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Dealer Funding and Market Liquidity

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Abstract

We consider a model in which dealers intermediate trades between clients and provide immediacy, or, market liquidity. Dealers can exert unobservable effort to improve the chance of intermediating profitably. This moral-hazard friction impairs dealers' ability to raise external finance and hence to compete aggressively with each other in providing liquidity. To alleviate the financing friction, dealers opt to finance with debt and intermediate in several markets simultaneously. Dealer leverage is therefore endogenous and related to variations in liquidity across otherwise unrelated markets. Our results shed light on the developments of intermediation in bond markets in response to post-crisis bank regulations.

Keywords: dealers, market-making, market liquidity, moral hazard, regulation, optimal contract
JEL classification: G23, G24

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1 Introduction

In many asset markets, especially in over-the-counter-markets, it is the intermediation by dealers that determines how easily or cheaply an asset can be traded. A dealer will allow a market participant to offload a position onto its balance sheet immediately and will then take over the task of finding another market participant to close the position later. In this sense, dealers provide immediacy. The terms on which dealers provide this immediacy determine the cost of immediacy for clients, or the market liquidity of the asset.

To intermediate, dealers typically need external funding. Frictions in the ability to raise such funding may limit dealers' ability to provide immediacy and therefore reduce market liquidity. Do the frictions suggest an optimal way to raise financing? Does this optimal way to raise financing explain why liquidity might co-move across markets, and does it explain why regulation (especially of bank-affiliated dealers) may have had adverse effects on market liquidity? To address these questions, we set up a model in which dealers need external funds to intermediate and exert a type of effort that is not observed by the financiers who provide these external funds.

This moral-hazard problem means that dealers can only raise a limited amount of external financing, because they need to preserve some "skin in the game" so that they still have incentives to exert effort after receiving financing. This limit on the amount of funds available for intermediation widens bid-ask spreads and reduces intermediation volumes. The problem is more severe for riskier assets and gets worse when dealers lose money. As in other models, *funding liquidity* (the ease with which dealers can raise funding) and *market liquidity* are linked.

The moral-hazard problem also implies that dealers must raise financing in a way that maximizes their incentives per dollar raised. This produces two key insights. First, dealers optimally choose to raise external finance in the form of debt, to retain the maximum upside. This means, for instance, that any regulation limiting dealer leverage is likely to harm market liquidity. Second, as we explain below, dealers optimally choose to intermediate across many markets simultaneously. The model therefore gives a rationale for why shocks to some dealers should lead to co-movement of liquidity across different markets. Moreover, together with the first insight, the second insight suggests that liquidity across different markets might co-vary with the *leverage* of dealers.

In the model, dealers acquire positions through the provision of immediacy, and can then exert effort in searching for a good counterparty, to increase the chance of closing the position at a profit. To acquire a position, a dealer needs cash. (Dealers either pay cash to buy a security or post cash as collateral to borrow a security to be able to sell it.) Dealers have some cash but can obtain additional cash from financiers against a promise of later repayment. Financiers cannot observe the dealers' search effort but can observe cash flows to dealers from intermediation and so make repayment contingent on these cash flows. The repayments need to be limited so as to leave some upside of effort to the dealer. But limited repayments also imply that there is a limit on the maximum amount of cash that financiers are willing to provide up-front. Following the optimal contracting literature, we will refer to this maximum amount that financiers can provide as dealers' "pledgeable income."

Consequently, the agency problem restricts dealers' balance-sheet capacity for intermediation and so has a negative effect on market liquidity. Financially unconstrained dealers would have plenty of funds to compete aggressively and drive down bid-ask spreads. However, financially constrained dealers lack the funds to compete aggressively, resulting in wider bid-ask spreads, especially for larger trades that require more financing. In the extreme, dealers may not be able to finance the largest trades at all. The effects are stronger for riskier assets for which the associated agency problem is more severe.

As in Innes (1990), external debt is part of the optimal financing arrangement. Debt leaves the maximum upside to the dealer, who then has a strong incentive to expend effort to close positions at a profit. Any regulation that restricts the use of debt financing effectively reduces the amount of external financing that dealers can raise. This prediction matches recent empirical findings regarding the impact of bank regulations on the liquidity of the U.S. corporate bond market: Our model suggests that increases in effective capital requirements or an imposition of maximum leverage ratios should reduce the balance-sheet capacity of (bank-affiliated) dealers, resulting in smaller average trade sizes, lower trading volumes, and a decline in capital committed by such dealers to intermediation (Bessembinder, Jacobsen, Maxwell, and Venkataraman, 2018; Bao, O'Hara, and Zhou, 2018; Schultz, 2017). To preserve balance-sheet capacity, dealers should broker (or "pre-arrange") more trades between clients (Choi and Huh, 2017). They should also charge higher prices for immediacy (Dick-Nielsen and Rossi, 2018).[1]

When dealers can intermediate in several separate markets simultaneously, this mitigates the agency problem and thus *increases* the amount of funds they can raise per unit of intermediated asset. Intuitively, earning an agency rent for successfully intermediating in one market can be made contingent on also successfully intermediating in all other markets. That is, it is as if the agency rent for intermediating in this one market is "cross pledged" as collateral for successful intermediation in all other markets. This improves incentives and relaxes the financing constraint.[2] Because of this effect, a dealer who intermediates in several markets will out-compete those who do not by offering better terms. Therefore, in equilibrium, all dealers will try to intermediate in several markets.

Finally, since dealers optimally use external debt financing and intermediate across many markets simultaneously, liquidity of different assets would co-vary with dealer leverage or the leverage constraints imposed on the dealers. Furthermore, to the extent that prices are affected by liquidity/

[1]Haselmann, Kick, Singla, and Vig (2019) find similar deterioration in liquidity provisions by German banks when they are required to hold more capital.

[2]The underlying optimal-contracting effect was first described by Cerasi and Daltung (2000) and Laux (2001). It is sometimes referred to as a "cross-pledging" effect (see Tirole, 2006, Section 4.2).

expected future transaction costs, or that our dealers are the marginal purchasers of certain assets, prices should co-move with dealer leverage (Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017).

Our paper is related to the theoretical literature that looks at limits to arbitrage or the role of intermediaries in asset pricing, initiated by Shleifer and Vishny (1997) (see Gromb and Vayanos, 2010, for a survey), in which dealers are needed to intermediate trades between clients across time or geography. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) assume that dealers have to finance with collateralized debt subject to some margin constraints and emphasize how asset characteristics, such as return volatility, affect margin constraints, and, in turn, market liquidity of the assets. Andersen, Duffie, and Song (2019) show that existing leverage causes dealers to widen bid-ask spreads to overcome a form of debt overhang problem. All these papers assume exogenous financing contracts.

Our key contribution to this literature is that we endogenize dealers' financing choices. As argued above, it delivers two new insights. First, to improve their incentives, dealers optimally use debt financing to expand their financing capacity. Consequently, the model provides a rationale for why *dealer leverage* should be tightly linked to the capacity of dealers to intermediate and hence market liquidity. Second, to improve their incentives, dealers also choose to intermediate in several markets simultaneously. Consequently, the model suggests that dealer leverage should be tightly linked to the liquidity of *several* markets and hence provides an underlying rationale for why liquidity co-moves across markets and for why liquidity shocks in one market can spill over to the other markets.[3]

Our paper is related to the intermediary asset pricing literature, which links the internal wealth

[3] Cespa and Foucault (2014) endogenize co-movement and spillovers across asset markets with information and learning about risky liquidity demands. Our mechanism does not require learning.

of intermediaries to asset prices.[4] In He and Krishnamurthy (2012, 2013), risk-averse households and fund managers try to share risks in the asset markets: Households can only indirectly invest in the risky assets by holding the equity in funds. Fund managers are subject to moral hazard, so the amount of equity they can issue is limited to a multiple of their own wealth. Insufficient wealth of intermediaries can hence affect prices of assets that have aggregate risks because it impedes efficient risk sharing.[5] Our model has fundamentally different setup and implications. In our model, all agents are risk-neutral and dealers exist to help finding the suitable counterparty of a trade. A moral-hazard friction also limits funding capacity of our dealers but less so if they use debt financing and intermediate in multiple assets. This implies that dealers optimally finance with debt and their internal wealth and leverage matter for the liquidity of many assets simultaneously, even if the assets are unrelated and have little fundamental risks. To the extent that prices are affected by liquidity/ expected future transaction costs (Amihud and Mendelson, 1986) (or that our dealers are the marginal purchasers of assets in certain markets) our model suggests that prices should co-move with dealer leverage. In that sense, our model also suggests an explanation of the empirical results of Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017), who show that dealer leverage is a priced factor for several assets, including U.S. government bonds.

Our paper also contributes to the theoretical market microstructure literature on the determinants of bid-ask spreads or, more broadly, market illiquidity. Existing works in the literature have shown that dealer's risk-aversion to holding inventory (Stoll, 1978), adverse selection (Kyle, 1985), search frictions in OTC markets (Duffie, Garleanu, and Pedersen, 2005), and dealers' ca-

[4]See He and Krishnamurthy (2018) for a survey of the literature. Glebkin, Gondhi, and Kuong (forthcoming) show that internal wealth of intermediaries can affect their incentive to acquire information and consequently the asset prices and information efficiency.

[5]Put differently, in He and Krishnamurthy (2012, 2013), if fund managers are risk-neutral and/ or the assets have little non-diversifiable risks so that no risk-sharing is needed, managers can borrow any amount with risk-free debt and hold all the assets and their internal wealth would not matter for asset prices.

pacity constraints (Green, 2007) can widen the bid-ask spread.[6] Our paper illustrates that agency frictions in market-making activities can limit dealers' balance-sheet capacity and hence lead to market illiquidity. Our result could be interpreted as a microfoundation of inventory costs.[7]

2 The model

There are two dates $t = \{1, 2\}$. There is no time-discounting. There are four types of risk-neutral agents: dealers, clients, financiers, and security lenders. Our focus is on dealers, so that clients, financiers, and security lenders will mostly play a passive role. We first consider the case of a single market, in which one type of asset is traded. For simplicity, in this case, we refer to this traded asset as "the asset." There is also always a risk-free asset with a zero net return, which we will call "cash." In Section 4, we consider the case of multiple markets.

There are two types of clients who differ in the value they attach to the asset, and also in the date at which they are in the market: The client "Earl" arrives early at $t = 1$ but is not present at $t = 2$, and the client "Laetitia" arrives late at $t = 2$ and is not present at $t = 1$. Earl can have either a high or low valuation of the asset. With probability $\frac{1}{2}$, Earl's valuation is high and equal to $V + \ell$, and he wants to buy a random quantity $\tilde{q}$ of the asset. With probability $\frac{1}{2}$, Earl's valuation is low and equal to $V - \ell$, and he wants to sell a random quantity $\tilde{q}$ of the asset. We assume that $\tilde{q}$ is uniformly distributed on $[0, 1]$. Laetitia always values the asset at V and is rich in both cash and assets.

The differential valuations between Earl and Laetitia generate potential gains from trade. How-

[6]See Foucault, Pagano, and Röell (2013) for a comprehensive survey on market liquidity.

[7]Biais, Glosten, and Spatt (2005) note that while individuals are likely to exhibit risk aversion, it is not obvious why financial institutions/ dealers should be averse to diversifiable risk. Our model proposes an explanation: Because of agency problems, dealers act *as if* they were averse to diversifiable risk. See the discussion on fundamental risks in Section 3.3 and the analysis of intermediating multiple assets in Section 4.

ever, since Earl and Laetitia are not present in the market at the same time, they cannot trade directly with each other. $N \geq 2$ competing dealers are present at both $t = 1$ and $t = 2$ and can intermediate, even though they attach no value to the asset per se. Dealers are initially all identical: They start with no position in the asset and all have the same, limited amount of cash w as internal capital. Dealers may need to obtain additional cash funding and/or borrow assets to intermediate. Financiers are rich in cash and do not own any units of the asset. They value the asset at $V - k$. Security lenders are rich in assets but have no cash. They value the asset at $V + k$ and require per-unit cash collateral of $V + k$ for lending the asset.[8]

Importantly, we assume $V > k > \ell > 0$ to ensure that the only available gains from trade are between Earl and Laetitia. The different valuations of the asset by various agents are summarized in Figure 1. The premia ($+k$ and $+\ell$) in valuations could be motivated by the agents' portfolio or hedging needs for the asset, whereas the discounts ($-k$ and $-\ell$) represent liquidity needs or the opportunity costs of cash.

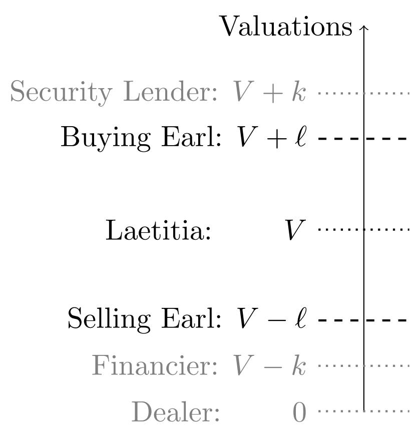

Figure 1. Valuations of the asset by different agents

[8]In this description, dealers post cash collateral in a securities lending transaction to obtain an asset for a short sale. In a formally equivalent description, dealers could issue securities and post these as collateral in securities lending transactions. Or dealers could acquire assets via reverse repos.

The timing is as follows. At $t = 1$, the dealers post bid price(s) $\{b(q)\}$ and ask price(s) $\{a(q)\}$ that are functions of trade size q. Earl arrives and will either have a valuation of $V - \ell$ per unit of the asset and want to sell q units or have a valuation of $V + \ell$ and want to buy q units. For simplicity, we assume that Earl cannot split up his order, and that he sells to the best bid or buys from the best ask.[9,10]

If Earl sells q units, then the chosen dealer has a financing need of $[qb(q) - w]^+$. If Earl buys q units, then the chosen dealer must borrow q units from the security lenders and provide them with cash collateral.Since Earl pays $qa(q)$ to the chosen dealer, the net financing need for the dealer in this case is $[q(V + k) - qa(q) - w]^+$. Financiers competitively supply funds. The dealer raises any cash that it needs by offering a contract with contingent repayments to financiers, as described in more detail below.[11]

At $t = 2$, a dealer who has opened a position chooses (unobservable) effort $e \in \{0, 1\}$, which affects the probability of finding Laetitia. Finding Laetitia is useful, as it allows closing the position at a more advantageous price – not finding Laetitia means that the dealer needs to close the position either by buying assets from security lenders at a high price, or selling to financiers at a low price. Once the position has been closed, repayments on the contract with financiers are made.

We assume that if the dealer exerts effort ($e = 1$), it finds Laetitia with probability 1. If the dealer shirks ($e = 0$), it finds Laetitia with probability $1 - \delta$ (where $1 > 1 - \delta > 0$). In terms of e, we can therefore write the probability of finding Laetitia as $1 - (1 - e)\delta$. We also assume that effort

[9]To break ties, we assume that dealer 1 has an effort cost that is infinitesimally smaller than the effort cost of all other dealers. This dealer will be able to slightly outbid all other dealers.

[10]Equivalently, we could allow Earl to split an order of size Q among our N identical and competitive dealers. In symmetric equilibrium, Earl will split the order equally, so that each dealer has to intermediate a quantity $q = Q/N$.

[11]In this description, a dealer posts bids and asks, is then potentially chosen by a client, and only then raises financing. Equivalently, a dealer could first obtain a funding commitment from financiers, and then request the funds under the funding commitment (e.g., draw down on a credit line) if it is chosen by a client. Also equivalently, a dealer could repo out the asset it is buying to finance the transaction after having bought from a client.

incurs a non-pecuniary cost proportional to the number of units of the asset, cq. This cost can be interpreted directly as the time and effort cost expended by the dealer to identify the right clients to offset the positions. More broadly, it represents the costs of all unobservable actions which can be taken by the dealers to increase the intermediation profits such as monitoring of public information (Foucault, Röell, and Sandås, 2003) and efficient unwinding of inventories.

Suppose Earl sells to a dealer's bid. If the dealer finds Laetitia and sells on to her, then the dealer will obtain a high gross cash flow of $x_b^H = qV$. If it does not find Laetitia and has to sell on to financiers, it would obtain a low gross cash flow of $x_b^L = q(V - k)$.

Similarly, suppose Earl buys from a dealer's ask, after the dealer has borrowed the assets from some security lenders. If the dealer finds Laetitia, buys back the assets from her at price qV, and then returns the assets to its security lenders and gets back the cash collateral $q(V + k)$, it will obtain a high gross cash flow of $x_a^H = q(V + k) - qV = qk$. If the dealer does not find Laetitia, buys back the assets from some other security lenders at a higher price $q(V + k)$, and then returns the asset to its security lenders and gets back the cash collateral $q(V + k)$, it will obtain a low gross cash flow of $x_a^L = q(V + k) - q(V + k) = 0$.

The repayments to financiers can be contingent on the gross cash flow, and on whether the dealer buys or sells from Earl. We denote these repayments as R_b^H, R_b^L, R_a^H, R_a^L for the case of high (superscript H) and low (superscript L) cash flows, in the case when the dealer buys (subscript b) and sells (subscript a), respectively.

We make the following assumptions on parameters. First, we assume that the cost of exerting effort is lower than both the gains from trade (ℓ) and the expected loss in value from shirking (δk). Hence it is efficient to exert effort.

Assumption 1. $c < \min\{\ell, \delta k\}$

We then also assume that the expected loss in value from shirking is larger than the gains from trade, implying that intermediation while shirking destroys value.

Assumption 2. *$\delta k > \ell$.*

We solve the single-market version of the model as outlined here in Section 3. In Section 4, we extend the model to multiple markets.

3 Intermediation in a single market

As a benchmark, we first discuss the case in which dealers' internal funds w are large, so that no external financing is required, and hence no agency problems arise. We then turn to the more interesting case in which w is small enough to produce agency frictions.

3.1 Frictionless benchmark

Suppose that internal funds w are so large that dealers do not need external financing. Assumption 1 then guarantees that they will always exert effort once they have opened a position. Dealers compete à la Bertrand in posting functions $b(q), a(q)$ that specify how much they would bid or ask for a given quantity q sold to them or bought from them. In equilibrium, they will make zero profits, net of the effort cost of intermediation. The asset can be sold to Laetitia (or bought back from her) at price V, implying that the zero-profit bid and ask are $b = V - c$ and $a = V + c$, respectively. The bid-ask spread is $2c$.

In equilibrium, utilities are as follows: The dealers' net expected utility is driven to zero by Bertrand competition. Earl either buys at price $V + c$ and values the asset at $V + \ell$ or sells at price $V - c$ and values the asset at $V - \ell$. Earl therefore realizes a utility gain of $\ell - c$ per intermediated unit. Laetitia's expected utility is zero. Per intermediated unit, the social surplus, defined as the unweighted sum of utilities of all agents, is therefore equal to $\ell - c$. Since all trades are intermediated, the average intermediated volume is equal to $E[q] = \frac{1}{2}$, and the total social surplus is therefore $E[q](\ell - c) = \frac{1}{2}(\ell - c)$.

We summarize this discussion in the following proposition:

Proposition 1 (Frictionless equilibrium with single market)**.** *Consider equilibrium when dealers' internal funds w are large, so that they do not require external financing to intermediate.*

The bid and ask are $b = V - c, \quad a = V + c$.

Orders for any size q are intermediated, and the expected intermediated volume is $E[q] = \frac{1}{2}$.

The social surplus, defined as the unweighted sum of utilities of all agents, is $\mathcal{S} = E[q](\ell - c) = \frac{1}{2}(\ell - c)$.

Proof. See preceding text. □

3.2 Equilibrium with agency frictions

Now consider the more interesting case in which dealers need to raise external finance to intermediate. The repayments to financiers under the contract are contingent on the gross cash flows to the dealer. The dealer will choose repayments that maximize its utility subject to some constraints. Any choice of repayments that maximize dealer utility represents an optimal contract.

Since dealers compete à la Bertrand, they will want to raise as much external funding as possible, so as to offer as competitive prices. The contract that maximizes the amount that can be raised is unique, and below, we will refer it as the *maximum-funding contract.* Also, following the literature, we will refer to the maximum possible amount that dealers can raise as *pledgeable income.*[12]

Recall that R_b^H, R_b^L, R_a^H, and R_a^L denote the repayments of dealers who buy at the bid (subscript b) or sell at the ask (subscript a), in the case when they have high (superscript H) or low

[12]In situations in which the pledgeable income is large enough for dealers to offer the zero-profit bids and asks, they will do so in equilibrium. In those cases, all contracts that raise at least the amount required for offering zero-profit bids and asks are optimal. Clearly, the maximum-funding contract is one of them.

(superscript L) gross cash flows from intermediation. Let $\{R_j\}$ denote the contract (R_j^H, R_j^L). Then the pledgeable income $\mathcal{P}_j(q)$ for the buying dealer and selling dealer is

$$\mathcal{P}_j(q) = \max_{\{R_j\}} R_j^H, \qquad \text{for } j \in \{b, a\}, \tag{PI}$$

$$\text{subject to} \quad x_j^H - R_j^H - cq \geq (1-\delta)(x_j^H - R_j^H) + \delta(x_j^L - R_j^L), \qquad \text{for } j \in \{b, a\}, \tag{IC}$$

$$R_j^i \leq x_j^i, \qquad \text{for } j \in \{b, a\} \text{ and } i \in \{H, L\}. \tag{LL}$$

To understand these expressions, first note that under Assumption 2, intermediation will destroy value unless effort is exerted. Contracts under which effort is not exerted can therefore never be optimal. If effort is exerted, Laetitia is always found, which in turn means that in equilibrium, the repayment to financiers is equal to R_j^H with probability 1. Since financiers break even the amount that can be raised at $t = 1$ is the amount financiers received at $t = 2$, that is, R_j^H.

This contract must satisfy some constraints. The incentive compatibility constraint (IC) ensures that the dealer will exert effort after financing: The dealer's utility from exerting the effort on the left-hand side of (IC) is weakly higher than that from shirking on the right-hand side. The limited liability constraint (LL) ensures that the residual payoffs to the dealer are not negative.

The maximum-funding contract is described in the next lemma.

Lemma 1 (Maximum-funding contract)**.** *The contract that raises the maximum amount from financiers is given by* $R_j^L = x_j^L, \quad R_j^H = x_j^L + q\left(k - \frac{c}{\delta}\right).$

Proof. Increasing R_j^L relaxes the constraint (IC) and therefore allows increasing the objective R_j^H. Thus, it is optimal to set R_j^L as high as possible. Therefore, the constraint (LL) binds. The maximum R_j^H is obtained when (IC) binds, given this level of R_j^L. □

Lemma 1 shows that under the maximum-funding contract, the dealer only receives positive payoffs when Laetitia is found. The intuition is as follows: To raise more external finance, the dealer has to promise a larger repayment to the financier when Laetitia is found. This however reduces its own payoffs and thus weakens its incentive to exert effort to find Laetitia. By retaining no payoffs when Laetitia is not found, the dealer preserves a strong incentive to exert effort, which in turn allows it to promise more repayment and to raise more finance.

Given the maximum-funding contract, the pledgeable income is

$$\mathcal{P}_b(q) = \max_{R_b^H} R_b^H = q\left(V - \frac{c}{\delta}\right) \quad \text{and} \quad \mathcal{P}_a(q) = \max_{R_a^H} R_a^H = q\left(k - \frac{c}{\delta}\right). \tag{1}$$

Agency frictions matter if and only if dealers cannot simply raise all the cash that they need to intermediate from financiers. We therefore make the following assumption:

Assumption 3. *Dealers must use some of their own cash for intermediation,* $\frac{c}{\delta} > \ell$.

To see why this assumption implies that dealers must use some of their own cash, consider a dealer who buys. Under Assumption 3, the per-unit pledgeable income, $\mathcal{P}(q)/q = V - \frac{c}{\delta}$, is smaller than the lowest valuation at which Earl is willing to sell $(V - \ell)$, implying that the dealer cannot raise all the required cash from financiers and must contribute at least $(\frac{c}{\delta} - \ell)$ of its own cash per intermediated unit. A similar argument applies for a selling dealer.

Assumption 3 implies that dealers may not be able to set high bids or low asks, because they may not be able to raise the cash required to finance these high bids or low asks. The maximum total amount that a dealer can bid is its internal funds w and the cash that can be raised from financiers $\mathcal{P}(q)$. In other words, $w + \mathcal{P}(q)$ is the dealer's *balance-sheet capacity.* This implies that the maximum bid that a dealer can finance (with the maximum-funding contract) is

$$b_{IC}(q, w) = \frac{w + \mathcal{P}(q)}{q} = \frac{w}{q} + \left(V - \frac{c}{\delta}\right). \tag{2}$$

Similarly, the maximum total amount that a dealer can raise to finance the cash collateral required for a sale (net of the ask price) is equal to the dealer's own cash, plus the cash that can be raised from financiers, or $q(V+k) - qa(q) = w + \mathcal{P}_a(q) = w + q\left(k - \frac{c}{\delta}\right)$. Hence the minimum ask price that a dealer can offer is

$$a_{IC}(q, w) = -\frac{w}{q} + \left(V + \frac{c}{\delta}\right). \tag{3}$$

In the absence of financial constraints, competitive dealers can raise bids and lower asks until they obtain zero utility, and bids and asks are equal to $b_c = V - c$ and $a_c = V + c$, respectively. Now, however, the maximum bid and minimum ask is constrained by pledgeable income, as described in Equations (2) and (3). Since more cash is required for intermediating trades of a larger size, there is a level of q above which these constraints bind. We denote this level as $\bar{q}(w)$. It is implicitly defined by $b_c \equiv b_{IC}(\bar{q}(w), w)$ and $a_c \equiv a_{IC}(\bar{q}(w), w)$, so that $\bar{q}(w) = \frac{w}{\frac{c}{\delta} - c}$. For any $q > \bar{q}(w)$, even dealers who are competing with each other cannot set a bid above $b_{IC}(q, w)$ or an ask below $a_{IC}(q, w)$. In other words, financial constraints widen the competitive bid-ask spreads and prevent profit from being driven to zero.

Also, since a dealer has to use $\left(\frac{c}{\delta} - \ell\right)$ units of its own cash per intermediated unit of the asset, the maximum number of units that a dealer can intermediate now cannot exceed $q_{\max}(w) = \frac{w}{\frac{c}{\delta} - \ell}$. If $q_{\max} < 1$, there can be realizations of q that exceeds $q_{\max}$ for which the trade is so large that it cannot be funded. For such realizations, trade will not occur and no gains from trade can be realized, so that intermediated volume and social surplus here can be lower than in the frictionless benchmark.[13] Below, we will refer to $q_{\max}(w)$ as the "market depth."

We note that $\bar{q}(w) < q_{\max}(w)$ and summarize this discussion in the following proposition.

Proposition 2 (Constrained equilibrium with a single market)**.** *Consider a situation in which*

[13] An alternative way to interpret $q_{\max}(w)$ is to note that for trades with $q > q_{\max}(w)$, a dealer would only be able to pay a bid below Earl's reservation value $V - \ell$ (would only be able to sell at an ask above Earl's reservation value $V + \ell$); so intermediation cannot occur.

dealers' internal funds w are small, so that dealers require external financing to intermediate.

In equilibrium, dealers have balance-sheet capacity of $w + \mathcal{P}_j(q)$ to offer the following bid and ask:

$$b(q,w) = \begin{cases} b_c = V - c & \textit{for} \quad q < \bar{q}(w) \\ b_{IC}(q,w) = \frac{w}{q} + \left(V - \frac{c}{\delta}\right) & \textit{for} \quad q \geq \bar{q}(w) \end{cases} \tag{4}$$

$$a(q,w) = \begin{cases} a_c = V + c & \textit{for} \quad q < \bar{q}(w) \\ a_{IC}(q,w) = \left(V + \frac{c}{\delta}\right) - \frac{w}{q} & \textit{for} \quad q \geq \bar{q}(w) \end{cases} \tag{5}$$

where $\bar{q}(w) = \frac{w}{\frac{c}{\delta} - c}$.

Only orders with size $q \leq q_{\max}(w) = \frac{w}{\frac{c}{\delta} - \ell}$ are intermediated. The expected intermediated volume is thus $E[q|q < q_{\max}]$, and the social surplus is $\mathcal{S} = E[q|q < q_{\max}](\ell - c)$.

Proof. See the preceding discussion. □

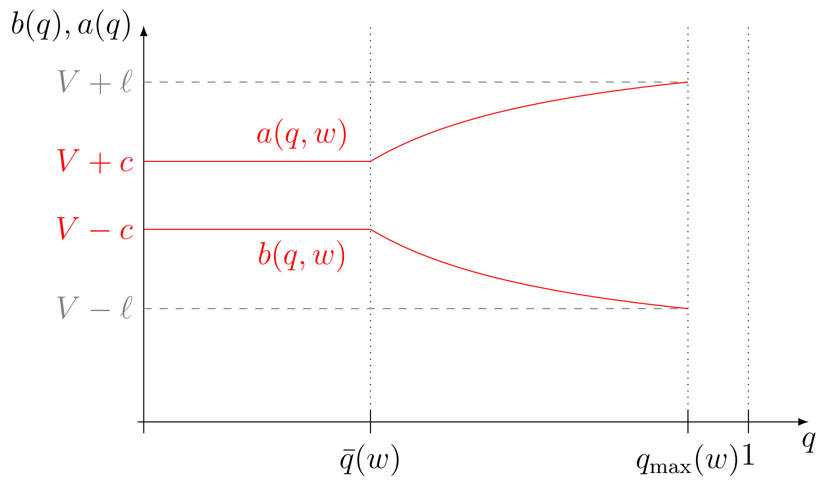

Figure 2. Bid-ask spreads with financial constraints
Competitive bid and asks $(b_c(q,w), a_c(q,w))$ as functions of size of trades q. Competition drives down bid-ask spreads to zero-profit level for smaller orders, with size up to $q \leq \bar{q}(w)$. For medium-sized orders, the limits to the ability to raise external finance, caused by agency problems, mean that dealers cannot compete aggressively, so competitive bid-ask spreads become wider. Large orders with $q > q_{\max}(w)$ are not intermediated because dealers cannot fund intermediation on terms at which clients find it profitable to trade.

Proposition 2 states that because of the dealers' limited balance-sheet capacity, the equilibrium bid-ask spreads, or, more precisely, cost of immediacy, increases in order sizes, while some large orders are not intermediated at all. We illustrate this result in Figure 2. For small orders with $q \leq \bar{q}(w)$, competition drives the bid-ask spread to the zero-profit level, as the dealers have enough balance-sheet capacity to post such narrow quotes. For medium-sized orders with $q \in (\bar{q}(w), q_{\max}(w))$, the dealers no longer have enough balance-sheet capacity to do so, causing the spread to widen. Large orders with $q > q_{\max}(w)$ are not intermediated because the quotes provided by dealers are so unattractive that clients do not find it profitable to trade.

It is worth emphasizing that our result should not be seen as contradictory to the well-documented findings that the average transaction costs decrease with trade size in corporate and municipal bond markets. First, such findings could be explained by the differences in sophistication and market power between large and small customers (Green, Hollifield, and Schürhoff (2007a,b))–elements that are absent in our model. More importantly, our result is primarily about the costs of immediacy, which, as we argue in the subsection below, are not necessarily the same as the average transaction costs.

3.3 Discussion

We end this section by discussing some implications already visible in the single-market model.

Leverage and the impact of leverage caps The maximum-funding contract can be implemented via debt, or a combination of debt and equity. The key insight is that some debt is necessary in the implementation because it gives the financiers priority in receiving payoffs when Laetitia is not found. As in Innes (1990), the use of debt expands the dealer's funding capacity in the presence of a moral-hazard problem. We summarize this in the following corollary to Proposition 2:

Corollary 1. *The maximum-funding contract in Lemma 1 can be implemented via a combination of a debt with a face value* $D \in [x_j^L, x_j^L + q\left(k - \frac{c}{\delta}\right)]$ *and external equity, which amount to a fraction* $\alpha = \frac{x_j^L + q\left(k - \frac{c}{\delta}\right) - D}{x_j^H - D}$ *of the total equity.*

For instance, consider the case when a dealer buys a quantity $q > \bar{q}(w)$ from Earl, and implements the maximum-funding contract with risky debt, with face value $D = x_b^L + q\left(k - \frac{c}{\delta}\right) = q\left(V - \frac{c}{\delta}\right) = \mathcal{P}_b(q)$. This dealer has total assets worth $w + \mathcal{P}_b(q)$. Leverage could be measured as the ratio of debt to total assets, which here would be $\frac{\mathcal{P}_b(q)}{w + \mathcal{P}_b(q)}$. It is clear that in our model, leverage is directly related to pledgeable income, the provision of immediacy, and bid-ask spreads. Our optimal contracting approach can therefore suggest an explanation for why dealer leverage in particular should be related to *funding liquidity*, that is, "the ease with which [dealers] can obtain funding,"[14] and *market liquidity* (Brunnermeier and Pedersen, 2009).

Consider, for example, unexpected trading losses that cause an unexpected decrease in internal funds w. Such losses increase leverage as defined above and also lead to worse market liquidity.[15]

Corollary 2 (Unexpected shocks to internal funds and market liquidity)**.** *Consider a situation in which financial constraints have an effect on market liquidity, in the sense that* $\bar{q}(w) < 1$, *and* $q_{\max}(w) < 1$. *Then an unexpected decrease in dealers' internal funds* w *produces a decrease in* $\bar{q}(w)$, *a decrease in market depth* $q_{\max}(w)$, *and a decrease in expected trading volume. The bid-ask spread for small trade sizes* ($q < \bar{q}(w)$) *is unaffected and remains at* $2c$. *For larger trade sizes, the bid-ask spreads widen.*

[14]The expression most closely associated with Brunnermeier and Pedersen's (2009) notion of funding liquidity would correspond to our pledgeable income per unit of asset $\mathcal{P}_j(q)/q$. Using the definition of $\mathcal{P}_j(q)$ (Equation (1)) shows that our definition differs from theirs in that ours contains a *dealer-specific* component $\left(\frac{c}{\delta}\right)$, which arises due to the agency friction inherent in the dealer's market making effort.

[15]While the implementation of the maximum-funding contract and thus leverage is not unique, the result that losses increase dealer leverage holds more generally. In Lemma A.1, we consider a dealer who chooses an implementation of the maximum-funding contract to minimize leverage. We show that this minimal leverage is decreasing in dealers' internal funds w.

When leverage increases because dealers lose money, market liquidity suffers: Bid-ask spreads for larger trades widen, market depth decreases, and volume decreases. This would be consistent, for instance, with the empirical finding of Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010), that spreads widen when NYSE specialists lose money. More generally, to the extent that market illiquidity of an asset increases its expected return or that dealers are the marginal purchasers of assets, and to the extent that changes in leverage are driven primarily by changes in retained earnings, it would also explain why the leverage of such dealers should negatively co-vary with asset returns, as shown in Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017).

It is not just unexpected losses that might produce changes in leverage, however. In response to the global financial crisis of 2008–2009, regulators around the world essentially forced bank-affiliated dealers to de-lever. There is some debate in the empirical literature about whether this has or has not had a negative effect on market liquidity.

We investigate the possible effects of a forced de-levering in Appendix A. In our model, debt is a necessary part of dealers' optimal financing arrangement. A cap on leverage decreases not only dealers' leverage, but also their balance-sheet capacity for the provision of immediacy. With a cap on bank-affiliated dealers, markets therefore become less liquid. To the extent that such dealers can broker or pre-arrange trades between clients, they will do so as an alternative to providing immediacy. Meanwhile, non-bank-affiliated dealers not subject to the cap become more competitive and intermediate more trades. We discuss these arguments in more detail and relate them to the existing empirical evidence in Appendix A.

The empirical predictions on the direction of the relationship between leverage and market liquidity therefore depend on the underlying cause of the change in leverage. When leverage *increases* because dealers lose money, market liquidity suffers (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010). But when leverage *decreases* because regulators force a de-leveraging, market liquidity also suffers, as shown in Haselmann, Kick, Singla, and Vig (2019) in the context of corporate bond markets and Bicu-Lieb, Chen, and Elliott (2020) in UK gilt markets.

Fundamental risk and liquidity Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010) also show that the effect of specialist losses on liquidity is more pronounced for high volatility stocks. In the Internet Appendix, we consider an extension of the model, in which the fundamental value V can change randomly from $t = 1$ until $t = 2$, while the dealer has an open position. With this risk of a change in the fundamental value, a dealer could lose out, for example, if it buys the asset from Earl, and the fundamental value of the drops from $t = 1$ until $t = 2$, while the dealer is looking for Laetitia. We show that for riskier assets, agency frictions are more severe. These assets thus have lower pledgeable income, or funding liquidity, resulting in larger bid-ask spreads, less market depth, and lower trading volume.

Costs of immediacy versus average transaction costs Our main results show that the cost of immediacy increases in trade size and that some larger trades may fail to take place when the cost is too high. This implies that the average transaction cost *conditional on a successful transaction* may underestimate the cost of immediacy. In a recent paper, Hendershott, Li, Livdan, and Schürhoff (2020) use auction data of collateralized loan obligations (CLO) to estimate what they call the "true cost of immediacy" (TCI). They find that the TCI is higher for larger trades and riskier tranches. Furthermore, they argue that traditional measures of transaction costs vastly underestimate the TCI, because the former does not take into account failed attempts to trade. Such failed attempts to trade are more likely for larger and riskier trades. All these findings are consistent with our results.[16]

[16] By making Earl's liquidity needs ℓ a random variable, we can achieve the results that the probability of trade failure increases in trade size and asset riskiness.

Choi and Huh (2017) point out a similar underestimation problem when dealers reduce immediacy provision in response to post-crisis regulations, and, as a result, a higher proportion of trades are brokered among clients. We discuss brokerage versus immediacy provision in Appendix A.

Microfoundation of dealers' inventory cost Our main result, illustrated in Figure 2, that bid-ask spreads increase in trade size can be seen as a microfoundation of dealers' inventory cost, dating back to the classic model of Stoll (1978). In most inventory models, it is costly for dealers to hold inventory because they are risk-averse. While it might not be obvious why large financial institutions like dealers are averse to diversifiable risk, our theory argues that it is not asset risk *per se* but the agency problem in market making that limit dealers' balance-sheet capacity, making them behave *as if* they are risk-averse.

Microfoundation of risk-based margins Our model can also be seen as a micro-foundation of risk-based margins for collateralized debt, which are exogenously assumed in Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). To see this, consider again the case when the dealer buys a larger quantity ($q > \bar{q}(w)$) from Earl and implements the maximum-funding contract with risky debt.

Corollary 1 states that the promised repayment (and amount raised) from financiers is equal to $R_b^H = x_b^L + q\left(k - \frac{c}{\delta}\right) = \mathcal{P}_b(q)$. The total funds used to finance the purchase from Earl are equal to $w + \mathcal{P}_b(q)$. The dealer's own funds that are put into the trade are w. Hence the "margin" that the dealer needs to put up is

$$\text{margin} = \frac{\text{own funds for trade}}{\text{total funds for trade}} = \frac{w}{w + \mathcal{P}_b(q)}. \tag{6}$$

If the dealer finds Laetitia, the financiers receive the promised repayment $\mathcal{P}_b(q)$. If the dealer does not find Laetitia, the financiers keep the collateral (the q units of the asset bought from Earl), to which they attach a per-unit value of $V-k$, so that they receive a payoff of $R_b^L = x_b^L = q(V-k)$, worth less than the promised repayment, as specified in Corollary 1.

From expression (6), it is immediately obvious that the margin increases in the risk of a change in fundamental value of the asset, since pledgeable income decreases in this risk (see the Internet Appendix).

4 Intermediation in multiple markets

In this section we consider a version of the model in which dealers can intermediate in two markets simultaneously. We assume that dealers need to exert independent search effort to find a good counterparty in each market. There are three key results: 1. Intermediating across markets relaxes incentive constraints and allows dealers to raise more financing per market; 2. Therefore, dealers who intermediate across markets can outbid dealers who specialize in only one market and will dominate in equilibrium; 3. As a consequence, when dealers are financially constrained, dealer leverage will co-move with the asset liquidity in all markets. These results are consequences of how dealers optimally raise financing to minimize the effect of the moral-hazard friction, and arise in spite of the fact that all agents are risk-neutral and that the asset payoffs are uncorrelated.

4.1 Setup

We consider a setup with two markets. We assume that each market has its separate, unrelated set of clients (Earls and Laetitias), so that search effort needs to be exerted independently in each market. In market A, an "Earl$_A$" arrives at $t=1$ to trade a quantity $q^A \in [0,1]$ of asset A. In market B, an "Earl$_B$" arrives at $t=1$ to trade a quantity $q^B \in [0,1]$ of asset B. A dealer may choose to post bid-ask spreads in only one market or in both markets. A dealer posting bid-ask spreads in market A may be chosen by the Earl$_A$ in market A and will then have to search for

Laetitia$_A$ in market A. A dealer posting bid-ask spreads in market B may be chosen by the Earl$_B$ in market B and will then have to search for the Laetitia$_B$ in market B.

Fundamental values (Laetitias' valuations) of the assets are fixed constants V_A and V_B and so are uncorrelated. To reduce notational clutter, we set $V_A = V_B = V$. Furthermore, we assume that a dealer intermediating in both markets has to make two separate search effort decisions to find the counterparties in the two different markets, with effort costs cq^A and cq^B, respectively. Conditional on the effort choices, the probabilities of finding a counterparty in either market are independent. The setting is intentionally stark to ensure that the only commonality between the two markets is that they are potentially both intermediated by the same dealer.

We consider $N \geq 3$ identical dealers who compete à la Bertrand to intermediate in the two markets.[17] (As before, we assume that Dealer 1 has infinitesimally smaller effort cost than the other dealers to break ties.)

Again, agency frictions matter if and only if dealers cannot simply raise all the cash that they need to intermediate from financiers. As we explain below, dealers who intermediate in both markets have higher pledgeable income per market. So to ensure that such dealers still need to use some of their internal capital w for intermediation, we need to replace Assumption 3 with the following, tighter assumption:

Assumption 4 (replaces Assumption 3)**.** *Even dealers intermediating in both markets must use some of their own cash for intermediation,* $\frac{c}{\delta(2-\delta)} > \ell$.

4.2 Equilibrium

Below, we will refer to a dealer who intermediates in both markets simultaneously as a *cross-market dealer*, and a dealer who only intermediates in one market as a *specialized dealer*. In

[17]With only two dealers, an equilibrium in which each dealer monopolizes intermediation in one asset is possible. Assuming at least three dealers rules out this equilibrium.

this subsection, we first show that a cross-market dealer has higher pledgeable income than the combined pledgeable income of two dealers who specialize and hence can intermediate larger trades. Effectively, there is a form of "financing-related economies of scope": It is easier to raise external finance when intermediating across many markets than when intermediating only in a single market. We then show that this also means that a cross-market dealer will win the Bertrand competition in equilibrium.

The first key result (the analogue of Lemma 1) is that a cross-market dealer can raise more external funds, as follows:

Lemma 2 (Maximum-funding contract and cross-pledging)**.** *The maximum-funding contract for a cross-market dealer rewards the dealer only when Laetitia is found in both markets. The total pledgeable income of the cross-market dealer in the case of bids and, respectively asks, is:*

$$\mathcal{P}_b(q^A, q^B) = \begin{cases} (q^A + q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) & \text{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A V + q^B\left(V - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B V + q^A\left(V - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta} \end{cases} \tag{7}$$

and

$$\mathcal{P}_a(q^A, q^B) = \begin{cases} (q^A + q^B)\left(k - \frac{c}{\delta(2-\delta)}\right) & \text{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A k + q^B\left(k - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B k + q^A\left(k - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta}. \end{cases} \tag{8}$$

which is strictly higher than the sum of the pledgeable incomes of two specialized dealers, that is, $\mathcal{P}_j(q^A, q^B) > \mathcal{P}_j(q^A) + \mathcal{P}_j(q^B)$ for $j \in \{b, a\}$. Finally, the contract can be implemented via a risky debt with a face value $D_j = \mathcal{P}_j(q^A, q^B)$ for $j \in \{b, a\}$.

Proof. See Appendix C. □

Lemma 2 shows that a dealer has larger pledgeable income and therefore balance-sheet capacity when it intermediates in multiple markets. Under the maximum-funding contract that raises this larger pledgeable income, a cross-market dealer only receives a positive payoff when intermediating successfully (i.e., when finding Laetitia) in *both* markets. As in the single-market case, the contract can be implemented with risky debt. However, now the face value of this debt has to be so high that the cross-market dealer will only be able to repay the debt if successfully intermediating in *both* markets.

The underlying effect that generates this result is known in the optimal contracting literature (Cerasi and Daltung, 2000; Laux, 2001), and sometimes called a "cross-pledging" effect (Tirole, 2006, Section 4.2). Intuitively, it is as if the dealer were pledging the rent obtained from intermediating in one market as collateral for intermediation in the other. Pledging this "collateral" strengthens the dealers incentives to exert effort, *ceteris paribus*, and hence allows the dealer to raise more money from financiers.

The key part of our assumptions that generates cross-pledging is that conditional on effort decisions, the events of finding Laetitia$_A$ and Laetitia$_B$ in the two markets are not perfectly dependent. That is, successfully intermediating in market A does not necessarily imply successfully intermediating in market B (or vice versa). The underlying effect is therefore a form of risk diversification. The moral hazard friction makes risk-neutral dealers behave *as if* they are averse to the risks that intermediation can turn out to be unprofitable.[18]

Because a cross-market dealer can raise more money from financiers than two specialized dealers combined, there are combinations of large trade sizes that can be intermediated by a cross-market dealer, but not by two specialized dealers, as follows:

Proposition 3 (Combination of trade sizes that can be intermediated by a cross-market dealer)**.**

[18]This is not the same as diversification of fundamental risk. In fact, there is no fundamental risk here, since V is constant. In the Internet Appendix, we consider a version of the model in which V is not constant and show that dealers are also *effectively* averse to fundamental risks.

Consider a pair of orders (q^A, q^B) for which $q^A \leq q^B$. A cross-market dealer can intermediate both assets when $q^B \leq q^B_{max}(q^A, w)$, where

$$q^B_{\max}(q^A, w) = \begin{cases} \dfrac{w + q^A \ell}{\frac{c}{\delta} - \ell} & \text{for} \quad q^A \in \left[0, \dfrac{(1-\delta)w}{\frac{c}{\delta} - \ell(2-\delta)}\right] \\ \dfrac{w}{\frac{c}{\delta(2-\delta)} - \ell} - q^A & \text{for} \quad q^A \in \left(\dfrac{(1-\delta)w}{\frac{c}{\delta} - \ell(2-\delta)}, \dfrac{\frac{1}{2}w}{\frac{c}{\delta(2-\delta)} - \ell}\right] \end{cases} \tag{9}$$

The expression for $q^A_{\max}(q^B, w)$, for trades with sizes $q^A \geq q^B$ is symmetric.

Any pair of orders (q^A, q^B) that can be intermediated by a pair of specialized dealers can also always be intermediated by a cross-market dealer. The converse is not true.

Proof. See Appendix C. □

Figure 3 illustrates how the larger pledgeable income of a cross-market dealer expands the set of order sizes (q^A, q^B) that can be intermediated by such a dealer, relative to the set of order sizes (q^A, q^B) that could be intermediated by two specialized dealers.

The figure also illustrates an aspect of the cross-pledging effect that is absent in the models of Cerasi and Daltung (2000) and Laux (2001). In their setting, project size is fixed, so the rent that can be cross pledged from one project to another is always of the same size. In our case, rent is proportional to trade size, and trade sizes can differ, so the size of the rent being cross pledged from one "project" to another can differ.

Lemma 2 and Proposition 3 both show that this matters. Suppose that q^A is much smaller than q^B. Then the potential rent from market A is small, while the potential rent from market B is large. Intuitively, the threat of losing the rent from market B gives very strong incentives to exert effort in market A. At the same time, the threat of losing the rent from market A does basically nothing for incentives to exert effort in market B.

This has implications for how pledgeable income changes as trade size changes. When q^A is

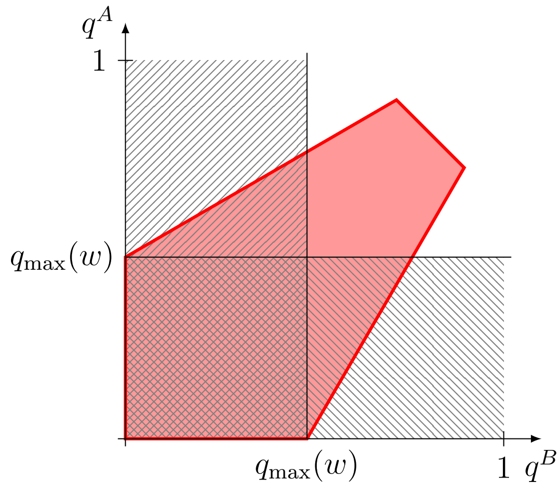

Figure 3. Trades sizes, cross-market dealers, and specialized dealers
The tuple of trades (q^A, q^B) can be intermediated by two specialized dealers if $q^A \leq q_{\max}(w)$ and $q^B \leq q_{\max}(w)$, as indicated by the cross-hatched region (see Proposition 2). The tuple of trades (q^A, q^B) can be intermediated by a single cross-market dealer if $q^A \leq q^A_{\max}(q^B, w)$ and $q^B \leq q^B_{\max}(q^A, w)$, as indicated by the dark shaded region in red (see Proposition 3).

much smaller than q^B, in the sense that $q^A < (1-\delta)q_B$, increasing q^A slightly expands pledgeable income of the cross-market dealer at a rate $\frac{\partial \mathcal{P}_b}{\partial q^A} = V$ (see Proposition 2). Because this dealer will exert effort in market A due to the threat of losing the rent in market B, the position in asset A becomes fully pledgeable. Contrast this with a specialized dealer intermediating in asset A: For this dealer, the rate at which pledgeable income expands is much lower and equal to $\frac{\partial \mathcal{P}_b(q^A)}{\partial q^A} = V - \frac{c}{\delta}$ (see Equation (1)). Here, this dealer will not automatically exert effort unless financiers leave some skin in the game, so the position in asset A cannot be fully pledgeable.

This argument implies that for a cross-market dealer, an increase in q^A can *improve* the ability to intermediate in market B. Imagine, for instance, that the cross-market dealer pays the competitive price $V - c$ in market A. This is a high price. So, as q^A increases, the cash that the dealer needs for intermediating in market A expands at a high rate equal to $\frac{\partial}{\partial q^A} q^A(V-c) = V - c$. At the same time, the rate at which pledgeable income expands is even higher and equal to $\frac{\partial \mathcal{P}_b(q^A, q^B)}{q^A} = V$. Therefore, the pledgeable income "left over" for intermediation in market B expands at rate $V-(V-c)=c$, and so *increases*. This explains, for example, why the maximum trade size in market B, that is, $q^B_{\max}(q^A, w)$, can be *increasing* in q^A (see Proposition 3 and Figure 3).

The fact that a cross-market dealer can raise more money from financiers than two specialized dealers combined also affects how Bertrand competition will play out between dealers. A cross-market dealer always wins.

Proposition 4 (Equilibrium with multiple markets)**.** *Consider a pair of orders (q^A, q^B) that can be intermediated by a cross-market dealer, as described in Proposition 3. In equilibrium, all such pairs will be intermediated by a cross-market dealer.*

Proof. See Appendix C. □

Proposition 4 is a central result of our paper. Our model provides a rationale for why dealers may want to intermediate across multiple markets simultaneously: This can alleviate financing frictions so that cross-market dealers can raise more external funding.

Proposition 4 is not an entirely obvious implication of Proposition 2. While a cross-market dealer can raise more external funds, two specialized dealers have more internal funds ($2w$ in total, vis-a-vis w for the cross-market dealer).

To give some intuition for the result in Proposition 4, consider first a situation in which w is large, so that the two specialized dealers have a large advantage in internal funds vis-a-vis the cross-market dealer (combined internal funds of $2w$ versus the cross-market dealer's w). In this situation, dealers are also less likely to be financially constrained. Hence prices are likely to be at the competitive level. Therefore, specialized dealers cannot take advantage of their advantage in internal funds to out-compete the cross-market dealer – because prices are at the competitive level already.

Now instead suppose w is small, so the advantage in internal funds of the two specialized dealers is small. Dealers are also more likely to be financially constrained. Hence prices are likely to be below the competitive level. Since the advantage of the specialized dealers in internal funds is small, the fact that a cross-market dealer can raise more external funds matters a lot, so that this dealer can out-compete the specialized dealers.

4.3 Positive correlation in liquidity across markets

Because a single dealer intermediates all markets in equilibrium in our model, liquidity will be positively correlated across all markets in our model. There are various aspects of liquidity we can describe. We start with market depth.

A straightforward implication of Propositions 3 and 4 is that market depth may co-move across fundamentally unrelated markets. For instance, an unexpected drop in dealers' internal funds w and the associated increase in leverage would mean that market depths would decrease in *both* otherwise unrelated markets, as follows:

Corollary 3 (Positive correlation in market depth across markets)**.** *In equilibrium, if w is small enough so that market depth is below the maximum possible trade size in both markets, then an unexpected decrease (increase) in w produces a decrease (increase) in market depth in both markets.*

Proof. See partial derivatives of market depth (in Proposition 3) with respect to w. □

The corresponding statement about bid-ask spreads is more complicated. Bertrand competition only pins down the level of profits that the winning cross-market dealer will earn in equilibrium: Either the quantities to be intermediated are small relative to balance-sheet capacity, and competition drives profits to zero, or the quantities to be intermediated are large relative to balance-sheet capacity, the winning dealer uses the maximum-funding contract and exhausts its balance-sheet capacity, which pins down a positive level of profits. In either case, there is one possible level of profits in equilibrium, determined by the parameters of the model.

Saying that in equilibrium, a dealer must make a certain level of profits does not pin down how much profit the dealer extracts from each of the two markets. A given level of profits can be achieved by large bid-ask spreads and high profitability in market A and small bid-ask spreads and low profitability in market B, or vice versa. This means that in general, bid-ask spreads are not pinned down; we can only determine these if we know how much of its balance sheet capacity the winning dealer allocates to each market.

However, if dealers do not change how they split balance-sheet capacity as w changes, an unexpected drop in dealers' internal funds w and the associated increase in leverage would mean that bid-ask spreads would widen in *both* otherwise unrelated markets, as follows:

Corollary 4 (Positive correlation in bid-ask spreads across markets)**.** *Define the* excess balance-sheet capacity *of the cross-market dealer as the difference between its balance-sheet capacity and the balance-sheet capacities of two (hypothetical) specialized dealers. Suppose the cross-market dealer splits this* excess balance-sheet capacity *across the two markets so that (i) both markets are allocated a positive amount of capacity, and (ii) how capacity is split does not change with w. Suppose furthermore that internal funds are low enough so that single-market dealers would be constrained in the bid-ask spreads that they can post and moral hazard matters ($w \leq \bar{w}(q^i)$ for $i \in A, B$). Then in an equilibrium in the two-market model, an unexpected decrease (increase) in w produces a widening (tightening) of the bid-ask spreads.*

Proof. See Appendix C. □

In Appendix B, we also show that under plausible assumptions about how dealers split balance-sheet capacity across markets, changes in trade size in one market can have a non-monotonic effect on bid-ask spreads in the other market. The underlying reasons for these effects are similar to the forces that can make maximum depth in one market increasing in trade size in the other market.

To sum up, unlike in Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), where a market-maker is assumed to intermediate across many markets, we have a market-maker who optimally chooses to do so. Intermediation across several markets relaxes the financing constraint, so that a dealer who does so will dominate in equilibrium. Our model can therefore provide a potential explanation for why a small number of large dealers dominate in many OTC markets in practice. Furthermore, as debt finance is optimal in our model, dealer leverage arises as a common determinant of liquidity across otherwise unrelated markets. Furthermore, to the extent that prices are affected by liquidity/ expected future transaction costs (Amihud and Mendelson, 1986) (or that our dealers are the marginal purchasers of assets in certain markets) our model suggests that prices should co-move with dealer leverage, as documented by Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017).

Our results could be used to investigate the liquidity–stability trade-offs in the regulation of dealer banks. We show that by adopting higher leverage, the cross-market dealer outbids specialized dealers and improves liquidity provisions across multiple markets. A regulator who is concerned with liquidity contagion and systemic risks in asset markets may want to promote specialized dealers by imposing a tighter leverage cap or additional capital requirement on cross-market dealers. This policy tool is not unlike the capital surcharges imposed on the global systemically important banks by the Federal Reserve Board, and our results suggest that worsening market liquidity would come as an expense of such regulations.

5 Concluding remarks

We present a model in which dealers need external finance to conduct their market-making activities, specifically to provide immediacy for their clients. Once they take over a position from a client, dealers need to exert unobservable effort in searching for a counterparty, to increase the chance of closing the position at a good intermediation profit. This moral-hazard problem affects how and how much external finance dealers can raise. It limits intermediation volume, softens competition between dealers, and widens bid-ask spreads. When dealers suffer losses, the problem becomes worse, especially for riskier assets and larger orders.

We show that dealers optimally raise finance in the form of external debt, explaining why dealer leverage matters and is related to market liquidity. Furthermore, we show that dealers can alleviate the agency frictions by intermediating across several markets. This provides a new microfoundation for why liquidity should co-move across markets.

The model can also provide a rationale for many of the observed changes in market liquidity during and after the financial crisis. External financing for dealers optimally takes the form of debt, to leave sufficient upside so that they have incentives to exert effort. Regulations that limit the use of debt, such as maximum leverage ratios or minimum capital requirements for bank affiliated dealers, will affect the ability of dealers to intermediate larger, riskier trades, and may prompt them to switch from providing immediacy with their balance sheet to brokering trades between clients.

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A Maximum leverage ratio and market liquidity

There is a debate around whether the tightened regulation in the aftermath of the global financial crisis has worsened bond market liquidity relative to pre-crisis levels.[19] In this appendix, we extend the model to study the effect of post-crisis regulations on market liquidity.[20] We show that in the context of our model, tightened leverage ratio and capital requirements should reduce bank-affiliated dealers' balance-sheet capacity, and hence, their ability to provide immediacy. Consequently, they fulfill proportionally more of their clients' liquidity demand via brokering between clients, and less so by directly taking on the position. Finally, they are more likely to be out-competed by non-bank-affiliated dealers.

A.1 Leverage regulation and market liquidity

As indicated by Corollary 1 (and also lemma 2), debt financing is necessary for maximizing the balance-sheet capacity of dealers. When regulators force (bank-affiliated) dealers to use less debt, for example, via a supplemental leverage ratio or tightened capital requirements, this could hurt their ability to provide immediacy. (In the context of our model, these two types of regulation are equivalent.) Hence market liquidity decreases. There is some evidence that, for example, tightened capital requirements have had this effect (Haselmann, Kick, Singla, and Vig, 2019).

Consider the single-market model of Section 3, and suppose the regulator were to impose a

[19]Trebbi and Xiao (forthcoming) argue that there is no deterioration in a variety of liquidity measures. Anderson and Stulz (2017) present a mixed view on price-based liquidity metrics but show that there has been a drop in post-crisis turnover. Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) point out that turnover, block trade frequency, and average trade size decreased post-crisis. They also point out that the decline in liquidity provision in the U.S. bond market is coming from a reduction in capital committed to intermediation by bank-affiliated dealers, whereas non-bank-affiliated dealers have increased their capital commitment.

[20]There are many regulatory changes that could have had an effect on the ability of bank-affiliated dealers in particular to intermediate. Potential candidates include the Volcker rule, net stable funding ratios, liquidity coverage ratios, supplementary leverage ratios, and tightened capital requirements.

maximum leverage ratio constraint. For simplicity, we only consider the case in which Earl sells to the dealer. In this case, the total assets of the dealer correspond to the asset bought from Earl. We assume that total assets are valued at the purchase price, which in this case is the bid price b, so that total assets would be worth qb.

Corollary 1 shows that the maximum-funding contract $\{R_H, R_L\}$ can be implemented with risky debt with a high face value or a combination of safe debt with a low face value $D(= x^L)$ and equity representing a fraction $\alpha = \frac{q\left(k-\frac{c}{\delta}\right)}{x_j^H - x_j^L}$ of the residual cash flows. A dealer subject to a leverage ratio constraint would like to minimize the amount of debt used, and so would choose the implementation with safe debt and equity. We illustrate the implementation of the minimum-funding contract in terms of safe debt and equity in Figure 4.

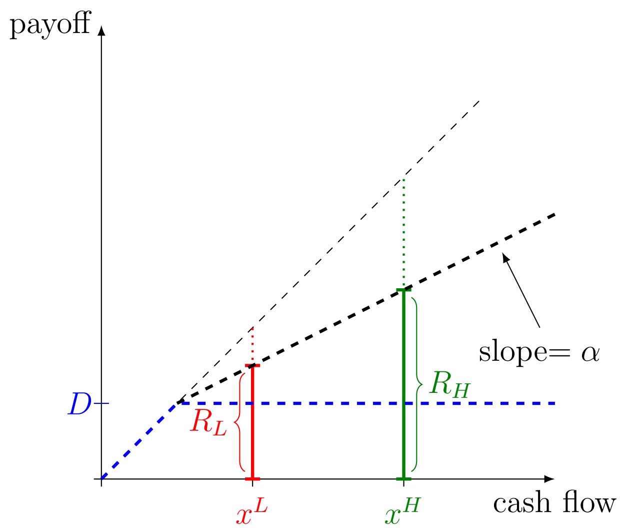


Figure 4. Repayments to financiers in terms of debt and equity
Any repayment to financiers $\{R_H, R_L\}$ can be implemented by a safe, standard debt contract that promises a payment D in both states $\{H, L\}$, and a standard equity contract that pays a fraction α of the remaining cash flows in the two states, $\{\alpha(x^H - D), \alpha(x^L - D)\}$. Therefore, $R_H = D + \alpha(x^H - D)$ and $R_L = D + \alpha(x^L - D)$.

We describe the leverage ratio resulting from choosing the implementation with the minimum possible amount of debt in the following lemma:

Lemma A.1. *Consider a competitive dealer. The dealer's leverage ratio $\Lambda(q,w)$ resulting from the optimal contract with minimum leverage is (weakly) increasing in q, and given by*

$$\Lambda(q,w) = \begin{cases} 0 & \text{if } q \leq q_{NL}(w) \\ \frac{V-\frac{\delta}{k}-\frac{w}{q}\frac{\delta k}{c}}{V-c} & \text{if } q_{NL}(w) < q \leq \bar{q}(w) \\ \frac{V-k}{\frac{w}{q}+V-\frac{c}{\delta}} & \text{if } \bar{q}(w) < q < q_{\max}(w), \end{cases} \tag{10}$$

where $q_{NL}(w) = \frac{w}{\frac{c}{\delta k}V-c}$, and $\bar{q}(w)$ and $q_{\max}(w)$ are as defined in Proposition 2.

Proof. See Appendix C. □

The leverage ratio $\Lambda(q,w)$ is increasing in q: When intermediating trades of small size q, a dealer can rely entirely on external equity, without causing incentive problems for the dealer, so that the leverage ratio can be zero. When the dealer wants to intermediate trades with larger and larger size q, it will have to use increasing amounts of debt (and therefore, leverage) so as to preserve incentives. The leverage ratio constraint will therefore start to bind for trades beyond some critical size $q_\Lambda(w)$.[21] We illustrate this in Figure 5.

Since orders with size $q > q_\Lambda(w)$ cannot be financed with the optimal contract, the competitive dealer has to offer a contract $\{D,\alpha\}$ (with $D \leq q(V-k)$) and solves the following problem:

$$\max_{\{D,\alpha\}} \mathcal{P} = D + \alpha(qV - D) \tag{11}$$

$$\text{subject to} \quad \Delta_R \equiv \underbrace{D + \alpha(qV-D)}_{R_H} - [\underbrace{D + \alpha(q(V-k)-D)}_{R_L}] \leq q(k - \frac{c}{\delta}) \tag{12}$$

$$\Lambda \equiv \frac{D}{D + \alpha(qV-D) + w} \leq \Lambda_{\max} \tag{13}$$

[21] This critical size is defined via $\Lambda\left(q_\Lambda(w), w\right) = \Lambda_{\max}$.

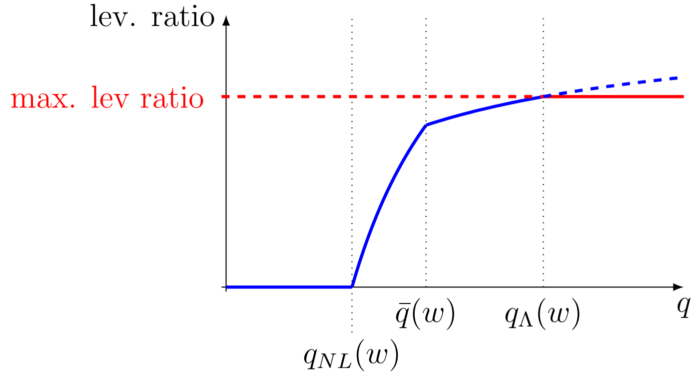


Figure 5. Leverage ratio constraint and trade size q
This figure describes Lemma A.1 in Appendix A. A dealer can finance the smallest trades ($q \leq q_{NL}(w)$) with equity only so that the resulting leverage ratio is zero. For slightly larger trades ($q_{NL}(w) < q \leq \bar{q}(w)$), the dealer will have to use some debt in the external financing mix, to improve incentives, so the leverage ratio becomes positive. For even larger trades $q \leq \bar{q}_\Lambda(w)$, the bids and asks, and hence the overall financing need, is affected by the incentive problem. For trades with a size that exceeds a critical level $q_\Lambda(w)$, the leverage ratio constraint will bind, which will increase bid-ask spreads. Market depth is reduced.

where the incentive compatibility constraint (12) can further be simplified to $\alpha \leq 1 - \frac{c}{\delta k}$. The characterization of the optimal debt and equity contract and thus the pledgeable income under leverage ratio requirement is straightforward, as follows:

Proposition A.1. *Suppose the regulator imposes a maximum leverage ratio requirement $\Lambda_{\max}$. For trade size $q > q_\Lambda(w)$, where $q_\Lambda(w)$ is defined via $\Lambda(q_\Lambda(w), w) = \Lambda_{\max}$, the dealer optimally issues share α^* of outside equity and issues debt with promised repayment $D^*(\Lambda_{\max})$,*

$$\alpha^* = \left(1 - \frac{c}{\delta k}\right); \quad D^*(\Lambda_{\max}) = \min\left\{\frac{w + (1 - \frac{c}{\delta k})qV}{\frac{1}{\Lambda_{\max}} - \frac{c}{\delta k}}, q(V-k)\right\}. \tag{14}$$

Therefore the pledgeable income under leverage requirement is

$$\mathcal{P}(q; \Lambda_{\max}) = \left(1 - \frac{c}{\delta k}\right) qV + \frac{c}{\delta k} D^*(\Lambda_{\max}). \tag{15}$$

Proof. See Appendix C. □

Here, since we focus only on the bid, we can think of a reduction of the bid as being synonymous with a reduction of market liquidity. In that sense, the following comparative statics results shows that tightening the leverage requirement harms market liquidity of the asset and thus social surplus, as follows:

Corollary A.1. *For $q > q_\Lambda(w)$, tightening leverage requirement (lowering $\Lambda_{\max}$)*

1. *reduces liquidity (= the bid), especially for trades of larger size;*
2. *reduces market depth (the maximum possible size of an intermediated order), and hence social surplus.*

Proof. See Appendix C. □

There is some supportive evidence for the results. Dick-Nielsen and Rossi (2018) use index exclusions as shocks to immediacy demands by uninformed index trackers and find that the cost of immediacy has more than doubled after the 2008 crisis, likely because of the more stringent regulatory environment.

A.2 Brokerage versus immediacy provision

There is empirical evidence on the effects of post-crisis banking regulations on how dealers make market in the U.S. corporate bond market. Goldstein and Hotchkiss (2018) and Schultz (2017)) show that dealers become more reluctant to provide immediacy by holding the assets and prefer intermediating via matching clients who have offsetting demand, or brokerage. Choi and Huh (2017) also document the increased bias for brokered trades and find that the bias is stronger for larger orders and riskier bonds.

Our model can be easily modified to incorporate brokerage and rationalize these empirical findings. Consider a version of the model in which with some probability $\pi \in (0, 1)$, Laetitia arrives at the market early, and hence the dealer can intermediate between Earl and Laetitia but without having to hold the asset over time. This would be "pre-arranged" or "brokered" trades. If a dealer is unconstrained, all trades will be intermediated and the proportion of brokered trades is simply π. As we have argued that the post-crisis regulations tend to reduce the dealers' balance-sheet capacity, some trades do not take place anymore (Proposition 2). Thus the proportion will increase, and more so for larger trades and riskier bonds because they require more dealers' balance-sheet capacity.

A.3 Bank versus non-bank dealers

Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) find that the decline in liquidity provision in the U.S. bond market in the post-crisis period is coming from reduction in capital committed to intermediation by bank-affiliated dealers, whereas non-bank-affiliated dealers have increased their capital commitment. Such an evolution of bond market liquidity provision can be rationalized in the context of our model, if bank-affiliated dealers are now subject to, for example, the maximum leverage ratio, but non-bank-affiliated dealers are not.

Suppose Dealer 1 is a bank-affiliated dealer and Dealer 2 is a non-bank dealer. Further assume that Dealer 1 is more efficient than Dealer 2 in liquidity provision, for example, $c_1 < c_2$. Without any additional regulatory constraint, Dealer 1 intermediates the asset in equilibrium. Suppose the equilibrium changes, as Dealer 1 is now subject to a maximum leverage ratio requirement. This can reduce the pledgeable income of Dealer 1 to the extent that Dealer 2, the non-bank dealer can now out-compete Dealer 1, the bank-affiliated dealer.

B Non-monotonic liquidity spillovers

In this appendix, we consider the version of the model introduced in Section 4, and examine the behavior of bid and ask prices in more detail. We show that, under some specific, plausible assumptions on splitting rules, in equilibrium, we can have non-monotonic liquidity spillovers from one market to the other market, as well as negative own-price impacts. The underlying mechanism is similar to the effect described in the main text, by which an increase in q^A can *increase* the maximum depth $q^B_{\max}(q^A, w)$ that a cross-market dealer can offer in market B.

As explained in the main text, Bertrand equilibrium pins down total profit, but not how the total profit is split across the two markets, and hence also does not pin down bid and ask prices in the two markets. Equivalently, Bertrand equilibrium pins down how much balance-sheet capacity dealers will use to intermediate, but not how that balance-sheet capacity is split across markets. Here, we need to make an assumption about how balance-sheet capacity is split.

Define the "excess balance-sheet capacity" Δ of a cross-market dealer (see also the proof of Proposition 4) as the difference between the balance-sheet capacity of the cross-market dealer and the amounts of cash employed by specialized dealers to intermediate, as follows:

$$\Delta_b := w + \mathcal{P}_b(q^A, q^B) - \left(q^A b^A_s + q^B b^B_s\right), \tag{16}$$

$$\Delta_a := w + \mathcal{P}_a(q^A, q^B) - \left(q^A(V+k) - q^A a^A_s + q^B(V+k) - q^B a^B_s\right). \tag{17}$$

Here, b^i_s and a^i_s are bids and asks, respectively, that would be posted by specialized dealers competing with each other. For the definition of $\mathcal{P}_j(q^A, q^B)$, see Lemma 2.

With this definition, we are ready to make the following assumption:

Assumption B.1. *Excess balance-sheet capacity is split across markets proportionally according to trade sizes, except if prices would exceed the competitive limit in one market ($V - c$ for bids,*

$V+c$ for asks). In this case, excess balance-sheet capacity is allocated so that prices in one market are pushed to the competitive limit, and any remaining excess capacity goes to the other market. (Competitive limits cannot be exceeded in both markets in equilibrium, as profits would then be negative.)

The following proposition summarizes the bid and ask prices that result from this assumption:

Proposition B.1. *Consider $q^A < q^B$. Under Assumption B.1, equilibrium bids and asks are as follows:*

1. *For $q^A < (1-\delta)q^B$,*

outcomes \ w	$w \in [0, w_1)$	$w \in [w_1, w_2)$	$w \geq w_2$
b^A	$V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^B}{q^A}\frac{1}{q^A+q^B}$	$V-c$	$V-c$
b^B	$V - \frac{c}{\delta}\frac{q^B}{q^A+q^B} + w\frac{q^A}{q^B}\frac{1}{q^A+q^B}$	$V - \frac{c}{\delta} + \frac{w}{q^B} + \frac{q^A}{q^B}c$	$V-c$
a^A	$V + \frac{c}{\delta}\frac{q^B}{q^A+q^B} - w\frac{q^B}{q^A}\frac{1}{q^A+q^B}$	$V+c$	$V+c$
a^B	$V + \frac{c}{\delta}\frac{q^B}{q^A+q^B} - w\frac{q^A}{q^B}\frac{1}{q^A+q^B}$	$V + \frac{c}{\delta} - \frac{w}{q^B} - \frac{q^A}{q^B}c$	$V+c$

2. *For $q^A \geq (1-\delta)q^B$,*

outcomes \ w	$w \in [0, w_1')$	$w \in [w_1', w_2')$	$w \geq w_2'$
b^A	$V - \frac{c}{\delta(2-\delta)} + w\frac{q^B}{q^A(q^A+q^B)}$	$V-c$	$V-c$
b^B	$V - \frac{c}{\delta(2-\delta)} + w\frac{q^A}{q^B(q^A+q^B)}$	$V - \frac{c}{\delta(2-\delta)} + \frac{w}{q^B} - c\frac{q^A(1-\delta)^2}{q^B\delta(2-\delta)}$	$V-c$
a^A	$V + \frac{c}{\delta(2-\delta)} - w\frac{q^B}{q^A(q^A+q^B)}$	$V+c$	$V+c$
a^B	$V + \frac{c}{\delta(2-\delta)} - w\frac{q^A}{q^B(q^A+q^B)}$	$V + \frac{c}{\delta(2-\delta)} - \frac{w}{q^B} + c\frac{q^A(1-\delta)^2}{q^B\delta(2-\delta)}$	$V+c$

for some thresholds w_1, w_2, w_1', w_2' specified below.

Proof. We start by considering bids.

Consider first the case where $q^A < (1-\delta)q^B$.

1. Suppose w is such that specialized dealers who exhaust pledgeable income would bid less than $V-c$ for both assets. This requires that $w \in [0, c\frac{1-\delta}{\delta}q^A)$.

   The excess balance-sheet capacity is $\Delta_b = q^A\frac{c}{\delta} - w$. In the equilibrium in which the winning (cross-market) dealer shares the excess balance-sheet capacity in proportion to trade sizes across markets, the bids are

$$b^i = \frac{1}{q^i} \times \left(w + q^i \left(V - \frac{c}{\delta} \right) + \frac{q^i}{q^A + q^B} \left(q^A \frac{c}{\delta} - w \right) \right) \qquad \text{for } i = \{A, B\}$$

The equilibrium bids are therefore

$$\begin{aligned} b^A &= V - \frac{c}{\delta} \frac{q^B}{q^A + q^B} + w \frac{q^B}{q^A} \frac{1}{q^A + q^B} \\ b^B &= V - \frac{c}{\delta} \frac{q^B}{q^A + q^B} + w \frac{q^A}{q^B} \frac{1}{q^A + q^B}, \end{aligned}$$

but we do require that $b^A < V - c$, so it must be the case that $w < w_1 \equiv c\frac{1-\delta}{\delta} q^A - c\frac{(q^A)^2}{q^B}$.

For $w \in [w_1, c\frac{1-\delta}{\delta} q^A]$, b^A is already at the zero-profit bid $V - c$. The excess balance-sheet capacity becomes $q^A c$, and all of it is allocated to market B. Hence in this case,

$$b^B = \frac{1}{q^B} \times \left(w + \mathcal{P}(q^A, q^B) - q^A (V - c) \right) = \frac{w}{q^B} + V - \frac{c}{\delta} + \frac{q^A}{q^B} c.$$

2. Suppose w is such that specialized dealers who exhausts pledgeable income would bid $V - c$ for asset A, but less than $V - c$ for asset B. This requires that $w \in \left[c\frac{1-\delta}{\delta} q^A, c\frac{1-\delta}{\delta} q^B \right)$.

   The excess balance-sheet capacity is $\Delta_b = q^A c$. Hence, the winning (cross-market) dealer will bid $b^A = (V - c)$ for asset A and use the remaining balance-sheet capacity for asset B, such that

   $$b^B = \min\{V - c, \frac{w}{q^B} + \frac{q^A}{q^B} c + V - \frac{c}{\delta}\}.$$

   We have that $b^B = \frac{w}{q^B} + \frac{q^A}{q^B} c + V - \frac{c}{\delta}$ when $w < w_2 \equiv c\frac{1-\delta}{\delta} q^B - q^A c$ and $b^B = V - c$ otherwise.

3. Suppose that w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for both assets. This requires that $w \geq c\frac{1-\delta}{\delta} q^B$. The equilibrium bids of the cross-market dealer are $b^A = b^B = V - c$.

Consider now the case where $q^A \geq (1-\delta)q^B$.

1. Suppose w is such that specialized dealers who exhaust pledgeable income would bid less than $V - c$ for both assets. This requires that $w \in \left[0, c\frac{1-\delta}{\delta}q^A\right)$.

   The excess balance-sheet capacity is $\Delta_b = \left((q^A + q^B)\frac{c}{\delta}\frac{1-\delta}{2-\delta} - w\right)$. In the equilibrium in which the winning (cross-market) dealer shares the excess balance-sheet capacity in proportion to trade sizes across markets, the bids are

   $$b^i = \frac{1}{q^i} \times \left(w + q^i\left(V - \frac{c}{\delta}\right) + \frac{q^i}{q^A + q^B}\left((q^A + q^B)\frac{c}{\delta}\frac{1-\delta}{2-\delta} - w\right)\right) \qquad \text{for } i = \{A, B\},$$

   or

   $$b^A = V - \frac{c}{\delta}\frac{1}{2-\delta} + w\frac{q^B}{q^A(q^A + q^B)},$$
   $$b^B = V - \frac{c}{\delta}\frac{1}{2-\delta} + w\frac{q^A}{q^B(q^A + q^B)},$$

   as long as $w < w_1' \equiv \frac{c(1-\delta)^2(q^A+q^B)q^A}{(2-\delta)\delta q^B}$.

   Define $w_2' \equiv \frac{c(1-\delta)^2(q^A+q^B)}{(2-\delta)\delta}$. For $w \in [w_1', w_2')$, b^A is already at the zero-profit bid $V - c$. The remaining balance-sheet capacity becomes $\frac{c}{\delta}\frac{1-\delta}{2-\delta}\left(q^B - (1-\delta)q^A\right)$, which is all allocated to market B. Hence, in this case, $b^B = \frac{w}{q^B} + V - \frac{c}{\delta}\frac{q^B + q^A(1-\delta)^2}{q^B(2-\delta)}$. And for $w \geq \left[w_2', c\frac{1-\delta}{\delta}q^A\right)$, $b^A = b^B = V - c$.

   It can be easily verified that all the above ranges of w are non-empty.

2. Suppose w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for asset A, but less than $V - c$ for asset B. This requires that $w \in [c\frac{1-\delta}{\delta}q^A, c\frac{1-\delta}{\delta}q^B)$. The cross-market dealer bids $V - c$ for asset A and allocates the remaining excess balance-sheet capacity to asset B. Since in this range of w, $w \geq w_2'$, the equilibrium bid for asset B is $V - c$.

3. Suppose w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for both assets. This requires $w \geq c\frac{1-\delta}{\delta}q^B$.

   The equilibrium bids are $b^A = b^B = V - c$.

The proof for ask prices is symmetric, and is omitted for brevity. $\square$

We can now consider how, for example, the bid price in one market changes when the trade sizes in the other market changes.

Corollary B.1 (Non-monotonic liquidity spillovers across markets)**.** *Consider an equilibrium in which $q_A \leq q_B$.*

For given q^B and $w \in \left(cq_B\frac{(1-\delta)^3}{\delta}, cq^B\frac{(1-\delta)^2}{\delta}\right)$, we have that

$$\lim_{q^A \uparrow (1-\delta)q_B} \frac{\partial b^B}{q^A} > 0$$
$$\lim_{q^A \downarrow (1-\delta)q_B} \frac{\partial b^B}{q^A} < 0,$$

so that b^B is non-monotonic in q^A in the vicinity of $q^A = (1-\delta)q_B$.

Proof. Compute expressions for w_1, w_2, w_1', and w_2' when $q^A = (1-\delta)q_B$ to find

$$w_1 = w_1' = cq_B\frac{(1-\delta)^3}{\delta} \quad \text{and} \quad w_2 = w_2' = cq_B\frac{(1-\delta)^2}{\delta}. \tag{18}$$

The interval $[w_1 = w_1', w_2 = w_2']$ is non-empty.

Compute partial derivatives of b^B in the given interval to obtain the result. $\square$

Intuitively, when q^A is much smaller than q^B in the sense that $q^A < (1-\delta)q^B$, and w is in the indicated range, an increase in q^A expands pledgeable income at rate V, while only using up cash at rate $b^A = V - c$ for intermediation, so that increasing q^A increases leftover pledgeable income for intermediation in market B at rate $V - (V - c) = c > 0$. Here, this means that the bid price in market B actually increases in q^A. (Once q^A exceeds $(1-\delta)q^B$, an increase in q^A generates pledgeable income at a smaller rate, so that this effect disappears.)

We note that Assumption B.1 is necessary to obtain this result. If excess balance-sheet capacity is simply split between markets according to the relative trade sizes, for instance, then the bid price in market A can exceed $V-c$. In this case, the leftover pledgeable income for intermediation in market B would expand at rate $V-b^A<0$ as q^A increases.

Corollary B.2 (Own-price impact). *Also, the own-price impact of q^A is negative, in the sense that $\frac{\partial b^A}{\partial q^A}>0$ and $\frac{\partial a^A}{\partial q^A}<0$ for $q^A<(1-\delta)q^B$ and $w<\min\left\{\frac{c(q^A)^2}{\delta(2q^A+q^B)}, w_1\right\}$.*

Again, for $q^A<(1-\delta)q^B$, an increase in q^A increases pledgeable income at rate V. Since the price b^A is a function of how much pledgeable income is available, expanding pledgeable at this high rate can actually lead to an *increase* in the price that is paid for asset A.

C Proofs

Proof of Proposition 1. See main text. □

Proof of Proposition 2. See main text. □

Proof of Lemma 2. Comparing to the single asset case, the two-market case entails a richer outcome space. There are four different possible outcomes, hence four contractible cash flows, and hence four possible repayments to investors. To preserve space, we use the notation $q = q^A + q^B$ in the proof.

First, consider the case in which both Earls sell and the dealer buys.

Scenarios	Contractible cash flow	Repayment to investors
Counterparty found for both assets	$q^A V + q^B V = qV$	R_1
Counterparty found for asset B only	$q^A(V-k) + q^B V = qV - q^A k$	R_2
Counterparty found for asset A only	$q^A V + q^B(V-k) = qV - q^B k$	R_3
No counterparty found	$q^A(V-k) + q^B(V-k) = q(V-k)$	R_4

Similarly, the possible effort decisions are richer in the two-market case. The dealer can choose to exert effort to search for a counterparty, in the first market, in the second market, in both markets, or in neither market. To ensure that the dealer exerts effort to search for both assets, which is by assumption the efficient action, the following three incentive-compatibility constraints have to be satisfied:

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) + \delta(qV - q^B k - R_3) - q^A c, \tag{IC1}$$

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) + \delta(qV - q^A k - R_2) - q^B c, \tag{IC2}$$

$$\begin{aligned} q(V-c) - R_1 \geq{} & (1-\delta)^2(qV - R_1) + \delta(1-\delta)(qV - q^A k - R_2) + \\ & \delta(1-\delta)(qV - q^B k - R_3) + \delta^2(qV - qk - R_4) \end{aligned} \tag{IC3}$$

The three incentive constraints ensure that exerting search effort in both markets is better than, respectively, i) only exerting search effort in the market A; ii) only exerting search effort in market B; and iii) not exerting any search effort. Therefore, the maximum-funding contract is

$$\begin{aligned}
&\max_{R_1,R_2,R_3,R_4} \mathcal{P}(q^A,q^B) = R_1 \\
&\text{subject to} \quad (IC1),(IC2),(IC3) \text{ and } (LL)
\end{aligned}$$

One immediately sees that it is optimal to set R_2, R_3, and R_4 as high as possible. Intuitively, in order to incentivize the dealer to exert effort to search in both asset, it should only be rewarded if it finds both counterparties. The incentive constraints then simplify to

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) - q^A c \Leftrightarrow R_1 \leq qV - \frac{q-q^A}{\delta}c, \tag{IC1}$$

$$q(V-c) - R_1 \geq (1-\delta)(qV - R_1) - q^B c \Leftrightarrow R_1 \leq qV - \frac{q-q^B}{\delta}c, \tag{IC2}$$

$$q(V-c) - R_1 \geq (1-\delta)^2(qV - R_1) \Leftrightarrow R_1 \leq q\left(V - \frac{c}{1-(1-\delta)^2}\right). \tag{IC3}$$

Setting R_1 to its maximum possible value produces the expression for $\mathcal{P}_b(q^A,q^B)$, as follows:

$$\mathcal{P}_b(q^A,q^B) = \begin{cases} (q^A+q^B)\left(V - \frac{c}{\delta(2-\delta)}\right) & \text{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A V + q^B\left(V - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B V + q^A\left(V - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta}. \end{cases}$$

Since the derivation in the case of asks is very similar, we state without proof that the pledgeable income in this case is

$$\mathcal{P}_a(q^A,q^B) = \begin{cases} (q^A+q^B)\left(k - \frac{c}{\delta(2-\delta)}\right) & \text{if} \quad \frac{q^A}{q^B} \in (1-\delta, \frac{1}{1-\delta}) \\ q^A k + q^B\left(k - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \leq 1-\delta \\ q^B k + q^A\left(k - \frac{c}{\delta}\right) & \text{if} \quad \frac{q^A}{q^B} \geq \frac{1}{1-\delta}. \end{cases}$$

Finally, it is easy to verify that $\mathcal{P}_j(q^A,q^B) > \mathcal{P}_j(q^A) + \mathcal{P}_j(q^B)$ (cf. Equation (1)). $\square$

Proof of Proposition 3. Consider orders $\{q^A, q^B\}$, where $q^A \leq q^B$. Trade can only take place if a dealer sets a bid and ask that are at least as high and as least as low as Earl's reservation values $V - \ell$ (when Earl sells) and $V + \ell$ (when Earl buys), respectively. For a cross-market dealer to be able to finance such trades requires

$$w + \mathcal{P}_b(q^A, q^B) \geq (q^A + q^B)(V - \ell) \tag{19}$$

and (after some algebra)

$$w + \mathcal{P}_a(q^A, q^B) \geq (q^A + q^B)(k - \ell) \tag{20}$$

for the case of bids and asks respectively. Using the expressions for pledgeable income from Lemma (2), we have, after some re-arranging,

$$\begin{cases} q^B \leq \dfrac{w + q^A \ell}{\frac{c}{\delta} - \ell} & \text{if} \quad q^A < (1-\delta)q^B \\ q^B \leq \dfrac{w}{\frac{c}{\delta(2-\delta)} - \ell} - q^A & \text{if} \quad q^A \in [(1-\delta)q^B, q^B] \end{cases} \tag{21}$$

In the first region $q^A < (1-\delta)q^B$, as q^A increases from 0 and approaches $(1-\delta)q^B$, the upper bound of q^B approaches $\frac{w}{\frac{c}{\delta} - (2-\delta)\ell}$ and thus the maximum value of q^A approaches $\frac{(1-\delta)w}{\frac{c}{\delta} - (2-\delta)\ell}$. As q^A starts at $\frac{(1-\delta)w}{\frac{c}{\delta} - (2-\delta)\ell}$, it reaches the second region and the upper bound of $q^B = \frac{w}{\frac{c}{\delta} - (2-\delta)\ell}$. As q^A approaches q^B, the upper bound of q^B approaches $\dfrac{\frac{1}{2}w}{\frac{c}{\delta(2-\delta)} - \ell}$. Hence, it is also the maximum value of q^A in this range. The expressions for $q^B_{\max}(q^A, w)$ in the proposition follow. $\square$

Proof of Proposition 4. First, we show that a cross-market dealer can always out-compete specialized dealers. We can then discuss competition between cross-market dealers.

Cross-market dealers out-compete specialized dealers: Define the "excess balance-sheet capacity" Δ of a cross-market dealer as the difference between the balance-sheet capacity of the cross-market dealer and the amounts of cash employed by specialized dealers to intermediate, as follows:

$$\Delta_b := w + \mathcal{P}_b(q^A, q^B) - \left(q^A b_s^A + q^B b_s^B\right), \tag{22}$$

$$\Delta_a := w + \mathcal{P}_a(q^A, q^B) - \left(q^A(V+k) - q^A a_s^A + q^B(V+k) - q^B a_s^B\right). \tag{23}$$

Here, b_s^i and a_s^i are bids and asks, respectively, that would be posted by specialized dealers competing with each other. For the definition of $\mathcal{P}_j(q^A, q^B)$, see Lemma 2.

To save space, we only present the proof for Δ_b. The argument for Δ_a is symmetric. We also only present the case in which $q^A < q^B$. The case for $q^A > q^B$ is symmetric.

When $q^A < q^B$, there are two cases to consider:

Case 2, $q^A < (1-\delta)q^B$: Cross-market dealer pledgeable income is $\mathcal{P}(q^A, q^B) = q^A V + q^B \left(V - \frac{c}{\delta}\right)$. The condition for a cross-market dealer to be able to outbid two specialized dealers is

$$\Delta_b = w + q^A V + q^B \left(V - \frac{c}{\delta}\right) - q^A b_s^A(w, q^A) - q^B b_s^B(w, q^B) > 0, \tag{24}$$

where $b_s^i = \min\{\frac{w}{q^i} + (V - \frac{c}{\delta}), V - c\}$. Whether this condition is satisfied depends on w.

1. Suppose w is such that specialized dealers who exhausts pledgeable income would bid less than $V - c$ for both assets. This requires $\bar{q}(w) < q^A < q^B$, or $w < q^A c \frac{1-\delta}{\delta}$.

   We have $q^i b_s^i = w + q^i \left(V - \frac{c}{\delta}\right)$, so that excess balance-sheet capacity is $\Delta_b = q^A \frac{c}{\delta} - w$. Condition (24) becomes $w < q^A \frac{c}{\delta}$, which is true in the proposed range of w.

2. Suppose w is such that specialized dealers who exhausts pledgeable income would bid $V - c$ for asset A but less than $V - c$ for asset B. This requires that $q^A \leq \bar{q}(w) < q^B$, or $q^A c \frac{1-\delta}{\delta} \leq w < q^B c \frac{1-\delta}{\delta}$.

   We have $q^A b_s^A = q^A(V - c)$ and $q^B b_s^B = w + q\left(V - \frac{c}{\delta}\right)$, so that excess balance-sheet capacity is $\Delta_b = q^A c$. Condition (24) becomes $q^A c > 0$, which is obviously true.

3. Suppose that w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for both assets. This requires that $q^B \leq \bar{q}(w)$, or $w \geq q^B c \frac{1-\delta}{\delta}$.

   We have $q^i b_s^i = q^i(V - c)$, so that excess balance-sheet capacity is $\Delta_b = w - q^B c \frac{1-\delta}{\delta} + q^A c$. Condition (24) becomes $w > q^B c \frac{1-\delta}{\delta} - q^A c$, which is true in the proposed range of w.

<u>Case 2, $q^A \geq (1-\delta) q^B$</u>: Cross-market dealer pledgeable income is $\mathcal{P}(q^A, q^B) = (q^A + q^B) \left(V - \frac{c}{\delta(2-\delta)} \right)$. The condition for a cross-market dealer to be able to outbid two specialized dealers is

$$\Delta_b = w + (q^A + q^B) \left(V - \frac{c}{\delta(2-\delta)} \right) - q^A b_s^A(w, q^A) - q^B b_s^B(w, q^B) > 0. \tag{25}$$

where $b_s^i = \min\{\frac{w}{q^i} + (V - \frac{c}{\delta}), V - c\}$. Whether this condition is satisfied depends on w.

1. Suppose w is such that specialized dealers who exhaust pledgeable income would bid less than $V - c$ for both assets. This requires $\bar{q}(w) < q^A < q^B$, or $w < q^A c \frac{1-\delta}{\delta}$.

   We have $q^i b_s^i = w + q^i \left(V - \frac{c}{\delta} \right)$, so that excess balance-sheet capacity is $\Delta_b = (q^A + q^B) \frac{c}{\delta} \frac{1-\delta}{2-\delta} - w$. Condition (25) becomes $w < (q^A + q^B) \frac{c}{\delta} \frac{1-\delta}{2-\delta}$, which is true since in the proposed range of w, we have $w < q^A c \frac{1-\delta}{\delta} < 2q^A \frac{c}{\delta} \frac{1-\delta}{2-\delta} < (q^A + q^B) \frac{c}{\delta} \frac{1-\delta}{2-\delta}$.

2. Suppose w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for asset A, but less than $V - c$ for asset B. This requires that $q^A < \bar{q}(w) \leq q^B$, or $q^A c \frac{1-\delta}{\delta} < w \leq q^B c \frac{1-\delta}{\delta}$.

   We have $q^A b_s^A = q^A (V - c)$ and $q^B b_s^B = w + q \left(V - \frac{c}{\delta} \right)$, so that excess balance-sheet capacity is (after some algebra) $\Delta_b = \frac{c}{\delta} \frac{1-\delta}{2-\delta} \left(q^B - q^A (1-\delta) \right)$. Condition (25) becomes $q^A < \frac{1}{1-\delta} q^B$, which is true since $q^A < q^B$.

3. Suppose w is such that specialized dealers who exhaust pledgeable income would bid $V - c$ for both assets. This requires that $q^B \leq \bar{q}(w)$, or $w \geq q^B c \frac{1-\delta}{\delta}$.

We have $q^i b_s^i = q^i(V-c)$, so that excess balance-sheet capacity is $\Delta_b = w-(q^A+q^B)c\frac{1-\delta}{\delta}\frac{1-\delta}{2-\delta}$. Condition (25) becomes $w > (q^A+q^B)c\frac{1-\delta}{\delta}\frac{1-\delta}{2-\delta}$. This is true, because in the proposed range of w, $w > c\frac{1-\delta}{\delta}q_B > c\frac{1-\delta}{\delta}\frac{q^B+q^A}{2} > c\frac{1-\delta}{\delta}(q^A+q^B)\frac{1-\delta}{2-\delta}$ (using $q^A < q^B$).

Competition between cross-market dealers: Since cross-market dealers can always outcompete specialized dealers, equilibrium will be determined by the competition between dealers who compete with each other as cross-market dealers.

In equilibrium, either the zero-profit constraint, or the constraint that all pledgeable income of a cross-market dealer is exhausted, is binding. To see this, suppose first that neither constraint binds. Then a cross-market dealer can deviate by raising slightly more financing, using this to slightly improve prices, and obtain positive profits doing so. Suppose now that the zero-profit constraint binds but that pledgeable income is not exhausted. A dealer could deviate by improving prices without exhausting pledgeable income. But such a deviation must produce negative profits. Suppose now that profits are positive but that cross-market dealer pledgeable income is exhausted. Since pledgeable income is exhausted, a dealer can only allocate more money to one market if simultaneously allocating less money to the other. A deviating dealer therefore has to worsen prices in one market, to improve them in the other. But such a deviation means that the dealer is not intermediating in both markets, and hence this dealer cannot raise the cross-market dealer pledgeable income.

Finally, our tie-breaking assumption implies that in equilibrium, Dealer 1 will be the winning cross-market dealer and will make infinitesimally small but positive profits in both markets. $\square$

Proof of Corollary 4. Let b_s^i, a_s^i denote the bids and asks of competitive specialized dealers in market i. Let b_c^i,a_c^i denote the bids and asks of competitive cross-market dealers in market i. As in the proof of Proposition 4, define the "excess balance-sheet capacity" Δ of a cross-market dealer as the difference between the balance-sheet capacity of the cross-market dealer and the amounts of cash employed by specialized dealers to intermediate, as follows:

$$\Delta_b := w + \mathcal{P}_b(q^A, q^B) - \left(q^A b_s^A + q^B b_s^B\right), \tag{26}$$

$$\Delta_a := w + \mathcal{P}_a(q^A, q^B) - \left(q^A(V+k) - q^A a_s^A + q^B(V+k) - q^B a_s^B\right). \tag{27}$$

We use the following notation to describe the dealers' rule for splitting (excess) balance-sheet capacity across markets: A fraction $\rho^i \in (0,1)$ is allocated to market i, where $\rho^A + \rho^B = 1$. Note that balance-sheet capacity must be exhausted in a competitive equilibrium. The cash used by the cross-market dealer in market i is equal to the amount used by a single-market dealer plus the amount $\rho^i \Delta_j$, as follows:

$$q_i b_c^i = \rho^i \Delta_b + q^i b_s^i, \tag{28}$$

$$q^i(V+k) - q_i a_c^i = \rho^i \Delta_a + \left(q^i(V+k) - q^i a_s^i\right). \tag{29}$$

Let $'$ denote a partial derivative w.r.t. w. The condition on the splitting rule in the lemma can be expressed as $(\rho^i)' = 0$.

Both single-market dealers post bids below $V - c$, so that $q^i b_s^i = w + \mathcal{P}(q^i)$, $(q^i b_s^i)' = +1$, and $(\Delta_b^i)' = -1$. Then $(q^i b_c^i)' = \left(\rho^i \Delta_b^i + q^i b_s^i\right)' = \rho_i (\Delta_b^i)' + (q^i b_s^i)' > 0$, since $\rho_i \in (0,1)$.

Both single-market dealers post asks above $V + c$, so that $q(V+k) - q^i a_s^i = w + \mathcal{P}(q^i)$, $(q(V+k) - q^i a_s^i)' = +1$, and $(\Delta_a^i)' = -1$. Then $(q(V+k) - q^i a_c^i)' = \left(\rho^i \Delta_a^i + q(V+k) - q^i a_s^i\right)' = \rho^i (\Delta_a^i)' + (q(V+k) - q^i a_s^i)' > 0$, since $\rho_i \in (0,1)$.

Taken together, this implies that $(b_c^i)' > 0$ and $(a_c^i)' < 0$. $\square$

Proof of Lemma A.1. With the definitions of R_H, R_L, the constraint (IC) can be rewritten as

$$\alpha \leq \left(1 - \frac{c}{\delta k}\right) \equiv \bar{\alpha}, \tag{30}$$

where $\bar{\alpha}$ is the maximum share of equity that can be issued to financiers, so that exertion of effort by the dealer is still incentive compatible.

A competitive dealer who bids $b_c(q,w) = \min\left\{\frac{w}{q} + \left(V - \frac{c}{\delta}\right)V - c\right\}$ must raise external finance $qb_c(q,w) - w$. The break-even constraint stipulates that $qb_c(q,w) - w = R_H$, so that the bid can be financed in this case as long as

$$R_H = \alpha(x^H - D) + D = \alpha x^H + (1-\alpha)D \geq \max(qb_c(q,w) - w, 0), \tag{31}$$

where we note that here, $x^H = qV$.

We can distinguish three cases, as follows:

1. The minimum amount of debt D_{min} can be zero if there exist an $\alpha \leq \bar{\alpha}$ such that $\alpha qV + (1-\alpha) \cdot \underbrace{D}_{=0} = \alpha qV = R_H$, where $R_H \geq qb_c - w$, that is, if

$$\bar{\alpha} qV \geq q(V - c) - w \tag{32}$$

or if

$$q \leq q_{NL}(w) = \frac{w}{\frac{c}{\delta k}V - c}. \tag{33}$$

2. For $q > q_{NL}(w)$, the minimum amount of debt $D_{\min}$ needed in the optimal contract is implicitly defined by

$$\bar{\alpha} qV + (1-\bar{\alpha})D_{\min} = q\,(V - c) - w \tag{34}$$

or

$$D_{\min} = q\left(V - \frac{\delta}{k}\right) - w\frac{\delta k}{c}. \tag{35}$$

Furthermore, debt should be risk-free, as assumed above, which requires

$$D_{\min} \leq x^L = q(V - k), \tag{36}$$

which is equivalent to requiring

$$q \leq \bar{q}(w). \tag{37}$$

3. Finally, for $q > \bar{q}(w)$, the maximum amount of (safe) debt is $D_{\min} = x^L = q(V - k)$, and the maximum amount that can be raised is $R_H = \bar{\alpha}x^H + (1 - \bar{\alpha}D_{\min} = q\left(V - \frac{c}{\delta}\right)$. Since $b_c(q, w) = \frac{w}{q} + \left(V - \frac{c}{\delta}\right)$, this is exactly the amount that must be raised, and intermediation can take place, as long as $q \leq q_{\max}(w)$ as defined above.

The leverage ratios in the lemma are calculated in the three cases with total assets as $D_{\min}/(qb_c(q, w))$.

□

Proof of Proposition A.1. First, notice that the incentive constraint (12) should be binding, as increasing α increases $\mathcal{P}$ and relaxes the leverage requirement. This pins down α^*. Similarly, the leverage constraint (13) should be binding, since increasing D increases $\mathcal{P}$ and relaxes the incentive constraint (12). This then pins down D^*. □

Proof of Corollary A.1. The maximum incentive compatible bid under the maximum leverage requirement is $b_{IC}(q; \Lambda_{\max}) = \frac{w}{q} + \frac{\mathcal{P}(q;\Lambda_{\max})}{q}$. The first result, $\frac{\partial b_{IC}(q;\Lambda_{\max})}{\partial \Lambda_{\max}} > 0$ and $\frac{\partial}{\partial q}\frac{\partial b_{IC}(q;\Lambda_{\max})}{\partial \Lambda_{\max}} > 0$, can be obtained by direct differentiation of the expression for $b_{IC}(q; \Lambda_{\max})$, using the results in Proposition A.1.

The second result comes from the definition of $q_{\max}$: The largest possible trade is the one for which the bid, $b_{IC}(q_{\max}; \Lambda_{\max})$, is just equal to Earl's reservation value of $V - \ell$, or for which $q_{\max} \cdot b_{IC}(q_{\max}; \Lambda_{\max}) = q_{\max}(V - \ell)$. Inserting the definitions of $b_{IC}(q_{\max}; \Lambda_{\max})$ and $\mathcal{P}(q; \Lambda_{\max})$, we find that $q_{\max} = \dfrac{w + \frac{c}{\delta k}D^*}{\frac{c}{\delta k} - \ell}$. Hence, sign $\left(\frac{\partial q_{\max}}{\partial \Lambda_{\max}}\right) =$ sign $\left(\frac{\partial D^*}{\partial \Lambda_{\max}}\right) > 0$. Reducing the maximum leverage ratio $\Lambda_{\max}$ decreases $q_{\max}$, and some larger orders are not intermediated. Hence social surplus decreases. □

Internet Appendix of "Dealer Funding and Market Liquidity"

In this Internet appendix, we extend the model in Section 3 to allow for random changes in fundamental value V during intermediation. Our main result is that the agency friction makes riskier assets less liquid. All proofs in the Internet Appendix are collected in Section IA.1.

We assume that after the effort decision at $t = 1$, the fundamental asset value will change. The fundamental value can either go up to $V + z$ or down to $V - z$, with equal probability. Correspondingly, Laetitia will now have a valuation of either $V + z$ or $V - z$, financiers will have a valuation of either $V + z - k$ or $V - z - k$, and security lenders will have a valuation of either $V + z + k$ or $V - z + k$.[22] We refer to z as the *risk* of an asset.

There are now four possible net cash flows to the dealer from intermediation, which depend on whether the fundamental value went up or down and whether the dealer found Laetitia or did not find Laetitia. We assume that contracts can be contingent on these four states. We use the subscripts U and D to refer to whether the fundamental value went up or down, respectively, and the subscripts H and L to refer to whether Laetitia was found or not found, respectively, as before.

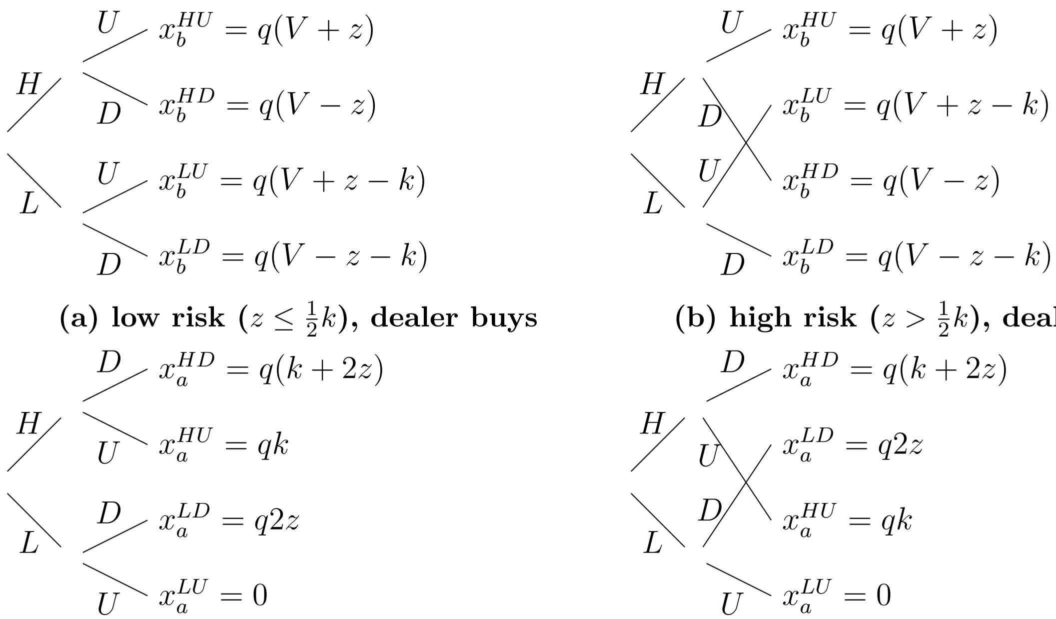
If the dealer buys from Earl, it can subsequently sell either at price $V + z$ or $V - z$ if it finds Laetitia and will have to make repayments R_b^{HU} or R_b^{HD}, or the dealer will be forced to sell at price $V + z - k$ or $V - z - k$ to financiers and make repayments R_b^{LU} or R_b^{LD} if it does not.

If the dealer sells to Earl, the dealer provides cash collateral $V + z + k$ to borrow the asset first, then sells, and can buy back later either at price $V + z$ or $V - z$ and make repayments R_a^{HU} or R_a^{HD} if the dealer finds Laetitia, or will be forced to buy back at price $V + z + k$ or $V - z + k$ from

[22] We have also considered an alternative description in which the dealer is only exposed to the risk of a change in fundamental value if Laetitia is not found. This would be consistent with the interpretation that search effort increases not just the chance of finding good counterparties, but also the speed with which they are found. Results in this version of the model are similar.

security lenders and make repayments R_a^{LU} or R_a^{LD} if it does not.

The possible cash flow orderings are illustrated in Figure IA.1.



(a) low risk ($z \leq \frac{1}{2}k$), dealer buys

(b) high risk ($z > \frac{1}{2}k$), dealer buys

(c) low risk ($z \leq \frac{1}{2}k$), dealer sells

(d) high risk ($z > \frac{1}{2}k$), dealer sells

Figure IA.1. Cash flows when fundamental value changes
We illustrate the cash flows to a dealer as a function of whether the fundamental value of the asset moves up (U) or down (D) and whether the dealer finds Laetitia (H) or not (L). There are four cases to consider: The dealer may buy from Earl and benefit from an up-move in fundamental value, or the dealer may sell to Earl and benefit from a down-move in fundamental value. Also, the effect of asset risk may dominate the effect of a successful search ($z > \frac{1}{2}k$), or it may not ($z \leq \frac{1}{2}k$).

For brevity, we again let $\{R_j\}$ denote the set of repayments R_j^{HU}, R_j^{HD}, R_j^{LU}, and R_j^{LD} for $j \in \{b, a\}$. We can then write the utility of the buying and selling dealer, respectively, as follows:

$$U_j(e, \{R_j\}) = (1 - (1 - e)\delta) \left[\frac{1}{2} \left(x_j^{HU} - R_j^{HU} \right) + \frac{1}{2} \left(x_j^{HD} - R_j^{HD} \right) \right]$$
$$+ \delta(1 - e) \left[\frac{1}{2} \left(x_j^{LU} - R_j^{LU} \right) + \frac{1}{2} \left(x_j^{LD} - R_j^{LD} \right) \right] - cqe \text{ for } j \in \{b, a\}. \quad \text{(IA.1)}$$

As before, the dealer's optimal contracting problem can be stated as

$$\max_{\{R_j\}} U_j(1, \{R_j\}), \text{ for } j \in \{b, a\}, \tag{P'}$$

subject to the incentive compatibility constraint

$$U_j(e = 1, \{R_j\}) \geq U_j(e = 0, \{R_j\}), \text{ for } j \in \{b, a\}. \tag{IA.2}$$

After some manipulation, this can be written as

$$\Delta R_j \leq q\left(k - \frac{c}{\delta}\right), \tag{IC'}$$

where ΔR_j is defined as the expected difference in repayments in case Laetitia is found versus the case in which Laetitia is not found, as follows:

$$\Delta R_j := \frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right) - \frac{1}{2}\left(R_j^{LU} + R_j^{LD}\right). \tag{IA.3}$$

In addition, there are limited liability constraints and the break-even constraint for financiers. We have

$$R_j^i \leq x_j^i, \text{ for } j \in \{b, a\}, \text{ and } i \in \{HU, HD, LU, LD\}, \tag{LL'}$$

$$\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right) = \begin{cases} qb(q) - w & \text{if } j = b, \\ q(V + k + z) - qa(q) - w & \text{if } j = a. \end{cases} \tag{BE'}$$

To obtain more plausible, monotone contracts, we now also impose two additional monotonicity constraints, which restrict the space of possible contracts that we consider, as follows:

Assumption IA.1 (Monotonicity)**.** *Net payoffs to the dealer must be non-decreasing in the underlying gross cash flows, and repayments to investors must be non-decreasing in the underlying gross* *cash flows:*

$$x_j^i - R_j^i \text{ is non-decreasing in } x_j^i, \tag{MCD}$$

$$R_j^i \text{ is non-decreasing in } x_j^i, \tag{MCF}$$

for $j \in \{b, a\}$, *and state* $i \in \{HU, HD, LU, LD\}$.

The first part of Assumption IA.1 could be motivated as follows: If the dealer can secretly burn cash, the dealer would have an incentive to do so with a payoff function that is decreasing in cash flow. The contract should not give the incentive to do this. The second part of Assumption IA.1 is as in Innes (1990) and can be motivated as follows: If the dealer can secretly inject cash, the dealer would have an incentive to do so when repayments to investors are decreasing in cash flow. The contract should not give the incentive to do this. Because of the monotonicity constraints, the cash flow ranking matters for the characterization of the optimal monotone contract.

The impact of Assumption IA.1 depends on the ranking of the gross cash flows, which in turn depend on the relative size of z and k. We can distinguish the following two cases:

1. The asset has relatively low risk, when $z \leq \frac{1}{2}k$.

2. The asset has relatively high risk, when $z > \frac{1}{2}k$.

In the high-risk case, what matters more for cash flows is whether the fundamental value moves up or down, and what matters less is whether Laetitia is found. Conversely, in the low-risk case, what matters more is whether the dealer does or does not find Laetitia, and what matters less is whether the fundamental value moves up or down.

We make the following additional assumption on parameters:

Assumption IA.2. $\delta k < 2c$

We can rewrite this assumption as

$$k - \frac{c}{\delta} < \frac{1}{2}k.$$

In this expression, the left-hand side relates to the NPV of effort, and the right-hand side relates to the critical cutoff for z, such that assets with a risk z that exceeds this number are high-risk in the above sense. The assumption ensures that the NPV of effort is relatively low, so that even for low-risk assets, the risk may still be large in relation to the NPV generated by the provision of effort. This ensures that prices will be affected by relatively low values of the asset risk z.

The optimal contract solves (P') subject to the constraints (IC'), (LL'), (BE'), (MCD), and (MCF). As before, there are potentially many optimal contracts. As before, rather than discussing the set of optimal contracts directly, we focus on pledgeable income $\mathcal{P}(q)$, defined as the maximum amount of cash that can be raised from financiers. Using the break-even constraint (BE'), we can define pledgeable income as the solution to the following problem:

$$\mathcal{P}_j(q) = \max_{\{R_j\}} \frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right), \tag{IA.4}$$

subject to the constraints (IC'), (LL'), (MCD), and (MCF).

Lemma IA.1. *For any contract that solves* (IA.4)*, the incentive compatibility* (IC') *constraint must bind.*

We can use this lemma to prove the following proposition, which states that the bid-ask spread is (weakly) increasing in asset risk.

Proposition IA.1. *The incentive compatible ask and bid are given by*

$$a_{IC}(q) = -\frac{w}{q} + V + \frac{c}{\delta} + \left(z - \left(k - \frac{c}{\delta}\right)\right)\mathbf{1}_{\left\{z \geq k - \frac{c}{\delta}\right\}} - \left(z - \frac{1}{2}k\right)\mathbf{1}_{\left\{z \geq \frac{1}{2}k\right\}}$$
$$b_{IC}(q) = \frac{w}{q} + V - \frac{c}{\delta} - \left(z - \left(k - \frac{c}{\delta}\right)\right)\mathbf{1}_{\left\{z \geq k - \frac{c}{\delta}\right\}} + \left(z - \frac{1}{2}k\right)\mathbf{1}_{\left\{z \geq \frac{1}{2}k\right\}}$$

Thus the equilibrium bid-ask spread is (weakly) increasing in asset risk z.

Figure IA.2 graphs the bid-ask spread as a function of asset risk z, as described by Proposition IA.1.

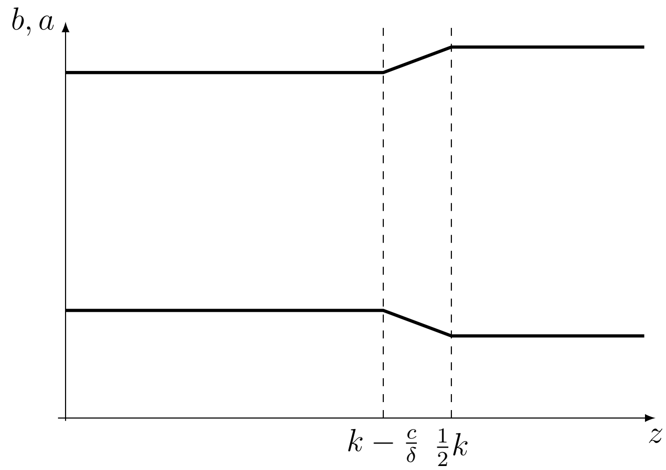


Figure IA.2. Bid-ask spread as a function of asset risk z
The bid-ask spread is weakly increasing in asset risk z, as described by the expression in Proposition IA.1.

IA.1 Proof

Proof of Lemma IA.1. We argue by contradiction. Suppose that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is maximized but that (IC') is slack.

Now suppose that both of the two limited liability constraints in (LL') relating to R_j^{HU} and R_j^{HD} bind, so that neither of the two repayments can be increased. It can be easily seen that in this case, (IC') cannot be satisfied. We have reached a contradiction.

Now suppose that one of the two limited liability constraints in (LL') relating to R_j^{HU} or R_j^{HD} binds, but the other does not. These repayments are associated with cash flows x^{HU} and x^{HD}. If the limited liability constraint binds for the repayment associated with the higher cash flow but does not bind for the repayment associated with the lower cash flow, then (MCD) cannot be satisfied. So this is a contradiction. If, instead, the limited liability constraint binds for the repayment associated with the lower cash flow but does not bind for the repayment associated with the higher cash flow, then it must be possible to raise the repayment associated with the higher cash flow: This does not violate the corresponding limited liability constraint, (MCD), (MCF), or (IC'). This implies that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is not maximized, and we have again reached a contradiction.

Finally, suppose that neither of the two limited liability constraints in (LL') relating to R_j^{HU} and R_j^{HD} bind. Then it must be possible to increase either one or the other or both, without violating (MCD), (MCF), or (IC'). This implies that $\frac{1}{2}\left(R_j^{HU} + R_j^{HD}\right)$ is not maximized, and we have again reached a contradiction. □

Proof of Proposition IA.1. We first consider the case in which Earl wants to sell and the dealer buys the asset from him, selling it on later. We then consider the case in which Earl wants to buy and the dealer borrows the asset and then sells it to him, buying it back later. For both cases, we need to consider the two subcases in which the asset is either relatively low-risk, $z \leq \frac{1}{2}k$, or relatively high-risk, $z > \frac{1}{2}k$, respectively.

Dealer buys; low-risk asset, $z \leq \frac{1}{2}k$. In this case, Assumption IA.1 implies the following constraints:

$$\begin{array}{lllll} R_b^{HU} \geq & R_b^{HD} \geq & R_b^{LU} \geq & R_b^{LD} (\geq 0), & (\mathrm{MCF}_{b,l}) \\ x_b^{HU} - R_b^{HU} \geq & x_b^{HD} - R_b^{HD} \geq & x_b^{LU} - R_b^{LU} \geq & x_b^{LD} - R_b^{LD} (\geq 0), & (\mathrm{MCD}_{b,l}) \end{array}$$

where $(\mathrm{MCF}_{b,l})$ stands for the monotonicity constraint of the financiers, when the dealer buys a low-risk asset, and $(\mathrm{MCD}_{b,l})$ stands for the monotonicity constraint of the dealer, when the dealer buys a low-risk asset.

By using Lemma IA.1, the pledgeable income can be written as

$$\mathcal{P}_b(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(R_b^{LU} + R_b^{LD}\right).$$

First, we argue that at the optimum, we must have $R_b^{LD} = x_b^{LD}$. Suppose this were not the case. Then, since the objective is increasing in R_b^{LD}, it must be that a constraint is binding. The only constraint that could bind is $R^{LD} \leq R_b^{LU}$ from $(\mathrm{MCF}_{b,l})$. But this implies that for all $i \in \{LD, LU, HD, HU\}$, we have $R_b^i < x_b^i$, due to the monotonicity of cash flows and $(\mathrm{MCD}_{b,l})$. This means that we could increase all R^i by the same small amount, and all constraints would still be satisfied. Therefore, we cannot be at an optimum, and we have reached a contradiction.

Now consider R_b^{LU}. It is optimal to set R_b^{LU} as high as possible. The two possible constraints that can bind here are either $R_b^{LU} = x_b^{LU}$, from (LL), or $R_b^{LU} = R_b^{HD} (< x_b^{LU})$, from $(\mathrm{MCF}_{b,l})$.

Consider the first the case in which $R_b^{LU} = x_b^{LU}$. In this case, using the fact that (IC') binds gives us

$$\frac{1}{2}\left(R_b^{HU} + R_b^{HD}\right) = q\left(V - \frac{c}{\delta}\right). \tag{IA.5}$$

The optimal values of R_b^{HU} and R_b^{HD} must furthermore satisfy (MCD$_{b,l}$) and (LL). Also, we require that $R_b^{HD} \geq R_b^{LU} = x_b^{LU}$, from (MCF$_{b,l}$). Conjecture that we set $R_b^{HD} = q\left(V - \frac{c}{\delta}\right)$ to relax this last constraint as much as possible, given $R_b^{HD} \leq R_b^{HU}$, from (MCF$_{b,l}$) and Equation (IA.5). Note that $R_b^{HD} \leq x_b^{HD}$ from (LL), as long as $z \leq k - \frac{c}{\delta}$, so that the proposed repayments maximize pledgeable income subject to all the constraints.

Now consider the case in which $R_b^{LU} = R_b^{HD} (< x_b^{LU})$. First, in this case, we can use the fact that (IC') binds to give us $R_b^{HU} = q\left(V + (k-z) - \frac{2c}{\delta}\right)$. Second, from the expression for pledgeable income, we can see that it is now optimal to set R_b^{HD} as high as possible. This would imply that $R_b^{HD} = R_b^{HU}$, from (MCF$_{b,l}$), would bind, so that $R_b^{LU} = R_b^{HD} = R_b^{HU} = q\left(V + (k-z) - \frac{2c}{\delta}\right)$. It remains to check that the constraint $R_b^{LU} \leq x_b^{LU} = q(V + z - k)$ is satisfied. It can be seen that this holds, as long as $z \geq k - \frac{c}{\delta}$.

To summarize, we have shown the following: If $z \leq k - \frac{c}{\delta}$, the repayments

$$\{R_b^{LD}, R_b^{LU}, R_b^{HD}, R_b^{HU}\} = \left\{q(V - z - k), q(V + z - k), q\left(V - \frac{c}{\delta}\right), q\left(V - \frac{c}{\delta}\right)\right\}$$

describe the optimal contract. The pledgeable income is $\mathcal{P}_b(q) = q\left(V - \frac{c}{\delta}\right)$ and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V - \frac{c}{q}\right)$.

If $z \geq k - \frac{c}{\delta}$, the repayments

$$\{R_b^{LD}, R_b^{LU}, R_b^{HD}, R_b^{HU}\} = \\ \left\{q(V - z - k), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + k - z - \frac{2c}{\delta}\right)\right\}$$

describe the optimal contract. The pledgeable income is $\mathcal{P}_b(q) = q\left(V + k - z - \frac{2c}{\delta}\right)$, and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V + k - z - \frac{2c}{q}\right)$.

Dealer buys; high-risk asset, $z > \frac{1}{2}k$**.** In this case, Assumption IA.1 implies the following constraints:

$$\begin{array}{ccccccccc} R_b^{HU} \geq & & R_b^{LU} \geq & & R_b^{HD} \geq & & R_b^{LD} \geq & 0, & (\mathrm{MCF}_{b,h}) \\ x_b^{HU} - R_b^{HU} \geq & & x_b^{LU} - R_b^{LU} \geq & & x_b^{HD} - R_b^{HD} \geq & & x_b^{LD} - R_b^{LD} \geq & 0. & (\mathrm{MCD}_{b,h}) \end{array}$$

Again, using the fact that (IC') binds from Lemma IA.1, we can write pledgeable income as

$$\mathcal{P}_b(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(R_b^{LU} + R_b^{LD}\right).$$

A similar argument to the one above shows that it is optimal to set $R_b^{LD} = x_b^{LD}$.

Now consider R_b^{LU}. It is optimal to set R_b^{LU} as high as possible. The three possible constraints that can bind here are either $R_b^{LU} = x_b^{LU}$, from (LL), or $R_b^{LU} = R_b^{HU}$, $R_b^{LU} = R_b^{HU} < x_b^{LU}$, from $(\mathrm{MCF}_{b,h})$, or $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$, from $(\mathrm{MCD}_{b,h})$.

Suppose first that $R_b^{LU} = x_b^{LU} = q(V + z - k)$. From $(\mathrm{MCD}_{b,h})$, it then follows that it must be the case that $R_b^{HD} = x_b^{HD}$. Finally, from the fact that (IC') binds, we can see that $R_b^{HU} = q\left(V + z - \frac{2c}{\delta}\right)$. However, for this candidate R_b^{HU}, we have that $R_b^{HU} < R_b^{LU}$, due to Assumption IA.2, so that $(\mathrm{MCF}_{b,h})$ is not satisfied. We have reached a contradiction. It follows that $R_b^{LU} < x_b^{LU}$.

Suppose next, therefore, that $R_b^{LU} = R_b^{HU} < x_b^{LU}$. From the fact that (IC') binds, we can then deduce that $R_b^{HD} = q\left(V + k - z - \frac{2c}{\delta}\right)$. (We can see that $R_b^{HD} < x_b^{HD}$, due to Assumption IA.2.) It is optimal to set $R_b^{LU}(= R_b^{HU})$ as high as possible; the constraint that will bind is $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$, from $(\mathrm{MCD}_{b,h})$. Therefore, $R_b^{LU} = R_b^{HU} = q\left(V + z - \frac{2c}{\delta}\right)$.

Finally, start by supposing $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$. In the previous subcase, we already worked out one solution that satisfied this condition. It remains to show that no other solution exists. Substitute the condition $R_b^{LU} = x_b^{LU} - x_b^{HD} + R_b^{HD}$ into the binding (IC') to obtain

$$\begin{aligned}\frac{1}{2}R_b^{HU} &= q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}\left(x_b^{LU} - x_b^{HD} + x_b^{LD}\right) \\ \Leftrightarrow R_b^{HU} &= q\left(V + z - \frac{2c}{\delta}\right)\end{aligned}$$

It is optimal to set R_b^{LU} as high as possible. Since the candidate R_b^{HU} satisfied $R_b^{LU} < x_b^{LU}$, we note that the binding constraint on R_b^{LU} must be $R_b^{LU} = R_b^{HU}$ from (MCF$_{b,h}$). We have established that there is only one solution, as worked out in the previous paragraph.

Therefore, for $z > \frac{1}{2}k$, the repayments

$$\{R_b^{LD}, R_b^{HD}, R_b^{LU}, R_b^{HU}\} = \left\{q(V - z - k), q\left(V + k - z - \frac{2c}{\delta}\right), q\left(V + z - \frac{2c}{\delta}\right), q\left(V + z - \frac{2c}{\delta}\right)\right\}$$

describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_b(q) = q\left(V + \frac{1}{2}k - \frac{2c}{\delta}\right)$, and the minimum bid for which effort can still be exerted is $b_{IC'}(q) = \frac{w}{q} + \left(V + \frac{1}{2}k - \frac{2c}{\delta}\right)$.

Dealer sells; low-risk asset, $z \leq \dfrac{1}{2}k$. We note that here, it is necessarily the case that $R_a^{LU} = 0$ by (LL), because $x_a^{LU} = 0$. (This is also the case in the high-risk asset case when the dealer sells.) By using Lemma IA.1, this implies that the pledgeable income can be written as

$$\mathcal{P}_a(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}R_a^{LD}.$$

Here, Assumption IA.1 implies the following constraints:

$$R_a^{HD} \geq \quad R_a^{HU} \geq \quad R_a^{LD} \geq \quad R_a^{LU} = 0, \qquad (\text{MCF}_{a,l})$$

$$x_a^{HD} - R_a^{HD} \geq \quad x_a^{HU} - R_a^{HU} \geq \quad x_a^{LD} - R_a^{LD} \geq \quad x_a^{LU} - R_a^{LU} = 0. \qquad (\text{MCD}_{a,l})$$

Increasing R_a^{LD} increases $\mathcal{P}_a(q)$ and relaxes (MCD$_{a,l}$) and (IC'). Hence the optimal R_a^{LD} will satisfy either $R_a^{LU} = x_a^{LU}$ due to (LL) or $R_a^{HU} = R_a^{LD}$ due to (MCF$_{a,l}$). We consider these two subcases below in turn.

Suppose first that the optimal $R_a^{LU} = x_a^{LU}$. It then remains to pin down R_a^{HD} and R_a^{HU}. Consider the choice of R_a^{HU}. It has to satisfy $R_a^{HU} \geq R_a^{LD} = x_a^{LD}$, from (MCF$_{a,l}$), and $R_a^{HU} = 2q(k+z-\frac{c}{\delta})-R_a^{HD}$, from (IC'). We conjecture and later verify that (LL) is slack for R_a^{HU}. The two constraints can be combined to state that $2q(k+z-\frac{c}{\delta})-R_a^{HD} \geq x_a^{LD}$. This combined constraint is most relaxed when R_a^{HD} is as small as possible, which would imply a choice of $R_a^{HD} = R_a^{HU}$, given that (MCF$_{a,l}$) has to be satisfied. Using the fact that (IC') binds, we then have $R_a^{HU} = R_a^{HD} = q(k+z-\frac{c}{\delta})$. The constraint $R_a^{HU} \geq x_a^{LD}$, from (MCF$_{a,l}$), then becomes $z \leq k - \frac{c}{\delta}$. By Assumption IA.2, $k - \frac{c}{\delta} < \frac{1}{2}k$, so that $z \leq k - \frac{c}{\delta}$ also implies $z < \frac{1}{2}$.)

Finally, it can easily be verified that R_a^{HU} satisfies (LL) as $R_a^{HU} = q(k+z-\frac{c}{\delta}) < x_a^{HU} = qk$. To sum up, for $z \leq \min\{\frac{1}{2}k, k-\frac{c}{\delta}\} = k-\frac{c}{\delta}$ (by Assumption IA.2), the repayments $\{R_a^{LU}, R_a^{LD}, R_a^{HU}, R_a^{HD}\} = \{0, qk, q(k+z-\frac{c}{\delta}), q(k+z-\frac{c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = q(k+z-\frac{c}{\delta})$, and the minimum ask for which effort can still be exerted is $a_{IC'}(q) = V + \frac{c}{\delta} - \frac{w}{q}$.

Next, suppose $R_a^{HU} = R_a^{LD} < x_a^{LD}$. The binding (IC') then implies that $R_a^{HD} = 2q(k-\frac{c}{\delta})$. The pledgeable income becomes $\mathcal{P}_a(q) = q(k-\frac{c}{\delta}) + \frac{1}{2}R_a^{HU}$ and is increasing in R_a^{HU}. Hence, it is optimal to set R_a^{HU} as high as possible until (MCF) binds; that is, $R_a^{HU} = R_a^{HD} = 2q(k-\frac{c}{\delta})$. The assumption that $R_a^{HU} = R_a^{LD} < x_a^{LD}$ is satisfied if and only if $z > k - \frac{c}{\delta}$. Thus, for $z \in (k-\frac{c}{\delta}, \frac{1}{2}k]$, the repayments $\{R_a^{LU}, R_a^{LD}, R_a^{HU}, R_a^{HD}\} = \{0, 2q(k-\frac{c}{\delta}), 2q(k-\frac{c}{\delta}), 2q(k-\frac{c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = 2q(k-\frac{c}{\delta})$, and the minimum ask for which effort can still be exerted is $a(q) = V - k + z + \frac{2c}{\delta} - \frac{w}{q}$.

Dealer sells; high-risk asset, $z > \dfrac{1}{2}k$**.** Again, it is necessarily the case that $R_a^{LU} = 0$, from (LL), because $x_a^{LU} = 0$, and by using Lemma IA.1, the pledgeable income can be written as

$$\mathcal{P}_a(q) = q\left(k - \frac{c}{\delta}\right) + \frac{1}{2}R_a^{LD}.$$

Here, Assumption IA.1 implies the following constraints:

$$R_a^{HD} \geq \quad R_a^{LD} \geq \quad R_a^{HU} \geq \quad R_a^{LU} = 0, \qquad (\mathrm{MCF}_{a,h})$$

$$x_a^{HD} - R_a^{HD} \geq \quad x_a^{LD} - R_a^{LD} \geq \quad x_a^{HU} - R_a^{HU} \geq \quad x_a^{LU} - R_a^{HU} = 0. \qquad (\mathrm{MCD}_{a,h})$$

As before, setting a higher R_a^{LD} increases $\mathcal{P}_a(q)$ and relaxes (IC'). Hence, the optimal R_a^{LD} is either $R_a^{LD} = R_a^{HD}$, so that the relevant part of ($\mathrm{MCF}_{a,h}$) binds, or $R_a^{LD} = x_a^{LD} - x_a^{HU} + R_a^{HU}$, so that the relevant part of ($\mathrm{MCD}_{a,h}$) binds, or both.

Suppose first that $R_a^{LD} < R_a^{HD}$, so that the relevant part of ($\mathrm{MCF}_{a,h}$) would not bind. Then it would have to be the case that $R_a^{LD} = x_A^{LD} - x_a^{HU} + R_a^{HU}$, so that ($\mathrm{MCD}_{a,h}$) would bind. Together with the fact that (IC') has to bind, this would imply that $R_a^{LD} - R_a^{HU} = 2q\left(z - \frac{1}{2}k\right)$. Since (IC') binds, this means that $R_a^{HD} = q\left(2z + k - \frac{c}{\delta}\right)$. But R_a^{LD} is not yet pinned down. The maximization of $\mathcal{P}_a$ requires that it is raised as far as possible given applicable constraints, which implies that it would have to be the case that $R_a^{LD} = R_a^{HD}$.

We now know that $R_a^{LD} = R_a^{HD}$. Then the fact that (IC') binds implies $R_a^{HU} = 2q\left(k - \frac{c}{\delta}\right)$. We can see that the constraint $R_a^{HU} \leq x_a^{HU}$, from (LL), is guaranteed to be satisfied because of Assumption IA.2. Next, it is optimal to increase R_a^{LD} (and hence also R_a^{HD}), but $R_a^{HU} = 2q\left(k - \frac{c}{\delta}\right)$ is fixed. This implies that the $x_a^{LD} - R_a^{LD} = x_a^{HU} - R_a^{HU}$, from ($\mathrm{MCD}_{a,h}$), must bind. In turn, this implies $R_a^{LD} = q\left(2z + k - \frac{2c}{\delta}\right)$.

Thus, for $z > \frac{1}{2}k$, the repayments $\{R_a^{LU}, R_a^{HU}, R_a^{LD}, R_a^{HD}\} = \{0, 2q(k - \frac{c}{\delta}), q(2z + k - \frac{2c}{\delta}), q(2z + k - \frac{2c}{\delta})\}$ describe the optimal contract. The corresponding pledgeable income is $\mathcal{P}_a(q) = q\left(\frac{3}{2}k - \frac{2c}{\delta} + z\right)$, and the minimum ask for which effort can still be exerted is $a_{IC'}(q) = V - \frac{k}{2} + \frac{2c}{\delta} - \frac{w}{q}$.

$\square$