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MONETARY POLICY AND TRADE GLOBALIZATION

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Monetary Policy and Trade Globalization*

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Abstract

I develop a two country general equilibrium model with heterogeneous price-setting firms to understand how shocks to monetary policy and aggregate labor productivity impact trade integration, which I capture through the (inverse) average productivity of exporting firms. A contractionary domestic monetary policy shock raises the average productivity of domestic exporting firms but lowers the average productivity of foreign exporting firms. The magnitude of these changes is greater when governments target domestic price inflation as opposed to consumer price inflation. A positive shock to domestic labor productivity generates positive - although quantitatively small - changes in the average productivity of all exporting firms when consumer price inflation is targeted. When domestic price inflation is targeted, the same shock causes a fall in the average productivity of domestic exporting firms, and a far larger rise in the productivity of foreign exporting firms.

Keywords: Monetary Policy, Heterogeneous Firms, Trade Globalization

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1. Introduction

The connection between deepening economic integration and the fall of world-wide inflation has generated considerable debate. One idea put forward is that globalization has led to increased levels of competition, which helped lower inflation - at least in the short to medium run (Rogoff, 2004) - and in the most open economies resulted in global excess capacity being the main driver of inflation dynamics (Rogoff, 2007). The alternative argument - that (trade) integration has had little impact on inflation dynamics - has been made by Ball (2006) and a number of others.[1] One interpretation of these results is that inflation fell over time because of a shift in policy at central banks. In the context of this debate, I develop a two country general equilibrium model which combines two key features: heterogeneous firms and inflation targeting central banks. In doing so, I provide a model based appraisal of the so-called "globalization-inflation hypothesis".

In my two country general equilibrium model, each country is associated with an industry in which differentiated varieties of a good are produced by monopolistically competitive price-setting firms that are heterogeneous in productivity. Because exporting is subject to fixed costs, some varieties of each good are not traded. The law of one price fails for varieties that are traded because when a firm decides to export it sets the local currency price of it's product in advance. This eliminates exchange rate pass through. Governments use the short-term nominal interest rate to target an inflation index. Because output price inflation generates resource costs, the allocation of resources within an industry depend on the inflation target set by the central bank. I consider two simple inflation targeting regimes. One in which consumer price inflation - which includes imported varieties of the foreign good - is targeted, and another in which domestic price inflation is targeted.[2] By specifying interest rate rules, the dynamics of inflation, the response of monetary policy, and trade integration - captured through variations in the size of the non-traded sector and average productivity of exporting firms - are jointly determined.

I allow for two sources of uncertainty; exogenous shocks to domestic monetary policy and domestic aggregate labor productivity. A contractionary domestic monetary policy shock generates a `cleansing out effect' in the domestic economy. When the shock hits, there is a rise in the domestic and foreign real interest rates, and, as a result, domestic and foreign consumption both temporarily fall. The shock also induces a fall in domestic consumer price inflation but foreign consumer price inflation rises. Since all firms that export set prices in local currency such correlations in consumption and inflation are contrary to those generated by a *fixed-varieties* model. However, the reaction of consumption and inflation are jointly determined with changes in the entry and export decisions of firms, so these correlations can be

[1] For example, see Mishkin (2009) and the conflicting evidence based on reduced form analysis of Phillips curves presented in Borio and Filardo (2007) and Ihrig *et al.* (2007).

[2] See Svensson (2000) for a discussion of the different implications of each policy regime in the context of a model with a fixed number of firms/varieties.

explained by the extent of the non-tradability (of varieties) of each good and the differential in levels of average productivity across each `sector' of the two economies.

Consider the domestic economy. First, costly price adjustment means that lower inflation is consistent with lower export costs. Reduced costs induce entry into the export market, but this raises the wage bill, pushing up marginal costs, eliminating weaker firms via resource reallocations. Thus a contractionary domestic monetary policy shock actually raises the average productivity of (the remaining) exporting firms. The reverse occurs in the foreign economy - more firms export but the average productivity of these firms is lower. Overall, the shock results in fewer varieties exported globally and a negative correlation between domestic and foreign consumer price inflation. Second, as entry costs are modeled as labor costs, firm entry falls in response to the shock, in line with VAR evidence (Bergin and Corsetti, 2008). In the closed economy, labor costs and sticky prices result in the same shock raising entry. My open economy model with heterogeneous firms generates a result more in line with the data because both domestic and foreign firms need to pay an additional fixed cost to export. This assumption also generates an endogenously determined non-traded sector, capturing the extent of trade integration. These results hold in both the consumer price and domestic price inflation targeting regimes I consider, and are magnified in the latter.

A rise in labor productivity generates only modest positive changes in average productivity for both sets of exporting firms when consumer price inflation is targeted. However, when domestic price inflation is targeted the shock causes a fall in the average productivity of home firms, and a far larger rise in the productivity of foreign firms. The reason for the differential response is that changes in domestic labor productivity have a direct impact on *all* domestic firm's pricing decisions. In particular, each firm lowers it's price in response to the fall in marginal costs. Lower marginal costs induce entry and exporting by preexisting firms. When consumer price inflation is targeted the foreign government alters it's policy to account for this change directly. When domestic price inflation is targeted each government is only concerned with domestically produced varieties. In this case, a rise in home labor productivity results in a shift of resources away from exports and towards domestic production. Consistent with this, there is a fall in the average productivity of domestic exporting firms.

A by-product of the analysis is that when each government targets consumer price inflation the real exchange rate is not independent of shocks to labor productivity. When the number of varieties are fixed and firms set prices in local currency (with the cost of adjustment in prices equal within and across countries) the real exchange rate is independent of technology shocks (Benigno, 2004). Once firms make entry decisions, the result no longer holds because firm entry and export decisions depend on fluctuations in relative prices. In particular, the total average profits of firms depend on relative prices, both through domestic sales (which compete with foreign exports and are imperfectly substitutable in utility for consumers) and export sales, the volume of which depends on a zero profit export cutoff. That

is, fluctuations in relative prices determine entry into export markets, and entry determines the number of varieties of a product available to consumers.

There is a growing literature on heterogenous firms in international trade to which this paper relates. The seminal paper of Melitz (2003) considers the impact of a reduction in trade costs on heterogeneous firms' decisions to enter, exit and export. Along similar lines, Bernard *et al.* (2007) study comparative advantage when firms are heterogeneous. More recently, however, there has been interest in heterogeneous firms and macroeconomic dynamics. Ghironi and Melitz (2005) study a dynamic version of Melitz's framework, Atkeson and Burnside (2009) consider the impact of a reduction in trade costs and the associated impact on aggregate productivity and Bergin and Glick (2009) focus on the behavior of international relative prices. DiGiovanni and Levchenko (2009) use a model in which firms are subject to idiosyncratic shocks to study the link between trade openness and macroeconomic volatility, focusing on the role of large exporters. Finally, Zlate (2008) studies offshore production and business cycles with heterogeneous firms in a DSGE setting and Neiman (2009) shows that prices associated with intra-firm trades are less sticky, less synchronized, and exhibit higher exchange rate pass-through, suggesting that firm boundaries also play a role in business cycle dynamics. This paper is the first to consider the role of monetary policy and macroeconomic dynamics in a heterogenous firms setting.

There is also a large literature on real exchange rate persistence and technology shocks in DSGE models. Analysis by Chari *et al.* (2002) points to the potential short-comings of DSGE models in replicating real exchange rate movements in the data. However, more recently, Dotsey and Duarte (2008) argue that non-traded goods are also important for international relative price movements and Carvalho and Necchio (2008) use a sticky-price model which generates sectoral real exchange rate dynamics in the presence of local currency pricing. Finally, Steinsson and Nakamura (2009) provide related empirical evidence on the role of product replacement bias and pricing-to-market. In my model, technology shocks hit all firms symmetrically, the non-traded sector of the economy arises endogenously, and when firms decide to export they price in local currency.

The remainder of the paper is organized as follows. In section two I describe the world economy. In section three I derive the linearized conditions and provide some intuition for the main mechanism in the model. Section four presents the simulation results for the different policy regimes and shocks. Section five concludes.

2. The World Economy

The world economy consists of a home and foreign economy each populated by a unit mass of atomistic households. Households consume home and foreign varieties of two specialized goods, supply labor and hold assets in terms of a mutual fund and non state-contingent domestic and foreign currency bonds.

Firms are heterogenous in their productivity. Each firm faces a sunk cost of entry, a fixed (per-period) cost of exporting, and a market-specific quadratic cost of price adjustment.

2.1 Domestic Households Intratemporal Consumption Decisions

A household's utility function for overall consumption takes the CES form,

$$C_t = \left(a_h^{1-\theta} C_{h,t}^{\theta} + a_f^{1-\theta} C_{f,t}^{\theta}\right)^{1/\theta} \tag{1}$$

where $C_{h,t}$ and $C_{f,t}$ are bundles of varieties of the home and foreign good and $1/(1-\theta)$ is the elasticity of substitution between the respective bundles. When $\theta = 0$, this upper-tier utility function is Cobb-Douglas and the elasticity of substitution between the two bundles is one. The consumption of each bundle of varieties depends on relative prices.

$$C_{h,t} = a_h \left(\frac{P_t}{P_{h,t}}\right)^{1/(1-\theta)} C_t \quad ; \quad C_{f,t} = a_f \left(\frac{P_t}{P_{f,t}}\right)^{1/(1-\theta)} C_t \tag{2}$$

where $P_t = \left(a_h P_{h,t}^{\theta/(\theta-1)} + a_f P_{f,t}^{\theta/(\theta-1)}\right)^{(\theta-1)/\theta}$ is the welfare-based consumer price index. The household's consumption of varieties of the home and foreign good are defined over a continuum, Ω_h and Ω_f^*, respectively. The lower tier of consumption is also of CES form,

$$C_{h,t} = \left[\int_{\omega\in\Omega_h} c_{h,t}(\omega)^{(\sigma-1)/\sigma} d\omega\right]^{\sigma/(\sigma-1)} \quad ; \quad C_{f,t} = \left[\int_{\omega^*\in\Omega_f^*} c_{h,t}(\omega^*)^{(\sigma-1)/\sigma} d\omega^*\right]^{\sigma/(\sigma-1)} \tag{3}$$

where $\sigma > 1$ is the elasticity of substitution between varieties of a particular good. In any period, only a subset of varieties are available; $\omega \in \Omega_{h,t} \in \Omega_h$ and $\omega^* \in \Omega_{f,t}^* \in \Omega_f^*$, for home and foreign goods respectively. Denoting $p_{h,t}(\omega)$ as the home currency price of a variety of the domestic good and $p_{f,t}(\omega^*)$ as the home currency price of a variety of the foreign good, I write the demand for these varieties in the following way,

$$c_{h,t}(\omega) = \left(\frac{p_{h,t}(\omega)}{P_{h,t}}\right)^{-\sigma} C_{h,t} \quad ; \quad c_{f,t}(\omega^*) = \left(\frac{p_{f,t}(\omega^*)}{P_{f,t}}\right)^{-\sigma} C_{f,t} \tag{4}$$

where $P_{h,t} = \left[\int_{\omega \in \Omega_{h,t}} p_{h,t}(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$ and $P_{f,t} = \left[\int_{\omega \in \Omega^*_{f,t}} p_{f,t}(\omega^*)^{1-\sigma} d\omega^*\right]^{1/(1-\sigma)}$. From here on, I will make the assumption that $\sigma > 1/(1-\theta)$, so that the elasticity of substitution between varieties of a particular good is always higher than the elasticity of substitution between (home and foreign) bundles.

2.2 Domestic Firms Pricing and Export Decisions

Firms technology is linear in labor, with A_t indexing aggregate labor productivity, and φ_h indexing relative productivity such that a firm with relative productivity φ_h produces $\varphi_h A_t$ units of output per unit of labor employed. Prior to entry, firms are identical, and face a sunk entry cost, f_e, equal to $w_t f_e / A_t$ units of the domestic consumption good. Upon entry, domestic firms draw their productivity level φ_h from a common distribution. Firms can serve the domestic and export market and because there are no fixed production costs all firms produce in every period until they are hit with a shock, which is assumed independent of the firm's productivity level. Firms also face nominal rigidity in the form of a quadratic cost of adjusting prices in the currency of the market being supplied. This cost is assumed to be proportional to the real revenue from output sales in that market. In period t, there is a mass $n_{h,t}$ of domestic firms producing and setting prices in the domestic economy. Of these, a fraction $n^*_{h,t}$ also serve the export market. When a firm sets the price of it's output (in either market) for the first time it does so consistent with the average product price in the market it serves.[3]

For an individual domestic firm, indexed φ_h, I decompose total profit, $d_t(\varphi_h)$, into portions earned from domestic sales, $d_{h,t}(\varphi_h)$, and from potential export sales, $d^*_{h,t}(\varphi_h)$. Thus, total profits in period t are given by $d_t(\varphi_h) = d_{h,t}(\varphi_h) + d^*_{h,t}(\varphi_h)$. Because production is linear, I separate the profit maximization problem for firms into the two markets they serve. Profits from domestic sales are,

$$d_{h,t}(\varphi_h) = \rho_{h,t}(\varphi_h) y_{h,t}(\varphi_h) - w_t l_{h,t}(\varphi_h) - \left(\frac{P_{h,t}}{P_t}\right) \cdot \xi_{h,t}(\varphi_h) \quad ; \quad y_{h,t}(\varphi_h) = \varphi_h A_t l_{h,t}(\varphi_h)$$

where $\rho_{h,t}(\varphi_h) \equiv p_{h,t}(\varphi_h)/P_t$, $\xi_{h,t}(\varphi_h)$ represents the cost of adjusting prices, and $w_t \equiv W_t/P_t$ is the real wage (in consumption units). The total demand for the output of firm φ_h is then,

[3] As in Bilbiie *et al.* (2007), entrants inherit the same price as pre-existing firms. Here, this price is an average (discussed below), associated with each market. Bilbiie *et al.* (2007) extend their framework and allow firms to enter with flexible prices and then set the price for the first time in the second period. This appears to have little importance for their quantitative results.

$$y_{h,t}(\varphi_h) = \left(\frac{p_{h,t}(\varphi_h)}{P_{h,t}}\right)^{-\sigma} Y_{h,t} \quad ; \quad Y_{h,t} \equiv C_{h,t} + \int_{n_{h,t}} \xi_{h,t}(\varphi_h)$$

where $Y_{h,t}$ is the output of domestic sales consumed. The real cost of movements in output-price inflation around a zero steady state level of inflation is given by the following.

$$\xi_{h,t}(\varphi_h) \equiv \frac{\xi_h}{2}\left(\frac{p_{h,t}(\varphi_h)}{p_{h,t-1}(\varphi_h)} - 1\right)^2 \frac{p_{h,t}(\varphi_h)}{P_{h,t}} y_{h,t}(\varphi_h)$$

Under this formulation of nominal rigidity, the demand for output comes from two sources - consumers and firms; that is, inflation reduces the level of output households access since firms also demand output to pay adjustment costs. Because these costs are deducted from profits, and are proportional to squared output price inflation, this is equivalent to assuming there is a tax on production. This tax distorts that allocation of resources and therefore the extent of product creation (firm entry). At time t, firm φ_h chooses $l_{h,t}(\varphi_h)$ and $\rho_{h,t}(\varphi_h)$ to maximize $d_{h,t}(\varphi_h) + \upsilon_{h,t}(\varphi_h)$ - where,

$$\upsilon_{h,t}(\varphi_h) \equiv \mathbb{E}_t \sum_{s=t+1}^{\infty} \mathcal{M}_{t,s} d_{h,t}(\varphi_h)$$

and $\mathcal{M}_{t,s}$ is the stochastic discount factor applied by households to future profits, adjusted for a constant probability δ of being hit by a death shock - subject to the demand for it's product, taking w_t, $P_{h,t}$, P_t and $Y_{h,t}$ as given. Profit maximization results in the following pricing equation,

$$\rho_{h,t}(\varphi_h) = \mu_{h,t}(\varphi_h)\frac{w_t}{A_t \varphi_h} \quad ;$$

$$\mu_{h,t}(\varphi_h) = \frac{\sigma}{(\sigma-1)\left[1 - \frac{\xi_h}{2}\left(\frac{p_{h,t}(\varphi_h)}{p_{h,t-1}(\varphi_h)} - 1\right)^2\right] + \xi_h \Omega_{h,t}(\varphi_h)} \quad ;$$

$$\Omega_{h,t}(\varphi_h) \equiv \left(\frac{p_{h,t}(\varphi_h)}{p_{h,t-1}(\varphi_h)} - 1\right)\left(\frac{p_{h,t}(\varphi_h)}{p_{h,t-1}(\varphi_h)}\right) - G_{h,t}(\varphi_h) \quad ;$$

$$G_{h,t}(\varphi_h) \equiv \frac{\mathcal{M}_{t,t+1} P_t}{P_{t+1}}\left(\frac{p_{h,t+1}(\varphi_h)}{p_{h,t}(\varphi_h)} - 1\right)\left[\frac{p_{h,t+1}(\varphi_h)}{p_{h,t}(\varphi_h)}\right]^2 \frac{y_{h,t+1}(\varphi_h)}{y_{h,t}(\varphi_h)} \tag{5}$$

where $\mu_{h,t}(\varphi_h)$ is a markup associated with the domestic sales of domestic firms and $w_t/A_t\varphi_h$ are real marginal costs. When the cost of adjusting prices is zero ($\xi_h = 0$), the markup is constant, and equal to $\sigma/(\sigma-1) > 1$. When $\xi_h > 0$ price adjustment is sluggish. Again, it it this parameter that affects the entry decision of domestic firms on the competitive fringe of the domestic market.

In a similar fashion, firms φ_h's potential profits from export sales are,

$$d^*_{h,t}(\varphi_h) = Q_t\rho^*_{h,t}(\varphi_h)y^*_{h,t}(\varphi_h) - w_t l^*_{h,t}(\varphi_h) - \frac{P^*_{h,t}}{P^*_t}Q_t\xi^*_{h,t}(\varphi_h) - \frac{W_t}{A_t}f^*_h \quad ;$$
$$y^*_{h,t}(\varphi_h) = \varphi A_t l^*_{h,t}(\varphi_h)$$

where $Q_t = e_t P^*_t/P_t$ is the welfare based consumer price real exchange rate and $\rho^*_{h,t}(\varphi_h) \equiv p^*_{h,t}(\varphi_h)/P^*_t$. Again, total demand for the product comes from two sources - foreign consumers and domestic firms - and is given by the following.

$$y^*_{h,t}(\varphi_h) = \left(\frac{p^*_{h,t}(\varphi_h)}{P^*_{h,t}}\right)^{-\sigma} Y^*_{h,t} \quad ; \quad \xi^*_{h,t}(\varphi_h) \equiv \frac{\xi^*_h}{2}\left(\frac{p^*_{h,t}(\varphi_h)}{p^*_{h,t-1}(\varphi_h)} - 1\right)^2 \frac{p^*_{h,t}(\varphi_h)}{P^*_{h,t}} y^*_{h,t}(\varphi_h)$$

where $Y^*_{h,t} \equiv C^*_{h,t} + \int_{n^*_{h,t}} \xi^*_{h,t}(\varphi_h)$. Optimization results in a similar dynamic pricing equation to (5), now with parameter ξ^*_h determining the extent of price adjustment. If we assume $\xi^*_h \neq \xi_h$, then the cost of adjusting prices is different in the domestic and export markets. In this case, there are two reasons why domestic firms change different prices across the two markets. First, exporting firms will be, on average, more productive, and hence face lower marginal costs. Second, it is simply less expensive to export when, for example, $\xi^*_h < \xi_h$, all else equal. This can give rise to local currency pricing, as emphasized in Benigno's (2004) analysis of the real exchange rate.

A firm will choose export if and only if it earns non-negative profit from doing so, which will only be the case if productivity φ_h is above a cut-off level $\varphi^*_{h,t}$, where $\varphi^*_{h,t} = \inf\{\varphi: d^*_{h,t}(\varphi_h) > 0\}$. I assume the lower bound to productivity is such that $\varphi^*_{h,t} > \underline{\varphi}_h$, and, in this case, there exists an endogenously determined non traded sector, where firms with productivity levels between $\underline{\varphi}_h$ and $\varphi^*_{h,t}$ only produce for the domestic market. This set of firms fluctuates over time with changes in the profitability of the export market, inducing changes in the productivity cutoff level. For simplicity, and following Ghironi and Melitz (2005), I assume away fixed costs of production such that all firms that enter produce.

2.3 Domestic Firm Averages, and Firm Entry and Exit

Each period a mass of $n_{h,t}$ firms produce in the domestic economy. I assume firms are distributed with a Pareto distribution over productivity levels, with shape parameter κ, and minimum productivity draw, $\underline{\varphi}_h$. As κ increases, dispersion decreases, and firm productivity levels are increasingly concentrated toward this lower bound. Average productivity levels are then defined as,

$$\overline{\varphi}_h \equiv \left[\int_{\underline{\varphi}}^{\infty} (1/\varphi_h)^{1-\sigma} dG(\varphi_h)\right]^{1/(\sigma-1)} \quad ; \quad \overline{\varphi}^*_{h,t}(\varphi^*_{h,t}) \equiv \left[\frac{1}{1-G(\varphi^*_{h,t})}\int_{\varphi^*_t}^{\infty} (1/\varphi_h)^{1-\sigma} dG(\varphi_h)\right]^{1/(\sigma-1)}$$

where $G(\varphi_h) = 1 - \left(\frac{\underline{\varphi}_h}{\varphi_h}\right)^{\kappa}$. These assumptions provide the following relationships between the minimum and average productivity of domestic firms and the productivity of the average export firm with the average productivity across all domestic firms,

$$\overline{\varphi}_h = \nu\underline{\varphi}_h \quad ; \quad \overline{\varphi}^*_{h,t} = \nu\varphi^*_{h,t}$$

where $\nu \equiv [\kappa/(\kappa-\sigma)]^{1/\sigma}$ (see Appendix for a derivation). This is interpreted such that the $n_{h,t}$ firms with productivity level $\overline{\varphi}_h$ produce in the home country and the $n^*_{h,t} \in n_{h,t}$ firms with productivity level $\overline{\varphi}^*_{h,t}$ export to the foreign market. Given this, we know the share of home exporting firms is, $n^*_{h,t}/n_{h,t} = 1 - G(\varphi_{h,t}) = \left(\nu\underline{\varphi}_h/\overline{\varphi}_{h,t}\right)^{\kappa}$. Total average profits for firm φ_h are then given by, $\overline{d}_t = \overline{d}_{h,t} + (n^*_{h,t}/n_{h,t})\overline{d}^*_{h,t}$, where the productivity averages $\overline{\varphi}_h$ and $\overline{\varphi}^*_{h,t}$ are constructed in such a way that $\overline{d}_{h,t} \equiv d_{h,t}(\overline{\varphi}_h)$ represents the average firm profit earned from domestic sales for all home producers and $\overline{d}^*_{h,t} \equiv d^*_{h,t}(\overline{\varphi}^*_{h,t})$ represents the average firm export profits for all home exporters. In this case, $\overline{d}_t = \overline{d}_{h,t} + \left[1 - G\left(\overline{\varphi}^*_{h,t}\right)\right]\overline{d}^*_{h,t}$ represents the average total profits of home firms, since $1 - G(\overline{\varphi}^*_{h,t})$ also represents the proportion of home firms that export and export profits from doing so.

Alongside preexisting firms, in every period there is an unbounded mass of forward-looking prospective entrants (a competitive fringe). They correctly anticipate future expected profits in every period, as well as the probability of incurring an exit-inducing shock. Entrants at time t start producing at time $t+1$ and the exogenous exit shock occurs after production, entry and price setting decisions are made. Prospective home entrants in period t compute their expected post-entry value which is given by the

present discounted value of their expected stream of profits. Using firm averages, I express the value of the average firm in the following way.

$$\overline{v}_t = \mathbb{E}_t \sum_{s=t+1}^{\infty} \mathcal{M}_{t,s} \overline{d}_s \tag{6}$$

Since both new entrants and incumbents face the same probability of survival the variable $\overline{v}_t$ also represents the average value of incumbent firms after production has occurred. Entry occurs until the average firm value is equalized with the entry cost, leading to the free entry condition, $\overline{v}_t = w_t f_{e,t}/A_t$. This condition holds so long as the mass of new entrants, $n_{e,t}$, is positive. Finally, the timing of entry and production implies that the number of home-producing firms during period t is given by $n_{d,t} = (1-\delta)(n_{h,t-1} + n_{e,t-1})$, where $n_{e,t-1}$ is the mass of new entrants in period t.

2.4 Domestic Household Intertemporal Decisions

The representative domestic household holds two types of assets: shares (in a mutual fund) and non state-contingent (domestic and foreign currency) bonds. Let s_t be the share of firms held by the representative household entering period t. The mutual fund pays a total profit in each period (in units of home currency) equal to the average total profit of all home firms that produce in that period, $P_t \overline{d}_t n_{h,t}$. During period t, the representative home household buys s_{t+1} shares of $n_{h,t} + n_{e,t}$ firms. Only $n_{h,t+1} \equiv (1-\delta)\left(n_{h,t} + n_{e,t}\right)$ firms will produce and pay dividends at time $t+1$. Since the household does not know which firms will be hit by the exit shock at the very end of period t, it finances the continuing operation of all pre-existing home firms and all new entrants during period t. The date t price (in units of home currency) of a claim to the future profit stream of the mutual fund of $n_{h,t} + n_{e,t}$ firms is equal to the average nominal price of claims to future profits of home firms, $P_t \overline{v}_t$. Finally, domestic residents can buy bonds, $A_{h,t}$, in domestic currency, and have access to foreign current bonds, $A_{f,t}$.

The household maximizes expected intertemporal utility, $\mathbb{U}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{\psi}{1+\sigma_l} L_t^{1+\sigma_l}\right]$, where $\beta \in (0,1)$ is the subjective discount factor and σ_l is the inverse of the Frisch elasticity of substitution, subject to the following period budget constraint.

$$\frac{A_{h,t}}{1+i_t} - A_{h,t-1} + e_t\left(\frac{A_{f,t}}{1+i_t^*} - A_{f,t-1}\right)$$
$$= s_t P_t\left(\overline{d}_t + \overline{v}_t\right)n_{h,t} + W_t L_t - s_{t+1}P_t\overline{v}_t\left(n_{h,t} + n_{e,t}\right) - P_t C_t$$

where i_t and i_t^* are the net nominal interest rates. The following are optimality conditions associated with the domestic households problem.

$$\frac{P_t}{C_t} = (1+i_t)\beta\mathbb{E}_t\left(\frac{P_{t+1}}{C_{t+1}}\right) \tag{7}$$

$$\frac{e_t P_t}{C_t} = (1+i_t^*)\beta\mathbb{E}_t\left(\frac{e_{t+1}P_{t+1}}{C_{t+1}}\right) \tag{8}$$

$$\overline{v}_t = (1-\delta)\beta\mathbb{E}_t\left[\left(\frac{P_t C_t}{P_{t+1}C_{t+1}}\right)\left(\overline{v}_{t+1} + \overline{d}_{t+1}\right)\right] \tag{9}$$

$$\frac{W_t}{P_t} = \psi C_t L_t^{\sigma_l} \tag{10}$$

Equation (7) is the consumption Euler equation, (8) is an uncovered interest rate parity (UIP) condition, (9) is an Euler equation for shares, and (10) is labor supply.

2.5 Foreign Economy

Foreign economic agents also have upper-tier CES preferences; i.e., $C_t^* = \left(a_h^{*1-\theta}C_{h,t}^{*\theta} + a_f^{*1-\theta}C_{f,t}^{*\theta}\right)^{1/\theta}$. The consumption allocations satisfy,

$$c_{h,t}^*(\varphi) = a_h^*\left(\frac{p_{h,t}^*(\varphi)}{P_{h,t}^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{h,t}^*}\right)^{1/(1-\theta)} C_t \quad ; \quad c_{f,t}^*(\varphi) = a_f^*\left(\frac{p_{f,t}^*(\varphi)}{P_{f,t}^*}\right)^{-\sigma}\left(\frac{P_t^*}{P_{f,t}^*}\right)^{1/(1-\theta)} C_t^*$$

Individual foreign firms set the price of their variety in the foreign and domestic markets in local currency terms and face sunk entry costs and fixed (per-period) entry costs to the export market. The parameters ξ_f^* and $\xi_f \neq \xi_f^*$ determine price adjustment costs in each market. Again, it is possible to generate genuine local currency pricing when within a country price rigidities are the same for imported and domestically produced goods; this requires $\xi_f = \xi_h$ and $\xi_h^* = \xi_f^*$. Firm relative productivity is drawn from a Pareto distribution, with dispersion parameter κ^*, such that, $G\left(\varphi_f^*\right) = 1 - \left(\frac{\underline{\varphi}_f^*}{\varphi_f^*}\right)^{\kappa^*}$. Foreign agents have access to $n_{f,t}^*$ varieties of the foreign good and $n_{h,t}^* \in n_{h,t}$ varieties of the home good. The

foreign household also supplies L_t^* units of labor in each period in the foreign labor market at the nominal wage rate W_t^*.

2.6 Equilibrium

Total labor used by home firms for production in the domestic and export markets is $L_{h,t} \equiv n_{h,t}\bar{l}_{h,t}$ and $L_{h,t}^* \equiv n_{h,t}^*\bar{l}_{h,t}^*$, respectively. In addition, new entrants hire f_e units of labor to cover the sunk entry cost and each exporter hires f_h^* units of labor to cover the fixed export cost. The total amount of labor hired to cover the costs associated with the creation of new products and their export is therefore $\left(n_{h,t}^* f_h^* + n_{e,t} f_e\right)/A_t$. The demand for labor is determined by the following condition.

$$L_t = L_{h,t} + L_{h,t}^* + \left(n_{h,t}^* f_h^* + n_{e,t} f_e\right)/A_t$$

$$\bar{l}_{h,t} = \frac{\bar{d}_{h,t} + \left(P_{h,t}/P_t\right)\bar{\xi}_{h,t}}{\left(\mu_{h,t} - 1\right)w_t} \quad ; \quad \bar{l}_{h,t}^* = \frac{\bar{d}_{h,t}^* + \left(e_t P_{h,t}^*/P_t\right)\bar{\xi}_{h,t}^* + f_h^* w_t/A_t}{\left(\mu_{h,t}^* - 1\right)w_t} \tag{11}$$

An analogous condition holds in the foreign economy.

Aggregating the household budget constraint (across symmetric agents) and imposing the equilibrium conditions ($A_{h,t} = A_{h,t+1} = 0$ and $s_t = s_{t+1} = 1$) yields the national budget constraint for the domestic economy.

$$e_t\left(\frac{A_{f,t}}{1+i_t^*} - A_{f,t-1}\right) = W_t L_t + P_t \bar{d}_t n_{h,t} - P_t\left(C_t + \bar{v}_t n_{e,t}\right) \tag{12}$$

The right-hand side of equation (12) can be interpreted as wage income plus profits, minus consumption and investment. Let F_t denote the domestic economy's trade balance. We know average total profits of domestic firms are, $\bar{d}_t = \bar{d}_{h,t} + \left(n_{h,t}^*/n_{h,t}\right)\bar{d}_{h,t}^*$, and that the free entry requirement equates the value of the firm with wages and the sunk costs of entry; $P_t\bar{v}_t = f_e W_t/A_t$. If we use labor demand and eliminate profits from domestic and foreign sales, it is possible to derive the following, $F_t \equiv \left(P_{h,t}/P_t\right)C_{h,t} + e_t\left(P_{h,t}^*/P_t\right)C_{h,t}^* - C_t$. Using the upper tier demands ($C_{h,t} = a_h\left(P_t/P_{h,t}\right)^{1/(1-\theta)}C_t$ and $C_{h,t}^* = a_h^*\left(P_t^*/P_{h,t}^*\right)^{1/(1-\theta)}C_t^*$) and the consumer price index I then write the trade balance, in units of consumption.

$$F_t = a_h^* \left(\frac{P_{h,t}^*}{P_t^*}\right)^{\theta/(\theta-1)} Q_t C_t^* - (1-a_h)\left(\frac{P_{f,t}}{P_t}\right)^{\theta/(\theta-1)} C_t \tag{13}$$

The first term on the right-hand side of (13) is exports, and the second term is imports. Although this appears to be a somewhat standard expression, it should be clear that I have adjusted for the number of varieties. In particular, I have also used,

$$\left(\frac{P_{f,t}}{P_t}\right)^{\theta/(\theta-1)} = n_{f,t}^{1+\Phi}\overline{\rho}_{f,t}^{\theta/(\theta-1)} \quad ; \quad \left(\frac{P_{h,t}^*}{P_t^*}\right)^{\theta/(\theta-1)} = n_{h,t}^{*1+\Phi}\overline{\rho}_{h,t}^{*\theta/(\theta-1)}$$

where $\Phi \equiv [\sigma(\theta-1)+1]/(\sigma-1)(1-\theta)$. Using this definition, note that when $\theta = 0$, the upper tier of utility is Cobb-Douglas, $\Phi = -1$. When the elasticity of substitution across varieties and is the same as across the two goods produced by each industry, then, $\Phi = 0$. Finally, also note that domestic resources used in domestic production are,

$$\overline{y}_{h,t} = a_h\left(1-\frac{\xi_h}{2}\overline{\pi}_{h,t}^2\right)^{-1} n_{h,t}^{\Phi}\overline{\rho}_{h,t}^{1/(\theta-1)} C_t \quad ; \quad \overline{y}_{h,t}^* = a_h\left(1-\frac{\xi_h^*}{2}\overline{\pi}_{h,t}^{*2}\right)^{-1} n_{h,t}^{*\Phi}\overline{\rho}_{h,t}^{*1/(\theta-1)}\tau_t C_t^*$$

respectively for the domestic and export market. Again, there are analogous conditions for the foreign economy.

3. Inflation Dynamics, Firm Heterogeneity and Monetary Policy

To simplify the exposition, I assume the distribution of firms size is the same in each country ($\kappa = \kappa^*$), the parameters governing costs of price adjustment are identical across and within countries ($\xi_h = \xi_h^* = \xi_f^* = \xi_h \equiv \xi$), the upper tier of utility is Cobb-Douglas ($\theta = 0$) and symmetric across countries ($a_h = a_h^* = 1/2$) and there is financial autarky ($A_{f,t} = 0$).

3.1 Inflation and Relative Price Dynamics

I begin by deriving expressions for the paths of inflation and relative prices. To do so, I focus on a linear approximation of the model around the steady state (see Appendix for details). For example, the deviation of a variable, say x_t, from it's steady state value, x, is denoted $\hat{x}_t$. I characterize the supply side of the world economy using firm pricing equations; that is, equation (5), along with the equivalent expression for domestic exported varieties, and their foreign counterparts. These expressions tie down

the dynamic paths of output price inflation, given a process for the markups. I convert these expressions into dynamic equations in relative and world welfare-based consumer price inflation; that is, $\widehat{\Pi}_t^R \equiv \widehat{\Pi}_t - \widehat{\Pi}_t^*$ and $\widehat{\Pi}_t^W \equiv \frac{1}{2}\widehat{\Pi}_t + \frac{1}{2}\widehat{\Pi}_t^*$, and consumer based relative prices; that is $\hat{T}_t^R \equiv \hat{T}_t - \hat{T}_t^*$ and $\hat{T}_t^W \equiv \frac{1}{2}\hat{T}_t + \frac{1}{2}\hat{T}_t^*$, where $\hat{T}_t \equiv \hat{P}_{f,t} - \hat{P}_{h,t}$ and $\hat{T}_t^* \equiv \hat{P}_{f,t}^* - \hat{P}_{h,t}^*$. An important part of constructing these equations therefore also lies in the adjustment for the number products/product quality. It is also worth noting that without entry and heterogeneity pinning down the path of relative prices is not always necessary - it depends on asset markets and consumer preferences. However, when there is entry into export markets relative prices play a dual role - they affect consumer decisions on consumption and firms decisions on entry and exporting.

In the case of relative inflation (and relative variables, in general), we need only account for the relative average productivity of export firms in each economy, defined as $\widehat{\overline{\varphi}}_t^R \equiv \widehat{\overline{\varphi}}_{h,t}^* - \widehat{\overline{\varphi}}_{f,t}$. In particular, and because $\kappa = \kappa^*$, we have,

$$\widehat{\Pi}_t^R = \widehat{\overline{\pi}}_t^R - \left(\frac{1}{1+\nu^{1-\sigma}}\right)\Delta\widehat{\overline{\varphi}}_t^R \quad ; \quad (1+\nu^{1-\sigma})^{-1} = \kappa/(\sigma-1) > 1 \quad ; \quad \sigma > 1$$

where $\widehat{\overline{\pi}}_t^R = \frac{1}{2}\left(\widehat{\overline{\pi}}_{h,t}^R + \widehat{\overline{\pi}}_{f,t}^R\right)$ and $\widehat{\overline{\pi}}_{h,t}^R$ and $\widehat{\overline{\pi}}_{f,t}^R$ are relative (domestic consumption versus foreign consumption) inflation rates in producer price terms of the home and foreign countries, in home and foreign currencies. Given this result, the path of welfare-based relative consumer price inflation is given by the following:

$$\widehat{\Pi}_t^R + \left(\frac{1/2}{1+\nu^{1-\sigma}}\right)\Delta\widehat{\overline{\varphi}}_t^R = \tilde{\beta}\mathbb{E}_t\widehat{\Pi}_{t+1}^R + \left(\frac{\tilde{\beta}/2}{1+\nu^{1-\sigma}}\right)\mathbb{E}_t\Delta\widehat{\overline{\varphi}}_{t+1}^R + \tilde{\xi}Q_t + \tilde{\xi}\left(\widehat{\overline{\varphi}}_t^R\right) \tag{14}$$

where $\Delta\widehat{\overline{\varphi}}_t^R \equiv \widehat{\overline{\varphi}}_t^R - \widehat{\overline{\varphi}}_{t-1}^R$ and $\tilde{\xi}\left(\widehat{\overline{\varphi}}_t^R\right)$ indicates that the forcing term is a function of $\widehat{\overline{\varphi}}_t^R$, dependent also on steady state parameters. Note that aggregate labor productivity shifters ($\hat{A}_t$ and $\hat{A}_t^*$) do not enter this expression. In constructing (14), I have used the labor demands of firms, and taken advantage of the fact that $2\hat{Q}_t = \widehat{\Psi}_{h,t} + \widehat{\Psi}_{f,t}$, where $\widehat{\Psi}_{h,t}$ and $\widehat{\Psi}_{f,t}$ are law-of-one-price (LOP) gaps (ex: $\widehat{\Psi}_{h,t} \equiv \hat{e}_t + \hat{P}_{h,t}^* - \hat{P}_{h,t}$), as in Monacelli (2004), unadjusted for product quality (or, $2\hat{Q}_t = \widehat{\overline{\psi}}_{h,t} + \widehat{\overline{\psi}}_{f,t} - \left(\frac{1}{1+\nu^{1-\sigma}}\right)\widehat{\overline{\varphi}}_t^R$, once I adjust for product quality). This formulation shows that absent heterogeneity, the path of relative inflation is the same as it would be in an economy without varieties. That is, because the two economies are identical, they produce the same number of varieties. Here, entry of firms into the

export market is also an important source of inertia to the dynamic adjustment of inflation as lag relative average productivity terms enter. Finally, note the special case, when $\nu^{1-\sigma} \to 0$. This is equivalent to assuming $\kappa \to (\sigma - 1)$, which is consistent with the 'granular' economy of DiGiovanni and Levchecnko (2009).[4] In my model, as $\nu^{1-\sigma} \to 0$, then the inertial terms on average productivity exert a relatively strong influence over the path of $\widehat{\Pi}_t^R$, all else equal.

In a similar way, the path of world welfare-based CPI inflation is given by the following:

$$\widehat{\Pi}_t^W - \left(\frac{1}{1+\nu^{1-\sigma}}\right)\left(\Delta\widehat{\overline{\varphi}}_t^W - \frac{\Delta\hat{n}_t^W}{\kappa}\right) = \tilde{\beta}\mathbb{E}_t\widehat{\Pi}_{t+1}^W - \left(\frac{\tilde{\beta}}{1+\nu^{1-\sigma}}\right)\mathbb{E}_t\left(\Delta\widehat{\overline{\varphi}}_{t+1}^W - \frac{\Delta\hat{n}_{t+1}^W}{\kappa}\right) + \tilde{\xi}\left(\hat{n}_t^W, \widehat{\overline{\varphi}}_t^W, \widehat{w}_t^W; A_t, \hat{A}_t^*\right) \tag{15}$$

where $\partial\tilde{\xi}(\cdot)/\,\partial\hat{A}_t < 0$ and $\partial\tilde{\xi}(\cdot)/\,\partial\hat{A}_t^* < 0$. In this case, world inflation is affected not only by the extent of firm heterogeneity, but also by the number of products in the global economy; i.e., the $\hat{n}_h + \hat{n}_{f,t}^*$ term. Moreover, unlike relative inflation, which is driven to a large extent by movements in the real exchange rate, world inflation also depends on conditions in the labor market explicitly. In this case, wages (essentially marginal costs) can be accounted for by individual labor supply decisions (and the marginal product of labor) when wages are assumed to be instantaneously adjusted in response to a shock. With entry and firm heterogeneity, labor supply is shifted between sectors (the creation of new firms, domestic supply from preexisting firms, exports by preexisting firms, see (11) when there is a shock, and is linked to the resource constraint of the economy. I discuss this in more detail below.

Using a similar approach there are two equations that determine the path of relative prices, $\hat{T}_t$ and $\hat{T}_t^*$. In relative terms,

$$\Delta\hat{T}_t^R - \left(\frac{2}{1+\nu^{1-\sigma}}\right)\Delta\widehat{\overline{\varphi}}_t^W = \tilde{\beta}\mathbb{E}_t\Delta\hat{T}_{t+1}^R - \left(\frac{2\tilde{\beta}}{1+\nu^{1-\sigma}}\right)\mathbb{E}_t\Delta\widehat{\overline{\varphi}}_{t+1}^W + \tilde{\xi}\left(\hat{T}_t^R, \widehat{\overline{\varphi}}_t^W\right) \tag{16}$$

where $\hat{\tilde{t}}_t^R = \hat{T}_t^R - \left(\frac{2}{1+\nu^{1-\sigma}}\right)\widehat{\overline{\varphi}}_t^W$, and in world terms,

[4] This analysis features firm idiosyncratic shocks. I only consider a shock to aggregate labor productivity in my analysis.

$$\Delta\hat{T}_t^W + \left(\frac{1}{1+\nu^{1-\sigma}}\right)\left(\frac{\Delta\widehat{\overline{\varphi}}_t^R}{2} - \frac{\Delta\hat{n}_t^R}{\kappa}\right)$$
$$= \tilde{\beta}\mathbb{E}_t\Delta\hat{T}_{t+1}^W + \left(\frac{\tilde{\beta}}{1+\nu^{1-\sigma}}\right)\mathbb{E}_t\left(\frac{\Delta\widehat{\overline{\varphi}}_{t+1}^R}{2} - \frac{\Delta\hat{n}_{t+1}^R}{\kappa}\right)$$
$$+ \tilde{\xi}\left(\hat{T}_t^W, \hat{Q}_t, \hat{n}_t^R, \widehat{\overline{\varphi}}_t^R, \hat{w}_t^R; \hat{A}_t, \hat{A}_t^*\right) \tag{17}$$

where $\hat{\tilde{t}}_t^W = \hat{T}_t^W + \left(\frac{1}{1+\nu^{1-\sigma}}\right)\left(\frac{\widehat{\overline{\varphi}}_t^R}{2} - \frac{\hat{n}_t^R}{\kappa}\right)$ and $\partial\tilde{\xi}(\cdot)/\,\partial\hat{A}_t > 0$ and $\partial\tilde{\xi}(\cdot)/\,\partial\hat{A}_t^* < 0$. These dynamic equations share features with those for inflation but there are some important differences. Because of the way in which relative prices have been defined, the combined world average export productivity variable affects the path of home versus foreign relative prices. In the same way, a world weighted average of relative prices is, in part, determined by differences in the average export productivity and the number of products between the two economies. The equivalent expression for inflation depends on the weighted average number of products, not the difference between them.

Finally, it is worth stressing that even in this simple case, when the two economies are essentially identical, fluctuations in world-relative prices play an important role in determining firm behavior. Whereas relative welfare-based consumer price inflation also plays a role in determining the relative patterns of consumption, relative prices affect firm profits in each economy, both for domestic sales and export sales, the latter through a zero-profit export cutoff. Note that if we were to drop the identical preferences assumption, made for simplicity, this result no longer holds, and world relative prices would also play a role in determining world consumption patterns (this can be easily shown in an economy with a fixed number of varieties).

3.2 Firm Heterogeneity

To determine the part of relative and world inflation and relative prices, I need only pin-down the paths of variables in $\tilde{\xi}(\cdot)$-terms in (14)-(17). To do so, I take advantage of a relative and world system of equations. That is, I solve a system in which, for any variable $\hat{x}_t$, $\hat{x}_t^R \equiv \hat{x}_t - \hat{x}_t^*$ and $\hat{x}_t^W \equiv \frac{1}{2}\hat{x}_t + \frac{1}{2}\hat{x}_t^*$. In each economy, there are dynamic equations that determine the evolution of the value of the average firm (a shares Euler equation) and a law of motion/transition equation for the number of products available. There are also static equations for resources (accounting for total labor supplied), total firm average profits, and a zero export-profit cutoff. These equations are exactly as they would be in an

environment of flexible prices. That is, if we temporarily assume $\xi = 0$, then these equations determine the evolution of the number of firms, the average productivity, and other endogenous variables. However, when prices are sticky, total average profits and the zero export-profit cutoff depend on markups, which are endogenous. The markups be accounted for in exactly the same way as in (14)-(17); that is, by using the pricing equations.

The solution of the model is therefore one of two interacting blocks, say, a 'sticky-price' block and a 'flex-price' block. Absent firm entry and heterogeneity, (14)-(17), consumption Euler equations and monetary policy rules provide a solution to the model (sticky-price block). Absent sticky prices, the consumption Euler equations are not needed and the firm monopoly markup is fixed. The means that relative price fluctuations, as described above, do not affect the entry and export decision of firms. A full list of these equilibrium conditions is given in the Appendix. Here I focus on total average profits and wages shed some light on how relative prices affect the two profit terms.

Differences in total average profits across the two economies, can be expressed in the following way:

$$\hat{\bar{d}}_t^R = \Psi \underbrace{\left[\hat{Q}_t - \hat{n}_t^R + (\sigma - 1)\left(\hat{\bar{\mu}}_{h,t} - \hat{\bar{\mu}}_{f,t}^*\right)\right]}_{\text{domestic sales}} + (1 - \Psi) \underbrace{\left(\hat{w}_t^R - \hat{A}_t^R - \kappa \hat{\bar{\varphi}}_t^R\right)}_{\text{export sales}} \tag{18}$$

where $\Psi \equiv \{\kappa + [\kappa - (\sigma - 1)]\}/\kappa \in (0,1)$. The only difference between this expression and that with flexible prices is a markup gap, specifically, $\left(\hat{\bar{\mu}}_{h,t} - \hat{\bar{\mu}}_{f,t}^*\right)$. Likewise, a weighted average of total firm profits, $\hat{\bar{d}}_t^W$, depends on weighted average of the same endogenous variables as in (18), and a weighted average of the same markups, $\hat{\bar{\mu}}_{h,t}$ and $\hat{\bar{\mu}}_{f,t}^*$. Both of these markups - and the relative and world composites - refer to domestic sales, for the home and foreign firms respectively. Given the pricing equations (cf. 5), it then becomes clear that $\hat{\bar{d}}_t^R$ must be a function of $\hat{T}_t^W$, and $\hat{\bar{d}}_t^W$ is a function of $\hat{T}_t^R$. In other words, relative and world profits depend on fluctuations in world-relative prices.

The second step is to use the zero-profit export cutoffs to determine wages. In relative terms, I find,

$$\hat{w}_t^R = \hat{Q}_t - \hat{n}_t^R + \kappa \hat{\bar{\varphi}}_t^R + (\sigma - 1)\left(\hat{\bar{\mu}}_{h,t}^* - \hat{\bar{\mu}}_{f,t}\right) + \hat{A}_t^R \tag{19}$$

Again, the key point is that a relative markup term, $\hat{\bar{\mu}}_{h,t}^* - \hat{\bar{\mu}}_{f,t}$, enters, and this means that relative price fluctuations, specifically, $\hat{T}_t^W$, affect the cut-off. Equally, there are an equivalent equations for the world

cut-off, which depends on the relative price $\hat{T}_t^R$. Finally, these static solutions for wages are connected to the free entry condition, which in linear term is, $\hat{\bar{v}}_t^j = \hat{w}_t^j - \hat{A}_t^j$, and, in turn, the path of wages is pined down through a shares Euler equation. In particular,

$$\hat{\bar{v}}_t^j = \hat{C}_t^j - \hat{C}_{t+1}^j + \beta(1-\delta)\hat{\bar{v}}_{t+1}^j + [1-\beta(1-\delta)]\hat{\bar{d}}_{t+1}^j \tag{20}$$

where $j \in \{R, W\}$. These conditions generate a link between the price of shares and bonds which also appear in the Phillips curves, (14)-(17), derived above. This channel also plays an important role in the closed economy model of Bilbiie *et al.* (2005) because it restores the Taylor Principle - with this particular form of capital accumulation - and gives rise to a secondary channel for the transmission of monetary policy. With heterogeneous firms, this channel is augmented because there is an additional demand for labor - from the export sector.

3.3 Monetary Policy

I assume monetary policy is conducted using the short-term interest rate. For simplicity, I focus on two inflation targeting regimes: welfare-based consumer price inflation targeting - which includes all imported varieties of the foreign good - and domestic price inflation targeting. Although the government cannot observe welfare based prices, CPI targeting seems a natural benchmark. There is no reason the analysis could not also be extended to incorporate a Taylor-type rule by adding an output term. However, this extension would apply to both regimes I consider and whilst arguably adding realism would detract from the main point of the analysis. Consider the first policy regime, in which each government targets welfare based consumer prices, with the same weight. Consistent with the discussion above, the relative and world interest rates are,

$$\hat{\imath}_t^R = \phi_\Pi \hat{\Pi}_t^R + \hat{v}_t \quad ; \quad \hat{\imath}_t^W = \phi_\Pi \hat{\Pi}_t^W + \frac{1}{2}\hat{v}_t$$

where $\hat{v}_t$ is a domestic monetary policy shock. Because each agent (home and foreign) only has access to a non-traded risk free bond denominated in domestic currency we can simply plug these equations into the relative and world consumption Euler equations to determine the path of $\hat{C}_t^R = \hat{Q}_t$ and $\hat{C}_t^W$.

When the governments use interest rate rules to target domestic price inflation ($\hat{\Pi}_{h,t}$ and $\hat{\Pi}_{f,t}^*$, for the domestic and foreign governments respectively), I rewrite the interest rate rules to reflect changes in

relative prices that the governments account for versus the case in which they target the welfare based CPI.[5] In particular,

$$\hat{\imath}_t^R = \phi_\Pi\left(\hat{\Pi}_t^R - \Delta\hat{T}_t^W\right) + \hat{\nu}_t \quad ; \quad \hat{\imath}_t^W = \phi_\Pi\left(\hat{\Pi}_t^W - \frac{1}{4}\Delta\hat{T}_t^R\right) + \frac{1}{2}\hat{\nu}_t$$

On thing is immediately clear from these expressions. Suppose that the number of varieties is fixed. If we use labor supply to eliminate wages in the dynamic equations for $\hat{\Pi}_t^W$ and $\hat{T}_t^W$ (I show the details of this in the Appendix) the latter system is self-contained. Moreover, when the number of varieties is fixed, it is immediate from (16) and (17) that $\hat{T}_t^W$ and $\hat{T}_t^R$ are orthogonal to the monetary policy shocks. That is, $\hat{T}_t^W$ and $\hat{T}_t^W$ only depend on their own lagged and lead values, as neither additional endogenous variables nor the shocks to the monetary rules enter. As such, a monetary policy shock will have the same effect under each policy rule, i.e., whether the governments target the GDP deflator or the consumer price index as the former is simply the latter adjusted by variables, which are independent from the more general system. When firms make entry decisions, subject to sunk costs, and export decisions, subject to period fixed costs, this result breaks down, for the reasons discussed above.

4. Quantitative Analysis

In this section I analyze the quantitative implications of the model through simulations. I describe the parametrization and present the quantitative results for the baseline specification described above. The shocks to domestic labor productivity and monetary policy are given by the following,

$$\hat{A}_t = \rho_A\hat{A}_{t-1} + \hat{\varepsilon}_{A,t} \quad ; \quad \hat{\nu}_t = \rho_\nu\hat{\nu}_{t-1} + \hat{\varepsilon}_{\nu,t}$$

where $\hat{\varepsilon}_{A,t}$ and $\hat{\varepsilon}_{\nu,t}$ are i.i.d. normal innovations with mean zero and variance $\sigma_{\varepsilon_A}^2$ and $\sigma_{\varepsilon_\nu}^2$.

4.1 Calibration

Because of the assumption of Cobb-Douglas preferences, many of the steady state parameters do not influence the dynamic behavior of the model around the steady state. Table 1 presents the parameters for the baseline calibration.

[5] That is, $\hat{\imath}_t^R = \phi_\Pi\left(\hat{\Pi}_{h,t} - \hat{\Pi}_{f,t}^*\right) + \hat{\nu}_t$ and $\hat{\imath}_t^W = \frac{\phi_\Pi}{2}\left(\hat{\Pi}_{h,t} + \hat{\Pi}_{f,t}^*\right) + \frac{1}{2}\hat{\nu}_t$.

I interpret periods as quarters and set $\beta = 0.99$. The size of the exogenous firm exit shock $\delta = 0.025$ is set to to match the U. S. empirical level of 10% job destruction per year. I set $\sigma = 3.8$, which is calibrated to fit US plant and macro trade data. Given the standard deviation of log US plant sales, $\sigma = 3.8$ implies that $\kappa = 3.4$ (which also satisfies the requirement $k > \sigma - 1$), the value of which is taken from Bernard *et al.* (2003). The Frisch elasticity of labor supply is $\sigma_l = 2$. The Rotemberg adjustment cost parameter is set at $\xi = 77$, as in Bilbiie *et al.* (2007). The parameter on inflation in the interest rate rule is set at $\phi_\Pi = 1.5$. Finally, the persistence parameters for the monetary policy and labor productivity shocks are set at $\rho_\nu = 0.3$ and $\rho_A = 0.979$, respectively.

To simulate the model, I use the method outlined in Binder and Pesaran (1996), which, in this case, transforms the system of equations into the following lead and lag system.

$$\mathbf{A}Z_t = \mathbf{B}Z_{t-1} + \mathbf{C}\mathbb{E}_t Z_{t+1} + \mathbf{D}X_t$$

where Z_t is a vector containing the endogenous variables of the model and X_t is a vector containing the monetary policy and aggregate labor productivity shocks. Turning off the shocks, the vector of endogenous variables can be reduced to

$$Z_t = \begin{bmatrix} \hat{Q}_t & \hat{C}_t^W & \hat{\Pi}_t^R & \hat{\Pi}_t^W & \hat{T}_t^R & \hat{T}_t^W & \hat{n}_t^R & \hat{n}_t^W & \hat{\bar{\varphi}}_t^R & \hat{\bar{\varphi}}_t^W \end{bmatrix}'$$

where it is worth noting again that, for example, a domestic economy variable is given by, $\hat{x}_t = \hat{x}_t^W + \frac{1}{2}\hat{x}_t^R$. The system can also be easily transformed into one containing $\left\{\hat{\bar{q}}_t, \hat{\bar{\pi}}_t^R, \hat{\bar{\pi}}_t^W, \hat{\bar{t}}_t^R, \hat{\bar{t}}_t^W\right\}$ because such a transformation only involves variables already in the system, specifically, $\left\{\hat{n}_t^R, \hat{n}_t^W, \hat{\bar{\varphi}}_t^R, \hat{\bar{\varphi}}_t^W\right\}$.

4.2 Monetary Policy Shocks

Consider the situation in which each government targets consumer price inflation. A contractionary domestic monetary policy shock (i.e., $\hat{\varepsilon}_{\nu,t} > 0$) generates a rise in the domestic and foreign real interest rates, and, as a result, domestic and foreign consumption both temporarily fall. Whilst the shock induces a fall in domestic consumer price inflation, foreign consumer price inflation rises. Since all firms that export set prices in local currency such correlations in consumption and inflation are contrary to those generated by the standard New Keynesian model with *fixed-varieties*. However, the reaction of consumption and inflation are jointly determined with changes in the entry and export decisions of firms,

so these correlations are explained by the extent of the non-tradability of varieties and the differential in levels of average productivity across each sector of the economies.

The reaction of macro aggregates to the monetary policy shock also has implications for the extent of trade in varieties of goods in the global economy. The monetary policy shock increases the average productivity of domestic exporting firms (note: this productivity is $\widehat{\overline{\varphi}}^{*}_{h,t} = \left(\hat{n}_{h,t} - \hat{n}^{*}_{h,t}\right)/\kappa > 0$ and determines the fraction of exported products) whilst reducing the average productivity of foreign exporting firms. In particular, lower inflation in the domestic economy reduces the costs associated with nominal rigidities and raises the profits of the average firm. This induces entry into the domestic and export markets. In the latter, as labor becomes more expensive, and marginal costs rise, weaker firms are eliminated. The net result is that fewer domestic varieties are sold abroad but the exporting firms are more productive. The opposite happens in the foreign economy. This is shown in figure 2, where the dashed line are the responses of the foreign economy's endogenous variables to the shock and the bold lines represent domestic economy variables.

A second result - that a contractionary shock lowers firm entry - is also in line with empirical VAR evidence (Bergin and Corsetti, 2008). However, many closed economy models with free entry and sticky prices fail to generate such a correlation. The reason is the following: the free entry condition implies the price of equity (value of the firm) enters the Phillips Curve (via wages). A no-arbitrage condition links bonds and equities. When the nominal interest rate rises so does the real return on bonds, generating an increase in the expected return on equity. This is equivalent to a rise in the expected return on investing in product creation, and implies that the value of the firm falls, today, relative to tomorrow. As such, the cost of creating new firms falls. When firms are heterogeneous in productivity, the link between wages and the value of the firms is unchanged. Rather, there is an extra dimension to average firms profits - i.e., potential export profits - which is driven by the average level of firm productivity. This generates an additional source of labor demand. When there is a contractionary domestic monetary policy shock, real wages fall, which induces entry, but since marginal costs are higher, more firms also decide to export. To export, firms need to pay additional labor costs per period and as firms enter the export market, this increases real wages. That is, in a sticky price model with heterogenous firms, a contractionary monetary policy shock induces a fall in entry - there is a fall in the real wage, which generates entry but there is also a countervailing rise in wages from firms wishing to export their product to earn extra profits overall (as shown in figure two). The net result is that there is a cleansing effect.

I now consider the alternative regime in which each government targets domestic prices. It is common in open economy New Keynesian analysis to consider such a possibility (see, for example, Clarida *et al.* (2002) and Monacelli (2004)) because, for certain specifications over preferences and specific shocks, optimal policy is consistent with targeting zero domestic price inflation. Here, and mostly as a comparison, I also study the impulse responses of average productivity in both regimes when the shock to interest

rates has an autoregressive parameter of $\rho_v = 0.8$. There is some debate about the value this parameter should take. For example, Smets and Wouters (2007) estimate of a value of 0.12 for the US (which they assume is a closed economy). However, they also document a significant amount of interest rate smoothing; one lag weighted at 0.81. Steinsson (2008) and Carvalho and Necchio (2009) work with calibrated two country models and both assume a persistence parameter closer to one. Figure 3 presents two sets of impulse responses. The dotted lines correspond to consumer price inflation targeting regime and the dashed lines are the domestic price inflation targeting regime.

It is clear that adding persistence to the shock adds persistence productivity process. It also increases the magnitude of the initial reaction of average productivity in both countries. Targeting domestic price inflation creates a larger initial reaction from productivity than targeting consumer price inflation but the persistence of the change in average productivity is less. When thinking about these differences it is important to recall that the costs associated with price changes arise from output price inflation. For example, the average home firm selling in the domestic market pays a cost in terms of $\overline{\pi}_{h,t}$. The home government is targeting domestic consumer prices, which are, $\widehat{\Pi}_{h,t} = \widehat{\overline{\pi}}_{h,t} - \left(\frac{1}{1-\sigma}\right)\Delta\hat{n}_{h,t}$. That is, once we account for the number/quality of products in the domestic economy - and all firms that produce sell in the domestic market - the government is using a rule that aims to reduce the level of inflation consistent with the costs of the average firm. When the government targets consumer prices, it is explicitly concerned with the price of imported goods, which represent only a fraction of the goods sold in the foreign economy by foreign firms. This target then implicitly accounts for the heterogeneity of foreign firms.

4.3 Labor Productivity Shocks

I now briefly consider a shock to domestic aggregate labor productivity - similar to the RBC literature - and how this impacts the average productivity of exporting firms in the two countries. I consider a relatively persistent shock consistent with most of these studies. The important point to note is that an exogenous rise in labor productivity has a direct impact on the firm's pricing decision and it directly affects *all* domestic firms marginal costs. Specifically, each firm will lower its price in response to the fall in marginal costs. As the number of producers (and also the relative price t) is predetermined, this results in a drop in the aggregate price level. The real wage rises as the price level falls. In my model, the reduction in marginal costs induces entry into the domestic market, but as firms also wish to export, the associated rise in the real wage also forces some firms out of the export market. However, overall, when consumer prices are targeted, the average productivity of both home and foreign exporting firms rises, albeit modestly. Figure 4 presents impulse response for the average productivity response of domestic and foreign exporting firms for the two regimes. As above, the dotted lines correspond to consumer price inflation targeting regime and the dashed lines are the domestic price inflation targeting regime.

It is clear that there is a large difference across regimes. When governments target consumer prices there is an increase in the average productivity of exporting firms. However, when each government targets domestic prices, and there a labor productivity shock in the home economy there is a larger (and still positive) effect on the average productivity of foreign exporter but a fall in the average productivity of domestic exporters. This result is related to the fact that the shock directly hits all firms marginal costs. The transmission of the shock differs across the two regimes and is compounded by the fact that exporting firms price to market. When the governments target domestic prices they are, in effect, neglecting the price of imported goods, despite that fact that prices are set in local currency terms. This situation creates a larger spillover effect, via labor markets, from the domestic sales sector to the export sector amongst preexisting firms.

5. Conclusion

I have developed a two country general equilibrium model with heterogeneous price-setting firms to understand how shocks to monetary policy and aggregate labor productivity impact macroeconomic interdependence. A contractionary shock to the domestic interest rate raises the average productivity of domestic exporting firms but lowers the productivity of foreign exporting firms - a cleansing out effect. The magnitude of these changes is greater when governments target domestic price inflation as opposed to consumer price inflation. A positive shock to domestic labor productivity generates only modest (positive) changes in average productivity for both sets of exporting firms when consumer prices are targeted. However, when domestic price inflation is targeted, the shock causes a fall in the average productivity of home exporting firms, and a larger rise in the average productivity of foreign exporting firms.

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Table 1. Baseline Calibration

Calibration of Parameters		
Parameter	Description	Base Value
σ	U. S. plant and macro-trade data	3.8
κ	Dispersion of firms	3.4
σ_l	Intertemporal elasticity of labor supply	2
ϕ_Π	Coefficient on inflation in interest rate rule	1.5
β	Subjective discount factor	0.99
ξ	Rotemberg weight/cost of price changes	77
δ	Exogenous firm exit shock	0.025
ρ_ν	Autoregressive parameter for monetary shock	0.3
ρ_A	Autoregressive parameter for labor productivity shock	0.979

Figure 2. Impulse Response Functions - Monetary Policy Shock

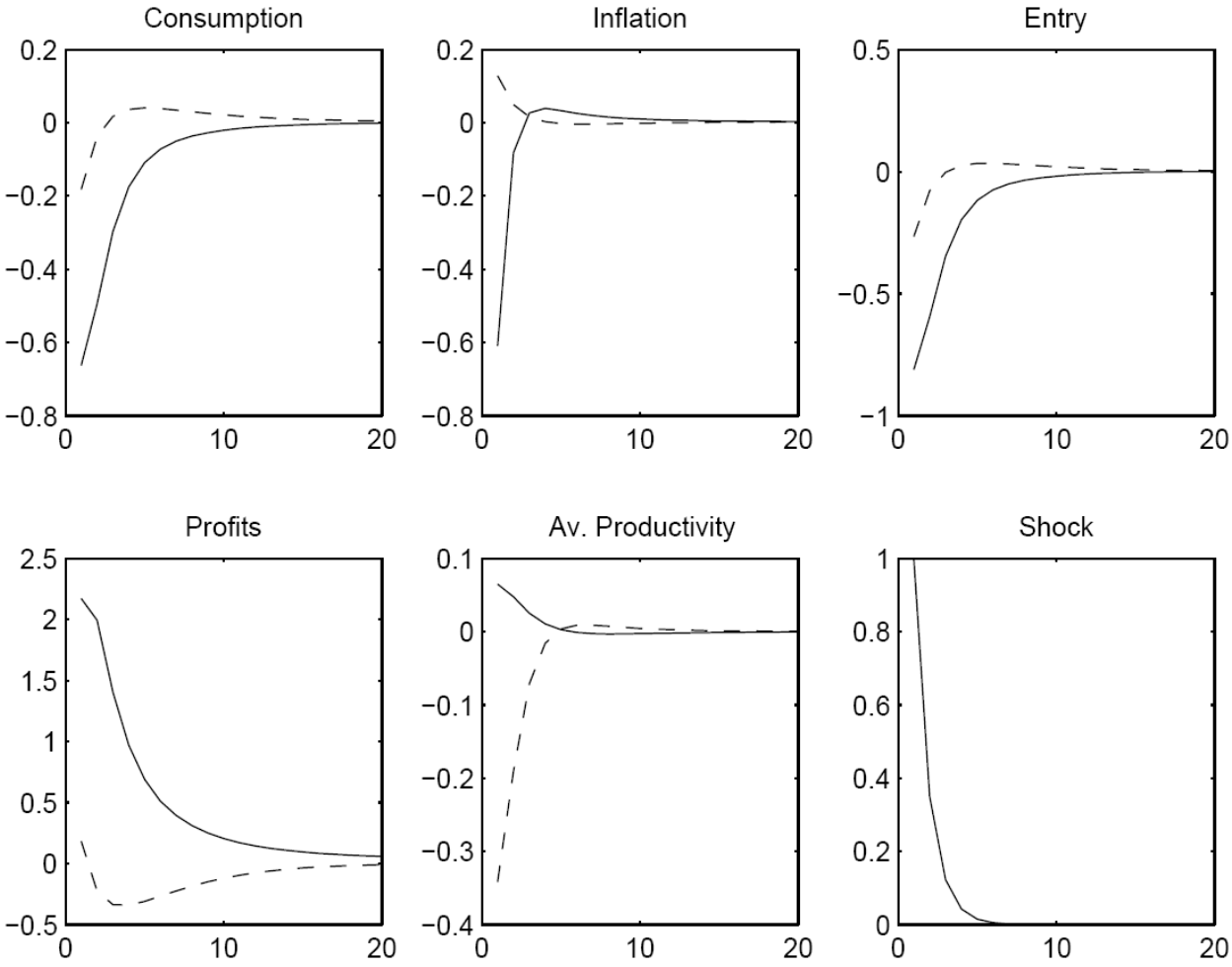


Figure 3. Av. Productivity IRFs for Two Policy Regimes - Monetary Policy Shock

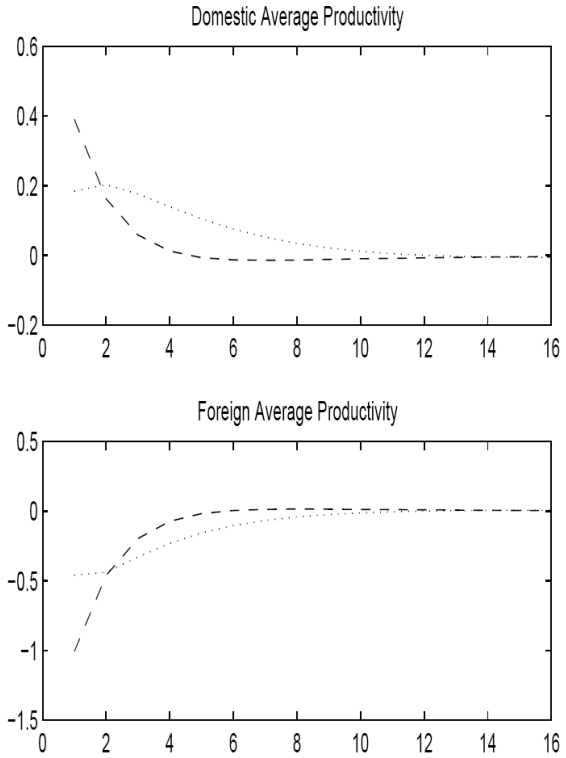
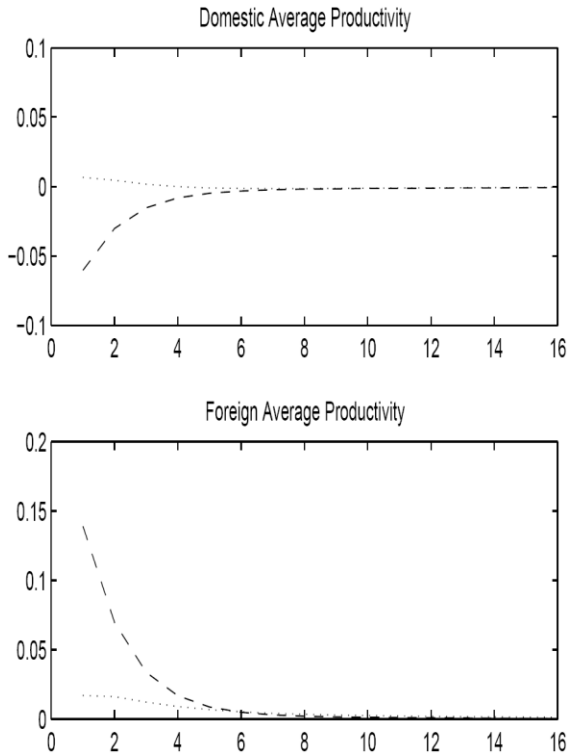


Figure 4. Av. Productivity IRFs for Two Policy Regimes - Labor Productivity Shock



Appendix A. Productivity Averages

Define the following cutoffs for the level of productivity of domestic firms as,

$$\varphi_t^* = \inf\{\varphi: d_{h,t}^*(\varphi) \geq 0\}$$

where $\varphi \equiv \varphi_h^* = \varphi_f$. Define two average levels of productivity; one for all domestic firms and one for all domestic exporters, as,

$$\overline{\varphi} \equiv \left[\int_{\underline{\varphi}}^{\infty} (1/\varphi)^{1-\sigma} dG(\varphi)\right]^{1/(\sigma-1)}$$

$$\overline{\varphi}_t^*(\varphi_t^*) \equiv \left[\frac{1}{1-G(\varphi_t^*)}\int_{\varphi_t^*}^{\infty} (1/\varphi)^{1-\sigma} dG(\varphi)\right]^{1/(\sigma-1)}$$

Note that $\overline{\varphi}$ does not vary over time and so has no time subscript. However, $\overline{\varphi}_t^*$ does vary over time. We will also assume that φ has a Pareto distribution, $G(\varphi) = 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^{\kappa}$, where $\kappa > 0$. Note that this implies, $\varphi = \underline{\varphi}[1 - G(\varphi)]^{-1/\kappa}$. Also note, $G\left(\underline{\varphi}\right) = 0$ and $G(\infty) = 1$. In this case,

$$\overline{\varphi} = \underline{\varphi}\left[\int_{\underline{\varphi}}^{\infty} [1 - G(\varphi)]^{(1-\sigma)/\kappa} dG(\varphi)\right]^{1/(\sigma-1)}$$

and so,

$$\int_{\underline{\varphi}}^{\infty} [1 - G(\varphi)]^{(1-\sigma)/\kappa} dG(\varphi) = \left| -\left(1 + \frac{1-\sigma}{\kappa}\right)^{-1} [1 - G(\varphi)]^{1+[(1-\sigma)/\kappa]} + const. \right|_{\underline{\varphi}_h}^{\infty}$$

$$= \left(1 + \frac{1-\sigma}{\kappa}\right)^{-1} = \frac{\kappa}{\kappa - (\sigma - 1)}$$

In this case,

$$\overline{\varphi} = \nu\underline{\varphi} \quad ; \quad \nu \equiv \left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{1/(\sigma-1)}$$

I perform the same analysis for $\overline{\varphi}_t^*$. Specifically,

$$\overline{\varphi}_t^* = \underline{\varphi}\left[\frac{1}{1 - G(\varphi_t^*)}\int_{\varphi_t}^{\infty}[1 - G(\varphi)]^{(1-\sigma)/\kappa}dG(\varphi)\right]^{1/(\sigma-1)}$$

and,

$$\int_{\varphi_{x,t}}^{\infty}[1 - G(\varphi)]^{(1-\sigma)/\kappa}dG(\varphi) = \left|-\left(1 + \frac{1-\sigma}{\kappa}\right)^{-1}[1 - G(\varphi)]^{1+[(1-\sigma)/\kappa]}\right|_{\varphi_{x,t}}^{\infty}$$

$$= \left(1 + \frac{1-\sigma}{\kappa}\right)^{-1}[1 - G(\varphi_t^*)]^{1+[(1-\sigma)/\kappa]}$$

which implies, $\overline{\varphi}_t^* = \left(\underline{\varphi}\nu\right) \cdot [1 - G(\varphi_t^*)]^{-1/\kappa}$, and as such,

$$\overline{\varphi}_t^* = \nu\varphi_t^* \quad ; \quad \left[\frac{\kappa}{\kappa - (\sigma - 1)}\right]^{1/(\sigma-1)}$$

Since $\overline{\varphi}_{h,t}^*$ is proportional to $\varphi_{h,t}^*$ we can study changes in either variable when analyzing the dynamics of the model.

Appendix B. Equilibrium

The demand for labor is determined by the following condition.

$$L_t = L_{h,t} + L^*_{h,t} + \left(n^*_{h,t} f^*_h + n_{Eh,t} f_E\right)/A_t$$

$$\bar{l}_{h,t} = \frac{\overline{d}_{h,t} + \left(P_{h,t}/P_t\right)\overline{\xi}_{h,t}}{\left(\mu_{h,t} - 1\right)w_t} \quad ; \quad \bar{l}^*_{h,t} = \frac{\overline{d}^*_{h,t} + \left(e_t P^*_{h,t}/P_t\right)\overline{\xi}^*_{h,t} + f^*_h w_t/A_t}{\left(\mu^*_{h,t} - 1\right)w_t}$$

Total labor used for production in the domestic and export markets is $L_{h,t} \equiv n_{h,t} l_{h,t}$ and $L^*_{h,t} \equiv n^*_{h,t} l^*_{h,t}$. In addition, new entrants hire f_E units of labor to cover the entry cost and each exporter hires f^*_h to cover the fixed export cost in each period. The total amount of labor hired to cover fixed export costs and new entrants is therefore $\left(n^*_{h,t} f^*_h + n_{Eh,t} f_E\right)/A_t$. We therefore have,

$$L_t = \frac{n_{h,t}}{\left(\mu_{h,t} - 1\right)w_t}\left[\overline{d}_{h,t} + \left(P_{h,t}/P_t\right)\overline{\xi}_{h,t}\right] + \frac{n^*_{h,t}}{\left(\mu^*_{h,t} - 1\right)w_t}\left[\overline{d}^*_{h,t} + \left(e_t P^*_{h,t}/P_t\right)\overline{\xi}^*_{h,t} + f^*_h w_t/A_t\right]$$
$$+ \frac{1}{A_t}\left(n^*_{h,t} f^*_h + n_{Eh,t} f_E\right)$$

Aggregating the household budget constraint (across symmetric agents) and imposing the equilibrium conditions ($A_{h,t} = A_{h,t+1} = 0$ and $s_t = s_{t+1} = 1$ and $n_{g,t} \equiv n_{h,t} + n_{e,t}$) yields,

$$e_t\left[\frac{A_{f,t}}{(1 + i^*_t)} - A_{f,t-1}\right] = W_t L_t + P_t \overline{d}_t n_{h,t} - P_t\left(C_t + \overline{v}_t n_{e,t}\right)$$

Now now eliminate profits. We know *average* total profits of domestic firms are, $\overline{d}_t = \overline{d}_{h,t} + \frac{n^*_{h,t}}{n_{h,t}}\overline{d}^*_{h,t}$, and that the free entry requirement equates the value of the firm with wages and the sunk costs of entry; $P_t \overline{v}_t = f_e W_t/A_t$. Once we impose labor demand, the trade account (in real terms) can be defined as,

$$F_t \equiv \left(\frac{\mu_{h,t}}{\mu_{h,t} - 1}\right) n_{h,t}\overline{d}_{h,t} + \left(\frac{\mu^*_{h,t}}{\mu^*_{h,t} - 1}\right) n^*_{h,t}\left(\overline{d}^*_{h,t} + f^*_h \frac{w_t}{A_t}\right) - C_t$$
$$+ \frac{n_{h,t}}{\mu_{h,t} - 1}\left[\left(P_{h,t}/P_t\right)\overline{\xi}_{h,t}\right] + \frac{n^*_{h,t}}{\mu^*_{h,t} - 1}\left[\left(Q_t P^*_{h,t}/P^*_t\right)\overline{\xi}^*_{h,t}\right]$$

where $Q_t = e_t P_t^* / P_t$. Note also that for domestic sales, average profits, in terms of consumption, are,

$$P_t \overline{d}_{h,t} = \overline{p}_{h,t} \overline{y}_{h,t} - W_t \overline{l}_{h,t} - P_{h,t} \overline{\xi}_{h,t} \quad ; \quad \overline{y}_{h,t} = \left(\frac{\overline{p}_{h,t}}{P_{h,t}} \right)^{-\sigma} \left(C_{h,t} + n_{h,t} \overline{\xi}_{h,t} \right)$$

$$\Leftrightarrow \overline{d}_{h,t} = \left(\frac{\mu_{h,t} - 1}{\mu_{h,t}} \right) \overline{\rho}_{h,t} n_{h,t}^{\sigma/(1-\sigma)} C_{h,t} - \frac{1}{\mu_{h,t}} \overline{\rho}_{h,t} n_{h,t}^{1/(1-\sigma)} \overline{\xi}_{h,t}$$

where $\overline{\rho}_{h,t} \equiv \overline{p}_{h,t} / P_t$, and for foreign sales, average profits, again in terms of consumption, are,

$$P_t \overline{d}_{h,t}^* = e_t \overline{p}_{h,t}^* \overline{y}_{h,t}^* - W_t \overline{l}_{h,t}^* - e_t P_{h,t}^* \overline{\xi}_{h,t}^* - f_h^* W_t / A_t \quad ; \quad \overline{y}_{h,t}^* = \left(\frac{\overline{p}_{h,t}^*}{P_{h,t}^*} \right)^{-\sigma} \left(C_{h,t}^* + n_{h,t}^* \overline{\xi}_{h,t}^* \right)$$

$$\overline{d}_{h,t}^* = \left(\frac{\mu_{h,t}^* - 1}{\mu_{h,t}^*} \right) Q_t \overline{\rho}_{h,t}^* n_{h,t}^{*\sigma/(1-\sigma)} C_{h,t}^* - \frac{1}{\mu_{h,t}^*} Q_t \overline{\rho}_{h,t}^* n_{h,t}^{*1/(1-\sigma)} \overline{\xi}_{h,t}^* - f_h^* w_t / A_t$$

where $\overline{\rho}_{h,t}^* \equiv \overline{p}_{h,t}^* / P_t^*$. These imply, $F_t \equiv \left(P_{h,t} / P_t \right) C_{h,t} + e_t \left(P_{h,t}^* / P_t \right) C_{h,t}^* - C_t$. Finally, using the upper tier demands and the CPI,

$$F_t = a_h \left(\frac{P_{h,t}^*}{P_t^*} \right)^{\theta/(\theta-1)} Q_t C_t^* - (1 - a_h) \left(\frac{P_{f,t}}{P_t} \right)^{\theta/(\theta-1)} C_t$$

where the right-hand side is exports, and the left-hand side imports.

Appendix C. Steady State

In the steady state there is balanced trade and zero inflation. In addition, $f_e = f_e^*$, $f_x = f_x^*$, $\tau = \tau^*$ and $A = A^* = 1$. Under these assumptions, the steady state of the model is symmetric, and $Q = 1$ and $C = C^*$. From the pricing equations,

$$\overline{\rho}_h = \overline{\mu}_h w/\overline{\varphi} \quad ; \quad \overline{\rho}_h^* = \overline{\mu}_h^* \tau w/\overline{\varphi}^* \quad ; \quad \overline{\mu}_h = \overline{\mu}_h^* = \frac{\sigma}{\sigma - 1} > 1$$

where $\overline{p}_h/P \equiv \overline{\rho}_h$ and $\overline{p}_h^*/P^* \equiv \overline{\rho}_h^*$. The CES price indexes reduce to, $P_h = n_h^{1/(1-\sigma)}\overline{p}_h$, $P_f = n_f^{1/(1-\sigma)}\overline{p}_f$, $P_h^* = n_h^{*1/(1-\sigma)}\overline{p}_h^*$, and consumption of products are,

$$\overline{c}_h = a_h n_h^{\Phi} \overline{\rho}_h^{1/(\theta-1)} C \quad ; \quad \overline{c}_f = (1 - a_h) n_f^{\Phi} \overline{\rho}_f^{1/(\theta-1)} C \quad ; \quad \overline{c}_h^* = (1 - a_h) n_h^{*\Phi} \overline{\rho}_h^{*1/(\theta-1)} C$$

where $\Phi \equiv [\sigma(\theta - 1) + 1]/(\sigma - 1)(1 - \theta)$. The expenditure share of domestic consumption on domestic goods is defined as $s_h \equiv a_h n_h^{1+\Phi} \overline{\rho}_h^{\theta/(\theta-1)}$. Note, when the elasticity of substitution is the same between goods and brands, $\sigma = 1/(1 - \theta) \Leftrightarrow \Phi = 0$, and so, $s_h \equiv a_h n_h \overline{\rho}_h^{1-\sigma}$; when the upper tier of utility is Cobb-Douglas, $\theta = 0 \Leftrightarrow \Phi = -1$, and, $s_h \equiv a_h$. In each case, the respective consumer price index is,

$$1 = a_h \left(n_h^{1/(1-\sigma)} \overline{\rho}_h\right)^{\theta/(\theta-1)} + a_f \left(n_f^{1/(1-\sigma)} \overline{\rho}_f\right)^{\theta/(\theta-1)} : \theta > 0$$
$$1 = \left(n_h^{1/(1-\sigma)} \overline{\rho}_h\right)^{a_h} \left(n_f^{1/(1-\sigma)} \overline{\rho}_f\right)^{a_f} : \theta = 0$$

And note, $\overline{y}_h = \overline{c}_h$ and $\overline{y}_h^* = \tau \overline{c}_h^*$. Total profits, the value of the firm, in terms of total profit, and the free entry condition are,

$$\overline{d}_t = \overline{d}_h + \left(\frac{n_h^*}{n_h}\right) \overline{d}_h^* \quad ; \quad \overline{v} = \left[\frac{(1 - \delta)\beta}{1 - (1 - \delta)\beta}\right] \overline{d} \quad ; \quad \overline{v} = w f_e$$

Profits from domestic and export sales, and the zero-profit export cutoff are,

$$\overline{d}_h = \overline{\rho}_h \overline{y}_h - w\overline{l}_h \quad ; \quad \overline{d}_h^* = \frac{1}{\tau}\overline{\rho}_h \overline{y}_h^* - w\overline{l}_h^* - f_h^* w \; ; \quad \overline{d}_h^* = (\nu^{\sigma-1} - 1) f_h^* w$$

where $\nu \equiv \{\kappa/[\kappa - (\sigma - 1)]\}^{1/(\sigma-1)}$. The share of exporting firms and number of firms are,

$$\frac{n_h^*}{n_h} = \left(\frac{\overline{\varphi}}{\overline{\varphi}^*}\right)^{\kappa} \quad ; \quad n_h = \left(\frac{1-\delta}{\delta}\right) n_e$$

and finally, labor demand, labor supply, and balanced trade are given by,

$$L = n_h l_h + n_h^* l_h^* + n_h^* f_h^* + n_e f_e \quad ; \quad w = \phi C L^{\varphi} \quad ; \quad C = wL + \overline{d} n_h - \overline{v} n_e$$

I first determine the *zero-profit export productivity level*, $\overline{\varphi}^*$. First, total profits, the value of the firm and the free entry condition imply,

$$\overline{d}_h + \left(\frac{n_h^*}{n_h}\right)\overline{d}_h^* = \left[\frac{1-(1-\delta)\beta}{(1-\delta)\beta}\right] w f_e$$

Second, profits from domestic and export sales - by apply the pricing equations to eliminate wages - can be expressed in terms of consumption as,

$$\overline{d}_h = \overline{\rho}_h \overline{y}_h - \frac{w}{\overline{\varphi}}\overline{y}_h \quad ; \quad \overline{\rho}_h = \frac{\sigma}{\sigma-1}\frac{w}{\overline{\varphi}} \Rightarrow \overline{d}_h = \left(\frac{a_h}{\sigma}\right) n_h^{\Phi} \overline{\rho}_h^{\theta/(\theta-1)} C$$

$$\overline{d}_h^* + f_h^* w = \frac{1}{\tau}\overline{\rho}_h \overline{y}_h^* - \frac{w}{\overline{\varphi}^*}\overline{y}_h^* \quad ; \quad \overline{\rho}_h^* = \frac{\sigma}{\sigma-1}\frac{\tau w}{\overline{\varphi}^*} \Rightarrow \overline{d}_h^* + f_h^* w = \left(\frac{a_h}{\sigma}\right) n_h^{*\Phi} \overline{\rho}_h^{*\theta/(\theta-1)} C$$

If we also use the zero-profit export cutoff, $\overline{d}_h^* = (\nu^{\sigma-1} - 1) f_h^* w$, we find an expression for profits from domestic sales only,

$$\overline{d}_h = \left(\frac{n_h}{n_h^*}\right)^{\Phi} \left(\frac{\overline{\rho}_h}{\overline{\rho}_h^*}\right)^{\theta/(\theta-1)} \nu^{\sigma-1} f_h^* w$$

where ν is defined above. Now use this condition, the expression for total profits and the zero-profit export cut off (again) to eliminate profits and wages.

$$\left[\frac{1-(1-\delta)\beta}{(1-\delta)\beta}\right]\frac{f_e}{f_h^*}=\left(\frac{\overline{\rho}_h^*}{\overline{\rho}_h}\right)^{\theta/(1-\theta)}\left(\frac{n_h^*}{n_h}\right)^{-\Phi}\nu^{\sigma-1}+\left(\frac{n_h^*}{n_h}\right)(\nu^{\sigma-1}-1)$$

Finally, the pricing equations and share of exporting firms are,

$$\frac{\overline{\rho}_h^*}{\overline{\rho}_h}=\frac{\tau\nu\underline{\varphi}}{\overline{\varphi}^*}\quad;\quad\frac{n_h^*}{n_h}=\left(\frac{\nu\underline{\varphi}}{\overline{\varphi}^*}\right)^{\kappa}$$

where we have also used, $\overline{\varphi}=\nu\underline{\varphi}$. Finally, we can determine $\overline{\varphi}^*$ through the following equation,

$$\xi_1\overline{\varphi}^{*\theta/(\theta-1)+\kappa\Phi}+\xi_2\overline{\varphi}^{*-\kappa}=\xi_3$$

where,

$$\xi_1\equiv\tau^{\theta/(1-\theta)}\underline{\varphi}^{\theta/(1-\theta)-\Phi\kappa}\nu^{\sigma-1+\theta/(1-\theta)-\Phi\kappa}$$

$$\xi_2\equiv\underline{\varphi}^{\kappa}(\nu^{\sigma-1}-1)\nu^{\kappa}\quad;\quad\xi_3\equiv\left[\frac{1-(1-\delta)\beta}{(1-\delta)\beta}\right]\frac{f_e}{f_h^*}$$

This derivation does not rely on the consumer price index.

I now determine the *expenditure share* when preferences are CES; that is $s_h=a_h n_h^{1+\Phi}\overline{\rho}_h^{\theta/(\theta-1)}$. Since I consider a symmetric steady state in which $n_f=n_h^*$ - that is, foreign varieties sold in the domestic economy are equal in number to domestic varieties sold in the foreign economy - and $\overline{\rho}_f=\overline{\rho}_h^*$, the consumer price index can be re-expressed as, $1=a_h n_h^{1+\Phi}\overline{\rho}_h^{\theta/(\theta-1)}+(1-a_h)n_h^{*1+\Phi}\overline{\rho}_h^{*\theta/(\theta-1)}$, and the expenditure share variable is given by the following.

$$s_h=a_h\left(\frac{\tau\nu\underline{\varphi}}{\overline{\varphi}^*}\right)^{\theta/(1-\theta)}\left[a_h\left(\frac{\tau\nu\underline{\varphi}}{\overline{\varphi}^*}\right)^{\theta/(1-\theta)}+(1-a_h)\left(\frac{\nu\underline{\varphi}}{\overline{\varphi}^*}\right)^{\kappa(1+\Phi)}\right]^{-1}$$

as $\overline{\rho}_h^*/\overline{\rho}_h=\tau\nu\underline{\varphi}/\overline{\varphi}^*$ and $n_h^*/n_h=\left(\nu\underline{\varphi}/\overline{\varphi}^*\right)^{\kappa}$. When the upper tier of utility is Cobb-Douglas,

$$\overline{\rho}_h n_h^{1/(1-\sigma)} = \left(\frac{\tau\nu\varphi}{\overline{\varphi}^*}\right)^{a_h-1}\left(\frac{\nu\varphi}{\overline{\varphi}^*}\right)^{\kappa(1-a_h)(\sigma-1)}$$

Once we have determined expenditure shares, we can use this to determine other variables. For *labor supply*, start from the aggregate accounting condition, but now use the shares Euler equation to eliminate $\overline{v}$ instead of $\overline{d}$ (we do not use the free entry condition),

$$C = wL + n_h\overline{d} - \overline{v}n_e \quad ; \quad n_e = \left(\frac{\delta}{1-\delta}\right)n_h \quad ; \quad \overline{v} = \overline{d}\left[\frac{(1-\delta)\beta}{1-(1-\delta)\beta}\right]$$

$$\Rightarrow C = wL + \left[\frac{1-\beta}{1-(1-\delta)\beta}\right]n_h\overline{d} \quad ; \quad \overline{d} = \overline{d}_h + \frac{n_h^*}{n_h}\overline{d}_h^*$$

We also have profits from domestic sales and profits from export sales (incorporating the cutoff for zero export profits).

$$n_h\overline{d}_h = \frac{s_h}{\sigma}C \quad ; \quad n_h^*\overline{d}_h^* = \left(\frac{s_h^*}{\sigma}\right)\left(1-\frac{1}{\nu^{\sigma-1}}\right)C$$

$$\Rightarrow \frac{w}{C} = \left\{1-\left[\frac{1-\beta}{1-(1-\delta)\beta}\right]\left[\frac{s_h}{\sigma}+\left(\frac{s_h^*}{\sigma}\right)\left(1-\frac{1}{\nu^{\sigma-1}}\right)\right]\right\}\frac{1}{L}$$

Finally, we have, $\psi L^{1/\eta} = w/C$, and so,

$$L^{(1+\eta)/\eta} = \left\{1-\left(\frac{r}{r+\delta}\right)\left[\frac{s_h}{\sigma}+\left(\frac{s_h^*}{\sigma}\right)\left(1-\frac{1}{\nu^{\sigma-1}}\right)\right]\right\}\frac{1}{\psi}$$

Notice also that if we set $s_h = 1$, then $s_h^* = 0$ (the closed economy), and we have a more familiar expression, $L = \left[\frac{(r+\delta)\sigma-r}{\psi(r+\delta)\sigma}\right]^{\eta/(1+\eta)}$.

Finally, I determine the *relative price*, $\overline{\rho}_h$. Given the definition of expenditure shares, this also determines the number of varieties in the economy. First, given the definition of s_h, we have,

$$\overline{\rho}_h^{1-\sigma} = \left(\frac{a_h}{s_h}\right)^{1/(1+\Phi)} n_h$$

where I have used $\theta/(\theta-1)(1+\Phi)=\sigma-1$. Second, use the trade balance, the number of firms, the value of the average firm (as a function of total profits) and the free entry condition. These imply,

$$\frac{C}{w}=L+\left[\frac{1-\beta}{(1-\delta)\beta}\right]f_e n_h \quad ; \quad L=\left(\phi\frac{C}{w}\right)^{-1/\varphi} \Rightarrow \frac{C}{w}=L+\left[\frac{1-\beta}{(1-\delta)\beta}\right]f_e n_h$$

where L has already been determined. Second, export profits and the zero-profit export cutoff (using $n_h^*=n_h\left(\underline{\varphi}\nu/\overline{\varphi}^*\right)^{\kappa}$) also yield a linear equation in the consumption-wages ratio, as used above.

$$\frac{C}{w}=\frac{n_h}{s_h^*}\left(\frac{\underline{\varphi}}{\overline{\varphi}^*}\right)^{\kappa}\sigma\nu^{\kappa+\sigma-1}f_h^*$$

Eliminating the consumption-wage ratio, we find,

$$n_h=\left\{\frac{1}{s_h^*}\left(\frac{\underline{\varphi}}{\overline{\varphi}^*}\right)^{\kappa}\sigma\nu^{\kappa+\sigma-1}f_h^*+\left[\frac{1-\beta}{(1-\delta)\beta}\right]f_e\right\}L^{-1}$$

which determines the number of varieties produced and the relative price, $\overline{\rho}_h$, given labor supply, L.

Appendix D. Fixed Varieties System

When the number of varieties is fixed and all goods are exported we can simply use labor supply and the goods market clearing conditions to eliminate $\widehat{w}_t^R$ and $\widehat{w}_t^W$ in the dynamic equations for $\widehat{\Pi}_t^W$ and $\widehat{T}_t^W$. This gives,

$$\widehat{\Pi}_t^W = \beta \mathbb{E}_t \widehat{\Pi}_{t+1}^W + \left[\frac{(1+\sigma_l)(1-\sigma)}{\xi}\right]\left(\hat{C}_t^W - \hat{A}_t^W\right)$$

$$\left\{1 + \beta + \left[\frac{(1+\sigma_l)(1-\sigma)}{\xi}\right]\right\}\widehat{T}_t^W - \left[\frac{(1+\sigma_l)(1-\sigma)}{\xi}\right]\hat{A}_t^R = \widehat{T}_{t-1}^W + \beta \mathbb{E}_t \widehat{T}_{t+1}^W$$

The remaining equations are as in the main text. That is,

$$\widehat{\Pi}_t^R = \beta \mathbb{E}_t \widehat{\Pi}_{t+1}^R + \left(\frac{1-\sigma}{\xi}\right)\hat{Q}_t$$

$$\left[(1+\beta) + \left(\frac{1-\sigma}{\xi}\right)\right]\widehat{T}_t^R = \widehat{T}_{t-1}^R + \beta \mathbb{E}_t \widehat{T}_{t+1}^R$$

and,

$$\hat{Q}_{t+1} - \hat{Q}_t = \hat{\imath}_t^R - \mathbb{E}_t \widehat{\Pi}_{t+1}^R \quad ; \quad \hat{C}_{t+1}^W - \hat{C}_t^W = \hat{\imath}_t^W - \mathbb{E}_t \widehat{\Pi}_{t+1}^W$$

where,

$$\hat{\imath}_t^R = \phi_\Pi\left[\widehat{\Pi}_t^R - \left(\widehat{T}_t^W - \widehat{T}_{t-1}^W\right)\right] + \hat{v}_t^R \quad ; \quad \hat{\imath}_t^W = \phi_\Pi\left[\widehat{\Pi}_t^W - \frac{1}{4}\left(\widehat{T}_t^R - \widehat{T}_{t-1}^R\right)\right] + \hat{v}_t^W$$

are the interest rate rules when each government targets GDP deflator inflation. These eight equations determine the path of $\{\hat{Q}_t, \hat{C}_t^W, \hat{\imath}_t^R, \hat{\imath}_t^W, \widehat{\Pi}_t^R, \widehat{\Pi}_t^W, \widehat{T}_t^R, \widehat{T}_t^W\}$, for shocks to $\hat{A}_t$ and $\hat{v}_t$; that is aggregate labor productivity and the nominal interest rate, that follow, $\hat{A}_t = \rho_A \hat{A}_{t-1} + \hat{\varepsilon}_{A,t}$ and $\hat{v}_t = \rho_v \hat{v}_{t-1} + \hat{\varepsilon}_{v,t}$, where $\hat{\varepsilon}_{A,t}$ and $\hat{\varepsilon}_{v,t}$ are i.i.d. It is worth noting that if we take relative output is, $\hat{y}_t^R = \widehat{T}_t^W$. That means, suppose we assume each country targets consumer price inflation and GDP, with parameter, $\phi_y > 0$. Then, ignoring the monetary policy shocks, we must have, $\hat{\imath}_t^R = \phi_\Pi \widehat{\Pi}_t^R + \phi_y \widehat{T}_t^W$ and $\hat{\imath}_t^W = \phi_\Pi \widehat{\Pi}_t^W + \phi_y \hat{C}_t^W$. Such a policy does not add any inertia to policy rules. When there is firm entry some of this

simplicity breaks down because we need to differentiate output that is consumed from that which is invested.

Appendix E. Heterogeneous Firms System

Recall that for any variable, say x_t, I defined $\Delta x_t \equiv x_t - x_{t-1}$, $x_t^R \equiv x_t - x_t^*$ and $x_t^W \equiv \frac{1}{2}x_t + \frac{1}{2}x_t^*$. Assuming that each government targets welfare-based consumer price inflation, I have the following system of ten dynamic equations that determine $\left\{\hat{Q}_t, \hat{C}_t^W, \hat{n}_t^R, \hat{n}_t^W, \hat{\Pi}_t^R, \hat{\Pi}_t^W, \hat{T}_t^R, \hat{T}_t^W, \hat{\bar{\varphi}}_t^R, \hat{\bar{\varphi}}_t^W\right\}$.

$$\hat{Q}_t + \left(\phi_\Pi \hat{\Pi}_t^R + \hat{v}_t\right) = \hat{Q}_{t+1} + \mathbb{E}_t \hat{\Pi}_{t+1}^R \quad ; \quad \hat{C}_t^W + \left(\phi_\Pi \hat{\Pi}_t^W + \frac{1}{2}\hat{v}_t\right) = \hat{C}_{t+1}^W + \mathbb{E}_t \hat{\Pi}_{t+1}^W$$

$$\hat{n}_t^R = (1-\delta)\hat{n}_{t-1}^R + \delta \hat{n}_{e,t-1}^R \quad ; \quad \hat{n}_t^W = (1-\delta)\hat{n}_{t-1}^W + \delta \hat{n}_{e,t-1}^W$$

$$-\tilde{\xi}\hat{Q}_t + \hat{\Pi}_t^R + A_2 \hat{\bar{\varphi}}_t^R = \tilde{\beta}\mathbb{E}_t \hat{\Pi}_{t+1}^R + \frac{\tilde{\kappa}\tilde{\beta}}{2}\mathbb{E}_t \hat{\bar{\varphi}}_{t+1}^R + \frac{\tilde{\kappa}}{2}\hat{\bar{\varphi}}_{t-1}^R$$

$$\hat{\Pi}_t^W - \tilde{\xi}\left(\hat{w}_t^W - \hat{A}_t^W\right) + A_1 \hat{n}_t^W - A_2 \hat{\bar{\varphi}}_t^W = \tilde{\beta}\mathbb{E}_t\left[\hat{\Pi}_{t+1}^W + \left(\frac{1}{\sigma-1}\right)\hat{n}_{t+1}^W - \frac{\tilde{\kappa}}{2}\hat{\bar{\varphi}}_{t+1}^W\right] + \left(\frac{1}{\sigma-1}\right)\hat{n}_{t-1}^W - \frac{\tilde{\kappa}}{2}\hat{\bar{\varphi}}_{t-1}^W$$

$$(\sigma-1)A_1 \hat{T}_t^R - \left[2\tilde{\kappa}\left(1+\tilde{\beta}\right) - \tilde{\xi}(1-\tilde{\kappa})\right]\hat{\bar{\varphi}}_t^W = \tilde{\beta}\mathbb{E}_t\left(\hat{T}_{t+1}^R - 2\tilde{\kappa}\hat{\bar{\varphi}}_{t+1}^W\right) + \hat{T}_{t-1}^R - 2\tilde{\kappa}\hat{\bar{\varphi}}_{t-1}^W$$

$$-\tilde{\xi}\hat{Q}_t + (\sigma-1)A_1 \hat{T}_t^W + \tilde{\xi}\left(\hat{w}_t^R - \hat{A}_t^R\right) - A_1 \hat{n}_t^R + A_2 \hat{\bar{\varphi}}_t^R = \tilde{\beta}\mathbb{E}_t\left[\hat{T}_{t+1}^W - \left(\frac{1}{\sigma-1}\right)\hat{n}_{t+1}^R + \frac{\tilde{\kappa}}{2}\hat{\bar{\varphi}}_{t+1}^R\right] + \hat{T}_{t-1}^W - \left(\frac{1}{\sigma-1}\right)\hat{n}_{t-1}^R + \frac{\tilde{\kappa}}{2}\hat{\bar{\varphi}}_{t-1}^R$$

$$-\hat{Q}_t + \hat{w}_t^R - \hat{A}_t^R = -\hat{Q}_{t+1} + \left(\frac{1-\delta}{1+r}\right)\left(\hat{w}_{t+1}^R - \hat{A}_{t+1}^R\right) + \left(\frac{r+\delta}{1+r}\right)\hat{\bar{d}}_{t+1}^R$$

$$-\hat{C}_t^W + \hat{w}_t^W - \hat{A}_t^W = -\hat{C}_{t+1}^W + \left(\frac{1-\delta}{1+r}\right)\left(\hat{w}_{t+1}^W - \hat{A}_{t+1}^W\right) + \left(\frac{r+\delta}{1+r}\right)\hat{\bar{d}}_{t+1}^W$$

These are consumption Euler equations, laws of motion for products, relative consumer price inflation, world price inflation, relative and world relative price Phillips Curves and shares Euler equations. The following six static conditions,

$$A_4 \hat{Q}_t - A_3 \hat{w}_t^R + \hat{n}_{e,t}^R - \left(\frac{n_d d}{n_E v}\right)\hat{n}_t^R - \left(\frac{n_d d}{n_E v}\right)\hat{\bar{d}}_t^R - \hat{A}_t^R = 0$$

$$A_4 \hat{C}_t^W - A_3 \hat{w}_t^W + \hat{n}_{e,t}^W - \left(\frac{n_d d}{n_E v}\right)\hat{n}_t^W - \left(\frac{n_d d}{n_E v}\right)\hat{\bar{d}}_t^W - \hat{A}_t^W = 0$$

$$-\Psi\hat{Q}_t + \Psi(\sigma-1)\hat{T}_t^W + A_5 \hat{w}_t^R + \hat{\bar{d}}_t^R - [\kappa - (\sigma-1)]\kappa\hat{\bar{\varphi}}_t^R - A_5 \hat{A}_t^R = 0$$

$$-\Psi\hat{C}_t^W + \Psi\left(\frac{\sigma-1}{4}\right)\hat{T}_t^R + A_5\hat{w}_t^W + \hat{\bar{d}}_t^W - [\kappa-(\sigma-1)]\kappa\hat{\bar{\varphi}}_t^W - \left(\frac{A_5}{2}\right)\hat{A}_t^W = 0$$

$$-[1+(\sigma-1)2]\hat{Q}_t + (\sigma-1)\hat{T}_t^W + \sigma\hat{w}_t^R - (\sigma-1)\hat{\bar{\varphi}}_t^R - \sigma\hat{A}_t^R = 0$$

$$-\hat{C}_t^W - \left(\frac{\sigma-1}{4}\right)\hat{T}_t^R + \sigma\hat{w}_t^W - (\sigma-1)\hat{\bar{\varphi}}_t^W - \sigma\hat{A}_t^W = 0$$

determine $\left\{\hat{w}_t^R, \hat{w}_t^W, \hat{n}_{e,t}^R, \hat{n}_{e,t}^W, \hat{\bar{d}}_t^R, \hat{\bar{d}}_t^W\right\}$. These are the resource constraints, total (average) firm profits, and zero-profit export cutoffs. The terms A_i, for $i = 1,\ldots,9$, are constants and determined by the underlying structural parameters of the model - see Appendix C.