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STOCHASTIC COOPERATION MODEL FOR MEASURING FIRMS' DEFAULT PROBABILITIES: AN APPLICATION TO CLIMATE TRANSITION RISK

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Stochastic cooperation model for measuring firms' default probabilities: An application to climate transition risk

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Abstract

To measure firms' default probabilities, we approach the problem of time-varying target leverage ratios from the perspective of ecological systems. In a particular industry, the existence of time-varying target leverage ratios results from the cooperation of firms in making decisions on their capital structure. This proposal yields the joint probability density function of the leverage ratios for an ensemble of firms in closed form so that a likelihood function can be constructed. We fit the model to market data to estimate the corresponding probability of default for the firms in our sample in two industries, the automotive industry manufacturing fossil fuel vehicles and the integrated oil industry in Europe and North America. These two sectors were significantly impacted by climate policies following the Paris Agreement in 2015. The calibration results demonstrate that the model parameters can be effectively estimated for the firms' leverage ratio dynamics, and that firms cooperate to adjust their leverage ratios towards their target level. In terms of the changes in the firms' default probabilities, an event study shows that firms face substantial transition risk caused by the climate mitigation policies.

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1. Introduction

How much debt and how much equity firms choose to finance their operations by balancing the costs and benefits is a fundamental question in corporate finance. According to the dynamic trade-off theory of capital structure, a firm tries to select an optimal ratio of its liabilities to asset value to balance the dead-weight costs of bankruptcy and the tax saving benefits of debt. In other words, in the presence of adjustment costs, firms try to set relatively stable targets of leverage ratios and keep these within the tolerable ranges. Such behaviour is inevitably expected to affect the dynamics of adjustment of leverage ratios to their desired level. Hence, much attention has been given to research on how corporate leverage ratios evolve and how this process is affected by capital structure decisions. For instance, based upon 111,106 firm-year observations between the years 1965-2001, Flannery and Rangan (2006) conclude that firms adjust rapidly towards time-varying target leverage ratios, suggesting that firms' current leverage ratios may be too far from their optimal level. They also observe that firms with high leverage narrow the gap between their current and target leverage ratio more quickly than those with lower leverage, suggesting that deviations from target are more costly for highly-leveraged firms.

In addition to the traditional trade-off between tax benefits and bankruptcy costs, studies find that market competition plays an important role in determining firms' investment and financing policies, i.e. firms' capital structure decisions. Matveyev and Zhdanov (2022) show that operationally identical firms optimally choose different debt ratios, which results in within-industry dispersion in leverage. They show that this dispersion can be economically significant, and depends on firms’ cash flow volatility, tax rates, and bankruptcy costs. These factors make firms optimally adopt different financing policies. A number of empirical studies find that the link between capital structure and competition is important. MacKay and Phillips (2005) show that the distribution of firms' leverage ratios depends on industry structure and the degree of competition among firms, and that new entrants within the sector have higher leverage compared to incumbents. Kovenock and Phillips (1997) demonstrate increased debt (leverage) makes competition more intensive. Other studies include Miao (2005) which examines the evolution of a competitive industry when firms experience idiosyncratic technology shocks by using a capital structure trade-off model. Fries, Miller and Perraudin (1997) and Zhdanov (2007) examine aggregate uncertainty and equilibrium financing strategies under perfect competition.

Given that competition among firms can lead them to optimally choose different debt ratios, this can result in significant within-industry dispersion of leverage with debt levels providing a strategic advantage over a firm's competitors. Therefore, a firm's optimal leverage is determined not only by its own characteristics, but also by the characteristics of the overall industry structure. To address this issue, in this paper we approach the problem of time-varying target leverage ratios from the perspective of ecological systems using a stochastic modified Lotka-Volterra Competition (LVC) model. We propose that, in a particular industry or financial sector, the existence of time-varying target leverage ratios results from cooperation and/or competition among firms in making decisions on their capital structure. Such a proposal implies that a firm's target leverage ratio is not necessarily confined to the neighbourhood of the industry average (or equivalently, its statistical average) and that the validity of the aforementioned partial adjustment approach is questionable. In addition, we are able to derive the joint probability density function (PDF) of leverage ratios for an ensemble of firms in closed form so that a likelihood function can be constructed, and model-fitting against the empirical data becomes feasible.

In recent research on the interaction between firms' leverage ratios and bond pricing (default risk), Flannery et al. (2012) find that target leverage ratios are an important consideration.[1] Feldhütter and Schaefer (2023) also investigate how the dynamics of corporate debt policy affect the pricing of corporate bonds, and find mean reversion in leverage gives more accurate predictions of credit spreads.[2] In relation to climate risk, Ginglinger and Quentin (2023) examine the impact of climate risk on capital structure and find that greater climate risk leads to lower leverage in the post-2015 period following the Paris Agreement. Several recent studies including Painter (2020) and Seltzer et al. (2024) emphasize that climate risk affects the pricing of corporate bonds. The impact of past major climate events on firms' financial decisions is studied by Hong et al. (2019) and Brown et al. (2021).

The proposed model in this paper links the model of time-varying target leverage ratios and the stochastic modified LVC model from the perspective of ecological systems. When a whole sector such as a high-emission sector is being challenged by new policies, such as carbon taxes or emissions quotas, arising from climate change mitigation, the model allows us to study whether firms in the sector will cooperate or compete during the transition caused by climate policy. Such analysis can potentially be useful to banks and credit rating agencies in assessing firms' transition plans to decarbonise.

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[1] Merton (1974) has been the pioneers in the development of the structural models for credit risk of corporates using a contingent-claims framework. Merton's model treats default risk equivalent to a European put option on a firm's asset value and the firm's liability is the option strike. To extend the Merton model, the structural models with different dynamic liability structures (where default boundaries are exogenous) have been considered by Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) and Hui et al. (2005, 2006). On the other hand, Leland and Toft (1996) model considers an endogenous-boundary model in which the firm issues debt of arbitrary maturity.

[2] Vo et al. (2022) show that firms tend to adjust their capital structure more rapidly in the period following the breakout of COVID 19.

Competition and mutualism are ubiquitous in the natural world. Organisms of the same or different species inevitably need to compete for the limited resources available. Competition within the same species is termed intraspecific competition, whereas competition between *different* species which have similar requirements is termed interspecific competition.

According to conventional wisdom, firms exhibit both intraspecific and interspecific competition. However, when a whole sector is being challenged as in the case of new climate policy, the firms may react differently – they may cooperate rather than compete. In a particular industry, the existence of time-varying target leverage ratios results from the cooperation of firms in making decisions on their capital structure. Given that the closed-form PDF of the set of stochastic variables in the model is known analytically in Lo (2023), all the model parameters can be calibrated for a sample of firms against market data by means of the maximum-likelihood method. We calibrate the model by using firms in the automotive industry which manufacture fossil fuel vehicles and in the integrated oil industry in Europe and North America. These two sectors are significantly impacted by climate policies following the 2015 Paris Agreement. In addition, the corresponding marginal PDF of each firm allows us to estimate its probability of default (PD). Thus, we can examine how the introduction of new climate policies affect the firms' PD empirically. If a firm succeeds in adapting to the new policy, its income will increase and its leverage ratio will be lowered.

As a core part of the financial system, banks should be proactive in managing climate risks and enhancing their risk management frameworks to address risks related to climate change and the transition to carbon neutrality. The Basel Committee on Banking Supervision (2022) published principles for the effective management and supervision of climate-related financial risks. One obvious area is continued work on how the physical and transition risks associated with climate change impact the lossgenerating process for banks. This is also central to an informed assessment of policy options and the calibration of any desired changes to the regulatory capital regime of banks. A firm-level study can help to assess potential impacts on firms' credit risk, ensure the appropriate treatment from an accounting perspective, and develop robust risk-weights that accurately represent the differential impact across assets.

While it is challenging to assess the effects of climate-related financial risks on firms' credit risk over a long time horizon given the lack of reliable forward-looking data, it remains important to develop consistent analytical frameworks to study the effects for two reasons. First, climate-related risks are the product of multiple interacting forces (e.g. natural, technological, societal and sectoral) which span a long time horizon. The associated effects are inherently uncertain. A consistent analytical model may be able to assess changes in a timely and analytically-trackable manner, and in particular how climate change and its associated policy response (i.e. transition risk) impacts firms' default risk and thus the desired capital requirements in the banking sector.

Suppose that firms in a particular industry (such as the fossil-fuel production industries and the automotive industry manufacturing fossil-fuel vehicles) are hit hard by the new climate policy with their expected incomes being drastically reduced [(see Kalkuhl and Wenz (2020) and Ho et al. (2022)]. This could result in a sharp increase in leverage ratios in accordance with the time-varying target-leverage-ratio model. This implies that the introduction of the new climate policy will change the model parameters governing the movement of firms' leverage ratios, and therefore their corresponding PDs. On the other hand, Lian et al. (2023) find evidence of positive economic consequences from focus on environmental performance including climate risk management from the perspective of corporate bond financing with reduced bond credit spreads.

The following section presents the derivation of our credit risk model and its associated PDs. In section 3, the model is calibrated using firms' data in the automotive industry and in the integrated oil industry, and their PDs are estimated. An event study in the fourth section analyses the impact of climate policies on the firms' PDs in the two industries. The final section concludes.

2. Credit risk model

The classical LVC model with N interacting species is a popular model which has been studied extensively owing to its theoretical and practical significance:

$$\frac{dx_i}{x} = \left[b_i - \sum_{j=1}^{N} a_{ij}x_j\right] \ dt + \widetilde{\sigma_\iota} dW_i \tag{1}$$

for i = 1, 2, 3, . . . , N, where x_i and b_i denote the population size and intrinsic growth rate of the i-th species at time t, respectively. Here a_{ii} is the intraspecific competition rate of the i-th species, and a_{ii} is the interspecific competition rate between distinct i-th and j-th species. The stochastic term $\widetilde{\sigma_\iota} dW_i$ is added to reflect the external randomness that affects the dynamical behaviour of the system. It should be noted that all model parameters are positive definite. A shortcoming of this model is that the population size can assume negative values. Furthermore, positive interaction may also exist among competing species; for instance, mutualism is generally believed to account for high production levels or 'overyielding' in communities with a greater variety of different plants. To similate mutualism, one may simply require the interspecific competition rates to be negative definite. Recently, in order to stave off negative population size, a modified version of the model has been proposed by Lo (2023).

By introducing $x_i = \ln(\frac{R_i}{R_0})$ as in Lo (2023), and $R_0 = 1$ which defines the leverage ratio R_i of each firm i at default, Eq.(1) is a generalized case of the dynamics of time-varying target leverage ratios in Lo and Hui (2012) with the correspondence: $\widetilde{\sigma_i} \leftrightarrow \sigma_i$, $b_i \leftrightarrow -\mu_i$, $a_{ii} \leftrightarrow \kappa_i(\gamma_i N^{-1} - 1)$, and $a_{ij} \leftrightarrow \kappa_i \gamma_i N^{-1}$ for $i \neq j$. μ_i is the drift of the leverage ratio of each firm i, the parameter $\gamma_i < 0$ determines how close the target leverage ratio of each firm i is set to coincide with the industry average, with the parameter $\kappa_i > 0$ being the corresponding speed of adjustment towards the target leverage ratio. The inter-firm competition rate a_{ij}, which specifies the inter-firm competition, must be negative definite. Also, the intraspecific competition rate a_{ii}, which specifies the default risk adjustment of each firm is greater than the interspecific competition rate. Hence, from the perspective of ecological systems, Lo and Hui (2012) argue that the existence of time-varying target leverage ratios results from cooperation of firms in a particular industry or financial sector as risk-averse firms make decisions on their capital structure.

Assuming the ensemble of firms share the common parameters $\{\sigma, \bar{\kappa}, \gamma\}$ for simplicity, the proposed model in this paper exhibits intraspecific competition as characterized by $\bar{\kappa}$. The parameter γ represents the nature of interactions. A positive γ indicates inter-specific competition, suggesting that firms strive for dominance in overlapping markets or resources. Conversely, a negative γ suggests inter-specific cooperation, where firms engage in collaborative efforts to reduce competitive pressures. The set of Ito's stochastic differential equations for the leverage ratio can be expressed as:

$$dx_i = \left[\bar{\kappa}\ln\theta_i - \bar{\kappa}\, x_i - \bar{\kappa}\gamma \frac{1}{N}\sum_{j=1}^{N} x_j\right]\ dt + \sigma_i dW_i \qquad (2)$$

where

$$\bar{\kappa} = \kappa \left[1 + \frac{\gamma}{1+(N-1)e^{-\kappa\gamma t}}\right] \quad (3)$$

$$\bar{\theta} = \left[\frac{1}{1+(N-1)e^{-\kappa\gamma t}}\right] \left\{-N\left(\frac{1}{2}\sigma^2 + \mu_i e^{-\kappa\gamma t}\right) + (e^{-\kappa\gamma t} - 1)\sum_{j=1}^{N}\mu_j + \kappa[e^{-\kappa\gamma t} - (1 + \gamma)]\sum_{j=1(j\neq i)}^{N} x_{j0}\right\} \quad (4)$$

Following Merton's (1974) structural model for credit risk of corporates using a contingent-claims framework, the default risk is equivalent to a European put option on a firm's asset value and the firm's liability is the option strike. The marginal probability density function (PDF) of each firm derived from Eq,(3) allows us to estimate the probability of default (PD) $P_i(x_{i0}, \tau)$ as

$$P_i(x_{i0}, \tau) = 1 - \int_{-\infty}^{0} p_i(x_i; \{x_{i0}\}, \tau) \quad dx_i \quad (5)$$

and $p_i(x_i; \{x_{i0}\}, \tau)$ is the corresponding marginal PDF of leverage ratio of firm i.

The PDF of the proposed leverage ratio model is known analytically in Lo and Ip (2021) and Lo (2023) and has the following form:

$$P(\{x_i\}; \{x_{i0}\}, \tau) = \frac{1}{\sqrt{(4\pi)^N \det\Omega}} \exp\left[-\frac{1}{4}(X_0 - x)^T \Omega^{-1} \ (X_0 - x)\right] \quad (6)$$

where X_0 is a $N \times 1$ column vector with

$$X_{i0} = \left(\ln\theta_i - \ln\bar{\theta}_G + \frac{1}{1+\gamma}\ln\bar{\theta}_G\right) + [(x_{i0} - \ln\theta_i) - (\bar{x}_0 - \ln\bar{\theta}_G)]e^{-\bar{\kappa}\tau} + \left(\bar{x}_0 - \frac{\ln\bar{\theta}_G}{1+\gamma}\right)e^{-\bar{\kappa}(1+\gamma)\tau} - \frac{\sigma^2}{2\bar{\kappa}(1+\gamma)}\left(1 - e^{-\bar{\kappa}(1+\gamma)\tau}\right) \quad (7)$$

with $\bar{\theta}_G = \left(\prod_j^n \theta_j\right)^{\frac{1}{N}}$ as the geometric mean of θ_i and $\bar{x}_0 = \sum_j^N x_{j0}$ is the arithmetic mean of x_0. Σ is a $N \times N$ matrix with the definition

$$\Sigma = b_1(\tau)I + b_2(\tau) \tag{8}$$

where I is a $N \times N$ identity matrix, and

$$b_1(\tau) = \frac{\sigma^2}{2}\frac{(1-e^{-2\bar{\kappa}\tau})}{2\bar{\kappa}},\ b_2(\tau) = \frac{\sigma^2}{2N}\left[\frac{(1-e^{-2\bar{\kappa}(1+\gamma)\tau})}{2\bar{\kappa}(1+\gamma)} - \frac{(1-e^{-2\bar{\kappa}\tau})}{2\bar{\kappa}}\right]. \tag{9}$$

With the availability of the PDF, all the model parameters can be calibrated against market data by means of the maximum-likelihood method. The marginal PDF is given by:

$$p(x_i; \{x_{i0}\}, \tau) = \frac{1}{\sqrt{4\pi[b_1(\tau)+b_2(\tau)]}} \exp\left[-\frac{1}{4}\frac{(X_{i0}-x_i)^2}{[b_1(\tau)+b_2(\tau)]}\right]. \tag{10}$$

3. Model calibration

We calibrate the model using data on firms in the automotive industry which manufactures fossil fuel vehicles and in the integrated oil industry in Europe and North America. These two sectors are expected to be significantly impacted by climate change policies arising from the 2015 Paris Agreement and adverse economic shocks such as the COVID 19 pandemic. Firms in the two sectors are expected to adjust their leverage ratios in response to such shocks. The leverage ratio of a firm i, R_i, is determined by its debt D_i and market capitalization M_i by:

$$R_i = \frac{D_i}{D_i + M_i} \tag{11}$$

Total debt of the firm is calculated by the product of the total debt per share and current shares outstanding. The financial metrics including total debt per share, current shares outstanding, and current market capitalization are obtained from Bloomberg. This highlights a key feature of our model which is that it uses input data readily available from the market. Due to the different updating frequencies of debt data and market capitalization, we perform a linear interpolation of the total debt data to smooth out the changes in debt over time. We adapt Bloomberg's Industry Classification Standard (BICS) to select representative firms in each of the two industries. This gives us eight firms in the fossil fuel automotive sector and thirteen firms in the integrated oil sector in Western Europe and North America. Tables 1 and 2 lists the names and tickers of the firms, while Figures 1 and 2 show their individual leverage ratios in natural logarithm scale. The sampling period for the automotive firms is from 17 November 2010 to 14 June 2024 and for the integrated oil firms is from 29 March 2007 to 14 June 2024.

The model parameters are calibrated using the maximum likelihood method for the closed-form joint PDF of Eq.(6). To account for the time-varying nature of the parameters, we perform the estimation using a rolling window of 750 days (approximately 3 years) for the automotive firms and 1500 days (approximately 6 years) for the integrated oil firms. For the integrated oil firms in the sample a longer window size is required to obtain robust results. This window size and data frame are chosen to ensure that we have sufficient data to investigate the impact of the 2015 Paris Agreement on the estimated PDs.

3.1 Automotive firms

Regarding the automotive firms manufacturing fossil fuel vehicles, Figure 3 reports significant estimates of the restoring drift (intraspecific competition) term $\bar{\kappa}$ (Panel A) with the z-statistic maintaining above the value of 1.96 (i.e., at the 5% significance level). $\bar{\kappa}$ drops from 2 in 2013 to 1 in 2015, suggesting that the restoring force pushing the leverage ratios towards its target level weakens during this period. Subsequently, $\bar{\kappa}$ increased from 1 to a peak of 5 in 2019, demonstrating a stronger mean-reverting force in the leverage ratio dynamics. The restoring force starts to drop from May 2019 and in general continues its downward trend until the end of 2023. This period covers the COVID 19 pandemic and substantial US interest rate hikes, suggesting that a weak economy and subsequent high-interest rate environment might have weakened the restoring force in the firms' leverage ratio dynamics. Overall, the statistical significance of $\bar{\kappa}$ illustrates the validity of the first model feature: that the mean reversion is present in the firms' leverage ratio dynamics.

Panel B shows the estimation of the interaction of the leverage ratios between the firms as characterized by $\bar{\kappa}\gamma$ which is negative and significant during 2015 - 2023. The significance of this term suggests cooperation (negative values of $\bar{\kappa}\gamma$) among firms. As $\bar{\kappa}\ \gamma$ is insignificant before September 2015, this indicates that firms begun to cooperate in 2015 when the Paris Agreement was adopted. The level of cooperation is relatively stable at a level of 3 until mid-2017 when it weakens. In May 2018, $\bar{\kappa}\gamma$ reached a local minimum of 2.54 before increasing to reach a value of -5.79 in May 2019. Subsequently, $\bar{\kappa}\gamma$ remained significant with negative values. This shows that firms cooperated during the COVID 19 pandemic in 2020-2022 and during the rise in US interest rates in 2022. The level of cooperation began to decrease with the estimated $\bar{\kappa}\gamma$ being insignificant in mid-2023. The significant $\bar{\kappa}\gamma$ during 2015-2023 illustrates the validity of the second feature of the model: that firms cooperate and adjust their leverage ratios towards their target level.

Panel C shows the calibrated parameter of the volatility σ with a value between 0.08 to 0.16. The significance of σ indicates the robustness of the stochastic term in the leverage ratio dynamics.

We include electric vehicle automotive firms in a separate calibration. This allows us to observe how interactions change with the inclusion of firms in the same sector but with very different business models. Due to limited market data on electric vehicle firms, only two firms, BYD Company and Tesla Inc, are included in the sample. The sampling period remains from 17 November 2010 to 14 June 2024. Figure 4 shows the natural logarithm of leverage ratios of the two firms.

Figure 5 shows the calibrated $[\bar{\kappa}, \bar{\kappa}\gamma, \sigma]$ including the two electric vehicle firms in the sample. $\bar{\kappa}$ in Panel A is mostly significant throughout the period, showing that the firms in the sample adjust their leverage ratios towards target. During July 2020 - January 2022, the insignificance of $\bar{\kappa}$ suggests that there is no mean reversion in the firms' leverage ratio dynamics. The generally insignificant $\bar{\kappa}\gamma$ in Panel B suggests that there is no competition or cooperation when the electric vehicle firms are included in the sample. In other words, the electric vehicle firms do not cooperate with the fossil fuel vehicle firms to adjust their leverage ratios towards common targets. The significance of σ in Panel C shows the robustness of the stochastic nature of the leverage ratio dynamics.

3.2 Integrated oil firms

Panels A and B of Figure 6 show the calibrated $\bar{\kappa}$ and $\bar{\kappa}\gamma$ of the integrated oil firms. $\bar{\kappa}$ is mostly significant throughout the calibration period, indicating that the firms adjust their leverage ratios towards their target. In 2013, the mean-reverting force in the firms' leverage ratio dynamics and their cooperation gradually weakens, as reflected by the decreasing magnitude of both $\bar{\kappa}$ and $\bar{\kappa}\gamma$. The intraspecific mean-reverting force in term of $\bar{\kappa}$ and interspecific interaction ($\bar{\kappa}\gamma$) is temporarily insignificant during May 2013 - January 2014. Subsequently, the parameter $\bar{\kappa}\gamma$ is significantly positive during January 2014 - July 2014, indicating competition among the firms. The firms shifted from competition to cooperation with $\bar{\kappa}\gamma$ turning significantly negative in July 2015 until November 2016. The cooperation might be due to the 2015 Paris Agreement. During these periods, the intraspecific mean-reversion force remained relatively stable with $\bar{\kappa}$ in the range of 0.25 to 0.4.

Between November 2016 and early 2019, the parameters representing the intraspecific competition ($\bar{\kappa}$) and interspecific cooperation ($\bar{\kappa}\gamma$) are both insignificant, suggesting that firms do not adjust their leverage ratios towards target. From early 2019, the two parameters are significant again. The mean reverting force increases to a peak level of $\bar{\kappa} = 1.6$ in March 2021, and enhanced cooperation is at a level of -1.5. This reflects a substantial reduction in oil demand during the COVID 19 pandemic which drove oil firms to cooperate and adjust their leverage ratios quickly towards their target. Both the mean-reverting force in the firms' leverage ratio dynamics and the level of cooperation reduces after 2021, while the estimates remain significant. Panel C illustrates that the calibrated σ is significant over the estimation period and the stochasticity in the firms' leverage ratio dynamics is robust.

The calibration results in these two subsections using firm-level data demonstrates that the model parameters of the PDF of Eq.(6) can be effectively estimated for firms' leverage ratio dynamics.

3.3 Estimation on default probability

To estimate the probability of default (PDs) of the firms in the sample, we rearrange Eq.(7) as follows:

$$X_{i0} = \left\{\ln\theta_i - \left[(\bar{x}_0 - \ln\bar{\theta}_G) - \left(\bar{x}_0 - \frac{\ln\bar{\theta}_G}{1+\gamma}\right)\right]\right\} + [(x_{i0} - \ln\theta_i) - (\bar{x}_0 - \ln\bar{\theta}_G)]e^{-\bar{\kappa}\tau} +$$

$$\left(\bar{x}_0 - \frac{\ln \bar{\theta}_G}{1+\gamma}\right) e^{-\bar{\kappa}(1+\gamma)\tau} - \frac{\sigma^2}{2\bar{\kappa}(1+\gamma)}\left(1 - e^{-\bar{\kappa}(1+\gamma)\tau}\right) \quad (12)$$

This expression has four components. The first component takes into account of the firm's target ratio $\ln \theta_i$, the difference between the sectoral mean leverage ratio and the naïve sector average target ratio $(\bar{x}_0 - \ln \bar{\theta}_G)$, and the difference between the sectoral mean leverage ratio and the interaction adjusted mean target ratio $\left(\bar{x}_0 - \frac{\ln \bar{\theta}_G}{1+\gamma}\right)$. The second component is the intraspecific competition $e^{-\bar{\kappa}\tau}$ adjusted by the difference between the firm's instantaneous leverage ratio and its target ratio at the firm level $(x_{i0} - \ln \theta_i)$ and sectoral level $(\bar{x}_0 - \ln \bar{\theta}_G)$ respectively. The third component is the interspecific competition/cooperation $e^{-\bar{\kappa}(1+\gamma)\tau}$ adjusted by the difference between the sectoral mean leverage ratio and interaction adjusted mean target ratio $\left(\bar{x}_0 - \frac{\ln \bar{\theta}_G}{1+\gamma}\right)$. The fourth component is the geometric Brownian motion correction term which appears when applying Ito's lemma.

Figures 7 and 8 show the estimation of $\log \theta_i$ of the individual firms in the automotive industry and the integrated oil industry respectively. As the intraspecific and interspecific interactions are assumed to be approximately equal, the figures show that changes in the firms in the same industry are qualitatively similar with differences in level.

With all parameters available, the PD is given by

$$P_i(x_{i0}, \tau) = 1 - N\left(\frac{-X_{i0}}{\sqrt{2[b_1(\tau) + b_2(\tau)]}}\right) \quad (13)$$

where $N(x)$ is the cumulative distribution function of the standard normal distribution, and $b_1(\tau)$ and $b_2(\tau)$ expressed in Eq.(9). The PDs are calculated using the parameters calibrated in the previous subsections. Panels A, B, and C of Figures 9 and 10 show the PDs with times $\tau = 0.5,1,2$ years for the fossil fuel automobile firms and the integrated oil firms respectively. The PDs are monthly averages. There are sharp peaks of the PDs in 2020 due to the outbreak of the COVID 19 pandemic.

4. Event study of the effect of climate policy on firms' default risk

To investigate the effect of climate policy on the two-year PDs of the fossil fuel automotive firms and integrated oil firms, an event study is presented in this section. Given that firms' PDs are determined by factors other than the expected transition risk arising from climate policy, the following control variables are included in the study:

- US dollar volatility (r_{USD}^2): the measure of the ex-post squared return of the US dollar index (DXY).
- Global risk appetite: CBOE VIX volatility index (VIX) and Euro Stoxx Volatility Index (VSTOXX) are used to proxy the market volatility of the US and European markets respectively.
- Funding liquidity constraint: US dollar, euro and Japanese yen (for automotive industry) TED spreads (TED), which is the difference between the 3-month interbank rate and the yield on a 3-month US Treasury bill, and euro-area government bonds and Japan sovereign bonds (for automotive industry) respectively, are used to proxy the funding liquidity constraint in the markets.
- Macro-financial conditions:
  - Stock market variables: Returns of S&P 500 index (SPX), Dow Jones EURO STOXX 600 index (STOXX) and Nikkei 225 Index (NKY, for the automotive industry).
  - Bond market variables: Term spreads between 10-year and 2-year yields of the US Treasuries (USTerm), euro-area government bonds (EUTerm) and Japan government bonds (JPTerm, for the automotive industry).

- Federal fund rate: Effective fed funds rate
- For the integrated oil industry, the crude oil price is included as a control variable.

All the data are from Bloomberg.

To capture the effect of climate policy in the study, a dummy variable D_1 is introduced at the time point t_1 = April 2016 when the Paris Agreement came into force[3]. For the fossil fuel automotive firms, three additional dummy variables at respective time points are introduced as follows:

- D_2 at t_2 = Jan 2019 when Regulation (EU) 2019/631EN was in force, setting CO2 emission performance standards for new passenger cars and vans.[4]
- D_3 at t_3 = Oct 2022 when the Parliament and EU countries reached an agreement on the ban on the sale of new petrol and diesel cars from 2035.[5]
- D_4 at t_4 = Apr 2023 when the above EU legislation was effective.[6]

After incorporating all these control variables, a regression with the dummy variables is conducted for the automotive firms using the following equations:

$$\Delta(\log PD_{t=1})_{t,i} = \alpha + \beta_1 \Delta r^2_{USD,t} + \beta_2 \Delta VIX_t + \beta_3 \Delta VSTOXX_t + \beta_4 \Delta TED_{US,t} +$$
$$\beta_5 \Delta TED_{EU,t} + \beta_6 \Delta TED_{JP,t} + \beta_7 \Delta SPX_t + \beta_8 \Delta STOXX_t + \beta_8 \Delta NKY +$$
$$\beta_{10} \Delta USTerm_t + \beta_{11} \Delta EUTerm_t + \beta_{12} \Delta JPTerm_t + \beta_{13} \Delta FundRate_t + \zeta_{1,i} D_1 +$$

[3] The Paris Agreement was adopted in December 2015 and came into force in November 2016.

[4] See https://climate.ec.europa.eu/eu-action/transport/road-transport-reducing-co2-emissions-vehicles/co2-emission-performance-standards-cars-and-vans_en and https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A02019R0631-20210301

[5] https://www.europarl.europa.eu/news/en/press-room/20221024IPR45734/deal-confirms-zero-emissions-target-for-new-cars-and-vans-in-2035

[6] See https://www.europarl.europa.eu/topics/en/article/20221019STO44572/eu-ban-on-sale-of-new-petrol-and-diesel-cars-from-2035-explained and https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:32023R0851

$$\zeta D_2 + \zeta D_3 + \zeta_{4,i} D_4 + \nu_t \quad (14)$$

and for the integrated oil firms:

$$\Delta(\log PD_{t=1})_{t,i} = \alpha + \beta_1 \Delta r^2_{USD,t} + \beta_2 \Delta VIX_t + \beta_3 \Delta VSTOXX_t + \beta_4 \Delta TED_{US,t} + \beta_5 \Delta TED_{EU,t} + \beta_6 \Delta SPX_t + \beta_7 \Delta STOXX_t + \beta_8 \Delta USTerm_t + \beta_9 \Delta EUTerm_t + \beta_{10} \Delta FundRate_t + \beta_{10} \Delta CrudeOilPrice_t + \zeta_i D_{1,i} + \nu_t \quad (15)$$

Cross section seemingly unrelated regressions (SUR) are employed using panel estimated generalized least squares to account for the fixed effect between firms in the sector. The PDs are sampled monthly by averaging the daily data, and converted to natural logarithms and differenced. Zero values of the PDs are entered as 10^{-18}. White cross-section standard errors and covariances are used in the estimations. Table 3 and Table 4 show that the SUR explains 7.56% and 7.78% of the total variance of the PDs for the automotive firms and the integrated oil firms respectively. Wald tests are conducted to examine whether the effects of climate policies are significant on the default risk by setting up a null hypothesis $H_0\colon \zeta_{n,i} = 0$ where n is the index of event and i represents firm i.

Table 3 shows that the PDs of all automotive firms increased significantly (at the 1% significance level) in April 2016, when the Paris Agreement came into force. The estimated coefficients ζ_{1i} range from 0.146 to 0.459 for the individual firms, showing that the Paris Agreement had a material negative impact on the automotive industry. The magnitude of the impact is around 10% of the monthly log-change in the PDs. Such an impact may incorporate the expected transition risk for the firms as anticipated by the market.

The default risk of the automotive firms falls significantly (negative coefficients ζ_{2i} at the 10% significance level for Ford Motor and at the 1% significance level for the other firms) in January 2019 when the stricter CO2 emission regulation were imposed. Similarly, in October 2022, the default risk of the firms (except Toyota Motor) dropped significantly with negative γ_{3i} at the 1% significance level, when the European parliament and EU countries reached an agreement to ban the sale of new petrol and diesel cars by 2035. The results seem counter intuitive. However, following the Paris Agreement and the Net Zero policy in the EU by 2050, those automotive firms had already embarked on transition plans to reduce the manufacture of fossil fuel vehicles and increase production of electric vehicles. Furthermore, the ban on fossil fuel vehicles was expected by the market and eliminated the uncertainty around more stringent future legislation. Such a positive effect is consistent with the news in June 2024 that the car industry warned EU leaders against reversing the 2035 combustion engine ban.[7] That said, in April 2023, the default risk for Mercedes-Benz, Ford Motor and General Motor is positively related to (with positive ζ_{4i} at the 1% significance level) legislation on the ban on the sale of new petrol and diesel cars from 2035, showing that the introduction of the legislation had an impact on some firms.

For the integrated oil industry, the SUR results find that the firms' PDs increased significantly at the 1% level (except for Total Energies) with ζ_i from 0.66 to 6.57, showing that the Paris Agreement had a significant impact on the integrated oil industry in Western Europe and North America. The material coefficients indicate that the integrated oil firms in the sample were expected to face substantial transition risk caused by climate policy.

[7] See https://www.reuters.com/business/autos-transportation/europe-automakers-will-not-challenge-2035-fossil-fuel-car-ban-industry-group-2024-02-26/ and https://www.euractiv.com/section/electricity/news/car-industry-warns-eu-leaders-against-reversing-2035-combustion-engine-ban/

5. Conclusions

Assuming that in a particular industry the existence of time-varying target leverage ratios result from cooperation among firms in making decisions on their capital structure, this paper proposes a credit risk model to measure firms' default probabilities by approaching the problem from the perspective of ecological systems. This proposal yields the joint probability density function of leverage ratios for a sample of firms in closed form so that a likelihood function can be constructed and model-fitting using market data becomes feasible. We calibrate the model using data on firms in the automotive industry which manufacture fossil fuel vehicles and in the integrated oil industry in Europe and North America. The calibration results show that the model parameters can be effectively estimated for firms' leverage ratio dynamics, and that firms cooperate to adjust their leverage ratios towards their target level. In terms of the changes in the firms' default probabilities, the event study illustrates that firms faced substantial transition risk caused by climate policies following the 2015 Paris Agreement.

We demonstrate that the model can assess changes in firms' default probabilities in a timely and analytically trackable manner, and in particular show how climate policy (i.e. transition risk) impacted firms' default risk following the 2015 Paris Agreement. The model can also be used to study whether firms in a particular sector will cooperate or compete during the transition caused by climate mitigation policies. Such analysis could help banks and credit rating agencies to assess the firms' transition plans arising from climate policies to deliver a Net Zero economy.

Annex

To analyze the default risk in the model, a heuristic argument on the breaching condition is provided in this annex. The steady state leverage ratio is given by

$$X_{i0}(\tau \to \infty) = \log \theta_i - \frac{\gamma}{1+\gamma} \ln \bar{\theta}_G \mathrm{s} \quad \text{(A1)}$$

in which $\frac{\gamma}{1+\gamma}$ serves as a penetrating factor to adjust the long-term mean in accordance with the sector's overall target ratio. $\frac{\gamma}{1+\gamma}$ is less than one if $\gamma \geq 0$. As γ becomes smaller and drops below zero, the interaction is cooperation. But the penetrating factor will then push the leverage ratio from the firms' target, and enhance default risk by increasing the value of $X_{i0}(\tau \to \infty)$. When γ approaches -1, we consider the interspecific cooperation term:

$$\left(\bar{x}_0 - \frac{\ln \bar{\theta}_G}{1+\gamma}\right) e^{-\bar{\kappa}(1+\gamma)\tau} \quad \text{(A2)}$$

Since the $\frac{1}{1+\gamma}$ factor diverges and dominates the exponential decay factor, this indicates an intermediate state when the interspecific cooperation $e^{-\bar{\kappa}(1+\gamma)\tau}$ and cooperation adjusted average $\frac{\ln \bar{\theta}_G}{1+\gamma}$ both dominate the mean-reversion process. Exceeding cooperation will alter the mean reversion of firms to adjust their leverage ratios towards the target. Finally, if $\gamma < -1$, the mean reversion breaks down and this significantly increases the target ratio X_{i0}. It is characterized by a default risk close to 1. As an illustration, Figure A1 shows the plot of $(\bar{\kappa} + \bar{\kappa}\gamma)$ and the half-year mean PDs of the automotive firms in our sample. The PDs surge whenever $\overline{(\kappa} + \bar{\kappa}\gamma)$ breaches below zero.

Figure 1. Leverage ratios in natural logarithm of fossil fuel automotive firms from 17 November 2010 to 14 June 2024.

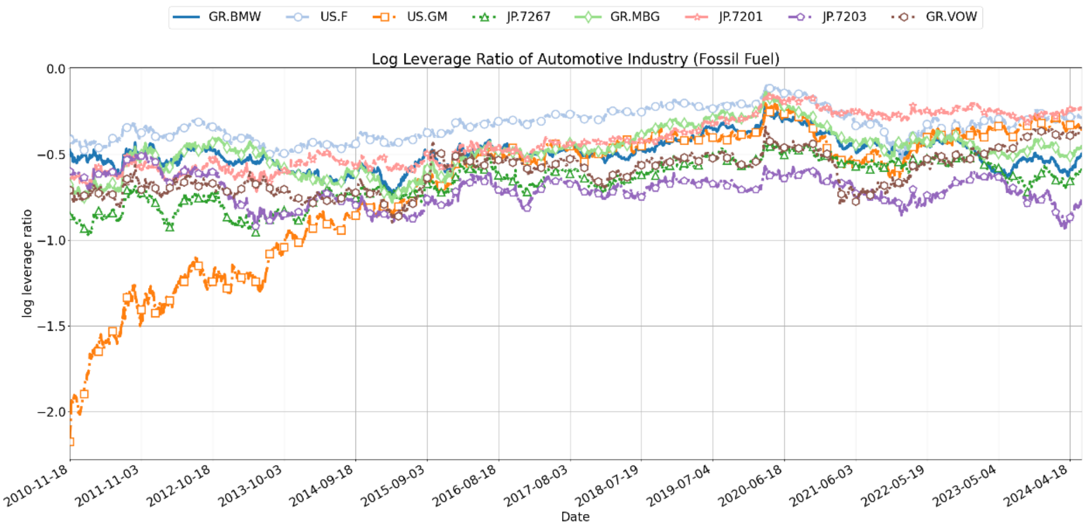

Figure 2. Leverage ratios in natural logarithm of integrated oil firms from 29 March 2007 to 14 June 2024.

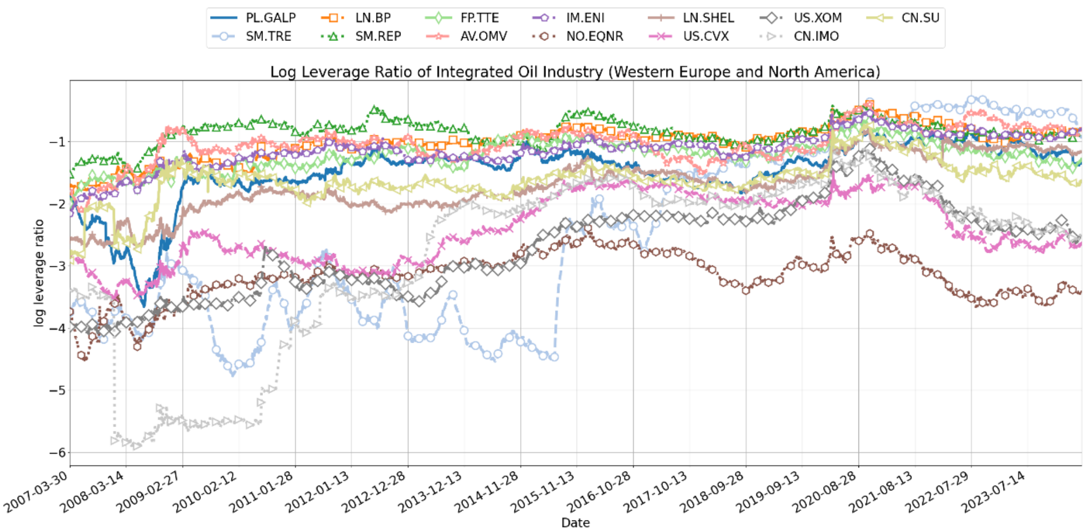

Figure 3: Estimated $\bar{\kappa}$ (Panel A), $\bar{\kappa}\ \gamma$ (Panel B), σ (Panel C), and their corresponding z-statistics, in leverage ratio dynamics of fossil fuel automotive firms using 3-year rolling window.

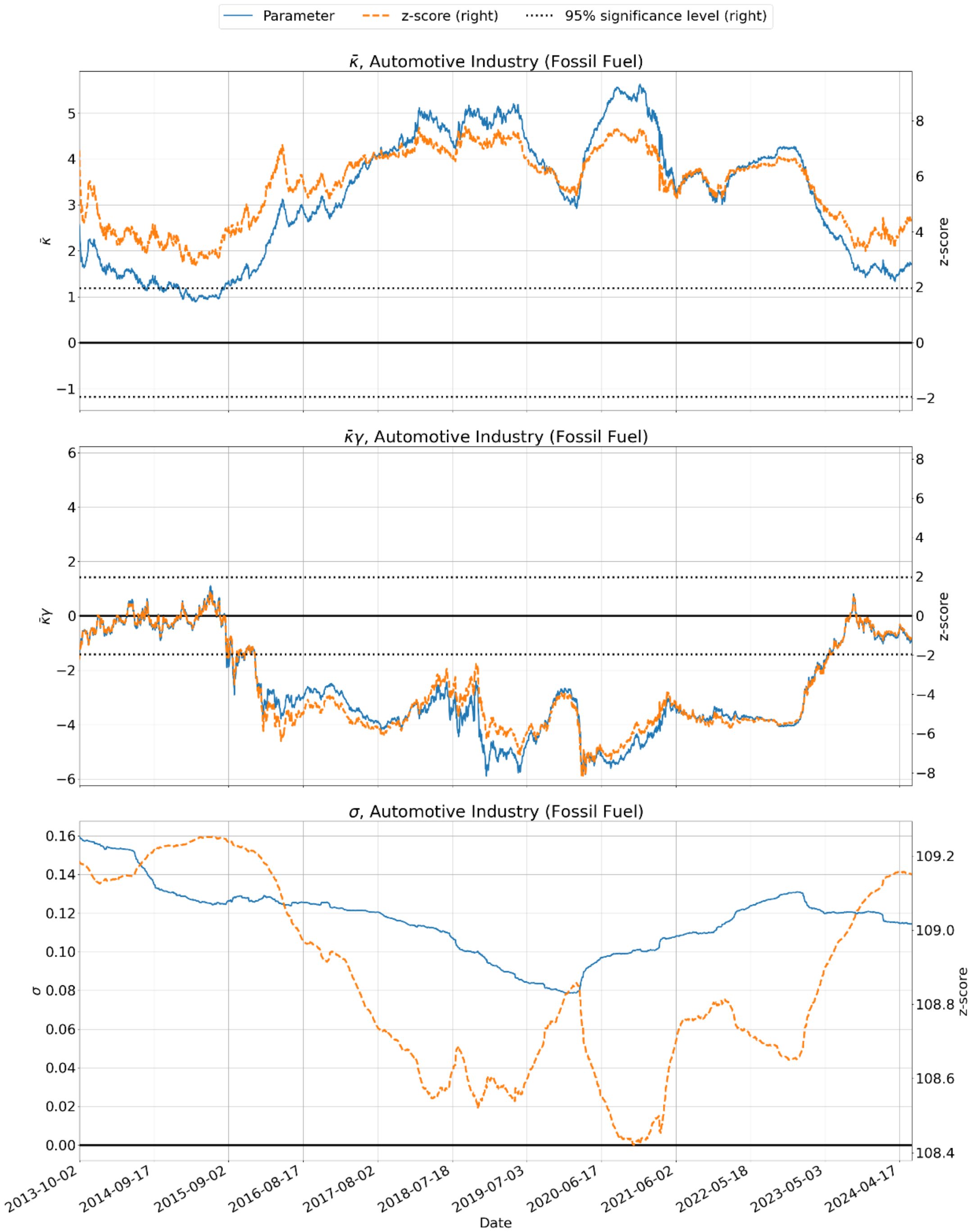

Figure 4. Natural logarithm of leverage ratios of two firms in the electric vehicle industry with a sampling period from 17 November 2010 to 14 June 2024.

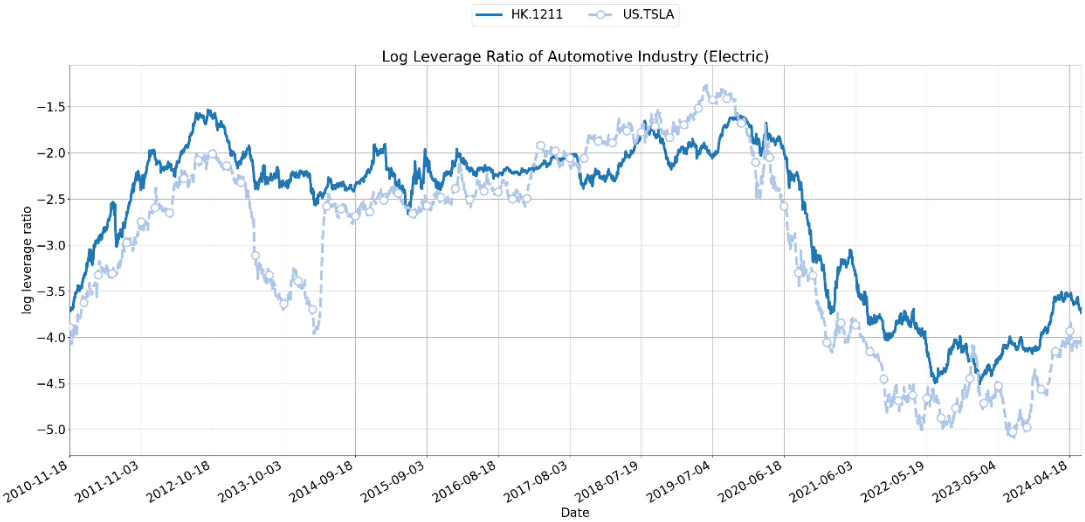

Figure 5: Estimated $\bar{\kappa}$ (Panel A), $\bar{\kappa}\ \gamma$ (Panel B), σ (Panel C), and corresponding z-statistics, in leverage ratio dynamics of fossil fuel and electric automotive firms using 3-year rolling window.

Figure 6: Estimated $\bar{\kappa}$ (Panel A), $\bar{\kappa}\ \gamma$ (Panel B), σ (Panel C), and corresponding z-statistics, in leverage ratio dynamics of integrated oil firms using 6-year rolling window.

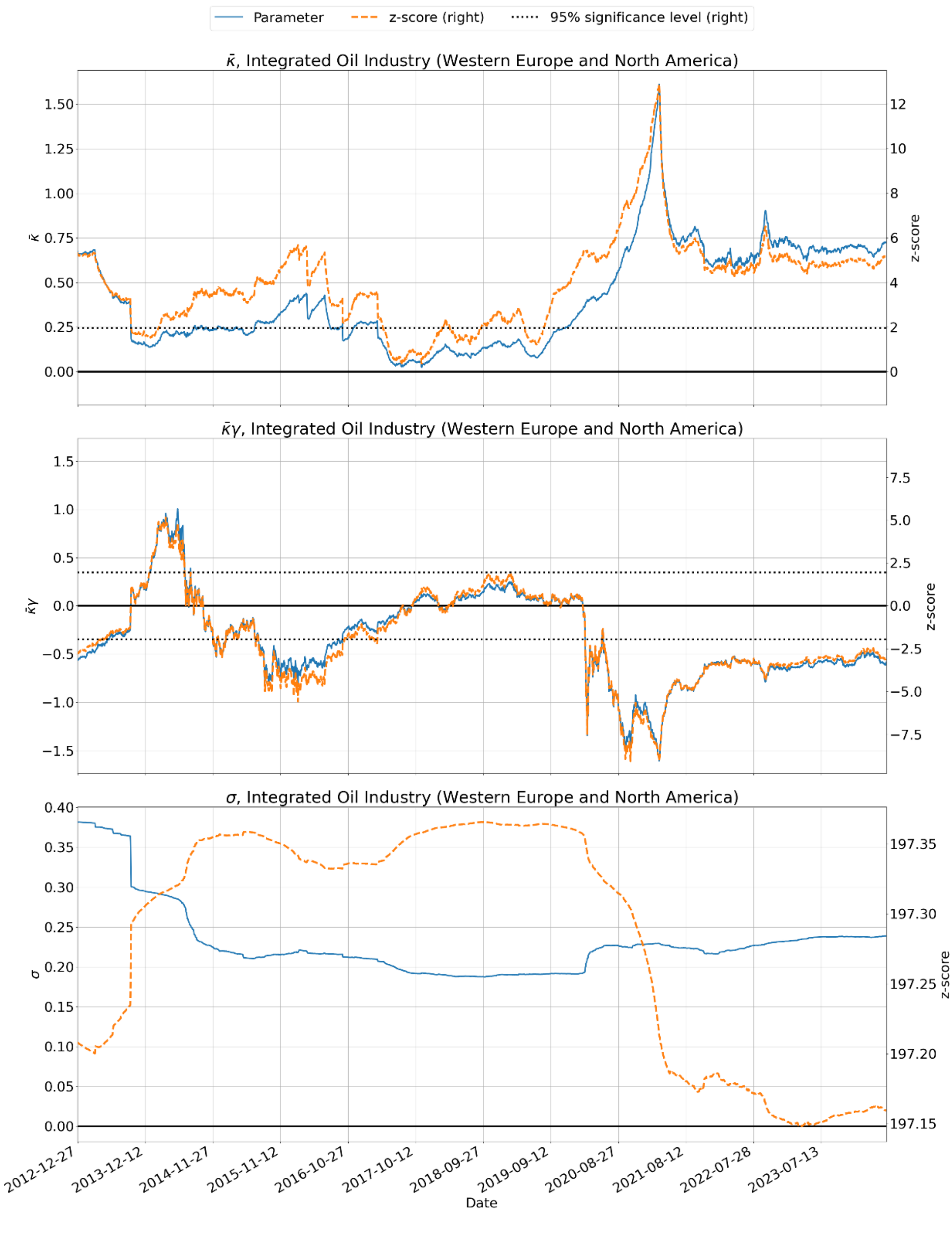

Figure 7. Estimated $\log \theta_i$ of leverage ratio of automotive firms using 3-year rolling window.

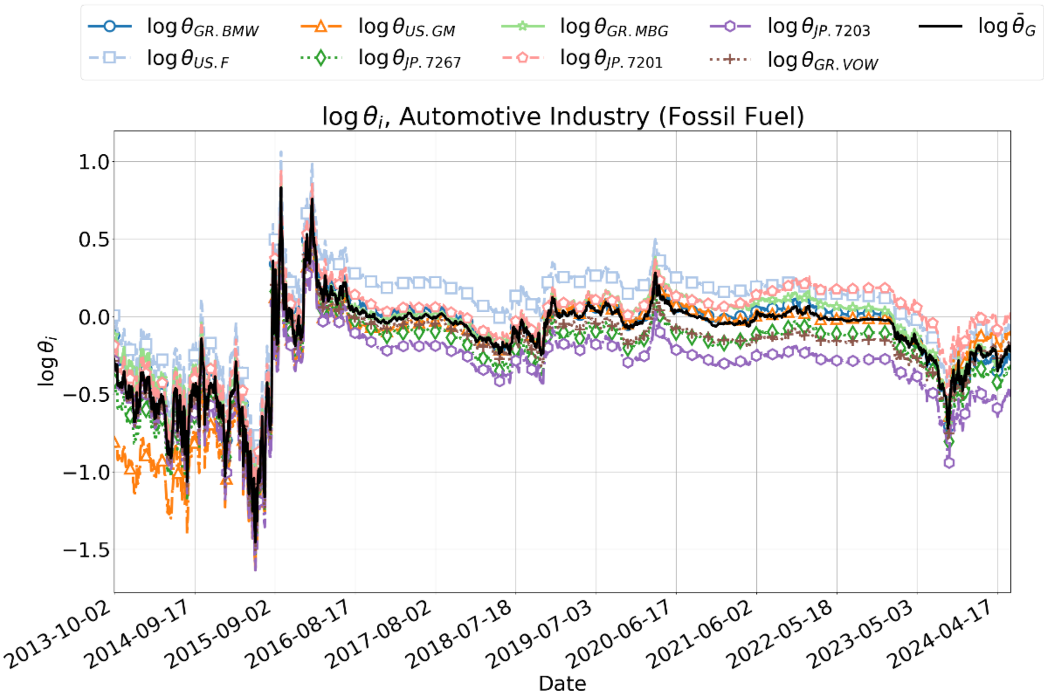

Figure 8. Estimated $\log \theta_i$ of leverage ratios of integrated oil firms using 6-year rolling window.

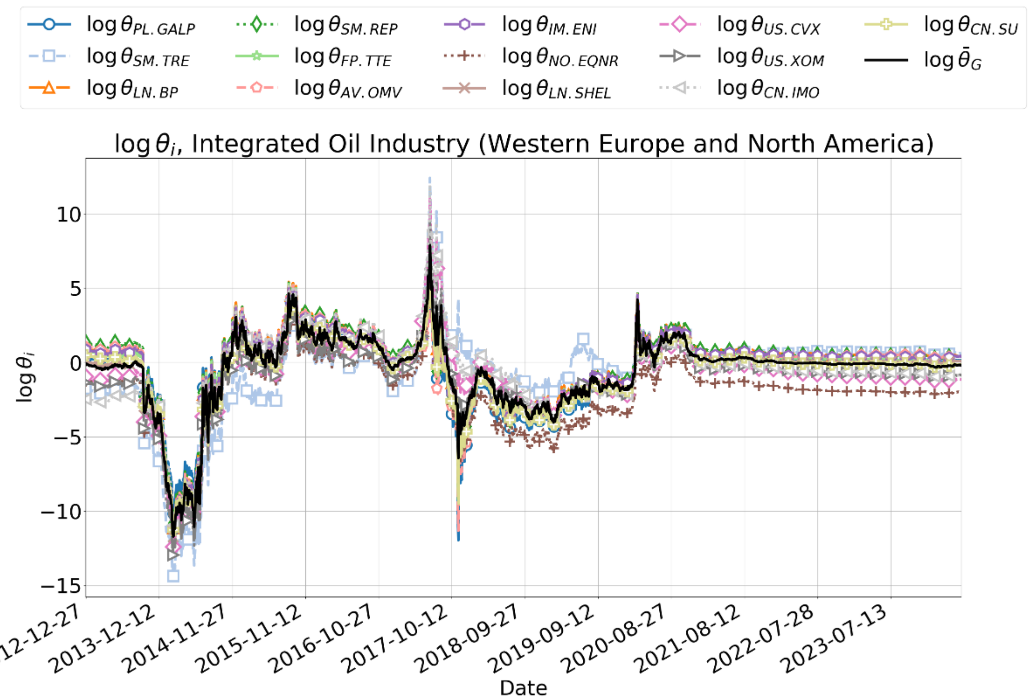

Figure 9. The monthly averaged probabilities of default in $0.5, 1, 2$ years of fossil fuel automotive firms.

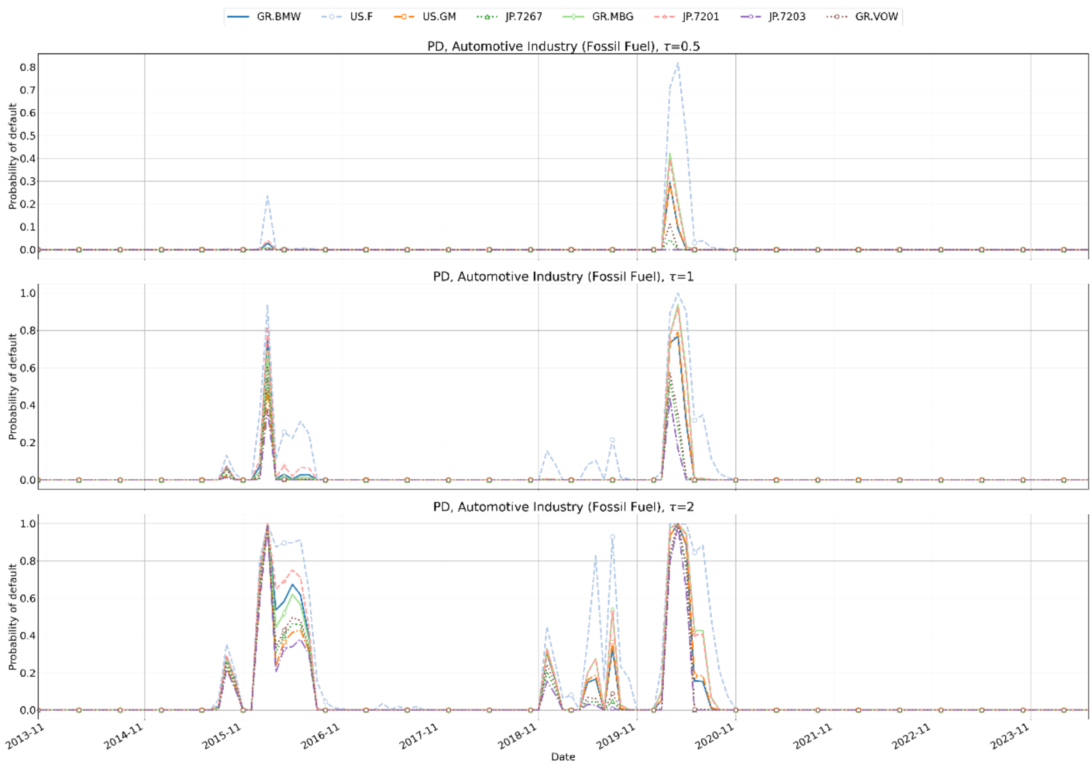

Figure 10. The monthly averaged probabilities of default in $0.5, 1, 2$ years of integrated oil firms.

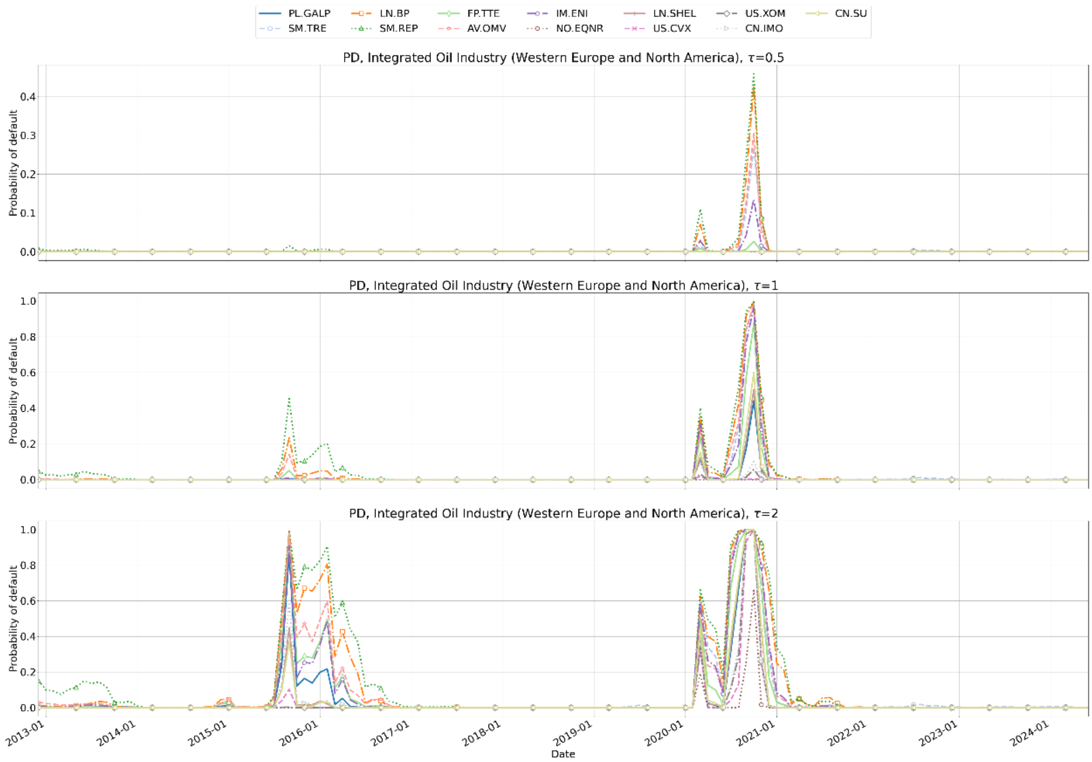

Figure A1. Plot of $\overline{(\kappa + \bar{\kappa}\gamma)}$ and log(Mean PD) of automotive industry using 3-year rolling window.

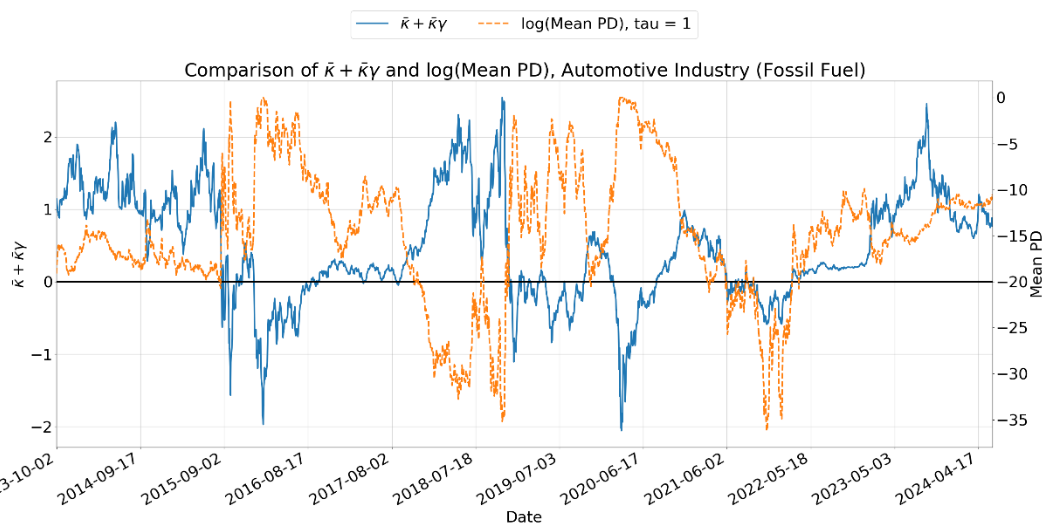

Table 1. Tickers and names of automotive firms.

Member Ticker	Member Companies
GR.BMW	BMW AG
US.F	FORD MOTOR CO
US.GM	GENERAL MOTORS C
JP.7267	HONDA MOTOR CO
GR.MBG	MERCEDES-BENZ GR
JP.7201	NISSAN MOTOR CO
JP.7203	TOYOTA MOTOR
GR.VOW	VOLKSWAGEN AG

Table 2. Tickers and names of integrated oil firms.

Member Ticker	Member Companies
PL.GALP	GALP ENERGIA
SM.TRE	TECNICAS REUNIDA
LN.BP	BP PLC
SM.REP	REPSOL SA
FP.TTE	TOTALENERGIES SE
AV.OMV	OMV AG
IM.ENI	ENI SPA
NO.EQNR	EQUINOR ASA
LN.SHEL	SHELL PLC
US.CVX	CHEVRON CORP
US.XOM	EXXON MOBIL CORP
CN.IMO	IMPERIAL OIL
CN.SU	SUNCOR ENERGY

Table 3. SUR results of 2-year default probabilities of automotive firms with the equation:

$$\Delta(\log PD_{t=1})_{t,i}$$
$$= \alpha + \beta_1 \Delta r^2_{USD,t} + \beta_2 \Delta VIX_t + \beta_3 \Delta VSTOXX_t + \beta_4 \Delta TED_{US,t} + \beta_5 \Delta TED_{EU,t} + \beta_6 \Delta TED_{JP,t} + \beta_7 \Delta SPX_t + \beta_8 \Delta STOXX_t$$
$$+ \beta_8 \Delta NKY + \beta_{10} \Delta USTerm_t + \beta_{11} \Delta EUTerm_t + \beta_{12} \Delta JPTerm_t + \beta_{13} \Delta FundRate_t + \zeta_{1,i} D_1 + \zeta_{2,i} D_2 + \zeta_{3,i} D_3 + \zeta_{4,i} D_4 + \nu_t$$

$\Delta \log(PD)$	GR.BMW	GR.MBG	GR.VOW	JP.7201	JP.7203	JP.7267	US.F	US.GM
Constant	0.015							
	(0.018)							
$\Delta r^2_{USD,t}$	-0.176							
	(0.129)							
ΔVIX_t	0.016*							
	(0.009)							
$\Delta VSTOXX_t$	-0.004							
	(0.009)							
$\Delta TED_{US,t}$	-0.304**							
	(0.13)							
$\Delta TED_{EU,t}$	0.445*							
	(0.229)							
$\Delta TED_{JP,t}$	-0.941**							
	(0.432)							
ΔSPX_t	-0.000248							
	(0.00024)							

$\Delta STOXX_t$	-0.0000442							
	(0.000175)							
ΔNKY_t	-0.0000256							
	(0.0000191)							
$\Delta USTerm_t$	-0.196*							
	(0.107)							
$\Delta EUTerm_t$	-0.122							
	(0.13)							
$\Delta JPTerm_t$	-0.224							
	(0.342)							
$\Delta FundRate_t$	0.023							
	(0.072)							
Apr 2016 (Wald test: $H_0: \zeta_{1,i} = 0$)	0.254***	0.294***	0.338***	0.197***	0.459***	0.362***	0.146***	0.43***
	(0.058)	(0.056)	(0.059)	(0.053)	(0.063)	(0.06)	(0.05)	(0.062)
Jan 2019 (Wald test: $H_0: \zeta_{2,i} = 0$)	-1.067***	-0.829***	-1.672***	-0.832***	-1.64***	-1.66***	-0.15*	-1.005***
	(0.085)	(0.084)	(0.082)	(0.08)	(0.086)	(0.085)	(0.079)	(0.088)
Oct 2022 (Wald test: $H_0: \zeta_{3,i} = 0$)	-0.276***	-0.265***	-0.322***	-0.198***	-0.109	-0.32***	-0.228***	-0.286***
	(0.059)	(0.059)	(0.065)	(0.058)	(0.067)	(0.065)	(0.053)	(0.063)

Apr 2023 (Wald test: $H_0: \zeta_{4,i} = 0$)	0.008	0.135***	-0.005	0.081	0.006	0	0.122***	0.377***
	(0.052)	(0.049)	(0.058)	(0.05)	(0.06)	(0.057)	(0.047)	(0.052)

R-squared	0.0756
Adjusted R-squared	0.0261
F-statistic	1.53
Total panel (balanced) observations	1024

Note: The *t*-statistics (in parentheses) are computed from the White cross-sectional standard errors. The Wald test of coefficient restriction with the null hypothesis $\zeta = 0$ indicates whether the estimated coefficients of the PDs are statistically different from 0. ***, **, and * represent significance at 1%, 5% and 10% levels respectively.

Table 4. SUR results of 2-year default probabilities of integrated oil firms with the equation:

$$\Delta(\log PD_{t=1})_{t,i} = \alpha + \beta_1 \Delta r^2_{USD,t} + \beta_2 \Delta VIX_t + \beta_3 \Delta VSTOXX_t + \beta_4 \Delta TED_{US,t} + \beta_5 \Delta TED_{EU,t} + \beta_6 \Delta SPX_t + \beta_7 \Delta STOXX_t + \beta_8 \Delta USTerm_t + \beta_9 \Delta EUTerm_t + \beta_{10} \Delta FundRate_t + \beta_{10} \Delta CrudeOilPrice_t + \zeta_i D_{1,i} + \nu_t$$

$\Delta \log(PD)$	PL. GALP	SM. TRE	LN. BP	SM. REP	FP. TTE	AV. OMV	IM. ENI	NO. EQNR	LN. SHEL	US. CVX	US. XOM	CN. IMO	CN. SU
Constant	-0.074												
	(0.083)												
$\Delta r^2_{USD,t}$	-0.003												
	(0.015)												
ΔVIX_t	0.095*												
	(0.051)												
$\Delta VSTOXX_t$	0.018)												
	(0.055)												
$\Delta TED_{US,t}$	0.342												
	(1.195)												
$\Delta TED_{EU,t}$	-0.954												
	(0.999)												
ΔSPX_t	0.004***												
	(0.001)												
$\Delta STOXX_t$	-0.004***												

	(0.001)												
$\Delta USTerm_t$	0.868												
	(0.730)												
$\Delta EUTerm_t$	-1.541*												
	(0.893)												
$\Delta FundRate_t$	-0.802												
	(0.507)												
$\Delta CrudeOilPrice_t$	-0.085***												
	(0.014)												
Apr 2016 (Wald test: $H_0: \zeta_i = 0$)	0.83 ***	5.96 ***	0.65 ***	0.42 ***	0.40 **	0.59 ***	0.66 ***	6.57 ***	2.74 ***	5.6 ***	5.31 ***	3.34 ***	2.26 ***
	(0.23)	(0.34)	(0.15)	(0.15)	(0.18)	(0.17)	(0.16)	(0.44)	(0.27)	(0.39)	(0.42)	(0.36)	(0.26)

R-squared	0.0778
Adjusted R-squared	0.0588
F-statistic	4.09
Total panel (balanced) observations	1781

Note: The *t*-statistics (in parentheses) are computed from the White cross-sectional standard errors. The Wald test of coefficient restriction with the null hypothesis $\zeta = 0$ indicates whether the estimated coefficients of the PDs are statistically different from 0. ***, **, and * represent significance at 1%, 5% and 10% levels respectively.

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