state. Instead, the new steady state is endogenously determined, jointly with the transition dynamics.

2.8 Calibration and Simulation Method

We study a temporary rise in patience in the foreign country. In other words, we assume that China has a 'savings-glut' shock. Specifically, we assume that the shock lasts for 60 periods -- or 15 years, and after that the discount factor falls back to the equilibrium level. We assume that this rise in patience is initially unanticipated, but after the first period, the length of the shock is known to the consumers. There are no stochastic shocks in the system. Consumers are assumed to have perfect foresight after the initial shock. As a result, expectation operators can be omitted in the solution of the model.

The calibration of parameters is summarised in Table 1 in Appendix 7.1. The values which we use are common in the literature. We assume that the world is initially symmetric so that both home and foreign economies inherit zero real debt in the beginning of the world. We use $\alpha = 0.7$ for the home bias. The elasticity of substitution between home and foreign goods, η , is set to 2. These parameters are the same as Obstfeld and Rogoff (2005). Each period in our model is a quarter. We use a home discount factor of $\beta_H = 0.99$, which implies that the annualised steady-state interest rate in the model is approximately 4.1%, a value which is commonly used in macroeconomic literature. The utility weight of disutility of labour, χ is set to 1. This keeps output in the initial steady state equal to unity for tractability. The inverse of Frisch elasticity of labour supply, φ , is set to 1, in line with evidence by Kimball and Shapiro (2008).

It is easy then to verify that in the initial steady state, before any shock occurs, output and consumption in the home and foreign country are unity $Y_{init} = Y_{init}^* = C_{init} = C_{init}^* = 1$. The symmetry of the model means that the real exchange rate S_{init} is also equal to unity initially.

The savings-glut shock is calibrated as follows:

$$\beta_{Ft} = \begin{cases} \bar{\beta}_F \times (1 + \varepsilon_{\beta t}), & t = 1, 2, \dots 60. \\ \bar{\beta}_F, & thereafter. \end{cases}$$
(33)

The magnitude of the shock is calibrated so that in 30 periods, (that is, as will be discussed in the next subsection, when the deleveraging shock sets in), the ratio of external debt to GDP is equal to 20%, following Fornaro (2012). This requires a shock to the foreign discount factor, $\varepsilon_{\beta t}$, of 0.37% above the steady-state level, at a quarterly frequency. This means that the value of the foreign discount factor

when the 'savings-glut' shock is in place is 0.9937.7

We simulate the model using the reverse shooting method. Given the random-walk property of debt discussed above, it is inappropriate to solve the log-linearised version of the model because we do not know the new steady state (and as our simulation will show the new steady state is far from the old one). For this reason, local approximation methods such as higher-order perturbation methods are not suitable. This means that we need to use non-linear solution methods. Since the external debt position is the only state variable in the system and there exists a steady state for any level of debt, the system reaches the steady state once the shock has dissipated. Under these circumstances, the reverse shooting method is simple to implement under perfect foresight. Furthermore, the method can be implemented in levels, preserving the non-linearities of the system. Detailed description of the solution algorithm is presented in Appendix 7.3.

3. The Global Savings Shock and Deleveraging

3.1 The Global Savings Shock

The simulation results for the savings-glut shock are displayed in the dashed line in Figure 1. When foreign consumers become more patient, they demand fewer goods initially. In response, interest rates fall to shift demand back to the present. Since home consumers are more willing to spend and their spending is biased towards home goods, the home terms of trade has to strengthen in order to clear the home and foreign goods markets. As a result the home country accumulates external debt for the periods which foreign consumers are more patient than home. The US real exchange rate in the new steady state is depreciated compared with the initial steady state, due to the accumulation of debt.

One feature of the impulse responses shown in Figure 1 is that consumption and output move in the same direction, as required by the data. Backus and Kehoe (1992), for instance, find that consumption is uniformly pro-cyclical using a dataset that covers ten advanced economy (including the US) for at least a century. Aguiar and Gopinath (2007) also find co-movement between these two variables in 13 emerging economies, with an average correlation of 0.72. To generate such co-movement, we use GHH preferences in the model. This is because additively separable preferences cannot generate positive co-movement between output and consumption in our open economy model, even although it can generate positive co-movement in a closed economy.⁸ In a closed economy, after a saving shock, consumption falls and so does output. Output must move in the same direction as consumption because the real wage falls by more than consumption so that labour supply falls

⁸ An example of additively separable preferences is $u(C_t, L_t) = \ln C_t - \chi L_t^{1+\varphi}/(1+\varphi)$.

⁷ The choice is consistent with calibrations in the zero lower bound literature. Fernandez-Villaverde et al. (2012) and Bernigno and Romei (2012), among others, calibrate their models so that the steady-state annual interest rate is 2.5%, which implies $\beta = 0.9938$. Those models, however, take the low world interest rates during the savings-glut periods as exogenous.

according to the labour supply condition. However, in our open-economy model, with additively separable preferences and reasonable calibrations, the fall in the real wage is lessened because it induces a real exchange rate depreciation which increases foreign demand for domestic goods by so much that output actually increases. With GHH preferences, the labour supply curve does not depend on the marginal utility of consumption, moderating the fall in the real wage, so when a saving shock occurs both consumption and output fall (as noted in Fernandez-Villaverde et al., 2011). Nevertheless, output falls by less than consumption because of the rise in foreign demand for home goods caused by the depreciation of the real exchange rate; it is therefore still the case that the 'savings-glut' country exhibits a rise in export surplus.

The simulation results are broadly consistent with what we observed between the US and China during the great moderation period. There was a fall in world interest rates,⁹ low saving rates in the advanced economies and a sustained current account deficit in the US.¹⁰ Our finding is in line with Bernanke's (2005) 'global savings glut' hypothesis. However, our simulation results suggest that the saving shock alone does not generate large movements in the real exchange rate of the magnitude observed in reality. Nor does it generate a rise in output in the foreign country.¹¹

Furthermore, Figure 1 shows that the level of net external debt in the new steady state is about 28% of annual output, a large number relative to the US net foreign debt-to-GDP ratio in 2007 of 17%¹². The prospect of accumulating such a large stock of debt could have provided a motivation for debt deleveraging.

3.2 Deleveraging

We study deleveraging in the following way. We suppose that, as a result of the accumulation of debt, capital markets force the home country to deleverage in an unanticipated manner. We assume that this occurs in period 30, i.e. in the middle of the savings-glut period, and that the deleveraging phase lasts for 30 periods at the end of which the debt of the home country has returned to zero. This assumption is arbitrary, but means that this phase ends exactly when the impatience shock in the foreign country also ends, so that the system returns to its pre-shock symmetric steady state in which there is no debt.

⁹ The US 10-year TIPS rate and the UK 10-year inflation-indexed government bond yields, common market-based proxies of the long-term real interest rate, declined steadily from around 3.5% in 1997 to around 2% in 2003 and stayed flat until 2008. After the crisis interest rates declined further to below zero. (Source: Global Financial Data)

¹⁰ The US current account deficit in 1998 was about 200 billion USD, or some 3% of US GDP. The deficit quadrupled to 800 billion USD, or 6% of USD GDP in 2006. During the same period, the current account surplus in developing East Asia (which includes China) went up from 2% of GDP in 2001 to 10% in 2007, according to IMF WDI data.

One explanation may be that developing countries such as China also experienced sustained rise in productivity during the same period. Zhu (2012) finds that the TFP growth in China in the post-1978 period is around 3.2% a year. US TFP growth is estimated to be 2.3% from 1995-2006. The narrowing of the productivity gap may account for some of the exchange rate depreciation in China.

¹² This number comes from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007).

Specifically, we impose an exogenous debt limit $\left(\frac{D_t}{P_t}\right)^{lim}$ to the overspent home country as follows:

$$\frac{D_t}{P_t} \le \overline{\left(\frac{D_t}{P_t}\right)}^{lim}.$$
(34)

The debt limit can be thought of as a collateral constraint in reduced form and is imposed directly as in other macro models with endogenous credit constraints such as Kiyotaki and Moore (1997), Aoki et al. (2010) and Mendoza (2010). Under limited enforcement, the borrower in a loan contract can only pledge an exogenous fraction of his future income and this will be the maximum amount the lender is willing to lend. However, the fraction of pledgable income in these models is not well microfounded and usually treated as an exogenous shock in the financial-frictions literature. The imposition of the debt limit in Equation (34) can be thought of as an unexpected fall in the fraction of pledgable income in a financial crisis.

For simplicity, we consider a case in which the debt limit evolves following an exogenous path similar to that in Benigno and Romei (2012):

$$\overline{\left(\frac{D_t}{P_t}\right)}^{lim} = \left(\frac{60-t}{30}\right)^{\varsigma} \frac{D_{30}}{P_{30}}, \quad \text{for } t = 30,31...60,$$
 (35)

where ς determines the speed of deleveraging. We assume the deleveraging shock is initially anticipated, but after the initial period, the subsequent path of debt evolution is known by the consumers.

Equation (35) implies that debt is decaying throughout the deleveraging period. During this period foreign consumers are more patient than home consumers, which implies that the debt limit Equation (34) is always binding. This means that consumers in the home country can no longer smooth their consumption according to their consumption Euler equation (10). Instead, consumption will be governed by the budget constraint so that when consumers are required to deleverage more quickly, they are forced to cut their consumption by more.

We assume debt decays more rapidly in the beginning and less rapidly towards the end (*i.e.* $\varsigma > 1$), contrary to Benigno and Romei (2012) and Fornaro (2012). We do this for two reasons. First, it is reasonable that creditors require debt to be deleveraged more quickly when the stock of debt is high. Second, a linear decay of debt as in Bernigno and Romei (2012) and Fornaro (2012) means that when the system exits the deleveraging phase, consumption of the deleveraging country is no longer constrained, which means that the interest rate has to jump up sharply for one period, as required by the consumption Euler equation; that is not the case here. We calibrate the speed of deleveraging parameter, ς , to be 2.38 to give appropriate behaviour in the case of a zero bound. (This calibration is discussed in the next section.)

The solid line in Figure 1 shows the adjustment of home and foreign consumption, output and the real interest rate together with the real exchange rate and net foreign debt, after imposing the deleveraging constraint. In response to the deleveraging shock, home consumption is forced to drop immediately. The home interest rate falls significantly, consistent with empirical data, because the shock is large. Because of consumption home bias, the relative price of home goods falls, *i.e.* there is an immediate and large depreciation of the home real exchange rate. Interest rates decrease abroad to stimulate demand by foreign consumers in response to the appreciation of the foreign exchange rate. In the long run, external debt returns to zero according to the exogenous deleveraging path given by Equation (35). The system returns to the initial symmetric steady state. The home interest rate must initially fall relative to the foreign interest rate. Home and foreign interest rates are linked by the uncovered interest parity, Equation (18). The home interest rate must fall initially relative to the foreign interest rate. Home and for home goods. As a result, the US real exchange rate will be appreciating along this path which means that the home interest rate has to fall initially by more than the foreign interest rate, to allow for exchange rate appreciation.

Output co-moves with consumption in response to the deleveraging shock because consumers have GHH preferences. The reason why additively separable preferences cannot produce the co-movements as required by the data has been explained in the previous section: when the deleveraging shock hits the home economy, consumption falls. But with reasonable parameterisation, the depreciation in the real exchange rate ensures that foreign consumers buy more home goods so that the real wage in the home country falls by less than consumption and home output rises. To produce the required co-movement between output and consumption with additively separable preferences, adjustments in the real exchange rate have to be limited. That is the case for Benigno and Romei (2012) and Fornaro (2012) who study the adjustments in fixed exchange rate regimes and monetary unions with nominal rigidities. We choose GHH preferences not only for simplicity, but also because these can produce co-movement between consumption and output for economies with flexible exchange rates in a way that is consistent with observed data.

The fall in the home interest rate during the deleveraging periods is on top of the reduction in the interest rate caused by the saving shock in the foreign economy. One can imagine that if such shocks are large enough, it is possible that the nominal interest rate is pushed to zero and cannot fall further. In the next section, we will extend this model to include nominal rigidity so as to analyse what happens when there is a zero bound in the home economy.

4. Nominal Rigidities and the Zero Bound

The previous section shows that when the world is hit by a saving shock and a deleveraging shock, the real interest rate falls to clear the goods markets. This gives rise to the possibility that the nominal interest rate may fall to zero. In this section, we study this situation explicitly by adding nominal rigidities to the model. We show that when the high-debt economy is hit by a large deleveraging shock,

the nominal interest rate may hit the zero lower bound. And, when this happens, the fall in output will be severe compared with a world with no zero bound.

In order to study nominal rigidities, we now assume that there are differentiated goods in each country and monopolistically competitive firms set prices in a staggered manner. Specifically, we assume on the supply side there is an infinite number of intermediate goods firms, indexed by $i \in (0,1)$, in each country. Home firms produce with a linear production function:

$$Y_t(i) = L_t(i). aga{36}$$

Aggregate labour is defined as $L_t \equiv \int_0^1 L_t(i) di$. The firms are monopolistically competitive. There is a final goods firm in each country which combines the varieties of goods into a final output using a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}.$$
(37)

The demand for each variety of goods is:

$$Y_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} Y_t,$$
(38)

and the price of final output is given by $P_{Ht} \equiv \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$. Hence, the aggregate production function is:

$$Y_{t} = L_{t} \times \left[\int_{0}^{1} \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} di \right]^{-1}.$$
 (39)

We model price setting following Calvo (1983) contracting. Specifically, in each period, there is a probability $(1 - \gamma)$ that an intermediate goods firm can re-optimise the price of its goods. This probability is independent across firms and time. With probability γ the price is not re-optimised and assumed to rise at steady-state rate of inflation $\overline{\Pi}_H = 1$, where $\Pi_{Ht} \equiv P_{Ht}/P_{Ht-1}$.

The profit maximisation problem for an intermediate goods firm i which is able to reset price in period t is:

$$Profit(i)_{t} = \max_{P_{Ht}(i)} E_{t} \sum_{s=0}^{\infty} \Lambda_{Ht,t+s} \gamma^{s} \left(\frac{P_{Ht}(i)}{P_{Ht+s}} Y_{t+s}(i) - \frac{1}{\mu_{w}} \frac{W_{t+s}}{P_{Ht+s}} L_{t+s}(i) \right), \tag{40}$$

where $\Lambda_{Ht,t+s} = (\Pi_{v=1}^{s} \beta_{Ht+v}) u_c(C_{t+s}, L_{t+s})/u_c(C_t, L_t)$ is the stochastic discount factor of the home economy. Firms maximise their profits subject to the production function and demand shown in Equation (38). The term $\mu_w = \theta/(\theta - 1)$ is a subsidy to ensure that the steady state is efficient and identical to that in the benchmark model in the last section so that these models are comparable. The subsidy is paid for by consumers with a lump-sum tax. Since the optimisation problem is entirely forward-looking, every optimising firm chooses the same price, denoted by \tilde{P}_{Ht} . The first order condition is given by:

$$\frac{\tilde{P}_{Ht}}{P_{Ht}} = \frac{K_t}{F_t},\tag{41}$$

where:

$$K_t = \frac{\theta}{\theta - 1} \frac{1}{\mu_w} E_t \sum_{s=0}^{\infty} \Lambda_{Ht,t+s} \gamma^s \frac{W_{t+s}}{P_{Ht+s}} \left(\frac{P_{Ht+s}}{P_{Ht}}\right)^{\theta} Y_{t+s},\tag{42}$$

$$F_t = E_t \sum_{s=0}^{\infty} \Lambda_{Ht,t+s} \gamma^s \left(\frac{P_{Ht+s}}{P_{Ht}}\right)^{\theta-1} Y_{t+s}.$$
(43)

Since only a fraction $(1 - \gamma)$ can re-optimise their prices, the aggregate price level can be written as follows

$$P_{Ht}^{1-\theta} = \gamma P_{Ht-1}^{1-\theta} + (1-\gamma) \tilde{P}_{Ht}^{1-\theta}.$$
(44)

Foreign firms are also subject to nominal rigidities in price setting. They have behaviour analogous to that of home firms.

In addition, we need to specify the monetary policy by the central banks. We suppose the two countries can use monetary policy to ensure zero inflation, with a real interest rate equal to that in the flexible price economy of the previous section, as long as nominal interest rates are not constrained by the zero lower bound. However, when the nominal interest rate implied by such a process would be below zero, then there is a zero lower bound, and the nominal interest rate is set to zero. Specifically, the monetary policy rules are:

$$(1+i_t) = \max(1+r_{flex,t}, 1), \tag{45}$$

$$(1+i_t^*) = \max(1+r_{flex,t}^*, 1).$$
(46)

where $r_{flex,t}$ and $r_{flex,t}^*$ are the interest rates that prevail in the flexible price economy. This is an extreme assumption, in order to make this model comparable to the model without price rigidity.

Finally, the path of the terms of trade adjusts to ensure that the uncovered interest parity holds so that:

$$(1+i_t^*) = (1+i_t)\frac{\prod_{t=1}^* S_{t+1}}{\prod_{Ht+1} S_t}.$$
(47)

The rest of the model comprises the home and foreign budget constraints, the first order conditions for home and foreign utility maximisation with respect to consumption and labour supply and the goods market clearing conditions. The equations are mostly identical to those given in the benchmark model presented in the previous section. Appendix 7.2 reports the full system with nominal rigidities.

The parameters are set to be the same as in the previous section. Two additional parameters related to nominal rigidities must be calibrated for this model. The price stickiness parameter, γ , is set to 0.75, which means that on average prices last for one year. We set the elasticity of substitution between varieties of goods, θ , equal to 6, implying a steady-state mark-up of 20%. These parameters are within the range of standard values in the literature. Given the assumptions on monetary policy and the calibration of the parameters, the only differences between this system and the system in the previous section are the nominal rigidities and the imposition of a zero lower bound for nominal interest rates.

Figure 2 shows the simulation of the model with nominal rigidities. As before, we assume initially that a savings-glut shock occurs in the foreign country according to Equation (33). The shock reduces interest rates in both the home and foreign economies. In the absence of deleveraging, the shock is not large enough to push the nominal interest rate down to the zero bound, given our calibration. Our assumption about monetary policies means that nominal interest rates are set so that the real interest rates coincide with those for the benchmark flex-price model. There is no change in prices and inflation. The behaviour of the system is therefore identical to that shown in the previous section. The dashed line in Figure 2 shows the effects of the savings-glut shock. The impulse responses before any deleveraging occurs are identical to those shown by the dashed-dotted line, which shows the impulse responses in the flexible price model. (Note that for the flex-price model we plot real interest rates.)

When we impose the second shock -- the deleveraging shock -- to this system we can see the effects of a zero bound. As in the previous section, the deleveraging shock in the US follows a process described by Equation (35). In particular, we choose the speed of deleveraging parameter, ς , so that the deleveraging country stays at the zero bound for a reasonable amount of time.¹³ In reality, since the outbreak of the global financial crisis in 2008, US interest rates have stayed at a level close to zero for more at least 4 years. We choose the speed of deleveraging parameter so that the zero

¹³ There is, however, little consensus as to how long a liquidity trap is likely to last. These numbers range from 4 quarters in Fornaro (2012) and 5 quarters on average in Christiano et al. (2010) to 12 quarters in Benigno and Romei (2012). Eggertsson and Woodford (2003) and Eggertsson and Krugman (2010) assume 10 quarters. Fernandez-Villaverde et al. (2012) analyses the length of a zero bound for a DSGE model and find that a zero bound on average lasts for 2.06 quarters with a standard deviation of 1.82 quarters.

bound lasts for 12 quarters in our simulation. We use $\varsigma = 2.38$. (We also use $\varsigma = 2.38$ in the case of the flexible price model.)

The solid line in Figure 2 shows the impulse responses when the deleveraging shock is imposed in period 30.¹⁴ In response to the deleveraging shock, consumption in the home country falls. The central bank lowers the nominal interest rate, in an attempt to bring inflation back to target. Given the size of the deleveraging shock, the central bank's attempt is not successful because it would require a negative nominal interest rate to hit the inflation target. As a result, the central bank brings the nominal interest rate down to zero, and the zero interest rate bound binds. Since the interest rate does not fall enough to equilibrate supply and demand in the goods markets, firms who can reset prices cut their prices, creating deflation in the home country.

Such deflation triggers a Fisher debt-deflation spiral: a combination of a zero bound in nominal interest rates and a fall in the home price level leads to a rise in the home real interest rate. This means that debt interest payments for home consumers rise, which tightens the deleveraging constraint and further depresses home consumption, demand and output, causing more deflation. The zero bound in the nominal interest rate also leads to an initial appreciation of the US real exchange rate immediately after the deleveraging shock in Figure 2. In such circumstance, one would normally expect the real exchange rate to depreciate to encourage demand for domestic goods. This happens eventually once the zero bound disappears. But immediately after the shock, deflation in the home country means that real interest rates are higher in the home country than abroad. This effect is large enough to cause the real exchange rate to initially appreciate. That strengthens the deflationary pressure. The overall effect on home output and inflation is substantial. In our simulation, the debt deflation spiral causes around a 4% fall in inflation in the home country on impact. Home output falls by more than 10%. Foreign output also falls initially, since the depreciation of the foreign real exchange rate causes a reduction in the foreign real wage and so in labour supply and output.¹⁵

Again, this initial appreciation of the real exchange rate again depends on GHH preferences. The reason is the following. As discussed previously, in the absence of a zero bound, deleveraging with normal preferences leads to a larger real exchange rate depreciation along the adjustment path, compared with the case with GHH preferences. In the presence of a zero bound, deleveraging causes this zero bound to bind in the home economy initially, which leads to deflation and an initial rise in the home real interest rate relative to the foreign real interest rate for both types of preferences. As a result, the real exchange rate must be initially depreciating over time, for a period of time which roughly corresponds to the periods in which the zero bound binds. After the zero bound ceases to bind, the real exchange rate must be appreciating over time, for reasons explained. But with additively

¹⁴ Details of the solution algorithm are presented in Appendix 7.3.

¹⁵ One feature of the financial crisis has been strong positive output correlation and consumption correlation across countries. Our model does not produce positive co-movement between domestic and foreign consumption because the Chinese interest rate falls to stimulate consumption when US consumption collapses. In reality, monetary policy may be less effective than our model suggests. Moreover, multiple equilibra (Bacchetta and van Wincoop, 2013) and credit channels may explain the observed comovement in reality. We leave these issues for future research.

separable preferences the real exchange rate will be more depreciated after the home interest rate leaves the zero bound than with GHH preferences, for reasons which we have also already explained. Such a 'floor' to the real exchange rate in the case of additively separable preferences prevents it from appreciating initially.

It is helpful to compare observed data with the simulation results. Figure 3 shows the observed US data.¹⁶ We use the date of the Lehman Brothers collapse in September 2008 as a proxy for the beginning of the deleveraging phase, indicated by the red dotted line. After the collapse, the nominal interest rate immediately dropped to zero, a clear indication of the binding zero bound. Sharp deflation was also immediate, with a quarter-to-quarter drop of more than 3%, which indicated a rise in the real interest rate. The deflation was short-lived, compared with our simulation. One possible reason is that commodity prices remained high after the global financial crisis. Quantitative easing might also have an effect on inflation (and the real exchange rate) which is not captured in this simple model. The output gap in the actual data also fell sharply, with a peak-to-trough drop of about 5%, but this was smaller than in our simulation. Given the simplicity of our setup, we can only explain part of what happened during that period.

Finally, there was an immediate exchange rate appreciation of about 10% in the actual data before a subsequent depreciation. Such behaviour of the exchange rate was in contrast to what many observers had predicted which was a steady fall in the dollar as a likely macroeconomic outcome in the process of the unwinding of global imbalances. See, for instance, Blanchard, Giavazzi and Sa (2005), Kuralbayeva and Vines (2009) and Krugman (2007). Various explanations have been put forward to explain this phenomenon. First, Blanchard and Milesi-Ferretti (2011) and Adam, Subacchi and Vines (2012) argue that emerging market economies followed a 'beggar-thy-neighbour' policy to keep a depreciated exchange rate, in order to achieve full employment in their own countries after the shock. Second, there was a sharp increase in demand for US government bonds -- the global safe asset -- as a result of the 'flight to safety' in a time of heightened volatility, something which is emphasised by the macro-finance literature (for instance Caballero and Krishnamurthy, 2008, Gourinchas, Rey and Govillot, 2010, and Maggiori, 2013). We provide an alternative explanation, based on a zero bound causing real interest rates to move in such a way as to provoke currency appreciation.

5. Conclusion

The model presented in this paper suggests that during the great moderation, a rise in saving in China led to a fall in the interest rates in both the US and China, together with a rise in the current account deficit in the US, features consistent with the description by Bernanke (2005) of the effects of the global savings glut. We have connected this analysis with the global financial crisis. We suggest

¹⁶ Data source: Bank of International Settlements, Bureau of Economic Analysis (US), Board of Governors of the Federal Reserve System and Federal Reserve Bank of Cleveland. The output gap shows the difference between the quarterly real GDP series and the hp-filtered series with a smoothing parameter λ =1600.

that as a consequence of debt accumulation, US consumers faced a deleveraging shock. We model this shock as having led to forced saving in the US, and to a further reduction in US interest rates in the US as far as the zero bound, leading to a collapse in output and to deflation. We argue that deflation raised US real interest rates and the real value of debt, further tightening the debt constraint. Moreover, using GHH preferences, we show how the resulting rise in the real interest rate might have caused an initial appreciation of the dollar, thereby leading to a further reduction in output, and to further deflation and to a tightening of the deleveraging constraint.

One major limitation of the analysis is its reliance on an exogenous debt deleveraging constraint, a limitation shared with other recent works in the field. In reality, the debt limit is likely to depend in part on economic fundamentals in ways that are not fully understood. But the transmission mechanisms are not clear.

Another direction for future research is related to the role of fiscal expenditure in countering the recession when monetary policy is stuck at the zero bound. The present analysis assumes no government, but could be extended to include one. Christiano, Eichenbaum and Rebelo (2010) and Eggertsson and Krugman (2012) have shown in the closed economy that the fiscal multiplier is significantly above unity when the interest rate is at the zero bound, so that fiscal support is powerful.¹⁷ Such work needs to be extended to an open economy.

¹⁷ See also Nakata (2012) who studies optimal fiscal and monetary policy with an occasionally binding zero bound constraint for a closed economy.

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Figure 3. US Data



Appendix

7.1 Parameter Values

The following table shows the parameter values we used to calibrate the model:

Table 1. Parameter Values

Definition	Parameter	Target/ Source
Consumption home bias	$\alpha = 0.7$	Obstfeld and Rogoff (2005)
Elasticity of substitution	$\eta = 2$	Obstfeld and Rogoff (2005)
between home and foreign goods		
Steady-state discount factor	$\beta_H = 0.99$	Interest rate $r = 4.1\%$
Utility weight of labour disutility	$\chi = 1$	Output in initial steady state $= 1$
Inverse of Frisch elasticity	$\varphi = 1$	Kimball and Shapiro (2008)
Magnitude of savings-glut shock	$\varepsilon_t = 0.0037$	Deleveraging shock begins in 7.5 years,
		at which home debt-to-GDP
		ratio = 20% (Fornaro, 2012)
Speed of deleveraging	$\varsigma = 2.38$	Zero bound at home binds for 3 years
Probability firm reoptimises prices	$\gamma = 0.75$	Prices fixed for 4 quarters on average
Elasticity of substitution between	$\theta = 6$	Steady-state mark-up = 20%
good varieties		

7.2 Full System with Nominal Rigidities

In this Appendix, we present the full system with nominal rigidities.

The home and foreign budget constraints are

$$C_{t} = \Theta_{t}^{-\frac{1}{1-\eta}} \Delta_{t} Y_{t} + \frac{1}{1+i_{t}} \frac{D_{t+1}}{P_{t+1}} \Pi_{Ht+1} \left(\frac{\Theta_{t+1}}{\Theta_{t}}\right)^{\frac{1}{1-\eta}} - \frac{D_{t}}{P_{t}},$$
(48)

$$C_t^* = \Xi_t^{-\frac{1}{1-\eta}} \Delta_t^* Y_t^* - Q_t \frac{1}{1+i_t} \frac{D_{t+1}}{P_{t+1}} \Pi_{Ht+1} \left(\frac{\Theta_{t+1}}{\Theta_t}\right)^{\frac{1}{1-\eta}} + Q_t \frac{D_t}{P_t}.$$
(49)

The utility maximisation conditions for the home and foreign consumers are

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$$1 = \Lambda_{Ht,t+1} (1+i_t) \frac{1}{\Pi_{Ht+1}} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{\frac{1}{1-\eta}},$$
(50)

$$\frac{W_t}{P_t} = \chi Y_t^{\varphi} \Delta_t^{\varphi}, \tag{51}$$

$$1 = \Lambda_{Ft,t+1}^{*} (1+i_t) \frac{1}{\Pi_{Ht+1}} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{\frac{1}{1-\eta}} \frac{Q_{t+1}}{Q_t},$$
(52)

$$\frac{W_t^*}{P_t^*} = \chi(Y_t^*)^{\varphi} (\Delta_t^*)^{\varphi}, \tag{53}$$

where the stochastic discount factors are

$$\Lambda_{Ht,t+1} = \beta_{Ht+1} \left(C_t - \chi \frac{(\Delta_t Y_t)^{1+\varphi}}{1+\varphi} \right) \left(C_{t+1} - \chi \frac{(\Delta_{t+1} Y_{t+1})^{1+\varphi}}{1+\varphi} \right)^{-1},$$

$$\Lambda_{Ft,t+1}^* = \beta_{Ft+1} \left(C_t^* - \chi \frac{(\Delta_t^* Y_t^*)^{1+\varphi}}{1+\varphi} \right) \left(C_{t+1}^* - \chi \frac{(\Delta_{t+1}^* Y_{t+1}^*)^{1+\varphi}}{1+\varphi} \right)^{-1}.$$

The price setting behaviour of home firms is given by

$$\frac{\tilde{P}_{Ht}}{P_{Ht}}F_t = K_t,\tag{54}$$

$$F_t = Y_t + \Lambda_{Ht,t+1} \gamma \Pi_{Ht+1}^{\theta-1} F_{t+1},$$
(55)

$$K_{t} = \frac{W_{t}}{P_{t}} \Theta_{t}^{\frac{1}{1-\eta}} Y_{t} + \Lambda_{Ht,t+1} \gamma \Pi_{Ht+1}^{\theta} K_{t+1},$$
(56)

$$1 = \gamma \Pi_{Ht}^{-(1-\theta)} + (1-\gamma) \left(\frac{\tilde{P}_{Ht}}{P_{Ht}}\right)^{1-\theta}.$$
(57)

The price setting behaviour of foreign firms is given by

$$\frac{\tilde{P}_{Ft}^{*}}{P_{Ft}^{*}}F_{t}^{*} = K_{t}^{*},$$
(58)

$$F_t^* = Y_t^* + \Lambda_{Ft,t+1}^* \gamma (\Pi_{Ft+1}^*)^{\theta - 1} F_{t+1}^*,$$
(59)

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$$K_t^* = \frac{W_t^*}{P_t^*} \Xi_t^{\frac{1}{1-\eta}} Y_t^* + \Lambda_{Ft,t+1}^* \gamma(\Pi_{Ft+1}^*)^{\theta} K_{t+1}^*,$$
(60)

$$1 = \gamma (\Pi_{Ft}^{*})^{-(1-\theta)} + (1-\gamma) \left(\frac{\tilde{P}_{Ft}^{*}}{P_{Ft}^{*}}\right)^{1-\theta},$$
(61)

and the evolution of the price dispersions Δ_t and Δ_t^* is

$$\Delta_t = \gamma \Delta_{t-1} \Pi_{Ht}^{\theta} + (1 - \gamma) \left(\frac{\tilde{P}_{Ht}}{P_{Ht}} \right)^{-\theta},$$
(62)

$$\Delta_{t}^{*} = \gamma \Delta_{t-1}^{*} (\Pi_{Ft}^{*})^{\theta} + (1 - \gamma) \left(\frac{\tilde{P}_{Ft}^{*}}{P_{Ft}^{*}}\right)^{-\theta},$$
(63)

where the price dispersion is defined by

$$\begin{split} \Delta_t &\equiv \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di, \\ \Delta_t^* &\equiv \int_0^1 \left(\frac{P_{Ft}^*(i)}{P_{Ft}^*}\right)^{-\theta} di. \end{split}$$

The home and foreign goods market clearing conditions are

$$\Delta_t Y_t = \alpha \Theta_t^{\frac{\eta}{1-\eta}} C_t + (1-\alpha) S_t^{-\eta} \Xi_t^{\frac{\eta}{1-\eta}} C_t^*, \tag{64}$$

$$\Delta_t^* Y_t^* = (1 - \alpha) S_t^\eta \Theta_t^{\frac{\eta}{1 - \eta}} C_t + \alpha \Xi_t^{\frac{\eta}{1 - \eta}} C_t^*, \tag{65}$$

The uncovered interest parity is written as

$$(1+i_t^*) = (1+i_t) \frac{\prod_{t=1}^* S_{t+1}}{\prod_{Ht+1} S_t}.$$
(66)

Lastly, the monetary policy rules for the home and foreign economies are described in Equation (45) and (46). In practice, we assume the monetary policies are Taylor rules as follows:

$$(1+i_t) = \max\left(\frac{1}{\beta_H} \left(\frac{\Pi_{Ht}}{\Pi_H}\right)^{\Phi_H}, 1\right),\tag{67}$$

$$(1+i_t^*) = \max\left(\frac{1}{\beta_F} \left(\frac{\Pi_{Ft}^*}{\Pi_F^*}\right)^{\Phi_F}, 1\right).$$
(68)

where we set the inflation elasticities to be high ($\phi_H = \phi_F = 10^8$). This ensures that in normal times inflation is close to zero and the nominal interest rates are set to replicate the flexible price economy.

Moreover, Θ_t , Ξ_t , Q_t are defined in Equations (29), (30) and (31). The process for β_{Ft} follows Equation (33).

The system contains 21 equations (Equations (48) - (68)) and solves for 20 variables

$$C_t, C_t^*, Y_t, Y_t^*, S_t, i_t, i_t^*, D_t/P_t, \Pi_{Ht}, \Pi_{Ft}^*, \frac{W_t}{P_t}, \frac{W_t^*}{P_t^*},$$

$$K_t, K_t^*, F_t, F_t^*, \frac{\tilde{P}_{Ht}}{P_{Ht}}, \frac{\tilde{P}_{Ft}^*}{P_{Ft}^*}, \Delta_t, \Delta_t^*.$$

given the initial level of debt D_0/P_0 and the price dispersions Δ_t , Δ_t^* and the exogenous shock process for β_{Ft} , which follows Equation (33). By Walras' law, one of the equations is redundant. After deleveraging, the home consumption Euler equation is dropped. Debt follows the exogenous debt deleveraging process (35).

After the deleveraging shock hits the home economy in period 30, home consumers are forced to deleverage. The consumption Euler equation (50) no longer applies, and is replaced with the exogenous debt deleveraging process (35).

7.3 Solution Algorithm

This appendix discusses the reverse shooting algorithm employed in this paper to solve the model. As consumers have perfect foresight, the shooting algorithm is a convenient technique to solve the model. In the following, we first describe the solution method in the flexible price model, and then the model with nominal rigidities. For each model, we first describe the simulation to the saving shock, and then the simulation to the deleveraging shock.

In the flexible price model, the only state variable is real debt D/P. In addition, it is known that debt follows a unit-root process in this model. Without any disturbances, for any given value of debt, there is an equilibrium associated with this level of debt. Steady-state values of other variables are found by solving the steady-state version of the system (Equation (20) -- (28)). We assume that the savings shock only lasts for 60 periods, and after that there are no further disturbances. The above discussion implies that at the end of period 60, debt reaches a new steady state and stays there forever. The reverse shooting algorithm involves guessing the level of debt in the new steady state and updating

the guess until convergence as follows (See Judd (1998) for further details):

- 1. Select an upper bound for the guess of the level of debt in the final steady state. This upper bound has to be sufficiently large. Call this upper bound $\left(\frac{D}{p}\right)^{high}$. Also set a lower bound at $\left(\frac{D}{p}\right)^{low} = 0$. The true level of debt in the final steady state has to be bounded by the two.
- 2. Guess the level of debt in the final steady state. Call this guess $\left(\frac{D}{P}\right)_{60}^{(j)}$ where the subscript means period 60 and the superscript denotes the *j*th guess.
- 3. Compute the steady-state values of $C_{60}^{(j)}$, $(C_{60}^*)^{(j)}$, $Y_{60}^{(j)}$, $(Y_{60}^*)^{(j)}$, $S_{60}^{(j)}$, $r_{60}^{(j)}$, $(r_{60}^*)^{(j)}$ implied by $\left(\frac{D}{P}\right)_{60}^{(j)}$, using the steady state version of Equation (20) -- (28).
- 4. Compute the values of $C_{59}^{(j)}$, $(C_{59}^*)^{(j)}$, $Y_{59}^{(j)}$, $(Y_{59}^*)^{(j)}$, $S_{59}^{(j)}$, $r_{59}^{(j)}$, $(r_{59}^*)^{(j)}$, $\left(\frac{D}{P}\right)_{59}^{(j)}$, using Equation (20) -- (28), where $C_{60}^{(j)}$, $(C_{60}^*)^{(j)}$, $Y_{60}^{(j)}$, $(Y_{60}^*)^{(j)}$, $S_{60}^{(j)}$, $\left(\frac{D}{P}\right)_{60}^{(j)}$ are known and $\beta_{F,59}$ is given by Equation (33).
- 5. Repeat the last step for t = 58,57 ... until we obtain $C_0^{(j)}$, $(C_0^*)^{(j)}$, $Y_0^{(j)}$, $(Y_0^*)^{(j)}$, $S_0^{(j)}$, $r_0^{(j)}$, $(r_0^*)^{(j)}$, $\left(\frac{D}{P}\right)_0^{(j)}$.
- 6. If $\left| \left(\frac{D}{p}\right)_{0}^{(j)} \right| < threshold$, we have found the transition path and stop. Otherwise, update the guess for the level of debt in the final steady state using method of bisection. Specifically, if $\left(\frac{D}{p}\right)_{0}^{(j)} > 0$, then $\left(\frac{D}{p}\right)_{60}^{(j+1)} = \left(\left(\frac{D}{p}\right)_{60}^{(j)} + \left(\frac{D}{p}\right)^{low}\right)/2$, and update the lower bound to $\left(\frac{D}{p}\right)^{low} = \left(\frac{D}{p}\right)_{60}^{(j)}$. Otherwise, if $\left(\frac{D}{p}\right)_{0}^{(j)} < 0$, then $\left(\frac{D}{p}\right)_{60}^{(j+1)} = \left(\left(\frac{D}{p}\right)_{60}^{(j)} + \left(\frac{D}{p}\right)_{60}^{high}\right)/2$, and update the upper bound to $\left(\frac{D}{p}\right)_{60}^{high} = \left(\frac{D}{p}\right)_{60}^{(j)}$.
- 7. Repeat from step 3 and iterate until convergence.

In the simulations we choose the upper bound $\left(\frac{D}{P}\right)^{high}$ to be 8, a debt-to-GDP ratio of 200%. The initial guess of the final steady state is $(D/P)_{60}^{(1)} = 0.1$. The stopping threshold is chosen to be 10^{-6} . The algorithm takes 21 iterations to converge.

The algorithm for the simulation of the deleveraging shock in the flexible price model is described as follows:

- 1. The dynamics of debt is given by Equation (34).
- 2. The system reaches the steady state at the end of period 60 with $\left(\frac{D}{P}\right)_{60} = 0$. The steady-state values of other variables are obtained by solving the steady state version of Equation (20) -- (28).
- 3. Use Equation (20), (21), (23) -- (28) to solve for C_{59} , C_{59}^* , Y_{59} , Y_{59} , S_{59} , r_{59} , r_{59}^* . Note that the dynamics of debt is known and $\beta_{F,59}$ is given by Equation (33).
- 4. Repeat the previous step for t = 58,57 ... until we reach period 30.

The system with nominal rigidities is more complicated because there are three state variables, namely $\Delta_t, \Delta_t^*, D_t/P_t$. With multiple state variables, this system does not reach the steady state immediately after period 60. We approximate this system by fixing the price dispersions, Δ_t and Δ_t^* , at unity and eliminate Equations (62) and (63). The benefit of the approximation is that this eliminates two predetermined variables so that external debt D/P is the only state variable remaining in the system. This makes it possible to reverse-shoot from period 60. And, according to Woodford (2003), the price dispersions are second-order terms, so the accuracy cost of this approximation is low. The alternative, multi-dimensional shooting method developed by Atolia and Buffie (2009) is computationally intensive and time-consuming. We do not pursue this approach in this paper.

We apply the same approach as in the flexible price model to solve for the dynamics for the saving shock in China, assuming the zero bound constraint does not bind in the home and foreign country. As before, we choose $\left(\frac{D}{P}\right)^{high} = 8$, $(D/P)_{60}^{(1)} = 0.1$, *threshold* = 10^{-6} . The algorithm converges at j = 23 iterations. The simulation result confirms that a zero bound does not bind with the saving shock alone.

In the simulation for the deleveraging shock, we have to allow for a zero bound. The algorithm is the same as the one used in the flexible price model, except for step 3, which is changed as follows:

Assume a zero bound does not bind in both the home and foreign countries. Use Equations (48), (49), (51) -- (61), (64) -- (68) to solve for

$$C_t, C_t^*, Y_t, Y_t^*, S_t, i_t, i_t^*, \Pi_{Ht}, \Pi_{Ft}^*, \frac{W_t}{P_t}, \frac{W_t^*}{P_t^*}, K_t, K_t^*, F_t, F_t^*, \frac{\tilde{P}_{Ht}}{P_{Ht}}, \frac{\tilde{P}_{Ft}^*}{P_{Ft}^*}.$$

If the solution is such that $i_t > 0$, $i_t^* > 0$ then go to the next step. If the solution is such that $i_t < 0$ then the zero bound constraint is binding, then set $i_t = 0$ and use the system equations to solve for the other variables. If the solution is such that $i_t^* > 0$, go to the next step. If this results in $i_t^* < 0$, impose the zero bound for the foreign country as well and use the system equations to solve for the other variables.