

**HONG KONG INSTITUTE FOR MONETARY RESEARCH**

**A QUASI-BOUNDED MODEL FOR SWISS FRANC'S  
ONE-SIDED TARGET ZONE DURING 2011-2015**

*C. H. Hui, C. F. Lo and T. Fong*

*HKIMR Working Paper No.15/2015*

July 2015



*Hong Kong Institute for Monetary Research*

香港金融研究中心

*(a company incorporated with limited liability)*

*All rights reserved.*

*Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.*

# A Quasi-Bounded Model for Swiss Franc's One-Sided Target Zone During 2011-2015

**C. H. Hui\***

Hong Kong Monetary Authority

and

**C. F. Lo\*\***

The Chinese University of Hong Kong

and

**T. Fong#**

Hong Kong Monetary Authority

July 2015

## Abstract

On 6 September 2011, a ceiling on the value of the Swiss franc was imposed, at CHF 1.2 per euro. With the continuous weakness of the euro area economy, this exchange rate limit was abandoned on 15 January 2015. This paper proposes a quasi-bounded process for the Swiss franc exchange rate dynamics under a one-sided target zone during this period, in which the exchange rate can breach the strong-side limit under a restricted condition of the relationship between the parameters of the drift term and stochastic part of the process. The empirical results using market data during 6 September 2011 – 14 January 2015 with a rolling one-year window suggest that this model can describe the dynamics of the Swiss franc under a one-sided target zone, where the drifting force is an increasing function of foreign reserves. While the exchange rate was bounded below the strong-side limit during most of the time, as indicated by its dynamics, the condition for breaching the limit was met in November 2014 using only information until that point, i.e., about two months before abandoning the limit.

**Keywords:** Exchange Rate Target Zone, Swiss Franc, Quasi-Bounded Process

**JEL Classification:** F31, G13

---

\* Correspondence. E-mail: [chhui@hkma.gov.hk](mailto:chhui@hkma.gov.hk). Phone: (852) 2878 1485. Fax: (852) 2878 2485. Research Department, Hong Kong Monetary Authority, 55/F, Two International Finance Centre, 8, Finance Street, Central, Hong Kong, China.

\*\* Institute of Theoretical Physics and Department of Physics, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, China. E-mail: [cflo@phy.cuhk.edu.hk](mailto:cflo@phy.cuhk.edu.hk)

# Research Department, Hong Kong Monetary Authority, 55/F, Two International Finance Centre, 8, Finance Street, Central, Hong Kong, China.

**Acknowledgements:** The authors gratefully acknowledge comments from the referee and assistance from Louis Chau.

The views expressed in this paper are those of the authors, and do not necessarily reflect those of the Hong Kong Monetary Authority, Hong Kong Institute for Monetary Research, its Council of Advisers, or the Board of Directors.

# 1. Introduction

As the European debt crisis deepened during the summer of 2011, a sharp risk re-appraisal triggered a selloff of risky assets and prompted investors to seek safe havens. As a result, the Swiss franc (CHF) came under tremendous upward pressure. In response, the Swiss National Bank (SNB) eased monetary policy several times in August 2011, lowering interest rates to practically zero. All these measures, however, met with little success in curbing the strength of the currency. The nominal exchange rate of the Swiss franc against the euro (EUR) shot up to a peak in August 2011 (Figure 1). In view of the potential impact on real activity and increasing risk of deflation, on 6 September 2011, the SNB put a ceiling on the value of the Swiss franc by imposing an explicit upper bound on the exchange rate of the Swiss franc at CHF 1.2 per euro (1/1.2 EUR/CHF) and vowed to enforce this limit "with utmost determination" and to buy foreign currency "in unlimited quantities".<sup>1</sup> This measure implemented by the SNB effectively established a one-sided target-zone system for the Swiss franc with a strong-side convertibility undertaking at 1/1.2 EUR/CHF. A series of interventions in the spot, forward, and option markets brought the Swiss franc below 1/1.2 EUR/CHF and forced the volatility of the exchange rate sharply lower.

The immediate outcome was welcomed by the SNB: the EUR/CHF exchange rate from 6 September 2011 onwards traded below the 1/1.2 limit, with the currency option-implied volatility suppressed markedly, suggesting that the policy measure was successful in diverting safe haven capital flows away from the Swiss franc. To the extent that the strong demand for safe havens continued, reflecting the European sovereign debt situation, the market kept the SNB busy defending the limit (see Figure 8 below about the increase in the SNB's foreign reserves). These foreign exchange market interventions would ultimately become a source of inflation concern over time. Hence, the sustainability of the mechanism depended on how quickly inflationary pressures built up and whether the European debt crisis deteriorated. With continuous weakness in the euro area economy, and anticipated further monetary policy easing (impending quantitative easing) by the European Central Bank versus normalization by the Federal Reserve, the SNB surprised the market by abandoning its exchange rate limit on 15 January 2015, sending the Swiss franc up by 23% against the euro in a single day.<sup>2</sup> The SNB considered that divergence between monetary policy in the major currency areas was likely to become even more pronounced. The euro depreciated considerably against the US dollar and this, in turn, caused the Swiss franc to weaken against the US dollar. The SNB therefore concluded that enforcing and maintaining the exchange rate cap for the Swiss franc against the euro was no longer justified.

<sup>1</sup> See the press release "Swiss National Bank sets minimum exchange rate at CHF 1.20 per euro" by the SNB on 6 September 2011, [http://www.snb.ch/en/mmr/reference/pre\\_20110906/source/pre\\_20110906.en.pdf](http://www.snb.ch/en/mmr/reference/pre_20110906/source/pre_20110906.en.pdf).

<sup>2</sup> At the same time, the SNB lowered the interest rate on sight deposit account balances that exceed a given exemption threshold by 0.5 percentage points, to -0.75%. It moved the target range for the three-month Libor further into negative territory, to between -1.25% and -0.25%, from the current range of between -0.75% and 0.25%. See the press release "Swiss National Bank discontinues minimum exchange rate and lowers interest rate to -0.75%" by the SNB on 15 January 2015, [http://www.snb.ch/en/mmr/reference/pre\\_20150115/source/pre\\_20150115.en.pdf](http://www.snb.ch/en/mmr/reference/pre_20150115/source/pre_20150115.en.pdf)

While several recent studies including Hetrich and Zimmermann (2014), Hanke et al. (2014 and 2015), and Jermann (2014) have investigated the credibility of the exchange rate limit from option-price or option-theoretic perspectives, to our knowledge, there has been only one study of the strong-side currency limit adapted by the SNB from a target-zone-model perspective by Studer-Suter and Janssen (2014). They find that, in line with the predictions of Krugman's (1991) model for the behavior of the exchange rate within a target zone, declines in the stock market and increases in stock options implied volatility are associated with a smaller appreciation of the Swiss franc against the euro as it moves towards its upper bound of 1/1.2. The exchange rate is an S-shaped function of the fundamentals. Krugman-type target-zone models suggest that the exchange rate function will appear to be less sensitive to changes in the fundamentals than the corresponding free-floating exchange rate. The effect of fundamentals on the exchange rate decreases as the exchange rate deviates from its central parity. This feature is called the 'honeymoon effect', which therefore implies that there is inherently stabilizing mechanism in a perfectly credible exchange rate target zone. As the SNB abandoned its exchange rate limit on 15 January 2015, such effects broke down and the exchange rate breached its imposed limit.

According to the target zone model of Krugman (1991), a Brownian motion with a reflecting boundary condition at the upper and lower limit can be used as the driving process to keep the exchange rate within the zone by mean-reverting towards its central parity.<sup>3</sup> The driving force behind this mean-reverting property has been widely debated.<sup>4</sup> Some attribute this to central bank intervention within the target zone. Others argue that credibility induces "stability speculation" by market participants, producing forces to pull the exchange rate back to the central parity whenever it drifts too far from it. However, section 3.1 below shows that the mean-reverting property of the Swiss franc was not strong and persistent. It has often been argued in the policy literature that within an exchange rate target zone, the mean-reverting properties of the currency can be taken as a sign that the market judges the zone to be credible. Many empirical studies attempt to investigate this theoretical prediction by examining the time-series properties of the European currencies within the Exchange Rate Mechanism (ERM).<sup>5</sup> The results are mixed, perhaps reflecting a violation of the assumption of a fully

<sup>3</sup> The reflecting condition in a target zone assumes that the intervention rule is based on a specific band for the fundamentals, and that, if necessary, the fundamentals will be regulated to remain within the band. Under such conditions, the fundamentals and the corresponding exchange rate follow a regulated Brownian motion such that the exchange rate displays reversion towards the central parity. See Christiansen et al. (1998) and De Jong et al. (2001) for reviews on modelling of exchange rates in target zones.

<sup>4</sup> For example, Svensson (1993), Rose and Svensson (1994), and Anthony and MacDonald (1998).

<sup>5</sup> The European Economic Community adopted this system which is also known as the ERM in March 1979, and it was replaced with the ERM II on 1 January 1999. Under the ERM of 1979 each member country was required to maintain its exchange rate with the European Currency Unit (ECU) within certain bands. After several realignments of currencies in the early 1980s, the ERM stabilized with bands of  $\pm 2.25\%$  around parity with the ECU for each currency. In September 1992, Italy and the UK were required to leave the ERM because they could not maintain their currencies within the bands. There were realignments of the bands of the depreciated Portuguese Escudo and Spanish Peseta in September and November 1992. In August 1993, the bands for six member countries (Belgium, France, Ireland, Portugal and Spain) were relaxed to  $\pm 15\%$ . Gómez-Puigand and Montalvo (1997) find that for the post-1993 period most of these countries' currencies improved their credibility, at least transitorily.

credible target zone. Some of the currencies are found to follow a random-walk process.<sup>6</sup> In particular, the exchange rate does not display reversion towards the central parity.

To improve the basic Krugman model, extensions of the basic model were developed to capture features of intra-marginal interventions and imperfect credibility. Froot and Obstfeld (1991) and Delgado and Dumas (1992) incorporate a simple way to model such interventions with imperfect credibility by specifying that the drift term of the fundamentals towards central parity is proportional to the deviation from central parity. Bertola and Svensson (1993) extend the basic target zone model by including a time-varying realignment risk with stochastic jumps in the central parity. This implies that the exchange rate is not necessarily mean-reverting to its original central parity. Given no central parity in a one-sided target zone, these extensions are not applicable to the exchange rate dynamics of the Swiss franc.

This paper proposes a simple and analytically tractable model of exchange rate dynamics in a one-sided target zone, and applies the model to the Swiss franc between the period of 6 September 2011 to 14 January 2015, i.e., before its exchange rate limit was abandoned. The proposed model uses a basic log linear model of the exchange rate on which most of the target-zone literature is based for a small open economy. The log exchange rate is equal to a 'fundamental' plus a term proportional to the expected change in the log exchange rate. When the exchange rate is "well below" the boundary, market participants behave as if they are in a comfort zone and do not feel particularly compelled or encouraged to pull the exchange rate further away from the boundary. However, when the exchange rate moves closer to the boundary, the market anticipates an intervention and acts to stabilize the exchange rate or even to push it away from the boundary. While the central bank can always keep the exchange rate below the upper bound by selling the domestic currency (i.e., the Swiss franc in this study) to maintain the exchange rate at or lower than the strong-side limit, the proposed exchange rate dynamics allow a quasi-bounded boundary condition at the limit.

The intervention policy of the central bank and the behaviour of market participants in the proposed model are modelled implicitly by specifying the drift and diffusion coefficients of the exchange rate dynamics that determine the quasi-bounded boundary condition at the limit. While the variance of the exchange rate declines towards the strong-side limit and vanishes at the limit, the exchange rate can breach the limit under particular conditions, i.e., realignment could occur.<sup>7</sup> The exchange rate is thus quasi-bounded (i.e., not completely bounded) at the limit. The possible realignment condition is determined by the value of the drift and diffusion coefficients of the exchange rate dynamics. The drift coefficient can be a function of foreign reserves or the monetary base. A quasi-bounded process for exchange rate dynamics within a two-sided target zone is applied to the Hong Kong dollar by Lo et al.

<sup>6</sup> For example, Ball and Roma (1994), Svensson (1993), Rose and Svensson (1994), Nieuwland et al. (1994), Anthony and MacDonald (1998, 1999), and Kanas (1998).

<sup>7</sup> Such a property is similar to the bounded exchange rate dynamics in Ingersoll (1996) and Larsen and Sørensen (2007) in which the variance of the exchange rate vanishes at both the weak-side and strong-side limits in a two-sided target zone. In their models the exchange rate is completely bounded under all circumstances determined by the model parameters. However, the exchange rate following the quasi-bounded process can breach the limit under particular conditions.

(2015). The dynamics are consistent with an exchange rate band in which the exchange rate can breach the weak-side limit under restricted conditions of the relationship between the parameters of the drift term and stochastic part of the process. The empirical results suggest that the model can describe the dynamics of the Hong Kong dollar against the US dollar under a two-sided target-zone system.

We discuss a proposed one-sided target-zone model and the associated quasi-bounded process of the exchange rate dynamics in the following section. The process and the conditions under which the exchange rate is no longer bounded at the strong-side limit are illustrated. Empirical estimates of the quasi-bounded process for the Swiss franc when the strong-side limit was in place during 6 September 2011 – 15 January 2015, the relationship between the model parameters and capital flows into the Swiss franc, and the interest rate differential are discussed in Section 3. The final section of the paper concludes.

## 2. Quasi-Bounded Exchange Rate Process in One-Sided Target Zone

### 2.1 A One-Sided Target-Zone Model

To develop a quasi-bounded one-sided target-zone model, we consider the exchange rate  $S$  defined as a foreign currency value (i.e., the euro) of a unit of a domestic currency (i.e., the Swiss franc), and let  $S_U = 1/1.2$  mark the strong-side limit. With no loss of generality, the normalized log exchange rate  $s$  is defined by:

$$s = \ln\left(\frac{S}{S_U}\right) \quad (1)$$

where  $-\infty < s \leq 0$  and  $s_U = 0$  corresponds to the strong-side limit. To establish the relationship between the exchange rate and fundamentals in the one-sided target-zone model, we use a basic log linear model of the exchange rate on which most of the target-zone literature is based for a small open economy. The exchange rate is equal to  $f(t)$  plus a term proportional to the expected change in the log exchange rate:

$$s(t) = f(t) + \alpha \frac{E[ds(t)]}{dt} \quad (2)$$

where  $\alpha$  is the absolute value of semi-elasticity of the exchange rate with respect to its expected rate of change, and  $E$  is the expectation operator.

The Krugman-type target-zone model conventionally assumes a monetary process of exchange rate determination, in which the 'fundamental' ( $f$ ) is the source of uncertainty. The 'fundamental' is a combination of the foreign and domestic money supplies, real incomes, money demand disturbance and real exchange rate movements. Along with this combination, we assume that the fundamentals are the sum of two components:

$$f(t) = m + v(t), \quad (3)$$

where  $m$  is the logarithm of the constant relative money supply of the foreign and domestic currencies, and  $v(t)$  which follows a stochastic process is the logarithm of a general purpose term encompassing changes in real output, money demand, and other factors in the two economies other than the money supply and expected currency depreciation or appreciation. The variable  $v(t)$  also includes exogenous determinants of the exchange rate that the authorities cannot influence, in particular those related to foreign currency. The exchange rate of the Swiss franc and euro is mainly affected by the economic and financial conditions in the euro area, as discussed in the previous section, and the press releases about the measures taken by the SNB (see footnotes 1 and 2), specify that  $v(t)$  is an increasing function of the positive fundamentals in the euro area. The SNB is prepared to change  $m$  by increasing the money supply to prevent  $s$  from rising above the announced cap level  $s_U$  (the strong-side limit) in the case of capital inflows. But as long as  $s$  lies below  $s_U$ , the money supply is unchanged.

The fundamentals are assumed to follow a stochastic process with drift  $\mu_v$  and instantaneous standard deviation  $\sigma_v$ :

$$df = dv = \mu_v dt + \sigma_v dZ, \quad (4)$$

where  $dZ$  is a Wiener process with  $E[dZ] = 0$  and  $E[dZ^2] = dt$ . To establish the relationship between the component  $v$  of the fundamental and the exchange rate  $s$ , we apply Ito's lemma to Eqs.(2) and (4), and have:

$$\frac{E[ds(t)]}{dt} = \mu_v \frac{ds}{dv} + \frac{1}{2} \sigma_v^2 \frac{d^2 s}{dv^2}. \quad (5)$$

Then substituting Eqs.(3) and (5) into Eq.(2) yields:



$$\frac{1}{2} \alpha \sigma_v^2 \frac{d^2 s}{dv^2} + \alpha \mu_v \frac{ds}{dv} - s = -v - m. \quad (6)$$

with the prescribed boundary condition:

$$\left. \frac{ds(v)}{dv} \right|_{v=0} = 0. \quad (7)$$

Eq.(7) is a smooth-pasting boundary condition at the strong-side limit implying an optimal boundary condition for the process, where interventions involve simply selling the domestic currency to the market. At the boundary, there is no foreseeable jump in the exchange rate, i.e., no arbitrage condition. Krugman and Rotemberg (1990) show that the smooth-pasting condition at the strong-side boundary ensures that the exchange rate does not cross the bound of the target zone.

In the basic target zone model, the drift term  $\mu_v$  of the fundamentals in Eq.(4) is a constant. Froot and Obstfeld (1991) and Delgado and Dumas (1992) extend the model to incorporate mean reversion of the fundamentals. The driving forces behind this mean-reverting property are either central bank intervention within the target zone or “stability speculation” by market participants, producing forces to pull the exchange rate back to its central parity whenever it drifts too far from it. Their effect on the expected exchange rate are captured by the mean-reverting drift of the fundamentals towards central parity. For the one-sided target zone, which is asymmetric because currency interventions are only carried out at the strong-side limit, we adopt the quasi-bounded process of the two-sided target zone model proposed by Lo et al. (2015) to have an asymmetric mean-reverting shock with:

$$\mu_v = \frac{1}{2} \left( -\kappa v + \frac{4\kappa\theta - \sigma^2}{4v} \right), \quad \kappa > 0 \quad ; \quad v > 0 \quad (8)$$

$$\sigma_v = \frac{1}{2} \sigma. \quad (9)$$

in Eq.(4) for the dynamics of the fundamental. Obviously, the second-order differential equation in Eq.(6) now becomes identical to its counterpart in Lo et al.’s two-sided target zone (2015). As a result, similar analyses and observations can be made.

As shown in Lo et al. (2015), the desired solution of Eq.(6) is given by [see the Appendix of Lo et al. (2015) for details]

$$s(v) = v^2 \sum_{n=0}^{\infty} A_n v^n \quad (10)$$

where

$$A_0 = -\frac{m}{\alpha\kappa\theta} < 0$$

$$A_1 = -\frac{1}{\alpha\kappa\theta} \left\{ \frac{2}{3} \left( 1 + \frac{\sigma^2}{4\kappa\theta} \right)^{-1} \right\}$$

$$A_{n+2} = \frac{1}{\alpha\kappa\theta} \left\{ \frac{2 + (n+2)\alpha\kappa}{n+4} \left[ 1 + \frac{(n+2)\sigma^2}{4\kappa\theta} \right]^{-1} \right\} A_n.$$

for  $n = 0, 1, 2, \dots$ . The series solution can be shown to be a convergent series for all  $v$  by means of the ratio test as  $\lim_{n \rightarrow \infty} |A_{n+2} / A_n| \rightarrow 0$ . As a result, the relationship between the exchange rate  $S$  and the fundamental  $v$  takes the form:

$$S(v) = S_U \exp \left( \sum_{n=0}^{\infty} A_n v^{n+2} \right). \quad (11)$$

Since all the  $A_n$ 's are negative real constants, the leading term provides an upper bound of the exact solution, namely:

$$S(v) < S_U \exp(A_0 v^2). \quad (12)$$

One can also estimate the total error associated with approximating the exact solution by this upper bound as follows:

$$\Delta S \equiv S_U \int_0^{\infty} \exp(A_0 v^2) \left[ 1 - \exp \left( \sum_{n=1}^{\infty} A_n v^{n+2} \right) \right] dv. \quad (13)$$

Moreover, a better approximate solution of the form:

$$\tilde{S}(v, \varepsilon) = S_U \exp(\varepsilon A_0 v^2) \quad (14)$$

can be determined by minimizing the total error:

$$\Delta\tilde{S}(\varepsilon) \equiv S_U \sqrt{\int_0^\infty \left[ \exp(\varepsilon A_0 v^2) - \exp\left(\sum_{n=0}^\infty A_n v^{n+2}\right) \right]^2 dv} \quad (15)$$

with respect to the positive real parameter  $\varepsilon$ . It is clear that by construction this optimal approximate solution should be better than the upper-bound solution which corresponds to the special case of  $\varepsilon = 1$ . Figure 2 shows the relationship between the exchange rate  $S$  and component  $v$  of the fundamentals using the series solution Eq.(10) with different numbers of  $n$  based on the estimated model parameters in section 3 below. The convergence of the series is fast as demonstrated numerically in the figure. Given that the exchange rate  $S$  ranged between 0.8333 and 0.7949 during the one-sided target zone regime, Figure 2 shows that the errors  $\Delta\tilde{S}(\varepsilon)$  are extremely small. Thus, the leading term of the expression:

$$\begin{aligned} s(v) &= A_0 v^2 \\ &= -mv^2 / (\alpha\kappa\theta) \end{aligned} \quad (16)$$

is a good approximation of the exact relationship between  $s$  and  $v$ .

Regarding the asymmetric mean-reverting shock, when  $v$  becomes small and approaches zero, the second term of the drift in Eq.(8) will push  $v$  to a higher value given that the term  $(4\kappa\theta - \sigma^2) > 0$ . With  $v$  an increasing function of the fundamentals of the foreign currency (i.e., the euro), the corresponding domestic (foreign) currency will depreciate (appreciate) and the exchange rate will move away from its strong-side limit. Conversely, when  $v$  moves too far away from the origin, the first term of the drift will push  $v$  towards the origin such that the domestic (foreign) currency will appreciate (depreciate) accordingly. It is noted that the mean-reverting force is not symmetric at the mean level  $\theta$ . The downward force (domestic currency depreciation) given by the second term with  $v$  close to zero is stronger than the upward force (domestic currency appreciation) provided by the first term. Such an asymmetric mean-reverting property with a strong force pushing the exchange rate away from the strong-side limit is consistent with the idea that the market judges the target zone to be credible at the strong-side limit.

The no leakage condition of  $4\kappa\theta - \sigma^2 > 0$  ensures that the exchange rate will not breach the strong-side limit and the one-sided target zone is credible; otherwise, the exchange rate may pass through the boundary, i.e., the one-side target zone is quasi-bounded at the strong-side limit. The smooth pasting condition of Eq.(7) thus breaks down in the proposed model if the no leakage condition does not hold at the boundary. The quasi-bounded process for the exchange rate will be studied in section

2.2 below, demonstrating that the analytical form of Eq.(16) is consistent with exchange rate dynamics under a quasi-bounded process in a one-sided target zone.

## 2.2 Quasi-Bounded Exchange Rate Dynamics

Based on the one-sided target-zone model developed in the previous section, the corresponding quasi-bounded process of the exchange rate is studied in this section. To illustrate the exchange rate dynamics, it is convenient to use the notation  $x \equiv -s$  so that the exchange rate is positive with  $x = 0$  corresponding to the strong-side limit. By applying Ito's lemma to Eq.(4) with  $\mu_v$ ,  $\sigma_v$  and  $s(v) = A_0 v^2$  given in Eq.(8), Eq.(9) and Eq.(16) respectively, the dynamics of  $x$  are shown to follow a mean-reverting square-root process:

$$dx = \kappa(\theta_x - x)dt + \sigma_x x^{1/2}dZ \quad (17)$$

where  $\theta_x = -A_0\theta > 0$  and  $\sigma_x = \sigma\sqrt{-A_0}$ .  $\sigma_x^2 x$  is the variance that depends upon the level of  $x$ , and  $\kappa$  determines the speed of the mean-reverting drift towards the long-term mean  $\theta_x$ . The empirical results in section 3 below demonstrate that the quasi-bounded process for the exchange rate under the proposed dynamics can adequately describe the movements of the Swiss franc in a one-sided target zone.

The instantaneous variance  $\sigma_x^2 / x$  of the fractional change in  $x$  (i.e.,  $dx/x$ ) is a decreasing function of  $x$ , especially in the presence of mean-reverting drift. There may be forces or incentives for market participants to drive the exchange rate away from its strong-side limit, not least that the probability of making money by holding a long position in the currency is almost zero when the exchange rate appreciates very close to its strong-side limit, usually corresponding to capital inflows into the currency. This inherent market mechanism which is characterized by the quasi-bounded boundary condition increases the credibility of the Swiss franc one-sided target zone. The presence of mean-reverting drift will help to move the exchange rate away from its limit, enhancing the 'honeymoon effect' as described by Studer-Suter and Janssen (2014) in the target-zone literature.

The boundary at the strong-side limit assumes that intervention policy involves simply selling the domestic currency to the market. However, it is noted that strong capital inflows would risk creating overheating (inflation) in the economy in general and in asset markets in particular, and increase uncertainty about the exchange rate level, which could call into question the credibility of the one-sided target zone adopted by the SNB. This model therefore implicitly assumes that much larger increases in foreign reserves by selling the domestic currency are necessary to ensure that the exchange rate is at its equilibrium level in the zone. The credibility of the central bank in defending the exchange rate regime is reflected in the drift coefficient  $\kappa$ . The central bank may engage in intra-marginal intervention in order to defend its currency whenever the currency appreciates towards the

strong-side limit. In addition to central bank interventions, market participants who believe that the exchange-rate band is credible engage in ‘stabilizing speculation’, which helps to push the exchange rate towards its mean level. Such effects increase the mean-reverting force, determined by the size of  $\kappa$ . Conversely, if market participants believe that the one-sided target zone system is not credible (probably due to high inflation arising from strong capital inflows), their speculative buying of the currency will weaken the restoring force (i.e., smaller  $\kappa$ ) towards its mean level. The empirical results in section 3.2 below show that the drift coefficient is an increasing function of foreign reserves, and that the accumulation of foreign reserves due to capital inflows is a critical element in the proposed framework to determine the credibility of the one-sided target zone adopted by the SNB.

The long-term mean  $\theta_x$  associated with the exchange rate dynamics is a time-varying equilibrium level, which can be determined either by the SNB through intra-marginal intervention or through action (or incentives) by market participants to drive the exchange rate towards its mean level. It is noted that when there are capital inflows, the Swiss franc will appreciate and the long-term mean  $\theta_x$  will move towards the strong-side limit. As the empirical results show that the drift coefficient  $\kappa$  is an increasing function of foreign reserves, an increase in foreign reserves due to capital inflows will strengthen the movement of the exchange rate towards the long-term mean.

### 2.3 Interest Rate Differential

The model has implications for the relation between exchange rate and interest rate differentials. Following Svensson (1991), in a narrow target zone, with and without devaluation risk, the foreign exchange risk premium is likely to be very small and can be assumed to be zero. By assuming uncovered interest rate parity (UIP) continually holds in the transformed exchange rate  $s(t)$ , we have:

$$r(t) - r^*(t) = \frac{E[ds(t)]}{dt}, \quad (18)$$

where  $r(t)$  and  $r^*(t)$  are the domestic (Swiss franc) and foreign (euro) interest rates of the currency pair based on the exchange rate  $s(t)$  respectively. As expressed by Eq.(11) in Svensson (1991), the interest rate differential  $\delta(f, t) = r(t) - r^*(t)$  of a target zone exchange rate is defined by:

$$\delta(f, t) = \frac{E[s(f(t)) | f(0) = f_0] - s(f_0)}{t} \quad \text{for } t > 0. \quad (19)$$

Eq.(19) can be solved under the proposed exchange rate dynamics. Eq.(17) shows that the variable  $x$  follows the dynamics of the mean-reverting square-root process given by Cox et al. (1985) under which the expected value of  $x$  is:

$$\bar{x}(t) \equiv E[x(t) | x(0) = x_0] = x_0 \exp(-\kappa t) + \theta_x [1 - \exp(-\kappa t)] . \quad (20)$$

In the quasi-bounded model, the expected value of  $v$  is given by:

$$\bar{v}(t) \equiv E[v(t) | v(0) = v_0] = \sqrt{\Delta + [v(0)^2 - \Delta] \exp(-\kappa t)} , \quad (21)$$

where:

$$\Delta = \theta - \frac{\sigma^2}{4\kappa} . \quad (22)$$

Combining Eq.(20) and Eq.(21), we have:

$$\frac{\bar{x}(t) - \theta_x}{x_0 - \theta_x} = \frac{\bar{v}(t)^2 - \Delta}{v_0^2 - \Delta} . \quad (23)$$

Svensson (1991) points out that the difficulty of evaluating  $\delta(f, t)$  in the Krugman-type target-zone model lies in computing the expected exchange rate at  $t > 0$ . This is because the exchange rate follows a complicated nonlinear stochastic process with variable drift and instantaneous standard deviation. However, in the quasi-bounded model we do not have this difficulty and the interest rate differential can be explicitly evaluated to yield the closed-form expression by putting back  $s \equiv -x$ :

$$\begin{aligned} \delta(f, t) &= \frac{s_0 \exp(-\kappa t) + \theta_s [1 - \exp(-\kappa t)] - s_0}{t} \\ &= (\theta_s - s_0) \left\{ \frac{1 - \exp(-\kappa t)}{t} \right\} \end{aligned} \quad (24)$$

where  $\theta_s \equiv -\theta_x$ .

Eq.(24) implies that when the spot exchange rate is momentarily weaker (stronger), relative to the mean level, the interest rate differential  $\delta(f, t)$  is negative (positive), i.e., the domestic 'Swiss franc' interest rate is lower (higher) than its foreign 'euro' counterpart. It is because market participants expect foreign exchange gains (losses) from the future exchange rate appreciation (depreciation) under the UIP. Therefore, negative (positive) interest rate differentials can occur when the exchange rate is stronger (weaker) than its central parity when  $\theta_s < s_0$  ( $\theta_s > s_0$ ). This contrasts with the Krugman-type target zone in which the interest rate differential is positive (negative) when the exchange rate is stronger (weaker) than its central parity, as the exchange rate in the Krugman model is stationary around its central parity. The empirical results on interest rate differentials for the Swiss

franc, which are presented in section 3.3 below, show that interest rate differentials implied by the quasi-bounded model are consistent with actual market value.

### 3. Empirical Test of Presence of Quasi-Bounded Process for Swiss Franc

#### 3.1 Estimations of Model Parameters

In this section, we investigate whether a quasi-bounded process for exchange rate dynamics within a one-sided target zone can describe movements in the Swiss franc. Figure 1 shows the EUR/CHF exchange rate in  $S$  and the transformed exchange rate in  $x$  from 6 September 2011 to 14 January 2015 (i.e., before its exchange rate limit was abandoned) when the exchange rate was capped at 0.8333 EUR/CHF. Under the proposed one-sided target-zone model, mean-reversion of the exchange rate is present. We first examine the dynamics of the daily EUR/CHF exchange rate using the Dickey-Fuller (DF) and the variance-ratio (VR) test. The DF test examines whether the time series is mean-stationary and always returns to its long-term mean within a very short period of time.<sup>8</sup> The rejection of mean stationarity under the DF test does not necessarily imply rejection of mean reversion over a longer period of time.<sup>9</sup> Therefore, the VR test is used to test whether the variance of five-day returns decreases with time.<sup>10</sup> In order to see how the significance of these two properties evolves over time, we employ a moving-window approach. Specifically, the two tests are conducted on each of the 628 windows, with the first window covering the period from 6 September 2011 to 12 August 2012 (i.e., 250 observations) and the last window covering the period from 30 January 2014 to 14 January 2015.

Figure 3 shows the two test statistics over time. At the 10% level of significance, few DF test statistics indicate the presence of a unit root, suggesting that the Swiss franc exchange rate is not mean-stationary in general. However, the VR test accepts the mean reversion hypothesis for a five-day

<sup>8</sup> The test is done by Dickey-Fuller test with GLS detrending. The Dickey Fuller test constructs a parametric correction for higher-order correlation by assuming that a time series process ( $y$ ) follows an  $AR(p)$  process and adds  $p$  lagged differences of  $y$  to the test regression of:

$$\Delta y_t = \gamma + \alpha y_{t-1} + \beta_1 \Delta y_{t-1} + \dots + \beta_p \Delta y_{t-p} + \varepsilon_t$$

We test the null hypothesis (the presence of a unit root) of  $\alpha = 0$  against the alternative hypothesis of  $\alpha < 0$ . The test is evaluated using the conventional  $t$ -ratio for  $\alpha$  ( $t_\alpha$ ) such that:

$$t_\alpha = \hat{\alpha} / (se(\hat{\alpha})),$$

where  $\hat{\alpha}$  is the estimate of  $\alpha$  and  $se(\hat{\alpha})$  is the coefficient standard error.

<sup>9</sup> For instance, a time series process, which is a sine-wave signal with white noise may not pass the test of stationarity. But it may pass the test of mean reversion because the time series process goes back to the mean regularly.

<sup>10</sup> The VR at lag  $K$  is defined as the ratio of the variance of the  $K$ -period return to the variance of the one-period return divided by  $K$ . A unity VR means that the time series is a random process. The time series is considered to be mean reverting if the VR is significantly smaller than one. If the VR is significantly larger than one, the time series is considered to be under a mean aversion. Since the VR test statistic asymptotically follows a normal distribution, a standardized VR test statistic is reported in this study for ease of comparison. Further details are in Kim et al. (1991).

horizon from December 2012 to January 2013 and most of the time in the first half of 2014, suggesting that the Swiss franc exchange rate could be mean-reverting over a five-day period during the period. The mixed results from the DF test and VR test taken together suggest that the Swiss franc was mean reverting at least for some, but not all, of the period when the strong-side limit was in place. This is also consistent with estimation of the model parameters below and the finding that some estimates of mean-reversion are not statistically significant.

The maximum likelihood estimation (MLE) is used to estimate the model parameters of the process specified in Eq.(17) based on a log-likelihood function which is constructed by the probability density function of Eq.(5) in Lo et al. (2015) of the process, which is:

$$G(x, t; x', t') = \frac{4}{\sigma_x^2 C_1(t-t')} \left( \frac{x}{x'} \right)^{\omega/2} \exp \left[ -\frac{\omega+2}{2} C_2(t-t') \right] \times \exp \left\{ -\frac{2x' + 2x \exp[-C_2(t-t')]}{\sigma_x^2 C_1(t-t')} \right\} \times I_\omega \left\{ \frac{4x^{1/2} x'^{1/2} \exp[-C_2(t-t')/2]}{\sigma_x^2 C_1(t-t')} \right\} \quad (25)$$

where  $\omega = 2\kappa\theta_x / \sigma_x^2 - 1$ ,  $C_1(\tau) = [\exp(\kappa\tau) - 1] / \kappa$ ,  $C_2(\tau) = -\kappa\tau$ , and  $I_\omega$  is the modified Bessel function of the first kind of order  $\omega$ .

The MLE on the daily exchange rate uses time series data from 6 September 2011 to 14 January 2015 for estimation. The estimation uses a rolling one-year window with the initial window covering the period between 6 September 2011 and 12 August 2012. The estimated  $\sigma_x$  shown in Figure 4 ranges between 0.010 and 0.019, which is relatively steady. The corresponding z-statistic is much higher than 1.96 (i.e., at 5% significance level) indicating that the estimated  $\sigma_x$  is highly significant. The results suggest that the estimation of the square-root-process part of the quasi-bounded dynamics is robust, and the changes of  $\sigma_x$  over time are within a relatively narrow range.

Figure 5 shows that the estimates of the drift term  $\kappa$  are significant in terms of the z-statistic (higher than the 5% significance level) when  $\kappa$  is higher than 0.02. In other words, the drift becomes weaker and less significant when  $\kappa$  is not significantly different from zero. The estimated  $\kappa$  increased from 0.02 to 0.06 in the second half of 2012, reflecting the initial effect of the exchange rate cap, which pushed the exchange rate away from the strong-side limit. The drift dropped below 0.02 in the beginning of 2013. The drift remained below 0.02 in the first half of 2013, then increased substantially from 0.02 to 0.1 in the last quarter of 2013, indicating that the one-sided target zone became more credible during second half of 2013 as reflected by the stronger mean-reverting force. Consistently, Figure 1 shows that the exchange rate was well (up to about 4%) below the strong-side limit in 2013



but was close to its limit in the second and third quarters of 2012. As a one-year rolling window is used in the estimations, the strong Swiss franc in 2012 is captured by the estimations in early 2013 showing weaker drift. When the Swiss franc weakened and moved away from its strong-side limit in 2013, the estimated mean-reverting force increased in the second half of 2013 accordingly. However,  $\kappa$  decreased continuously from 0.1 in early 2014 to about 0.01 in the last quarter of 2014. Subsequently, the SNB removed the cap on the Swiss franc exchange rate in 15 January 2015. As the exchange rate moved towards its strong-side limit during this period, as shown in Figure 1, the mean-reverting force weakened with the appreciation of the Swiss franc.

Figure 6 shows that the estimated mean  $\theta_x$  decreased to the level of 0.005 in the second half of 2012 and then increased to 0.025 in 2013. A comparison of the changes of  $\kappa$  in Figure 5 and  $\theta_x$ , shows that they are generally in opposite directions. This implies that when the mean moves towards (away from) the strong-side limit ( $x = 0$ ), the restoring force from the drift term will increase (decrease) to push the exchange rate back to its mean level. Therefore, the mean-reverting force relative to the distance between the exchange rate and the strong-side limit is generally steady in 2013 even though the two parameters  $\kappa$  and  $\theta_x$  are time varying. However, both parameters declined continuously in 2014, showing realignment pressure building on the strong-side limit with the mean-reverting force weakening during 2014.

Given that the condition of  $\sigma_x^2 / 4\kappa\theta_x < 1$  indicates no probability leakage at the strong-side limit, Figure 7 shows the measure  $\sigma_x^2 / 4\kappa\theta_x$  to study the credibility of the one-sided target zone. The measure was below 1 from August 2012 to November 2014, in particular it declined from about 0.4 in early 2013 to close to zero in mid-2014. This suggests that there was no concern about probability leakage indicating that the exchange rate was well bounded at the strong-side limit and the one-sided target zone was adequately credible. However, the measure began to rise in August 2014 and surged above 1 in November 2014 and, subsequently, fluctuated between 0.5 and 1. Such changes coincided with declines in the two parameters  $\kappa$  and  $\theta_x$  in 2014 as the mean-reverting force weakened. The exchange rate dynamics suggest that, from November 2014, probability leakage was possible and there was a risk that the exchange rate would breach the strong-side limit. The existence of the leakage condition (i.e.,  $\sigma_x^2 / 4\kappa\theta_x \geq 1$ ) in November 2014 indicates that the credibility of the one-sided target zone deteriorated substantially because the exchange rate was no longer bounded at the strong-side limit as shown by the exchange rate dynamics. The deterioration of the credibility measured by the leakage condition is consistent with the abandonment of the exchange rate limit by the SNB on 15 January 2015, sending the Swiss franc appreciating by 23% against the euro (well above the limit) in a single day.

In summary, the estimation results using market data during 6 September 2011 – 14 January 2015 based on the MLE shown in Figures 4-7 provide evidence that the quasi-bounded process adequately fits the data on the Swiss franc exchange rate assuming a one-sided target zone. The figures show

that the estimated model parameters are time varying. The existence of the leakage condition (i.e.,  $\sigma_x^2 / 4\kappa\theta_x \geq 1$ ) in November 2014 using only information until that point indicates that the dynamics of the exchange rate are informative and suggest an erosion of the credibility of the target zone.

### 3.2 Relationship between Mean-Reverting Drift and Foreign Reserves

As shown in the previous section the credibility of the one-sided target zone adopted by the SNB is related to the mean-reverting force and the leakage condition for the Swiss franc exchange rate dynamics, in which the speed of the drift ( $\kappa$ ) towards its long-term mean is the key parameter in determining the exchange rate movements relative to the strong-side limit. To maintain the credibility of the exchange rate limit when there were capital inflows into the Swiss franc, the SNB sold Swiss francs in the foreign exchange market to keep the exchange rate below the strong-side limit. These interventions could affect the exchange rate dynamics in relation to the parameter  $\kappa$ , as well as increasing the bank's foreign reserves. To identify the relationship between changes in foreign reserves arising from capital flows and the speed of the drift  $\kappa$ , we estimate the following simple linear regression:

$$\ln(\kappa_t) = \alpha + \beta \ln(\text{Reserves}_t) \quad (26)$$

where  $\text{Reserves}_t$  is the SNB's foreign reserves reported in US dollars at time  $t$ ,  $\alpha$  and  $\beta$  are the coefficients to be estimated.<sup>11</sup> Figure 8 shows that the SNB's reserves increased as the bank bought euros to maintain its exchange rate cap. Given the reverses reported in US dollars, comprising 46% in the euro, 28% in the US dollar and the rest in the Japanese yen, sterling and others in 2014, their decline in the second half of 2014 was mainly due to an appreciation of the euro against the dollar. When the SNB abandoned the strong-side limit in January 2015, it was concerned about the substantial depreciation of the euro against the dollar that caused the Swiss franc to weaken against the dollar.<sup>12</sup> The reserves reported in US dollars partially reflected such concerns, which could have affected the Swiss franc's exchange rate dynamics.

Since only monthly information of the reserves is available, we use the month-end  $\kappa$  for estimating the regression. Figures 8 and 9 show the two time series in monthly frequency and their scatter plots respectively. Based on the 29 observations (covering the period from August 2012 to December 2014), the coefficients  $\alpha$  and  $\beta$  are estimated to be -117.68 and 8.72 respectively.<sup>13</sup> The t-statistics (p-values) of these two estimates are -2.53 (0.018) and 2.45 (0.021) respectively, which suggests that

<sup>11</sup> The data of the foreign currency reserves are from the SNB.

<sup>12</sup> See the press release "Swiss National Bank discontinues minimum exchange rate and lowers interest rate to -0.75%" by the SNB on 15 January 2015, [http://www.snb.ch/en/mmr/reference/pre\\_20150115/source/pre\\_20150115.en.pdf](http://www.snb.ch/en/mmr/reference/pre_20150115/source/pre_20150115.en.pdf)

<sup>13</sup> When monthly average  $\kappa$  is used in the regression, the coefficients  $\alpha$  and  $\beta$  are estimated to be -132.45 and 9.85 respectively, with the R-squared being 0.2300.

the two coefficients are statistically significant at a level of 5%. The positive coefficient  $\beta$  suggests a positive relationship between  $\kappa$  and the reserves. When there are capital inflows and the SNB sells the Swiss franc,  $\kappa$  will increase, indicating an increase in the restoring force towards the exchange rate's long-term mean. When the exchange rate moves towards its strong-side limit as a result of capital inflows, the tendency to mean-reversion can act as a stabilizing force limiting the upward movement of the exchange rate. This implies that the SNB's interventions enhanced the credibility of the one-sided target zone as reflected by the exchange rate dynamics. This result is consistent with the model proposed by Cook and Yetman (2014) based on the central bank balance sheet, which shows that an expectation of exchange rate appreciation will cause foreign exchange reserves to swell reflecting the increase in the Monetary Base. This lowers the likelihood of the fixed exchange rate being abandoned.

### 3.3 Interest Rate Differential

Figure 10 presents interest rate differentials between the Swiss franc and euro implied by the model Eq.(24), which is compared with actual market interest rate differentials between the 1-month Libor of the two currencies.<sup>14</sup> The figure shows that negative market interest rate differentials occurred during the period when the strong-side limit was in place. The model-implied interest rate differentials, which were negative during the period between November 2013 and January 2015 when the exchange rate  $s_0$  was weaker than the long-term mean  $\theta_s$ , are consistent with the market observations. In particular, while the differences between the actual market and model-implied values are about 5 to 10 basis points, their changes are quite similar during the period. The differences narrow substantially after August 2014.

The analysis of interest rate differentials presented in Figure 10 provides further evidence of consistency between the observed dynamics of the Swiss franc exchange rate and the quasi-bounded process. Given that interest rate differentials are affected by other factors (e.g., liquidity in the banking system especially those European banks) in addition to exchange rate expectations, the interest rate differentials derived from the expected exchange rate according to the UIP condition assumed in the quasi-bounded model is not able to match exactly the corresponding market value.

## 4. Conclusion

On 6 September 2011, the SNB imposed a ceiling on the value of the Swiss franc at CHF 1.2 per euro. Following continuous weakness of the euro area economy, this exchange rate limit was eventually abandoned on 15 January 2015. This paper proposes a simple and analytically tractable one-sided target-zone model in which the strong-side limit is quasi-bounded, and applies the model to the Swiss franc during the period with the limit in place. In the model, the exchange rate can breach

<sup>14</sup> The data are from Bloomberg.

the strong-side limit under a restricted condition of the relationship between the parameters of the drift term and stochastic part of the process. The results of our empirical analysis using the market data during 6 September 2011 – 14 January 2015 with a rolling one-year window before the exchange rate limit was abandoned suggest that the Swiss franc follows a quasi-bounded process. The estimated model parameters of the process adequately fit the data on the Swiss franc exchange rate. The negative interest rate differentials between the Swiss franc and euro implied by the model are consistent with actual market interest rate differentials.

The speed of the mean-reverting drift is estimated as an increasing function of foreign reserves. As foreign reserves increased due to capital inflows into the Swiss franc, the speed of the mean-reverting drift increased, which pushed the exchange rate towards its long-term mean and away from the strong-side limit. While the empirical results show that the exchange rate was bounded below the strong-side limit during most of the period as indicated by its dynamics, the condition for breaching the limit was met in November 2014 using only information until that point, i.e., about two months before the SNB abandoned the limit. The dynamics of the exchange rate suggest an erosion of the credibility of the target zone.

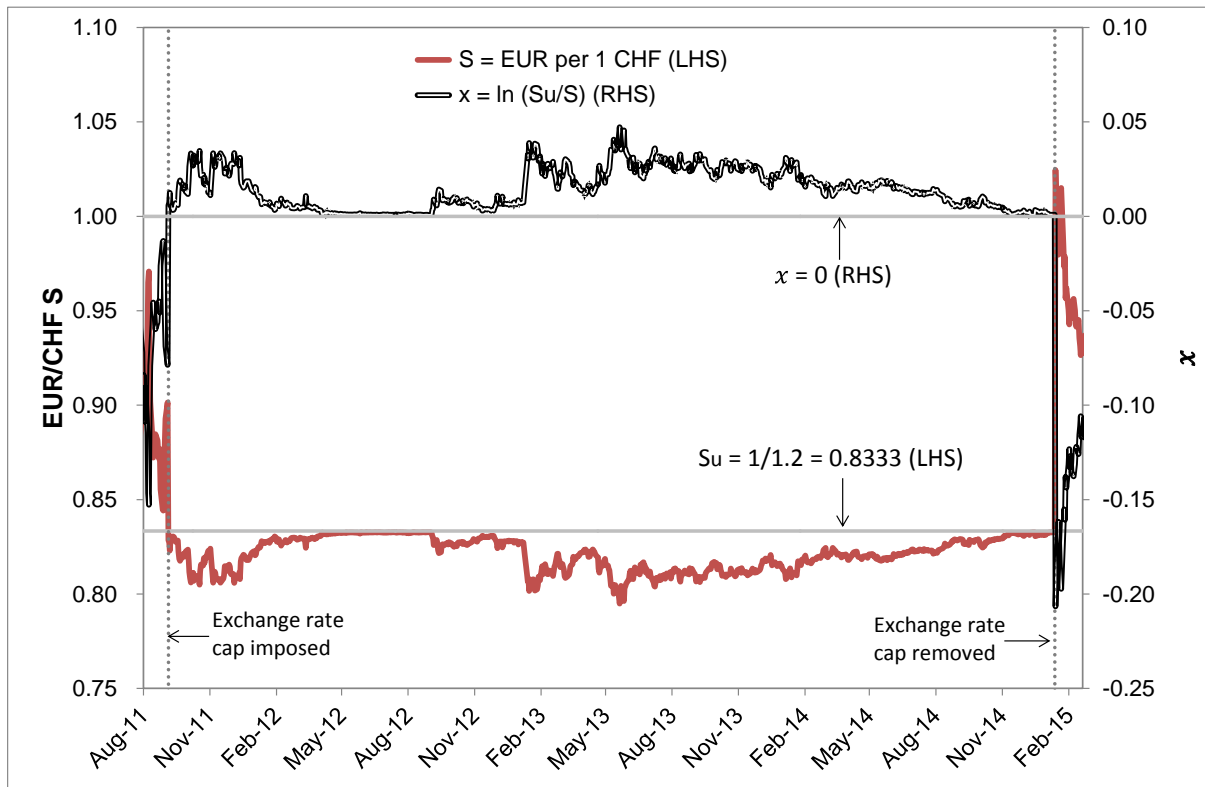
## References

- Anthony, M. and R. MacDonald (1998), "On the Mean Reverting Properties of Target Zone Exchange Rates: Some Evidence from the ERM," *European Economic Review*, 42: 1493-523.
- Anthony, M. and R. MacDonald (1999), "The Width of the Band and Exchange Rate Mean Reversion: Some Further ERM-Based Results," *Journal of International Money and Finance*, 18: 411-28.
- Ball, C. A. and A. Roma (1994), "Target Zone Modelling and Estimation for European Monetary System Exchange Rates," *Journal of Empirical Finance*, 1: 385-420.
- Bertola, G. and L. Svensson (1993), "Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models," *Review of Economic Studies*, 60: 689-712.
- Christensen, P. O., D. Lando and K. R. Miltersen (1998), "State Dependent Realalignments in Target Zone Currency Regimes," *Review of Derivatives Research*, 1: 295-323.
- Cook, D. and J. Yetman (2014), "Currency Boards when Interest Rates are Zero," *Pacific Economic Review*, 19: 135-51.
- Cox, J. C., J. E. Ingersoll and S. A. Ross (1985), "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53: 385-407.
- De Jong, F., F. C. Drost and B. J. M. Werker (2001), "A Jump-Diffusion Model for Exchange Rates in a Target Zone," *Statistica Neerlandica*, 55: 270-300.
- Delgado, F. and B. Dumas (1992), "Target Zones, Broad and Narrow," in P. Krugman and M. Miller, eds., *Exchange Rate Targets and Currency Bands*: 35-56, Cambridge: Cambridge University Press.
- Froot, K. and M. Obstfeld (1991), "Exchange Rate Dynamics under Stochastic Regime Shifts: A Unified Approach," *Journal of International Economics*, 31: 203-29.
- Gómez-Puiga, M. and J. G. Montalvo (1997), "A New Indicator to Assess the Credibility of the EMS," *European Economic Review*, 41: 1511-35.
- Hanke, M., R. Poulsen and A. Weissensteiner (2014), "Analyzing the Swiss National Bank's Euro Exchange Rate Policy: A Latent Likelihood Approach," Working Paper, University of Liechtenstein.

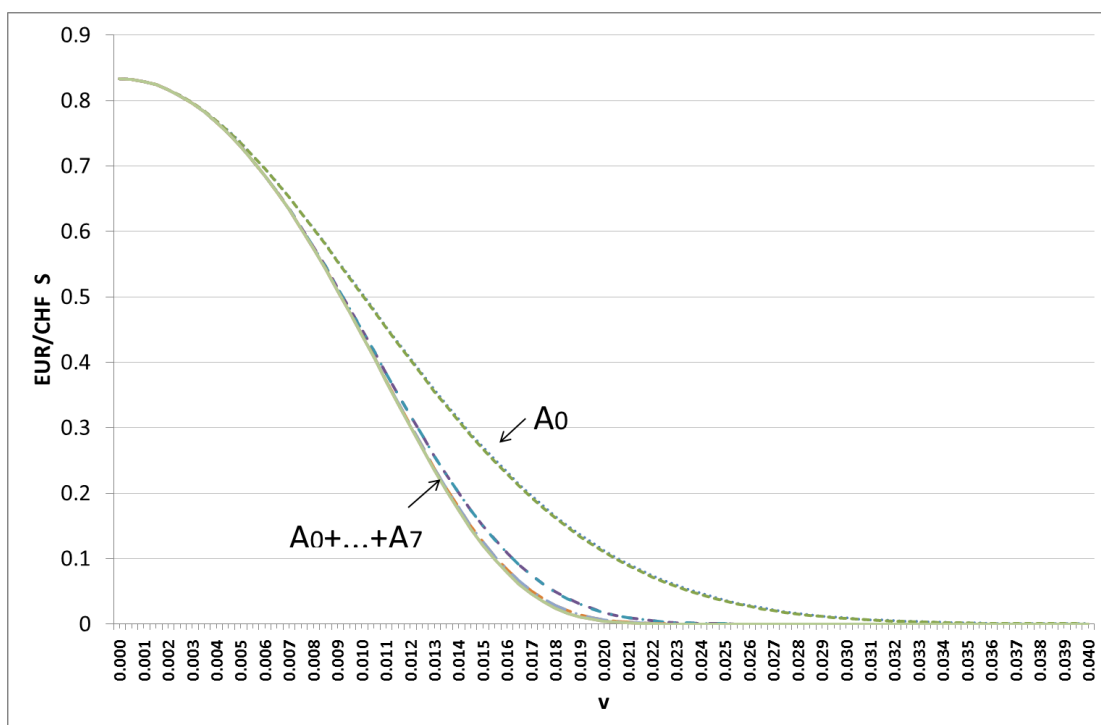
- Hanke, M., R. Poulsen and A. Weissensteiner (2015), "Where Would the EUR/CHF Exchange Rate Be Without the SNB's Minimum Exchange Rate Policy?" *Journal of Futures Markets*, forthcoming.
- Hetrich, M. and H. Zimmermann (2014), "On the Credibility of the Euro/Swiss Franc Floor: A Financial Market Perspective," Working Paper, University of Basel.
- Ingersoll, J. E., Jr. (1997), "Valuing Foreign Exchange Rate Derivatives with a Bounded Exchange Process," *Review of Derivatives Research*, 1: 159-81.
- Jermann, U. (2014), "Financial Markets' Views about the Euro-Swiss Franc Floor," Working Paper, University of Pennsylvania.
- Kanas, A. (1998), "Testing for a Unit Root in ERM Exchange Rates in the Presence of Structural Breaks: Evidence from the Boot-Strap," *Applied Economics Letters*, 5: 407-10.
- Kim, M. J., C. R. Nelson and R. Startz (1991), "Mean Reversion in Stock Prices? A Reappraisal of The Empirical Evidence," *Review of Economic Studies*, 58: 515-28.
- Krugman, P. and J. Rotemberg (1990), "Target Zones with Limited Reserves," NBER Working Paper No.3418.
- Krugman, P. (1991), "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 106: 669-82.
- Larsen, K. S. and M. Sørensen (2007), "A Diffusion Model for Exchange Rates in a Target Zone," *Mathematical Finance*, 17: 285-306.
- Lo, C. F., C. H. Hui, T. Fong and S. W. Chu (2015), "A Quasi-Bounded Target Zone Model – Theory and Application to Hong Kong Dollar," *International Review of Economics and Finance*, 37: 1-17.
- Nieuwland, F. G. M. C., W. F. C. Verschoor and C. C. P. Wolff (1994), "Stochastic Jumps in EMS Exchange Rates," *Journal of International Money and Finance*, 13: 699-727.
- Rose, A. K. and L. E. O. Svensson (1994), "European Exchange Rate Credibility before the Fall," *European Economics Review*, 38: 1185-216.
- Studer-Suter R. and A. Janssen (2014), "The Swiss Franc's Honeymoon," Working Paper No.170, University of Zurich.

- Svensson, L. E. O. (1991), "The Term Structure of Interest Rate Differentials in a Target Zone: Theory and Swedish Data," *Journal of Monetary Economics*, 28: 87-116.
- Svensson, L. E. O. (1993), "Assessing Target Zone Credibility," *European Economic Review*, 37: 763-802.

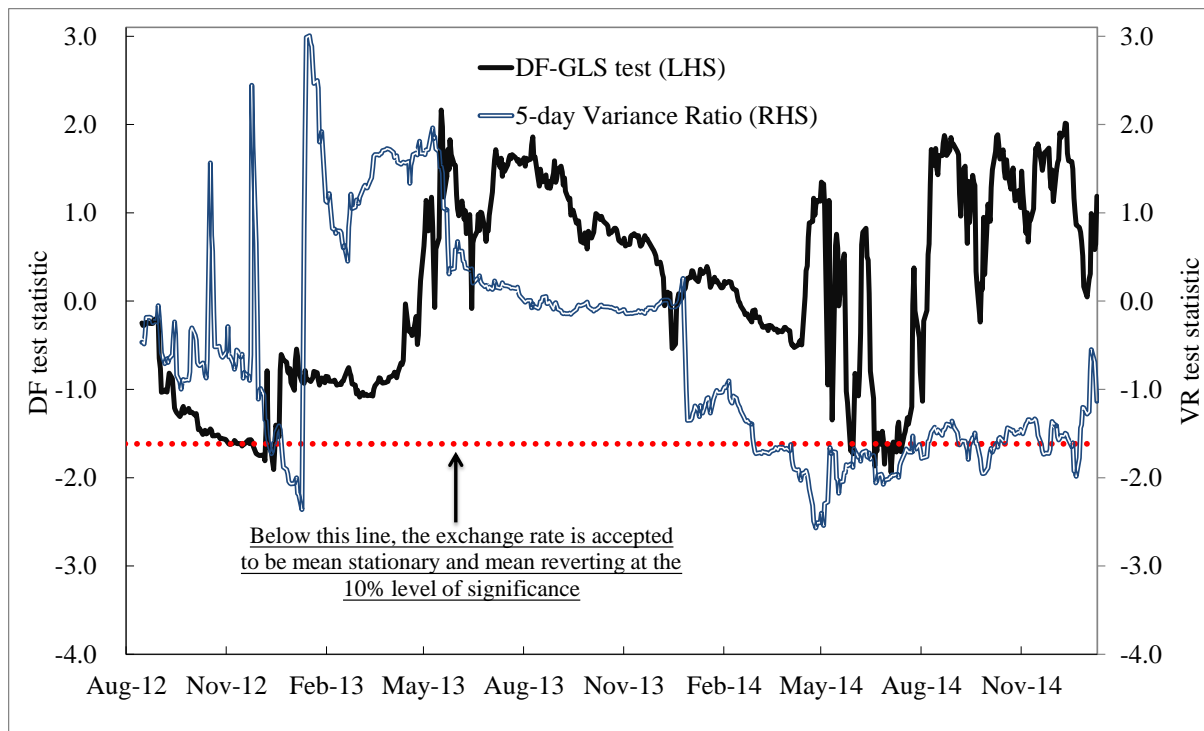
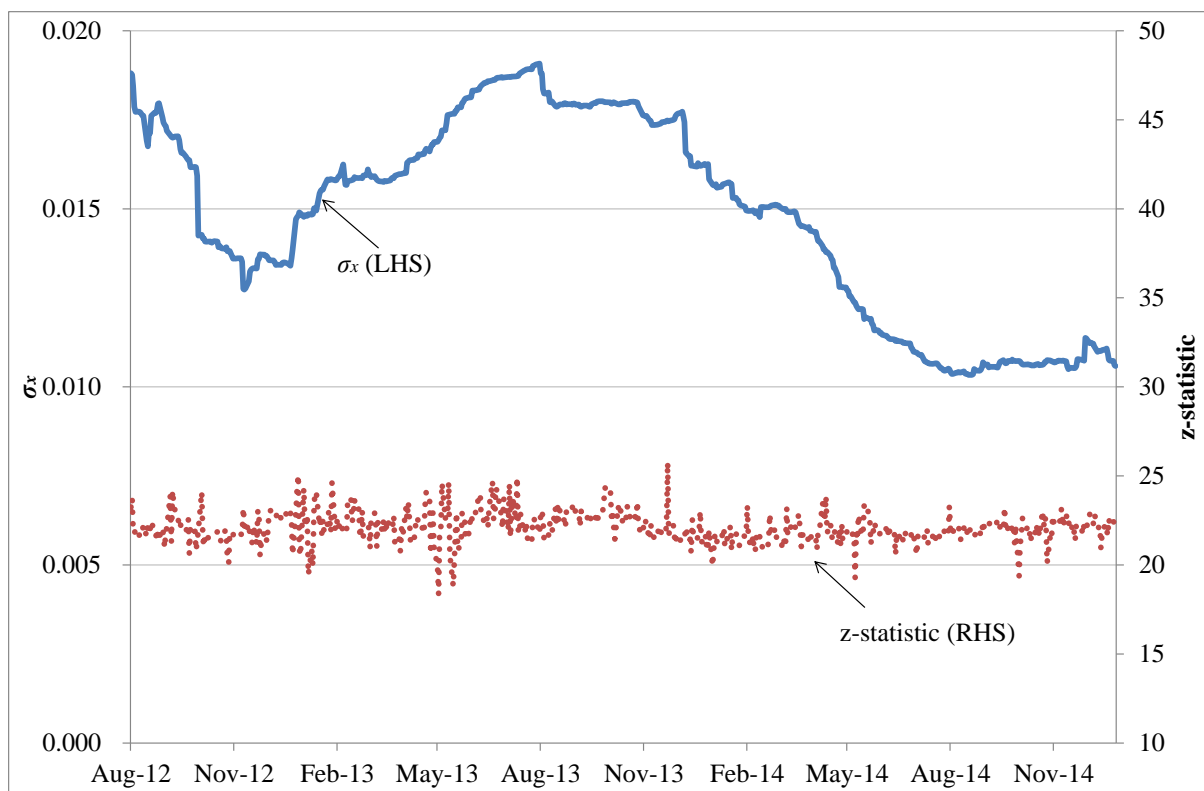
**Figure 1. EUR-CHF Exchange Rate in S- and X-Scale with Strong-Side Limit at  $S = 0.8333$  and  $x = 0$**

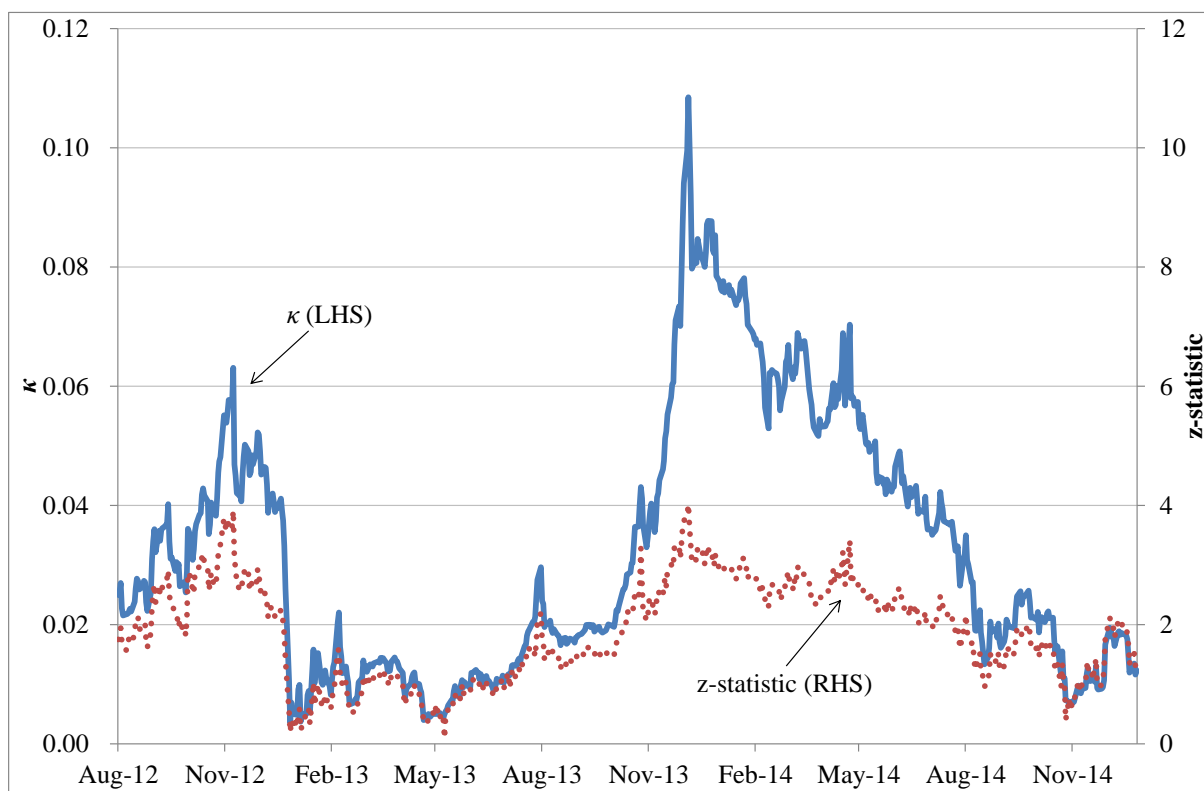
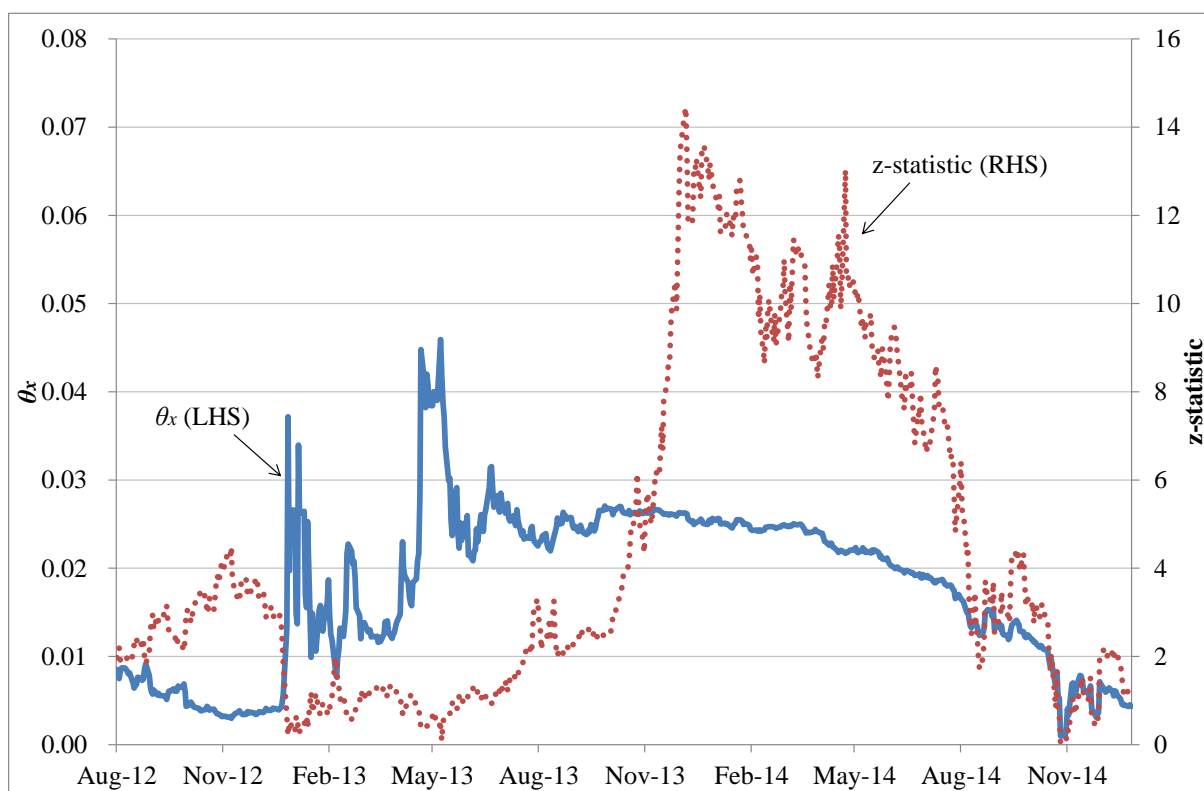


**Figure 2. Relationship between the Exchange Rate  $S$  and the Component  $v$  of the Fundamental:  $S = S_v \exp[v^2 g(v)]$  using Different Numbers of Terms ( $A_0, A_0 + A_1, \dots, A_0 + \dots + A_7$ ) of the Series Solution  $g(v)$  and Parameters  $m = 1$ ,  $\alpha = 0.2$ ,  $\kappa = 0.05$ ,  $\sigma = 0.015$  and  $\theta = 0.02$**

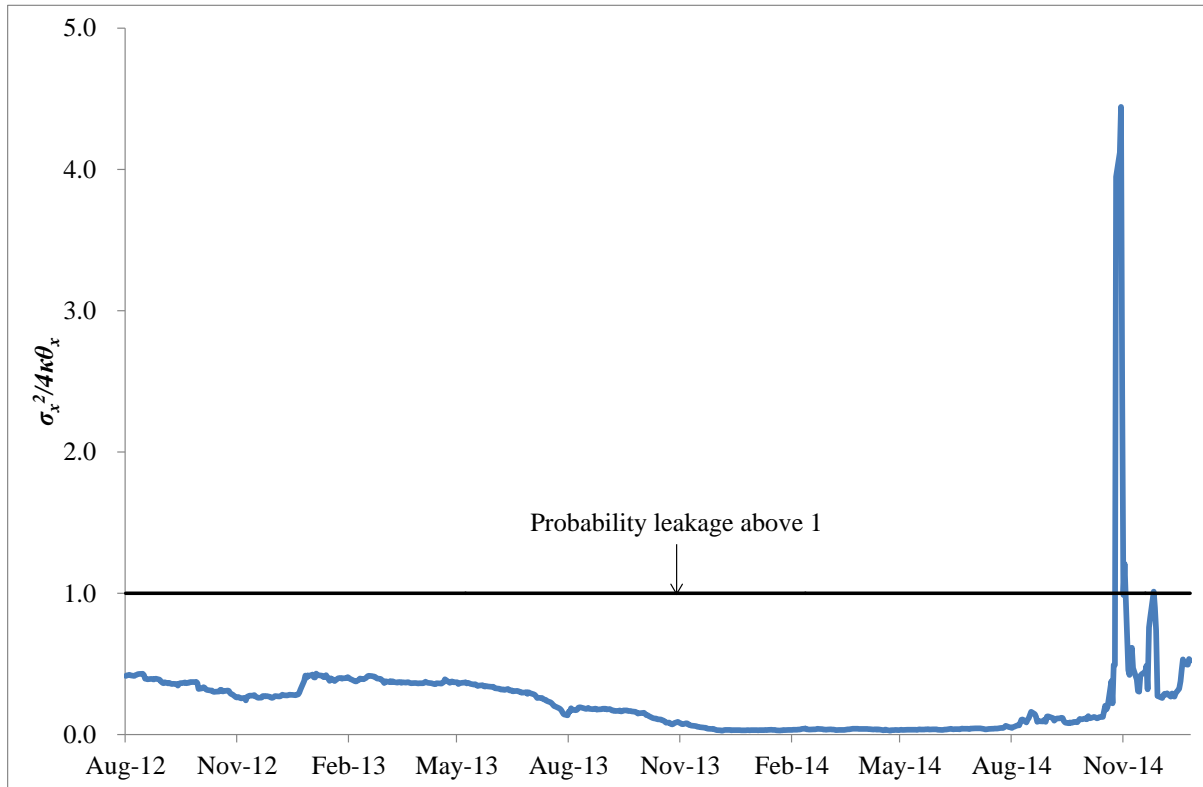




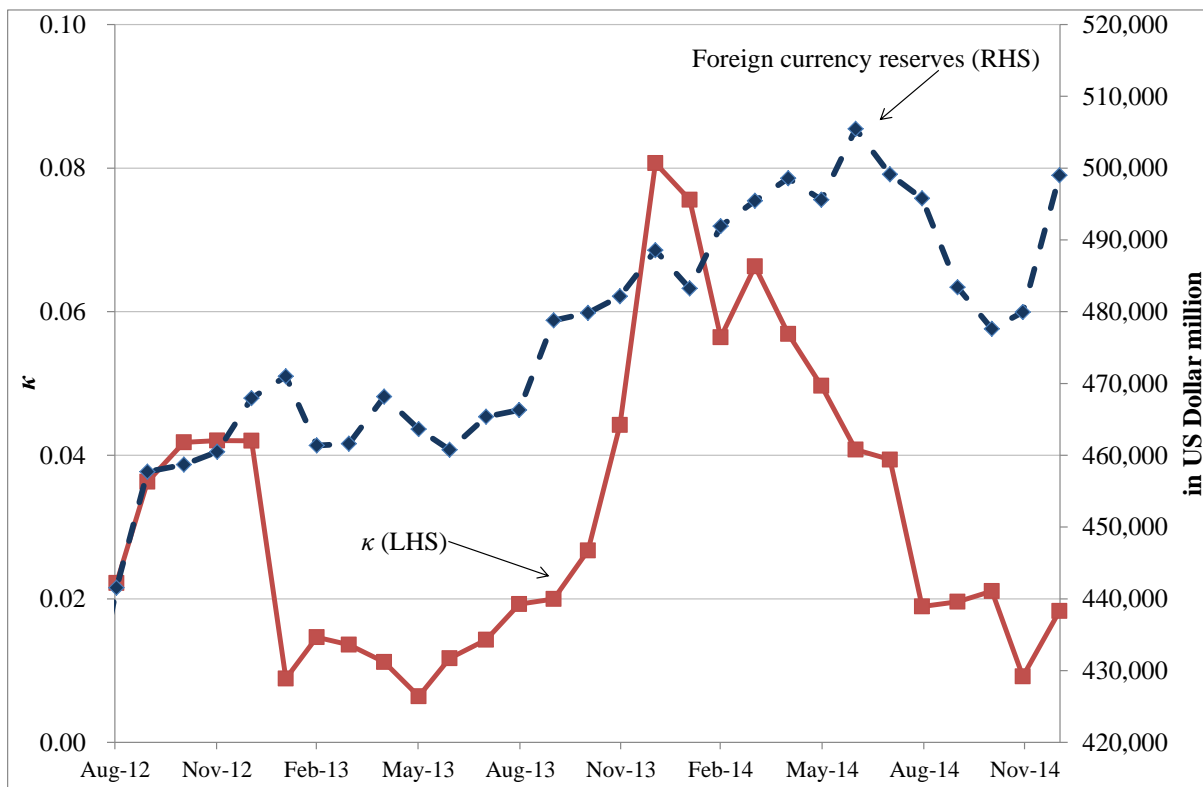
**Figure 3. DF Test Statistics and VR Test Statistics Using a Moving-Window Approach****Figure 4. Estimated  $\sigma_x$  and Corresponding Z-Statistic Using a One-Year Rolling Window**

**Figure 5. Estimated  $\kappa$  and Corresponding Z-Statistic Using a One-Year Rolling Window****Figure 6. Estimated  $\theta_x$  and Corresponding Z-Statistic Using a One-Year Rolling Window**

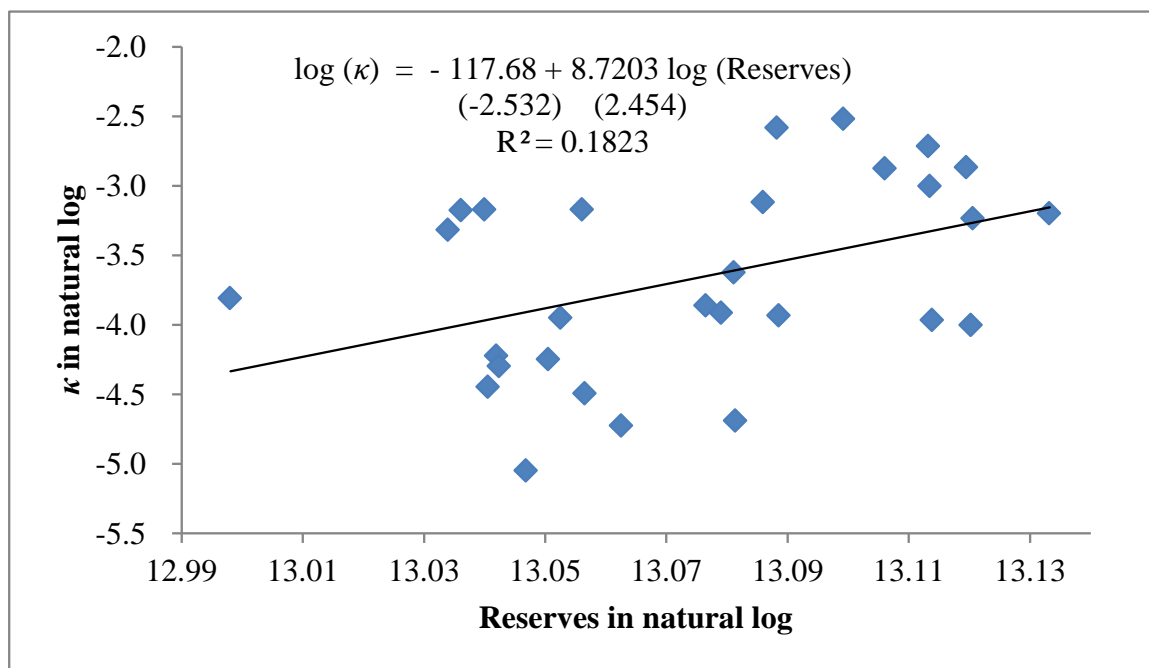
**Figure 7. Estimated Values of the Critical Condition  $\sigma_x^2 / 4\kappa\theta_x$  at the Strong-Side Limit Using a One-Year Rolling Window**



**Figure 8. Estimated  $\kappa$  and SBN's Foreign Reserves**



**Figure 9. A Simple Regression of  $\ln(\kappa_t) = \alpha + \beta \ln(\text{Reserves}_t)$  (With the t-Statistic in the Parenthesis)**



**Figure 10. Model-Implied and Actual Market Interest Rate Differentials (i.e., The Difference between 1-Month Libors of Swiss Franc and Euro)**

