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# CREDIBILITY AND FLEXIBILITY WITH MONETARY POLICY COMMITTEES

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#### **Abstract**

We consider independent monetary policy committees as a simple way of attaining relatively low inflation without completely sacrificing an activist role of monetary policy. If central banker's types are unknown, then for a wide range of parameters an independent monetary policy committee is better than either a mandated zero-inflation rule or discretionary policy conducted by an opportunistic central banker.

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#### 1. Introduction

The purpose of this paper is to consider independent monetary policy committees as a simple way of attaining relatively low inflation without completely sacrificing an activist role for monetary policy. We show that if central bankers' types are their private information, then an independent committee made up of central bankers with overlapping terms will inflate less than a policy maker who minimises social welfare loss. The likelihood it responds to shocks is increasing in the size of the shock; it is relatively likely to pursue low inflation in normal times, but will react to extraordinary shocks.

Distortions in the labour market give the monetary authority an incentive to increase output. Rational expectations and nominal wage contracting or a Lucas supply curve ensure output is increasing in unanticipated inflation. Thus, a policy maker wants to generate unexpected inflation, while disliking actual inflation. If the private sector knows the policy maker's objectives and has no informational disadvantage, then in equilibrium its expectations are correct. Thus, at the equilibrium inflation rate, the central bank has no incentive to cause surprise inflation. Under the reasonable assumption that the marginal benefit of an increase in unexpected inflation exceeds the marginal cost of actual inflation when inflation and expected inflation equal the socially optimal rate, the outcome is too high inflation, but no unexpected inflation. This is the familiar time-inconsistency problem of monetary policy.

A solution to the time-inconsistency problem is to legally bind the central bank to follow a zero-inflation rule. As a result of the growing belief that using monetary policy to increase employment and output systematically leads solely to excess inflation, many countries have recently done this. For example, the Reserve Bank of New Zealand Act of 1989 statutorily binds the Reserve Bank to price stability, the Bank of England Act of 1997 imposes an inflation target on the Bank of England and the Bank of Japan Act of 1997 orders the pursuit of low inflation.

If central banks could be legally bound to zero inflation or if monetary policy were made by independent central banks that cared solely about inflation, the time-inconsistency problem is solved and there would be no systematic inflation bias. However, society cares about output as well as inflation, and this complicates matters. Stochastic shocks realised after the public's expectations are formed, but before monetary policy is made, provide an activist role for the central bank. An effective zero-inflation rule removes the inflation bias at the expense of the central bank not responding to shocks. This is the tradeoff described by Rogoff (1985).

One proposed solution to Rogoff's problem is a contingent inflation rule or contract. Walsh (1995) proposes a contract where the government imposes a linear penalty on a central bank in excess of its target and pays a reward for inflation below its target. If the contract is properly specified, the tradeoff problem is solved. Svensson (1997) shows that a suboptimally low inflation target can achieve the same goal.

These solutions are problematic. The government must know the central bank's preferences to pick the optimal rule and it only works if the government is fully credible that it will enforce the contract or target. But, the time-inconsistency problem arises precisely because the policy makers are not credible. McCallum (1995) and Briault, Haldane and King (1997) point out that giving the government the responsibility for monitoring the central bank and punishing deviations merely shifts the time-inconsistency

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problem from the central bank to the enforcing government. Possibly as a result of this, actual examples of Walsh (1995) contracts or suboptimally low inflation targets are hard to find.<sup>1</sup>

Another solution is proposed by Lohmann (1992). She suggests that in normal times, a central bank that is "conservative", in the sense of caring more about inflation than society, should be allowed independence. In times of large shocks, the government should threaten to override the central bank if it does not stabilise. Lohmann demonstrates that if the government can choose both the preferences of the central banker and the cost of overriding him, this institutional arrangement dominates appointing either an independent central banker with the same preferences as society or one who is more conservative.

Real world examples of attempts at Lohmann's solution exist. For example, in the face of an economic crisis, the New Zealand government can override the Reserve Bank and the Reserve Bank is allowed to accommodate the first-round effect of the shock on prices. In 'extreme economic circumstances', the UK Treasury is allowed to instruct the Bank of England on monetary policy for a limited time.

Lohmann's strong welfare results depend on the implausible assumption that the government can precisely pick the preferences of the central bank, as well as the size of the cost of overriding the central bank. It also requires a significant amount of government credibility. It must be believable that the government would intervene in extraordinary times if the central bank does not stabilise, but would refrain from intervention in normal times.

Real world escape clause rules suffer from the problem that it is not possible to precisely define the state that triggers intervention. There is unlikely to be agreement over what constitutes, for example 'extreme economic circumstances'. This admits the possibility that opportunistic governments will override the central bank in less than extreme circumstances.

This paper suggests that if central banker's types are unknown, independent monetary policy committees are an arrangement that requires no government interference, produces a nonlinear policy rule similar Lohmann's, and can lead to higher expected social welfare than either a zero-inflation rule or discretion.

We assume that inflation is set by a two-person committee of policy makers who serve overlapping two-period terms. There are two sorts of central bankers. The first is opportunistic, attempting to use surprise inflation to raise output. The second is mechanistic, always voting for zero inflation. McCallum (1995) claims that some central bankers recognize the futility of opportunistic behavior and simply refrain from it. This second type may also be viewed as caring solely about inflation. Perhaps Hans Tietmeyer and Paul Volcker represent examples of this type of central banker. Adopting the avian terminology of the British press, we will call the first type a *dove* and the second type a *hawk*. A policy maker's type is his private information.

<sup>&</sup>lt;sup>1</sup> See Briault, Haldane and King (1997) for a discussion of this.

In this setup, doves may be deterred from voting for inflation in their first period in office by an incentive to gain a reputation for inflationary toughness. If a dove does not vote for inflation in his first period in office, then the likelihood the public attaches to his being a hawk goes up. This lowers future expected inflation, making future inflationary surprises less costly. Thus, doves may masquerade as hawks when they first take office to lower expected inflation later on.<sup>2</sup>

A dove's incentive to act like a hawk will depend on the size of the current stochastic shock. Doves will find it relatively attractive to pretend to be hawks when shocks are small. However, the reputational gain is less likely to be worth not responding to large shocks.

We establish analytically some sufficient conditions for a committee to be better than a rule or discretionary policy making by a single dove. If the variance of the shocks is small enough, then a zero-inflation rule is better than discretionary policy making. If the variance is not too small and if the proportion of hawks in the candidate policy maker population is sufficiently high, we show that the committee produces a lower expected welfare loss than a zero-inflation rule. If the variance of the shocks is sufficiently large, discretion is better than rule. We show that unless the variance is very large, the committee is better than discretion as long as the fraction of doves in the candidate policy maker population is not too small.

More precise results require specification of the shock's distribution and a numerical solution. We assume that the shock is lognormally distributed. When the variance is such that discretion and a zero-inflation rule yield the same expected welfare loss, committees are better no matter what the ratio of hawks to doves. For higher variances, the committee is still better if the ratio of hawks to doves in the policy maker population is low enough. This cutoff level falls as the variance rises. With a lower variance, there is a minimum proportion of hawks that is necessary to ensure committees are better than a rule. As the variance falls, this minimum proportion rises. For a large range of variances of the shock, committees are better than rules or discretion for a wide range of policy maker populations.

This welfare result suggests that the independent committees described here provide an attractive way of trading off low inflation and responding to shocks. They also require less government commitment to work than does a contingent rule or Lohmann's (1992) escape-clause solution. There is no need for the government to monitor or influence the central bank's behavior, it needs only to set up an independent central bank. This appears possible; it may be that opportunistic governments can sometimes commit themselves to a broad constitutional or quasi-constitutional arrangement.<sup>3</sup> The solution does not require or allow any intervention to punish a deviating government. Thus, a government cannot use the excuse of punishing a deviation or invoking an escape clause to intervene inappropriately.

Backus and Driffill (1985) adopt Kreps and Wilson's (1982) reputation model to provide the original monetary policy model of hawks and doves. Sibert (2002a) extends this framework to a committee structure, similar to the one here. Neither Backus and Driffill (1985) nor Sibert (2002a) consider an activist role for monetary policy.

<sup>&</sup>lt;sup>3</sup> This is discussed in more detail in Buiter and Sibert (2001).

The solution does not require the unrealistic assumption that the government knows - let alone can *choose* - the preferences of the central bank. The government can improve welfare, however, by making a more concerted attempt at appointing hawks or doves, thus changing the public's prior beliefs about the composition of the potential policy maker population.

The model is presented in Section 2. In Section 3, we look at expected inflation when policy is made by a committee. In Section 4, we compare the expected welfare loss under a committee with that with a zero-inflation rule and discretion. Section 5 is the conclusion.

#### 2. The Model

The underlying macroeconomic framework is a variant of the Barro-Gordon (1983) model. Society's within-period welfare loss is increasing in inflation and decreasing in output. Inflation is disliked because it leads to shoe-leather and menu costs; it makes the domestic currency an inconvenient unit of account; it may redistribute income in a way that is perceived as unfair; and, in the presence of staggered nominal price contracts, it distorts relative prices. Either nominal wage contracting and rational expectations, as in the Barro and Gordon (1983) model, or a Lucas (1976) expectations view of aggregate supply ensure output is increasing in unanticipated inflation.

The loss to society in period t is represented by

$$\pi_t^2/2 - \chi(\pi_t - \pi_t^e)\varepsilon_t, \tag{1}$$

where  $\pi_t$  is period-t inflation,  $\pi_t^e$  is the public's expectation of period-t inflation, conditional on variables dated t- 1 and earlier,  $\varepsilon_t$  is an i.i.d. output shock with positive function  $f(\varepsilon)$  on  $\mathbb{R}_+$ , and  $\chi > 0$  is the weight society places on output loss relative to inflation.

As in Barro and Gordon (1983), the loss function is assumed to be linear in output and can be interpreted as representing social preferences over inflation and unemployment with a standard expectations-augmented Phillips curve.<sup>4</sup> This assumption, which is also made by Backus and Driffill (1985) and Cukierman and Meltzer (1986), is necessary for tractability here because it ensures the policy maker has a dominant strategy in his final period in office and allows first-period expected inflation to be taken as a constant.<sup>5</sup>

Drazen (2000) provides an extensive discussion of this and other loss functions used in the literature.

<sup>5</sup> The alternative quadratic specification implies booms are disliked as much as recessions. However, it has the attractive feature, missing here, that society dislikes output volatility.

The multiplicative, rather than additive, form of the output shock in equation (1) is somewhat unusual.<sup>6</sup> With a nominal-wage contracting story, it can be viewed as a technological shock. For a Cobb-Douglas production function it is a shock to labour's share of output. In the Lucas (1973) model it is a shock to the slope of the (expectations-augmented) Phillips curve. An example would be a decrease in the volatility of aggregate demand that lowers the variance of the general price level and flattens the Phillips curve. Consistent with the particular specification, we assume that the shock has mean one, implying that the expected slope of the Phillips curve is constant. The interpretation of the multiplicative shock to the slope of the Phillips curve is particularly intuitive because a large shock flattens the curve and, as we show, increases the incentive to inflate. There may be an additive component to the shock as well, but this is not made explicit as it can be treated as a constant in the central bank's optimisation.

Monetary policy is made by a committee of members with overlapping terms. This structure is chosen for two reasons. First, it replicates the way monetary policy is actually made in many countries. Second, reputation models with a single policy maker (for example, Backus and Driffill (1985)) have the empirically unattractive result that inflation tends to be low in the first part of a dove's tenure and high at the end.

Choosing the simplest scenario, we suppose the committee has two members and they serve two periods. Policy makers come in two types. Hawks always vote for zero inflation. They can be viewed as mechanistic or as caring solely about inflation. Doves are opportunistic and benevolent, wanting to minimise social welfare loss.<sup>7</sup> A policy maker's type is his private information and it is common knowledge that a fraction  $\rho \in (0,1)$  of policy makers are hawks.<sup>8</sup>

The policy maker taking office at time t is denoted by  $\theta_t$ . At the beginning of period t, the private sector forms its expectation of inflation,  $\pi_t^e$ , then the shock is observed by all, and then  $\theta_{t-1}$  and  $\theta_t$  jointly choose inflation and their individual votes are published.<sup>9</sup>

Both Dixit and Lambertini (1999) and Dixit and Jensen (2000) consider multiplicative shocks. The alternative specification of an additive shock, coupled with quadratic preferences, provides a stabilisation role for the central bank. The activist role for monetary policy that is analysed here is the multiplicative shock analogue of the stabilisation role for monetary policy that is more commonly discussed in the literature.

Assuming that some agents are opportunistic, while others are mechanistic, is standard in reputation models. (See, for example, Backus and Driffill (1985)). The presence of hawks gives doves an incentive to build a reputation for inflationary toughness by emulating hawks. Alternatively, Vickers (1986) and Sibert (2002b) suppose all agents are strategic and care about both inflation and output, but some care more about inflation than others. They show that relatively inflation-averse types inflate less than they otherwise would to signal they are not less inflation averse.

It is usual in reputation models to imagine that  $\rho$  is small. This need not be the case here. Blinder (1997) suggests that central bankers tend to be inflation averse, saying, "... the noun 'central banker' practically cries out for the adjective 'conservative' (page 14)."

<sup>&</sup>lt;sup>9</sup> The Bank of England, Bank of Japan and the Federal Reserve all publish members'votes.

Suppose there were a single opportunistic policy maker. If he held office for only one period, he would minimise social welfare loss. Minimising equation (1), taking  $\pi_t^e$  as given, he would choose inflation to be  $\chi \varepsilon_{t}$ . Note that this choice of inflation leads to higher inflation at times when the Phillips curve is flatter and the output gain is higher. If his tenure lasts two periods, then in his second period in office, he would choose inflation to be  $\chi \varepsilon_{t+1}$ . But, in period one he might choose zero inflation to increase the private sector's belief that he might be a hawk. By strengthening this belief, he would increase the benefit of inflating unexpectedly in period two.

If there are two policy makers, then at time t each will vote for inflation of either  $\chi \varepsilon_t$  or zero.<sup>10</sup> If both prefer the same policy, that policy is implemented. If one policy maker prefers zero and one policy maker prefers  $\chi \varepsilon_t$ , then a compromise inflation rate,  $\alpha \chi \varepsilon_t$  is enacted.<sup>11</sup> We will assume that  $\alpha$  is such that the loss for the dove is equal to the average of the loss if his preferred inflation rate is implemented and the loss if the hawk's choice of zero is chosen. Then,  $\alpha$  must satisfy

$$(\alpha \chi \varepsilon_t)^2 / 2 - \chi (\alpha \chi \varepsilon_t - \pi_t - \pi_t^e) \varepsilon_t = (1/2) \left[ (\chi \varepsilon)^2 / 2 - \chi (\chi \varepsilon_t - \pi_t^e) \varepsilon_t \right] + (1/2) \chi \pi_t^e \varepsilon_t. \tag{2}$$

Solving yields  $2\alpha^2 - 4\alpha + 1 = 0$ . Thus the above expressions equal  $\chi^2 \mathcal{E}_t^2 / 4 + \chi \pi_t^e \mathcal{E}_t$  and  $\alpha = 1 - 1/\sqrt{2} < 1/2$ .

Consider the scenario in period t. The senior policy maker votes for zero inflation if he is a hawk and for inflation of  $\chi \varepsilon_t$  if he is a dove. The new policy maker votes for zero inflation if he is a hawk and solves a two-period problem by backwards recursion if he is a dove<sup>13</sup>. The solution is the probability he does *not* vote for inflation in period t. We refer to this probability as his strategy and we allow mixed strategies, where the probability is between zero and one. We suppose that doves and the public have the common discount factor  $\delta \in (0,1)$ .

If the policy maker taking office at t is a dove, he knows he will vote for inflation in period t+1. Expected social welfare loss in t+1 depends on the likelihood  $\theta_{t+1}$  is a dove and the conjectured probability that  $\theta_{t+1}$  votes for inflation in period t+1 if he is a dove. Thus, we must specify how  $\theta_t$  and the private sector believe  $\theta_t$ 's actions and strategy affect the strategy of  $\theta_{t+1}$ , if he is a dove.

<sup>&</sup>lt;sup>10</sup> No one can credibly claim they want anything else.

Another possibility would be to have an odd-numbered committee with the median voter's preferred inflation chosen. However, it appears common for monetary policy groups to vote for a consensus view. Blinder (1997, p.16) claims that the United States' FOMC makes decisions by consensus, not by majority rule.

The assumption that the policy makers compromise this way is admittedly *ad hoc*. The way that the central bank reaches a decision might be written into its constitution and need not be the result of a bargaining process. Sibert (2002a) examines the effect of varying  $\alpha$ .

<sup>13</sup> The policy maker may care about what happens after he leaves office, but he cannot influence inflationary expectations beyond his tenure. Therefore, he solves a two-period problem.

Following Prescott and Townsend (1980), we restrict attention to equilibria in minimal state or memoryless Markov strategies. How we assume that the strategy of a dove who has just taken office is a time-invariant function of the current shock. Our selection of compromise inflation ensures that a junior dove's strategy does not depend on the senior policy maker's type. By equation (1), the current gain to voting for, rather than against inflation is  $(\chi \alpha \varepsilon_t)^2/2 - \chi^2 \alpha \varepsilon_t^2$  if the senior policy maker is a hawk and  $(\chi \varepsilon_t)^2/2 - \chi^2 \varepsilon_t^2 - (\chi \alpha \varepsilon_t)^2/2 - \chi^2 \alpha \varepsilon_t^2$  if he is a dove. By equation (2), both these gains equal  $-\chi^2 \varepsilon_t^2/4$ .

Let  $\phi^*(\varepsilon_{t+1})$  denote the public's and  $\theta_t$ 's conjectured probability that  $\theta_{t+1}$  votes for zero inflation if he is a dove. Then, the probability that an *arbitrary* policy maker taking office at time t+1 votes for zero inflation at t+1 is conjectured to be  $y^*(\varepsilon_{t+1}) \equiv \rho + (1-\rho) \phi^*(\varepsilon_{t+1})$ .

Let  $p_{t+1}$  denote the private sector's beginning of period t+1 probability assessment that the policy maker who took office at time t is a hawk. Then, the private sector's and  $\theta_t$ 's expectation of inflation in period t+1 is

$$\pi_{t+1}^{e} = \pi^{e}(p_{t+1}) \equiv$$

$$\chi \int_{0}^{\infty} \{ (1 - p_{t-1}) y^{*}(\varepsilon) \alpha + p_{t+1} [1 - y^{*}(\varepsilon)] \alpha + (1 - p_{t+1}) [1 - y^{*}(\varepsilon)] \varepsilon f(\varepsilon) d\varepsilon =$$

$$\chi \int_{0}^{\infty} [y^{*}(\varepsilon) \alpha + 1 - y^{*}(\varepsilon)] \varepsilon f(\varepsilon) d\varepsilon - \chi p_{t-1} A^{*}$$

$$\text{where } A^{*} \equiv 1 - \alpha - (1 - 2\alpha) \int_{0}^{\infty} y^{*}(\varepsilon) \varepsilon f(\varepsilon) d\varepsilon.$$

$$(3)$$

By (3) and the definition of  $y^*$ ,  $A^*$  is decreasing in  $\rho$ . An increased belief that  $\theta_t$  is a hawk increases the likelihood of compromise inflation, rather than within-period optimal inflation, if  $\theta_{t+1}$  is a dove, and increases the likelihood of zero inflation rather than compromise inflation if  $\theta_{t+1}$  is a hawk. Because  $\alpha < 1/2$ , the former gain exceeds the latter. Thus, an increase in the likelihood that  $\theta_{t+1}$  is a hawk lowers the value of a gain in reputation.

The private sector updates its beliefs with Bayes' rule. Thus,

$$p_{t+1} = \begin{cases} P(y^*(\varepsilon_t)) \equiv \rho/y^*(\varepsilon_t) & \text{if } \theta_t \text{ votes for zero inflation at t,} \\ 0 & \text{otherwise.} \end{cases}$$
 (4)

The Markov restriction rules out repeated-game equilibria where past strategies influence current play, not because they influence the state of the economy, but solely because players believe that past strategies matter. See Maskin and Tirole (1988) for a discussion of the relative merits of Markov and repeated-game equilibria.

By (4) and the definition of  $y^*$ , P is increasing in  $\rho$ . If the junior dove votes for inflation, he is revealed to be a dove. If he does not, the public is unsure whether he is a hawk or a dove masquerading as a hawk. The greater the proportion of hawks, the more likely it considers the former scenario to be. Thus, an increase in  $\rho$  makes it easier to gain a reputation for inflationary toughness.

By equation (1) and the Markov restriction, social welfare loss at time t+1 depends on the action of the time-t policy maker solely through  $\pi_{t+1}^e$ . By equation (1),  $\pi_{t+1}^e$  depends on the action of the time-t policy maker, if he is a dove, can be written as

$$-\phi_{t}\chi^{2}\varepsilon_{t}^{2}/4 + \delta\chi\left[\phi_{t}\pi^{e}(P(y^{*}(\varepsilon_{t})) + (1-\phi_{t})\pi^{e}(0)\right]$$
(5)

where unimportant constants are ignored.

By equation (3), each term in equation (5) is multiplied by  $\chi^2$ ; hence,  $\chi$  is an unimportant constant in the optimisation problem. The intuition is that an increase in  $\chi$  raises the cost of lower current inflation and the benefit of higher expected inflation proportionately. Using equations (3) and (4) and eliminating unimportant constants, the policy maker minimises

$$\phi_t[\varepsilon_t^2/4 - \delta P(y^*(\varepsilon_t))A^*]. \tag{6}$$

If the junior dove does not vote for inflation, inflation will be further below its within-period optimal level than it would have been if he had voted for inflation. The first term in the brackets represents this cost. The second term represents the discounted gain from not voting for inflation. This gain is equal to the increase in reputation (P) multiplied by the value of an enhanced reputation  $(A^*)$ .

Given conjectures, an increase in  $\delta$  increases the current benefit to voting against inflation. If the future becomes more important, so does gaining a reputation and the incentive to inflate falls. An increase in  $\rho$ , however, has an ambiguous effect. It increases P and lowers  $A^*$ .

Equation (6) is linear in  $\phi_i$ ; hence a solution has

$$\varepsilon_t^2 - 4\delta P(y^*(\varepsilon_t))A^* \begin{cases} > \\ = \\ < \end{cases} 0 \text{ and } y_t \begin{cases} = \rho \\ \in [\rho, 1] \\ = 1 \end{cases}. \tag{7}$$

If the current cost to zero inflation exceeds the expected future benefit, then junior doves never vote for zero inflation. If the expected future benefit exceeds the current cost, then they always do. If the cost equals the benefit, then junior doves are indifferent over randomisations between inflating and not inflating.

Consistency requires conjectures are correct. The Markov restriction implies strategies are not time-varying. Thus,  $\phi_t = \phi^*(\varepsilon_t) = \phi(\varepsilon_t)$  and  $y_t = y^*(\varepsilon_t) = y(\varepsilon_t)$  where  $y(\varepsilon_t) = \rho + (1 - \rho)\phi(\varepsilon_t)$  for every t. Substituting equation (4) into equation (7) and using the definition of  $A^*$  from equation (3) yields that an equilibrium must satisfy

$$\varepsilon_{t}^{2}y(\varepsilon_{t}) - 4\delta\rho \left[ 1 - \alpha - (1 - 2\alpha) \int_{0}^{\infty} y(\varepsilon)\varepsilon f(\varepsilon)d\varepsilon \right] \begin{cases} > \\ = \\ < \end{cases} \text{ o and } y(\varepsilon_{t}) \begin{cases} = \rho \\ \in [\rho, 1] \\ = 1 \end{cases}. \tag{8}$$

The left-hand side of equation (8) is continuous in the shock and is strictly negative at  $\varepsilon_{t}=0$ . Thus, there exists a right-hand-side neighbourhood of zero such that doves never vote for inflation if the shock is in that interval. If the shock is sufficiently large, junior doves always vote for inflation. Thus, solving (8) requires finding two cutoff values of  $\varepsilon_{t}$ . If  $\varepsilon_{t}$  is below the lower cutoff value,  $y(\varepsilon_{t})=1$  and junior policy makers never vote for inflation. If  $\varepsilon_{t}$  is above the higher cutoff value,  $y(\varepsilon_{t})=\rho$  and hawks never vote for inflation, while doves always do. If  $\varepsilon_{t}$  is between the two cutoff values, then  $y(\varepsilon_{t})$  is a function of  $\varepsilon_{t}$  that takes values on  $(\rho,1)$ .

The second term of equation (8) does not depend on the realisation of the shock. Thus, when (8) holds with equality, the realisation of the shock cannot affect the first term either. Thus, between the two cutoff values,  $y(\varepsilon_t)$  must take the form  $c/\varepsilon_t^2$ , where c is a strictly positive constant. Then, by equation (8), the lower cutoff value is the  $\varepsilon_t$  that satisfies  $c/\varepsilon_t^2 = 1$  and the higher cutoff value is the  $\varepsilon_t$  that satisfies  $c/\varepsilon_t^2 = \rho$ . Thus, the two cutoff values are  $\sqrt{c}$  and  $\sqrt{c/\rho}$  and

$$y(\varepsilon_{t}) = \begin{cases} \rho \text{ if } \varepsilon_{t} > \sqrt{c/\rho} \\ c/\varepsilon_{t}^{2}, \text{ if } \sqrt{c} < \varepsilon_{t} < \sqrt{c/\rho}. \\ 1 \text{ if } \varepsilon_{t} < \sqrt{c} \end{cases}$$
(9)

Substituting equation (9) into equation (8) when equality holds yields

$$c = 4\delta\rho(1 - \alpha) - 4\delta\rho(1 - 2\alpha)H(c,\rho)$$
where  $H(c,\rho) := \int_{0}^{\infty} \varepsilon f(\varepsilon)d\varepsilon + c \int_{\sqrt{c}}^{\infty} f(\varepsilon)/\varepsilon d\varepsilon + \rho \int_{\sqrt{c/\rho}}^{\infty} \varepsilon f(\varepsilon)d\varepsilon \in (\rho,1).$  (10)

**Definition 1.** An equilibrium is a strictly positive constant c such that equation (10) is satisfied. **Proposition 1.** There exists a unique equilibrium and it is an element of  $(4\alpha\delta\rho, 4\delta\rho (1-\alpha-\rho (1-2\alpha)))$ . **Proof.** See the Appendix.

For small values of the shock, there is a pooling equilibrium where no junior policy maker votes for inflation. For large values of the shock, there is a separating equilibrium where hawks never vote for inflation and doves always do. For intermediate values of the shock, there is a semi-separating equilibrium where junior doves randomise between voting for and voting against inflation.

#### 3. Inflation

In this section, we look at equilibrium inflation and show how it varies with the model's parameters. We demonstrate that expected inflation (conditional on the shock) is a nonlinear function of the shock. Committees react more to large shocks than they do to small ones.

How do changes in the parameters,  $\delta$  and  $\rho$  affect the likelihood a junior dove or an arbitrary policy maker votes for zero inflation?

**Proposition 2**. An increase in  $\delta$  increases the likelihood either a junior dove or an arbitrary policy maker votes for zero inflation. For every  $\delta \in (0,1)$ , there exists  $\rho^* \in (0,1)$  such that an increase in  $\rho$  increases (decreases) the likelihood an arbitrary policy maker votes for zero inflation if  $\rho < (>) \rho^*$ . For  $\delta$  sufficiently small,  $\rho^* < 1$ .

**Proof**. See the Appendix.

An increase in  $\delta$  has no effect on hawks. It increases c; hence, it strictly increases the likelihood a junior dove votes for zero inflation in the interval of shocks where doves randomise. It also increases the two cutoff points, enlarging the interval of shocks where junior doves always vote for zero inflation and shrinking the interval where they never do. The intuition is that if the future is more important, gaining a reputation is more valuable.

As previously discussed, an increase in  $\rho$  makes gaining a reputation for inflationary toughness easier, but it lowers the value of having a reputation. As a result, the effect of  $\rho$  on the likelihood either a junior dove or even an arbitrary junior policy maker votes for inflation is ambiguous.

Be equation (9), expected inflation, given that the current shock is  $\varepsilon_{\rm t}$  is

$$\{(1 - \rho)y(\varepsilon_{t}) \alpha + \rho [1 - y(\varepsilon_{t})] \alpha + (1 - \rho)[1 - y(\varepsilon_{t})]\}\chi\varepsilon_{t}$$

$$= \begin{cases} (1 - \rho)(2\rho\alpha + 1 - \rho)\chi\varepsilon_{t} & \text{if } \varepsilon_{t} \geq \sqrt{c/\rho} \\ (1 - \rho + \rho\alpha)\varepsilon_{t} - [\rho\alpha + (1 - \rho)(1 - \alpha)]c/\varepsilon_{t}\}\chi & \text{if } \sqrt{c} < \varepsilon_{t} < \sqrt{c/\rho}. \end{cases}$$

$$(11)$$

$$(1 - \rho)\alpha\chi\varepsilon_{t} & \text{if } \varepsilon_{t} < \sqrt{c}$$

The expected inflation curve is continuous in the shock with kinks at the cutoff values of the shocks,  $\sqrt{c}$  and  $\sqrt{c/\rho}$ . Above the higher cutoff value and below the lower cutoff value, expected inflation rises linearly in the shock. The slope is higher for shocks above  $\sqrt{c/\rho}$  than for shocks below  $\sqrt{c}$ . It is important to stress that in both cases it is lower than  $\chi$  - the value of the slope under discretion. Between the two cutoff points, inflation rises at a decreasing rate that is always greater than the slope of the curve below  $\sqrt{c}$ . To provide a graphic illustration of the reaction function we have to assume specific values for the key parameters and for the stochastic properties of the shock. In Figure 1 we present the relationship between inflation and the underlying shocks for  $\chi=1$ ,  $\rho=0.3$ ,  $\delta=0.8$  and a mean-one lognormal density function.

So far we have established the link between expected inflation and the model's key parameters under the assumption that we observe the value of the current shock. But to evaluate the advantages of having a committee we need to establish the dependence of the unconditional expectation of inflation on the underlying structural characteristics of the economy.

By equations (10) and (11), unconditional expected inflation is

$$E(\pi) = \chi \{ 1 - \rho + \rho \alpha - [\rho \alpha + (1 - \rho)(1 - \alpha)]H(c, \rho) \}. \tag{12}$$

The following result establishes that expected inflation is decreasing in both  $\delta$  and  $\rho$ .

**Proposition 3**. An increase in either  $\delta$  or  $\rho$  lowers unconditional expected inflation. **Proof**. See the Appendix.

An obvious consequence of Proposition 3 is that as more weight is put on the future, the lower is inflation. While an increase in  $\rho$  may lower expected inflation for some values of  $\varepsilon$ , if  $\rho$  is close enough to one, an increase in  $\rho$  always lowers unconditional expected inflation.

#### 4. Welfare

In this section, we look at the welfare implications of having inflation chosen by a committee. We compare the expected welfare loss with that under a zero-inflation rule and with that with an opportunistic central banker making policy at his discretion.

The social (command) optimum would be achieved if inflation were set equal to  $\chi \varepsilon_t$  -  $\chi$  in period t. In this case expected inflation equals zero and the central bank responds optimally to the shocks. Unfortunately, central bankers cannot commit themselves to doing this and it is not likely that the government can credibly impose a state-contingent inflation rule on the central bank. If society could appoint a completely conservative central banker (a hawk) or credibly impose a zero-inflation rule, then the expected welfare loss in any period would be zero.

If it appointed a dove and could not impose a rule, inflation in period t would be  $\chi\varepsilon_t$  and the expected welfare loss would be  $\chi^2(1-\sigma^2)/2$ , where  $\sigma^2$  is the variance of the shock. As long as this variance is less than one, it would be better if society could appoint a hawk or to follow a zero inflation rule. If society could pick a central banker with preference parameter  $\chi^*$ , then the expected social welfare loss would be  $\chi^{*2}(\sigma^2+1)/2-\chi\chi^*\sigma^2$ . The optimal choice of  $\chi^*$  is  $\chi\sigma^2/(1+\sigma^2)$ . It is optimal to appoint a central banker who is more conservative than society, but not so conservative as to place no weight on output. This is Rogoff's (1985) result.

Suppose monetary policy is the discretionary choice of a dove. The expected welfare loss is  $\int_S \chi^2(\varepsilon - \varepsilon^2/2) f(\varepsilon) d\varepsilon$ , where S is the support of  $\varepsilon$ . The integrand is positive for small shocks, but it is decreasing at an increasing rate and becomes negative if  $\varepsilon > 2$ . This suggests that a regime mandating zero inflation for small shocks, but allowing discretion for large shocks, would be better than either a rule or discretion. This is a rationale for the escape clauses in central banking legislation and the intuition behind Lohmann's (1992) proposal.

Figure 2 (which is drawn for  $\chi=1,\;\rho=0.3,\;\delta=0.8$  and a lognormal density function), depicts optimal inflation, discretionary inflation, inflation with Rogoff's optimally conservative central banker, and (conditional) expected inflation with a committee. From the above discussion and the figure, it is clear the committee is somewhat similar to Rogoff's conservative central banker solution. By having both hawks and doves on the committee, the committee is more conservative than society, but not completely conservative. It also looks somewhat like a rule with an escape clause (not shown). Because strategic doves are more willing to sacrifice their reputations for big shocks than for small shocks, committees are more likely to inflate when faced with big shocks than with small shocks. This suggests that committees may share some of the welfare-enhancing qualities of both Rogoff's conservative central banker and a rule with an escape clause.

It is possible to obtain simple analytical results relating the welfare with a committee to welfare with a zero-inflation rule and welfare with (opportunistic) discretionary policy making. Using equation (1) and the definition of  $\alpha$ , the expected welfare loss with a committee is

$$-\frac{\chi^2}{4}\int_0^\infty \{\rho[1-y(\varepsilon)] + (1-\rho)y(\varepsilon) + 2(1-\rho)[1-y(\varepsilon)]\}\varepsilon^2 f(\varepsilon)d\varepsilon + \chi E(\pi_t). \tag{13}$$

By equation (9), this equals

$$-\frac{\chi^{2}}{4}[(2-\rho)(\sigma^{2}+1)-G(c,\rho)]+\chi E(\pi_{t}), \tag{14}$$

where 
$$G(c,\rho) \equiv \int_{0}^{\sqrt{c}} \varepsilon^{2} f(\varepsilon) d\varepsilon + c \int_{\sqrt{c}}^{\sqrt{c/\rho}} f(\varepsilon) d\varepsilon + \rho \int_{\sqrt{c/\rho}}^{\infty} \varepsilon^{2} f(\varepsilon) d\varepsilon$$

and  $\sigma^2$  is the variance of the shock.

**Proposition 4.** If  $\sigma^2 > 2\alpha^2$ , then there exists  $a \rho^* \in (0,1)$  such that committees are better than a zero-inflation rule if  $\rho > \rho^*$ .<sup>15</sup>

**Proof**. See the Appendix.

**Proposition 5.** If  $\sigma^2 < 2(1 - \alpha^2)$ , then there exists  $a \rho^* \in (0,1)$  such that committees are better than discretionary policy making by a dove if  $\rho < \rho^*$ .<sup>16</sup>

**Proof.** See the Appendix.

 $<sup>^{15}</sup>$   $2\alpha^2$  is approximately equal to .17.

 $<sup>^{16}</sup>$   $2(1-\alpha^2)$  is approximately equal to 1.83.

Propositions 4 and 5 are in the spirit of Rogoff's (1985) result. If all potential policy makers are hawks ( $\rho$  = 1), the outcome is zero inflation, but no response to shocks. If all potential policy makers are doves ( $\rho$  = 0), the outcome is an optimal response to shocks, but too high inflation. By increasing the ratio of hawks to doves in the policy maker population, society can attain lower inflation at the cost of less activism. As long at the variance of the shock is not very small, some doves are better than none. Thus, a committee is better than a zero-inflation rule as long as  $\rho$  is not too small. If the variance of the shock is not too big, some hawks are better than none. A committee is better than discretionary policy making by a dove if  $\rho$  is not too big.

Over a wide range of variances, a committee can be better than either a rule or discretion, as long as the government has sufficient control over  $\rho$ . While it is unlikely that the government can reliably pick the preferences of a policy maker, it can easily affect the makeup of the potential policy maker population. It can, for example, appoint people from the financial sector in an attempt to get hawks or representatives from the manufacturing sector in an attempt to get doves.

The above results give *sufficient* conditions for a committee to dominate rules and discretion. An important aspect of reputation building was not exploited in the proofs, however. As noted above, when a dove makes policy at his discretion, the loss in any period decreases at an increasing rate in the size of the shock. When deciding between the benefits of current inflation or an enhanced reputation as a hawk, a junior dove is more likely to pick the former when shocks are large. Thus, he is especially flexible, preferring price stability when shocks are small and activism when shocks are large.

To capture this feature, and evaluate the benefits of a committee more precisely, it appears necessary to specify the distribution function of the shocks and to solve the model numerically. We assume that the shock is lognormally distributed.<sup>17</sup> The expected welfare loss relative to that of a zero-inflation rule and discretion for different variances is shown in Figures 3 - 6. We compute the losses for  $\delta = 0.0$ , 0.6, 0.8 and 1.0 and for the entire range of  $\rho$  (evaluated at intervals of 0.05.)<sup>18</sup>

Figure 3 depicts the case where the variance equals one. This is the knife-edge case where the expected welfare loss is the same for both the rule and discretion. It is seen that the committee does better than either a rule or discretion for all three values of  $\delta$ , no matter what the ratio of hawks to doves is. The optimal policy maker population has somewhere between 25 and 35 percent hawks.

Also shown in Figure 3 is the case of  $\delta$  equal to zero. This is corresponds to a committee of hawks and doves where there is no reputation building on the part of doves. The inclusion of this case is to illustrate an important mechanism of the model. Including both hawks and doves in the policy maker population allows society to trade off low inflation and activism in a similar fashion to Rogoff's (1985) conservative banker. However, the reputation-building confers an additional advantage by making the junior doves more apt to inflate when shocks are large than when they are small.

<sup>&</sup>lt;sup>17</sup> The experiments were replicated with the assumption of a gamma distribution with little change in the results.

The integrals in H and G were solved with a 16-point Gauss-Legendre quadrature rule.

Figures 4 depicts the cases where the variance equals 0.5. In this case, the rule is now better than discretion and so is the committee, for every possible composition of the policy maker population. For the plausible case of  $\delta \ge 0.6$ , the committee does better as long as at least about thirty-five percent of the candidate policy makers are hawks. When  $\delta$  equals one, the committee is better as long as at least about a fourth of the policy makers are hawks.

Figure 5 depicts the case where the variance is 1.5 and both discretion and a committee are preferred to a rule. In this case, for  $\delta \ge 0.6$ , the committee is better than discretion as long as at least about forty percent of the policy maker population is made up of doves.

Figures 6 and 7 depict give the range of policy maker populations such that committees dominate rules and discretion for different variances. Figure 6 depicts gives the compositions for variances less than one. It is seen that the minimum fraction of hawks rises as the variance falls and that even as the variance becomes close to one, there is some composition that makes a committee better than a rule. Figure 7 depicts the composition for variances greater than one. The maximum fraction of hawks falls as the variances rises and even for very large variances, a committee is better than discretion for some types of policy maker populations.

#### 5. Conclusion

If central bankers' types are unknown, this paper suggests that an independent monetary policy committee is – for a wide range of parameters – better than either a mandated zero-inflation rule or a discretionary policy making by an opportunistic central banker. This arrangement requires limited credibility on the part of the government. The government need not monitor the committee or attempt to influence its behavior.

The result is obtained for two reasons. First, if candidate central bankers can be inflation hawks or inflation doves, then a typical committee will be less conservative than society and this, by itself, will improve welfare. Second, the less conservative central bankers have an incentive to masquerade as more conservative central bankers in their first term in office. Their desire to gain a reputation will cause them to vote for zero inflation as long as shocks are small. However, in the face of a sufficiently large shock, the reputational gain will not be worth the cost of lost output. Thus, the central bank will tend to have low inflation in normal times, but high inflation in times of large shocks.

There are certainly other features of the decision-making process in monetary policy committees that our model does not capture – internal group dynamics, the building up of consensus in the process of debating, or the use of personal 'credibility' capital within the committee to insist on one action or another. Similarly, on the basis of experimental analysis Blinder and Morgan (2000) argue that groups solve problems better than individuals. Our model does not have the machinery to analyse why committees are superior to single policy makers in data-processing or making the right decision more often. However, the model does describe a powerful and realistic mechanism – the willingness to forego reputation-building at times when it is costly while fighting inflation as a hawk when the economic environment is relatively stable.

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Figure. 1 (Conditional) Expected Inflation

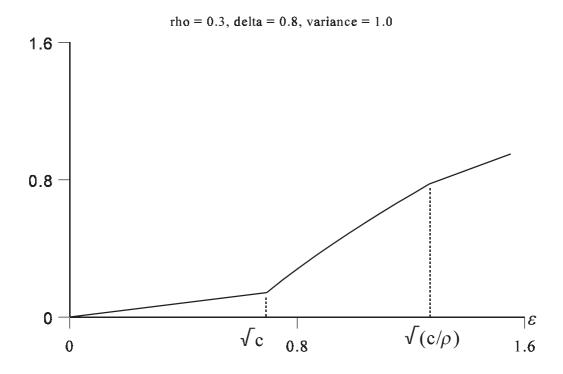


Figure. 2 Inflation under Various Regimes

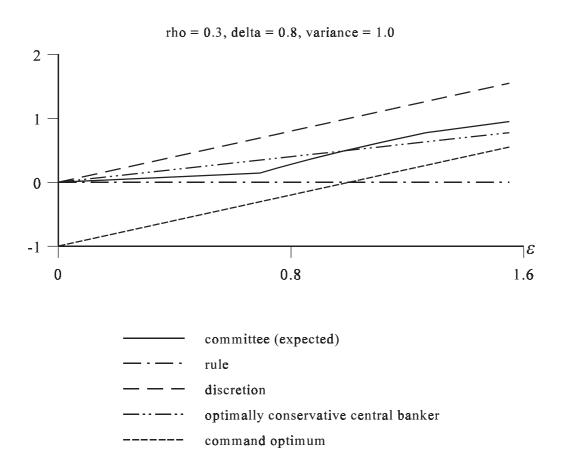


Figure. 3 Welfare Loss with Variance = 1.0

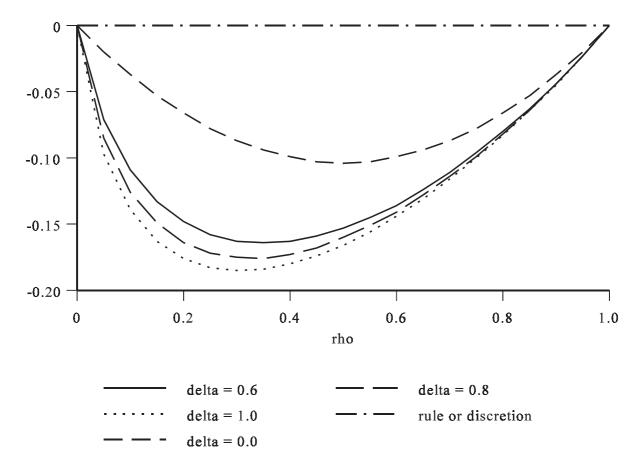
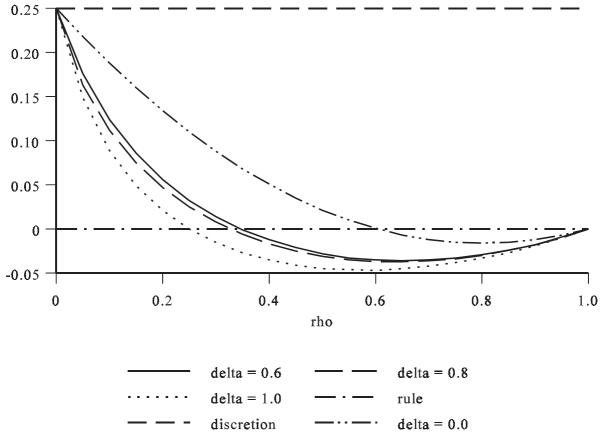


Figure. 4 Welfare Loss with Variance = 0.5





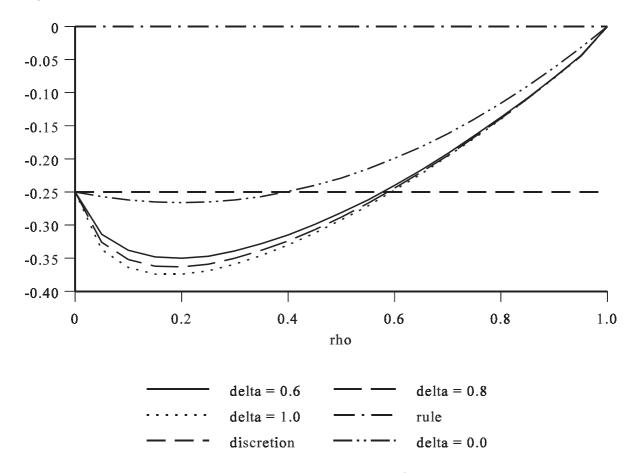


Figure. 6 Minimum Fraction of Hawks for a Committee to Be at Least as Good as a Rule

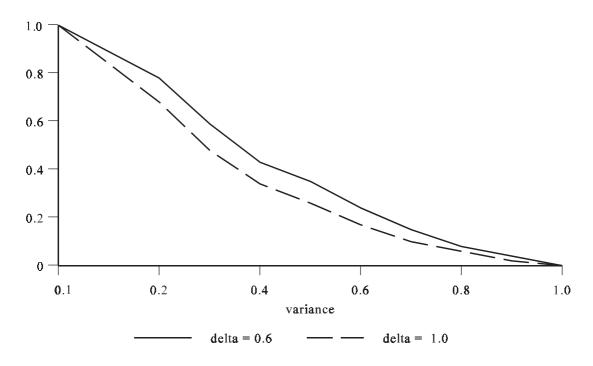
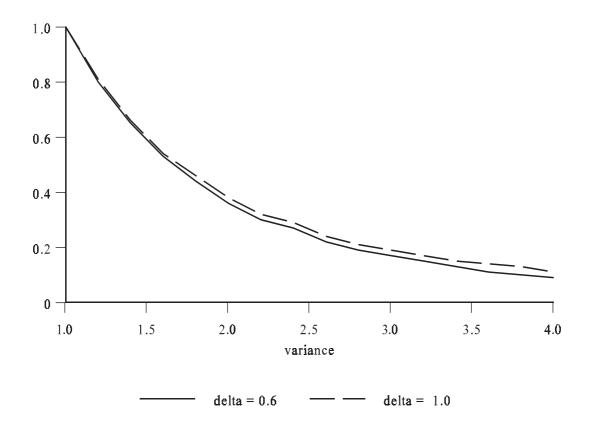


Figure. 7 Maximum Fraction of Hawks for a Committee to Be at Least as Good as Discretion



### **Appendix**

Proof of Proposition 1. By Liebnitz's rule, partial  $\partial H/\partial c=\int\limits_{\sqrt{c}}^{\sqrt{c}/\rho}f(\varepsilon)^{\sqrt{c}}/\varepsilon d\varepsilon>0$  . (The terms involving

derivatives of the limits of integration cancel out.) Hence, the right-hand side of equation (10) is strictly decreasing in c. As  $c \to 0$ ,  $H \to \rho$  and the right-hand side of equation (10) goes to  $4\delta\rho[(1-\alpha)-\rho(1-\alpha)]>4\delta\rho>0$ . As  $c\to\infty$ ,  $H\to 1$  and the right-hand side of equation (10) goes to  $4\delta\rho\alpha$ . The left-hand side of equation (10) is strictly positive and strictly increasing on  $\mathbb{R}_+$ ; hence, equation (10) has a unique solution contained in  $(4\delta\rho\alpha,4\delta\rho[1-\alpha-\rho(1-2\alpha)]$ .

Proof of Proposition 2. By equation (10),

$$\frac{\partial c}{\partial \delta} = \frac{c/\delta}{1 + 4\delta\rho(1 - 2\alpha)\partial H/\partial c}.$$
 (15)

The result in the proof of proposition 1 ensures this is strictly positive. By equation (10) and the result the proof of proposition 1,  $dc/d\rho > 0$  iff

$$H + \rho \int_{\sqrt{c/\rho}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon < \frac{1 - \alpha}{1 - 2\alpha}.$$
 (16)

The left-hand side of (16) is strictly increasing in  $\rho$ . As  $\rho \to 0$ ,  $c \to 0$  and thus,  $H \to 0$ . Hence, (16) holds. This implies that for every  $\delta$ , there exists  $\rho^* \in \to (0,1)$  such that  $dc/d\rho > (<) 0$  as  $\rho < (>) \rho^*$ . As  $\rho \to 1$ ,  $H \to 1$ , and thus  $c \to 4\delta\alpha$ . Thus, we must have  $\int\limits_{2\sqrt{\delta\alpha}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon < \alpha/(1-2\alpha) < 1 \text{ for equation (16) to hold as } \rho \to 1$ . This is not true if  $\delta$  is sufficiently small. This implies that for  $\delta$  sufficiently small,  $\rho^* < 1$ .

*Proof of Proposition 3*. The effect of  $\delta$  is obvious. By (12),  $E(\pi_*)$  is decreasing in  $\rho$  iff

$$1 - \alpha - (1 - 2\alpha)H + [\rho\alpha + (1 - \rho)(1 - \alpha)]dH/d\rho > 0.$$

 $H \in [0,1]$ ; hence this is true if  $dH/d\rho > 0$ .

By (10),

$$\frac{dH}{d\rho} = \frac{\partial H}{\partial c} \frac{\partial c}{\partial \rho} + \frac{\partial H}{\partial \rho} = \frac{\frac{c}{\rho} \frac{\partial H}{\partial c} + \frac{\partial H}{\partial \rho}}{1 + 4\delta\rho(1 - 2\alpha)\frac{\partial H}{\partial c}} > 0.$$
 (17)

Proof of Proposition 4. If  $\rho \to 1$ , then by (10),  $H \to 1$  and  $c \to 4\delta\alpha$ . By (12),  $E(\pi_t) \to 0$ . By (14),  $G \to 1 + \sigma^2$ . Hence, by (14), the expected welfare loss goes to zero - the loss associated with the rule. By (10), (12) and (14), the expected welfare loss is increasing in  $\rho$  iff

$$\sigma^{2} + 1 + \frac{\partial c}{\partial \rho} \int_{\sqrt{c}}^{\sqrt{c/\rho}} f(\varepsilon) d\varepsilon + \int_{\sqrt{c/\rho}}^{\infty} \varepsilon^{2} f(\varepsilon) d\varepsilon - \left\{ 1 - \alpha - (1 - 2\alpha)H + [\rho\alpha + (1 - \rho)(1 - \alpha)] \left[ \frac{\partial c}{\partial \rho} \int_{\sqrt{c}}^{\sqrt{c/\rho}} \frac{f(\varepsilon)}{\varepsilon} d\varepsilon + \int_{\sqrt{c/\rho}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon \right] \right\}$$

$$(18)$$

By (10) and (14),

$$\frac{\partial c}{\partial \rho} = \frac{c/\rho - 4\delta\rho(1 - 2\alpha) \int_{-\infty}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{1 + 4\delta\rho(1 - 2\alpha) \int_{-\infty}^{\infty} \frac{f(\varepsilon)}{\varepsilon} d\varepsilon}.$$
(19)

As  $\rho \to 1$ ,  $\partial c/\partial \rho$  remains finite; hence, the loss is strictly increasing as  $\rho \to 1$  if

$$\sigma^{2} + 1 + \int_{\sqrt{4\delta\alpha}}^{\infty} \varepsilon^{2} f(\varepsilon) d\varepsilon - 4\alpha \left( 1 + \int_{\sqrt{4\delta\alpha}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon \right) > 0.$$
 (20)

The left-hand side of (20) is increasing in  $\delta$ ; hence, this is true if  $2(\sigma^2 + 1) - 8\alpha > 0$ . By  $4\alpha - 1 = 2\alpha^2$  and a continuity argument, the result is true.

*Proof of Proposition 5.* By (10), as  $\rho \to 0$ , H and c go to zero and  $c/\rho \to 4(1 - \alpha)\delta =: s$ . By (19),  $\partial c/\partial \rho \to s$ . Hence, by (18), the expected welfare loss is decreasing in  $\rho$  as  $\rho \to 0$  iff

$$\sigma^{2} + 1 + s \int_{0}^{\sqrt{s}} f(\varepsilon) d\varepsilon + \int_{\sqrt{s}}^{\infty} \varepsilon^{2} f(\varepsilon) d\varepsilon - 4(1 - \alpha) \left[ 1 + s \int_{0}^{\sqrt{s}} \frac{f(\varepsilon)}{\varepsilon} d\varepsilon + \int_{\sqrt{s}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon \right] < 0.$$
 (21)

This is true iff

$$2(\sigma^2 + 1) - 8(1 - \alpha) + \int_0^{\sqrt{s}} [\varepsilon - 4(1 - \alpha)] \frac{s - \varepsilon^2}{\varepsilon} f(\varepsilon) d\varepsilon < 0.$$
 (22)

The integral in (22) is strictly negative, hence (22) is true if  $\sigma^2 + 1 < 4$  (1 -  $\alpha$ ). By the definition of  $\alpha$ , this is true if  $\sigma^2 < 2(1 - \alpha)$ .