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Abstract

We study the two-way interaction between central banks and financial markets using a beauty contest framework. The analysis identifies when asset prices reveal useful information about fundamentals and when they reflect back the central bank's pronouncements. In equilibrium, the central bank is overly dependent on financial market signals and the information value of asset prices is diminished. Our results highlight the need to guard against giving undue prominence to market signals during monetary policy deliberations, but they can be specific to the mathematical model employed in the paper.

Keywords: Reflection problem, beauty contests, market expectations, central bank independence
JEL classification: E58, E44, D82

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How aloof should central banks be from financial markets? In his influential work on central banking, Blinder (1998, 2004) observes that while considerable progress has been made on the issue of central bank independence from political influence, scant attention has been paid to central bank independence from financial markets. He highlights the risk that monetary policy may become privatised – central banks may be enraptured by financial markets to such an extent that they dutifully deliver on the policy implied by the signals from asset prices.[4] But when central banks follow market forecasts which are, in turn, based on the central bank's own assessments of the economic outlook, the potential circularity can result in excessively volatile monetary policy. In recent work, Morris & Shin (2018) dub this two-way flow between market prices and monetary policy the "reflection problem" and highlight its relevance for the debate on central bank forward guidance.

In this paper, we formalise Blinder's concern that modern central banks may have become too market-friendly and analyse the consequences for the signal value of financial market prices. In our model, the central bank optimally displays excess dependency on financial market signals, beyond the level that a social planner concerned with ensuring that financial prices closely match economic fundamentals would exhibit. But the reliance on financial market signals is self-defeating. In trying to match the central bank's actions, market participants over-weight public information and underweight their own private information about economic fundamentals. As a result of this exaggerated "beauty contest" effect, the information value of financial prices in equilibrium is diminished.

In formulating its monetary policy rule, the central bank does not place any weight on its public forecast and downplays its own confidential intelligence on the economic outlook in equilibrium. The information embodied in the public forecast is captured in market prices and so the central bank does not draw on this directly in deriving its own forecast of economic fundamentals. But since the central bank's objective requires it to set its action equal to a target, rather than ensuring that the market economy properly aggregates diverse information, the central bank underweights its own private information relative to the socially desired weight.

Our model identifies the circumstances under which financial market prices reveal useful information about fundamentals or merely reflect back the central bank's own assessments. When the central bank's dependency on the market-based signal is relatively low, monetary policy is based on the market signal, as well as the public forecast and confidential information of the central bank. Since the public forecast is used in the monetary policy rule, market participants place greater emphasis on it and downplay their own private signals about fundamentals. This distortionary effect towards the public signal outweighs the consequences of the beauty contest effect. As a result, there is no reflection problem. The

[4] "...Central bankers are only human; they want to earn high marks – from whomever is handing out the grades. . . the markets provide a giant biofeedback machine that monitors and publicly evaluates the central bank's performance in real time. So central bankers naturally turn to the markets for instant evaluation..." (Blinder, 1998, p.36).

central bank can therefore induce greater informativeness from market prices by decreasing its emphasis on public information in the policy rule. There is, thus, a critical threshold of financial market dependency at which the informational content of financial prices is maximised, and this is associated with a monetary policy rule that only uses the market-based signal and the central bank's private signal.

Beyond the critical threshold, a reflection problem is induced as the central bank increases its reliance on the market signal. Since the monetary policy rule no longer hinges on the public forecast, there is only a beauty contest effect. So, when the central bank relies more on the market signal, the attempts by market participants to match the central bank's actions leads them to place greater weight on the public forecast, diminishing the signal value of financial market prices.

From a normative perspective, our theoretical model suggests that society could benefit from safeguards which ensure that financial markets do not play an out-sized role in policy deliberations. This may be achieved by requiring central banks to pre-commit to norms and an institutional culture that discourages policymakers from becoming too close to market participants.[5] Alternatively, much like Rogoff's (1985) argument for a conservative central banker to mitigate a time inconsistency problem, it may be welfare-improving to appoint a market-insensitive central banker to counteract the effects of the beauty contest.

1. Related Literature

Our paper builds on the recent work of Morris & Shin (2018). Using a beauty contest framework, they show how a reflection problem can bedevil a central bank attempting to engage in forward guidance.[6] But in their model, the monetary policy rule of the central bank is based only on a market signal and central bank private information. By contrast, we allow for a generalised monetary policy rule – that depends on the market signal as well as "in-house" analysis – balancing both the central bank's public forecast and information confidential to the policymaker. The choice of a zero weight on public information, exogenously assumed by Morris & Shin (2018), emerges endogenously in the equilibrium of our model.[7] The generalised monetary policy rule also allows us to make clear the conditions under which the reflection problem operates, and we provide a normative benchmark against which to judge the central banker's optimally preferred weight on the financial market signal.

[5] For example, the Bank of England has recently established a Market Intelligence Charter outlining the standards it expects from its staff when liaising with market participants (Jeffery et al., 2017). Cieślak et al. (2018) provide evidence of financial institutions receiving systematic preferential access to the Federal Reserve.

[6] There is a large literature studying beauty contests and the social value of public information. A non-exhaustive list includes Morris & Shin (2002), Angeletos & Pavan (2007), Cornand & Heinemann (2008), James & Lawler (2011), Chahrour (2014), Colombo et al. (2014), Myatt & Wallace (2014), and Ui & Yoshizawa (2015).

[7] Public information in Morris & Shin (2018) is semi-public in the sense that information is known to the market but not the central bank (see also Morris & Shin (2007)). In our model, by contrast, public information is known to market participants and the central bank.

Our paper contributes to the literature on the two-way interaction between policymakers and financial markets. Söderlind & Svensson (1997) provide an early discussion of the reflection problem, cautioning central banks against reacting mechanically to measures of market expectations in monetary policy. Bernanke & Woodford (1997) show formally that if the central bank tries to implement a policy that is based on market participants' inflation forecasts, market signals may be uninformative about the underlying state and there may be non-existence of equilibrium. As in our model, they emphasise the importance of the central bank relying on its own analysis rather than depending too heavily on market signals.[8] A more general treatment of the reflection problem is provided by Bond et al. (2010). In their equilibrium characterisation, the ability of the policymaker to extract information from the market depends on the quality of information. When the policymaker's information is precise, informative market signals allow implementation of the preferred policy. But when information is less precise, additional equilibria can exist in which the policymaker intervenes too much or too little.[9]

The normative implications of our model share common ground with Stein & Sunderam (2018) who present a model where the central bank is averse to bond market volatility and has private information. The desire to curb bond market volatility leads the policymaker to pursue a gradualist monetary policy, the effects of which are undone when long-term rates react more than one-for-one to the change in short rates. In their model, a time-inconsistency issue arises because the central bank cannot commit to not smooth the private information it communicates to the market via its changes to the policy rate. Stein and Sunderam suggest that the appointment of a central banker who is less averse to bond-market volatility may ameliorate this issue. Although the results in our model also point to the benefits of appointing a market-insensitive central banker, they are cast in terms of a beauty contest mechanism rather than a commitment problem.

2. Model

A central bank, as a Stackelberg leader, faces a continuum of financial market participants indexed on a unit interval $[0, 1]$ endowed with the Lebesgue measure. Each market participant chooses an action $a_i \in \mathbb{R}$, so that $\bar{a} = \int_{[0,1]} a_i \mathrm{d}i$ is the average action in the population. Economic fundamentals are given by the state θ, which is drawn from an (improper) uniform distribution over the real line.[10]

[8] Goldstein et al. (2011) and Bond & Goldstein (2015) show how the reflection problem emerges in the context of currency crises and financial bailouts. See also Baeriswyl et al. (2018) for a recent application to monetary policy in the context of a micro-founded model of the macroeconomy.

[9] Nimark (2008) presents a general equilibrium model of monetary policy in which the central bank uses information in the term structure to gauge economic fundamentals. But his model bypasses the pitfalls of an "expectation targeting" regime since the information in the term structure is connected to the underlying structural model.

[10] The assumption of an improper uniform prior allows for an algebraically tractable solution. Morris & Shin (2018) show that the improper prior corresponds to the limit of a proper normal prior as the variance tends to

The central bank bases its monetary policy rule on three elements: a market-based signal, information about fundamentals in the public domain, and information about the economy that is private to the policymaker. The market-based signal arises from the average action of market participants, $\bar{a}$, while the central bank learns about the state of fundamentals from a public signal

$$y = \theta + \eta, \tag{1}$$

and a private signal

$$z = \theta + \nu. \tag{2}$$

The noise terms η and ν are normally distributed with mean 0, and are independent of each other and θ. We denote the precision of the public and private signals by α and γ respectively.

The public signal y is common knowledge among all agents. It can be interpreted as the economic forecast and *Inflation Report* typically required of an inflation targeting central bank (Bernanke, 2011). But y can also be viewed as the publication of economic statistics, such as GDP, earnings, and trade data, or as a fashionable narrative that takes hold among all agents in the economy (Morris & Shin, 2018).

The private signal, z, reflects confidential intelligence from bank supervision activity and international policy networks that gives the central bank a temporary lead over market participants in recognizing developments in the economy (Romer & Romer, 2000; Goodfriend, 2008). Such intelligence is available only on a "need to know" basis within the central bank, with many of the staff responsible for generating the publicly available forecasts being unaware of this information (Peek et al., 1999).[11] As a result, y and z can be viewed as being independent of each other.[12]

Let $\lambda \in (0, 1)$ denote the weight placed by the central bank on the market-based signal in its monetary policy rule. The weight on its own analysis, which comprises the public forecast y and the confidential signal z, is therefore $(1 - \lambda)$. Suppose further that, in balancing between public and confidential analysis, the central bank places weight μ on z. We restrict ourselves to the case $\mu \in [0, 1]$. This assumption seems plausible. Values of $\mu < 0$ would correspond to the central bank placing a negative weight on the private information it receives about developments in the financial sector and the macroeconomy, which seems unrealistic. Likewise, values of $\mu > 1$ imply that the central bank places negative weight

infinity.

[11] Peek et al. (1999) also provide evidence suggesting that confidential bank supervisory information improves forecasts of inflation and unemployment in the US, and that FOMC members draw on such private information when voting on monetary policy.

[12] In our model we assume that the central bank does not engage in intentional leaks of information. See, for example, the transcript of the 15 October 2010 FOMC conference call, in which Chairman Bernanke expresses concern about leaks to market participants. See https://www.federalreserve.gov/monetarypolicy/files/FOMC20101015confcall.pdf. The issue of strategic leaks is beyond the scope of our paper.

on the information that it chooses to make public. The monetary policy rule is thus

$$r = \lambda \bar{a} + (1-\lambda)\big[(1-\mu)y + \mu z\big]. \tag{3}$$

In what follows, we initially treat λ as exogenous, returning later to endogenise it in Section 5. The central bank therefore determines the extent to which its private information shapes the interest rate decision. It chooses μ to minimise the quadratic loss in the distance between the underlying fundamentals θ and its action, r, i.e.,

$$\begin{aligned} L_{CB} &= (r-\theta)^2 \\ &= \big\{\lambda \bar{a} + (1-\lambda)\big[(1-\mu)y + \mu z\big] - \theta\big\}^2. \end{aligned} \tag{4}$$

In keeping with Blinder (1998, 2006), we interpret λ as the degree of *dependence* of the central bank on financial markets. Central banks vary in their reliance on financial market information. Blinder (2006) distinguishes between two stereotypes: the *old-fashioned* central banker who largely disregards financial markets and the *new-fangled* central banker who, by contrast, is deeply respectful of financial markets, routinely using asset prices to "read" what the market expects and is loath to depart from that expectation. Accordingly, the higher is λ, the more reliant is the central bank on the market-based signal.

Market participant i selects an action, a_i, to match both the central bank's action as well as fundamentals. Each market participant maximises

$$u_i = -\omega(a_i - r)^2 - (1-\omega)(a_i - \theta)^2, \tag{5}$$

where $\omega \in (0,1)$ is the weight placed on matching the central bank's action, and $(1-\omega)$ is the weight on matching fundamentals.[13] The decision is based on the public signal y, and a signal that is private to market participant i,

$$x_i = \theta + \varepsilon_i, \tag{6}$$

where ε_i is an idiosyncratic noise term that is normally distributed with mean 0 and precision β. The noise terms (ε_i) are i.i.d. and also independent of θ, η and ν. Following the realisation of y and x_i, market participant i's conditional expectation of θ is

$$\mathbb{E}_i[\theta] = \frac{\alpha y + \beta x_i}{\alpha + \beta}. \tag{7}$$

We proceed by backward induction, solving the Stackelberg game in which the central bank first commits to its monetary policy rule by choosing μ, and market participants follow by choosing their actions, a_i.

[13] Under complete information, the central bank chooses $r = \theta$, and market participants choose $a_i = \theta$.

3. Equilibrium

The optimal action for each market participant, i, is obtained by maximizing the utility function (5). The first-order condition is

$$a_i = \omega \mathbb{E}_i[r] + (1-\omega)\mathbb{E}_i[\theta]. \tag{8}$$

Since the monetary policy rule, r, depends on the market-based signal, $\bar{a}$, market participants' actions exhibit strategic complementarity, i.e., the optimal choice a_i increases in the average action $\bar{a}$. Substituting (3) and (7) into (8) yields

$$a_i = \omega\lambda \mathbb{E}_i[\bar{a}] + (1-\omega+\omega(1-\lambda)\mu)\mathbb{E}_i[\theta] + \omega(1-\lambda)(1-\mu)y. \tag{9}$$

From (9), it is clear the optimal action is linear in expectations of $\bar{a}$ and θ, as well as the public signal y. Moreover, since the expectation of θ is also linear in the signals x_i and y, we can conjecture that the equilibrium action of any market participant is of the form

$$a_i = \xi x_i + (1-\xi)y, \tag{10}$$

where $\xi \in (0,1)$ reflects the *information value* of market participants' actions. Higher values of ξ imply that market participants place more weight on their own "window on the world", so that the market indicator $\bar{a}$ provides the central bank with a better gauge of economic conditions to come. The average action thus becomes

$$\bar{a} = \xi\theta + (1-\xi)y, \tag{11}$$

and substituting (7) and (11) into (9) yields

$$a_i = \frac{\beta - \beta\omega\big[1-\lambda\xi-(1-\lambda)\mu\big]}{\alpha+\beta}x_i + \frac{\alpha + \beta\omega\big[1-\lambda\xi-(1-\lambda)\mu\big]}{\alpha+\beta}y. \tag{12}$$

Matching coefficients between (10) and (12) gives the best response choice of ξ by the market participant to μ, the emphasis placed by the central bank on its own private information:[14]

$$\xi = \frac{\beta\big[1-\omega+\omega(1-\lambda)\mu\big]}{\alpha+\beta(1-\lambda\omega)}. \tag{13}$$

A Stackelberg equilibrium obtains when the central bank commits to an optimal choice of μ^*, taking the best response (13) as given. The ex ante expected loss of the central bank is

$$\begin{aligned}\mathbb{E}[L_{CB}] &= \mathbb{E}[(r-\theta)^2] \\ &= \big[\lambda(1-\xi)+(1-\lambda)(1-\mu)\big]^2\frac{1}{\alpha} + (1-\lambda)^2\mu^2\frac{1}{\gamma},\end{aligned} \tag{14}$$

and μ^* is given by the first-order condition, $\mathrm{d}\mathbb{E}[L_{CB}]/\mathrm{d}\mu = 0$.

[14] This best response can also be established by considering higher-order expectations of θ (see Morris & Shin (2002) and Angeletos & Pavan (2007)).

Proposition 1. There exists a critical threshold $\widehat{\lambda} = \widehat{\lambda}(\alpha, \beta, \gamma, \omega) \in (0, 1)$ such that:
(i) when $\lambda \geq \widehat{\lambda}$, the equilibrium weight placed by the central bank on its private information corresponds to a boundary optimum, $\mu^* = 1$.
(ii) when $\lambda < \widehat{\lambda}$, the equilibrium weight is

$$\mu^* = \frac{\gamma(\alpha+\beta)[\alpha+\beta(1-\lambda)]}{(1-\lambda)\left\{\gamma(\alpha+\beta)^2+\alpha[\alpha+\beta(1-\lambda\omega)]^2\right\}} \in (0,1). \tag{15}$$

Proof. See Appendix. □

Proposition 1 establishes the conditions under which the central bank bases its monetary policy on the public forecast, y, and/or its private information, z. If the central bank's dependence on the market-based signal equals or exceeds the threshold $\widehat{\lambda}$ then, in equilibrium, it relies exclusively on its private information. For values of λ smaller than $\widehat{\lambda}$, the central bank is less dependent on financial markets for guidance, and it is optimal for it to base monetary policy on both public and private signals.

Corollary 1. The optimal weight on central bank private information, $\mu^* \in (0, 1)$, decreases with the precision of the public signal, α. It increases with the precision of the central bank's private signal, γ, and the weight on the market signal, λ.

Proof. See Appendix. □

Intuitively, as the public signal becomes more precise, the central bank emphasises it more and increases its weight, $1 - \mu^*$, to better match fundamentals. Similarly, the central bank places greater weight on its private information as its quality improves. We clarify why μ^* increases with the degree of financial market dependence in Section 4.

Corollary 2. The critical threshold, $\widehat{\lambda}$, increases with the precision of the public signal, α, and decreases with the precision of central bank's private signal, γ.

Proof. See Appendix. □

A more precise public signal induces the central bank to place less weight on its private signal, z. There is, thus, a smaller range of values of λ over which the central bank exclusively relies on its confidential information. The converse occurs as the precision of central bank private information, γ, increases.

Given the central bank's choice of μ^*, the information value of market participants' actions in equilibrium when $\lambda \geq \widehat{\lambda}$ is:[15]

$$\xi^* = \frac{\beta(1-\lambda\omega)}{\alpha+\beta(1-\lambda\omega)}. \tag{16}$$

Note that $\xi^* \to 0$ as $\beta \to 0$ or $\alpha \to \infty$. When the private signals of market participants are completely imprecise or, conversely, when the public signal is extremely precise, market

[15] Note that ξ^* is continuous but not differentiable with respect to λ at $\widehat{\lambda}$.

participants ignore their private information in equilibrium. Similarly, $\xi^* \to 1$ as $\beta \to \infty$ or $\alpha \to 0$, i.e., if market participants' information is extremely precise or public information is completely garbled, then the public signal y is ignored by financial markets.

For the case $\lambda < \widehat{\lambda}$, the information value of market participants' actions in equilibrium is

$$\xi^* = \frac{(\alpha+\beta)\beta\gamma + \alpha\beta(1-\omega)[\alpha+\beta(1-\lambda\omega)]}{(\alpha+\beta)^2\gamma + \alpha[\alpha+\beta(1-\lambda\omega)]^2}. \tag{17}$$

Again, $\xi^* \to 0$ as $\beta \to 0$ or $\alpha \to \infty$. Note, however, that as $\beta \to \infty$,

$$\xi^* \to \frac{\gamma + \alpha(1-\omega)(1-\lambda\omega)}{\gamma + \alpha(1-\lambda\omega)^2} \in (0,1),$$

which corresponds to a 'distortion' towards y. Since the monetary policy rule depends on the public signal, y, when $\lambda < \widehat{\lambda}$, the attempts by market participants to match the central bank's action induces them to place extra weight on y and de-emphasise their own information. To see this more clearly, suppose that $\lambda = 0$ and each market participant plays a single-person game of matching both the central bank's action and the fundamentals. It follows that, when $\beta \to \infty$, market participants place a positive weight (and hence a distortion) on the public signal, as indicated by

$$1 - \xi^* \to 1 - \frac{\gamma + \alpha(1-\omega)}{\gamma+\alpha} = \frac{\alpha\omega}{\gamma+\alpha} > 0.$$

However, if market participants only cared about matching fundamentals, i.e., $\omega = 0$, $\xi^* \to 1$ as $\beta \to \infty$, implying no distortion. Similarly, when $\alpha \to 0$ so that the public signal is fully noisy, market participants are not biased away from their private signal and hence $\xi^* \to 1$. Market participants do not place extra emphasis on public information in the case $\lambda \geq \widehat{\lambda}$ since the monetary policy rule does not depend on y.

4. Information value of financial market prices

Market prices reveal the collective wisdom of *all* agents in the economy by aggregating the diverse information they possess individually. A key insight of the beauty contest literature (Morris & Shin, 2002, 2018) is that the more the central bank tries to steer market expectations, the less likely it is that the market outcome will serve as an aggregator of the dispersed knowledge of market participants. In this section, we first establish when financial market prices reveal useful information about fundamentals, or merely reflect back the central bank's pronouncements. We then identify the maximum amount of information that market prices can reveal.

Definition 1. The central bank's monetary policy rule induces a *reflection problem* if the information value, ξ^*, is decreasing in λ, the weight placed by the central bank on the average market action, i.e., $\partial\xi^*/\partial\lambda < 0$.

Proposition 2. If $\lambda > \widehat{\lambda}$, then $\partial\xi^*/\partial\lambda < 0$ and there is a reflection problem. But if $\lambda < \widehat{\lambda}$, then $\partial\xi^*/\partial\lambda > 0$ and there is no reflection problem.

The attempts by market participants to match the central bank's action, together with the emphasis by the central bank on the market-based signal, $\bar{a}$, induce a 'beauty contest' in the monetary policy rule. As in Morris & Shin (2002), the public signal y plays a dual role – it updates information about fundamentals and serves as a focal point for market participants to match the average action. When $\lambda > \widehat{\lambda}$, a reflection problem arises from the increased attention paid by market participants to the public signal. So, when the central bank is more dependent on the market-based signal, market participants place greater weight on the public signal and the effect of the beauty contest is exaggerated.

To see this more clearly, we combine (12) and (16) to obtain

$$a_i = \underbrace{\left(1 - \frac{\alpha\lambda\omega}{\alpha + \beta(1-\lambda\omega)}\right)}_{=\,1-\kappa_1} \mathbb{E}_i[\theta] + \underbrace{\left(\frac{\alpha\lambda\omega}{\alpha + \beta(1-\lambda\omega)}\right)}_{=\,\kappa_1} y, \tag{18}$$

where κ_1 measures the extent to which the public signal, y, is over-weighted.[16] In the absence of a beauty contest, $\kappa_1 = 0$, and market participants care only about matching fundamentals. But in the presence of a beauty contest,

$$\frac{\partial \kappa_1}{\partial \lambda} = \frac{\alpha\omega(\alpha+\beta)}{[\alpha + \beta(1-\lambda\omega)]^2} > 0. \tag{19}$$

Thus, as the central bank depends more on the financial markets for guidance, market participants place greater emphasis on the public signal.

For the case $\lambda < \widehat{\lambda}$, the monetary policy rule depends both on the public signal y and the central bank's private signal z. As discussed in Section 3, the presence of y in the policy rule creates a distortion as market participants downplay their own information in favour of the public signal. This distortion, together with the beauty contest effect, leads market participants to over-weight the public signal y. When the central bank increases its dependence on the market-based signal there is a dual effect – the focal role of y is reduced since $(1-\mu^*)$ is lowered while, at the same time, the beauty contest effect is exaggerated. But since the former effect dominates there is no reflection problem.[17] Recall from Corollary 1, the weight placed by the central bank on the public signal, $1-\mu^*$, decreases with its degree of dependence on financial markets. When λ increases, an exaggerated beauty contest makes market participants place more weight on y and shifts the central bank's action, r, further away from fundamentals. To restore r as a better match, the central bank chooses to decrease its weight on y in the monetary policy rule and hence reduces the distortion.

[16] Let $\sigma = \beta/(\alpha+\beta)$. Then, in this case, $\xi^* = (1-\kappa_1)\sigma$.

[17] In a sense, the reflection problem arises in the case $\lambda > \widehat{\lambda}$ because the central bank cannot offset the exaggerated beauty contest effect by putting a negative weight on the public signal. If the central bank were able to do so, the negative weight on y would reverse the distortion towards y and hence correct the reflection problem. Morris & Shin (2018) also highlight this issue.

To see why there is no reflection problem, we combine (12) and (17) to give

$$a_i = \underbrace{\left\{1 - \frac{\alpha\omega[\alpha + \beta(1-\lambda)][\alpha + \beta(1-\lambda\omega)]}{(\alpha+\beta)^2\gamma + \alpha[\alpha + \beta(1-\lambda\omega)]^2}\right\}}_{=1-\kappa_0} \mathbb{E}_i[\theta] + \underbrace{\left\{\frac{\alpha\omega[\alpha + \beta(1-\lambda)][\alpha + \beta(1-\lambda\omega)]}{(\alpha+\beta)^2\gamma + \alpha[\alpha + \beta(1-\lambda\omega)]^2}\right\}}_{=\kappa_0} y. \quad (20)$$

Then substituting (7) into (20) yields

$$a_i = (1-\kappa_0)\frac{\alpha y + \beta x_i}{\alpha + \beta} + \kappa_0 y = \frac{\beta(1-\kappa_0)}{\alpha+\beta} x_i + \frac{\alpha + \beta\kappa_0}{\alpha+\beta} y. \quad (21)$$

Since $\partial\kappa_0/\partial\lambda < 0$, it follows that

$$\frac{\partial}{\partial\lambda}\left[\frac{\beta(1-\kappa_0)}{\alpha+\beta}\right] = -\left(\frac{\beta}{\alpha+\beta}\right)\frac{\partial\kappa_0}{\partial\lambda} > 0, \quad (22)$$

implying that, when the central bank increases its dependence on financial markets, market participants place more weight on their own private information in order that their actions better match fundamentals.

The results described above shed additional light on the information value of financial market prices. Unlike our model, the policy rule in Morris & Shin (2018) does not depend on y. As a result, a reflection problem operates because market participants overweight public information and private signals are not fully revealed. They suggest that if the central bank could condition on public information, it could correct for this bias. But our analysis shows that a policy rule that depends on y only partially corrects the bias – there is no reflection problem when $\lambda < \widehat{\lambda}$.

Proposition 3. The maximum informational content that market participants can reveal is $\widehat{\xi} = \xi^*(\widehat{\lambda})$.

Proposition 3 follows directly from Proposition 2. Figure 1 illustrates. To the left of $\widehat{\lambda}$, a higher degree of dependence by the central bank on financial market information is associated with a higher information value of financial market prices. But to the right of $\widehat{\lambda}$, the presence of the reflection problem means that the actions of market participants reveal more information when the degree of dependence is lower. Thus, the maximum information value occurs when $\lambda = \widehat{\lambda}$.

Corollary 3. The maximum information value of market participants' actions, $\widehat{\xi}$, decreases in α, the precision of public information, and increases with γ, the precision of the central bank's private information.

Proof. See Appendix. □

Figure 2 shows how $\widehat{\xi}$ shifts as α and γ change. An increase in α enhances the focal role of y and hence decreases the maximum information value. But the increase in α does

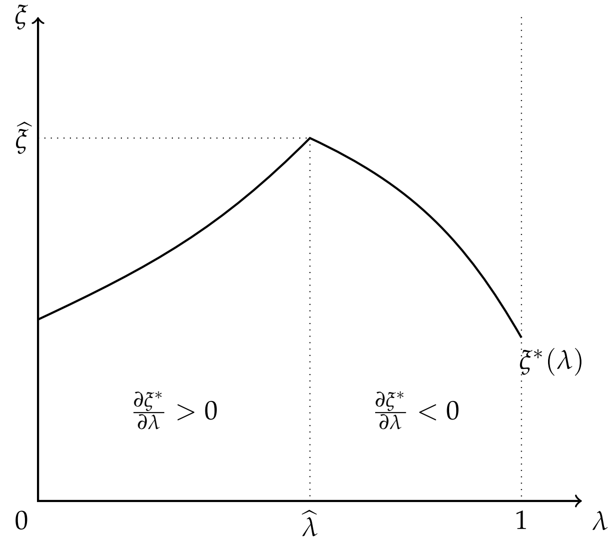

Figure 1: The solid curve shows how the maximum information value, ξ^*, changes with the central bank's degree of dependence on financial markets, λ, with other parameters given. At $\lambda = \widehat{\lambda}$, ξ^* reaches the maximum information value, $\widehat{\xi}$, that the central bank can learn from financial markets.

not correspond to a parallel, downward shift of the curve $\xi^*(\lambda)$ because it also implies an increase in the critical threshold, $\widehat{\lambda}$. An increase in γ makes the central bank place less emphasis on public information y, which, in turn, decreases the distortion towards it and increases the information value of financial market prices. Since equation (16) does not depend on γ, an increase in γ always moves $\widehat{\xi}$ along the curve $\xi^*(\lambda)$ to the left of $\widehat{\lambda}$ as $\widehat{\lambda}$ is increasing in γ.

5. Central bank dependence on financial markets

So far, we have treated the degree of dependence, λ, as an exogenous parameter. In a world where central bank actions work *through* markets and many of a policymaker's indicators *are* market prices, such an assumption would not be appropriate. We therefore consider a prior stage to the game described in Section 2 and contrast the optimal choice of λ by the central bank with an informationally efficient "Hayekian" benchmark where a planner chooses λ in order to ensure that financial market prices reveal the maximum information possible.[18]

[18] We borrow the term "Hayekian benchmark" from Hellwig & Venkateswaran (2014). Note that our informationally efficient benchmark differs substantially from Angeletos & Pavan (2007). In their analysis, the normative benchmark is the best that society can do under the constraint that information cannot be centralised or otherwise communicated between agents. Since the central bank acts as a "large" player in our setup, it is not clear how social welfare should be defined properly. Therefore, applying the benchmark of Angeletos and Pavan is beyond the scope of this paper.

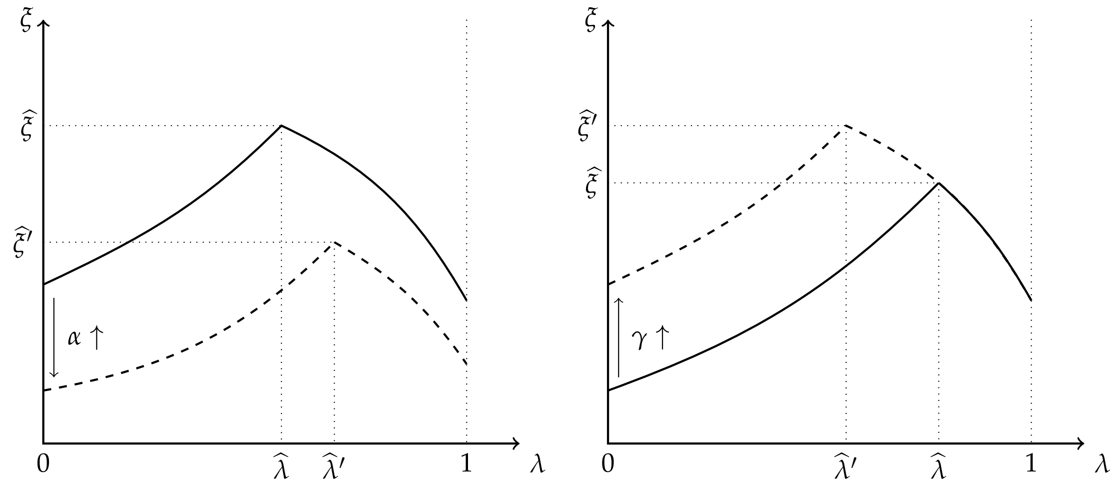

Figure 2: The maximum information value, $\widehat{\xi}$, decreases in α (left panel) and increases with γ (right panel).

Proposition 4. Given the central bank's choice of μ^* and market participants' choices of ξ^*, the optimal degree of dependence on financial markets chosen by the central bank exceeds the critical threshold, i.e., $\lambda^* > \widehat{\lambda}$.

Proof. See Appendix. □

The optimal extent of financial market dependency selected by the central bank implies that, in equilibrium, the central bank ignores the public signal, y, in its monetary policy rule. It only uses its confidential intelligence, i.e., $\mu^* = 1$. Note, however, that this does not mean that public information is entirely discarded by the central bank – it instead enters policy deliberations indirectly through the market-based signal.

Next consider a planner who seeks to ensure that the collective wisdom of market participants is as closely aligned to the fundamentals of the economy as possible, in the classic way suggested by Hayek (1945). As Blinder (2006) observes, financial markets tend to run in herds and adopt excessively short time horizons for investment decisions. So this benchmark, through minimizing the beauty contest effect, can also be thought of as reflecting a desire to avoid unnecessary financial market volatility that could impinge on economic stability.

From Proposition 3, we obtain our next proposition.

Proposition 5. The degree of dependence on financial markets in the Hayekian benchmark case is $\widehat{\lambda}$.

Proof. See Appendix. □

Propositions 4 and 5 make clear that the central bank optimally chooses a degree of financial dependence that exceeds the Hayekian level. Moreover, at λ^*, the information value of financial market prices is not maximised and the central bank places less weight on its own analysis, z, than is socially desirable.

The intuition for this result is as follows. For low values of λ, the monetary policy rule depends on both y and z. In this situation, market participants downplay their own information in favour of the public signal. So the central bank decreases its emphasis on the public signal (in favour of its private signal), to guide market participants away from public information and towards their private signals, ξ. At the threshold $\lambda = \widehat{\lambda}$, $\mu^* = 1$ – the central bank no longer uses y in its monetary policy rule and the informational value of market participants' actions is maximised. But since the central bank's objective differs from the Hayekian benchmark, it continues to under-weight its own information z, and increases its dependency on the market-based signal beyond $\widehat{\lambda}$, in order that its policy action, r, closely aligns with fundamentals, θ. For their part, market participants try to match the central bank's action and over-weight the public signal, y, thereby exaggerating the beauty contest effect and diminishing the information value of market prices in the process.

Our analysis thus suggests that a policymaker who wishes to ensure that asset prices are as closely aligned to fundamental values as possible would wish to ensure that the central bank commits to a set of norms and an institutional culture that is more market-insensitive and places more weight on valuable central bank private information.[19] This result is analogous to Rogoff (1985) who argues that society is better off employing a conservative central banker to eliminate the time inconsistency problem. It also echoes Stein & Sunderam (2018) who advocate the appointment of central bankers who are less market-sensitive than society. But unlike Stein & Sunderam (2018), our conclusion is based on beauty contest considerations rather than a commitment problem on the part of the central bank.

Finally, our model sheds light on whether increased precision of public information can guard against the central bank becoming overly dependent on financial markets. Blinder (2004) observes that a central bank may be able to lower its dependency on financial markets and assume a leadership role by being transparent about its goals and methods. Following Svensson (2006), we can view the precision of public information, α, as the degree of central bank transparency. Our results then suggest that improved public information (in the form of a more precise economic forecast and *Inflation Report*) leads the central bank to rely *more*, rather than less, on the market-based signal.[20] Since the public signal only enters the monetary policy rule via the market-based signal, more precise public information

[19] Such policy considerations are more than just theoretical curiosities. In many countries (e.g., Australia, Canada, New Zealand and UK), policymakers have expressed concern about misaligned house prices and are actively examining measures to deal with the deviations of house prices from their fundamental values (Hördahl & Packer, 2007).

[20] Svensson (2006) argues that the precision of public information is more likely to be higher than that of any market participant. We think this is the case, particularly in the context of central bank providing its view of the economic outlook.

encourages the central bank to be more attentive to financial markets. In the limit, as public information becomes extremely precise and there is complete information, the central bank as well as market participants put full reliance on public information and ignore their private information.

6. Conclusion

The link between monetary policy and financial market prices is a two-way street. Central bankers pay close attention to the information about future economic developments contained in market prices. Market participants, for their part, scrutinise central banks intently since monetary policy actions strongly affect the opportunities to win or lose money. If the central bank relies too much on market information when formulating monetary policy, it risks being trapped in an echo chamber – acting on market signals that merely reflect back its own pronouncements. And the information value of market prices decreases to the extent that they no longer reflect an independent evaluation by thousands of market participants about future economic conditions.

Our paper identifies the circumstances under which financial market prices reveal useful information about economic fundamentals or merely reflect back the central bank's own assessments. We show, moreover, that the central bank optimally chooses to over-emphasise financial market signals, relative to a benchmark in which financial prices reveal the maximum information possible. Since the central bank cares only about aligning the policy rate with fundamentals, it downplays its own private information in favour of the market signal. In trying to match the central bank's action, market participants over-weight public information inducing a reflection problem that diminishes the information value of prices.

An implication of our analysis is that a policymaker concerned with ensuring that asset prices match fundamentals as closely as possible may wish to appoint a more market-insensitive central banker or, more broadly, commit the central bank to policies which ensure that market-signals are not overly prominent in monetary policy. Our analysis also suggests that more precise public information – perhaps in the form of a more detailed economic forecast or even quantitative forward guidance – leads the central bank to rely more, rather than less, on market prices. In the limit, as the central bank becomes highly transparent, financial market prices will increasingly reflect the central bank's own expectations about the future. A more detailed examination of the precise ways in which a central bank can maintain its independence from financial markets is an important area for future research.

Finally, our model is silent about the issue of strategic information leakage. If the central bank wishes to reveal its confidential information to a small subset of market participants, then the signal z becomes semi-public information. Extending the analysis to allow for such a possibility raises potentially interesting trade-offs between conveying an information advantage to some members of the public against refining the central bank's view of the economy.

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Appendix A. Appendix

Proof of Proposition 1. Fix $\alpha, \beta, \gamma, \omega$ and λ. We prove this proposition in two steps. Step 1 shows that there exists a $\widehat{\gamma} = \widehat{\gamma}(\alpha, \beta, \omega, \lambda)$ such that if $\gamma < \widehat{\gamma}$, then the equilibrium weight placed by the central bank on its private information, μ^*, has the form of (15), and if $\gamma \geq \widehat{\gamma}$, then the equilibrium weight is a boundary optimum, namely, $\mu^* = 1$. Step 2 derives $\widehat{\lambda}$ from $\widehat{\gamma}$ so that $\lambda < (\geq)\ \widehat{\lambda}$ is equivalent to $\gamma < (\geq)\ \widehat{\gamma}$.

Step 1: First note that

$$\frac{\mathrm{d}\mathbb{E}[L_{CB}]}{\mathrm{d}\mu} = 2(1-\lambda)\left\{\frac{(1-\lambda)\mu}{\gamma} - \frac{(\alpha+\beta)[\beta(1-\lambda)(1-\mu)+\alpha(1-(1-\lambda)\mu)]}{\alpha[\alpha+\beta(1-\lambda\omega)]^2}\right\}.$$

Then the solving the first-order condition, $\mathrm{d}\mathbb{E}[L_{CB}]/\mathrm{d}\mu = 0$, gives equation (15):

$$\mu^* = \frac{\gamma(\alpha+\beta)[\alpha+\beta(1-\lambda)]}{(1-\lambda)\left\{\gamma(\alpha+\beta)^2+\alpha[\alpha+\beta(1-\lambda\omega)]^2\right\}} > 0.$$

Let

$$\widehat{\gamma} = \left(\frac{1-\lambda}{\lambda}\right)\frac{[\alpha+\beta(1-\lambda\omega)]^2}{\alpha+\beta}$$

and observe that $\mu^* = 1$ when $\gamma = \widehat{\gamma}$. But rewriting μ^* as

$$\mu^* = \frac{(\alpha+\beta)[\alpha+\beta(1-\lambda)]}{(1-\lambda)\left\{(\alpha+\beta)^2+(\alpha/\gamma)[\alpha+\beta(1-\lambda\omega)]^2\right\}}$$

implies that μ^* is increasing in λ; therefore we need to have $\gamma \leq \widehat{\gamma}$ to ensure that $\mu^* \leq 1$. Now whenever $\gamma > \widehat{\gamma}$,

$$\begin{aligned}\frac{\mathrm{d}\mathbb{E}[L_{CB}]}{\mathrm{d}\mu} &< \frac{2(1-\lambda)(\alpha+\beta)}{[\alpha+\beta(1-\lambda\omega)]^2}\left\{\lambda\mu - \frac{1}{\alpha}\left[\beta(1-\lambda)(1-\mu)+\alpha(1-(1-\lambda)\mu)\right]\right\}\\ &= \frac{2(1-\lambda)(\alpha+\beta)}{[\alpha+\beta(1-\lambda\omega)]^2}\left[-(1-\mu)-\frac{\beta}{\alpha}(1-\lambda)(1-\mu)\right] < 0.\end{aligned}$$

Thus, we have a boundary optimum $\mu^* = 1$.

Step 2: Let $\Lambda = \{\lambda' \in (0,1) : \widehat{\gamma}(\lambda') = \gamma\}$. Since we can show that $\partial\widehat{\gamma}/\partial\lambda < 0$, $\lim_{\lambda\to 0^+} \widehat{\gamma}(\lambda) = +\infty$, and $\lim_{\lambda\to 1^-} \widehat{\gamma}(\lambda) = 0$, the set Λ corresponds to a singleton whose element lies between 0 and 1 (see Figure A.3). We denote by $\widehat{\lambda}$ the element of Λ. So for a fixed λ, whenever $\lambda > \widehat{\lambda}$, we have $\widehat{\gamma} = \widehat{\gamma}(\lambda) < \widehat{\gamma}(\widehat{\lambda}) = \gamma$. Similarly, we can show that $\lambda < \widehat{\lambda}$ if and only if $\gamma < \widehat{\gamma}$.

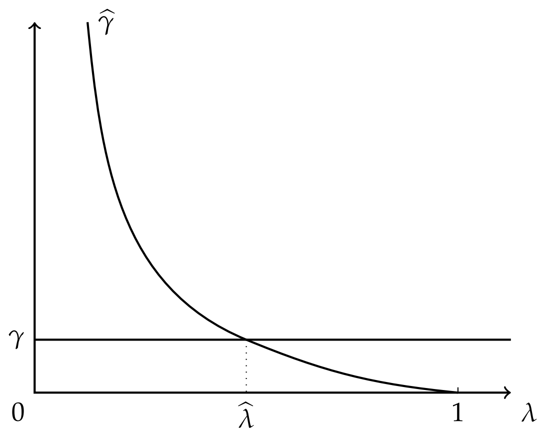

Figure A.3: There exists a unique $\widehat{\lambda}$ such that $\lambda > (\leq)\ \widehat{\lambda}$ if and only if $\gamma > (\leq)\ \widehat{\gamma}$.

□

Proof of Corollary 1. Let f be the numerator of μ^* and g the denominator. To show $\partial\mu^*/\partial\alpha < 0$, it suffices to verify that $(\partial f/\partial\alpha)\cdot g - (\partial g/\partial\alpha)\cdot f < 0$. It is true because

$$\left(\frac{\partial f}{\partial \alpha}\right) g - f\left(\frac{\partial g}{\partial \alpha}\right) < -\alpha\gamma(1-\lambda)\left[\alpha + \beta(1-\lambda)\right]\left[\alpha^2 + 2\alpha\beta + \beta^2(1-\lambda^2\omega^2)\right] < 0.$$

Similarly, we have

$$\begin{aligned}\left(\frac{\partial f}{\partial \lambda}\right) g - f\left(\frac{\partial g}{\partial \lambda}\right) = \alpha\gamma(\alpha+\beta)\{\alpha\beta^2[&\underbrace{1 + 2\omega(\lambda^2 - 4\lambda + 2) - \lambda\omega^2(2-3\lambda)}_{=\psi}]\\ &+ 2\alpha^2\beta(1+\omega-2\lambda\omega) + 2\beta^3\omega(1-\lambda)^2(1-\lambda\omega)\\ &+ \alpha^3 + (\alpha+\beta)^2\gamma\} > 0,\end{aligned}$$

because

$$\psi > \lambda\omega^2(2\lambda^2 - 5\lambda + 3) = \lambda\omega^2\left[2\left(\lambda - \frac{5}{4}\right)^2 - \frac{1}{8}\right] > 0.$$

Thus $\partial\mu^*/\partial\lambda > 0$. We have already shown in the proof of Proposition 1 that $\partial\mu^*/\partial\gamma > 0$. □

Proof of Corollary 2. Let $\phi = \widehat{\gamma}(\lambda) - \gamma$ and recall that $\widehat{\lambda}$ is the only solution to $\phi = 0$. Then by the implicit function theorem, we have

$$\frac{\partial\widehat{\lambda}}{\partial\alpha} = -\frac{\partial\phi/\partial\alpha}{\partial\phi/\partial\lambda} = -\frac{\partial\widehat{\gamma}/\partial\alpha}{\partial\widehat{\gamma}/\partial\lambda}.$$

Since

$$\frac{\partial \widehat{\gamma}}{\partial \alpha} = \frac{(1-\lambda)}{(\alpha+\beta)^2\lambda}\left[\alpha+\beta(1-\lambda\omega)\right]\left[\alpha+\beta(1+\lambda\omega)\right] > 0,$$

and

$$\begin{aligned}\frac{\partial \widehat{\gamma}}{\partial \lambda} = &-\frac{2\beta\omega(1-\lambda)[\alpha+\beta(1-\lambda\omega)]}{(\alpha+\beta)\lambda} \\ &-\frac{(1-\lambda)[\alpha+\beta(1-\lambda\omega)]^2}{(\alpha+\beta)\lambda^2} - \frac{[\alpha+\beta(1-\lambda\omega)]^2}{(\alpha+\beta)\lambda} < 0,\end{aligned}$$

it follows that $\partial\widehat{\lambda}/\partial\alpha > 0$. Similarly,

$$\frac{\partial \widehat{\lambda}}{\partial \gamma} = -\frac{\partial\phi/\partial\gamma}{\partial\phi/\partial\lambda} = \frac{1}{\partial\widehat{\gamma}/\partial\lambda} < 0.$$

Figure A.4 gives an intuitive illustration of the proof. □

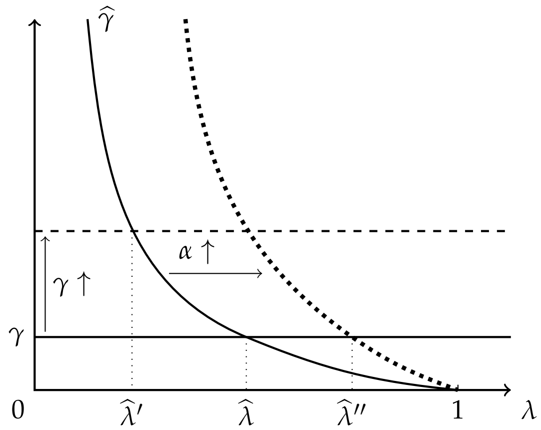

Figure A.4: The critical threshold $\widehat{\lambda}$ increases with α but decreases with γ.

Proof of Corollary 3. Since ξ^* is continuous in λ, we can simply write

$$\widehat{\xi} = \xi^*(\widehat{\lambda}) = \frac{\beta(1-\widehat{\lambda}\omega)}{\alpha+\beta(1-\widehat{\lambda}\omega)}.$$

It follows that

$$\frac{\partial\widehat{\xi}}{\partial\alpha} = \frac{\partial\xi^*}{\partial\alpha} + \frac{\partial\xi^*}{\partial\lambda}\frac{\partial\widehat{\lambda}}{\partial\alpha}.$$

Since

$$\frac{\partial\xi^*}{\partial\alpha} = -\frac{\beta(1-\widehat{\lambda}\omega)}{[\alpha+\beta(1-\widehat{\lambda}\omega)]^2} < 0$$

and

$$\frac{\partial\xi^*}{\partial\gamma} = -\frac{\alpha\beta\omega}{[\alpha+\beta(1-\widehat{\lambda}\omega)]^2} < 0,$$

by Corollary 2 we have $\partial\widehat{\lambda}/\partial\alpha > 0$ and hence $\partial\widehat{\xi}/\partial\alpha < 0$. Similarly, we can show that

$$\frac{\partial\widehat{\xi}}{\partial\gamma} = \frac{\partial\xi^*}{\partial\lambda}\frac{\partial\widehat{\lambda}}{\partial\gamma} > 0.$$

Thus the proof is complete. □

Proof of Propositions 4 and 5. When $\lambda \leq \widehat{\lambda}$, we have

$$\mathbb{E}[L_{CB}] = \frac{[\alpha+\beta(1-\lambda)]^2}{\gamma(\alpha+\beta)^2 + \alpha[\alpha+\beta(1-\lambda\omega)]^2}.$$

It follows that

$$\frac{\partial\mathbb{E}[L_{CB}]}{\partial\lambda} = -\frac{2\beta(\alpha+\beta)[\alpha+\beta(1-\lambda)]\{\gamma(\alpha+\beta)+\alpha(1-\omega)[\alpha+\beta(1-\lambda\omega)]\}}{\{\gamma(\alpha+\beta)^2+\alpha[\alpha+\beta(1-\lambda\omega)]^2\}^2} < 0,$$

thus $\mathbb{E}[L_{CB}]$ is decreasing in λ. When $\lambda > \widehat{\lambda}$, the central bank's expected loss yields

$$\mathbb{E}[L_{CB}] = \frac{(1-\lambda)^2}{\gamma} + \frac{\alpha\lambda^2}{[\alpha+\beta(1-\lambda\omega)]^2},$$

implying that

$$\frac{\partial\mathbb{E}[L_{CB}]}{\partial\lambda} = -\frac{2(1-\lambda)}{\gamma} + \frac{2\alpha(\alpha+\beta)\lambda}{[\alpha+\beta(1-\lambda\omega)]^3} = 2(1-\lambda)\left(\frac{\gamma-\widetilde{\gamma}}{\gamma\widetilde{\gamma}}\right),$$

where

$$\widetilde{\gamma} = \frac{(1-\lambda)[\alpha+\beta(1-\lambda\omega)]^3}{\alpha(\alpha+\beta)\lambda} = \widehat{\gamma}\left(\frac{\alpha+\beta(1-\lambda\omega)}{\alpha}\right) > \widehat{\gamma}.$$

It is then straightforward to see that $\mathbb{E}[L_{CB}]$ decreases with λ when $\gamma < \widetilde{\gamma}$, increases with λ when $\gamma > \widetilde{\gamma}$ and attains its minimum when $\gamma = \widetilde{\gamma}$. Also note that $\lim_{\lambda\to 0^+}\widetilde{\gamma} = +\infty$ and $\lim_{\lambda\to 1^-}\widetilde{\gamma} = 0$. Then we can show, by a similar argument as presented above, that there exists a λ^*, which solves $\gamma = \widetilde{\gamma}$, so that $\lambda > (<)\ \lambda^*$ if and only if $\gamma > (<)\ \widetilde{\gamma}$ (see also Figure A.5). But since $\widehat{\gamma} < \widetilde{\gamma}$, $\lambda^* > \widehat{\lambda}$. Now we have that $\mathbb{E}[L_{CB}]$ decreases with λ whenever $\lambda \leq \lambda^*$ and increases with λ whenever $\lambda > \lambda^*$. Thus, the central bank's expected loss is minimised at $\lambda = \lambda^*$.

□

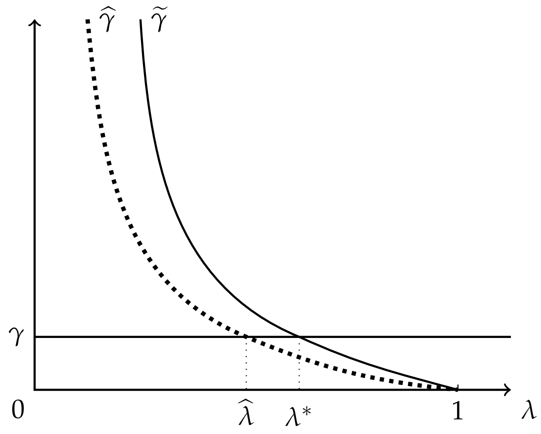

Figure A.5: There exists λ^* at which the central bank's expected loss $\mathbb{E}[L_{CB}]$ is minimised. Also λ^* is greater than $\widehat{\lambda}$.