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Exchange Rate Dynamics and US Dollar-denominated Sovereign Bond Prices in Emerging Markets

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Abstract

Based on an analogy between an economy's currency price and a firm's stock price, this paper develops a two-factor pricing model with closed-form solutions for US dollar-denominated sovereign bonds in which foreign exchange rates and US risk-free interest rates are the stochastic factors to study the dynamic linkage between the sovereign bond spreads and exchange rates in emerging markets. The numerical results during the pre-crisis (2003 - 2007) and post-crisis (2009 - 2014) periods and the associated error analysis show that the model credit spreads can broadly track the market credit spreads of the sovereign bonds of Brazil, Colombia, Mexico, the Philippines, Russia and Turkey. The results are consistent with empirical evidence of a connection between sovereign credit

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spreads and exchange rates, and the well-documented studies about twin sovereign debt and currency crises in emerging markets.

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1. Introduction

A common measure of a country's borrowing costs in international capital markets is its credit spread, which is defined as the difference between the interest rate the government pays on its US dollar-denominated debt and the corresponding yield of US Treasury bonds of comparable maturity. The credit spread is considered an indicator of the country's (issuer) economic-political stability, which is linked to country-specific macro-economic variables, such as output growth, foreign exchange reserves, budget deficit, real effective exchange rate deviation, and foreign direct investment. When financial markets are operating efficiently, anticipated changes in the credit quality of a sovereign borrower should be reflected in the prices of credit-sensitive instruments such as sovereign bonds. A substantial increase in sovereign risk due to economic-political instability would lead investors to sell securities denominated in both the US dollar and the country's own currency, hence putting upward pressure on sovereign credit spreads and downward pressure on its currency and increasing its volatility.

The relationship between sovereign risk and exchange rate stability has long been a subject of interest in international finance including papers by Eichergreen et al. (1996), Frankel and Rose (1996), Kaminsky et al. (1998), and Kumar et al. (2003), who use macro-economic indicators to estimate the probability of currency crashes. On the empirical side, Reinhart (2002), Herz and Tong (2004) as well as Dreher et al. (2006) provide empirical analyses of twin debt and currency crises. Reinhart (2002) finds that 84 percent of the defaults in her emerging markets sample are connected with currency crises and almost half of the currency crises in the sample are related to defaults. Herz and Tong (2004) analyze emerging markets and find that 32 percent of all debt crises are linked to currency crises, while 20 percent of currency crises are associated with debt crises. Dreher et al. (2006) study the empirical relationship between currency and sovereign debt crisis covering 80 countries over the period from 1975 to 2000 and find that currency crisis are more likely to occur with a contemporaneous debt crisis and vice versa. Empirically, twin debt and currency crises occur more frequently than twin banking and currency crises.

While sovereign risk and exchange rate stability have been studied in the context of sovereign debt crises, the literature on the empirical determinants of sovereign yield spreads usually focuses on other variables instead of exchange rates. Some papers including Edwards (1986), Eichengreen and Mody (2000), Min (1998), Beck (2001), and Ferrucci (2003) concentrate on reduced form regressions of spreads on a large set of macroeconomic variables. GDP growth, inflation and US Treasury yields are found to be important explanatory variables. Duffie et al. (2003) develop a flexible reduced form model of sovereign yield spreads. They estimate their model using weekly data on Russian dollar-denominated debt and US swap yields between 1994 and 1998. The study relates spreads implied by the model to political factors, foreign exchange reserves, oil prices, and the CBOE VIX volatility index.[1] Longstaff et al. (2011) demonstrate that US stock, bond market returns, and global volatility, can explain a large portion of the variation in sovereign credit default swap (CDS) spreads. Hilscher and Nosbusch (2010) investigate the effects of macroeconomic fundamentals on emerging market sovereign credit spreads. They find that the volatility of terms of trade data in particular has a statistically and economically significant effect on spreads.

Until more recently, the linkage between sovereign credit risk and exchange rate dynamics has not? been studied for emerging markets. Carr and Wu (2007) investigate the connection between currency option-implied volatilities and sovereign creditworthiness for Mexico and Brazil from 2002 to 2005. They find that the level and skew of the option-implied volatility display significant co-movement with the sovereign CDS spreads of the two countries. Pan and Singleton (2008) explore the term structure of CDS spreads for Mexico, Turkey, and Korea from 2001 to 2006 and consider the risk-neutral credit event intensities and loss rates that best describe the CDS data. Their results suggest that currency option volatilities may have served as a proxy for the fundamental macroeconomic and event risks embodied in VIX. Pavlova and de Boyrie (2015) find information flows between currency carry-trade returns of nine Asian-Pacific economies and changes in the Markit iTraxx SovX Asia Pacific index from 2008 to 2011, which are negatively correlated.[2]

In view of the linkage between a country’s credit risk and the strength of its currency, this paper studies the dynamic linkage between US dollar-denominated sovereign bonds and exchange rates in

1 VIX is the market volatility of the US S&P 500 index which gauges the global risk appetite in the financial market.

2 Carry trades are speculative investment strategies in the foreign exchange market, where investors borrow low yielding (funding) currencies and invest in high yielding (investment) currencies.

emerging markets. We derive a two-factor risky bond pricing model with closed-form solutions in which the exchange rate and its option-implied volatility are the respective underlying factor and its associated model parameter. The other factor is the stochastic US risk-free interest rate. Currency option markets have the desirable property of being forward-looking in nature and are thus useful sources of information for gauging market sentiment about future exchange rates. Market sentiment of the 'crash risk' of a currency can be inferred from its out-of-the-money option-implied volatility. Kamin and von Kleist (1999), Eichengreen and Mody (2000) and Cantor and Packer (1996) find that market sentiment is important in determining sovereign spreads of emerging markets in addition to country-specific fundamentals and external factors.[3] The use of out-of-the-money option volatility is consistent with a number of recent studies on currency crashes using information on currency option prices, including Brunnermeier et al. (2009), Farhi et al. (2009), and Hui and Chung (2011). The empirical results presented in this paper also demonstrate that exchange rates and US interest rates emerge as significant determinants of credit spreads on dollar-denominated sovereign bonds of Brazil, Colombia, Mexico, the Philippines, Russia and Turkey.

The risky bond pricing model used in this paper follows a structural framework for pricing corporate bonds proposed by Black and Scholes (1973) and Merton (1974). In the Black-Scholes-Merton structural framework, the capital structure is explicitly considered and default happens if the total asset value is lower than the value of liabilities at the bond's maturity. Default risk is therefore equivalent to a European put option on a firm's asset value. From a microeconomic perspective, such structural framework can be applied to price sovereign dollar- denominated bonds. A firm can be considered as an entity which issues two different classes of securities – a single homogeneous debt consisting of a zero-coupon bond and a residual claim, i.e., equity. In the balance sheet of the firm, the total value of its assets is equal to the sum of the values of a zero-coupon bond and equity. Under the Modigliani and Miller (1958) proposition, corporate debt and equity sum to the present value of future earnings, and the firm's total asset value is independent of this capital structure decision. Sims (1999) and Cochrane (2005) propose an analogy between corporate valuation and budget constraints for an economy.

3 Gray et al. (2007) use exchange rate volatility as a fundamental factor associated with the dynamics of an economy's asset, which is the underlying variable estimated from the public balance sheet, in their contingent claims analysis to price sovereign credit risk.

The analogy is as follows. On the balance sheet of an economy, foreign and domestic debt sum to the present value of the future budget surplus. Foreign debt of the economy is the “actual” debt while domestic debt and fiat money act like equity in a firm. Given that the government promises only to pay the domestic debt in local currency in the future, the function of domestic debt is to absorb fiscal risk by adjustment of its foreign currency (e.g., US dollar) equivalent value. In other words, solvency can be restored through devaluation of nominal debt, created by currency depreciation. As long as there is some probability that the government will run a primary surplus in the future and/or will engage in the repurchase of domestic currency debt then such debt has value. Furthermore, the currency price, e.g., the US dollar price of the local currency, is analogous to the stock price. Similar to a firm facing risk of insolvency as its equity value declines substantially, when instability is anticipated in the economy, the currency devalues with rising volatility and the credit quality of the economy deteriorates. Assuming efficient markets, this analogy suggests a positive linkage between sovereign foreign debt credit spreads and currency return/volatilities, analogous to the linkage identified between corporate credit spreads and stock return/volatilities. The proposed risky bond pricing model in which the exchange rate is the underlying factor can therefore be considered as a semi-structural approach. In contrast to previous studies on emerging markets sovereign bonds, our focus is to explore directly through the bond pricing model the extent to which exchange rate dynamics incorporated with option-implied volatility determine dollar-denominated sovereign bond spreads.

Following the Black-Scholes-Merton's corporate bond pricing framework, subsequent studies extend the framework to incorporate a general default triggering mechanism which is considered as the first time the value of a firm’s assets are lower than a default (constant or deterministic) barrier.[4] Applying the above analogy between a firm and an economy, the US dollar price of local currency should indicate the credit quality of the economy, i.e., the ability to repay the foreign debt owners. Similarly, there is a default barrier associated with the currency value in the proposed semi-structural model. As in the framework developed by Duffie and Lando (2001), if the dynamics of the exchange rate are

4 To have more accurate measures of the default probability, subsequent studies mainly focus on the liability structure such that models with more complex and dynamic liability structures including Jones et al. (1984), Leland and Toft (1996), Longstaff and Schwartz (1995), Colin-Dufresne and Goldstein (2001), and Hui et al. (2003) have been developed.

treated as a hazard rate process, which governs an inaccessible default stopping time, the proposed model can be recast as a special case of "reduced-form models".

Another factor in the risky bond pricing model is the required rate of return on default-free debt. An important concern is how to incorporate stochastic risk-free interest rates into the model. To address this concern, general equilibrium term-structure models of risk-free interest rates such as the Vasicek (1977) model and Cox–Ingersoll–Ross (CIR) model (1985) are conventionally used in the two-factor corporate bond pricing framework. The term-structure models assume that short-term interest rates are derived from general assumptions about the state variables, which describe the overall economy. The stochastic risk-free interest rate in the proposed model for pricing dollar-denominated sovereign bonds is assumed to follow the double square-root (DSR) process proposed by Longstaff (1989). One important characteristic of the DSR model is that it has a nonlinear restoring force in its drift term such that the interest rate is sticky downward. It is therefore particular relevant to the low interest rate environment since the global financial crisis in 2008, in which the short-term interest rate has tended to persist near the zero bound instead of moving back towards higher levels in a short time as implied by the CIR and Vasicek models.[5] There are some empirical findings supporting the DSR model. The empirical results in Longstaff (1989) suggest that by estimating the model parameters the DSR model is more successful in capturing the level and variation of 6- to 12-month Treasury bill yields during the 1964-1986 period compared with the CIR model. The results also suggest that yields are nonlinearly related to the risk-free interest rate as the model implies. Ahn et al. (2002) test the empirical performance of the quadratic term structure models including the DSR model in explaining historical bond price behaviour in the US during the period December 1946 – February 1991. They find the quadratic term structure models outperform the affine term structure models including the CIR and Vasicek models. Aït-Sahalia (1996) shows that there is evidence of nonlinearities in the drift function of the interest rate term structure using a nonparametric approach.

The scheme of this paper is as follows. In the following section we study the empirical relationship

5 Following the bankruptcy of Lehman Brothers in mid-September 2008, developments took a dramatic turn and spilled over to other economies. During 2008, the US Fed lowered the policy interest rate from the 4% level to 0-0.25% to provide monetary support for the economy. Subsequently, it has taken unprecedented measures including quantitative easing policies that have lowered long-term borrowing costs and fostered economic activity. As the interest-rate term structure was affected by the Federal Reserve's ultra-accommodative monetary policy, the 3-month US Treasury-bill yield has fallen to near zero for extended periods of time and the 10-year Treasury yield has been falling and hit the historical low of 1.39% in July 2012.

among the exchanges rates and US dollar-denominated sovereign bond spreads of emerging markets including Brazil, Colombia, Mexico, the Philippines, Russia and Turkey, and the US Treasury yields. In section 3, we develop the US dollar-denominated sovereign bond pricing model under a semi-structural-model framework. In section 4, we study the credit spreads of our sample of emerging markets calculated from the model and compare them with actual market data. The final section concludes.

2. Relations among dollar-denominated sovereign bond spreads, foreign exchange rates and US Treasury yields

In this section, we conduct an empirical test on dollar-denominated sovereign credit spreads in emerging markets, including Brazil, Colombia, Mexico, the Philippines, Russia and Turkey to study their relationship with each country's exchange rate and the US Treasury yields. These countries are selected because of their relatively liquid dollar-denominated bond markets. We obtain daily data on sovereign bond yields from 1 June 2003 to 29 September 2014. Based on data availability, the tenors of the bonds are as follows: Brazil, Mexico and Turkey (10-year, 15-year, 20-year and 30-year); Colombia (10-year and 30-year); the Philippines (10-year, 15-year and 20-year); and Russia (15-year). Bond credit spreads, which are the difference between bond yields and US Treasury yields with a corresponding tenors, are illustrated in Figure 1. The credit spreads of sample countries declined from their relatively high level at the end of the regional financial crises in 2003 but then surged during the global financial crisis in 2008. They subsequently fell substantially in 2009 and then were traded in the range of about 0.5% - 4%. Given the illiquid sovereign bond markets during the global financial crisis in 2008 and the structural differences before and after the onset of the crisis, we split the sample into two periods. The first period is from 1 June 2003 to 31 December 2007 (i.e., pre-crisis period), and the second period is from 1 January 2009 to 29 September 2014 (i.e., post-crisis period).[6]

As a factor for changes in interest rates, we use changes in Treasury yields of a corresponding tenor.

6 The data are from Bloomberg. For the Philippines, data are only available for the bonds with 10-year and 20-year tenors in the post-crisis period.

Let ΔCS denote the change in the credit spread, ΔY denote the change in the Treasury yield and *I* denote the change in the exchange rate of the corresponding country. The regression equation is given by:

$$\Delta CS = a + b\Delta I + c\Delta Y + \varepsilon$$

where *a*, *b* and *c* are regression coefficients. The descriptive statistics of the variables are reported in Table 1. Movements in the exchange rate of the sample of countries shown in Figure 2 show are quite similar to changes in their sovereign bond spreads.

Table 2 reported the regression results. The coefficients *b* for all the countries are statistically significant at the 1% level and suggest a positive relationship between credit spreads and exchange rates. This is consistent with the expected sign: that credit spreads increase with weaker currencies (i.e., higher exchange rates per dollar). The magnitude of the estimates of *b* shows that the relation between credit spreads and interest rates is economically significant. For example, the regression results for Brazil imply that a 0.1 rise in the USD/BRL exchange rate (which has the mean of 2.44 in the pre-crisis period) increases the 30-year bond spread by 26 basis points in the pre-crisis period and 10 basis points in the post-crisis period. The difference between the effects of the exchange rate in the two periods is due to lower credit spreads in the post-crisis period (the mean of the credit spreads is 3.64% and 1.96% in the pre- and post-crisis periods as shown in Table 1). There are similar observations for the other countries. The empirical results show a strong linkage between the sovereign credit spreads and exchange rates.

The coefficients *c* are all statistically significant at the 1% level and indicate a negative relationship between credit spreads and US Treasury yields. This finding supports the argument that investors, in particular risk-averse ones, sell risky asset (i.e., sovereign bonds in emerging markets) and buy US Treasuries which are treated as safe-haven assets in stressed markets. The magnitude of the estimates of *c* shows that the relation between credit spreads and US Treasury yields is economically significant. Regarding the 10-year Brazilian bond, a 100-basis point increase in the 10-year Treasury yield decreases the Brazilian bond spread by 42 basis points in the pre-crisis period and 76 basis points in the post-crisis period. Similar effects in terms of the magnitude are found for the other countries' sovereign bonds. Comparing the coefficients *c* in the two sample periods, the effects are generally stronger for the sample countries in the post-crisis period, reflecting a more important role of US Treasuries as safe-haven assets after the crisis. One explanation is that during an extended period of low interest rates and volatility caused by the accommodative monetary policies adopted by the US and other developed economies, market participants have displayed a tendency to seek higher returns by investing in securities that carry higher credit risk such as emerging market sovereign bonds. This 'search for yield' behavior has caused sovereign bond spreads to be more sensitive to US Treasury yields in the post-crisis period.

The estimated coefficients on exchange rates and Treasury yields suggest high explanatory power ranging from 0.20 to 0.71 (in terms of the adjusted R-squared) for the sample sovereign bonds. This suggests that these two factors are adequate determinants of the countries' sovereign credit spreads. The empirical results support the view that the exchange rates of emerging market currencies can be used as the state variable with adequate explanatory power in a semi-structural model for pricing their own US dollar-denominated sovereign bonds. Comparing the adjusted R-squared in the two sample periods, the explanatory power of exchange rates in the post-crisis period (about 0.31-0.71) is stronger than that in the pre-crisis period (about 0.2-0.51). This indicates that the link between sovereign credit spreads and the dynamics of the exchange rates and US interest rates has become stronger in the post-crisis period.

3. Dollar-denominated sovereign bond pricing model

The dynamics of the US risk-free short-term interest rate *r* in the dollar-denominated sovereign bond pricing model is drawn from the term structure model governed by the DSR process which is introduced by Longstaff (1989):[7]

$$dr = \kappa_r \left(\theta_r - \sqrt{r}\right) dt + \sigma_r \sqrt{r} dz_r \tag{1}$$

7 It is a log utility general equilibrium model.

where $\kappa_r, \sigma_r > 0$ and $\theta_r = \sigma_r^2 / 4\kappa_r > 0$. Incorporating the market price of risk ${}_r$ with respect to *r*, the risk-adjusted Eq.(1) becomes

$$dr = \left(\frac{\sigma_r^2}{4} - \kappa_r \sqrt{r} - 2\lambda_r r \right) dt + \sigma_r \sqrt{r} dz_r . \tag{2}$$

The drift term in Eq.(1) is a nonlinear restoring force which makes the dynamics of the interest rate different from those in the CIR and Vasicek models which have a linear restoring force. While the DSR model and the CIR model have a number of common empirically relevant characteristics such as negative interest rates being precluded and having a stationary distribution, the DSR model has two particular features due to the nonlinear restoring force.[8] First, only two parameters κ_r and σ_r^2 are required to determine the interest-rate dynamics. It is because θ_r^2 which is the long-run interest rate, is a function of the other two parameters such that $\theta_r^2 = \sigma_r^4 / 16\kappa_r^2$. Second, the interest rate is sticky downward as illustrated by Longstaff (1989).

Without an explicit boundary condition at $r = 0$, the associated risk-free bond price function $\Phi(r, \tau)$ with time to maturity is given by Longstaff (1989):[9]

$$\Phi(r, \tau) = A(\tau) \exp\left\{ C(\tau)\sqrt{r} + B(\tau) r \right\}, \tag{3}$$

where

$$A(\tau) = \sqrt{\frac{1 - C_0}{1 - C_0 \exp\{\gamma\tau\}}} \exp\left(\alpha_1 + \alpha_2 \tau + \frac{\alpha_3 + \alpha_4 \exp\left\{ \frac{1}{2}\gamma\tau \right\}}{1 - C_0 \exp\{\gamma\tau\}} \right) \tag{4}$$

8 The other two common characteristics include: (i) the interest rate returns to positive values if it approaches zero; and (ii) the instantaneous variance $\sigma_r^2 r$ is directly related to level of the interest rate. More detailed analyses and empirical evidence of stochastic interest rates following the DSR process, and the boundary behaviour of the process are in Longstaff (1989) and (1992), and Karlin and Taylor (1981. ch. 15).

9 This is analogous to the unrestricted equilibrium discussed in section 3 in Longstaff (1992). Beaglehole and Tenney (1992) point out that Longstaff's (1989) bond pricing equation is not the solution for a reflecting boundary condition.

$$B(\tau) = \frac{2\lambda_r - \gamma}{\sigma_r^2} + \frac{2\gamma}{\sigma_r^2 [1 - C_0 \exp\{\gamma\tau\}]} \tag{5}$$

$$C(\tau) = \frac{2\kappa_r (2\lambda_r + \gamma)\left(1 - \exp\left\{\frac{1}{2}\gamma\tau\right\}\right)^2}{\gamma\sigma_r^2 [1 - C_0 \exp\{\gamma\tau\}]} \tag{6}$$

with

$$\gamma = \sqrt{4\lambda_r^2 + 2\sigma_r^2} \quad , \quad C_0 = \frac{2\lambda_r + \gamma}{2\lambda_r - \gamma}$$

$$\alpha_1 = -\frac{\kappa_r^2}{\gamma^3 \sigma_r^2}(4\lambda_r + \gamma)(2\lambda_r - \gamma) \quad , \quad \alpha_2 = \frac{2\lambda_r + \gamma}{4} - \frac{\kappa_r^2}{\gamma^2}$$

$$\alpha_3 = \frac{4\kappa_r^2}{\gamma^3 \sigma_r^2}\left(2\lambda_r^2 - \sigma_r^2\right) \quad , \quad \alpha_4 = -\frac{8\kappa_r^2 \lambda_r}{\gamma^3 \sigma_r^2}(2\lambda_r + \gamma) \quad .$$

In the valuation of US dollar-denominated sovereign bonds, we assume a semi-structural framework, and let the exchange rate be a stochastic variable. The risk-adjusted dynamic of the exchange rate *S*, which determines the probability of default of a sovereign bond, is assumed to follow a lognormal diffusion process, which is commonly used for pricing exchange rate options and derivatives. Its continuous stochastic movement is modelled by the following stochastic differential equation:

$$\frac{dS}{S} = \alpha_S dt + \sigma_S dz_S \tag{7}$$

where ${}_S$ is the volatility, ${}_S$ is the drift rate, and dz_S denotes a standard Wiener process.

With using the DSR model of the risk-free interest rate in the pricing framework, the Wiener processes dz_r and dz_S in Eqs.(2) and (7) are correlated with:

$$dz_S dz_r = \rho_{Sr} dt. \tag{8}$$

We apply Ito's lemma to derive the partial differential equation (PDE) governing a sovereign discount bond $P(S, r, \tau)$ with the time-to-maturity of as follows:

$$\begin{aligned} \frac{\partial P}{\partial \tau} &= \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 P}{\partial S^2} + \rho_{Sr}\sigma_S \sigma_r S\sqrt{r}\frac{\partial^2 P}{\partial r \partial S} + \frac{1}{2}\sigma_r^2 r \frac{\partial^2 P}{\partial r^2} + \alpha S \frac{\partial P}{\partial S} \\ &+ \left(\frac{1}{4}\sigma_r^2 - \kappa_r \sqrt{r} - 2\lambda_r r\right)\frac{\partial P}{\partial r} - rP \end{aligned} . \tag{9}$$

In terms of the new variables and parameters:

$$x = \sqrt{2r} \quad , \quad y = \ln\left(\frac{S}{S_0}\right) \quad , \quad y_0 = \ln S_0 \quad , \quad \tilde{\sigma}_r = \frac{\sigma_r}{\sqrt{2}} \ ,$$

$$\tilde{\kappa}_r = \frac{\kappa_r}{\sqrt{2}} \quad , \quad \tilde{\alpha} = \alpha - \frac{\alpha^2}{2} \quad ,$$

Eq.(9) becomes:

$$\begin{aligned} \frac{\partial P}{\partial \tau} &= \frac{1}{2}\sigma_S^2 \frac{\partial^2 P}{\partial y^2} + \rho_{Sr}\sigma_L \tilde{\sigma}_r \frac{\partial^2 P}{\partial x \partial y} + \frac{1}{2}\tilde{\sigma}_r^2 \frac{\partial^2 P}{\partial x^2} + \tilde{\alpha}\frac{\partial P}{\partial y} \\ &\quad - \left(\tilde{\kappa}_r + \lambda_r x\right)\frac{\partial P}{\partial x} - \frac{1}{2}x^2 P \end{aligned} . \tag{10}$$

Without loss of generality, we assume $P(y,x,\tau)$ to be of the product form: $P(y,x,\tau) = \Phi(x,\tau)\tilde{P}(x,y,\tau)$, where the unknown function $\tilde{P}(x,y,\tau)$ denotes the discount factor of the risk-free bond price function $\Phi(x,\tau)$ due to the possibility of default. It is not difficult to show by direct substitution that $\tilde{P}(x,y,\tau)$ satisfies the PDE:

$$\begin{aligned} \frac{\partial \tilde{P}}{\partial \tau} &= \frac{1}{2}\sigma_S^2 \frac{\partial^2 \tilde{P}}{\partial y^2} + \rho_{Sr}\sigma_S \tilde{\sigma}_r \frac{\partial^2 \tilde{P}}{\partial x \partial y} + \frac{1}{2}\tilde{\sigma}_r^2 \frac{\partial^2 \tilde{P}}{\partial x^2} \\ &\quad + \left\{\frac{1}{\sqrt{2}}\rho_{Sr}\sigma_S\tilde{\sigma}_r C(\tau) + \rho_{Sr}\sigma_S\tilde{\sigma}_r B(\tau)x + \alpha\right\}\frac{\partial \tilde{P}}{\partial y} . \\ &\quad + \left\{\frac{1}{\sqrt{2}}\tilde{\sigma}_r^2 C(\tau) - \tilde{\kappa}_r - \left[\lambda_r - \tilde{\sigma}_r^2 B(\tau)\right]x\right\}\frac{\partial \tilde{P}}{\partial x} \end{aligned} \tag{11}$$

In order to eliminate all the terms involving first derivatives with respect to x and y, we rewrite $\tilde{P}(x,y,\tau)$ as:

$$\begin{aligned} \tilde{P}(x,y,\tau) &= \exp\left\{\zeta(\tau)x\frac{\partial}{\partial x}\right\}\exp\left\{\xi(\tau)y\frac{\partial}{\partial y}\right\}\exp\left\{\omega(\tau)x\frac{\partial}{\partial y}\right\}\times \\ &\quad \exp\left\{\Omega_1(\tau)\frac{\partial}{\partial x}\right\}\exp\left\{\Omega_2(\tau)\frac{\partial}{\partial y}\right\}Q(x,y,\tau) \quad , \\ &= Q\left(\exp\{\zeta(\tau)\}x + \Omega_1(\tau), \exp\{\xi(\tau)\}y + \omega(\tau)\exp\{\zeta(\tau)\}x + \Omega_2(\tau), \tau\right) \end{aligned} \tag{12}$$

where:

$$\zeta(\tau) = \frac{1}{2}\gamma\tau + \ln\left|\frac{1-C_0}{1-C_0\exp(\gamma\tau)}\right|$$

$$\omega(\tau) = \rho_{Sr}\sigma_S\tilde{\sigma}_r\int_0^\tau B(\tau')\exp\{-\zeta(\tau')\}d\tau'$$

$$\Omega_1(\tau) = \int_0^\tau\left[\frac{1}{\sqrt{2}}\tilde{\sigma}_r^2 C(\tau') - \tilde{\kappa}_r\right]\exp\{\zeta(\tau')\}d\tau'$$

$$\Omega_2(\tau) = \int_0^\tau\left[\frac{1}{\sqrt{2}}\rho_{Sr}\sigma_S\tilde{\sigma}_r C(\tau') + \tilde{\alpha}\right]d\tau' + \int_0^\tau\left[\frac{1}{\sqrt{2}}\tilde{\sigma}_r^2 C(\tau') - \tilde{\kappa}_r\right]\omega(\tau')\exp\{\zeta(\tau')\}d\tau' \quad .$$

Then substituting Eq.(12) into Eq.(11), we can show that $Q(x,y,\tau)$ satisfies the two-dimensional diffusion equation:

$$\frac{\partial Q}{\partial \tau} = \frac{1}{2}\sigma_x^2(\tau)\frac{\partial^2 Q}{\partial x^2} + \rho_{xy}(\tau)\sigma_x(\tau)\sigma_y(\tau)\frac{\partial^2 Q}{\partial x \partial y} + \frac{1}{2}\sigma_y^2(\tau)\frac{\partial^2 Q}{\partial y^2} \tag{13}$$

where:

$$\sigma_x^2(\tau) = \tilde{\sigma}_r^2\exp\{2\zeta(\tau)\}$$

$$\sigma_y^2(\tau) = \tilde{\sigma}_r^2\omega^2(\tau)\exp\{2\zeta(\tau)\} + \sigma_S^2 + 2\rho_{Sr}\sigma_S\tilde{\sigma}_r\omega(\tau)\exp\{\zeta(\tau)\}$$

$$\rho_{xy}(\tau)\sigma_x(\tau)\sigma_y(\tau) = \rho_{Sr}\sigma_S\tilde{\sigma}_r\exp\{\zeta(\tau)\} + \tilde{\sigma}_r^2\omega(\tau)\exp\{2\zeta(\tau)\} \quad .$$

It should be noted that since the final payoff condition is independent of the interest rate *r* (or equivalently x), i.e., $P(S,r,0)$ is a function of S only, it is obvious that $Q(x,y,0)$ does not depend upon x. Thus, by defining $f(y) = Q(x,y,0)$, we can readily obtain the solution of Eq.(13) as follows:

$$Q(x,y,\tau) = \exp\left\{\frac{1}{2}\Delta(\tau)\frac{\partial^2}{\partial y^2}\right\}Q(x,y,0) = \frac{1}{\sqrt{2\pi\Delta(\tau)}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(y-y')^2}{2\Delta(\tau)}\right\}f(y')dy' \tag{14}$$

where

$$\Delta(\tau) = \int_0^\tau \sigma_y^2(\tau')d\tau' \quad .$$

This solution satisfies the natural boundary condition. Hence, the corresponding solution $\widetilde{P}(x, y, \tau)$ is given by:

$$\widetilde{P}(x, y, \tau) = \frac{1}{\sqrt{2\pi\Delta(\tau)}} \int_{-\infty}^{\infty} \exp\left\{ -\frac{[Y(x, y, \tau) - y']^2}{2\Delta(\tau)} \right\} f(y')dy', \tag{15}$$

where $Y(x, y, \tau) = y + \omega(\tau)\exp\{\zeta(\tau)\}x + \Omega_2(\tau)$.

Under a scenario in which default can only occur at maturity and the recovery rate is *R* at default, the final payoff condition of the bond with the face value of 1 is:

$$P(S, r, 0) = 1 \text{ if } S \le S_0;$$

$$P(S, r, 0) = R \text{ if } S > S_0. \tag{16}$$

where S_0 is the default-triggering level above which default occurs and bondholders receive an exogenously given number of default-free bonds. The integral in Eq.(15) can be straightforwardly evaluated to give the bond price solution:

$$\begin{aligned} P(x, y, \tau) &= \Phi(x, \tau)\widetilde{P}(x, y, \tau) \\ &= \frac{\Phi(x, \tau)}{\sqrt{2\pi\Delta(\tau)}} \int_{-\infty}^{0} \exp\left\{ -\frac{[Y(x, y, \tau) - y']^2}{2\Delta(\tau)} \right\} dy + \\ &\quad \frac{R\Phi(x, \tau)}{\sqrt{2\pi\Delta(\tau)}} \int_{0}^{\infty} \exp\left\{ -\frac{[Y(x, y, \tau) - y']^2}{2\Delta(\tau)} \right\} dy \\ &= \Phi(x, \tau)\left\{ (1 - R)N\left(-\frac{Y(x, y, \tau)}{\sqrt{\Delta(\tau)}} \right) + R \right\} \end{aligned}, \tag{17}$$

where *N*(.) denotes the cumulative distribution function of a standard normal distribution. The discount factor of the sovereign bond price can be identified as the measure of default probability associated with the exchange rate *S*; that is, $P_{def}(S, r, \tau) = 1 - \widetilde{P}(x, y, \tau)$.

In order to allow default before maturity, a fixed or constant absorbing boundary (default barrier) is incorporated into the pricing model. Regarding pricing corporate bonds, Black and Cox (1976) assume a default-triggering level for the firm's assets whereby default can occur at any time. This trigger level is introduced by considering a safety covenant that protects bondholders. However, in the absence of a supranational legal authority to enforce any safety covenant, a country may have more discretion whether to pay its debt obligations in the case of default.[10] Given such sovereign immunity, we assume that currency depreciation is the only factor that a country will consider in deciding whether to meet its obligations on US dollar-donominated bonds. When the exchange rate devalues and breaches a predefined level, default occurs before maturity and the issuer (government) is unable to repay its sovereign foreign currency debt. However, with a constant default barrier, no closed-form pricing solution is available. On the other hand, by the method of images, we can derive the closed-form pricing solution $\tilde{P}(x, y, \tau)$ which has a moving absorbing boundary specified by $y = y^*(x,\tau) \equiv \{\chi\Delta(\tau) - \Omega_2(\tau) - \omega(\tau)\exp[\varsigma(\tau)]x\}$ for some adjustable real parameter χ as follows:

$$\tilde{P}(x,y,\tau) = \frac{1}{\sqrt{2\pi\Delta(\tau)}}\int_{-\infty}^{0}\left[\exp\left\{-\frac{[Y(x,y,\tau)-y']^2}{2\Delta(\tau)}\right\} - \exp(2\chi y')\exp\left\{-\frac{[Y(x,y,\tau)+y']^2}{2\Delta(\tau)}\right\}\right] f(y')dy' \quad (18)$$

Since $f(y)$ is equal to unity, the integral can be straightforwardly evaluated to give:

$$\tilde{P}(x,y,\tau) = N\left(-\frac{Y(x,y,\tau)}{\sqrt{\Delta(\tau)}}\right) - N\left(\frac{Y(x,y,\tau)}{\sqrt{\Delta(\tau)}} - 2\chi\sqrt{\Delta(\tau)}\right)\times \exp\left[-2\chi Y(x,y,\tau) + 2\chi^2\Delta(\tau)\right] \quad (19)$$

The sovereign bond price with default allowed before bond maturity and a recovery rate of R is:

$$P(S,r,\tau) = \Phi(x,\tau)\tilde{P}(x,y,\tau) + R\left(1-\tilde{P}(x,y,\tau)\right)\Phi(x,\tau) \quad (20)$$

As the movement of the absorbing boundary is adjustable by tuning the parameter χ, the default barrier can thus be adjusted such that the solution in Eq.(20) provides a good approximation to the exact result for a constant default barrier by using the methodology developed in Lo et al. (2003) for solving barrier option values with time-dependent model parameters. In addition, such a dynamic default barrier is flexible to incorporate different default scenarios as demonstrated in Hui et al. (2003). For example, default could be triggered even though the exchange rate is below S_0 because of the liquidity problem (such as repayments of short-term debts) faced by the government. In this case, a

10 Discussions on sovereign immunity and debt crisis can be found in Eaton (1996) and Kletzer and Wright (2000).

default scenario of higher short-term default probability can be incorporated into the valuation model by adjusting the default barrier lower than S_0 in the early period (say one to two years) of the time to maturity of the bond.

4. Market and model-implied sovereign bond credit spreads

4.1 Parameters for pricing bonds

Using the sovereign bond pricing model developed in the previous section, we obtain the corresponding daily model-implied bond prices for Brazil, Mexico and Turkey (with tenors of 10-year, 15-year, 20-year and 30-year); Colombia (with tenors of 10-year and 30-year); the Philippines (with tenors of 10-year, 15-year and 20-year); and Russia (with a tenor of 15-year) in the pre- and post-crisis periods. The model-implied credit spread C_S of a sovereign discount bond price $P(S, r, T)$ with a default barrier (default allowed before bond maturity) based on Eq.(20) is given as:

$$C_S(S,r,\tau) = -\frac{1}{\tau}\ln\left(\frac{P(S,r,\tau)}{\Phi(r,\tau)}\right) = -\frac{1}{\tau}\ln\left(\tilde{P}(x,y,\tau)\right). \tag{21}$$

The input parameters for the model are α_s and σ_s for the exchange rate S and κ_r, σ_r and λ_r for the US dollar interest rate r. To make the pricing framework simple for analysing the performance of the model, we assume the drift α_s to be zero.[11] Given that the bond pricing model needs to capture forward-looking market information and currency crash risk which has an impact on credit spreads, we use the 3-month currency option-implied volatility of the 25-delta out-of-the-money call (US dollar) for volatility σ_s which are illustrated in Figure 2.[12] While the option-implied volatilities of the currencies surged during the crisis in 2008 as expected, they also varied substantially during other times in particular in the post-crisis period. The sensitivity of dollar-denominated sovereign bond spreads to exchange rate risk anticipated by market participants is incorporated into the model through the model

11 The expected future exchange rate can be incorporated into the model by specifying a drift for the exchange rate dynamics. For example, assuming that uncovered interest rate parity holds, the drift is the interest rate differential between an emerging market currency and the US dollar.

12 The Black-Scholes delta provides a normalised measure of option moneyness where the delta of a European option increases monotonically from 0 to 100, with the moneyness moving from out-of-the-money to in-the-money.

parameter of the exchange rate volatility.

For the interest rate, r is the daily US dollar 3-month Libor. The values of the model parameters κ_r, σ_r, and λ_r of the DSR interest rate model are shown in Table 3. The DSR model parameters are estimated by using Hansen's (1982) generalized method of moments technique which is also used by Longstaff (1989) for estimation of the DSR term structure model.[13,14] The daily correlation ρ_{Sr} is estimated by the dynamic conditional correlation multivariate GARCH model proposed by Engle and Sheppard (2001). The recovery rate R is 0.25 for Brazil, Turkey, Mexico, Colombia, and Russia, and 0.4 for the Philippines.[15] Given that default barriers are not observable, we use two simple methods to set the barriers to test the to calibrate the barriers (denoted as calibrated default barriers) by minimizing the differences between the market and model-implied credit spreads. The levels of default barriers based on these two methods in the pre- and post-crisis periods for the pricing of the corresponding sovereign bonds are in Table 4.

4.2 Predicted spreads from the model

The model and market credit spreads of the sovereign bonds of Brazil, Mexico, Turkey, the Philippines and Russia with a tenor of 15 years; and Colombia with the tenor of 10 years are illustrated in Figure 3. The results in Panels A and B show that the model credit spreads based on the calibrated and reference default barriers of the Brazilian sovereign bond track the market credit spreads closely, particularly during 2007 before the global financial crisis and in 2009-2010 after the crisis. Given that the calibrated barriers are higher than the reference barrier, the model credit spreads obtained from the calibrated barriers are lower than those obtained from the reference barrier and closer to the market credit spreads as expected. The trend in the model credit spreads generated from the different default barriers are qualitatively similar, indicating that the default barriers affect mainly the level of the spreads. The correlations between the market and model credit spreads of the bonds with different

13 Both the estimations in Table 3 and Longstaff (1989) have large standard errors which are primarily due to the high correlations among the individual parameters.

14 The market yields are the US-treasury yield of time to maturities of 1-month, 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year. The estimations are using month-end zero-coupon yield to maturity data of 3-month, 12-month and 10-year US Treasury bills and notes during the January 2000 – September 2014 period.

15 The data of option-implied volatility are from JP Morgan. The recovery rates are from Bloomberg.

tenors are reported in Table 5. The high positive correlations in level (higher than 0.80) and in log change (between 0.30 and 0.54) are consistent with the observations in Panels A and B.

Regarding the Mexican sovereign bond in Panel C, the model credit spreads based on the calibrated and reference default barriers have similar trends in the market credit spreads in the pre-crisis period. For example, both the market and model spreads jumped in mid-2006 and mid-2007. The correlations between their spreads at different tenors are above 0.40 in level and 0.28 in log change. During the post-crisis period as shown in Panel C, the model credit spreads generated from the calibrated barriers are quite close to the market credit spreads with the correlations for different tenors above 0.65 in level and 0.21 in log change. While the model credit spreads based on the reference barrier are higher than the market values, their changes broadly follow those of the market and model (based on the calibrated barrier) credit spreads, as indicated by the high positive correlations in level and log change. The results demonstrate that the calibrated and reference default barriers determine the levels of the bond spreads but in general do not affect their changes substantially.

Panel E in Figure 3 shows that the market and model spreads of the 15-year Turkish sovereign bond are quite close to each other during 2004 and 2005 but substantially different in 2003 and 2006. Therefore, their correlations shown in Table 5 are lower than those of the Brazilian and Mexican sovereign bonds in the pre-crisis period. Similar to the Mexican sovereign bond, the model spreads with the reference barrier are higher than the market spreads and model spreads with the calibrated barriers in the post-crisis period (see Panel F). Regarding the 10-year Colombian sovereign bond in Panels G and H, the model spreads with the calibrated barriers broadly track the market spreads in particular in the post-crisis period with high correlations in levels. The tracking performance of the model spreads with the reference barrier is also better in the post-crisis period. The model spreads of the 15-year Philippine sovereign bond shown in Panels I and J exhibit similar movements of the market spreads with quite high positive correlations among the bonds with different tenors. Panel K shows that the model spreads of the 15-year Russian sovereign bond do not fit the market spreads well in the pre-crisis period. The tracking performance however improves significantly in the post-crisis period as shown in Panel L, and the correlations between the model and market spreads increase accordingly.

In summary, the comparison between the market and model credit spreads of the sovereign bonds of Brazil, Mexico, Turkey, Colombia, the Philippines and Russia as illustrated in Figure 3 and their correlations in Table 5 show that the proposed model can generate credit spreads which track the changes of the market credit spreads. The results are consistent with the regression results in section 2 that exchange rates in the emerging markets are related to their sovereign credit spreads. The calibrated and reference default barriers used in the model mainly shift the levels of model spreads but do not make their changes different materially.

4.3 Error analysis

Table 6 summarizes the pricing errors of the model in terms of credit spreads. There are three error measures including: (i) root-mean-square errors (RMS) in basis points (bps); (ii) percentage errors; and (iii) absolute percentage errors. The percentage errors, as well as their absolute values, are calculated as the predicated (model) spread minus the market spread divided by the market spread. Their means are reported and the numbers in parentheses are the standard deviation of the errors. For pricing the Brazilian sovereign bonds using the reference barrier, the RMS errors are about 72 – 263 bps (with the aggregate of 162 bps) and 32 – 116 bps (with the aggregate of 103 bps) for different tenors in the pre- and post-crisis periods respectively. The corresponding absolute percentage errors are 18 – 34% (with the aggregate of 21%) and 17 – 29% (with the aggregate of 24%). Using the calibrated barriers, the RMS errors drop to the ranges of 56 – 81 bps (with the aggregate of 69 bps) and 32 – 51 bps (with the aggregate of 39 bps) in the pre- and post-crisis periods respectively. The corresponding absolute percentage errors are in the ranges of 13 – 26% (with the aggregate of 17%) and 15 – 34% (with the aggregate of 23%). The results show that the RMS errors are smaller in the post-crisis period than those in the pre-crisis period. However, the absolute percentage errors are larger in the post-crisis period, indicating that the differences in the RMS errors are mainly due to the lower credit spreads in the post-crisis period (with the mean about 2%) compared with the pre-crisis period (with the mean about 3.5%) as shown in Table 1. As expected, the performance of the model based on the calibrated barriers is better than that based on the reference barrier. However, if we compare the ranges and aggregates of errors for the two types of barriers in post-crisis period, the use of the calibrated barriers does not out-perform substantially compared with the reference barrier. The percentage errors indicate that the model with the calibrated barrier tends to generate lower credit spreads in both the pre- and post-crisis periods at -2.6% and -6.2% in aggregate. While the magnitudes are relatively small as compared with the standard deviations, this indicates a marginal systematic negative (positive) bias of the model for the pricing of the bond spreads (prices).

Regarding the Mexican sovereign bonds, the RMS errors with the reference barrier are in the ranges of 58 – 89 bps (with the aggregate of 69 bps) and 89 – 444 bps (with the aggregate of 279 bps) in the pre- and post-crisis periods respectively. The corresponding absolute percentage errors are 28 – 46% (with the aggregate of 37%) and 56 – 312% (with the aggregate of 171%). The poorer performance of the model in the post-crisis period reflects the fact that the reference barrier is very different from the level of the exchange rate triggering default. Using the calibrated barriers, the RMS errors in the pre- and post-crisis periods drop to the ranges of 38 – 45 bps (with the aggregate of 42 bps) and 38 – 60 bps (with the aggregate of 50 bps) respectively. The corresponding absolute percentage errors thus fall in the ranges of 15 – 25% (with the aggregate of 21%) and 17 – 37% (with the aggregate of 27%). The performance of the model for the Mexican sovereign bonds is not very different in the pre- and post-crisis periods, while the magnitudes of their errors are similar to those of the Brazilian sovereign bonds. In addition, the aggregate percentage errors indicate that the model with the calibrated barriers tends to generate lower credit spreads in the post-crisis period, but the error magnitude of -3.7% is lower than that of the Brazilian bonds.

The performance results for the sovereign bonds of Turkey, Colombia, the Philippines and Russia are qualitatively similar to those for Brazil and Mexico in general. Using the calibrated barriers, their aggregate absolute percentage errors are in the range of 16 – 32% and 19 – 35% in the pre- and post-crisis periods respectively. The errors are not much different from those for Brazil (17% and 23%) and Mexico (21% and 27%). Their percentage errors using the calibrated barriers are mostly negative in the range of 0.3% – -14.7%, reflecting a marginal systematic negative (positive) bias of the model for the pricing of the bond spreads (prices) particularly in the post-crisis period. The generally poorer performance of the model using the reference barriers (in particular for Turkey in the post-crisis period) demonstrates that the performance is sensitive to the setting of the default barriers.

In order to have a broad assessment of the performance of the proposed model for pricing sovereign bonds, its pricing errors are comparing with those of the structural models for corporate bonds. Eom et al. (2004) empirically tests five structural models of corporate bond pricing which are considered as typical models, including those of Merton (1974), Geske (1977), Longstaff and Schwartz (LS) (1995), Leland and Toft (LT) (1996), and Collin-Dufresne and Goldstein (CDG) (2001). They implement the models using a sample of 182 bond prices from firms with simple capital structures during the period 1986–1997. The mean absolute percentage errors of model credit spreads in the study are 78% for the Merton model, 66% for the Geske model, 97% for the LS model, 146% for the LT model, and 170% for the CDG model. While pricing sovereign and corporate bonds based on the structural approach is different, in terms of the pricing errors, the performance of the proposed sovereign bond pricing model is no worse than that of the structural models for the pricing of corporate bonds.

5. Conclusion

Based on an analogy between an economy's currency price and a firm's stock price, this paper studies the dynamic linkage between exchange rates and US dollar-denominated sovereign bonds in emerging markets. Using data on emerging markets, including Brazil, Colombia, Mexico, the Philippines, Russia and Turkey, our empirical results support the view that the exchange rates of their currencies have adequate explanatory power in explaining their US dollar-denominated sovereign bonds. We develop a two-factor risky bond pricing model based on a semi-structural approach in which the stochastic exchange rates and US risk-free interest rates are the underlying factors. To incorporate forward looking market information into the model, the currency option-implied volatility is used as the associated model parameter of the exchange rate. The closed-form solutions of risky bond prices are derived from the model with default at maturity and a default barrier (default prior to maturity) respectively.

Using US dollar-denominated sovereign bonds with different tenors, the numerical results from the closed-form solution with default prior to maturity show that the credit spreads generated from the pricing model broadly track changes in the market credit spreads in both pre- and post-crisis periods. The correlations between them are positively high especially when the calibrated default barriers are used for pricing the bonds. The corresponding absolute percentage errors vary among the bonds. For example the aggregate absolute percentage errors for Brazil in the pre- and post-crisis periods are 17% and 23% respectively, while those for Turkey are 27% and 35%. The percentage errors show a marginal systematic negative (positive) bias of the model for pricing bond spreads (prices) particularly in the post-crisis period. The magnitude of the errors is lower than that of conventional structural models for the pricing of corporate bonds. Our numerical results are consistent with empirical results that the exchange rate dynamics of the emerging market currencies are significantly related to their sovereign credit spreads.

Our results support the findings of a strong relationship between emerging markets' sovereign risk and exchange rate stability in the literature on international finance and studies about twin sovereign debt and currency crises. This paper's findings suggest that dollar-denominated sovereign bonds are directly influenced by exchange rate dynamics. This suggests that both governments and investors might be better served by issuing debt in local currency, and letting investors hedge these risks in currency markets.

Given that the model simply captures the contributions due to exchange rate dynamics, future research could develop reduced-form models augmented to allow for the possible sensitivity of bond credit spreads to exchange rates and exchange rate volatility, and to depend on observable country-specific or macro-economic variables such as foreign reverses which affect sovereign risk.

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Figure 1: Dollar-denominated sovereign bond credit spreads of emerging market countries.

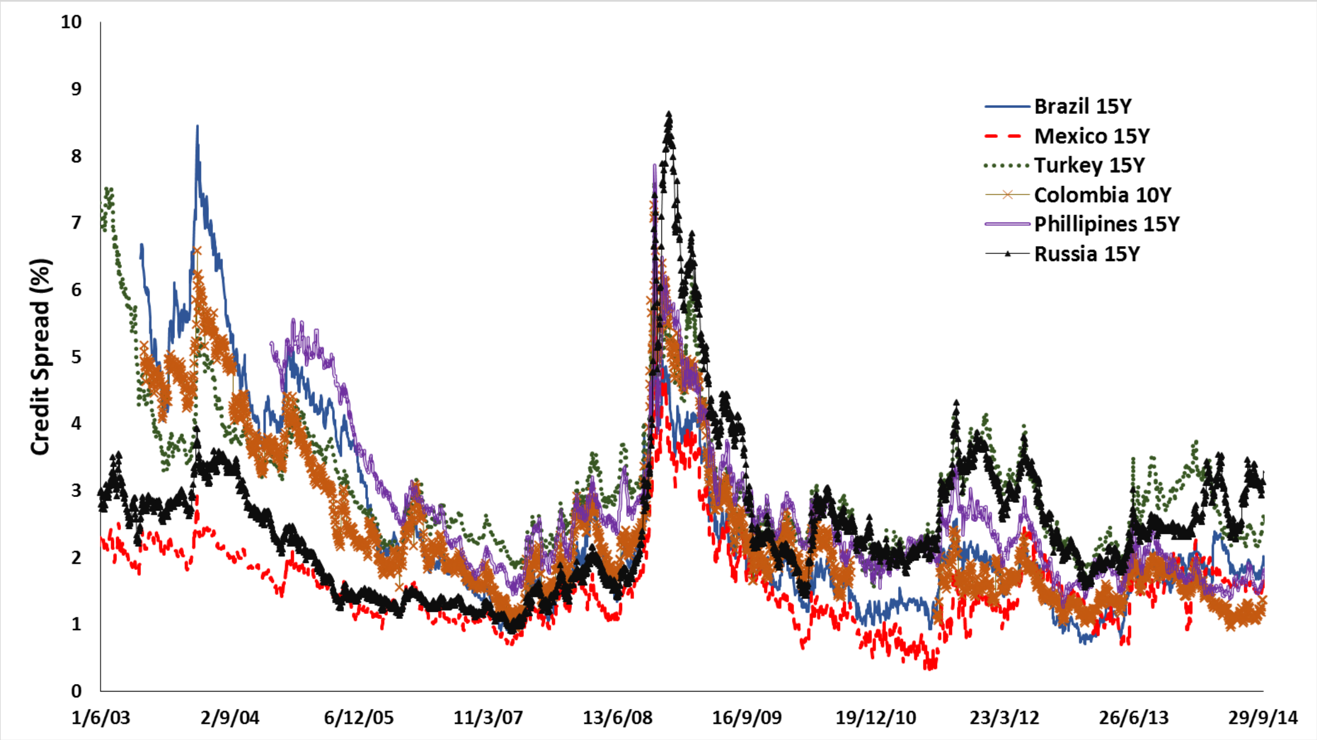
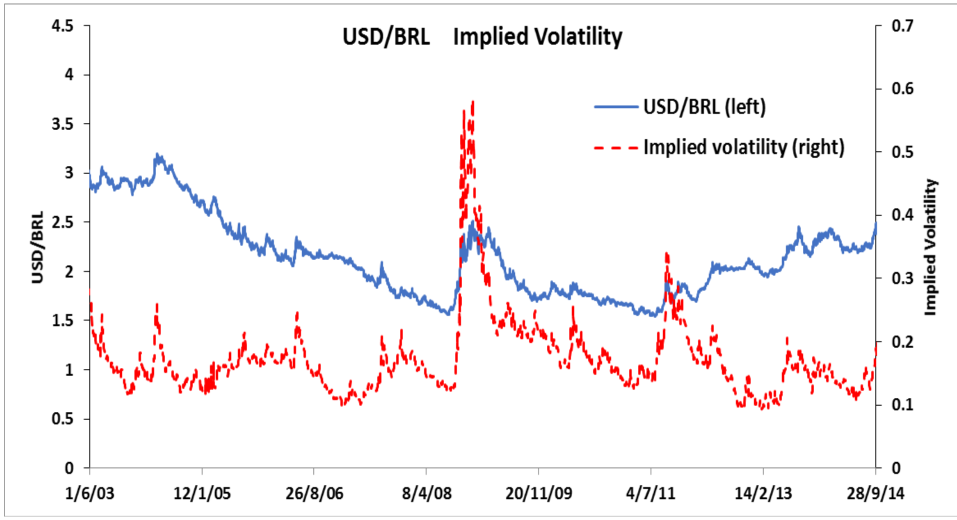
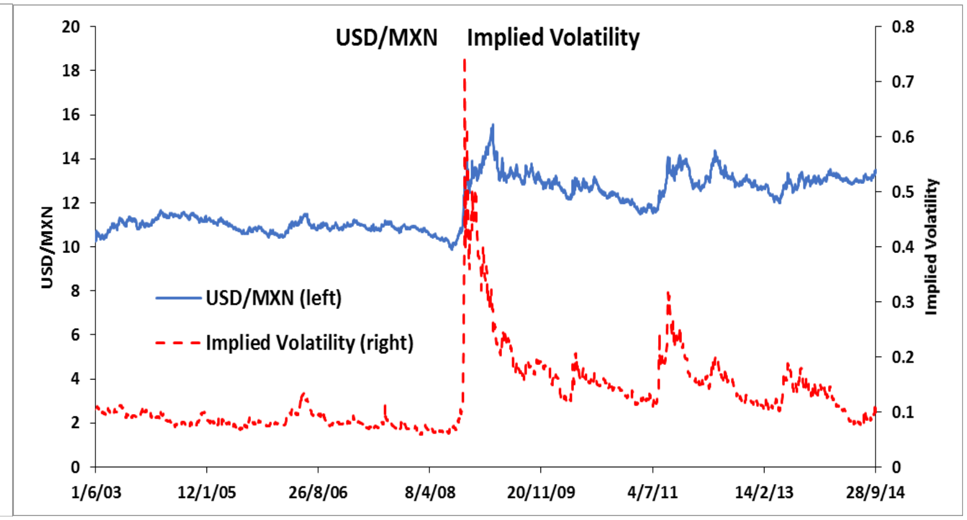


Figure 2: Exchange rates and option-implied volatility of emerging market currencies.



(Panel A)



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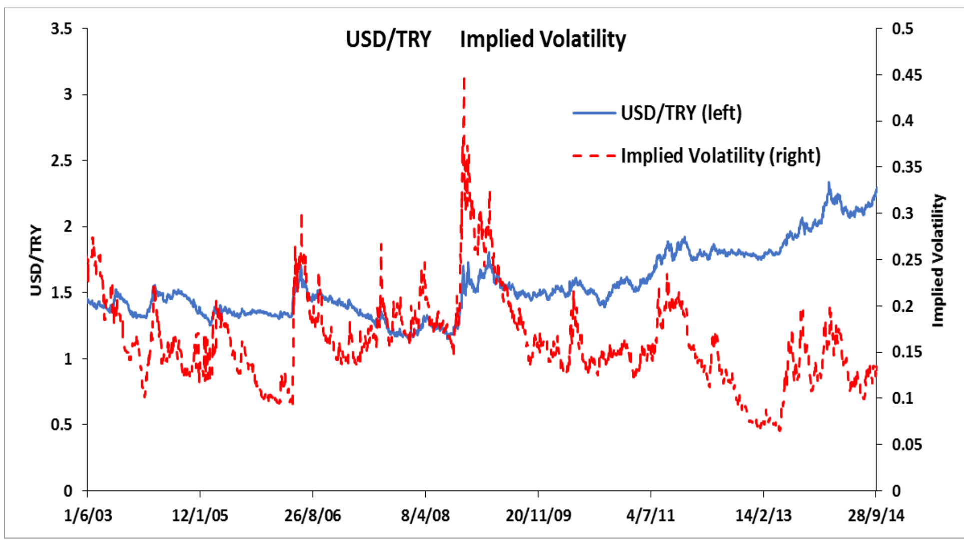

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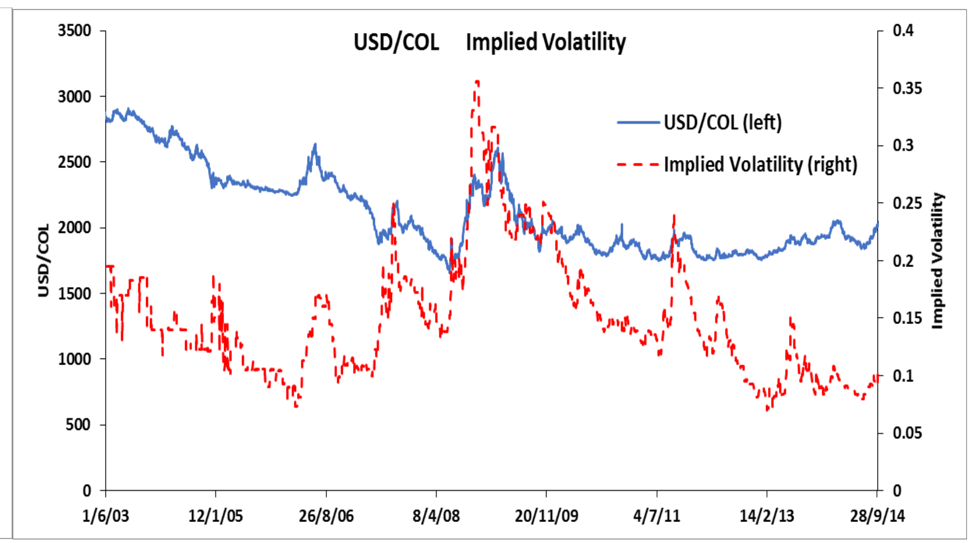
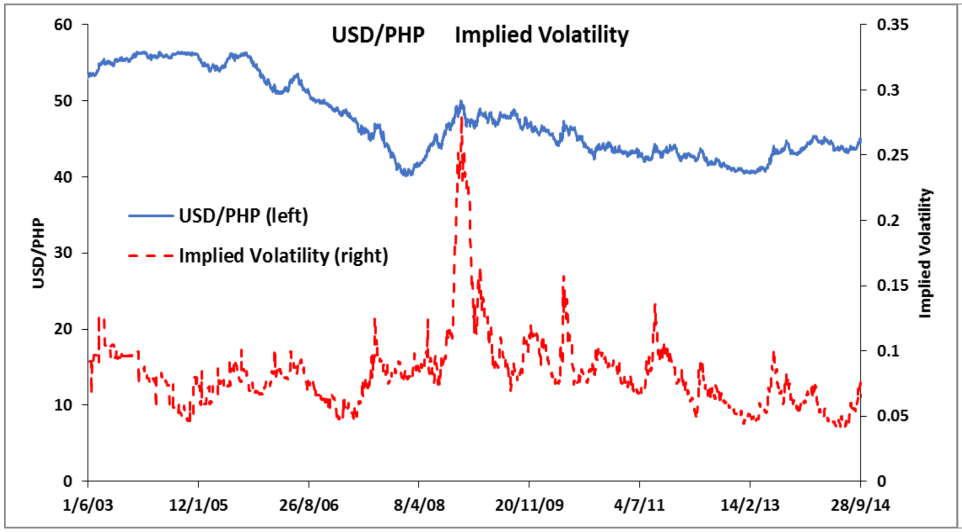
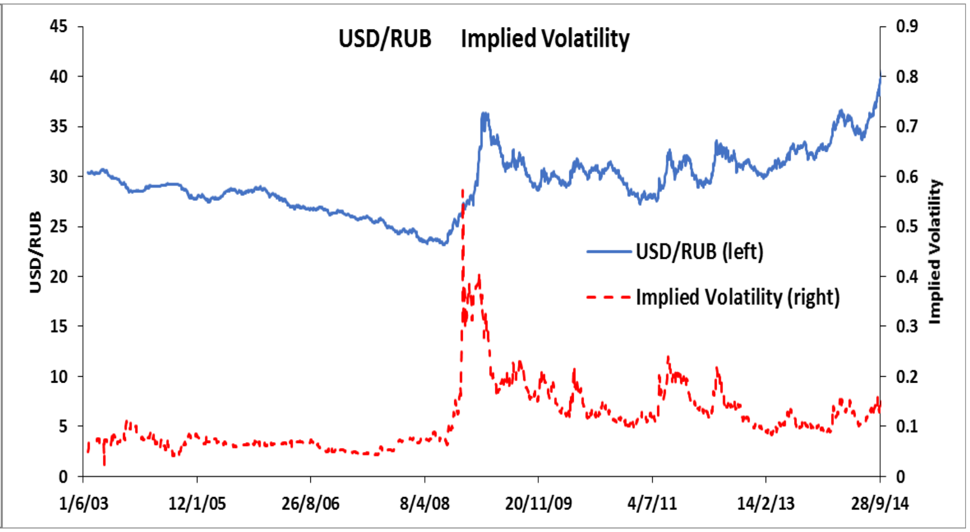


(Panel D)

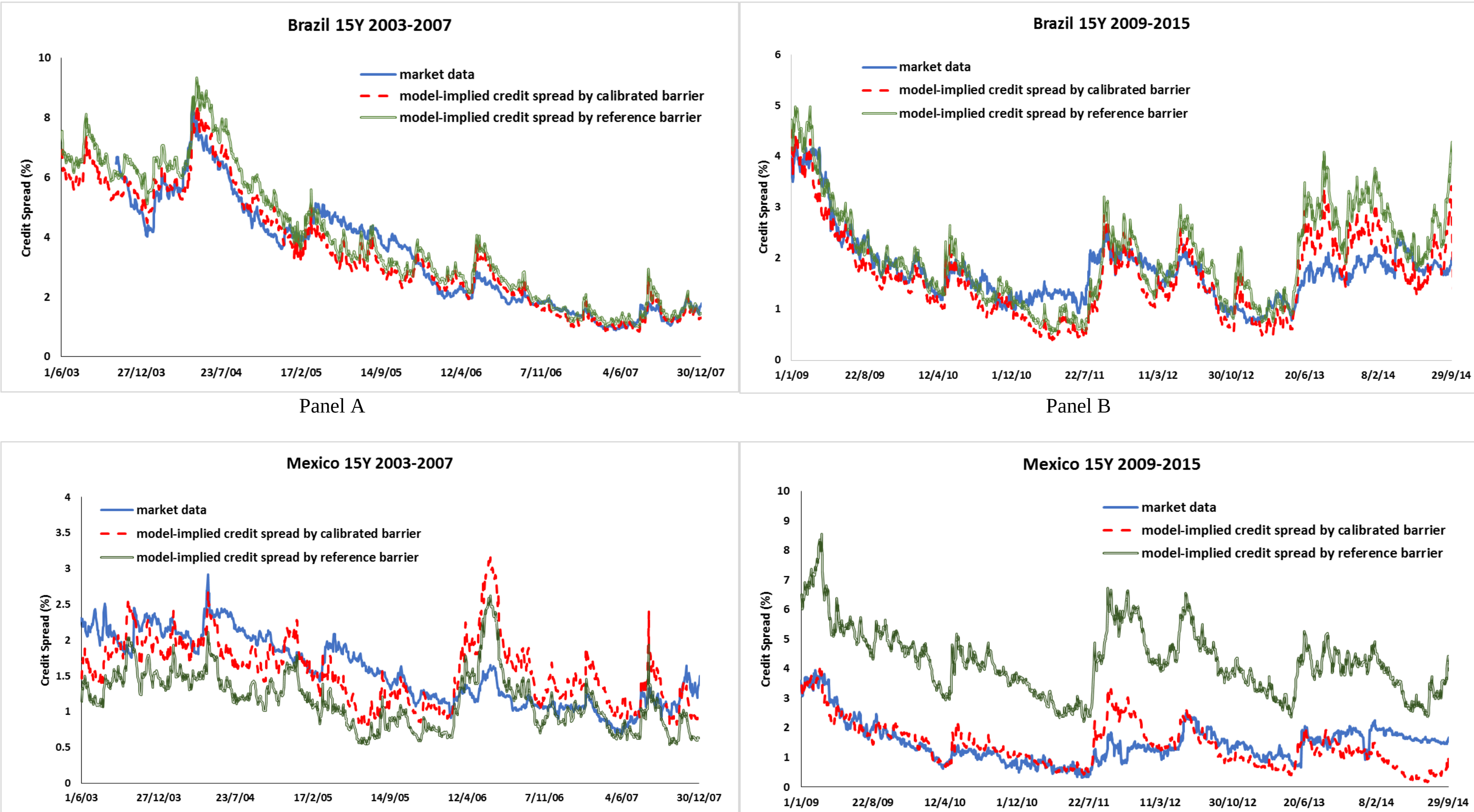


(Panel E)



(Panel F)

Figure 3: Comparison of model and market sovereign bond credit spreads.

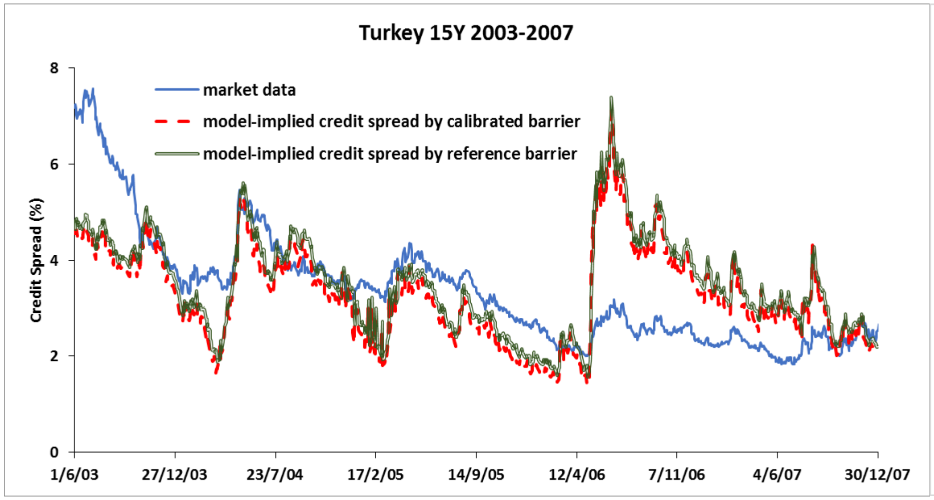


Panel A

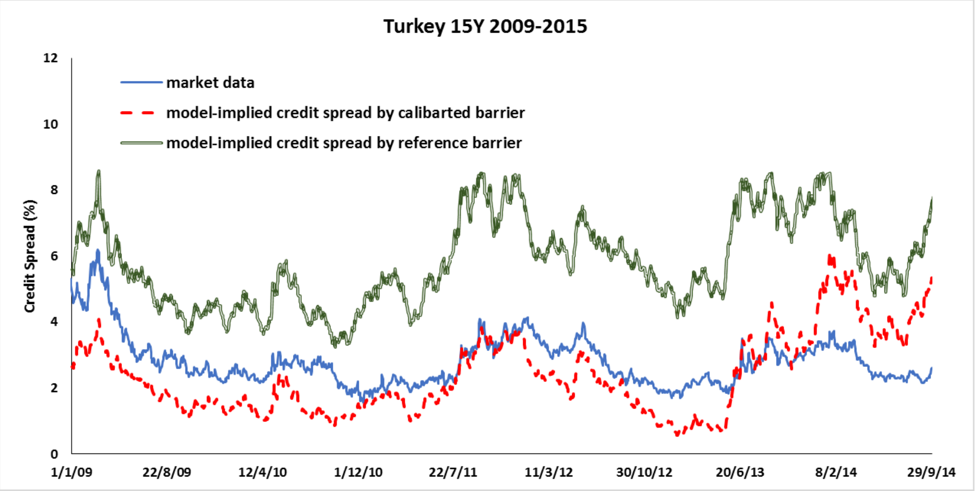
Panel B

Panel C

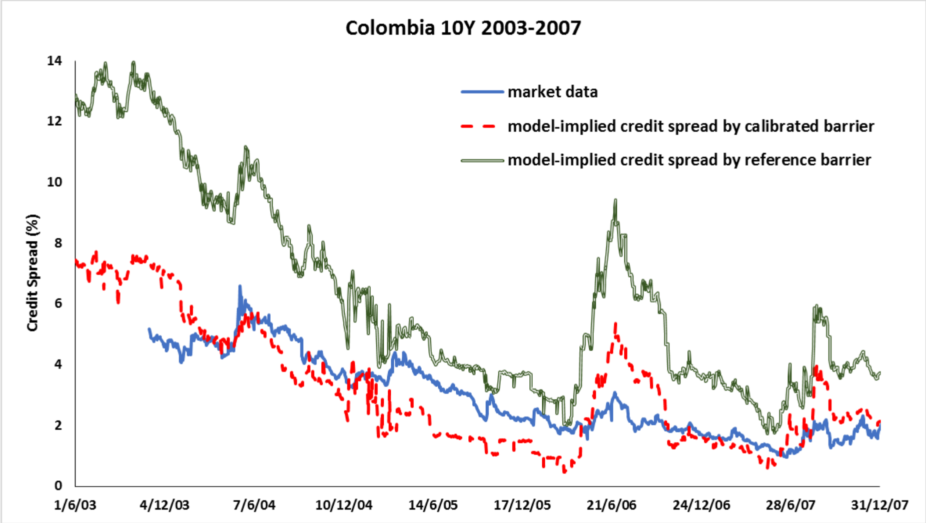
Panel D



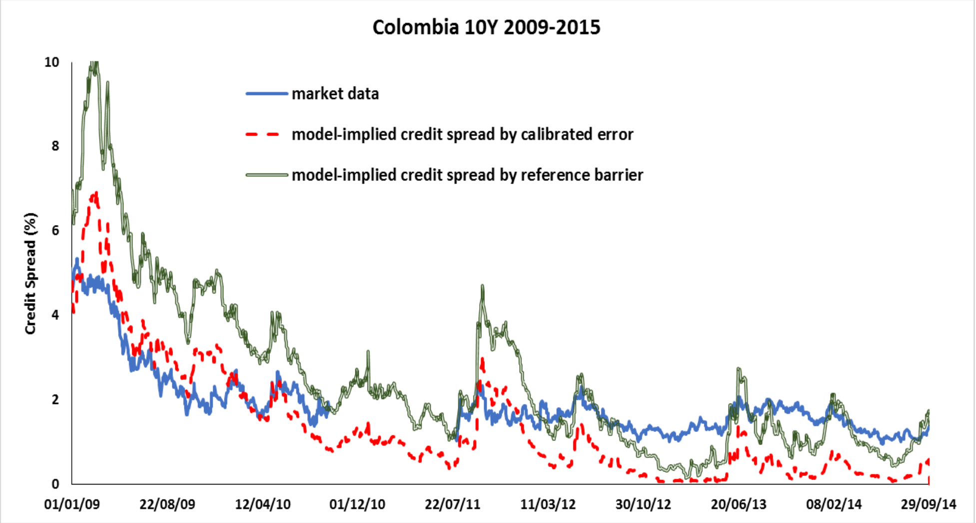
Panel E



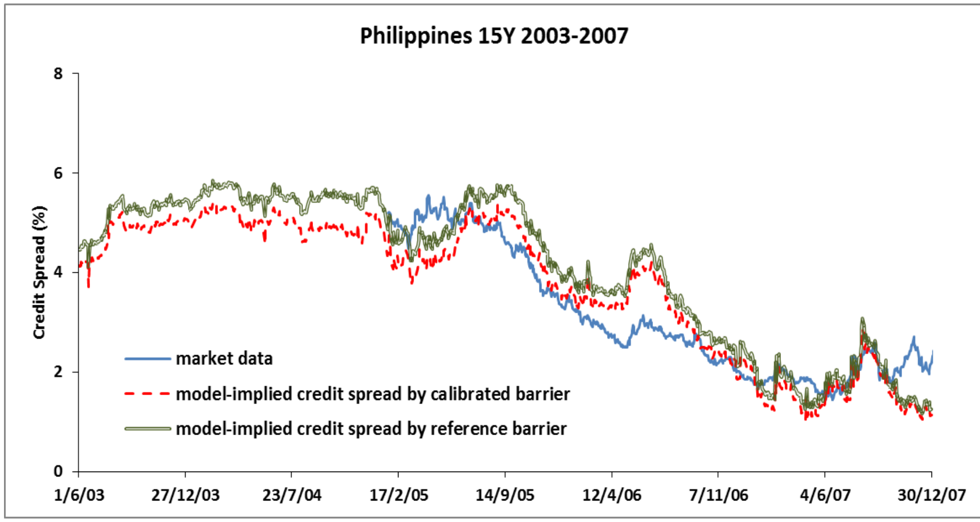
Panel F



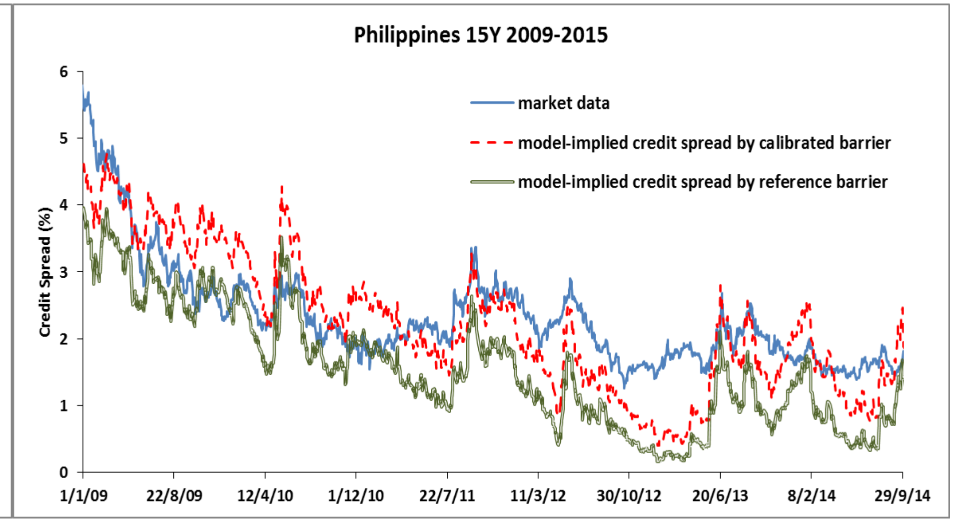
Panel G



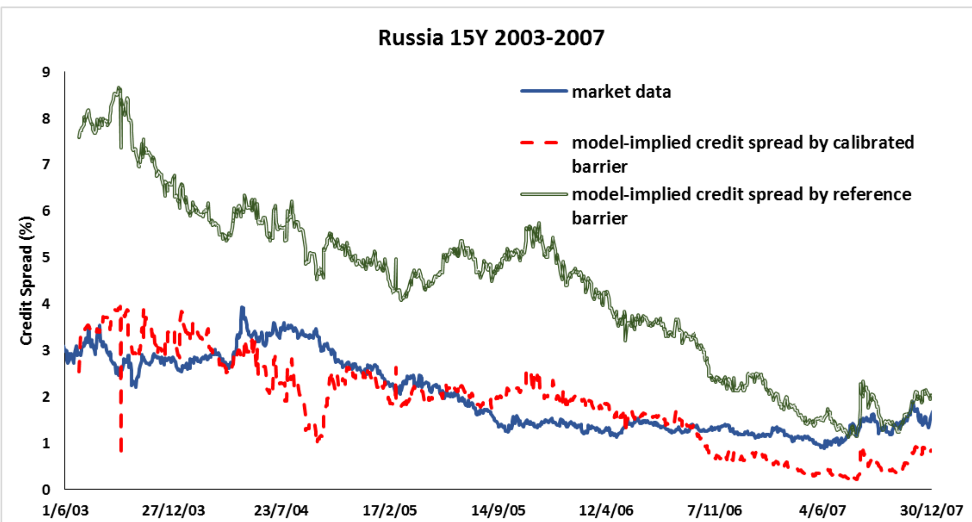
Panel H



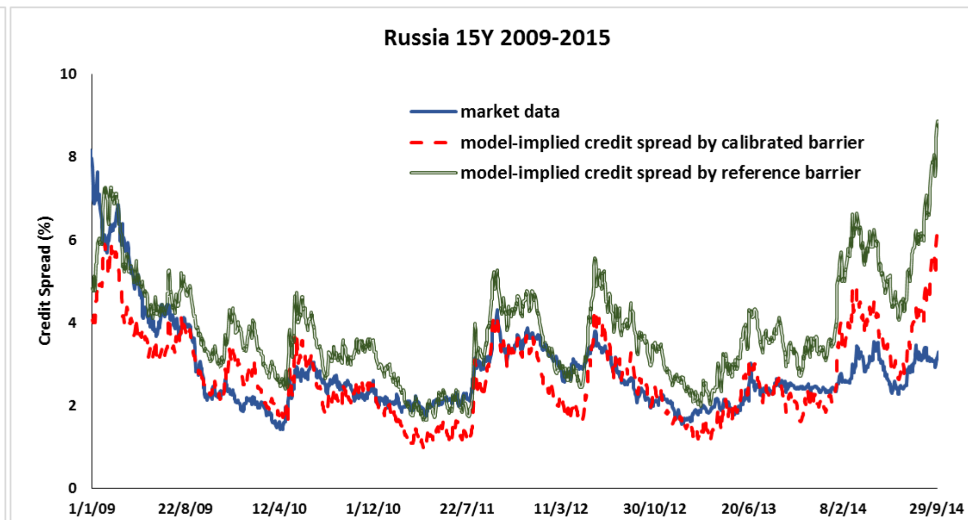
Panel I



Panel J



Panel K



Panel L

Table 1: Summary statistics on the bonds in the sample, exchange rates and option-implied volatility in 1 June 2003 - 31 December 2007 (pre-crisis period) and 1 January 2009 - 29 September 2014 (post-crisis period).

	Max		Min		Mean		Standard-Deviation	
Brazil	Pre-crisis	Post-crisis	Pre-crisis	Post-crisis	Pre-crisis	Post-crisis	Pre-crisis	Post-crisis
Bond spread 10Y (%)	8.8577	4.4195	0.9635	0.6473	3.5827	1.7503	1.9283	0.7182
Bond spread 15Y (%)	8.4530	4.2563	0.8996	0.7070	3.4004	1.8832	1.8107	0.6844
Bond spread 20Y (%)	8.7270	4.4560	0.8730	0.9810	3.5393	2.0319	1.8349	0.7148
Bond spread 30Y (%)	8.6911	3.9394	1.1196	0.6178	3.6430	1.9588	1.7895	0.7245
Exchange rate USD/BRL	3.2118	3.8415	1.7320	1.5387	2.4431	2.0954	0.4014	0.4282
Implied volatility (%)	26.125	41.425	9.5000	9.0500	15.6212	17.6596	3.0692	4.9606
Mexico								
Bond spread 10Y (%)	2.3667	4.2861	0.6882	0.6349	1.3393	1.4691	0.3307	0.6967
Bond spread 15Y (%)	2.9160	3.9521	0.6916	0.3119	1.5674	1.5189	0.4808	0.6294
Bond spread 20Y (%)	2.6470	4.1130	0.9480	0.9880	1.6564	1.7462	0.3848	0.5078
Bond spread 30Y (%)	2.8581	4.2509	0.8760	0.8778	1.8034	1.6958	0.4330	0.5407
Exchange rate USD/MEX	11.664	17.201	10.247	11.493	10.987	13.216	0.2719	0.9856
Implied volatility (%)	13.430	39.900	6.6503	7.2500	8.6008	15.237	1.2371	4.9398
Turkey								
Bond spread 10Y (%)	7.5787	6.8843	1.5221	1.1503	3.1713	2.4952	1.2667	0.8759
Bond spread 15Y (%)	7.5601	6.1952	1.8266	1.5637	3.3764	2.7566	1.2055	0.7108
Bond spread 20Y (%)	3.5580	5.7610	1.7760	1.1170	2.4748	2.5389	0.4280	0.6876
Bond spread 30Y (%)	7.0480	6.5044	1.9971	1.0967	3.1377	2.5457	0.9674	0.8164
Exchange rate USD/TRY	1.7063	3.0305	1.1682	1.3944	1.3840	1.8587	0.0877	0.3512
Implied volatility (%)	30.274	32.453	9.1150	6.5065	16.327	15.184	4.0406	4.3012
Colombia								
Bond spread 10Y (%)	6.5799	5.3527	0.9426	0.9568	2.9517	1.8914	1.3107	0.8027
Bond spread 20Y (%)		5.1400		1.3310		2.5187		0.6780
Bond spread 30Y (%)	6.3961	5.3048	1.1533	0.8428	3.4375	1.9067	1.2717	0.7755
Exchange rate USD/COL	2,904.6	3,259.0	1,874.2	1,748.0	2,410.3	1,995.3	261.5	263.4
Implied volatility (%)	24.980	31.600	7.2789	7.0000	13.648	15.083	3.3262	5.4264
Philippines								
Bond spread 10Y (%)		3.3160		0.8343		1.6766		0.5018
Bond spread 15Y (%)	5.5588	5.6873	1.4410	0.9749	3.0935	2.2001	1.2251	0.7913
Bond spread 20Y (%)		2.8680		0.7480		1.6341		0.4185
Exchange rate USD/PHP	56.420	48.925	40.950	40.450	52.411	44.195	3.9561	1.9480
Implied volatility (%)	12.680	22.201	4.6280	4.1127	7.7496	7.5778	1.6558	2.4954
Russia								
Bond spread 15Y (%)	3.9306	7.7536	0.8840	1.4204	2.0109	3.0320	0.7949	1.1416
Exchange rate USD/RUB	30.740	70.808	24.280	27.201	27.801	34.869	1.5546	9.1571
Implied volatility (%)	11.000	76.556	2.4000	8.2126	6.4438	17.489	1.2677	10.410
United States								
Treasury yield 10Y (%)	5.2928	3.9859	3.1121	1.3875	4.4369	2.5855	0.3660	0.6400
Treasury yield 15Y (%)	5.3664	4.3478	3.6211	1.7488	4.6777	2.9555	0.2968	0.6725
Treasury yield 20Y (%)	5.6100	4.7500	4.1300	2.0400	4.9180	3.3237	0.2856	0.7137
Treasury yield 30Y (%)	5.5606	4.8395	4.1730	2.2222	4.8515	3.5645	0.2783	0.6415
Short term interest rate	5.7250	1.4213	1.0000	0.2229	3.5596	0.3725	1.7198	0.2209

Table 2: Results from regressing daily changes in credit spreads on changes in Treasury yields and exchange rates during pre-crisis (1 June 2003 - 31 December 2007) and post-crisis (1 January 2009 - 29 September 2014) periods. $\Delta CS_t = a + b\Delta I_t + c\Delta Y_t + \varepsilon_t$; ΔCS: change in credit spread; ΔI: change in exchange rate;
ΔY: change in USD Treasury bond yield

	A	b	c	Adj. R^2	N
Brazil pre-crisis					
10Y	-1.79E-05	0.032579***	-0.423821***	0.301740	1069
15Y	-2.28E-05	0.022915***	-0.456325***	0.247755	1091
20Y	-2.63E-05	0.024404***	-0.434065***	0.269502	1194
30Y	-2.39E-05	0.026017***	-0.377374***	0.274398	1193
Brazil post-crisis					
10Y	-1.51E-05	0.008872***	-0.759033***	0.564520	1365
15Y	-1.12E-05	0.008278***	-0.908440***	0.714829	1495
20Y	-1.18E-05	0.008134***	-0.795108***	0.559443	1495
30Y	-163E-05	0.009925***	-0.777030***	0.474237	988
Mexico pre-crisis					
10Y	-8.88E-05	0.002285***	-0.388730***	0.250256	1083
15Y	-7.67E-05	0.002536***	-0.438689***	0.264938	1148
20Y	-1.38E-05	0.002025***	-0.351745***	0.199819	921
30Y	-8.88E-06	0.002381***	-0.422331***	0.254845	1193
Mexico post-crisis					
10Y	-1.66E-05	0.001129***	-0.757392***	0.305870	1425
15Y	-1.15E-05	0.000752***	-0.874257***	0.321954	1438
20Y	-1.16E-05	0.001270***	-0.856517***	0.546522	1494
30Y	-1.02E-05	0.001863***	-0.717229***	0.538340	1493
Turkey pre-crisis					
10Y	-4.60E-05	0.037080***	-0.725606***	0.312478	931
15Y	-3.04E-05	0.032201***	-0.704508***	0.437636	1148
20Y	-4.84E-05	0.020255***	-0.750334***	0.506729	768
30Y	-2.79E-05	0.027035***	-0.705168***	0.380692	1195
Turkey post-crisis					
10Y	-2.88E-05	0.024269***	-0.988732***	0.475544	1494
15Y	-2.79E-05	0.021272***	-1.043532***	0.589325	1438
20Y	-2.44E-05	0.020266***	-0.999196***	0.559800	1491
30Y	-2.88E-05	0.019635***	-1.022787***	0.548374	1497
Colombia pre-crisis					
10Y	-2.69E-05	2.00E-05***	-0.878571***	0.226331	1082
30Y	-8.41E-06	1.78E-05***	-0.671917***	0.241309	1192
Colombia post-crisis					
10Y	-2.07E-05	1.60E-05***	-0.857612***	0.517760	1280
20Y	-1.04E-05	1.04E-05***	-0.858695***	0.536560	1135
30Y	-1.59E-05	1.57E-05***	-0.796530***	0.579839	1470
Philippines pre-crisis					
15Y	-3.16E-05	0.000519***	-0.897891***	0.453476	761
Philippines post-crisis					
10Y	-1.83E-05	1.44E-05***	-0.973819***	0.528773	919
15Y	-2.46E-05	0.000486***	-0.999035***	0.652513	1498
20Y	-1.45E-05	0.000621***	-0.964019***	0.601138	1290
Russia pre-crisis					
15Y	-5.22E-06	0.001060***	-0.650849***	0.304584	1128
Russia post-crisis					
15Y	-3.96E-05	0.000980***	-0.991912***	0.564024	1438

Note: *** indicates significant at 1% level.

Table 3: Generalized method of moments estimates of the DSR model using month-end zero-coupon yield to maturity data of 3-month, 12-month and 10-year US Treasury bills and notes from January 2000 to September 2014

Parameters	κ_r	σ_r^2	λ_r
Point estimate	0.0176	0.0072	-0.0686
Standard errors	0.0073	0.0003	0.00276

Note: The data are from the US Federal Reserve. The standard errors are computed by the Newey and West heteroskedasticity and autocorrelation-consistent estimate of the covariance matrix of the yields.

Table 4: Levels of default barriers.

	Pre-crisis period		Post-crisis period	
	Calibrated Barrier	Reference Barrier	Calibrated Barrier	Reference Barrier
Brazil				
10Y	3.64	3.2	3.68	3.2
15Y	3.31	3.2	3.64	3.2
20Y	3.11	3.2	3.17	3.2
30Y	2.77	3.2	2.91	3.2
Mexico				
10Y	15.7	16	23.7	16
15Y	15.2	16	22.6	16
20Y	14.94	16	20.4	16
30Y	13.77	16	18.7	16
Turkey				
10Y	2.1	1.86	2.86	1.86/Change
15Y	1.89	1.86	2.61	1.86/Change
20Y	1.93	1.86	2.52	1.86/Change
30Y	1.6	1.86	2.29	1.86/Change
Colombia				
10Y	3400	2900	3410	2900
20Y			2650	2900
30Y	2690	2900	2600	2900
Philippines				
10Y			53.7	57
15Y	58	57	53.3	57
20Y			53.3	57
Russia				
15Y	35.1	31	44.1	40

Note: For Turkey, 1.86/Change denotes that the reference barrier is at level of 1.86 but changes during the sample period as the exchange rate exceeds 1.86.

Table 5: Correlations between market and model credit spreads in pre-crisis (1 June 2003 - 31 December 2007) and post-crisis (1 January 2009 - 29 September 2014) periods. Log change is ln(CS_t/CS_{t-1}).

	Pre-crisis Period				Post-crisis Period			
	Calibrated Barrier		Reference Barrier		Calibrated Barrier		Reference Barrier	
	Level	Log change	Level	Log change	Level	Log change	Level	Log change
Brazil								
10Y	0.959539	0.523275	0.959598	0.520982	0.908860	0.379767	0.868848	0.397125
15Y	0.953949	0.482760	0.952600	0.487191	0.830241	0.303421	0.800445	0.309539
20Y	0.921730	0.508187	0.926139	0.501518	0.893335	0.320706	0.901480	0.316430
30Y	0.882142	0.536708	0.899338	0.514740	0.847727	0.390204	0.871172	0.384474
Mexico								
10Y	0.552880	0.438373	0.549319	0.437605	0.847623	0.199990	0.799435	0.234820
15Y	0.527560	0.326232	0.511315	0.325294	0.650370	0.210405	0.721163	0.240200
20Y	0.418776	0.275607	0.397264	0.275836	0.785107	0.341452	0.785477	0.374332
30Y	0.504708	0.350846	0.489280	0.359946	0.747097	0.427725	0.746418	0.457096
Turkey								
10Y	0.514546	0.089379	0.483176	0.129808	0.521127	0.140926	0.482898	0.318999
15Y	0.418965	0.424573	0.419699	0.272989	0.507177	0.234234	0.568609	0.372451
20Y	0.277704	0.426114	0.284289	0.243198	0.546733	0.259238	0.645104	0.374980
30Y	0.371978	0.443446	0.443260	0.256475	0.269549	0.301469	0.405902	0.352574
Colombia								
10Y	0.749448	0.108970	0.820612	0.163759	0.905720	0.208497	0.90707	0.237631
20Y					0.862308	0.216809	0.842845	0.202812
30Y	0.680392	0.285348	0.643841	0.217847	0.908509	0.298449	0.887793	0.267469
Philippines								
10Y					0.733676	0.225239	0.756206	0.217695
15Y	0.892079	0.266252	0.897575	0.272526	0.807720	0.224934	0.829884	0.211198
20Y					0.748286	0.244271	0.767511	0.228892
Russia								
15Y	0.735777	0.03452	0.762646	0.072708	0.772496	0.588978	0.696061	0.579182

Table 6: Performances of the model in pre-crisis (1 June 2003 - 31 December 2007) and post-crisis (1 January 2009 - 29 September 2014) periods.

	Pre-crisis period			Post-crisis period		
	RMS error (basis points)	Percentage error	Absolute percentage error	RMS error (basis points)	Percentage error	Absolute Percentage error
Brazil		(Reference barrier)			(Reference barrier)	
10Y	263.43	48.47%	34.63%	115.78	14.92%	28.67%
		(49.75%)	(32.76%)		(31.94%)	(20.52%)
15Y	72.03	10.34%	17.69%	61.7	14.74%	27.03%
		(17.53%)	(10.06%)		(33.75%)	(25.01%)
20Y	78.95	-6.596%	13.37%	32.33	7.099%	22.45%
		(15.62%)	(10.43%)		(25.78%)	(14.52%)
30Y	164.57	-33.67%	33.67%	49.39	-19.71%	16.74%
		(10.81%)	(10.81%)		(17.66%)	(13.64%)
Aggregate	162.18	1.064%	21.49%	102.66	6.267%	24.27%
		(27.39%)	(18.32%)		(28.68%)	(19.39%)
Brazil		(Calibrated barrier)			(Calibrated barrier)	
10Y	67.33	-11.18%	25.95%	51.03	-10.63%	34.01%
		(22.50%)	(17.09%)		(44.77%)	(30.99%)
15Y	56.18	-1.792%	13.21%	46.37	-8.041%	23.18%
		(15.93%)	(9.078%)		(27.79%)	(17.32%)
20Y	71.6	2.039%	14.81%	32.01	-4.019%	15.11%
		(17.38%)	(9.326%)		(19.23%)	(12.56%)
30Y	80.69	-0.267%	14.33%	34.47	-0.692%	23.27%
		(17.27%)	(9.651%)		(21.59%)	(15.28%)
Aggregate	69.84	-2.592%	16.92%	38.93	-6.217%	23.43%
		(18.37%)	(11.65%)		(35.03%)	(20.42%)
Mexico		(Reference barrier)			(Reference barrier)	
10Y	58.05	-18.48%	27.86%	443.51	311.8%	311.8%
		(28.37%)	(19.25%)		(97.14%)	(97.14%)
15Y	59.7	-22.62%	30.21%	287.54	222.0%	222.0%
		(25.54%)	(15.87%)		(110.3%)	(110.3%)
20Y	60.8	-25.97%	30.79%	176.3	103.6%	103.6%
		(21.47%)	(13.70%)		(34.86%)	(34.86%)
30Y	89.11	-45.89%	45.89%	89.23	55.08%	55.55%
		(12.66%)	(12.66%)		(29.03%)	(28.12%)
Aggregate	68.79	-28.62%	36.99%	279.01	173.9%	171.2%
		(22.72%)	(15.58%)		(78.19%)	(76.51%)
Mexico		(Calibrated barrier)			(Calibrated barrier)	
10Y	42.87	-5.848%	24.79%	56.94	-17.89%	36.76%
		(30.75%)	(19.10%)		(44.10%)	(20.24%)
15Y	45.49	4.349%	24.76%	60.48	0.884%	33.96%
		(31.37%)	(19.75%)		(26.26%)	(31.42%)
20Y	40.41	2.398%	19.29%	39.6	-1.700%	19.28%
		(26.33%)	(18.07%)		(24.17%)	(14.68%)
30Y	37.85	-3.782%	15.03%	37.75	3.046%	16.77%
		(18.09%)	(10.76%)		(20.85%)	(12.76%)
Aggregate	41.80	-0.784%	20.97%	49.62	-3.701%	26.55%
		(27.54%)	(17.21%)		(29.96%)	(22.68%)

	Pre-crisis period			Post-crisis period		
	RMS error (basis points)	Percentage error	Absolute percentage error	RMS error (basis points)	Percentage error	Absolute Percentage error
Turkey		(Reference barrier)			(Reference barrier)	
10Y	108.8	53.61%	56.19%	650.17	264.5%	264.5%
		(62.85%)	(60.55%)		(103.2%)	(103.2%)
15Y	120.55	10.63%	30.29%	359.45	127.7%	127.7%
		(39.20%)	(27.05%)		(54.73%)	(54.73%)
20Y	77.16	7.874%	25.75%	235.41	92.63%	92.64%
		(29.54%)	(16.48%)		(38.05%)	(38.04%)
30Y	137.95	-31.22%	31.44%	112.72	36.97%	41.98%
		(16.20%)	(15.77%)		(33.10%)	(26.46%)
Aggregate	116.69	7.670%	35.67%	393.7	130.4%	131.7%
		(39.86%)	(34.24%)		(63.60%)	(62.83%)
Turkey		(Calibrated barrier)			(Calibrated barrier)	
10Y	133.55	-2.822%	36.08%	126.95	-22.55%	46.40%
		(48.29%)	(32.21%)		(47.54%)	(24.80%)
15Y	120.22	3.425%	30.09%	112.3	-15.91%	35.89%
		(37.57%)	(22.74%)		(38.37%)	(20.89%)
20Y	73.97	-2.844%	24.71%	88.71	-10.87%	28.54%
		(28.13%)	(13.72%)		(31.61%)	(17.39%)
30Y	93.54	1.549%	18.91%	107.86	-9.26%	29.27%
		(23.07%)	(13.31%)		(33.02%)	(17.88%)
Aggregate	108.9	0.275%	27.18%	109.82	-14.65%	35.03%
		(35.32%)	(21.78%)		(38.15%)	(20.45%)
Colombia		(Reference barrier)			(Reference barrier)	
10Y	325.88	96.95%	96.95%	153.48	20.55%	52.21%
		(54.26%)	(54.26%)		(60.43%)	(36.73%)
20Y	NIL	NIL	NIL	91.16	-30.98%	34.74%
					(26.87%)	(21.79%)
30Y	93.69	-18.54%	23.02%	58.86	-22.78%	24.03%
		(21.22%)	(16.26%)		(16.63%)	(14.76%)
Aggregate	234.7	36.35%	58.16%	107.25	-10.89%	36.45%
		(40.44%)	(39.22%)		(38.98%)	(25.80%)
Colombia		(Calibrated barrier)			(Calibrated barrier)	
10Y	109.68	-2.256%	31.49%	95.3	-37.32%	52.12%
		(38.71%)	(22.61%)		(46.69%)	(29.28%)
20Y	NIL	NIL	NIL	70.72	-8.014%	24.99%
					(27.75%)	(14.47%)
30Y	86.91	2.431%	21.98%	33.9	4.364%	14.51%
		(25.03%)	(12.21%)		(18.09%)	(11.65%)
Aggregate	98.39	0.203%	26.50%	69.93	-12.99%	29.97%
		(32.26%)	(17.92%)		(32.67%)	(19.88%)
Philippines		(Reference barrier)			(Reference barrier)	
10Y	NIL	NIL	NIL	84.05	-42.38%	46.06%
					(31.29%)	(25.56%)
15Y	67.73	9.111%	19.55%	87.93	-36.16%	37.73%
		(21.83%)	(13.32%)		(25.29%)	(22.89%)
20Y	NIL	NIL	NIL	59.77	-31.09%	32.57%
					(22.97%)	(20.82%)
Aggregate	67.73	9.111%	19.55%	78.23	-35.94%	38.00%
		(21.83%)	(13.32%)		(26.17%)	(22.90%)

	Pre-crisis period			Post-crisis period		
	RMS error (basis points)	Percentage error	Absolute percentage error	RMS error (basis points)	Percentage error	Absolute Percentage error
Philippines		(Calibrated barrier)			(Calibrated barrier)	
10Y	NIL	NIL	NIL	61.29	-24.04%	34.67%
					(34.04%)	(23.12%)
15Y	58.68	-1.219%	16.49%	62.19	-6.698%	24.55%
		(20.78%)	(12.70%)		(29.42%)	(17.54%)
20Y	NIL	NIL	NIL	41.71	-3.570%	21.32%
					(25.86%)	(15.07%)
Aggregate	58.68	-1.219%	16.49%	55.68	-9.911%	25.94%
		(20.78%)	(12.70%)		(29.49%)	(18.33%)
Russia		(Reference barrier)			(Reference barrier)	
15Y	272.45	121.8%	122.1%	135.12	38.52%	40.59%
		(73.15%)	(72.63%)		(33.16%)	(30.58%)
Russia		(Calibrated barrier)			(Calibrated barrier)	
15Y	68.74	-8.773%	32.31%	71.48	-3.048%	19.00%
		(38.42%)	(22.57%)		(23.77%)	(14.61%)

Note: The numbers in parentheses are the standard deviations of the errors.