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BETWEEN iTRAXX EUROPE FINANCIAL AND  
CORPORATE SUB-INDEXES**

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# Estimating Option-Implied Correlation between iTraxx Europe Financial and Corporate Sub-Indexes

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## Abstract

This paper proposes a model to estimate option-implied correlation embedded in options on the iTraxx Europe indexes as a measure of the spillover effect of default risk between the financial and corporate sectors in Europe. The correlation structure between the iTraxx Financials and Non-Financials sub-indexes are reflected in the option on the iTraxx Main Index, which is simultaneously considered as a basket option with the two sub-indexes as two correlated underlyings. The spillover effect was found to be more severe during the second half of year 2011, and the abrupt changes of the realized correlation between the CDS indexes of the financial and corporate sectors anticipated information of the corresponding option prices. The sovereign default risk, funding liquidity risk, level of risk aversion, and equity market performance were found to be significant determinants of the option-implied correlation, implying inter-dependence amongst various markets during the European debt crisis.

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# 1. Introduction

A reason for the emergence of studies on correlation between financial variables over the past few years is that correlation has been widely used as an important measure to assess the possible spillover effect between two markets or sectors. However, such correlation estimated by using historical data (realized correlation) is backward looking. To overcome this deficiency, options can be used given that they may provide interesting additional information not contained in the historical data. Options have the desirable property of being forward-looking in nature and thus are a useful source of information for gauging market sentiment about future prices of financial assets and their dynamics. Studies on implied correlation measures are relatively limited and they focus on currency options [see Bodurtha and Shen (1995), Siegel (1997), Campa and Chang (1998), and Walter and Lopez (2000)] and equity index options [see Skintzi and Refenes (2005) and Driessen et al. (2009)]. Some of their findings show that implied correlations are useful for gauging future correlations. This paper extends the studies to the credit derivatives market and proposes a model to estimate the option-implied correlation from credit default swap (CDS) index options to measure the spillover effect of default risk between the financial and corporate sectors in Europe.

A CDS index contract is an insurance contract covering default risk of the pool of names in the index and traded over-the-counter (OTC). The CDS index used in this paper is the iTraxx Europe Main Index which consists of 125 equally weighted underlying CDS contracts with the highest liquidity on selected European entities.<sup>1</sup> The Main Index is further divided into two sub-indexes with 25 financial entities in the Financials Index, and the other 100 corporate entities in the Non-Financials Index. In effect the two sub-indexes reflect market perspectives about the financial and corporate sectors in Europe. The daily transactions of the Main Index and Financials Index at the 5-year tenor were about 8-17bn euros and 2-5bn euros respectively in January 2012.<sup>2</sup> Options on these two indexes are also traded in the OTC market, with a daily trading volume of about 2bn euros in 2011.<sup>3</sup> The transaction figures show that the markets of the iTraxx Indexes and the corresponding options are both adequately liquid.

While the options on the iTraxx Main and Financials Indexes have daily quotes from the market based on the Black (1976) option pricing model, the price information of options on the Non-Financials Index and the option-implied correlation between the two sub-indexes are missing. Following the Black model, this paper proposes a simple stochastic model for the three iTraxx Indexes and derives closed-form formulae to estimate those two missing quantities. We assume that the correlation

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<sup>1</sup> A new series of indexes is established every six months in March and September with a new underlying portfolio and maturity date, to reflect changes in the credit market and to help investors maintain a relatively constant duration if they wish. Priority is given to credits that appeared in the previous series in order to minimize composition differences between consecutive series. If there is a credit event in an underlying CDS, the credit is effectively removed from the indexes in which it is included.

<sup>2</sup> The figures are reported by Markit. The iTraxx indexes are owned, managed, compiled, and published by Markit Group Limited.

<sup>3</sup> Private communication with JP Morgan.

structure between the Financials and Non-Financials sub-indexes are being reflected in the option on the Main Index, which is simultaneously considered as a basket option with the two sub-indexes as two correlated underlyings. The derived option-implied correlation is then considered as a measure to gauge the spillover effect of default risk between the financial and corporate sectors in Europe. Then we further examine whether the realized correlations between the two sub-indexes anticipate information of their options. And finally, we investigate the determinants of the option-implied correlation and its movements during the European debt crisis.

In 2011, the European debt crisis intensified the negative feedback loop between the troubled sovereigns and the banking sector.<sup>4</sup> European banks are major holders of sovereign government bonds. The decline in the value of these bonds has weakened banks' capital positions, forcing many out of the funding market. CDS spreads for many European banks surpassed the highs reached post the Lehman collapse reflecting concerns about their health. The shrinking capital base would ultimately see some of these banks seeking bailout from their governments, bringing more debt onto the governments' balance sheets. Such a vicious cycle of debt-infused credit deterioration between the sovereigns and banks became more apparent in the third quarter of 2011 and was seen as a key channel through which the crisis could spread into a banking crisis.

It is widely accepted that banking crises have adverse consequences for the economy. If the banking system is the key institution allowing credit constraints to be relaxed during normal periods as argued by Rajan and Zingales (1998), then a sudden negative shock to the banking system should have a disproportionately contractionary impact on the corporate sector whose growth depends on the funding provided by banks. Kroszner et al. (2007) provide evidence on the mechanisms linking the financial and real (corporate) sectors in a financial crisis. Therefore, such spillover of default risk between the two sectors may trigger systemic defaults within Europe. The iTraxx option-implied correlation derived in this paper could provide a useful measure to gauge such spillover effect, if the realized correlations between the two sub-indexes is found to anticipate information of their options.

This paper is structured as follows. Section 2 develops the model to estimate option-implied correlation between the Financials and Non-Financials Indexes. Section 3 examines the information transmission from the option-implied correlation to the realized correlation between the two sub-indexes. Section 4 identifies the major determinants of the option-implied correlation and studies their contemporaneous interactions during the European debt crisis. Section 5 contains the conclusion.

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<sup>4</sup> After the new Greek government took office in October 2009, the size of the deficit was revealed to be at 12.7% of the GDP in 2009, with public debts projected to rise to 135% of the GDP by 2011. Greece's problems have laid bare the dangers of divergent fiscal policies in the eurozone. Such dangers may induce economic-political events (e.g., substantial restructuring or even default of sovereign debt) and may also cause contagion to the other members with weaker fiscal positions such as Portugal, Italy, Ireland, and Spain. While Greece accounts for only 1.4% of foreign claims in European banks, economies such as Portugal, Ireland, Italy, and Spain, which have had similar fiscal problems as a whole, accounted for 15.4% in September 2009. The figures are from the Bank for International Settlements. European banks refer to domestically owned banks of Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, and the UK.

## 2. Implied Correlation from iTraxx Options

As only the implied volatilities of the iTraxx Europe Main Index and Financials Index have daily quotes from the market, one needs to adopt an appropriate stochastic model to estimate the implied volatilities of iTraxx Non-Financials Index as well as the implied correlations between Financials and Non-Financials. Based on the Black (1976) pricing formula from which market iTraxx options are priced (see JP Morgan (2006) for details), it is natural to assume the underlying CDS spreads are lognormally distributed with zero drift under the risk-neutral probability measure.<sup>5</sup> This assumption is consistent with that used in Hull and White (2003) and Pan and Singleton (2008). Hull and White assume a lognormal distribution of corporate CDS spreads for pricing CDS options. Pan and Singleton use the conditional distribution of the sovereign CDS spreads derived from the conditional lognormal distribution of the state to estimate the corresponding term structures. We denote the CDS spreads of iTraxx Main, Financials and Non-Financials by  $I$ ,  $A$  and  $B$  respectively. They are approximately related by:

$$D_I I \simeq \omega_A D_A A + \omega_B D_B B$$

where  $D_{I/A/B}$  are the quoted durations and  $\omega_{A/B}$  are the weightings of Financials and Non-Financials sub-indexes within the Main Index. For the iTraxx Europe case, we have  $\omega_A = 0.2$  and  $\omega_B = 0.8$ . We further introduce the effective weightings,  $\tilde{\omega}_A = \omega_A D_A / D_I$  and  $\tilde{\omega}_B = \omega_B D_B / D_I$ , such that the above relation takes a simpler form:

$$I = \tilde{\omega}_A A + \tilde{\omega}_B B \quad (1)$$

Since the implied volatilities of  $I$  and  $A$  are directly quoted from the market (to be labeled as  $\sigma_I$  and  $\sigma_A$  respectively), we assume the following zero drift lognormal processes:

$$\begin{aligned} dI &= \sigma_I I dW_I \\ dA &= \sigma_A A dW_A \end{aligned} \quad (2)$$

where  $dW_{I/A}$  are two standard Wiener processes. Then, we further assume:

$$dB = \sigma_B (\alpha \tilde{\omega}_A A + \tilde{\omega}_B B) dW_B \quad (3)$$

<sup>5</sup> Theoretically, the underlying of Black's (1976) model is forward price rather than spot price. But in our case the tenors of quoted options (typically two months) are much shorter than that of the underlying CDS contracts (5-years), and therefore the 2M5Y forward CDS spread is effectively the same as the spot CDS spread.

where  $dW_B$  is another Wiener process, which is correlated to  $dW_A$  through  $\mathbb{E}[dW_A dW_B] = \rho dt$ . This assumption is based upon Lo (2012), which has shown that the dynamics of the difference of two lognormal stochastic variables is described by a 'shifted lognormal process'. In this case the shifting is governed by an arbitrary parameter  $\alpha$ , which serves for the purpose of calibration. And  $\rho$  is the key unknown parameter, namely the implied correlation that we aim to estimate in this study.

Following the derivations in the appendix, one obtains closed-form approximate formulae for  $\sigma_B$  and  $\rho$ , as given by:

$$\sigma_B = \frac{\sqrt{\tilde{\omega}_A^2 \sigma_A^2 + [(2\alpha - 1)\tilde{\omega}_A + \tilde{\omega}_B] \sigma_I^2}}{\tilde{\omega}_B (\alpha \tilde{\omega}_A + \tilde{\omega}_B)} \quad (4)$$

$$\rho = \frac{(1 - \alpha) \sigma_I^2 - \tilde{\omega}_A^2 \sigma_A^2}{\tilde{\omega}_B \sigma_A \sigma_B (\alpha \tilde{\omega}_A + \tilde{\omega}_B)} \quad (5)$$

where the optimal value of  $\alpha$  is calibrated by the following procedures. First, we attempt to compare the implied volatilities of  $B$  with the market quoted implied volatilities of  $A$  and  $I$  to see if the estimates are reasonable. However, the estimates of  $\sigma_B$  are not directly comparable with  $\sigma_{I/A}$  as they do not play the same role within their respective stochastic processes. In view of the stochastic differential equation defined for  $B$  in equation (3), we introduce a scaled implied volatility of  $B$ :

$$\sigma_B^{scaled} = \sigma_B \left( \frac{\alpha \tilde{\omega}_A A + \tilde{\omega}_B B}{B} \right) \quad (6)$$

to facilitate a more sensible comparison with  $\sigma_{I/A}$ . It is reasonable to have  $\sigma_B^{scaled}$  staying in between  $\sigma_A$  and  $\sigma_I$  as iTraxx Main is a more diversified portfolio of CDS contracts which must have smaller volatilities than its sub-indexes (see Markowitz(1952) for risk diversification). Also, the volatilities of  $B$  were expected to be smaller than those of  $A$ , as anticipated by the market when default risk of financial institutions has increased during the European debt crisis. By enforcing the inequality  $\sigma_I < \sigma_B^{scaled} < \sigma_A$  to hold true, one may solve two non-linear equations:

$$\sigma_B^2 \left( \frac{\alpha \tilde{\omega}_A A + \tilde{\omega}_B B}{B} \right)^2 = \sigma_{I/A}^2$$

for the unknown parameter  $\alpha$ , to obtain a lower and upper bound estimate. In our study, the lower and upper bounds were found to be 0.08 and 0.28 respectively, and taking a simple average yields our final estimate of  $\alpha = 0.18$ . In fact, one may solve the two non-linear equations on a daily basis to

obtain different estimates for  $\alpha$ . However, the fluctuations of the resultant  $\alpha$  are minimal and its choice is relatively robust against changes in the market environment. One may confidently adopt the same  $\alpha$  value for a considerable period once it is determined. Despite the allowed arbitrariness of  $\alpha$ , its actual choice only has a translational effect to the overall level of the implied correlation. In our subsequent analysis, we mainly focus on changes instead of levels of the implied correlation.

The estimates of  $\sigma_B^{scaled}$  using  $\alpha = 0.18$  compared with the market quotes of  $\sigma_I$  and  $\sigma_A$  are presented in Figure 1. It appears that the estimates of  $\sigma_B^{scaled}$  contain no more market information than those already being reflected in the market quotes of  $\sigma_I$  and  $\sigma_A$ . In fact, the primary purpose of introducing  $\sigma_B^{scaled}$  is to connect the three stochastic variables  $A, B$  and  $I$  in a consistent manner, such that the implied correlation between  $A$  and  $B$  may be estimated on a daily basis. It is necessary because the daily information quoted from the iTraxx option market are limited. The key rationale was to assume that the correlation structure between  $A$  and  $B$  are being reflected in the option on  $I$ , which is simultaneously considered as a basket option with  $A$  and  $B$  as two correlated underlyings.

In Figure 2, the daily implied correlations using equation (5) with  $\alpha = 0.18$  are compared with the realized correlations. We adopt the dynamic conditional correlation multivariate GARCH model (to be labelled as DCC\_GARCH hereafter) proposed by Engle and Sheppard (2001) to estimate the realized correlations between the time series of  $A$  and  $B$ . It appears that the implied correlations do more or less capture the approximate level and trend of the realized correlations. The descriptive statistics of the two time series are presented in Table 1. However, it also exhibits some lead-lag relationships between the two series, which is studied in the next section.

### 3. Lead-lag Information Flow from Option-implied Correlation to Realized Correlation

In this section, we first perform statistical tests to study the lead-lag relation between the iTraxx option-implied correlations derived in the previous section and the realized correlations estimated by the DCC\_GARCH model. Then we investigate the information transmission between the iTraxx Europe CDS market and the corresponding index option market to examine whether the realized correlations between the underlying CDS indexes anticipate information of the implied correlations from the option prices.

We obtain the daily CDS spreads of the iTraxx Europe Main Index ( $I$ ) and the CDS spreads of its two compositions, namely the Financials ( $A$ ) and the Non-Financials ( $B$ ) Indexes, from 3 January 2011 to 16 March 2012.<sup>6</sup> All the CDS spreads are quoted for the 5-year on-the-run iTraxx series as they are commonly used as benchmarks of the European CDS market. Next we obtain the daily option-implied

<sup>6</sup> All iTraxx data are collected from JP Morgan DataQuery.

volatilities of at-the-money (ATM) options on the iTraxx Europe Main Index ( $\sigma_I$ ) and Financials Index ( $\sigma_A$ ). The tenors of all the quoted options are typically 2-months, which last much shorter than the 5-years tenors of their underlying CDS index. We choose the options with relatively short tenors because they tend to contain more direct information of market expectation in the forthcoming period, yet our estimation for the implied correlation will be more accurate for shorter tenors too.

We perform the Granger causality test (GC test) to assess preliminarily the lead-lag relationship between the option-implied correlations and the realized correlations estimated by the DCC\_GARCH model. The GC test is a statistical hypothesis test for determining whether one time series is useful in forecasting another time series. Specifically, it uses an F-test to check if the coefficients of one time series in the preceding periods are significant in the regression of the other in the current period. If the lead-lag relationship is clear, one would have rejected the null hypothesis that 'the coefficients are jointly equal to zero' and conclude that one time series 'Granger causes' the other. We use weekly data to perform the GC test to avoid abrupt movements in the daily time series. And more importantly, the typical 2-months tenors of quoted options contain leading information, if any, of the forthcoming weeks rather than days. The GC test results with up to 4-week lags are presented in Table 2. The results suggest that the option-implied correlations 'Granger cause' the realized correlations, but not the other way round.

We further examine the sample cross correlation function (XCF) between the two time series. The XCF is a direct measure of the correlations between one time series in the preceding periods and the other in the current period. The XCF results in Table 3 confirm that the lead-lag relationship is significantly stronger in one way than the other, implying one-way information flow from the option-implied correlations to the realized correlations.

The preliminary results above reveal potential information flow from the option-implied correlations to the realized correlations. Following the methodology employed by Acharya and Johnson (2007) to examine information flow from the corporate CDS market to the equity market, we attempt to address a similar issue between the iTraxx Europe CDS market and the corresponding option market, using the two correlation time series mentioned above. The first step is to regress option-implied correlations on contemporaneous realized correlations to filter out the residual components. Lags of the option-implied correlations are also included in the regression to absorb any lagged information transmission within the iTraxx option market. The regression reads:

$$\rho_t^{imp} = a + b\rho_t^{real} + \sum_{k=1}^4 c_k \rho_{t-k}^{imp} + u_t \quad (7)$$

where  $u_t$  are the residuals of the regression, being interpreted as the innovations of the iTraxx option market about the correlations between the financial and corporate sectors in Europe, that is not

appreciated by the actual iTraxx CDS market at the time. Next we regress the realized correlations onto the innovations terms  $u_t$  and their lags to study the proposed information flow:

$$\rho_t^{real} = \alpha + \sum_{k=1}^4 \beta_k u_{t-k} + \sum_{k=1}^4 \gamma_k \rho_{t-k}^{real} + \varepsilon_t \quad (8)$$

in which the lags of  $\rho^{real}$  are also included to isolate any lagged influences within the iTraxx CDS market itself. The point estimate  $\mathbf{I} = \sum_{k=1}^4 \beta_k$  is used as a measure of permanent information flow from the iTraxx option market to its underlying iTraxx CDS market. If the information flow is large and significant,  $\mathbf{I}$  should be significantly positive. Similarly, one may swap the positions of  $\rho_t^{imp}$  and  $\rho_t^{real}$  to examine the reverse information flow from the iTraxx CDS market to the iTraxx option market. The estimated value of  $\mathbf{I}$ , and its statistical significance checked by the Wald test are presented in Table 4.

The results from Table 4 further confirm that there is a robust one-way information flow from the iTraxx option market to its underlying iTraxx CDS market, as the measure of  $\mathbf{I}$  is significantly positive for one way but not the other way round. And the leading time is about four weeks. This means that the option-implied correlation embeds information about future movements of the iTraxx CDS indexes, and thus is a useful forward-looking indicator to gauge the spillover effect of default risk between the financial and corporate sectors in Europe.

#### 4. Determinants of Option-implied Correlation and its Dynamics in the European Debt Crisis

The previous section provides statistical evidence of information flow from the iTraxx Europe option market to its underlying iTraxx CDS market, and in particular the information about correlation between the financial and corporate sectors within the iTraxx Europe Main Index. As pointed out by Heidorn and Kahlert (2010), the implied base correlations of iTraxx tranches provide a measure of market systemic risk within the European credit markets. Here we use the correlation implied from iTraxx options to investigate a similar issue related to the market systemic risk, namely the spillover effect of default risk between the financial the corporate sectors.

Recent research finds that the CDS markets interact strongly with the equity and fixed income markets. Acharya and Johnson (2007) find asymmetric information flow from the corporate CDS market to the equity market due to insider trading. Norden and Weber (2004) study the interactions amongst CDS, bond and equity markets in the period 2000-2002. In contrast to Acharya and Johnson (2007), they find leading information in the stock market. Longstaff et al.(2003) study the lead-lag relations amongst the changes in CDS spreads, corporate bond spreads and stock returns in a vector

autoregression (VAR) framework. They find lags in the corporate bond market behind the stock and CDS markets. More recently, Pan and Singleton (2008) find that the sovereign CDS spreads are related to global risk appetite, market volatility, and macroeconomic policy. Acharya and Schaefer (2006) provide a link between liquidity risk and correlation risk. Longstaff et al. (2011) show that sovereign CDS spreads are primarily driven by the equity and high-yield bond markets.

To understand better the contemporaneous interactions between the iTraxx CDS index option market and other markets, we use a regression analysis to identify the major determinants of the iTraxx option-implied correlation. As inspired by previous research, we choose to examine four macro-financial variables:

**i) Sovereign Default Risk** We use the CDS spread of the iTraxx SovX Western Europe Index to measure the credit default risk of European sovereign debt. The iTraxx SovX Western Europe Index is another iTraxx CDS index covering the sovereign CDS of major countries in Western Europe.<sup>7</sup> During the European debt crisis, the sovereign CDS spread is one of the key determinants driving all other markets within the eurozone because it is a direct measure of expected fiscal conditions of the corresponding European countries.

**ii) Funding Liquidity Risk** We use the swap spread to measure the level of funding liquidity risk, which is defined as the difference between the interbank swap rate and the yield of government bonds.<sup>8</sup> As studied by Duffie and Singleton (1997), the market prices of the swap spread contain information about the funding liquidity risk which is determined by both default and market liquidity risks. In times of tight funding environments, the cost of interbank borrowing increases and therefore banks are more reluctant to finance the corporate sector, or they simply require a higher compensation from the corporations to provide funding. As a result, the liquidity risk within the financial sector, being gauged by the swap spread, would potentially have spillover effects on the corporate sector. So it is reasonable to include the swap spread as another explanatory variable to the iTraxx option-implied correlation.

**iii) Risk Aversion** We use the VSTOXX volatility index to gauge the level of risk aversion within the eurozone, which is based on equity option-implied volatilities and is primarily designed to measure market expectations of volatility in the equity market.<sup>9</sup> The iTraxx option-implied correlation shares commonality with the VSTOXX index as a measure of investors' aversion to systemic risk and hence their willingness to put capital at risk.

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<sup>7</sup> The iTraxx SovX Western Europe is a tradable index composed of 15 equally weighted constituents selected from the pool of eurozone countries plus Denmark, Norway, Sweden and the United Kingdom. The selection of constituents is done every six months based on their sum of weekly trading activity.

<sup>8</sup> In particular we use the euro 5-year term swap spot rate (vs 6-month Euribor) to subtract the euro 5-year government benchmark bond yield. All rates are collected from JP Morgan DataQuery.

<sup>9</sup> VSTOXX data are collected from Bloomberg. The VSTOXX index is based on the EURO STOXX 50 index options traded at the Eurex and is designed to reflect market expectations of near-term up to long-term volatility in the eurozone equity market. It measures implied volatility on equity options across all maturities.

**iv) Equity Market** Regarding the equity market variable, we use the STOXX Europe 600 Index (STOXX 600).<sup>10</sup> The performance of the equity market reflects directly the economic outlook within the eurozone. The equity market index normally moves in opposite direction to the volatility index (i.e., risk aversion in our context), so we expect a negative relation between the STOXX 600 Index and the iTraxx option-implied correlation.

Figure 4 illustrates the trajectories of the iTraxx option-implied correlation and the four control variables mentioned above. After incorporating the above variables into the regression, we propose:

$$\Delta\rho_t^{imp} = a + \beta_1\Delta\text{SovX}_t + \beta_2\Delta\text{SS}_t + \beta_3\Delta\text{VSTOXX}_t + \beta_4\Delta\text{Equity}_t + \sum_{k=1}^2 \gamma_k \Delta\rho_{t-k}^{imp} + \varepsilon_t \quad (9)$$

where  $\varepsilon_t$  is the residual of the regression. We use different specifications of equation (9) to isolate the potential effects of multicollinearity. The regression analysis for weekly changes of the implied correlation is presented in Table 5. Two lags of the dependent variable are included to account for any information transmission within the iTraxx option market.<sup>11</sup>

It was found that the CDS spread of the iTraxx SovX Western Europe Index and the swap spread are the major determinants of the option-implied correlation. When they are regressed with the weekly changes of option-implied correlation (right column on Table 5), their coefficients both turn out to be 5% significant and attribute an adjusted  $R^2$  of 31.2% to the regression. Not surprisingly, the sovereign default risk and funding liquidity risk are the key driving factors behind the spillover effect during the European debt crisis period of our sample.

When the equity and VSTOXX variables are considered separately (the two columns in the middle of Table 5), they are both identified as significant determinants, while the equity attains a negative coefficient and the VSTOXX attains a positive coefficient. This aligns with our expectations. However, when the equity and VSTOXX variables are pooled into a single regression, the equity variable turns out to be insignificant. This can be explained by their strong collinearity within the sample period.<sup>12</sup> Altogether the four determinants attribute an adjusted  $R^2$  of 35.5% to the regression of weekly changes.

<sup>10</sup> The STOXX Europe 600 index is derived from the STOXX Europe Total Market Index (TMI) and is a subset of the STOXX Global 1800 index. The 600 components are fixed and represent large, medium and small capitalisation companies across 18 countries of the European region.

<sup>11</sup> The second lag of the dependent variable was found to be 1% significant for all cases, whereas the third lag onwards were insignificant to the regression.

<sup>12</sup> The correlation between STOXX 600 and VSTOXX is estimated to be as large as -0.78 for the sample period.

In summary, the regression results support the significant explanatory powers of the four determinants. The spillover effect of default risk between the financial and corporate sectors is evidently driven by the funding liquidity risk, risk aversion level and the development of sovereign default risk. Below we attempt to analyze the dynamics of the option-implied correlation in response to the development of the European debt crisis.

In mid-June 2011, it was expected that Greece might default on its debt and become the first country to leave the eurozone. Figure 2 shows that the iTraxx option-implied correlation jumped from about 0.6 to over 0.7. However, the realized correlation fell, while the CDS spreads of iTraxx Europe Financials and Non-Financials Indexes did not move much as shown in Figure 3. They only reacted to the development of the crisis in July. This reflects that the iTraxx option market anticipated differently the spillover effect between the financial and corporate sectors compared with the corresponding iTraxx CDS market.

Then in early August, there were concerns about the spread of sovereign default to the larger economies like Italy and Spain. The worry was transmitted to global equity markets causing a sharp fall in share prices. At the same time, the iTraxx option-implied correlation surged to 0.8 and then hovered around that level until early November. This shows that the spillover of solvency risk between the financial and corporate sectors deepened rapidly. Figure 2 shows that the realized correlation broadly followed the movements of the option-implied correlation from August to October 2011.

In October, eurozone ministers approved another bailout loan for Greece, potentially rescuing the country from default. European leaders also agreed on new measures to boost the main bailout fund to 1 trillion euros, and to raise capital requirements of banks to minimize the negative effect under potential stressed scenarios. The option-implied correlation dropped from 0.85 in early November to below 0.7 in one month, indicating that the measures taken by the euro-area countries could mitigate to a certain extent the contagion risk of the financial and corporate sectors.<sup>13</sup> The option-implied correlation then fluctuated around the level of 0.7, which remains higher than most of the estimates in the first half of 2011, however.

## 5. Conclusion

This paper proposes a model to estimate option-implied correlation embedded in options on the European CDS indexes as a measure of the spillover effect of default risk between the financial and corporate sectors in Europe. We consider the option on the iTraxx Europe Main Index as a basket option with two correlated underlyings, namely the Financials and Non-Financials sub-indexes. While the implied volatilities of options on the Main and Financials Indexes can be directly quoted from the

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<sup>13</sup> The ECB on 8 December 2011 decided on additional enhanced credit support measures to support bank lending and liquidity in the euro area money market, in particular, to conduct two longer-term refinancing operations (LTROs) in December 2011 and February 2012. The option-implied correlation declined in tandem with the swap spread, reflecting the effectiveness of the LTROs to support funding liquidity in the banking system.

market, the implied volatilities of option on the Non-Financials Index and the implied correlation between the Financials and Non-Financials Indexes are missing. Closed-form formulae based on a simple stochastic model are proposed to estimate those two missing quantities.

The option-implied correlation is used to study the information transmission between the iTraxx CDS indexes and their corresponding options. Statistical results show that the option-implied correlation moves with a four-week leading time compared with the actual movements of the indexes. It suggests that the option-implied correlation is a useful forward-looking indicator to gauge the contagion risk between the financial and corporate sectors Europe.

Four macro-financial variables are then examined as major determinants of the option-implied correlation based on a regression analysis. The sovereign default risk, funding liquidity risk, level of risk aversion, and equity market performance were found to be significant determinants, implying that the option-implied correlation incorporates information about various markets during the European debt crisis.

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**Table 1. Descriptive Statistics of Realized and Option-Implied Correlations**

	Realized correlation	Option-implied correlation
Mean	0.7130	0.7077
Median	0.7156	0.7073
Maximum	0.8102	0.8556
Minimum	0.6114	0.5852
Std. Dev.	0.0508	0.0694
Skewness	-0.0163	0.0981
Kurtosis	2.0324	2.0800
Observations	301	301
Pairwise-correlation	0.4364	

**Table 2. Granger Causality**

A - Option-implied correlations					
B - Realized correlations by DCC_GARCH					
Weekly Lags	# observations	GC Test	F-stat	p-value	
1	62	A leads B	8.0503***	0.0062	
		B leads A	1.9623	0.1665	
2	61	A leads B	3.9653**	0.0245	
		B leads A	2.8341*	0.0672	
3	60	A leads B	3.3389**	0.0260	
		B leads A	0.8101	0.4940	
4	59	A leads B	3.5793**	0.0121	
		B leads A	1.0826	0.3791	

\*\*\*/\*\*/\* significant at 1/5/10% level

**Table 3. Sample Cross-Correlation between Realized and Option-Implied Correlations up to  $\pm 8$  Week Lags**

A - Option-implied correlations									
B - Realized correlations by DCC_GARCH									
Weekly lags	1	2	3	4	5	6	7	8	
A leads B	0.4526**	0.4821**	0.5116**	0.5593**	0.5443**	0.4806**	0.4290	0.4114	
B leads A	0.3770	0.3863	0.3758	0.2961	0.2341	0.1851	0.1032	0.0022	

\*\* significant at 95% confidence interval

**Table 4. Information Flow between iTraxx Europe CDS Market and its Option Market**

A - Option-implied correlations						
B - Realized correlations by DCC_GARCH						
<b>I</b> - sum of $\beta$ 's (measure of information flow)						
	Flow from A to B			Flow from B to A		
	Value	Std. Err.		Value	Std. Err.	
<b>I</b>	0.4050	0.1821	<b>I</b>	0.9298	0.4981	
$\chi^2$ -stat	4.9446**		$\chi^2$ -stat	2.4987		
p-value	0.0262		p-value	0.1589		

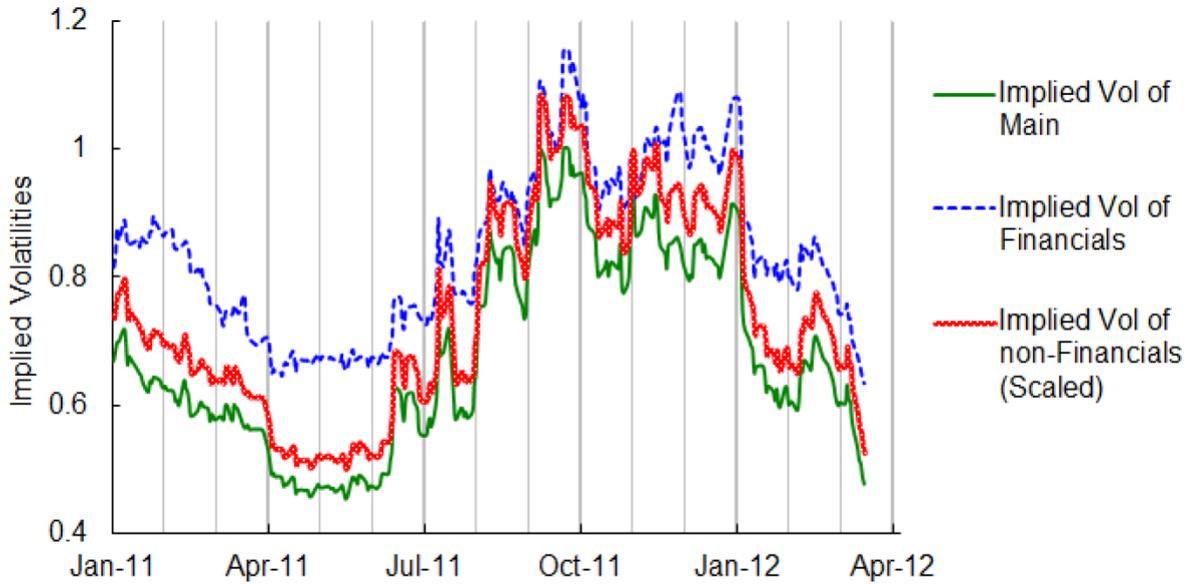
\*\*significant at 5% level

**Table 5. Determinants of Weekly Changes in Option-Implied Correlations**

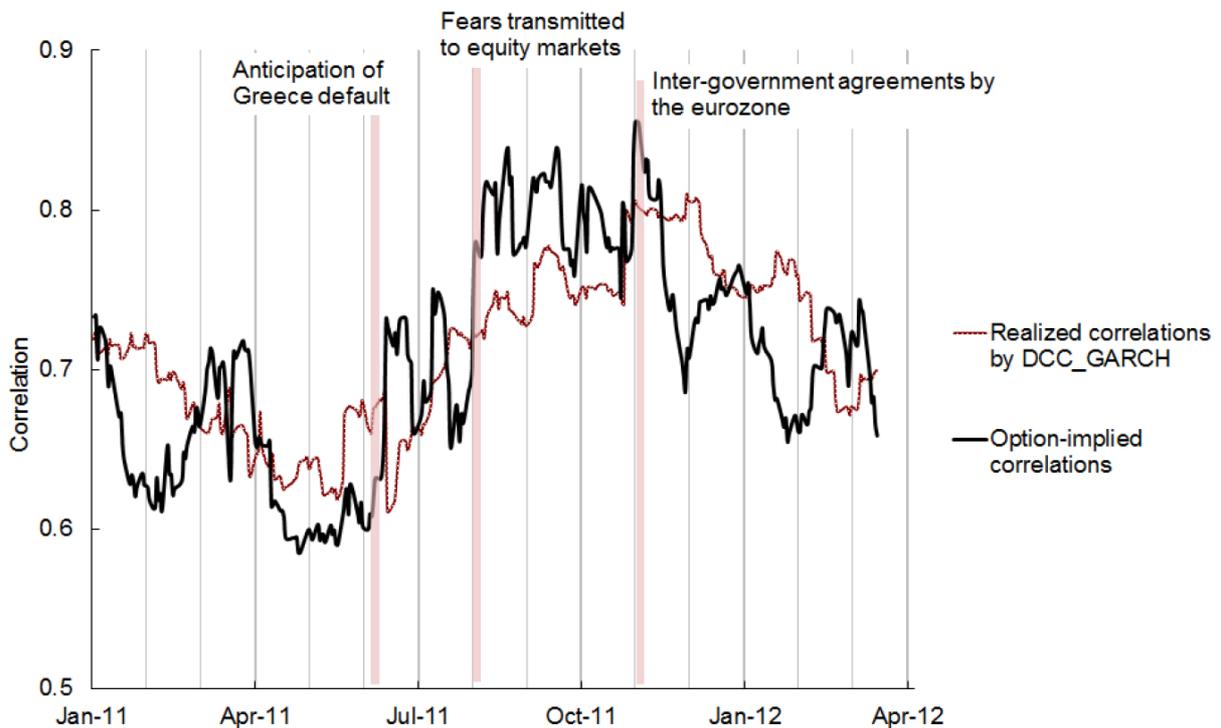
Variable	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
constant	-0.0012	-0.286	-0.0007	-0.173	-0.0003	-0.079	-0.0020	-0.632
iTraxx SovX	0.0004	1.439*					0.0005	0.019**
Swap Spread	0.1399	1.842*	0.1839	2.379**	0.1506	2.011**	0.1946	0.011**
VSTOXX	0.0032	2.101**			0.0033	3.125***		
Equity	0.0005	0.543	-0.0013	-2.129**				
Lags of dependent variable								
Implied Corr (lag1)	0.0843	0.775	0.0529	0.475	0.0629	0.593	0.0953	0.870
Implied Corr (lag2)	-0.3150	-2.872***	-0.3902	-3.552***	-0.3248	-3.026***	-0.3732	-3.427***
$R^2$	42.1%		34.5%		39.8%		35.9%	
Adjusted $R^2$	35.5%		29.7%		35.4%		31.2%	
Durbin-Watson stat	1.91		1.94		1.96		1.82	
observations	60		60		60		60	

\*\*\*/\*\*/\* significant at 1/5/10% level

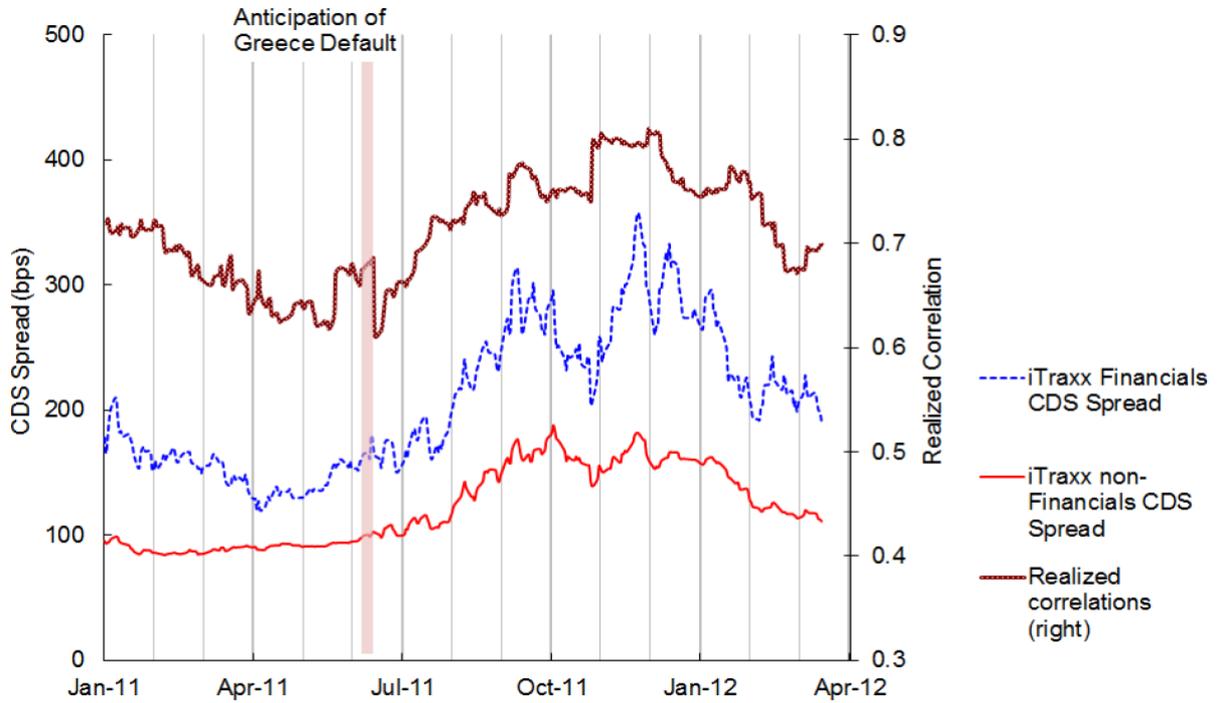
**Figure 1. Estimates of scaled  $\sigma_B$  using equations (6) and (4) compared with market quotes of  $\sigma_I$  and  $\sigma_A$ , i.e. comparisons of implied volatilities amongst the CDS spreads of iTraxx Main, Financials and Non-Financials Indexes.**



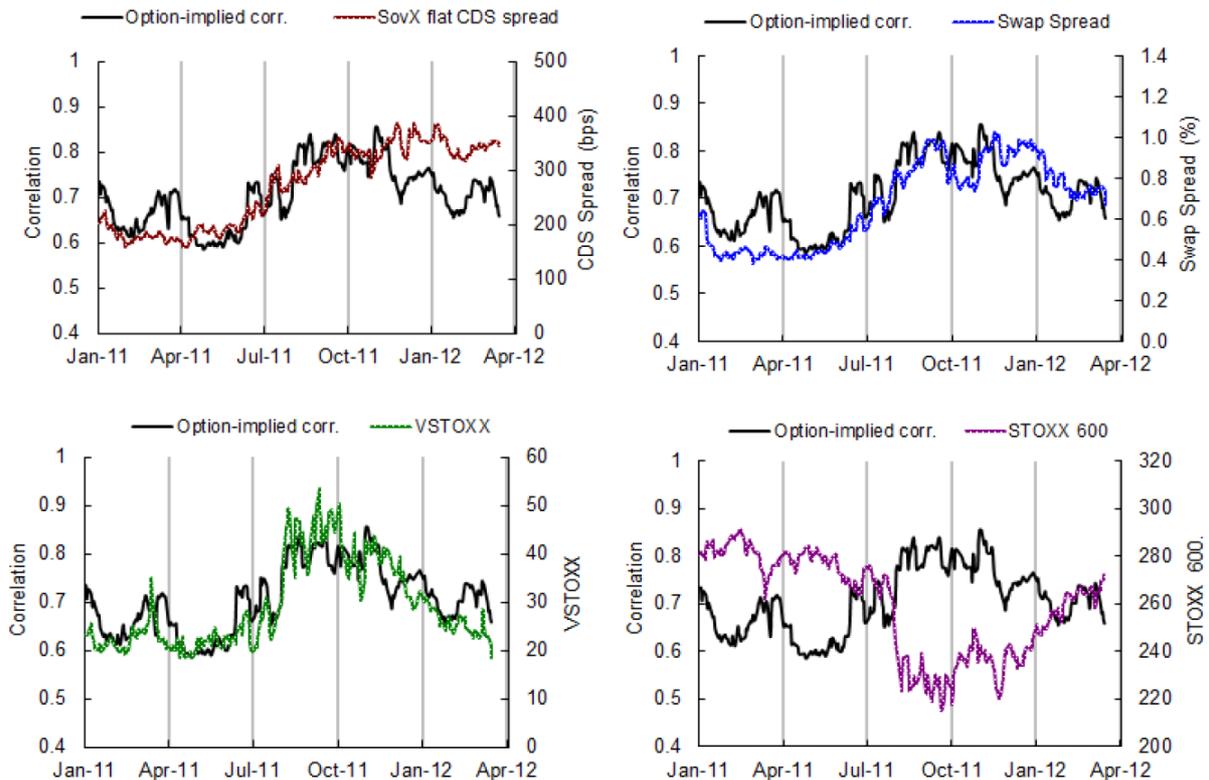
**Figure 2. Daily option-implied correlations using equation (5) compared with realized correlations estimated by DCC\_GARCH model.**



**Figure 3. Daily CDS spreads of iTraxx Europe Financials and Non-Financials Indexes from January 2011 to March 2012 (left), and their realized correlations estimated by DCC\_GARCH model (right).**



**Figure 4. Time series of the iTraxx option-implied correlation and its four determinants, namely iTraxx SovX CDS spread, swap spread, VSTOXX (risk aversion level) and STOXX 600 (equity market index), from January 2011 to March 2012.**



## Appendix

From equation (2), the stochastic processes for  $I$  and  $A$  are assumed to follow two zero drift lognormal processes:

$$\begin{aligned}dI &= \sigma_I I dW_I \\dA &= \sigma_A A dW_A\end{aligned}$$

and  $I$  consists of two weighted components:

$$I = \tilde{\omega}_A A + \tilde{\omega}_B B$$

Since  $I$  and  $A$  are lognormal variables, it is well-known in the financial literature that the weighted difference between them, namely  $B$ , cannot not be a lognormal variable (see Carmona and Durrleman (2003) for a review on spread option). We assume  $B$  to follow:

$$dB = \sigma_B (\alpha \tilde{\omega}_A A + \tilde{\omega}_B B) dW_B$$

with  $A$  and  $B$  correlated through  $\mathbf{E}[dW_A dW_B] = \rho dt$ . The assumption is based upon Lo (2012), which has shown that the dynamics of the spread between two lognormal variables is described by a 'shifted lognormal process'. The shifting is governed by the arbitrary parameter  $\alpha$ , whose optimal value is determined by calibration to option market data, which is discussed in section 2.

By Ito's lemma, the price of a European option with  $I$  being its single underlying, to be denoted by  $P_I(I, t)$ , can be shown to satisfy the backward partial differential equation (PDE):

$$-\frac{\partial P_I(I, t)}{\partial t} = \left[ \frac{1}{2} \sigma_I^2 I^2 \frac{\partial^2}{\partial I^2} - r \right] P_I(I, t) \quad (\text{A1})$$

subject to appropriate terminal condition at  $t = T$ , such as  $P_I(I, T) = \max(K - I, 0)$  for a standard put option on  $I$ .<sup>14</sup> One may alternatively consider  $P_I$  as a basket option with two correlated underlyings  $A$  and  $B$ . And under such case,  $P_I$  would satisfy a backward PDE with two spatial dimensions:

<sup>14</sup> A put option on CDS spread ( $I$ ) corresponds to a call option on the CDS contract (iTraxx Europe Main Index).

$$-\frac{\partial P_t(A, B, t)}{\partial t} = [\hat{L}(A, B) - r]P_t(A, B, t) \quad (A2)$$

where

$$\begin{aligned} \hat{L}(A, B) &= \frac{1}{2}\sigma_A^2 A^2 \frac{\partial^2}{\partial A^2} + \rho\sigma_A\sigma_B A(\alpha\tilde{\omega}_A A + \tilde{\omega}_B B) \frac{\partial^2}{\partial A\partial B} \\ &\quad + \frac{1}{2}\sigma_B^2 (\alpha\tilde{\omega}_A A + \tilde{\omega}_B B)^2 \frac{\partial^2}{\partial B^2} \end{aligned}$$

subject to terminal condition at  $t = T$ , such as  $P_t(A, B, T) = \max(K - \tilde{\omega}_A A - \tilde{\omega}_B B, 0)$  for a basket put option. Then we perform change of variables:

$$I = \tilde{\omega}_A A + \tilde{\omega}_B B ; R_A = A - I$$

and re-write the partial derivatives as:

$$\frac{\partial}{\partial A} = \tilde{\omega}_A \frac{\partial}{\partial I} + (1 - \tilde{\omega}_A) \frac{\partial}{\partial R_A} ; \frac{\partial}{\partial B} = \tilde{\omega}_B \frac{\partial}{\partial I} - \tilde{\omega}_B \frac{\partial}{\partial R_A}$$

Upon collecting all terms that involve  $\partial^2/\partial I^2$  only, one may split the operator  $\hat{L}(A, B)$  from equation (A2) into two parts:

$$\hat{L} = \hat{L}_I + \hat{L}_R$$

with all  $\partial^2/\partial I^2$  terms carried by the  $\hat{L}_I$  operator alone:

$$\begin{aligned} \hat{L}_I &= \frac{1}{2} \left[ \tilde{\omega}_A^2 \sigma_A^2 + 2\tilde{\omega}_A \tilde{\omega}_B \rho_{AB} \sigma_A \sigma_B (\alpha\tilde{\omega}_A + \tilde{\omega}_B) + \tilde{\omega}_B^2 \sigma_B^2 (\alpha\tilde{\omega}_A + \tilde{\omega}_B)^2 \right] I^2 \frac{\partial^2}{\partial I^2} \\ &\quad + \left[ \begin{aligned} &\tilde{\omega}_A^2 \sigma_A^2 + \tilde{\omega}_A \tilde{\omega}_B \rho_{AB} \sigma_A \sigma_B ((2\alpha - 1)\tilde{\omega}_A + \tilde{\omega}_B) \\ &\quad + \tilde{\omega}_A \tilde{\omega}_B^2 \sigma_B^2 (\alpha - 1)(\alpha\tilde{\omega}_A + \tilde{\omega}_B) \end{aligned} \right] I R_A \frac{\partial^2}{\partial I^2} \\ &\quad + \frac{1}{2} \left[ \tilde{\omega}_A^2 \sigma_A^2 + 2\tilde{\omega}_A \tilde{\omega}_B \rho_{AB} \sigma_A \sigma_B (\alpha - 1) + \tilde{\omega}_A^2 \tilde{\omega}_B^2 \sigma_B^2 (\alpha - 1)^2 \right] R_A^2 \frac{\partial^2}{\partial I^2} \end{aligned}$$

and the remaining terms carried by  $\hat{L}_R$  (see Lo (2012) for details). Having the original operator split up in this form, one can approximate the formal solution of equation (A2) by Lie-Trotter (1959) splitting method:

$$\begin{aligned}
P_I(A, B, t) &= \exp\{(T-t)\hat{L}(A, B)\}P_I(A, B, T) \\
&\simeq \exp\left\{-\frac{(T-t)^2}{2}[\hat{L}_R, \hat{L}_I]\right\} \exp\{(T-t)\hat{L}_I\} \exp\{(T-t)\hat{L}_R\}P_I(A, B, T) \\
&\simeq \exp\{(T-t)\hat{L}_I\}P_I(A, B, T)
\end{aligned}$$

in which the relation  $e^{(T-t)\hat{L}_R}P_I(A, B, T) = P_I(A, B, T)$  is utilized. This holds true because the final payoff of a standard vanilla option is only a function of  $I$ . As a result, the original PDE problem simplifies into:

$$-\frac{\partial P_I(A, B, t)}{\partial t} \simeq [\hat{L}_I - r]P_I(A, B, t)$$

with the truncated errors scale in the order of  $o(T-t)^2$ . This explains why the proposed approximation scheme is more accurate for shorter tenors.<sup>15</sup> By further assuming  $\left|\frac{R_A}{I}\right| \ll 1$ , which is generally valid unless  $R_A$  and  $I$  are both very close to zero, and setting

$$\tilde{\omega}_A^2\sigma_A^2 + 2\tilde{\omega}_A\tilde{\omega}_B\rho\sigma_A\sigma_B W + \tilde{\omega}_B^2\sigma_B^2 W^2 = \sigma_I^2 \quad (\text{A3})$$

$$\tilde{\omega}_A^2\sigma_A^2 + \tilde{\omega}_A\tilde{\omega}_B\rho\sigma_A\sigma_B [W + (\alpha - 1)\tilde{\omega}_A] + \tilde{\omega}_A\tilde{\omega}_B^2\sigma_B^2(\alpha - 1)W = 0 \quad (\text{A4})$$

where  $W = \alpha\tilde{\omega}_A + \tilde{\omega}_B$ , the operator  $\hat{L}_I$  may be simplified into:

$$\hat{L}_I \simeq \frac{1}{2}\sigma_I^2 I^2 \frac{\partial^2}{\partial I^2}$$

which is in exact agreement with equation (A1). Hence, having put down the coupled equations in (A3) and (A4), one has effectively bridged the two option pricing problems together, namely the standard option with  $I$  as single underlying and the basket option with  $A$  and  $B$  as two underlyings, into a unified PDE framework. The coupled equations (A3) and (A4) can be solved analytically to yield closed-form formulae for the two unknown parameters,  $\sigma_B$  and  $\rho$  (i.e. equations 4 and 5):

<sup>15</sup> This achieves negligible errors for the case of  $T-t = 1/6$  (2 months) in our study on the iTraxx options.

$$\sigma_B = \frac{\sqrt{\tilde{\omega}_A^2 \sigma_A^2 + [(2\alpha - 1)\tilde{\omega}_A + \tilde{\omega}_B] \sigma_I^2}}{\tilde{\omega}_B (\alpha \tilde{\omega}_A + \tilde{\omega}_B)}$$

$$\rho = \frac{(1 - \alpha) \sigma_I^2 - \tilde{\omega}_A^2 \sigma_A^2}{\tilde{\omega}_B \sigma_A \sigma_B (\alpha \tilde{\omega}_A + \tilde{\omega}_B)}$$

One should note that they are only approximations to the exact values of the proposed stochastic model, although they were shown to be consistent with historical estimates.