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Asset Price and Monetary Policy – The Effect of Expectation Formation*

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Abstract

This paper is a theoretical study of the effects of monetary policy reacting to fluctuations in asset price, accounting for the expectation formation effect of policy regime shift in a DSGE model calibrated to the U.S. economy. We find that the effect of expectation formation can substantially influence the movement of asset price. In contrast to the linear policy rule, under the regime switching policy rule reacting to asset price can generate substantial stabilization effect: the "expected" inflation-output volatility frontier shifts downward, thereby lowering both the volatilities of inflation and output for all possible policy choices. The trade-off between the expected volatility of inflation and that of output, as demonstrated by the "Taylor curve," greatly diminishes, implying that the Taylor rule which considers expectation formation effect and asset price movement expands the set of monetary policy choices available for monetary authority.

Keywords: Asset Price, Monetary Policy, Regime Switching, DSGE

JEL Classification: E3, E52 G1

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1. Introduction

The large fluctuations in asset prices in Nordic and east Asian countries during late 1980s and 1990s, during the "lost decade" of Japan, and in the recent subprime crisis have raised concerns among academics and policy makers regarding whether and how monetary policy should respond to fluctuations in asset prices. These episodes raise concerns because large fluctuations in asset prices exert considerable real effects on the economic activity.¹

This paper is a theoretical study of the effects of monetary policy reacting to fluctuations in asset price in a dynamic, stochastic, general equilibrium (DSGE) model, accounting for the expectation formation effect, i.e., the impact arising from expectations of possible change in policy regime, à la Leeper and Zha (2003) and Davig and Leeper (2007). The model allows monetary policy reaction function to shift between "Hawkish" and "Dovish" regimes, according to the Markov process. We calibrate the model to the U.S. economy and investigate the significance of expectation formation effect on the volatilities of stock price, inflation, and output. We then study whether this policy rule, which reacts to asset price and accounts for expectation formation effect, performs better than a conventional linear policy rule in stabilizing economic aggregates.

Previous works that characterize the responses of monetary policy to asset price specify the parameter values of the reaction function as constant, implying that the central bank will not shift its policy rule between different regimes. In contrast, this paper places a particular emphasis on the role of expectation formation effect in characterizing the responses of monetary policy to asset price for the following reasons. A large empirical literature has confirmed the existence of regime switching in monetary policy across central banks (Clarida *et al.*, 2000; Sargent *et al.*, 2006; Sims and Zha, 2006). Thus, assuming constant responsiveness of reaction function in monetary policy is inconsistent with the behavior of monetary authorities. This assumption is also inconsistent with the notion of rational expectations in which agents form expectations based on all available information. Since empirical evidence strongly suggests that monetary policy should switch regimes from time to time, upon observing the pattern of monetary policy, rational agents will form expectations of future policy shifts. As discussed in Leeper and Zha (2003) and Davig and Leeper (2007), expectations that future policy might switch to an alternative regime, i.e., the "expectation formation effect," will affect equilibrium under the current regime. Finally, since asset prices are sensitive to expectations of economic fundamentals and policy changes in the future and expectation formation effect will affect the current equilibrium, reacting to asset prices may affect the strength of

¹ For example, Case *et al.* (2005) and Campbell and Cocco (2007) found that changes in asset prices may have significant wealth effect in consumption. Kiyotaki and Moore (1997) and Mishkin (2001, 2007) emphasize the role of asset prices in collateralized lending and in monetary transmission mechanism: a large decline in asset prices may cause a deterioration in the quality and value of collateral, leading to a severe credit crunch and subsequent sharp rise in bankruptcy. Large swings in asset prices may cause financial instability, leading to systemic risk and further amplifying the impact on the economy as a whole (Borio and Lowe (2002), Cecchetti *et al.* (2000), Detken and Smets (2003), and Mishkin (2008)).

expectation formation effect in different regimes and the effectiveness of monetary policy in stabilizing output gap and inflation.

The main findings of the paper are as follows. First, the effect of expectation formation can substantially influence the movement of asset price. This means that, aside from those factors studied in the literature, expectation formation effect also plays an important role in asset pricing. Second, under the linear policy rule, the model calibrated to the U.S. economy generates very limited stabilization effect in responding to asset prices in terms of output and inflation volatilities. The negligible gains from responding to asset prices are reminiscent of the findings of Bernanke and Gertler (1999, 2001) and Iacoviello (2005).

Third, under the regime switching policy rule, reacting to asset price significantly shifts the "expected" inflation-output volatility frontier (the "Taylor curve") downward, thereby lowering both the volatilities of inflation and output for all possible policy choices. More importantly, the trade-off between the expected volatility of inflation and that of output, as demonstrated by the Taylor curve, greatly diminishes under the regime switching policy rule. As discussed in Friedman (2006) and Taylor (2006), the Taylor curve represents an efficiency frontier showing the trade-off between inflation-output volatility for optimal monetary policy. We find that given a linear policy rule, the model exactly exhibits this trade-off. However, the regime switching policy rule that includes asset price greatly reduces this trade-off. A certain parameter range exists where the trade-off between the volatilities of output and inflation vanishes. This means that the policy rule that accounts for expectation formation effect and asset price movement expands the set of monetary policy choices available for a monetary authority, by allowing the expected volatility of inflation to remain low for an extended range of parameter values when the monetary policy stabilizes the output gap.

Finally, given a measure of weighted loss function, we find that overall gains from reacting to asset price can be substantial when accounting for expectation formation, while reacting to asset price tends to raise weighted losses under the linear policy rule. This suggests that ignoring the effect of expectation formation may underestimate the stabilization effect of reacting to asset price.

Existing works are divided on the issue of whether monetary policy should systematically react to change in asset prices. Borio and Lowe (2002) observed that sustained rapid credit growth combined with large increases in asset prices appears to increase the probability of financial instability episodes. Thus a monetary response to credit and asset markets may be appropriate to preserve both financial and monetary stability. Mishkin (2008) argued that it is more likely for financial regulators and central banks to identify a credit-fueled bubble in real time because "they might have information that lenders have weakened their underwriting standards and that credit extension is rising at abnormally high rates." In a similar view, Cecchetti (1998) and Cecchetti *et al.* (2000) contended that asset price misalignments are no more difficult to identify than other components of the Taylor rule, such as output gaps, and thus monetary policy should react when asset prices become misaligned with fundamentals. Goodhart (1995)

and Goodhart and Hofmann (2000) further recommended that central banks replace conventional inflation measures with a broader measure that includes weighted housing and stock market prices.²

In contrast, Assenmacher-Wesche and Gerlach (2008) used data from 1986 to 2006 of 17 countries to study the responses of residential property and equity prices, inflation, and economic activity to monetary policy shocks. They found that using monetary policy to offset asset price movements to guard against financial instability is likely to induce pronounced macroeconomic fluctuations. Bernanke and Gertler (1999, 2001) expressed doubt that policymakers can judge reliably whether asset prices are driven by "irrational exuberance" or whether an asset price collapse is imminent. They argue that central banks should consider fluctuations in stock price only to the extent that they affect primary monetary policy goals of price stability and output growth. Greenspan (2002) expressed similar views that bubbles in asset prices are very difficult to identify as they build up and monetary policy cannot successfully deflate asset price bubbles without triggering a recession. Based on Bernanke *et al.* (1999) together with collateral constraints, Iacoviello (2005) found that the gains from reacting to asset price are negligible in terms of output and inflation stabilization. Similarly, Faia and Monacelli (2006) found that when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes.

All the above works have investigated this issue with *linear* monetary policy rules. Bordo and Jeanne (2002), on the other hand, concluded that optimal monetary policy is contingent on the economic conditions in a complex, *non-linear* way, and cannot be summarized in a simple rule-type policy.³ This paper responds to this view by investigating the effect of reacting to asset price in a non-linear monetary policy rule.

The rest of the paper is organized as follows. Section 2 describes the environment of the model and the behaviors of the agents. Section 3 solves the equilibrium and examines the effect of expectation formation on asset price movements. Section 4 evaluates whether reacting to asset price performs better in lowering volatilities of output gap and inflation in a regime-switching monetary rule. Section 5 concludes.

2. Model

We consider a model in which time is discrete and the economy is populated with households, final goods-producing firms, intermediate goods-producing firms, and a central bank. Each intermediate goods-producing firm produces distinct intermediate goods, indexed by $i \in [0,1]$.

² However, empirical analysis of Filardo (2000) found little evidence that this recommendation would reliably improve economic outcomes.

³ Similarly, Gruen *et al.* (2005) show that the reaction function of a central banker sensitively depends on the detailed stochastic properties of the asset bubble. This result highlights the need for stringent informational requirements for a central bank to react to asset-price bubbles.

2.1 Households and Final Goods Producers

At the beginning of period t , a household has dividends $D_t S_{t-1}$ from holding the shares of intermediate goods firms and wage income $W_t L_t$ from supplying labor to intermediate goods-producing firms, where W_t denotes the nominal wage rate. With these incomes, the household purchases composite final goods C_t given the general price level P_t , and decides holdings of equities S_t , given the nominal equity price Q_t .

The household's problem is to maximum the expected lifetime utility function

$$\max E_0 \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\eta}}{1+\eta} \right),$$

subject to the budget constraint (in real terms)

$$C_t + q_t (S_t - S_{t-1}) = w_t L_t + d_t S_{t-1},$$

where $w_t \equiv W_t/P_t$ is the real wage rate, $q_t \equiv Q_t/P_t$ is the real equity price, and $d_t \equiv D_t/P_t$ is the dividend in real terms. The term σ is the inverse of elasticity of substitution, η is the inverse elasticity of labor supply, and χ is the relative weight of leisure.⁴

The final consumption good is a Dixit-Stiglitz aggregation of differentiated intermediate goods,

$$C_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \text{ where } Y_{j,t} \text{ denotes the type-}j \text{ intermediate good and } \theta > 1 \text{ measures the elasticity}$$

of substitution between the differentiated intermediate goods.

Given the level of C_t , cost minimization by the households to achieve this level of composite consumption implies that the demand for intermediate good j is

⁴ Note that we abstract from the liquidity services of real money balances. This is done to simplify the exposition. This model can be viewed as a money-in-utility function model in which the household's utility function includes a third term, \mathcal{M}_t/P_t , and we take \mathcal{G} to be arbitrarily small. A justification is the "cash-less" limit of a monetary economy, in which the cashless limit is a sufficiently good approximation (Woodford (1998)). For example, if financial innovation has proceeded enough to make cash balances of sufficiently small importance in carrying out transactions, then fluctuations in money demand have only negligible effects upon the equilibrium price level under a Wicksellian policy regime. Since we will be interested in formulating monetary policy by interest rate rules, explicit reference to the existence of money is not necessary.

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} C_t, \quad (1)$$

where $P_{j,t}$ is price of good j . Zero profit implies that these prices of differentiated goods are related to the aggregate price level P_t as follows,

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

The households then choose the level of composite consumption, asset demands, and labor supply, given goods prices, equity prices, and the wage rate. Solving the maximization problem yields a set of first order conditions. Together with the equity market clearing condition $S_t = 1$, for all t , we have

$$C_t^{-\sigma} = \beta R_t E_t \left(\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right), \quad (2)$$

$$C_t^{-\sigma} = \beta E_t \left(\frac{d_{t+1} + q_{t+1}}{q_t} C_{t+1}^{-\sigma} \right), \quad (3)$$

$$\frac{\chi L_t^\eta}{C_t^{-\sigma}} = w_t, \quad (4)$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate.

2.2 Intermediate Goods Producers

The intermediate goods producers hire labor from households to produce intermediate goods using a linear technology as follows,

$$Y_{i,t} = a_t L_{i,t},$$

where a_t represents an aggregate productivity disturbance. These firms set prices for their differentiated products in a staggered fashion. Following Calvo (1983), we assume that in each period, a firm cannot adjust its price with a probability τ . By the law of large numbers, a fraction $1-\tau$ of firms in a given period can re-optimize their pricing decisions.

The maximization problem of intermediate goods firm j is to choose $P_{j,t}$ to maximize the expected discounted dividend flows

$$E_t \sum_{i=0}^{\infty} \tau^i \Theta_{i,t+i} \left[\left(\frac{P_{j,t}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{P_{j,t}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},$$

subject to the demand schedule (1). The discount factor $\Theta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$ and φ_t equals the firm's real marginal cost. Since all firms that are able to adjust prices in period t face the same problem, these firms will set the same price. The optimal price setting P_t^* satisfies,

$$\frac{P_t^*}{P_t} = \left(\frac{\theta}{\theta-1} \right) \frac{E_t \sum_{i=0}^{\infty} \tau^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \tau^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \quad (5)$$

The price level of economy at period t satisfies

$$P_t^{1-\theta} = (1-\tau)(P_t^*)^{1-\theta} + \tau P_{t-1}^{1-\theta}. \quad (6)$$

Clarida *et al.* (2001) suggested that one way to include a stochastic shock in the inflation adjustment equation is to add a stochastic wage markup u_t^w , which represents deviations between the real wage and the marginal rate of substitution between leisure and consumption. Thus the labor supply condition (4) becomes

$$\left(\frac{\chi L_t^\eta}{C_t^{-\sigma}} \right) e^{u_t^w} = w_t, \quad (7)$$

where u_t^w follows an i.i.d. normal process with mean zero and variance σ_w^2 .

2.3 The Central Bank

The crucial specification of this model is the reaction function of the central bank. To study the implications of regime shifts in monetary policy, we allow for the coefficients in the Taylor rule to vary with policy regime

$$\left(\frac{R_t}{R}\right) = \left(\frac{\pi_t}{\pi^*}\right)^{\alpha_{\pi,s_t}} \left(\frac{x_t}{x}\right)^{\alpha_{x,s_t}} \left(\frac{q_t}{q}\right)^{\alpha_{q,s_t}} e^{u_t^M}, \quad (8)$$

where α_{π,s_t} , α_{x,s_t} and α_{q,s_t} are regime-dependent policy parameters that measure the aggressiveness of monetary policy against inflation π_t , output gap x_t , and the real equity price q_t . The term π^* is the inflation target, R , x , and q are the steady states of R_t , x_t , and q_t , respectively, and u_t^M is the shock to monetary policy which follows an i.i.d. normal process with mean zero and variance σ_M^2 .

Monetary policy regime follows a Markov switching process between two states, a Hawkish regime ($s_t = 1$) and a Dovish regime ($s_t = 2$), that differ in the responsiveness to inflation, i.e., $\alpha_{\pi,1} > \alpha_{\pi,2}$. The transition probabilities for the regime switching process are given by the following matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where $p_{ij} = \text{Prob}(s_{t+1} = i | s_t = j)$. Each column of \mathbf{P} sums up to 1 so that $p_{21} = 1 - p_{11}$ and $p_{12} = 1 - p_{22}$. After identifying the two states, we compute the smoothed probability:

$$P_{j,t} = \Pr(s_t = j | \Psi_T), \quad j = \{1,2\},$$

where Ψ_T is all the information available in our sample periods. The idea is to compute the probability of a certain state of the economy from an ex-post point of view, thus utilizing the full set of information. If $\Pr(s_t = j | \Psi_T) > 0.5$, then we identify the economy most likely in state j , $j = 1,2$.

2.4 Equilibrium

The final goods market clears when $C_t = Y_t$. The labor market clear when the demand from intermediate goods producers equals the supply from households, $L_t^D = L_t^S$. Finally, the equity market equilibrium requires $S_t = 1$.

The equilibrium consists of an allocation $\{Y_t, C_t, L_t, D_t, S_t\}$ and a sequence of prices $\{P_t, Q_t, W_t, R_t\}$ that satisfy the optimality conditions of households, intermediate goods producers, and final goods producers. Finally, all markets clear.

2.5 Linearization of the System

The following presents log-linearized equilibrium conditions around the deterministic steady state for the model. Let a variable with hat \hat{z}_t denote the percentage deviation for a variable z_t around the steady state and a variable with superscript f denote the equilibrium under the environment with flexible prices.

Log-linearizing the optimal pricing rule (5), (6), and the labor supply (7), yields the expectations-augmented Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t^S, \quad (9)$$

where $\kappa \equiv (1-\tau)(1-\beta\tau)/\tau$, $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^f$ is the output gap, \hat{Y}_t^f is the output when prices are fully flexible ($\tau = 0$), and $u_t^S = \kappa u_t^w$ represents the cost-push shock.

Log-linearizing the consumption Euler equation (2) leads to the forward-looking IS curve

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \hat{\pi}_{t+1}) + u_t^D, \quad (10)$$

where $1/\sigma$ is the elasticity of substitution and $u_t^D = E_t \hat{Y}_{t+1}^f - \hat{Y}_t^f$ represents the demand side shock.

Log-linearizing (2) and (3), together with (10), we obtain the equation for equity price determination,

$$\hat{q}_t = \beta E_t \hat{q}_{t+1} - \varpi_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \varpi_3 u_t^D - \varpi_4 u_t^S, \quad (11)$$

where ϖ_2 , ϖ_3 and ϖ_4 are complex functions of model parameters, detailed in the appendix.

Finally, log-linearizing the interest rate rule (8), we have

$$\hat{R}_t = \alpha_{\pi, s_t} \hat{\pi}_t + \alpha_{x, s_t} \hat{x}_t + \alpha_{q, s_t} \hat{q}_t + u_t^M. \quad (12)$$

Recall that the central bank reacts more strongly to inflation in a Hawkish regime ($s_t = 1$) than in a Dovish regime ($s_t = 2$), i.e., $\alpha_{\pi,1} > \alpha_{\pi,2}$.

The disturbance terms u_t^S , u_t^D , and u_t^M , for (9), (10), and (12), respectively, are assumed to follow a first order autoregressive process

$$u_t^i = \rho_i u_{t-1}^i + \varepsilon_t^i, i = S, D, M,$$

where ε_t^i is an i.i.d. normally distributed random variable with zero mean and unity variance.

In sum, the system of economy can be summarized by four equations, (9), (10), (11), and (12), that characterize the dynamics of inflation, output gap, equity price, and interest rate.

3. Solving the Equilibrium with a Regime-Switching Policy Rule

The system of the economy can be expressed as

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{x}_t + u_t^S, \quad (13)$$

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \hat{\pi}_{t+1}) + u_t^D, \quad (14)$$

$$\hat{R}_t = \alpha_{\pi, s_t} \hat{\pi}_t + \alpha_{x, s_t} \hat{x}_t + \alpha_{q, s_t} \hat{q}_t + u_t^M, \quad (15)$$

$$\hat{q}_t = \beta E_t \hat{q}_{t+1} - \varpi_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \varpi_3 u_t^D - \varpi_4 u_t^S. \quad (16)$$

To rewrite the model in a more compact form, we substitute (15) into (14) and re-arrange (13), (14) and (16) to characterize the economy by a three-variable system of inflation, output gap, and equity price, $(\hat{\pi}_t, \hat{x}_t, \hat{q}_t)$.

Define $\hat{\pi}_{i,t} \equiv \hat{\pi}_t(s_t = i)$, $\hat{x}_{i,t} \equiv \hat{x}_t(s_t = i)$, and $\hat{q}_{i,t} \equiv \hat{q}_t(s_t = i)$, $i = 1, 2$, which denote the state-dependent inflation, output gap and equity price. To handle the expectations of $\hat{\pi}_{t+1}$, \hat{x}_{t+1} , and \hat{q}_{t+1} in the system, we define the forecasting errors:

$$\eta_{i,t+1}^\pi \equiv \hat{\pi}_{i,t+1} - E_t \hat{\pi}_{i,t+1},$$

$$\begin{aligned}\eta_{i,t+1}^x &\equiv \hat{x}_{i,t+1} - E_t \hat{x}_{i,t+1}, \\ \eta_{i,t+1}^q &\equiv \hat{q}_{i,t+1} - E_t \hat{q}_{i,t+1}, i = 1, 2,\end{aligned}$$

where $E_t \hat{\pi}_{i,t+1}$ is the one-step-ahead conditional expectation of $\hat{\pi}_{t+1}$ for a given state $s_t = i$ at time t , which can be computed as follows,

$$E_t \hat{\pi}_{i,t+1} \equiv E\{\hat{\pi}_{t+1} | s_t = i, \Gamma^{-s}\} = p_{ii} E_t \hat{\pi}_{i,t+1} + p_{ji} E_t \hat{\pi}_{j,t+1},$$

where $j \neq i$, and Γ^{-s} is the information set of time t which excludes the current regime. The conditional expectations of \hat{x}_{t+1} , and \hat{q}_{t+1} can be calculated in the same way.

We can then express the system in the following form,

$$AY_{t+1} = BY_t + A\eta_{t+1} - Cu_t, \quad (17)$$

where $Y_t = [\hat{\pi}_{1,t}, \hat{\pi}_{2,t}, \hat{x}_{1,t}, \hat{x}_{2,t}, \hat{q}_{1,t}, \hat{q}_{2,t}]'$, $\eta_t = [\eta_{1,t}^\pi, \eta_{2,t}^\pi, \eta_{1,t}^x, \eta_{2,t}^x, \eta_{1,t}^q, \eta_{2,t}^q]'$, and $u_t = [u_t^S, u_t^D, u_t^M]'$, and the matrices, A , B and C , are defined in the Appendix. A unique equilibrium requires six unstable roots to generate six linear restrictions that determine the regime-dependent forecasting errors for inflation, output gap and asset price. We compute the generalized eigenvalues for the system to check the existence and uniqueness of the model's equilibrium following Sims (2002).

To solve the model, we employ the method of undetermined coefficients that delivers solutions as functions of the smallest set of state variables, u_t^S , u_t^D , u_t^M , and s_t (more detail can be found in appendix). We posit the equilibrium solution with a regime-switching policy rule as follows:

$$\begin{aligned}\hat{\pi}_t &= a^S(s_t)u_t^S + a^D(s_t)u_t^D + a^M(s_t)u_t^M, \\ \hat{x}_t &= b^S(s_t)u_t^S + b^D(s_t)u_t^D + b^M(s_t)u_t^M, \\ \hat{q}_t &= c^S(s_t)u_t^S + c^D(s_t)u_t^D + c^M(s_t)u_t^M,\end{aligned}$$

which can be compactly expressed as,

$$Z_t = G_{1,s_t}^A Z_{t-1} + G_{2,s_t}^A \varepsilon_t, \quad (18)$$

where $Z_t = [\hat{x}_t^A, \hat{\pi}_t^A, \hat{q}_t^A, u_t^S, u_t^D, u_t^M]'$, $\varepsilon_t = [\varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^M]'$, with the covariance matrix $\Omega_{s_t}^A = E[Z_t Z_t']$, and the superscript A denotes the solution reacting to asset price.

3.1 The Equilibrium with a Fixed-Regime Policy Rule

We also solve the counterpart of (18) under which the central bank follows a specific regime j indefinitely. When the current regime of monetary policy is expected to stick to a specific regime, which is equivalent to imposing $p_{jj} = 1$ for $j \in \{1, 2\}$, $\alpha_{\pi, j}$, $\alpha_{x, j}$, and $\alpha_{q, j}$ ($j = 1, 2$) in the reaction function are time-invariant. In this case, we can solve for the model analytically. The expressions of the solution are stated in the appendix. Here we rewrite the solution, currently given at regime j , as the same form in (18):

$$Z_{t,j} = \bar{G}_{1,j}^A Z_{t-1,j} + \bar{G}_{2,j}^A \varepsilon_t, \quad j \in \{1, 2\}, \quad (19)$$

where $Z_{t,j} = [x_{t,j}^A, \pi_{t,j}^A, q_{t,j}^A, u_t^S, u_t^D, u_t^M]'$, $\varepsilon_t = [\varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^M]'$ with the covariance matrix given the policy regime j , $\bar{\Omega}_j^A$, and the upperbar denotes the solution under a fixed regime.

3.2 Parameters Values

This section empirically estimates the structural parameter values in the reaction function (15), and then choose the remaining parameter values from the literature.

Before estimating the reaction function, note that there is a feedback effect in the system (13)-(16), so that $\hat{\pi}_t$, \hat{x}_t , or \hat{q}_t may also react to changes in the interest rate. To check for this potential endogeneity problem, we conduct the Hausman (1978) test to see whether any of these three variables is correlated with the error term. Let $X_t \in [\hat{\pi}_t, \hat{x}_t, \hat{q}_t]$ be the variable that may correlates with the error term. Given an instrumental vector V_t that contains the lags of X_t , we define $\tilde{X}_t = P_z X_t$, where $P_z = V_t(V_t' V_t)^{-1} V_t'$, and estimate the following model

$$\hat{R}_t = \delta_0 X_t + \delta_1 \tilde{X}_t + \varepsilon_t.$$

We test the null hypothesis that the coefficient δ_1 is zero, i.e., an omitted variable test for the variables \tilde{X}_t . We estimate this equation using quarterly U.S. data from 1975Q1 to 2008Q4. For the explanatory

variable, the inflation rates measured by changes in the GDP deflator, output gap is measured by detrended real GDP, \hat{q}_t is represented by the de-trended stock price index of S&P500, and the interest rate is represented by the federal funds rate.

From Table 1, inflation rate is the only variable that is endogenous to the federal funds rate. To account for the endogeneity problem, we follow Kim (2009) to estimate $\hat{\pi}_t = z_t' \delta_\pi + u_t$ by OLS, where the vector z_t includes the instrumental variables, selected to be the lags of inflation rate, and δ_π is the vector of parameters for these lagged inflation rate. According to the AIC criterion, we select four instruments with five to eight lag periods. After estimating this equation, we incorporate the residual term \hat{u}_t into (15), and then estimate the following modified policy rule:

$$\hat{R}_t = \alpha_{\pi, s_t} \hat{\pi}_t + \alpha_{x, s_t} \hat{x}_t + \alpha_{q, s_t} \hat{q}_t + \psi_{\pi, s_t} \hat{u}_t + u_t^M,$$

Table 2 report our estimation results using U.S. quarterly data from 1975Q1 to 2008Q4. The estimated transition probabilities are $p_{11} = 0.97$ and $p_{22} = 0.96$. The parameter values of the policy rule are $\alpha_{\pi, 1} = 2.6$, $\alpha_{x, 1} = 0.07$, and $\alpha_{q, 1} = 0.02$ in the Hawkish regime, and $\alpha_{\pi, 2} = 1.5$, $\alpha_{x, 2} = 0.01$, and $\alpha_{q, 2} = 0.03$ in the Dovish regime. Note that the reactions to inflation under both regimes are significant at the one percent level, and the reaction under the Hawkish regime ($\alpha_{\pi, 1} = 2.6$) is clearly more active than that under the Dovish regime ($\alpha_{\pi, 2} = 1.5$). The reactions to output gap and stock price are significant under the Hawkish regime, while insignificant under the Dovish regime, suggesting that the Fed has been actively reacting to output gap and asset price in the Hawkish regime, while remaining passive in the Dovish regime.

We also estimate a linear model for comparison, which yields $\alpha_\pi = 1.9$, $\alpha_x = 0.07$, and $\alpha_q = 0.03$.⁵ where the reactions to inflation and output gap are both significant, while the reaction to asset price is not. The reaction to asset price appears insignificant under linear specification, implying that the linear policy rule fails to detect the Fed' active response to asset price in the Hawkish regime, albeit small in

⁵ The estimate of α_q by Rigobon and Sack (2003) in a linear model is round 0.02. Our estimation result for the linear model is close to Rigobon and Sack's (2003) findings.

magnitude ($\alpha_{q,1} = 0.02$).⁶

As listed in Table 2, the model allowing regime switching attains a higher log-likelihood value (-543.52) than the linear model (-578.90). The LR statistic, proposed by Garcia (1988), is given by $LR = 2(L_U - L_R) = 70$, which is significant at the 1% level (with a critical value 14.2), suggesting that the Markov Switching model performs better than the linear model.

Other structural parameters in this model are standard in the literature. We select their values from previous works. Table 3 reports these parameter values.

Figure 1 shows the computed smoothed probability together with the federal funds rate. The shaded areas represent those periods dated as recessions by the NBER.⁷ Clearly, the monetary policy switched to a Hawkish regime after the early 1980s recession, and then shifted back to a Dovish regime in response to the aftermath of the savings and loans crisis in the early 1990s. This is in line with literature findings on the monetary policy environment in the pre-Volcker and Volcker-Greenspan eras (Clarida *et al.* (2000) and Lubik and Schorfheide (2004)). In approximately 2002, the monetary policy shifted once more to the Dovish regime, ending the Hawkish regime since 1994. As Taylor (2007) and others forcefully argued, the lax monetary environment was one of the major causes leading to the run-up in house price in the early 2000s before it collapsed in late 2006.

3.3 Effect of Expectation Formation on the Asset Price

This section investigates how the effect of expectation formation influences the movement of the asset price. We denote the asset price under regime-switching, \hat{q}_{t,s_t}^A , and that under fixed regime, $\bar{q}_{t,j}^A$, from systems (18) and (19), respectively. The simulation of the asset prices consists of three steps. Given the benchmark values of structural parameters with the Fed reacting to asset price, we randomly draw from the three disturbance terms for each system, (18) and (19), respectively, and obtain a pair of asset prices under regime-switching, \hat{q}_{t,s_t}^A , and the other under fixed-regime j , $\bar{q}_{t,j}^A$. Then we draw 1000 times to generate two series of asset prices. Given a regime j , we compute the following,

⁶ The result here is relevant for the debate as to whether central banks have been reacting to asset prices. Empirical evidence has shown mixed results. For example, Bernanke and Gertler (1999) found that the Fed has focused its attention on expected inflation and the output gap and has not actively attempted to stabilize stock prices during the Volcker and Greenspan eras (1979M10–1997M12). However, Rigobon and Sack (2003) found that the Fed has systematically responded significantly to stock price movements. For the cross-country evidence, see, for example, Bohl *et al.* (2007) and Furlanetto (2008).

⁷ We also measure the probability that a regime occurs by computing the filtered probability:

$$P_{j,t} = \Pr(s_t = j | \Psi_t), j = \{1,2\},$$

where only the information up to time t (Ψ_t) is used. We find that the regimes identified by both methods are almost identical.

$$\frac{1}{N} \sum_{t=1}^N \left(\hat{q}_{t,s_t}^A - \bar{q}_{t,j}^A \right)^2, \quad (20)$$

where $N = 1000$. This measures the average quadratic difference between the asset price generated under the model allowing for regime shift and that under the model given a fixed regime j , i.e., the difference in these two asset prices caused by the effect of expectation formation. Finally, we repeat step 1 and 2 for 3000 times to obtain 3000 observations for each regime j , and then plot their distributions in Figure 2.

If expectation formation effect is substantial, then the difference between the distributions of these two simulated measures in (20) for the Hawkish regime and the Dovish regime should be sizeable. If expectation formation effect is getting negligible, these two distributions will move closer to each other and eventually collapse into a single point at the mean value.

The solid line in Figure 2 represents the distribution of (20) under the Dovish regime and the dashed line represents that under the Hawkish regime. The distribution under the Dovish regime has a higher mean than that under the Hawkish regime, suggesting that the expectation formation has a larger effect on stock price in the Dovish regime than in the Hawkish regime.⁸ Furthermore, these two distributions are located so far from each other that they are completely separated, implying that expectation formation effect substantially influences asset price dynamics.

The results suggest that, aside from those factors studied in the literature, expectation formation effect also plays an important role in asset pricing.

4. Can Responding to Asset Price Lower Volatilities of Inflation and Output?

To evaluate the stabilization effect of the monetary policy actively reacting to asset price and accounting for expectation formation effect, we compute the following measure:

$$\frac{\Omega_{ii,s_t}^A - \bar{\Omega}_{ii,j}}{\bar{\Omega}_{ii,j}}, s_t = 1, 2, i = 1, 2, 3, \quad (21)$$

⁸ Note that the dispersion of the distribution will surely increase with its mean because of the quadratic metric in (20).

where Ω_{ii,s_t}^A is the diagonal element of the matrix $\Omega_{s_t}^A$, i.e., the variance of each element in Z_t when the regime-switching policy rule reacts to asset price. $\bar{\Omega}_{ii,j}$ is the diagonal element of the matrix $\bar{\Omega}_j$, where $\bar{\Omega}_j$ is the corresponding covariance matrix under the fixed regime j when the policy rule does not react to asset price. To obtain this covariance matrix, we repeat the above procedure for solving the equilibrium by imposing $\alpha_{q,s_t} = 0$, for all s_t , in (15) and $p_{jj} = 1$ for $j \in \{1,2\}$ so that $\alpha_{\pi,j}$ and $\alpha_{x,j}$ ($j=1,2$) in (15) are time-invariant. The appendix states the analytical solution of the system. Here we denote the system, currently given at regime j , as follows:

$$Z_{t,j} = \bar{G}_{1,j}Z_{t-1,j} + \bar{G}_{2,j}\varepsilon_t, j \in \{1,2\}, \quad (22)$$

where $Z_{t,j} = [\bar{x}_t, \bar{\pi}_t, \bar{q}_t, u_t^S, u_t^D, u_t^M]'$, $\varepsilon_t = [\varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^M]'$, and the covariance matrix denoted as $\bar{\Omega}_j = E[Z_{t,j}Z_{t,j}']$. The variables with the upperbar but without superscript A denote the solution under fixed regime j without reacting to asset price.

We also compute the relative volatilities under the linear monetary policy rule. Let Ω^A be the covariance matrix of the equilibrium solution under the linear model when the Fed reacts to asset price, while Ω corresponds to that when the Fed does not react to asset price. Thus, the effect of reacting to asset price under the linear policy rule is

$$\frac{\Omega_{ii}^A - \Omega_{ii}}{\Omega_{ii}}, i = 1,2,3, \quad (23)$$

where Ω_{ii}^A and Ω_{ii} are the corresponding diagonal element of the matrix Ω^A and Ω , respectively. Table 4 and 5 display the results. A positive (negative) entry means that reacting to asset price combined with the expectation formation effect raises (lowers) the volatility of a variable proportionally relative to the fixed regime equilibrium, in which each variable is measured in terms of percentage deviation from the steady state.

Examining the overall effects of reacting to asset under these two different policy rules in Table 4 and 5 yields the following notable findings. Table 5 shows that reacting to asset price under the linear monetary rule reduces all the volatilities of inflation, output gap, and asset price. However, gains from responding to asset prices are small in terms of output and inflation stabilization under the linear monetary rule. The

negligible gains from responding to asset prices are reminiscent of the findings of Bernanke and Gertler (1999, 2001) and Iacoviello (2005).

In contrast, Table 4 shows that introducing the possibility of regime shift *can* amplify the gains from reacting to asset price by significantly lowering the volatilities of output gap in both regimes. Furthermore, asymmetric responses of inflation and asset price exist across regimes, with a larger stabilizing effect under the Dovish regime than the destabilizing effect under the Hawkish regime. For example, the *total effect* of reacting to asset price significantly lowers inflation volatility in the Dovish regime by 37.05% , while it only moderately raises inflation volatility in the Hawkish regime by 16.25% .

To understand the intuition behind the result, we decompose the total effect of reacting to asset price under regime switching into two components:

$$\underbrace{\Omega_{ii,s_t}^A - \bar{\Omega}_{ii,j}}_{\text{Total Effect}} = \underbrace{(\bar{\Omega}_{ii,j}^A - \bar{\Omega}_{ii,j})}_{\text{Direct Effect}} + \underbrace{(\Omega_{ii,s_t}^A - \bar{\Omega}_{ii,j}^A)}_{\text{Expectation Formation Effect}} ,$$

where the former is the direct effect of reacting to asset price for a given fixed regime, and the latter is the effect of expectation formation.

The direct effect, shown in the bottom row of Table 4, appears to be volatility-stabilizing under both regimes, but differs in its magnitude across regimes. For example, the relative volatility of inflation declines by 13.52% when the current regime is Hawkish and the regime is expected to prevail indefinitely, while it declines by only 9.68% when the current regime is Dovish. On the contrary, the relative volatility of output gap declines by 20.32% in the currently fixed Hawkish regime, less than the decline of 53.82% in the currently fixed Dovish regime. The reason for this result is as follows. In our model, the stock price is the sum of discounted future dividends (i.e., there is no "bubble" in the stock price), and a change in dividends reflects output fluctuation in that period. Therefore, stabilizing stock price helps to stabilize output. Recall that in Table 2, the Fed reacts to stock prices more actively in the Dovish regime ($\alpha_{q,1} = 0.02$ and $\alpha_{q,2} = 0.03$). Thus, reacting to asset price is much more effective in stabilizing output gap in the Dovish regime than in the Hawkish regime. Since there is a trade-off between stabilizing inflation and output gap, reacting to asset price can stabilize inflation volatility more in the Hawkish, than in the Dovish regime.

However, expectation formation effect introduces a volatility trade-off between the two regimes for all variables: the anticipation of regime shift raises the volatilities of inflation and stock price in the Hawkish regime and lowers them in the Dovish regime, while the effects reverse for the output gap. Table 4 shows

that expectation formation effect raises inflation volatility by 29.37% in the Hawkish regime and lowers it by 27.37% in the Dovish regime, while the effect lowers the volatility of output gap by 31.44% in the Hawkish regime and raises it by 3.88% in the Dovish regime. The results are intuitive: anticipating a regime shift from the Hawkish to the Dovish regime amplifies inflation volatility, but it helps stabilize output gap, and vice versa.⁹

The total combined effect of reacting to asset price generates asymmetric responses of volatilities of inflation and output gap across regimes, which introduces a trade-off for volatility stabilization.

4.1 Taylor Curve and the Trade-off in "Expected" Volatilities

The trade-off for stabilizing the volatilities of inflation and output involves the probability that a regime occurs at a certain point in time. Therefore, we consider the following measures of "expected" volatilities for output gap and inflation under the regime switching policy rule,

$$\Sigma_x^{RS} \equiv \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^2 \Pr(s_t = j | \Psi_T) \frac{\left[(\sigma_{x,s_t}^A)^2 - \sigma_{x,j}^2 \right]}{\sigma_{x,j}^2}, \quad (24)$$

$$\Sigma_\pi^{RS} \equiv \frac{1}{T} \sum_{t=1}^T \sum_{j=1}^2 \Pr(s_t = j | \Psi_T) \frac{\left[(\sigma_{\pi,s_t}^A)^2 - \sigma_{\pi,j}^2 \right]}{\sigma_{\pi,j}^2}, \quad (25)$$

where the proportional deviation from the variance under a fixed regime j is weighted by the smoothed probabilities over the two regimes and then averaged over the whole sample period ($T = 127$).

The corresponding measures for the linear policy rule are given by

$$\Sigma_x \equiv \frac{\left[(\sigma_x^A)^2 - \sigma_x^2 \right]}{\sigma_x^2}, \quad \Sigma_\pi \equiv \frac{\left[(\sigma_\pi^A)^2 - \sigma_\pi^2 \right]}{\sigma_\pi^2}. \quad (26)$$

We then allow the coefficients of monetary policy rule of asset price in (15) under the Hawkish regime, $\alpha_{q,1}$, to vary within the range [0.02,0.12], and alternatively, under the Dovish regime, $\alpha_{q,2}$, to vary

⁹ We experiment with different combinations of parameter values to see how the effect of expectation formation changes. We find that when $p_{22} = 0$ (i.e., when the policy regime falls into Dovish, it is believed that the regime will surely shifts back to Hawkish), while everything else remains the same, the expectation formation effect attains its maximum. The expectation formation effect appears larger for inflation and stock price, working more in the direction of reducing volatilities when the current regime is Dovish. This means that if a central bank has a reputation of maintaining low inflation and strictly abides by the Hawkish regime, i.e., p_{11} is close to unity and p_{22} is close to zero, the expectation formation effect will work most effectively when the current regime is Dovish, because the current regime is only transitory.

within the range $[0.03, 0.12]$. The corresponding coefficient for the linear policy rule, α_q , varies within the range $[0.03, 0.12]$. Note that $\alpha_{q,1} = 0.02$, $\alpha_{q,2} = 0.03$, and $\alpha_q = 0.03$, are the benchmark estimated values of parameters listed in Table 2.

Figure 3 shows the inflation-output volatility frontiers (the "Taylor Curve") for (24), (25), and (26). The horizontal axis is the volatility of output gap under the regime switching policy rule and under the linear rule, $(\Sigma_x^{RS}, \Sigma_x)$; and the vertical axis is the volatility of inflation under the regime switching policy rule and under the linear rule, $(\Sigma_\pi^{RS}, \Sigma_\pi)$. Several remarks on these volatility frontiers are as follows.

Consider first the linear policy rule. As α_q rises from 0.03 to 0.12, the inflation-output volatility frontier, represented by the dashed line, stretches from the southeast end, where $\alpha_q = 0.03$, toward the northwest end, which means that when the Fed reacts to asset price more actively, it stabilizes the output gap at the expense of price stabilization. The trade-off between the expected volatilities of inflation and output is a standard feature of the Taylor curve, representing an efficiency frontier of fluctuations in output and inflation for policy choice derived from a specific model (Friedman, 2006).

Under the regime-switching policy rule, we allow $\alpha_{q,1}$ or $\alpha_{q,2}$ to vary, starting with their benchmark values, 0.02 and 0.03, at which the solid line (varying $\alpha_{q,1}$) and the dotted line (varying $\alpha_{q,2}$) meet. Clearly the frontiers for the regime switching policy rule lie far below the frontier for the linear policy rule, and reacting to asset price in Hawkish regime (the solid line) produces a larger decline in inflation volatility than in the Dovish regime (the dotted line), given a certain level of output volatility. This suggests that ignoring the effect of expectation formation may underestimate the stabilization effect of reacting to asset price.

Interestingly, a certain parameter range exists where the trade-off between the volatilities of output and inflation vanishes, i.e., $\alpha_{q,1} \in [0.02, 0.06]$ and $\alpha_{q,2} \in [0.03, 0.06]$. The expected inflation-output volatility frontiers are positively sloped within this range. Thus, reacting to asset price strictly lowers both the expected volatilities of inflation and output. When the strength of reaction to asset price is strong enough, i.e., $\alpha_{q,1} \geq 0.06$ and $\alpha_{q,2} \geq 0.06$, output stabilization begins to take its toll on inflation volatility, returning the trade-off between the expected volatilities of output gap and inflation returns. This means that accounting for expectation formation effect and asset price movement greatly diminishes the trade-off implied by the Taylor curve and enlarges the set of monetary policy choices available for the monetary authority.

4.2 The Loss Function

We further consider a loss function for overall evaluation of the effect of reacting to asset price. Suppose the monetary authority is concerned with the volatilities of inflation and output gap. Define the loss function L^{RS} to be the weighted declines in the volatilities of output gap and inflation, based on (24) and (25):

$$L^{RS} = \omega \Sigma_x^{RS} + \Sigma_\pi^{RS},$$

where ω is the relative weight on the output gap stabilization. We also compute the loss function under the linear policy rule, which measures the weighted decline in volatilities of inflation and output gap when the Fed reacts to the asset price,

$$L = \omega \Sigma_x + \Sigma_\pi.$$

The calibrated value of the relative weight on the output gap stabilization ω in the literature varies from 0.05 to one-third (Jensen, 2002). We follow Jensen (2002) and McCallum and Nelson (2004b) by setting $\omega = 0.2$. Given our benchmark parameter values listed in Table 2 and 3, we plot the loss functions of L^{RS} and L , varying the reaction coefficient to asset price under the linear policy rule (α_q) and that under the regime switching policy rule ($\alpha_{q,1}$ or $\alpha_{q,2}$) in Figure 4.

For the linear policy rule, the value of loss function L is negative only for a very limited range of α_q and turns positive as the policy responds more actively to asset price, implying that strongly reacting to asset price raises weighted losses under the linear policy rule. However, reacting to asset price lowers weighted losses under the regime switching policy rule: the value of loss function L^{RS} , no matter whether varying $\alpha_{q,1}$ or $\alpha_{q,2}$, is far below zero throughout. Corresponding to Figure 3, strongly responding to asset price in the Hawkish regime (higher $\alpha_{q,1}$), represented by the solid line in Figure 4, generates the lowest schedule of loss function.

We also find that a minimum point for the loss function exists under the regime switching policy rule, given the parameter value of reacting to asset price, $\alpha_{q,1} = 0.07$ and $\alpha_{q,2} = 0.08$, respectively. In particular, reduced weighted volatility could reach 24.88% by setting $\alpha_{q,1} = 0.07$. In sum, accounting for expectation formation effect greatly strengthens the gains from reacting to asset price. Among those concerns reviewed above, this result provides an alternative rationale for asset price stabilization.

5. Concluding Remarks

This paper sheds light on recent debates about whether the central bank should react to fluctuations in asset prices by accounting for expectations that possible regime shifts in monetary policy may occur in the future. As discussed in Leeper and Zha (2003) and Davig and Leeper (2007), expectations that future policy might switch to an alternative regime will affect equilibrium under the current regime. Therefore, by incorporating expectation formation effect, this paper analyzes the issue of monetary policy and asset prices in non-linear policy rules to precisely evaluate the gains or losses in reacting to asset prices.

We find that expectations of possible change in policy regime have a potentially large effect on asset price movements, implying the importance of considering the effect of expectation formation in pricing assets. Contrasted to the linear policy rule that generates negligible stabilization effect from responding to asset prices, the regime switching policy rule significantly shifts the "expected" inflation-output volatility frontier (the Taylor curve) downward, suggesting that expectation formation effect reinforces the gains from reacting to asset price. The trade-off between expected volatility of inflation and that of output, as implied by the Taylor curve, greatly reduces under the regime switching policy rule. This implies that the policy rule accounting for expectation formation effect and reacting to asset price expands the set of policy choices available for the monetary authority.

Finally, given a measure of weighted loss function, we find that overall gains from reacting to asset price could be substantial when accounting for expectation formation effect, while reacting to asset price raises weighted losses under the linear policy rule. Given our calibrated economy, we also find an interior optimal monetary policy in reacting to asset price.

Our results, however, requires cautious interpretation in terms of policy implications. This paper intentionally maintains the model as parsimonious as possible, by abstracting from several features concerning monetary policy and asset price. For example, we do not consider the existence of asset price "bubbles." The model also abstracts from financial market frictions, and thus the transmission mechanism of asset prices in affecting economic aggregates is primitive. A more thorough assessment of the problem calls for a more elaborate model. Incorporating any of these features, in our view, can only provide a stronger rationale for reacting to asset price and therefore strengthen our results.

Nevertheless, our model serves as a preliminary step to evaluate how monetary policy should respond to asset prices when the expectation formation effect is important. The bottom line is that the significance of expectation formation effect can strengthen net gains for reacting to asset price, and thus provide an alternative perspective for policymakers in how to respond to fluctuations in asset prices.

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Table 1. Hausman Test for Endogeneity

Variable X_t	Description	δ_1	t-value
π_t	Inflation Rate	1.7	8.7
x_t	Detrended GDP	0.006	0.05
q_t	Detrended stock price	0.05	0.5

Table 2. Estimated Parameter Values of Monetary Policy Rule When the Fed React to Asset Price ($\alpha_q > 0$)

	Linear Model	Markov Switching Model	
		Hawkish regime	Dovish regime
α_π	1.9***	2.6***	1.5***
	0.1	0.06	0.0553
α_x	0.07*	0.07***	0.01
	0.04	0.0166	0.0280
α_q	0.03	0.02**	0.03
	0.02	0.01	0.02
ψ_π	-1.7***	-2.1***	-1.6***
	0.2	0.22	0.18
σ^M	0.6	0.11***	0.23***
		0.02	0.04
p_{11}		0.97***	
		0.02	
p_{22}		0.96***	
		0.03	
LogLik	-578.90	-543.52	

Note: $2(L_u - L_c) = 70$ is larger than 99% critical value 14.2, *** is significant at 1%, ** is significant at 5% and * is significant at 10%.

Table 3. Model Parameter Values

<i>Other Preference Parameters</i>			
Discount Rate	$\beta = 0.99$		
Relative Risk Aversion	$\sigma = 1$		
<i>Labor Supply</i>			
Elasticity of Labor Supply	$\eta = 0.1$		
Weight of Leisure	$\chi = 0.2$		
<i>Price Setting</i>			
Price Sticky	$\tau = 2/3$		
<i>Aggregate Shocks</i>			
Persistence	$\rho_S = 0.85$	$\rho_D = 0.85$	$\rho_M = 0.6$
Standard dev.	$\sigma_S = 1$	$\sigma_D = 1$	$\sigma_M = 1$

Table 4. The Effect of Reacting to Asset Price Taken Into Account of the Expectation Format Effect

	π_t		x_t		q_t	
	Hawkish	Dovish	Hawkish	Dovish	Hawkish	Dovish
Regime						
Total Effect	0.1625	-0.3705	-0.5176	-0.4994	0.0123	-0.3067
Expectation Format Effect	0.2977	-0.2737	-0.3144	0.0388	0.0808	-0.1147
Direct Effect	-0.1352	-0.0968	-0.2032	-0.5382	-0.0685	-0.1920

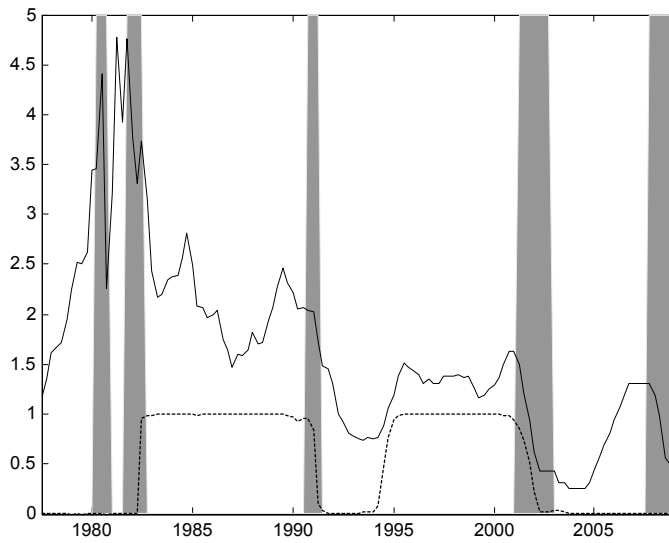
Note : The entries are given by the estimated parameter values in Table 2, which are $p_{11} = 0.97$, $p_{22} = 0.96$, $\alpha_{\pi,1} = 2.6$, $\alpha_{x,1} = 0.07$, $\alpha_{q,1} = 0.02$, $\alpha_{\pi,2} = 1.5$, $\alpha_{x,2} = 0.01$ and $\alpha_{q,2} = 0.03$. It is measured by the difference between the two sets of solutions, one allowing for regime shift and the other under fixed regime, as the volatility of a variable deviating proportionally from a given regime j . A positive(negative) entry means that the expectation formation effect raises (lowers) the volatility of a variable, given a fixed policy regime.

Table 5. The Effect of Reacting to Stock Price under Linear Model

π_t	x_t	q_t
-0.0029	-0.0523	-0.0440

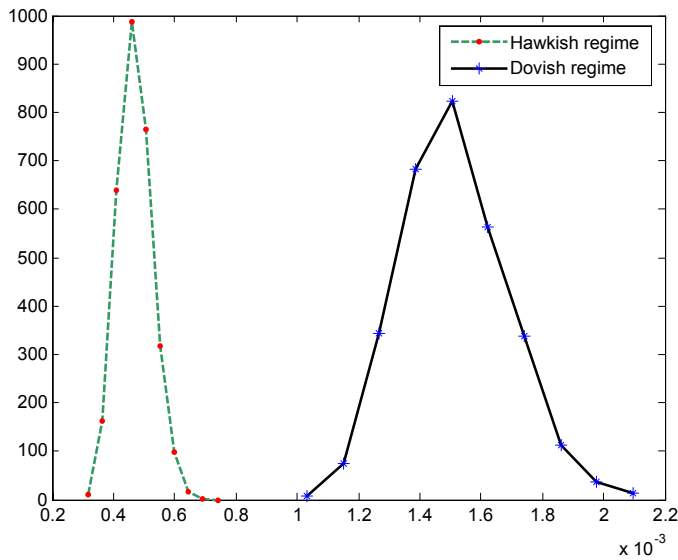
Note : The entries are given by the benchmark estimated parameter values, which are $\alpha_x = 0.07$, $\alpha_\pi = 1.9$ and $\alpha_q = 0.03$.

Figure 1. The Federal Fund Rate and Smoothed Probability of Hawkish Regime



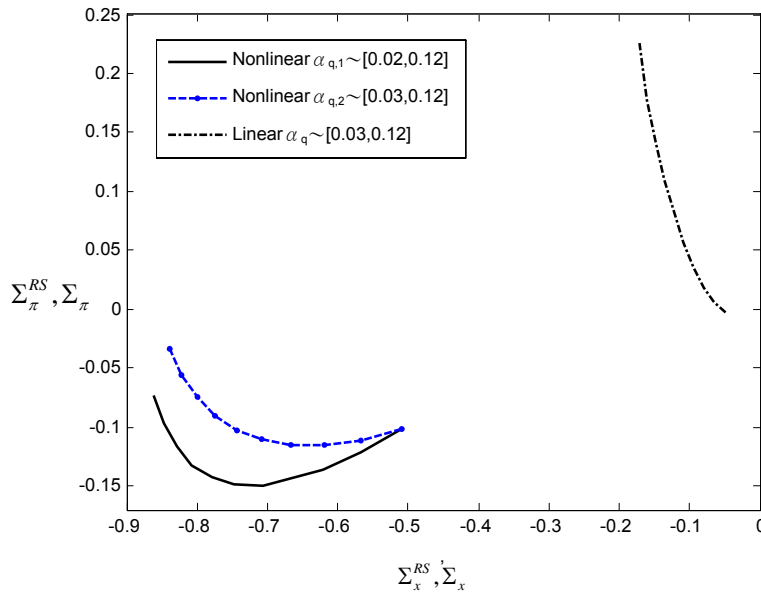
Note: The solid line represents the federal funds rate, the dashed line represents the smoothed probability of the Hawkish regime, and the shaded areas are periods of recession identified by NBER.

Figure 2. The Effect of Expectation Formation on Asset Price



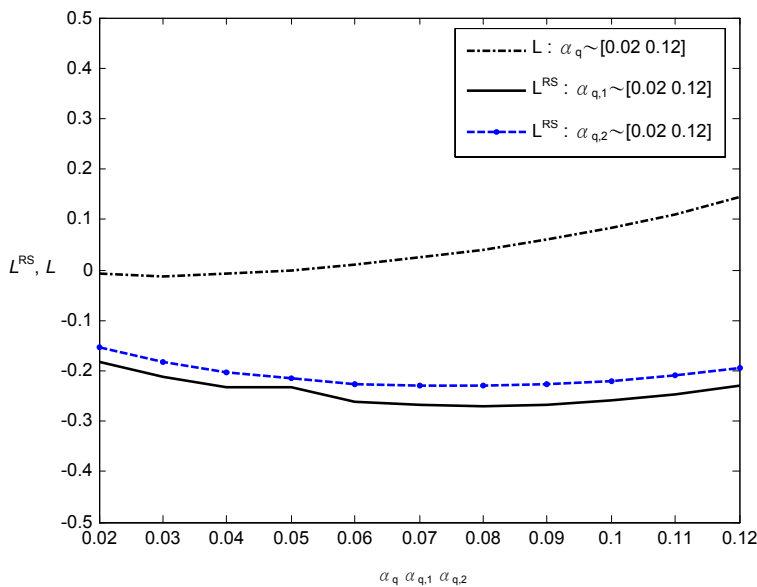
Note: Each observation on a distribution comes from 1000 random drawings, measuring the average quadratic difference between the asset price generated under the model allowing for regime shift and that under the model given a fixed regime j , i.e., the difference in these two asset prices caused by the effect of expectation formation under each regime. We then repeat the procedure for 3000 times to obtain 3000 observations for each given regime j .

Figure 3. Inflation-Output Volatility Frontiers



Note: The horizontal axis is the expected volatility of output gap under the regime switching policy rule and the volatility of output gap under the linear rule; and the vertical axis is the expected volatility of inflation under the regime switching policy rule and the volatility of inflation under the linear rule.

Figure 4. Loss Functions



Note: The horizontal axis is the coefficient of reaction to asset price under the linear policy rule (α_q) and that under the regime switching policy rule ($\alpha_{q,1}$ or $\alpha_{q,2}$), and the vertical axis is the loss function under the linear policy rule (L) and that under the regime switching policy rule (L^{RS}) varying $\alpha_{q,1}$ or $\alpha_{q,2}$, respectively. The relative weight on the output gap stabilization, ω , is taken to be 0.2.

Appendix A.1 Derivation of (10) and (11)

First, we assume that intermediate goods price setting is flexible and the superscript f denotes a variable with flexible prices. Linearizing the first order condition of labor supply $\chi(L_t^f)^\eta/(C_t^f)^{-\sigma} = a_t$ and the intermediate goods production function $Y_t^f = a_t L_t^f$, we have $\eta \hat{L}_t^f + \sigma \hat{C}_t^f = a_t$ and $\hat{Y}_t^f = \hat{L}_t^f + a_t$.

Using the goods market clearing condition $\hat{Y}_t^f = \hat{C}_t^f$, combining with the above two equations, we have

the flexible of price equilibrium output $\hat{Y}_t^f = \frac{1+\eta}{\sigma+\eta} a_t$. Define u_t^D as

$$\begin{aligned} u_t^D &\equiv E_t \hat{Y}_{t+1}^f - \hat{Y}_t^f \\ &= \frac{1+\eta}{\sigma+\eta} (\rho_a - 1) a_t. \end{aligned}$$

The relationship between the aggregate productivity disturbance a_t and u_t^D can be rewritten as

$$a_t = u_t^D \frac{\sigma+\eta}{(1+\eta)(\rho_a-1)}.$$

Turning to the staggered price model, linearizing (2) and using the goods market clearing condition $\hat{Y}_t = \hat{C}_t$, we have

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \hat{\pi}_{t+1}).$$

Expressing this in terms of the output gap $\hat{x}_t \equiv \hat{Y}_t - \hat{Y}_t^f$, we have

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \hat{\pi}_{t+1}) + u_t^D,$$

which is (10) in the main text.

The real dividends from intermediate good firm are

$$d_t \equiv \frac{D_t}{P_t} = Y_t - w_t L_t$$

$$= \left[1 - \frac{\chi Y_t^{\eta+\sigma}}{a_t^{\eta+1}} e^{\frac{u_t^S}{k}} \right] Y_t.$$

Obviously, the dividends are negatively associated with cost-push shocks. Log-linearizing the above equation and normalizing $Y = 1$, we obtain

$$\hat{d}_t = \frac{(1-\chi) - (\eta+\sigma)\chi}{1-\chi} \hat{Y}_t + \frac{(1+\eta)\chi}{1-\chi} a_t - \frac{\chi}{k} u_t^S.$$

Let $\varepsilon = \frac{\chi}{1-\chi}$ and use $\hat{Y}_t \equiv \hat{x}_t + \hat{Y}_t^f$, then \hat{d}_t can be written as:

$$\hat{d}_t = (1 - (\eta + \sigma)\varepsilon)\hat{x}_t + \left[(1 + \eta)\varepsilon + (1 - (\eta + \sigma)\varepsilon)\frac{1 + \eta}{\sigma + \eta} \right] a_t - \frac{\chi}{k} u_t^S$$

$$= \varpi_0 \hat{x}_t + \varpi_1 a_t - \frac{\chi}{k} u_t^S,$$

where $\varpi_0 \equiv (1 - (\eta + \sigma)\varepsilon) > 0$, $\varpi_1 \equiv (1 + \eta)\varepsilon + \varpi_0 \left(\frac{1 + \eta}{\sigma + \eta} \right) > 0$, which guarantees that dividends are positively correlated with output gap and the technology shock. Linearizing (2) and (3) and using (10), we obtain

$$\hat{q}_t = \beta E_t \hat{q}_{t+1} - [\sigma - (1 - \beta)\varpi_0] E_t \hat{x}_{t+1} + \sigma \hat{x}_t - \frac{\chi \rho_s}{k} (1 - \beta) u_t^S$$

$$+ \left[\sigma + (1 - \beta)\varpi_1 \rho_a \frac{\sigma + \eta}{(1 + \eta)(\rho_a - 1)} \right] u_t^D,$$

$$= \beta E_t \hat{q}_{t+1} - \varpi_2 E_t \hat{x}_{t+1} + \sigma \hat{x}_t + \varpi_3 u_t^D - \varpi_4 u_t^S.$$

where $\varpi_2 \equiv \sigma - (1 - \beta)\varpi_0$, $\varpi_3 \equiv \sigma + (1 - \beta)\varpi_1 \rho_a \frac{\sigma + \eta}{(1 + \eta)(\rho_a - 1)}$ and $\varpi_4 \equiv \frac{\chi \rho_s}{k} (1 - \beta)$. This is the equation (11) in the main text.

Appendix A.2 The System of the Model

Given the system (13)-(16), we rewrite the model in a more compact form by substituting (15) into (14) and re-arrange (13), (14) and (16) to characterize the economy by a three-variable system of inflation, output gap, and equity price, $(\hat{\pi}_t, \hat{x}_t, \hat{q}_t)$.

Define $\hat{\pi}_{i,t} \equiv \hat{\pi}_t(s_t = i)$, $\hat{x}_{i,t} \equiv \hat{x}_t(s_t = i)$, and $\hat{q}_{i,t} \equiv \hat{q}_t(s_t = i)$, $i = 1, 2$, which denote the state-dependent inflation, output gap and equity price. To handle the expectations of $\hat{\pi}_{t+1}$, \hat{x}_{t+1} , and \hat{q}_{t+1} in the system, we define the forecasting errors:

$$\begin{aligned}\eta_{i,t+1}^\pi &\equiv \hat{\pi}_{i,t+1} - E_t \hat{\pi}_{i,t+1}, \\ \eta_{i,t+1}^x &\equiv \hat{x}_{i,t+1} - E_t \hat{x}_{i,t+1}, \\ \eta_{i,t+1}^q &\equiv \hat{q}_{i,t+1} - E_t \hat{q}_{i,t+1}, i = 1, 2.\end{aligned}$$

for a given state $s_t = i$ at time t , the one-step-ahead conditional expectations of $\hat{\pi}_{t+1}$, \hat{x}_{t+1} , and \hat{q}_{t+1} are therefore

$$\begin{aligned}E\{\hat{\pi}_{t+1} | s_t = i, \Gamma^{-s}\} &= p_{ii} E_t \hat{\pi}_{i,t+1} + p_{ji} E_t \hat{\pi}_{j,t+1}, \\ E\{\hat{x}_{t+1} | s_t = i, \Gamma^{-s}\} &= p_{ii} E_t \hat{x}_{i,t+1} + p_{ji} E_t \hat{x}_{j,t+1}, \\ E\{\hat{q}_{t+1} | s_t = i, \Gamma^{-s}\} &= p_{ii} E_t \hat{q}_{i,t+1} + p_{ji} E_t \hat{q}_{j,t+1},\end{aligned}$$

where $j \neq i$ and Γ^{-s} is the information set of time t which excludes the current regime.

The system of the model can be expressed by:

$$\begin{aligned}BY_t &= AE_t Y_{t+1} + Cu_t \\ &= AY_{t+1} - A\eta_{t+1} + Cu_t\end{aligned}\tag{A1}$$

where $\eta_{t+1} = Y_{t+1} - E_t Y_{t+1}$. Rearranging the equation, we have

$$AY_{t+1} = BY_t + A\eta_{t+1} - Cu_t, u_t = \rho u_{t-1} + \varepsilon_t$$

$$\text{where } Y_t = \begin{bmatrix} \hat{\pi}_{1,t} \\ \hat{\pi}_{2,t} \\ \hat{x}_{1,t} \\ \hat{x}_{2,t} \\ \hat{q}_{1,t} \\ \hat{q}_{2,t} \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \eta_{1,t}^\pi \\ \eta_{2,t}^\pi \\ \eta_{1,t}^x \\ \eta_{2,t}^x \\ \eta_{1,t}^q \\ \eta_{2,t}^q \end{bmatrix}, \quad u_t = \begin{bmatrix} u_t^S \\ u_t^D \\ u_t^M \end{bmatrix},$$

$$A = \begin{bmatrix} \beta \otimes \mathbf{P} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \sigma^{-1} \otimes \mathbf{P} & \mathbf{P} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & -\varpi_2 \otimes \mathbf{P} & \beta \otimes \mathbf{P} \end{bmatrix}, \quad \rho = \begin{bmatrix} \rho_S & 0 & 0 \\ 0 & \rho_D & 0 \\ 0 & 0 & \rho_M \end{bmatrix},$$

$$B = \begin{bmatrix} \mathbf{I}_{2 \times 2} & & & & & & \\ \sigma^{-1} \alpha_{\pi,1} & & & & & & \\ 0 & & & & & & \\ & \mathbf{I}_{2 \times 2} & & & & & \\ & 0 & 1 + \sigma^{-1} \alpha_{\pi,1} & & 0 & \sigma^{-1} \alpha_{q,1} & 0 \\ & \sigma^{-1} \alpha_{\pi,2} & 0 & & 1 + \sigma^{-1} \alpha_{\pi,2} & 0 & \sigma^{-1} \alpha_{q,2} \\ \mathbf{0}_{2 \times 2} & & & -\sigma \mathbf{I}_{2 \times 2} & & & \mathbf{I}_{2 \times 2} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -\sigma^{-1} \\ 0 & 1 & -\sigma^{-1} \\ -\varpi_4 & \varpi_3 & 0 \\ -\varpi_4 & \varpi_3 & 0 \end{bmatrix}.$$

Appendix A.3 Undetermined Coefficient Method

Following McCallum (2004a), the solution of the Undetermined Coefficient method in our context is given by

$$\begin{aligned}\hat{\pi}_t &= a^S(s_t)u_t^S + a^D(s_t)u_t^D + a^M(s_t)u_t^M, \\ \hat{x}_t &= b^S(s_t)u_t^S + b^D(s_t)u_t^D + b^M(s_t)u_t^M, \\ \hat{q}_t &= c^S(s_t)u_t^S + c^D(s_t)u_t^D + c^M(s_t)u_t^M,\end{aligned}$$

where $\hat{\pi}_t$, \hat{x}_t and \hat{q}_t are functions of the smallest set of state variables. Rewriting the system as the following form:

$$Y_t = Sol^m u_t, \text{ where } Sol^m = \begin{bmatrix} a_1^S & a_1^D & a_1^M \\ a_2^S & a_2^D & a_2^M \\ b_1^S & b_1^D & b_1^M \\ b_2^S & b_2^D & b_2^M \\ c_1^S & c_1^D & c_1^M \\ c_2^S & c_2^D & c_2^M \end{bmatrix}.$$

Plugging $Sol^m u_t$ into Y_t in (A1), we have

$$BSol^m u_t = ASol^m \rho u_t + C u_t.$$

Rewriting the equation as $Sol^m = B^{-1}ASol^m \rho + B^{-1}C$ we thus have

$$vec(Sol^m) = (\mathbf{I} - \rho \otimes B^{-1}A)^{-1}vec(B^{-1}C).$$

Redefining $Z_t = [\hat{\pi}_t, \hat{x}_t, \hat{q}_t, u_t^S, u_t^D, u_t^M]'$, $s_t = 1, 2$, the solution of the system can be expressed by:

$$Z_t = G_{1,s_t} Z_{t-1} + G_{2,s_t} \varepsilon_t,$$

where $\varepsilon_t = [\varepsilon_t^S, \varepsilon_t^D, \varepsilon_t^M]'$. The solution implies that the covariance matrix of Z_t , measured by $\Omega_{s_t} = E[Z_t Z_t']$, which is a 6×6 matrix, can be expressed as the stacked column vector given as follows,

$$\text{vec}[\Omega_{s_t}] = (\mathbf{I} - G_{1,s_t} \otimes G_{1,s_t})^{-1} \text{vec}[G_{2,s_t} G_{2,s_t}'].$$

Similarly, the covariance matrix of Z_t at fixed regime j , given by $\bar{\Omega}_j$, can be expressed as

$$\text{vec}(\bar{\Omega}_j) = (\mathbf{I} - \bar{G}_{1,j} \otimes \bar{G}_{1,j})^{-1} \text{vec}(\bar{G}_{2,j} \bar{G}_{2,j}').$$

Appendix A.4 Fixed Regime Equilibrium

Case 1: Without Responding to Asset Price

Let $p_{jj} = 1$, $j \in \{1,2\}$, we obtain the fixed regime solution using the undetermined coefficient method

$$\begin{aligned}\hat{\pi}_{j,t} &= \frac{\alpha_{x,j} + \sigma(1 - \rho_S)}{\Delta_{j,S}} u_t^S + \frac{\kappa\sigma}{\Delta_{j,D}} u_t^D + \frac{-\kappa}{\Delta_{j,M}} u_t^M, \\ \hat{x}_{j,t} &= \frac{(\rho_S - \alpha_{\pi,j})}{\Delta_{j,S}} u_t^S + \frac{(1 - \beta\rho_D)\sigma}{\Delta_{j,D}} u_t^D + \frac{(\beta\rho_M - 1)}{\Delta_{j,M}} u_t^M, \\ \hat{q}_{j,t} &= \frac{\psi_j}{\Delta_{j,S}} u_t^S + \frac{\phi_j}{\Delta_{j,D}} u_t^D + \frac{(\varpi_2\rho_M - \sigma)}{\Delta_{j,M}} u_t^M,\end{aligned}$$

where $\Delta_{j,c} \equiv \kappa(\alpha_{\pi,j} - \rho_c) + (\alpha_{x,j} + \sigma(1 - \rho_c))(1 - \beta\rho_c) > 0$,

$\psi_j \equiv [(\rho_S\varpi_2 - \sigma - \varpi_4\kappa)\frac{(\alpha_{\pi,j} - \rho_S)}{1 - \rho_S\beta}] - \varpi_4 [\alpha_{x,j} + \sigma(1 - \rho_S)] < 0$ and $\phi_j \equiv \{ \varpi_3[\frac{(\alpha_{\pi,j} - \rho_D)}{(1 - \beta\rho_D)}\kappa + (1 - \rho_D)\sigma + \alpha_{x,j}] + (\sigma - \rho_D\varpi_2)\sigma \} > 0$, $(\varpi_2\rho_M - \sigma) < 0$, $c = S, D, M$ and $j = 1, 2$. To understand the solution, we first consider the responses of inflation, output gap, and asset price when the Fed reacts more rigorously to inflation (a higher $\alpha_{\pi,j}$) or output gap (a higher $\alpha_{x,j}$). We have checked that $\rho_S - \alpha_{\pi,j} < 0$ (if $\alpha_{\pi,j} > 1$), $1 - \beta\rho_D > 0$, $\beta\rho_M - 1 < 0$, and $\partial\Delta_{j,k}/\partial\alpha_{\pi,j} > 0$, for all k .

To examine the signs of those coefficients in the system, consider $z_t = A(\gamma)u_t$, where A is a function of structural parameters γ and u_t following an AR(1) process. Since $\sigma_z^2 = \frac{A^2}{1 - \rho_u^2}\sigma_u^2$, $\partial A^2/\partial\gamma > 0$ implies that the volatility of z_t resulting from u_t will rise. Thus, we have

$$\begin{aligned}\frac{\partial(\psi_j/\Delta_{j,S})^2}{\partial\alpha_{\pi,j}} &= \frac{2(\sigma - \varpi_2\rho_S)[\alpha_{x,j} + (1 - \rho_S)\sigma]\Xi_1}{\Delta_{j,S}^3(1 - \rho_S\beta)} > 0, \\ \frac{\partial(\phi_j/\Delta_{j,D})^2}{\partial\alpha_{\pi,j}} &= \frac{2\kappa\sigma(\varpi_2\rho_D - \sigma)\Xi_2}{\Delta_{j,D}^3(1 - \rho_D\beta)} < 0,\end{aligned}$$

$$\frac{\partial(\psi_j/\Delta_{j,S})^2}{\partial\alpha_{x,j}} = \frac{2(\varpi_2\rho_S - \sigma)(\alpha_{\pi,j} - \rho_S)\Xi_1}{\Delta_{j,S}^3(1 - \rho_S\beta)} < 0,$$

$$\frac{\partial(\phi_j/\Delta_{j,D})^2}{\partial\alpha_{x,j}} = \frac{2\sigma(\varpi_2\rho_D - \sigma)\Xi_2}{\Delta_{j,D}^3} < 0,$$

where $\Xi_1 = (\alpha_{\pi,j} - \rho_S) (\sigma - \varpi_2\rho_S + \varpi_4\kappa) + \varpi_4(1 - \rho_S\beta) (\alpha_{x,j} + \sigma(1 - \rho_S)) > 0$ and $\Xi_2 = (\alpha_{\pi,j} - \rho_D)\kappa\varpi_3 + (1 - \rho_D\beta)[\varpi_3(\alpha_{x,j} + (1 - \rho_D)\sigma) + \sigma(\sigma - \varpi_2\rho_D)] > 0$. Next, we have

$$\frac{\partial((\alpha_{x,j} + \sigma(1 - \rho_S))/\Delta_{j,S})^2}{\partial\alpha_{x,j}} = \frac{2\kappa(\alpha_{\pi,j} - \rho_S)(\alpha_{x,j} + (1 - \rho_S)\sigma)}{\Delta_{j,S}^3} > 0,$$

$$\frac{\partial((\rho_S - \alpha_{\pi,j})/\Delta_{j,S})^2}{\partial\alpha_{\pi,j}} = \frac{2(\alpha_{\pi,j} - \rho_S)(\alpha_{x,j} + (1 - \rho_S)\sigma)(1 - \rho_S\beta)}{\Delta_{j,S}^3} > 0.$$

finally, it is straightforward to check that $\partial[(\varpi_2\rho_M - \sigma)/\Delta_{j,M}]^2/\partial\alpha_{\pi,j} < 0$ and $\partial[(\varpi_2\rho_M - \sigma)/\Delta_{j,M}]^2/\partial\alpha_{x,j} < 0$. We summarize the results in Table A1.

As the table shows, when $\alpha_{\pi,j}$ is higher, a trade-off exists in supply shocks: a more active policy in reacting to inflation reduces the elasticities of inflation, but it raises the responsiveness of output and stock price. An increase in $\alpha_{\pi,j}$ reduces the responsiveness of all variables in response to both demand shock and monetary shock. A more active policy in reacting to output gap (a higher $\alpha_{x,j}$) reduces the responsiveness of output and stock price, but it raises the elasticities of inflation, which mirrors the effect of an increase in $\alpha_{\pi,j}$. However, an increase in $\alpha_{x,j}$ reduces the responsiveness of all variables in response to both demand shock and monetary shock.

Case 2: Responding to Asset Price

When the central bank responds to asset price, we can obtain the fixed regime solution with $p_{jj} = 1$,

$j \in \{1, 2\}$:

$$\hat{\pi}_{j,t} = \frac{\alpha_{x,j} + (1 - \rho_S)\sigma + \frac{(\varpi_4\kappa + \sigma - \rho_S\varpi_2)}{(1 - \beta\rho_S)}\alpha_{q,j}}{\tilde{\Delta}_{2,S}} u_t^S + \frac{\kappa[\sigma - \frac{\varpi_3\alpha_{q,j}}{(1 - \beta\rho_D)}]}{\tilde{\Delta}_{2,D}} u_t^D + \frac{-\kappa}{\tilde{\Delta}_{2,M}} u_t^M,$$

$$\hat{x}_{j,t} = \frac{\varpi_4 \alpha_{q,j} + \rho_S - \alpha_{\pi,j}}{\tilde{\Delta}_{2,S}} u_t^S + \frac{(1 - \beta \rho_D) \sigma - \varpi_3 \alpha_{q,j}}{\tilde{\Delta}_{2,D}} u_t^D + \frac{(\beta \rho_M - 1)}{\tilde{\Delta}_{2,M}} u_t^M,$$

$$\hat{q}_{j,t} = \frac{\psi_j}{\tilde{\Delta}_{2,S}} u_t^S + \frac{\phi_j}{\tilde{\Delta}_{2,D}} u_t^D + \frac{(\varpi_2 \rho_M - \sigma)}{\tilde{\Delta}_{2,M}} u_t^M,$$

where $\tilde{\Delta}_{2,c} = \kappa(\alpha_{\pi,j} - \rho_c) + (\alpha_{x,j} + \sigma(1 - \rho_c))(1 - \beta \rho_c) + \alpha_{q,j}(\sigma - \varpi_2 \rho_c)$, $c = S, D, M$. The responses of endogenous variables to various disturbances can be analyzed as in Case 1.

Table A1. The Effects of $\alpha_{\pi,j}$ and $\alpha_{x,j}$ on Inflation, Output Gap, and Stock Price Given Various Sources of Disturbances

$\alpha_{\pi,j}$	Source of Disturbance			$\alpha_{x,j}$	Source of Disturbance		
	u_t^S	u_t^D	u_t^M		u_t^S	u_t^D	u_t^M
$\hat{\pi}_{j,t}$	-	-	-	$\hat{\pi}_{j,t}$	+	-	-
$\hat{x}_{j,t}$	+	-	-	$\hat{x}_{j,t}$	-	-	-
$\hat{q}_{j,t}$	+	-	-	$\hat{q}_{j,t}$	-	-	-