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China's Capital and "Hot" Money Flows: An Empirical Investigation

Tao Cai*

Sun Yat-sen University

and

Vinh Q. T. Dang**

University of Macau

and

Jennifer T. Lai[#]

Guangdong University of Foreign Studies

Hong Kong Institute for Monetary Research

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Abstract

We examine time-series characteristics of China's capital flows during 1998-2014. More specifically, we employ Kalman filtering state-space models to gauge relative importance of permanent and transitory components in China's overall FDI, equity, bond, other investment, and bank credit flows. Our results show that only in the case of FDI are both gross inflow and net flow dominated by a permanent stochastic level, suggesting that this source of capital is largely permanent. Incorporating covariates into the state-space models, we find that larger difference between RMB onshore and offshore interest rates encourages capital inflows that are dominated by a transitory component. Greater global risk perception, proxied by S&P 500's volatility index, on the other hand, discourages them. These covariates imply that capital control may not be effective in stemming volatile and speculative flows. Our results on bilateral capital flows between China and US also suggest that these flows are less persistent and more volatile during 1998-2014 than previously found based on 1988-1997 data. Our results bear important policy implications as China engages in further reforms in its domestic financial system and greater integration with the world financial system.

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* School of Mathematics and Computational Science, Sun Yat-sen University, China

** Department of Economics, University of Macau, Macau SAR, China

Corresponding author: Jennifer T. Lai. Email address: econlai@gmail.com. School of Finance, Guangdong University of Foreign Studies, China. We would like to thank Kenneth Chan, Isabel Yan, Fred Y. K. Kwan, Vikas Kakkar, Mingming Jiang, Iikka Korhonen and the anonymous referee for helpful comments. The authors have also benefited from discussions with Shangjin Wei, Dong He, Wei Liao, and other participants at the Reserve bank of Australia 2010 workshop: China and East Asia in the Global Economy. Xiaoling Li and Qianjun Lyu provided excellent research assistance. Jennifer T. Lai acknowledges financial support from the National Natural Science Foundation of China (Project no. 71403061) and from Hong Kong Institute for Monetary Research.

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1. Introduction

Since economic reform was initiated in 1978, China has made great strides in integrating its economy in terms of trade with the world economy; it is now the largest trading partner of many developed and developing countries. Financial integration[1], however, is much more limited. As China is poised to become the world's largest economy, there is a push for much further and more systemic financial reforms, arising from the need to find more sustainable domestic-driven and balanced growth. Major trading partners, seeking better access to financial investment opportunities in China, have also called for its greater participation in the global financial system.

Financial crisis in the last two decades have provided cautionary motive for China's piece-meal financial reform and gradual opening of its capital account to the rest of the world[2]. Strict capital control measures have been put in place to govern capital flows in and out of China. Yet, capital flows have increased significantly over the past decades, especially after mid 2000s. They are depicted in Figure 1 for 1998-2014 data[3]. In Figure 1a, FDI gross inflow to China rose from 8.9 billion USD in 1998Q1 to 66.1 billion USD in 2014Q1 in nominal terms. Equity gross inflow (Figure 1b) increased from practically zero to over 20 billion USD in mid 2000s and remained around 17 billion USD at the end of 2013. There is a rise in bond and bank credit flows, as shown in Figures 1c and 1d, as well.

In Figure 2, we scale China's capital flows against its GDP. The ratios of equity, bond, and other investment flows to GDP are well below that of FDI, reflecting capital control policies that are tilted heavily in favor of the latter. From Figures 1 and 2, there are large and more frequent swings in the gross and net flows after 2005.

The increase in both the volume and volatility of China's international capital flows has attracted some attention in the literature. Ma and Sun (2007) argue that larger financial flows may bring the so-called hot money, which may cause instability in the exchange rate regime. Ljungwall and Wang (2008) show that capital flight from China rose significantly from late 1980s to early 2000s and was mainly financed by foreign liabilities, which corroborates the results in Gunter (1996, 2004). In a more recent study, Cheung and Qian (2010) demonstrate that the momentum of capital flight accelerated in the 1990s and exhibited large fluctuation in the 2000s; they also suggest that the dynamics of capital flight could be well explained by its own history and covered interest rate differentials.

1 Here, financial integration refers to that with the rest of the world. Jiang (2014), Lai et al. (2013), and Chan et al. (2011, 2014) examine financial integration among different regions within China.

2 Aizenman (2013) finds that China's trilemma configuration is unique as it puts much emphasis on exchange rate stability and at the same time compromises its financial integration and monetary independence.

3 Gross inflow of a particular capital account item is defined as net purchases of domestic assets by foreign residents; gross outflow is net purchases of foreign assets by domestic residents. Similar definition is used in Forbes and Warnock (2012) and Broner et al. (2013).

The rising tide of cross-border capital flows is not only restricted to China, it has been a world-wide phenomenon since 1980s, spurring both theoretical and empirical work that attempts to understand its dynamics. For example, Tille and van Wincoop (2010) incorporate portfolio choice into a two-country dynamic stochastic general equilibrium model and find that endogenous time variation in expected returns and risk are key determinants of portfolio choice under full capital mobility. They also show that persistence of capital flows is mainly driven by expected excess return and portfolio growth.

In a real business cycle model with portfolio choice, Evan and Hnatkovska (2014) argue that capital flows exhibit different degree of volatility across different stages of financial integration of the domestic economy with the world financial market. The volume and volatility are large during the early stage; both then decline along with increasing integration. Although G-7 data over 1980-2010 are used for empirical validation, the implications of their model are also relevant for China. Currently, China's position can be considered as "partial integration", defined in Evan and Hnatkovska (2014) as the case in which households have access to domestic equities and international bond, but cannot hold foreign equities. China is close to this description, as one could treat the central bank in China as an agent for domestic residents in international financial markets (Bacchetta et al., 2014). China can therefore be expected to experience capital flows with high volume and volatility based on their model prediction. Evan and Hnatkovska (2014) also show that along increasing financial integration, global factors become more important determinants of equity returns. This idea seems to be corroborated by Ahmed and Zlate (2013), who show that global risk appetite is one of the statistically and economically significant determinants of net private capital flows to emerging markets.

Against the backdrop of recent surges in China's capital flows, there have been some studies evaluating the effectiveness of its capital control. Ma and McCauley (2008) argue that although there might be loopholes in the measures and implementation of capital control, there is still significant gap between onshore and offshore Renminbi (RMB) interest rates. Cheung et al. (2008) suggest that Chinese interest rate is not driven by the US rate due to capital controls that are designed to retain some monetary independence.[4] In a more recent paper, Cheung and Herrala (2014) show that the covered interest rate differentials are not shrinking over time but actually have become larger after the 2008 global financial crisis. The sustained wedge between the onshore-offshore RMB interest rates is interpreted in their paper as evidence of effective capital control.

Our paper has the following objectives. First, we perform Kalman filter on China's capital flows to see if they are more permanent or transitory in dynamics. The results have important implications on the design and implementation of capital control as well as formulation of policy response to adverse capital flows. In this regard, our paper is closely related to Sarno and Taylor (1999), in which they examine bilateral capital flows from the US to Latin American and Asian emerging markets, including China, during 1988-1997. In our analysis, however, we look at China's flows to and from the rest of

[4] Jeanne (2011, 2012) argues that capital control in China could be exercised by the government to keep the currency undervalued and to stimulate growth in the tradable sector.

the world since financial transactions with the US, for most of the capital account items, only account for a small fraction of China's overall external transactions.

Second, we examine whether China's capital flows have become more volatile when compared to previous decades during which China was much less integrated financially with the rest of the world (Jonhansson, 2010). To this aim, we also analyze more recent (1998-2014) China-US bilateral flows data and compare the results to those in Sarno and Taylor (1999). If Evan and Hnatkovska's (2014) theoretical prediction of heightened volatility is confirmed empirically for the case of China, prudential policies are necessary to guard against potential sudden stop or reversal in these financial flows.

Third, we also investigate whether capital control measures in China are effective in preventing speculative capital flows. If they are, then these flows should not be sensitive to movements in short-term arbitrage profit opportunities or short-term market volatility. We will incorporate some covariates into the state-space models to see if what relationship, if any, holds between them. The results also provide a robustness check on the results from Kalman filter.

In addition, earlier studies on China's capital flows focus mainly on net flows whereas we extend our investigation to gross inflows and outflows. Although net flows, taken as the difference between gross inflows and outflows, are used in Forbes and Warnock (2012), these authors also argue that they become increasingly insufficient at describing cross-border capital flows in emerging-market economies because the dynamics are offset in the net flows. Broner et al. (2013) provide a synthesized study of gross capital flows in both developed and developing countries, in which they find that gross flows are very large and volatile compared to net flows. In Figure 1, China's gross inflows and outflows for some capital items started to diverge around 2005.

Kalman state-space models are presented in Section 2 and data description follows in Section 3. We discuss the results in Section 4 and offer some concluding remarks in Section 5.

2. State Space Models

In order to gauge the relative importance of permanent and transitory component in a capital flow series, we employ the following state space model suggested by Harvey (1981, 1989):

$$f_t = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{pmatrix} + \varepsilon_t, \qquad t = 1, 2, ..., T, \; \varepsilon_t \sim N(0, \sigma_*^2) \tag{1}$$

$$\begin{bmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_v \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \\ \upsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \zeta_t \\ \xi_t \end{bmatrix}, \tag{2}$$

Equation (1) is called the measurement equation, in which we decompose an observable capital flow series f_t into several unobserved components such as μ_t, υ_t and ε_t. μ_t is a trend component; β_t is a slope component for μ_t. υ_t is a first-order autoregressive AR(1) component, and the absolute value of ρ_v is constrained to be less than 1 in order to ensure stationarity of the component. ε_t is an irregular component and is approximately normally independently distributed with mean zero and constant variance σ_*^2. Equation (2) is called the transition equations, which describe the evolvement of the unobservable state vector $(\mu_t \quad \beta_t \quad \upsilon_t)^T$. All the three error terms in Equation (2) are independently, identically, and normally distributed with mean zero and variances σ_η^2, σ_ς^2, and σ_ξ^2 respectively.

The above model can be put in a more compact form:

$$y_t = \mathbf{Z}\boldsymbol{\alpha}_\mathbf{t} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_*^2) \tag{3}$$

$$\boldsymbol{\alpha}_\mathbf{t} = \mathbf{T}\boldsymbol{\alpha}_\mathbf{t-1} + \mathbf{R}\boldsymbol{\eta}_\mathbf{t}, \quad \boldsymbol{\eta}_\mathbf{t} \sim N(0, \mathbf{Q}_{\boldsymbol{\eta}}) \tag{4}$$

where,

$$\mathbf{Z} = \begin{pmatrix}1 & 0 & 1\end{pmatrix}, \quad \boldsymbol{\alpha}_t = \begin{pmatrix}\mu_t & \beta_t & \upsilon_t\end{pmatrix}^T,$$

$$\mathbf{T} = \begin{bmatrix}1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_v\end{bmatrix}, \ \mathbf{R} = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}, \ \mathbf{Q}_{\boldsymbol{\eta}} = \begin{bmatrix}\sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_\varsigma^2 & 0 \\ 0 & 0 & \sigma_\xi^2\end{bmatrix}, \boldsymbol{\eta}_\mathbf{t} = \begin{bmatrix}\eta_t \\ \zeta_t \\ \xi_t\end{bmatrix}$$

Sarno and Taylor (1999) employ this model to study the persistence of capital flows from US to various emerging markets in Asia and in Latin America, including China. They, however, do not account for seasonal effects. Seasonality has long been considered an integral feature of macroeconomic time series; moreover, it has recently been suggested as an important driving factor of fund flows in the US (Kamstra et al., 2013). Failure to account for seasonality may lead to conclusions that tend to overemphasize the importance of temporary component in a time series. Hence, we incorporate seasonality into the above model[5]:

[5] *s* is equal to12 and 4 for, respectively, monthly and quarterly data.

$$f_t = \begin{bmatrix} \mathbf{Z} & \underset{(1\times(s-1))}{\mathbf{C_t}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_\mathbf{t} \\ \underset{((s-1)\times 1)}{\boldsymbol{\gamma}_\mathbf{t}} \end{bmatrix} + \varepsilon_t, \quad t = 1, 2, ..., T, \ s = 12 \tag{5}$$

$$\begin{bmatrix} \boldsymbol{\alpha}_\mathbf{t} \\ \underset{((s-1)\times 1)}{\boldsymbol{\gamma}_\mathbf{t}} \end{bmatrix} = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{0} & \underset{((s-1)\times(s-1))}{\mathbf{T_s}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_{\mathbf{t}-1} \\ \underset{((s-1)\times 1)}{\boldsymbol{\gamma}_\mathbf{t-1}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_\mathbf{t} \\ \underset{((s-1)\times 1)}{\boldsymbol{\omega}_\mathbf{t}} \end{bmatrix}, \tag{6}$$

For trigonometric seasonality:

$$\underset{(1\times(s-1))}{\mathbf{C_t}} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 \end{bmatrix}, \quad s = 12 \tag{7}$$

$$\underset{((s-1)\times 1)}{\boldsymbol{\gamma}_\mathbf{t}} = \begin{bmatrix} \gamma_{1t} & \gamma_{1t}^* & \gamma_{2t} & \gamma_{2t}^* & \cdots & \gamma_{\left(\frac{s}{2}\right)t} \end{bmatrix}^T \tag{8}$$

$$\underset{((s-1)\times(s-1))}{\mathbf{T_s}} = \begin{bmatrix} \cos\lambda_1 & \sin\lambda_1 & & & & \\ -\sin\lambda_1 & \cos\lambda_1 & & & & \\ & & \cos\lambda_2 & \sin\lambda_2 & & \\ & & -\sin\lambda_2 & \cos\lambda_2 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}, \ \lambda_j = \frac{2\pi j}{s}, j = 1, ... \frac{s}{2} \tag{9}$$

$$\boldsymbol{\omega}_\mathbf{t} = \begin{bmatrix} \omega_{1t} & \omega_{1t}^* & \omega_{2t} & \omega_{2t}^* & ... & \omega_{\left(\frac{s}{2}\right)t} \end{bmatrix}^T, \ \boldsymbol{\omega}_\mathbf{t} \sim N(\mathbf{0}, \underset{((s-1)\times(s-1))}{\mathbf{Q}_{\boldsymbol{\omega}}}), \ \mathbf{Q}_{\boldsymbol{\omega}} = \sigma_\omega^2 \mathbf{I} \tag{10}$$

where it is assumed that each element in $\boldsymbol{\omega}_\mathbf{t}$ is approximately normally independently distributed with mean zero and common variance σ_ω^2.

If we use seasonal dummy variables instead:

$$\underset{(1\times(s-1))}{\mathbf{C_t}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}, \quad s = 12 \tag{7'}$$

$$\underset{((s-1)\times 1)}{\boldsymbol{\gamma}_\mathbf{t}} = \begin{bmatrix} \gamma_t & \gamma_{t-1} & \cdots & \gamma_{t-s+2} \end{bmatrix}^T \tag{8'}$$

$$\underset{((s-1)\times(s-1))}{\mathbf{T_s}} = \begin{bmatrix} -1 & -1 & \cdots & -1 & -1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \tag{9'}$$

$$\boldsymbol{\omega}_\mathbf{t} = \begin{bmatrix} \omega_t & 0 & \dots & 0 \end{bmatrix}^T, \tag{10'}$$

$$\boldsymbol{\omega}_\mathbf{t} \sim N(\mathbf{0}, \underset{((s-1)\times(s-1))}{\mathbf{Q}_{\boldsymbol{\omega}}}),\ \mathbf{Q}_{\boldsymbol{\omega}} = \begin{bmatrix} \sigma_\omega^2 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

The state-space models can also accommodate covariates:

$$f_t = \begin{bmatrix} \mathbf{Z} & \underset{(1\times k)}{\mathbf{Z}_{\boldsymbol{\lambda},\mathbf{t}}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_\mathbf{t} \\ \underset{(k\times 1)}{\boldsymbol{\lambda}_t} \end{bmatrix} + \varepsilon_t, \quad t = 1, 2, \dots, T,\ \varepsilon_t \sim N(0, \sigma_*^2) \tag{11}$$

$$\begin{bmatrix} \boldsymbol{\alpha}_\mathbf{t} \\ \underset{(k\times 1)}{\boldsymbol{\lambda}_\mathbf{t}} \end{bmatrix} = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{0} & \underset{(k\times k)}{\mathbf{I}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha}_\mathbf{t-1} \\ \underset{(k\times 1)}{\boldsymbol{\lambda}_\mathbf{t-1}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_\mathbf{t} \\ \underset{(k\times 1)}{\mathbf{0}} \end{bmatrix},\ \boldsymbol{\eta}_\mathbf{t} \sim N\left(\mathbf{0},\ \mathbf{Q}_{\boldsymbol{\eta}}\right) \tag{12}$$

where:

$$\mathbf{Z}_{\boldsymbol{\lambda},\mathbf{t}} = \begin{bmatrix} X_{1t} & \cdots & X_{jt} & \cdots & X_{kt} \end{bmatrix},\ j = 1, \dots, k \tag{13}$$

$$\boldsymbol{\lambda}_\mathbf{t} = \begin{bmatrix} \lambda_{1t} & \lambda_{2t} & \cdots & \lambda_{kt} \end{bmatrix}^T \tag{14}$$

Any intervention or explanatory variables are contained in $\mathbf{Z}_{\boldsymbol{\lambda},\mathbf{t}}$; their parameters are contained in $\boldsymbol{\lambda}_\mathbf{t}$. $\mathbf{Z}$, $\boldsymbol{\alpha}_\mathbf{t}$, $\mathbf{T}$ and $\boldsymbol{\eta}_\mathbf{t}$ are defined in Equations (3) and (4). Here we include the parameters for variables in $\mathbf{Z}_{\boldsymbol{\lambda},\mathbf{t}}$ in the state vector to get their estimates. Their corresponding error terms are suppressed to have zero variances in order to get time-invariant estimation.

Once a state space model has been set up, Kalman filter technique could be employed to compute the optimal estimator up to time *t*-1 (the prediction equations), information up to time *t* (the updating equations), and information for the whole period *T* (the smoothing equations) (Harvey, 1981 and1989; Durbin and Koopman, 2001).[6]

6 For algorithm details, see Sarno and Taylor (1999) and Durbin and Koopman (2001).

We employ Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton algorithm (Harvey, 1981;Sarno and Taylor, 1999). When choosing the starting value for the algorithm, we resort to a method of exact initialization developed by Koopman (1997) and Durbin and Koopman (2001). This method could significantly improve the precision of the algorithm over the traditional method of diffused initialization used by Harvey (1989) and Sarno and Taylor (1999). We present the algorithm briefly in Appendix.

The estimated variance parameters indicate the relative contribution of each component in the state vector in explaining the total variation in the time series under consideration (Durbin and Koopman, 2001). By using the information provided by the estimated variances regarding the size of the nonstationary and the stationary components in the series, we would be able to quantify the degree of persistence of the series in question. If a large and statistically significant proportion of the variation in a capital flow is attributed to the stochastic level component μ_t , then one may expect that a large part of that flow will not easily reverse itself and will remain in the recipient country for some time. However, if a large portion of the variation in that flow is attributed to a temporary component such as the irregular component ε_t or AR(1) component υ_t , then it has low permanence and therefore can be easily reversed.

Table 1 contains 6 model specifications. Each of these specification can be augmented with trigonometric seasonality (let it be version b) or seasonal dummy variables (version c). For each flow series, we rely on Bayesian information criterion (BIC) to select an appropriate specification (Harvey, 1989; Sarno and Taylor, 1999).[7]

3. Data

Quarterly data (1998Q1 to 2014Q1) of China's overall capital flows are obtained from China's State Administration of Foreign Exchange. In particular, we examine gross inflows, gross outflows, and net flows of foreign direct investment (FDI), portfolio investment comprising of equity (EQY) and bond (BOND), and other investment (OI), in which bank credit (BC) is one of the main components. As shown in Figure 1, both the volume and volatility of China's various capital flows increase over the sample period.

Data sample of China-US bilateral flows also spans from 1998 to 2014. These items are displayed in Figure 3. Quarterly FDI gross inflow to China for 1998Q1-2014Q1 is collected from US Bureau of Economic Analysis.FDI gross outflow from China to the US is not available. Equity, bond, and bank credit gross and net flows are obtained from US Department of Treasury. There are no equivalent bilateral "other investment" flows data. We analyze equity and bond series in monthly frequency (1998M1-2014M5) to take advantage of larger number of observations.

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[7] $BIC = \log(PEV * \exp(\log T * (n+d)/T)$, where *PEV* is prediction error variance, *T* is time dimension of the sample, *n* is the number of hyperparameters, and d is the number of non-stationary elements in the state vector (Harvey, 1989, p. 270).

US FDI outbound for China increased significantly, from 308 million USD at the beginning of 1998 to 7.9 billion USD at the end of 2008 (Figure 3a). It then declined sharply in 2009 and fluctuated considerably thereafter. The volume and volatility of equity and bond flows also rise over the sample period (Figures 3b and 3c). In more recent years, net equity flow displays large swing and net bond flow is driven by the outflow. The value of bank credit is very large and greater than the sum of other flows. The sharp decline in bank credit net flow to China in the third quarter of 2008 was driven by gross outflow of over 100 billion USD from China.

We incorporate RMB interest rate differential between onshore and offshore markets as a covariate in the state space models. Following Cheung and Herrala (2014), the onshore rate is the one-month Shanghai interbank offer rate (SHIBOR) and the offshore rate is the one month RMB deposit rate implied by the covered interest rate parity of the RMB and USD. As China integrates further with the world financial market, global factors are expected to become important determinants of capital flows (Ahmend and Zlate, 2013; Evan and Hnatkovska, 2014). We include S&P 500's Volatility Index (VIX) computed by the Chicago Board Options Exchange to proxy for global risk factor as another covariate (Forbes and Warnock, 2012). Higher values of VIX indicate higher risk perception and therefore lower risk tolerance. These explanatory variables are used to test whether capital control is effective and to check the robustness of the results obtained from Kalman filter. If the capital control is effective, then capital flows, particularly those that are transitory, should not be sensitive to movements in the short-term return differential or market volatility.

4. Empirical Results

4.1 China's Overall Capital Flows: Basic Models

In Table 2, we present the results of Kalman filter of each of China's overall capital items (Column 1). The specification selected for the series from 6 models in Table 1 is shown in Column 2. When the model is augmented with trigonometric seasonality or seasonal dummy variables, it is indicated by a suffix "b" or "c", respectively. Columns 3-6 show the Q-ratios and statistical significance of the estimated coefficients of the components in the final state vector. The Q-ratios are the ratios of the estimated standard deviation (SD) of each component to the largest SD among the components. Therefore, a value of 1 indicates this is the dominant component in the model. Column 7 contains the estimated AR(1) coefficient ρ_v. The last two columns show R-squared and p-value of the Ljung-Box test statistic. The result for gross inflows, gross outflows, and net flows are contained, respectively, in Panels A, B, and C.

Model 2 augmented with trigonometric seasonality (2c) is selected for the FDI gross inflow series. The estimated Q-ratios indicate that this capital item is dominated by a permanent component, as the stochastic level has much larger SD compared to the seasonal and irregular components. This is consistent with our prior expectation; FDI tends to gravitate toward long-term investment opportunities in the host countries, and is less speculative compared to other flows. In the final state vector, the estimated coefficient is statistically significant for all three components in the model.

For equity (EQY) gross inflow, the irregular component has a much larger SD than the stochastic level component. This is consistent with our expectation; portfolio equity flows, unlike FDI, tend to explore short-term financial investment opportunities, rather than long-term real investment opportunities. Hence, this equity flow is prone to short-term volatility and reversibility. On the other hand, bond gross inflow appears to be dominated by a statistically-significant stochastic level component, implying this capital inflow is quite persistent, more so than the equity series. A closer look at the data, however, suggests otherwise. Figure 2c shows that the value of bond inflow is close to zero and it remains dormant for most of the sample period, only showing some increases in activities starting 2012. The result is likely driven by negligible foreign investment in China’s bond market, which had been small, underdeveloped, and inactive until the last few years.

Most of the variation in the other investment (OI) item is explained by the irregular component whereas for bank credit (BC), a constituent of OI, the stochastic level is the dominant component. Bank credits arise from recurring need to finance transactions in the current accounts and to maintain liquidity facility between financial institutions in the trading countries. This capital item is therefore affected by such long-term factors as the country’s credit rating, macroeconomic condition, and growth prospect. Moreover, in China, strict government supervision of state-owned banks and capital control imposed on all banks ought to keep this flow less prone to short-term speculative inbound movements of funds. Beyond bank credits, other items such as non-bank loans and off-balance sheet transactions that make up the rest of OI are less transparent and more speculative; therefore they can contribute more to variation in the series. It can also explain how the stochastic level component is statistically significant in the OI flow, even though it does not account for a large part of the variation. Similar finding in the case of bank credit is observed in Sarno and Taylor (1999) for China-US bilateral flow.

In Panel B, all outflows, except equity, are dominated by irregular component, indicating that these outward investments are quite volatile and transitory in nature. Figure 1b helps explain why a large part of variation in the equity series is derived from a permanent component. China’s equity gross outflow, as bond gross inflow, had been non-active due to capital control measures until the introduction of government-sanctioned Qualified Domestic Institutional Investor (QDII) scheme in 2006, which allows domestic institutional investors to invest in offshore capital markets. In 2007, the restriction of outbound investment to only foreign fixed-income and money-market instruments was relaxed to allow for equity investment as well (Yao and Wang, 2012). The result was a sharp increase in equity outflows starting from 2007.

Among the net capital flows in Panel C, only in FDI series is there evidence of permanence; the stochastic level is also statistically significant. The dynamics of FDI gross inflow seem to drive those of the net flow since China is the largest recipient among emerging markets of this type of capital for many years, including the sample period, but it has made significant outward FDI only recently. The stationary AR(1) component dominates the stochastic level component in the equity and other investment flows although it is only statistically significant in the latter. The estimated AR(1) coefficient is 0.278, implying that the half life of shocks affecting OI net flow to China is 0.541 or approximately 1.5 months.

To recapitulate, China's FDI gross inflow and net flow are both dominated by a permanent component and therefore are expected to be rooted in the recipient country for a considerable amount of time. Bank credit gross inflow also displays some degree of permanence. The rest of capital flows appear quite transitory and are likely subject to sudden stop and/or reversal.

4.2 China's Overall Capital Flows: Interest Rate Differential and Volatility Index

Table 3 contains results from including a covariate in the state-space models. Except bank credit gross inflow, Model 2, or its seasonality-augmented version, is selected for all the flow items. When interest rate differential is included, its estimated coefficient is positive and statistically significant for OI gross inflow and net flow, and bank credit net flow, all of which are transitory based on Kalman filter results discussed earlier. From Table 2, in these three flows, the permanent component account for negligible, and in fact smallest, fraction of total variation. It appears that some of China's very volatile gross inflows and net flows respond to interest rate differentials. Larger spread between onshore and offshore interest rates attracts more inflows.

When volatility index is included, its coefficient is negative and statistically significant for equity gross and net flows and bank credit net flow. Higher global volatility discourages some volatile gross inflow and net flow into China. It also appears that higher global volatility discourages Chinese domestic residents from investing abroad. This finding is consistent with what is suggested in Evan Hnatkovska (2014) and Ahmed Zlate (2013) in that as a country is increasingly integrated with the world financial market, such global factors as global risk perception are important determinants of its capital flows.

Neither interest rate differential nor market volatility affects FDI gross inflow, FDI net flow, bank credit gross inflow, all of which are found to contain a dominating and statistically-significant permanent component. This indicates some robustness to the findings from Kalman filter above, at least for these flow data series. The results from including the covariates in the state-space models also suggest that China's capital control measures may not be effective because some volatile inflows, both on the gross and net bases, do respond to fluctuations in these two short-term variables, which are themselves transitory.

4.3 China-US Bilateral Capital Flows: Comparison with Sarno and Taylor (1999)

We repeat Kalman filtering algorithm on China-US bilateral capital flows for 1998-2014 data. The variation in each series is dominated by either a transitory or irregular component. We will discuss the results in Table 4 in the context of comparison with those obtained from 1988-1997 data in Sarno and Taylor (1999). Similar model specification is selected for most of the capital items in both studies.

In Sarno and Taylor (1999), FDI and bank credit inflows are dominated by a statistically significant stochastic level, suggesting a considerable amount of permanence in them. In our study, they are dominated by AR(1) component. In addition, the estimated AR(1) coefficient is very small, 0.207 and -0.061 respectively, suggesting small degree of persistence.

Equity net flow from US to China is dominated by AR(1) component in both data samples. The estimated AR(1) coefficient in our sample, 0.095, is much smaller than that in their sample (0.214). This implies that shocks affecting equity net flow have a much shorter half life during 1998-2014 than during 1988-1997. Even for transitory AR(1) component, its persistence declines over the two sample periods. Similar finding is observed for bond gross inflow.

There is evidence that bilateral capital flows from US to China have become less persistent and more volatile over time. It is consistent with prediction from Evan and Hnatkovska (2014) in that as an economy goes through the process of financial integration with the rest of the world, its capital flows are more volatile during the early stage of integration.

5. Conclusion

While the financial markets and institutions across countries are increasingly integrated, there have been frequent crises that wreak increasingly severe havoc on the global economy as well. Although China has escaped financial crises in the last two decades relatively unscathed, it must exercise great caution as to how to proceed through the inevitable process of integration with the world financial market.

There are important policy implications from our findings. There is robust evidence affirming FDI as a steady source of capital that is not sensitive to movement in short-term return differential or market volatility. Being the largest recipient of FDI in emerging markets for the last two decades, China has benefited tremendously from this productive source of capital. Policy makers should continue to cultivate the incentives that have helped attracting FDI. There is also some evidence that bank credit gross inflow is quite steady; it should be supported as well[8]. Greater attention should be paid to the dynamics of other capital flows as they are volatile and therefore susceptible to sudden stop and/or reversal. Instead of relying on various capital control measures to restrict these flows as these measures may not be effective in light of active attempts to circumvent them (Kar and Freitas, 2012),

[8] Jeanne (2011, 2012), however, provide a caveat. The time series features of bank credit flows might be endogenous when China further opens its capital account and gradually remove its capital controls. Specifically, if bank credit flows are mainly related to cross-border trade activities (which have experienced sustained growth over the past several decades), then once the government relaxes its control on capital flows, real exchange rate may no longer be subdued to stimulate the growth of the tradable sector. Then it is not clear if bank credit flows would still be dominated by a permanent component.

Chinese authorities should focus on deeper and more systemic reforms in the domestic financial system as to eliminate distortions and motives for speculative capital flows.

Finally, regardless of whether a capital flow is dominated by a permanent or transitory component, it appears to be less persistent and more volatile over time. In this regard, macro-prudential policies and proactive financial supervisory oversight should be put in place to safeguard against increasing volatility of capital flows and therefore vulnerability of the domestic financial system.

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Table 1. Structural Time-Series Models

Model 1: Stochastic level (no slope) + AR(1)

$$f_t = \mu_t + \upsilon_t$$
$$\begin{bmatrix} \mu_t \\ \upsilon_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho_v \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \upsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \xi_t \end{bmatrix}$$

Model 2: Stochastic level (no slope) + irregular component

$$f_t = \mu_t + \varepsilon_t$$
$$\mu_t = \mu_{t-1} + \eta_t$$

Model 3: Stochastic level (no slope) + AR(1) + irregular component

$$f_t = \mu_t + \upsilon_t + \varepsilon_t$$
$$\begin{bmatrix} \mu_t \\ \upsilon_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho_\upsilon \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \upsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \xi_t \end{bmatrix}$$

Model 4: Stochastic level (fixed slope) + AR(1) + irregular component

$$f_t = \mu_t + \upsilon_t + \varepsilon_t$$
$$\begin{bmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_v \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \\ \upsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ 0 \\ \xi_t \end{bmatrix}$$

Model 5: Stochastic level (fixed slope) + irregular component

$$f_t = \mu_t + \varepsilon_t$$
$$\begin{bmatrix} \mu_t \\ \beta_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ 0 \end{bmatrix}$$

Model 6: Stochastic level (fixed slope) + AR(1)

$$f_t = \mu_t + \upsilon_t$$
$$\begin{bmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_v \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \\ \upsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ 0 \\ \xi_t \end{bmatrix}$$

Table 2. Kalman Filter Results of China's Overall Capital Flows (1998Q1-2014Q1)

Series	Model[a]	Q-ratio from the SD of the error term of the components[b] & statistical significance of the components[c]				ρ_v	R^2	*p*-value of LB[d]
		Level	AR(1)	Seasonal	Irregular			
Panel A: Gross inflows								
FDI_{in}	2c	1.000**	.	0.360**	0.670**	.	0.954	0.552
EQY_{in}	2	0.140**	.	.	1.000	.	0.442	0.115
$BOND_{in}$	2	1.000**	.	.	0.836	.	0.847	0.296
OI_{in}	2	0.136**	.	.	1.000	.	0.221	0.000**
BC_{in}	5b	1.000**		0.523*	0.031*	.	0.357	0.427
Panel B: Gross outflows								
FDI_{out}	2	0.284**	.	.	1.000	.	0.799	0.254
EQY_{out}	2	1.000**	.	.	0.803	.	0.706	0.136
$BOND_{out}$	2	0.685	.	.	1.000	.	0.102	0.070
OI_{out}	2	0.204**	.	.	1.000	.	0.374	0.165
BC_{out}	2	0.123**	.	.	1.000	.	0.181	0.298
Panel C: Net flows								
FDI_{net}	2c	1.000**		0.281**	0.749**	.	0.929	0.299
EQY_{net}	1	0.667	1.000	.	.	-0.364	0.149	0.132
$BOND_{net}$	2	0.684*	.	.	1.000	.	0.157	0.067
OI_{net}	1	0.000	1.000**			0.278	0.046	0.031*
BC_{net}	5b	0.000	.	.	1.000	.	-0.011	0.002**

Notes:

[a]See model specifications in Table 1. It is chosen based on BIC.
Model #b is the model with the corresponding number in Table 1 augmented with trigonometric seasonality.
Model #c is the model with the corresponding number in Table 1 augmented with seasonal dummy variables.

[b]Q-ratio is the ratio of the standard deviation (SD) of each component to the largest SD across components included in the model. Therefore, a value of 1 indicates this is the dominant component in the model.

[c] ** and * indicate statistical significance at 1% and 5% levels.

[d]*p*-value of LB is the p-value of Ljung-Box statistic obtained in the null hypothesis of no serial correlation in the residuals.

Table 3. Kalman Filter Results of China's Overall Capital Flows with Interest Rate Differential and Volatility Index (1998Q1-2014Q1)

Series	Model	Interest rate differential		Volatility index	
		Coefficient	R^2/ *p*-value of LB	Coefficient	R^2/ *p*-value of LB
Panel A: Gross inflows					
FDI_{in}	2c	2.857 [6.741]	0.954 / 0.549	-1.479 [1.573]	0.953 / 0.389
EQY_{in}	2	-5.862 [3.253]	0.452 / 0.181	-2.012 [0.834]*	0.478/ 0.086
$BOND_{in}$	2	-1.059 [1.073]	0.852 / 0.310	0.116 [0.271][a]	0.840 / 0.308
OI_{in}	2	37.778 [18.364]*	0.328 / 0.021*	-5.743 [5.317]	0.274/ 0.014*
BC_{in}	5b	8.011 [7.894]	0.368 / 0.513	-4.585 [2.642]	0.377 / 0.439
Panel B: Gross outflows					
FDI_{out}	2	2.905 [3.304]	0.797 / 0.285	0.453 [0.843]	0.788/ 0.274
EQY_{out}	2	-0.944 [2.658]	0.715 / 0.128	-1.214 [0.675]*	0.710/ 0.121
$BOND_{out}$	2	-2.101 [8.339]	0.106 / 0.098	3.293 [2.039]	-0.023 / 0.022*
OI_{out}	2	6.325 [18.508]	0.364 / 0.173	2.986[4.820]	0.340/ 0.189
BC_{out}	2	3.181 [6.197]	0.160/ 0.358	-1.151 [1.661]	0.125 / 0.279
Panel C: Net flows					
FDI_{net}	2c	2.857 [6.741]	0.954/ 0.549	-1.476 [1.573]	0.953 / 0.389
EQY_{net}	2	-4.992 [3.361]	0.444/ 0.044*	-2.176 [0.971]*	0.144 / 0.287
$BOND_{net}$	2	-1.059 [1.073][a]	0.852 / 0.310	0.117 [0.269]	0.840 /0.305
OI_{net}	2	37.379 [16.843]*	0.270 / 0.0003**	-7.105 [4.840]	0.208 / 0.0001**
BC_{net}	2	20.623 [9.612]*	0.215 / 0.0003**	-7.311 [3.061]*	0.186 /0.001*

Note: See notes in Table 2.

Table 4. Kalman Filter Results of China-US Bilateral Capital Flows (1998-2014)

Series	Model[a]	Q-ratio from the SD of the error term of the components[b] & statistical significance of the components[c]				ρ_v	R^2	*p*-value of LB[d]
		Level	AR(1)	Seasonal	Irregular			
Panel A: Gross inflows								
FDI_{in}	3	0.101**	1.000**	.	0.026	0.207	0.449	0.606
EQY_{in}	5	0.081	.	.	1.000	.	0.016	0.980
$BOND_{in}$	3	0.064**	1.000**	.	0.018	-0.160	0.273	0.000**
BC_{in}	3	0.048	1.000**	.	0.034	-0.061	0.007	0.000**
Panel B: Gross outflows								
EQY_{out}	3	0.101	1.000	.	0.080	0.082	0.113	0.610
$BOND_{out}$	1	0.000*	1.000**	.	.	-0.076	-0.001	0.998
BC_{out}	1	0.000	1.000	.	.	0.286	0.057	0.134
Panel C: Net flows								
EQY_{net}	3	0.090	1.000	.	0.084	0.095	0.097	0.857
$BOND_{net}$	2	0.064**	.	.	1.000	.	0.168	0.000**
BC_{net}	1	0.000	1.000*	.	.	0.295	0.060	0.387

Notes: See notes in Table 2.

Data on equity and bond are available in monthly frequency (1998M1-2014M5); FDI and bank credit are available in quarterly frequency (1998Q1-2014Q1).

.

Figure 1. China’s Overall Capital Flows (1998Q1-2014Q1, in 100 Millions USD)

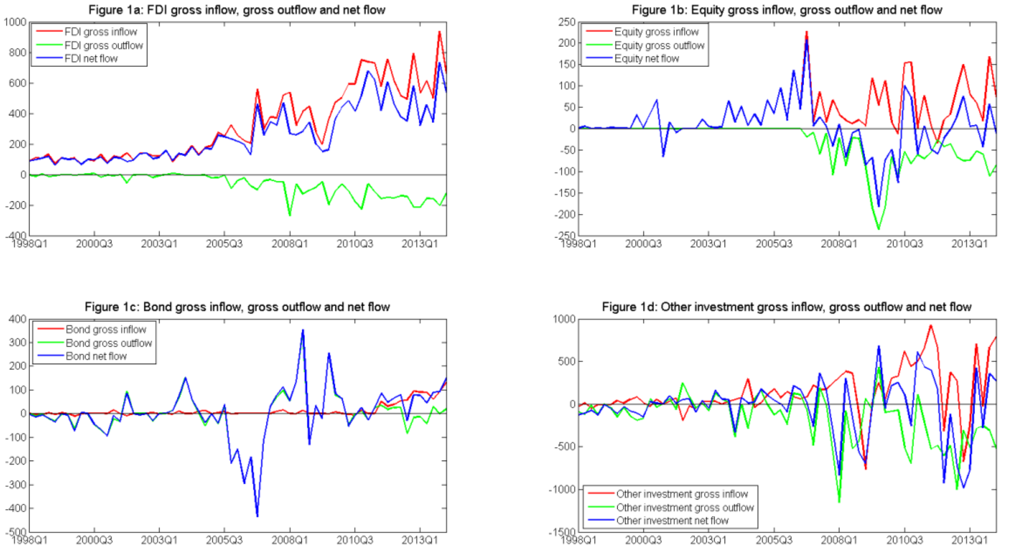


Figure 2. China’s Overall Capital Flows to GDP Ratios

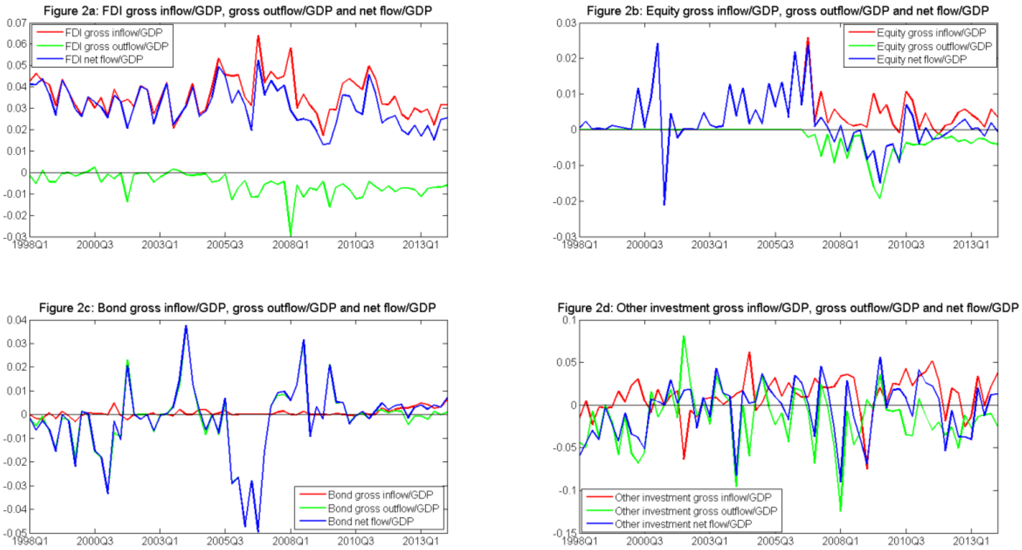
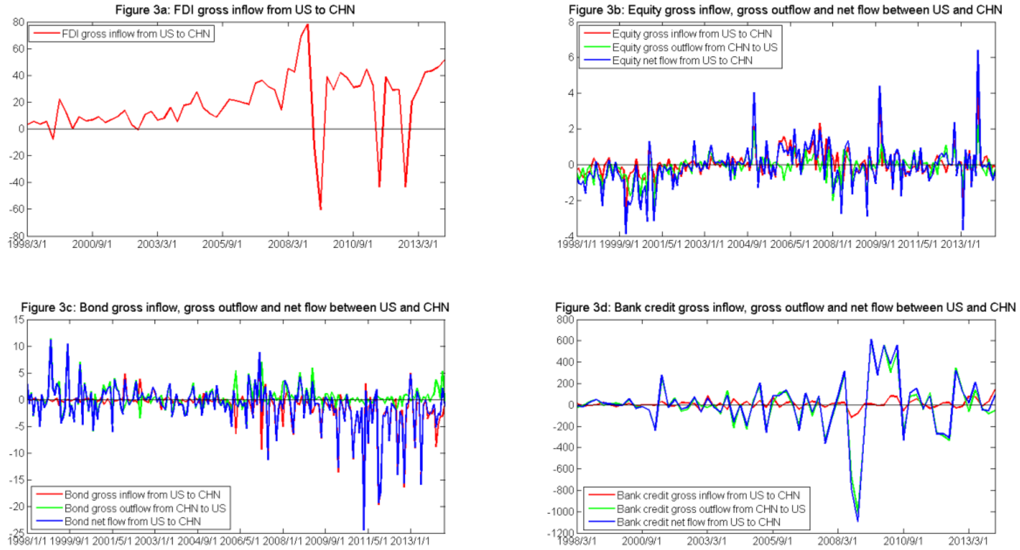


Figure 3. Bilateral Capital Flows between US and China (1998-2014, in 100 Millions USD)



Appendix. Exact Initialization for Kalman Filter Algorithm of State Space Models

For simplicity, a model without any covariate is used for illustration here. The algorithm, however, could straightforwardly be extended to incorporate an explanatory variable by treating the parameters as non-stationary component with variance constrained to be zero.

The general model we use is written as:

$$y_t = \mathbf{Z_t}\boldsymbol{\alpha}_\mathbf{t} + \varepsilon_t, \qquad Var(\varepsilon_t) = h_t \tag{A1}$$

$$\boldsymbol{\alpha}_\mathbf{t} = \mathbf{T_t}\boldsymbol{\alpha}_\mathbf{t-1} + \mathbf{R_t}\boldsymbol{\eta}_\mathbf{t}, \quad Var(\boldsymbol{\eta}_\mathbf{t}) = Q_t \tag{A2}$$

It includes a random walk part, an AR(1) part, and a slope β:

$$y_t = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{pmatrix} + \varepsilon_t, \tag{A3}$$

$$\begin{pmatrix} \mu_t \\ \beta_t \\ \upsilon_t \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_{t-1} \\ \beta_{t-1} \\ \upsilon_{t-1} \end{pmatrix} + \begin{pmatrix} \eta_t \\ \zeta_t \\ \xi_t \end{pmatrix} \tag{A4}$$

Applying the usual Kalman filtering technique on this system yields:

1. Prediction equations:

$$\begin{aligned} \mathbf{a_{t/t-1}} &= \mathbf{T_t a_{t-1}}, \\ \mathbf{P_{t/t-1}} &= \mathbf{T_t P_{t-1} T_t'} + \mathbf{R_t Q_t R_t'}, \quad t = 1, \dots, T \end{aligned} \tag{A5}$$

Here $\mathbf{a_{t-1}} = E(\boldsymbol{\alpha}_\mathbf{t-1} \mid y_{t-1}), \quad \mathbf{P_{t-1}} = E\left[\left(\boldsymbol{\alpha}_\mathbf{t-1} - \mathbf{a_{t-1}}\right)\left(\boldsymbol{\alpha}_\mathbf{t-1} - \mathbf{a_{t-1}}\right)^T\right]$ from the step just before (A5).

2. Updating equations:

$$\mathbf{a_t} = \mathbf{a_{t/t-1}} + \mathbf{P_{t/t-1} Z_t F_t^{-1}} v_t, \quad \mathbf{P}_t = \mathbf{P_{t/t-1}} - \mathbf{P_{t/t-1} Z_t F_t^{-1} Z_t P_{t/t-1}} \tag{A6}$$

$$v_t = y_t - \mathbf{Z_t a_{t/t-1}}, \quad \mathbf{F_t} = \mathbf{Z_t P_{t/t-1} Z_t'} + \mathbf{H_t} \tag{A7}$$

When selecting the initial values for the algorithm, first we divide the system into stationary and non-stationary parts:

$$\boldsymbol{\alpha}_0 = \mathbf{a} + \mathbf{A}\boldsymbol{\delta} + \mathbf{R}_0\gamma_0, \quad \gamma_0 \sim N(0, q_0),\ \boldsymbol{\delta} \sim N(\mathbf{0}, \kappa\mathbf{I}_{(2\times 2)}) \qquad \text{(A8)},$$

Here κ is an arbitrary large number (usually 1e7). $\boldsymbol{\delta}$ contains non-stationary components, and γ_0 contains stationary component.

$$\gamma_0 = \upsilon_0,\ \boldsymbol{\delta} = \begin{pmatrix} \mu_0 \\ \beta_0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \mathbf{R}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ \mathbf{a} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{(A9)},$$

$$\begin{aligned} &\mathbf{a}_0 = E(\boldsymbol{\alpha}_0) = \mathbf{a} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}', \\ &\mathbf{P}_0 = Var(\boldsymbol{\alpha}_0) = \mathbf{A}\mathbf{A}'\kappa\mathbf{I} + \mathbf{R}_0 q_0 \mathbf{R}_0' = \kappa\mathbf{P}_{\infty,0u} + \mathbf{P}_{*,0u}, \qquad \text{(A10)} \\ &\mathbf{P}_{\infty,0u} = \mathbf{A}\mathbf{A}',\ \mathbf{P}_{*,0u} = \mathbf{R}_0 q_0 \mathbf{R}_0'. \end{aligned}$$

The diffused initialization originally used by Harvey (1989) and Sarno and Taylor (1999) is to replace κ in (A10) by an arbitrary large number and then use the standard Kalman filter prediction and updating equations from (A5) to (A7). This approach can be useful for approximate exploratory work. However, it can lead to large rounding errors. In our paper, we use the exact initial Kalman filter treatment developed by Koopman (1997) and Durbin and Koopman (2001). We apply their technique to our models where both prediction and updating equations are needed, while in the original model of Koopman (1997) and Durbin and Koopman (2001), only updating equations are included for the recursion.

Following (A10) we decompose $\mathbf{P}_t$ as

$$\mathbf{P}_t = \kappa\mathbf{P}_{\infty,tu} + \mathbf{P}_{*,tu} + O(\kappa^{-1}),\ \text{t=1,...T} \qquad \text{(A11)}$$

According to Durbin and Koopman (2001, p102, eq.5.5), $O(\kappa^{-1})$ is a function $f(\kappa)$ such that the limit of $\kappa^j f(\kappa)$ as $\kappa \to \infty$ is finite for $j = 1, 2$.

This leads to the similar decomposition as follows:

$$\begin{aligned} &\mathbf{F}_t = \kappa\mathbf{F}_{\infty,t} + \mathbf{F}_{*,t} + O(\kappa^{-1}),\ \text{t=1,...T} \qquad \text{(A12)} \\ &\text{where } \mathbf{F}_{\infty,t} = \mathbf{Z}_t\mathbf{P}_{\infty,t}\mathbf{Z}_t',\ \mathbf{F}_{*,t} = \mathbf{Z}_t\mathbf{P}_{*,t}\mathbf{Z}_t' + h_t \end{aligned}$$

We write $\mathbf{F}_t^{-1}$ as a power series in κ^{-1}:

$$\mathbf{F}_t^{-1} = \mathbf{F}_\mathbf{t}^{(0)} + \kappa^{-1}\mathbf{F}_\mathbf{t}^{(1)} + \kappa^{-2}\mathbf{F}_\mathbf{t}^{(2)} + O(\kappa^{-3}) \qquad \text{(A13)}$$

And by utilizing $\mathbf{I_p} = \mathbf{F_t F_t^{-1}}$, we obtain:

$$\mathbf{F}_\mathbf{t}^{(0)} = 0, \quad \mathbf{F}_\mathbf{t}^{(1)} = \mathbf{F}_{\infty,\mathbf{t}}^{-1}, \quad \mathbf{F}_\mathbf{t}^{(2)} = -\mathbf{F}_{\infty,\mathbf{t}}^{-1}\mathbf{F}_{\infty,\mathbf{t}}\mathbf{F}_{\infty,\mathbf{t}}^{-1} \qquad \text{(A14)}$$

By using equations (A11) to (A14), we have the following algorithm for the exact initialization of the state space models:

1. The prediction equations:

$$\begin{aligned}
&\mathbf{a}_{\mathbf{t/t-1}} = \mathbf{T_t a_{t-1}}, \qquad \mathbf{P}_{\mathbf{t/t-1}} = \kappa\mathbf{P}_{\infty,\mathbf{t}} + \mathbf{P}_{*,\mathbf{t}} \\
&\mathbf{P}_{\infty,\mathbf{t}} = \mathbf{T_t}\mathbf{P}_{\infty,\mathbf{(t-1)u}}\mathbf{T}_\mathbf{t}', \quad \mathbf{P}_{*,\mathbf{t}} = \mathbf{T_t}\mathbf{P}_{*,\mathbf{(t-1)u}}\mathbf{T}_\mathbf{t}' + \mathbf{R_t Q_t R}_\mathbf{t}'
\end{aligned}$$

2. The updating equations:

$$\begin{aligned}
&\mathbf{a_t} = \mathbf{a_{t/t-1}} + \mathbf{K}_\mathbf{t}^{(0)}v_t^{(0)}, \quad \mathbf{K}_\mathbf{t}^{(0)} = \mathbf{P}_{\infty,\mathbf{t}}\mathbf{Z}_\mathbf{t}'\mathbf{F}_{\infty,\mathbf{t}}^{-1}, \quad v_t^{(0)} = y_t - \mathbf{Z_t a_{t/t-1}} \\
&\mathbf{P_t} = \kappa\mathbf{P}_{\infty,\mathbf{tu}} + \mathbf{P}_{*,\mathbf{tu}}, \\
&\mathbf{P}_{\infty,\mathbf{tu}} = (\mathbf{I_t} - \mathbf{Z}_\mathbf{t}'\mathbf{F}_{\infty,\mathbf{t}}^{-1}\mathbf{Z_t}\mathbf{P}_{\infty,\mathbf{t}})\mathbf{P}_{\infty,\mathbf{t}} \\
&\mathbf{P}_{*,\mathbf{tu}} = (\mathbf{I_t} - \mathbf{Z}_\mathbf{t}'\mathbf{F}_{\infty,\mathbf{t}}^{-1}\mathbf{Z_t}\mathbf{P}_{\infty,\mathbf{t}})\mathbf{P}_{*,\mathbf{t}} - (\mathbf{P}_{\infty,\mathbf{t}}\mathbf{Z}_\mathbf{t}'\mathbf{F}_\mathbf{t}^{(2)}\mathbf{Z_t} - \mathbf{P}_{*,\mathbf{t}}\mathbf{Z}_\mathbf{t}'\mathbf{F}_\mathbf{t}^{(1)}\mathbf{Z_t})\mathbf{P}_{\infty,\mathbf{t}}
\end{aligned}$$

We have 2 non-stationary components here; after two steps of algorithm, $\mathbf{P}_{\infty,\mathbf{2u}} = \mathbf{0}$, the original Kalman filter equations (A6) and (A7) take over.