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Abstract

This paper studies corporate debt structure over the business cycle and its implications for aggregate macroeconomic dynamics. We develop a tractable macro-finance model featuring debt heterogeneity with both secured and unsecured debt. Unlike secured debt, unsecured debt gives the lenders no access to the borrowers' assets in the event of default, and borrowers keep their assets at the cost of losing future access to the unsecured debt market. The difference in the nature of debt contracts leads to different risk taking behavior in the two debt markets. Our model generates strongly procyclical unsecured debt and weakly procyclical secured debt, in line with the stylized facts in US data. Moreover, we show that the inclusion of heterogeneous debt structures creates additional amplification effects relative to Bernanke, Gertler and Gilchrist (1999).

Keywords: Secured debt, unsecured debt, corporate debt structure, financial accelerator
JEL classification: E32, E44, G32

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1. Introduction

Standard macro-finance models often assume a uniform debt structure where collateralized credit is the main channel for the propagation and amplification of economic shocks. While this approach may help in building tractable theoretical models, it ignores the fact that firms are financed by different types of debt, and, importantly, these debts present different cyclical patterns across the business cycle. In this paper, we aim to explain the driving forces behind the dynamics of different debt instruments and study whether, and how, the conclusions of standard models of financial frictions change when firms have access to different types of debt.

We start by documenting the empirical patterns of firms' debt structures using U.S. firm level data. We find that firms operate with different debt structures. High credit quality firms rely almost exclusively on unsecured debt, while low credit quality firms use a large share of secured debt, confirming the finding by Rauh and Sufi (2010). We also find that unsecured and secured debt have different dynamics during the business cycle. Unsecured debt is strongly procyclical while secured debt is at best weakly procyclical, consistent with the empirical findings by Azariadis, Kaas and Wen (2016).

To explain the cyclicality of secured and unsecured debt, we build a tractable dynamic stochastic general equilibrium model with debt heterogeneity. In the model, firms borrow secured or unsecured debt from perfectly-competitive lenders subject to idiosyncratic productivity shocks and costly-state-verification problems similar to Bernanke, Gertler and Gilchrist (1999) (henceforth BGG). In the secured debt contract, the lender takes over the firm's assets in the event of default, and the borrower exits with nothing left. In the unsecured debt contract, the lender receives no payment in the event of default and the borrower keeps a fraction of revenue and keeps operating.[1] Firm's track record of default evolves endogenously over time and can be observed by lenders. Specifically, firms with a default record will be punished by being excluded from using unsecured credit in the future and will only borrow in the secured debt market. This is undesirable because we assume secured debt is costly to initiate, and borrowers prefer unsecured debt contracts.[2]

[1] In the data, secured and unsecured debt do have very different recovery rates. Acharya, Bharath and Srinivasan (2007) use Standard and Poors Credit Pro data and find that collateralized instruments have mean recovery rates ranging from 63% to 94%. For un-collateralized bonds, the mean recovery rate is 39%. Based on Moodys Default Risk Service data, Mora (2012) finds that the recovery rate for senior secured debt is 56%, whereas unsecured and subordinated debt has recovery rates ranging from 27% to 37% only.

[2] The initialization cost for secured debt has some justifications. In a secured debt contract, the lender gets the collateral in case of default, so it is reasonable for the lender to monitor the quality of the collateral. For example, loans and mortgages which constitute a large part of secured debt are routinely screened and monitored *ex ante*, usually by banks. In fact, in our model, what is needed is that *ex ante* monitoring is related to higher recovery rate *ex post*. This is supported by data. Mora (2012) finds that bank loans (which are usually secured) have higher recovery rates than bonds (which are usually unsecured).

The characteristics of secured and unsecured debt contracts affect firms' incentives to borrow. Similar to BGG, the optimal secured (unsecured) debt contract can be characterized by a threshold level of idiosyncratic productivity below which the borrower defaults. Under this setup, borrowers are subject to a moral hazard problem. They reap the benefits of potential upside risk, *i.e.*, when idiosyncratic productivity is above the threshold, and do not need to bear the full costs of downside risk, *i.e.*, when idiosyncratic productivity is below the threshold and default takes place. Importantly, the moral hazard problem is more severe for secured debt borrowers. In the downside risk scenario, secured debt borrower has "less skin in the game" compared to unsecured debt borrower. Secured debt borrower goes bankrupt with zero continuation value regardless of the indebtedness, whereas unsecured debt borrower stays in business with a penalty on their track record and a loss of continuation value. As a result, secured borrowers care less about downside risk compared to unsecured borrowers.

The contractual framework also implies that lenders face different incentives when they supply credit in the two debt markets. A lender in the secured debt market worries less about default, simply because the lender is able to recover a fraction of secured borrowers' asset in the event of default, but cannot recover anything from unsecured borrowers.

The above reasoning holds over the business cycle. In response to a negative productivity shock, the capital stock in the economy decreases, and the expected return on capital increases, which drives up borrowers' demand for credit.[3] Secured borrowers, who worry less about downside risks, increase their credit demand more than unsecured borrowers. Meanwhile, with higher expected returns, secured debt lenders are happy to increase credit supply more than unsecured debt lenders. Therefore, during economic downturns, the leverage ratio of secured borrowers increases more than that of unsecured borrowers.[4] For secured borrowers, a rise in the leverage ratio together with a fall in net worth in the economic downturn results in a relatively stable level of secured debt over the business cycle. In contrast, the increase in the leverage ratio for unsecured borrowers is more limited. Together with a fall in net worth, it implies a larger fall in unsecured debt in a downturn.

To explore the quantitative significance of this channel, we embed our contractual framework into a standard dynamic stochastic general equilibrium model and calibrate the model based on US corporate debt data. With a TFP shock and a shock to the cross-sectional dispersion of idiosyncratic firm productivity similar to Christiano, Motto and Rostagno (2014), our model-simulated moments feature strongly procyclical unsecured debt and weakly pro-

[3] A negative productivity shock reduces the price of capital and the return on capital on impact. Subsequently, the price of capital gradually rises back to the steady state from below, and this 'capital gains' effect associated with rising price of capital increases the expected return on capital after the negative shock.

[4] Leverage ratio is defined as asset per unit net worth.

cyclical secured debt, consistent with US data.

A key implication of this paper is that the introduction of unsecured debt contracts amplifies the financial accelerator effect in BGG. In the one-sector BGG model, the financial accelerator effect exists because debt falls just when a firm's net worth falls, which amplifies the effects of the original shock. By contrast, in our model, in response to a negative shock, the increase in the leverage ratio in the unsecured debt market is rather limited, so the fall in debt is more pronounced. This leads to a quantitatively important amplification effect relative to BGG. Our results suggest that the standard one-sector BGG model may underestimate the amplification effects of the financial accelerator mechanism.

Finally, we consider several extensions of the model with more realistic features in the firm sector. We allow for (1) positive recovery ratios for lenders of unsecured debt; (2) exogenous upgrading of credit ratings; (3) predetermined productivity differences in the firm sector; and (4) a mixed debt structure in low-credit-rating firms. We find that our key finding is robust, that unsecured debt is more procyclical than secured debt in each of these extensions.

Our paper is related to two strands of literature. First, this paper is related to a vast literature incorporating financial frictions into macroeconomic models. This paper adopts a costly state verification approach because it is straightforward to endogenize default. See Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2014) and Nuno and Thomas (2017). By contrast, default is eliminated as an equilibrium outcome in models in which financial frictions arise due to limited enforcement problems (see for example Kiyotaki and Moore (1997), Meh and Moran (2010), Jermann and Quadrini (2012) and Gertler and Karadi (2011)). Moreover, the theory of Kiyotaki and Moore (1997) implies that secured debt is strongly procyclical, which cannot explain the cyclicality of secured and unsecured debt in the data.

Second, there is a large theoretical literature on corporate debt structure, following Diamond (1991), Besanko and Kanatas (1993) and Boot and Thakor (1997). This literature focuses on the determinants of a firm's financing based on bank debt versus corporate bonds. For instance, Diamond (1991) argues that high credit quality firms have good reputations allowing them to avoid the additional costs of bank debt associated with monitoring. Our model is in this spirit. Chemmanur and Fulghieri (1994), Bolton and Freixas (2000) and De Fiore and Uhlig (2011) argue that banks have an information advantage about a firm's profitability. Such information is particularly useful for assessing the risk of low-quality borrowers. Empirically, Denis and Mihov (2003) find that credit quality is a major determinant of a firm's debt structure, with higher credit quality firms choosing public debt and lower quality firms choosing bank loans. Rauh and Sufi (2010) show that high credit quality firms rely exclusively on unsecured debt; whereas low credit quality firms rely more on secured debt. This literature, however, does not study the macroeconomic effects of corporate debt structure.

A few papers discuss debt structure and its relation to the macroeconomy. De Fiore and Uhlig (2015) assume that bank monitoring yields useful information about relatively low productivity firms. They find that the flexibility in substituting alternative debt instruments by firms reduces macroeconomic volatility. In Crouzet (2017), firms borrow partly through banks because banks are more flexible in debt restructuring. The paper argues that since bond finance cannot be restructured in the future, firms switching from bank finance to bond finance will deleverage, which increases the negative macroeconomics effects of a shock to the banking sector. Our paper addresses different aspects of a firm's debt choice by studying secured versus unsecured debt to explain the cyclicality of these different types of debt contracts. In terms of aggregate implications, we argue that unsecured borrowers have less volatile leverage ratios, so fluctuations in debt are amplified. Therefore, the amplification due to the financial accelerator mechanism is stronger in an economy with a large fraction of unsecured debt.

The work of Azariadis, Kaas and Wen (2016) is most relevant to ours. Their model features multiple equilibria driven by unsecured debt and relies on sunspot shocks to generate persistent and highly volatile dynamics of macroeconomic variables. They argue that fluctuations in unsecured debt, but not in secured debt, are driven by sunspot shocks, and that sunspot shocks account for around half of output volatility. In this paper, we show that the nature of secured and unsecured debt contracts implies that borrowers and lenders of unsecured debt are more cautious, and therefore the leverage ratios of unsecured borrowers are less volatile. Our simulation results demonstrate that even with only fundamental shocks, our endogenous mechanism can account for the relative procyclicality of unsecured debt observed in US data.

The rest of the paper is organized as follows. Section two provides empirical analysis. Section three presents the full model with debt heterogeneity. Section four explores the key properties of credit contracts and explains theoretically why they result in different cyclical movements in secured and unsecured debt. Section five describes calibration of the model. Section six discusses the model properties and quantitative results. Section seven compares the benchmark model with a standard BGG model. Section eight discusses four extensions to our benchmark models. Section nine concludes.

2. Empirical analysis

In this section, we review important stylized facts about credit ratings and debt structures. Our main findings can be summarized as follows:

1. Debt structure is closely related with a firm's credit quality. High credit quality firms rely almost exclusively on unsecured debt while low credit quality firms have a substantial share of secured debt.
2. A firm's leverage is countercyclical and there is huge heterogeneity among leverage ratios across credit quality distributions. In particular, high credit quality firms operate with relatively low leverage while low credit quality firm use higher leverage.
3. Unsecured and secured debt show different dynamics along the business cycle: unsecured debt is strongly procyclical, while secured debt is at best weakly procyclical.

We begin with a description of the data and variables in our sample. The sampling universe includes public traded non-financial and non-utility U.S. firms included in Compustat with a long-term issuer credit rating in the last one year from 1981 to 2017.[5] There are 1142 rated firms in the sample. In line with Azariadis, Kaas and Wen (2016), we use the item "mortgages and other secured debt" to measure secured debt. We then attribute the difference between "long term debt + total current debt" and "mortgages and other secured debt" to unsecured debt. To clean the data, we remove firm-year observations where any of the variables are missing, negative, or for which secured debt exceeds total debt. We also winsorized all firm-level variables at the 1% and 99% levels to remove outliers.

We measure leverage as total assets divided by total assets net of the sum of long-term debt and total current debt. Panel A of Table 1 shows summary statistics for the leverage of firms in the sample. Rated firm-year observations have a mean leverage ratio of 1.74 and a negative correlation with contemporaneous GDP -0.15. The dynamics of observed leverage for all observations over the business cycle is summarized in Column 4. The results show counter-cyclical dynamics for the average firm, with a correlation between leverage and GDP of -0.37, consistent with the findings of Halling, Yu and Zechner (2016). Panel B shows the leverage ratios across credit quality distributions. Interestingly, we observe that leverage stays low for firms with high credit ratings and jumps to more than 2.0 for firms rated CCC and below, implying a big difference in a firm's financing choice and capital structure.

Next we focus on how debt structure varies across the credit quality distribution. Figure 1 plots the time series of unsecured debt share by credit rating. On average, 75% of rated firms' total debt financing comes from unsecured debt, implying a non-negligible role of unsecured debt in firms' credit. Moreover, there is debt heterogeneity. Unsecured debt constitutes a substantial part of high credit quality firms' debt financing and is much lower for firms with low credit ratings. In particular, the share of unsecured debt for BBB and

[5]Coverage by Capital IQ is comprehensive only from 2001 onwards, therefore we restrict our main sample to Compustat. This allows us to have long enough sample periods to calculate correlations and other business cycle moments.

above rated firms ranges from 0.75 to 0.90. In contrast, it drops down to around 0.6 for BB+ and below rated firms. Note that the difference in the unsecured debt share between high and low credit ratd firms is smaller than that found by Rauh and Sufi (2010). One reason for this is that Compustat is biased towards large public firms which have greater access to bond markets and other forms of unsecured debt financing. Therefore we explore the debt information of private firms in Capital IQ as well. Figure 1 shows the time series of the unsecured debt share for samples obtained from Compustat and Capital IQ. Once private firms are included, the disparity in unsecured debt shares based on credit ratings increases substantially. For instance, the average unsecured debt share in Capital IQ for BBB firms is 0.81, which is higher than 0.49 for B- firms. Moreover, the differences in debt structure widens over time after 2000, represented by a sharply declining use of unsecured debt by low credit rating firms and a steadily increasing use of unsecured debt by high credit rating firms.[6]

In line with the previous literature, the time series variation shows that unsecured debt plays a much stronger role in output dynamics than secured debt. We deflate the annual time series from Compustat by the gross value added index for business (a price index constructed by the Bureau of Economic Analysis), and detrend all series using HP filter (smoothing parameter = 100). As shown in Table 2, the contemporaneous correlation between output and unsecured debt is 0.48 and but only 0.06 for secured debt. While our sample focuses on firms that are credit-rated, the vast majority of U.S. firms are not. To complement, we also compute the cyclical properties for all firms regardless of credit rating. The correlation between output and unsecured debt is 0.50 and 0.15 for secured debt, similar to the results obtained from our main sample, suggesting that our results are robust.

The empirical findings above confirm Azariadis, Kaas and Wen (2016)'s key result that unsecured firm credit is more procyclical than secured credit.[7] This finding suggests that macro-finance models should not only analyze secured credit, but also look at unsecured credit. In the next session, we build a model that features both secured and unsecured debt contracts. We show that by taking into account debt heterogeneity, the model is able to explain the stylized facts in US data.

[6]We check that high credit quality firms still rely more on unsecured debt than low credit quality firms after controlling for firm size, age and industry.

[7]Azariadis, Kaas and Wen (2016) use linear detrending and find $Corr(\text{Secured debt}, GDP) = -15 - 15\%$ and $Corr(\text{Unsecured debt}, GDP) = 70 - 75\%$ with different winsorization. These correlations are close to what we find.

Table 1
Summary statistics on leverage.

Panel A: Sample Summary Statistics on Leverage			
Rated Only		All Observations	
Mean	Correlation with GDP	Mean	Correlation with GDP
1.78	-0.15	1.83	-0.37

Panel B: Leverage Ratios Across Quality Distribution			
	Leverage Ratio		Leverage Ratio
AA and above	1.53	B- and below	1.95
BBB and above	1.62	CCC and below	2.13
BBB- and above	1.65	CC and below	2.31

This table reports summary statistics of firm leverage. Statistics are calculated for the Compustat sample of U.S. rated firms and all firms (both rated and non-rated) in Panel A. Panel B summarizes the leverage ratios across credit ratings.

Table 2
Debt volatilities and correlations with GDP.

	Rated Only		All Observations	
	Std. Deviation	Corr. with GDP	Std. Deviation	Corr. with GDP
Secured Debt	10.16	0.06	10.07	0.15
Unsecured Debt	13.94	0.48	15.29	0.50

This table reports the standard deviations and contemporaneous correlations of debt with GDP. The left panel shows rated firms only. The right panel shows all firms (both rated and non-rated). GDP is deflated by the GDP deflator. Debt is deflated by business gross valued index. All series have been logged and HP filtered with $\lambda = 100$.

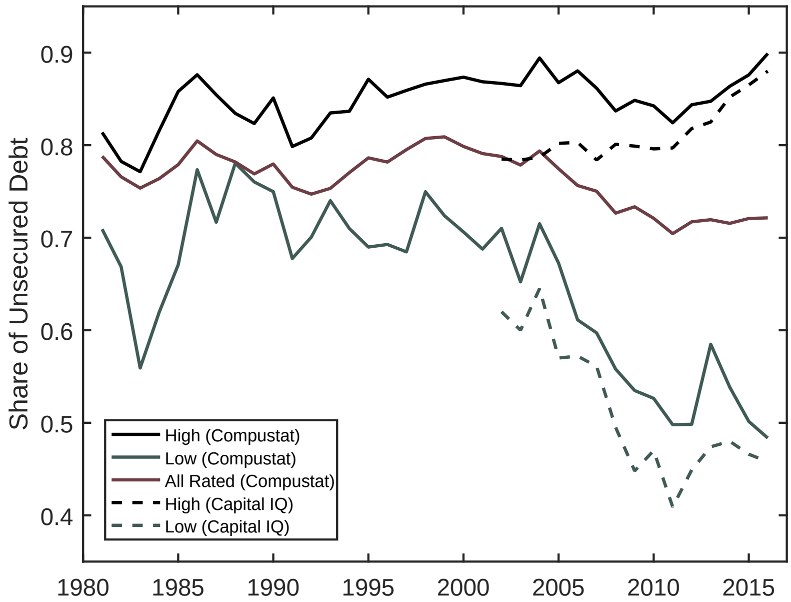

Fig. 1. This figure shows the share of unsecured debt for public and private U.S. firms by credit ratings. (Compustat, 1981-2016 and Capital IQ, 2001-2016)

3. Model

The model economy is composed of four types of agents: households, investors, capital goods producers and firms. Households lend to investors in the form of deposits without any frictions. Risk-neutral investors use the funds to lend to a continuum of firms, subject to credit frictions. Specifically, each firm is subject to an idiosyncratic shock to its capital quality, but the lender cannot observe its realization without cost. This gives rise to a costly-state-verification problem similar to Bernanke, Gertler and Gilchrist (1999). The lender, however, has a record-keeping technology. A firm has either high or low credit quality. If the firm has low credit quality, the lender does not lend unless the credit is secured. Setting up a secured debt contract requires an initialization cost, and if the firm defaults, the lender has access to the firm's assets. On the other hand, setting up an unsecured debt contract does not require an initialization cost, but if the firm defaults, the lender has no access to the firm's asset.[8] Moreover, there is a positive probability that the firm can walk away with its assets, but the firm's credit quality will become low in the future. Investors lend to many firms and diversify perfectly.

[8]The assumption that lenders in an unsecured debt contract receive no payment and borrowers lose reputation in a default event follows from Azariadis, Kaas and Wen (2016) and Cui and Kaas (forthcoming). One interpretation is that a defaulting G firm liquidates its assets and the owner starts a new firm.

The real side of the model is standard. In each period, after production takes place, firms use their credit to buy physical capital from capital goods producers. In the beginning of the next period, the idiosyncratic shock realises, and firms use their stock of effective capital and labor from households to produce a final good. The good is purchased by households for consumption and by capital goods producers, who combine undepreciated capital and investment goods to produce new capital. The markets for labor, physical capital and final good clear every period.

The following discusses the behavior of each type of agent and the market clearing conditions.

3.1. Households

Infinite-lived representative households derive utility from consumption and disutility from supplying labor. The problem of the representative household is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

subject to the following budget constraint:

$$w_t L_t + R_{t-1} D_{t-1} + \Pi_t^K = C_t + D_t + tr_t, \tag{2}$$

where χ governs the weight on labor disutility, $h < 1$ captures the habit persistence in consumption, and φ governs the inverse of Frisch labor elasticity.[9] In each period, a representative household receives wage income $w_t L_t$, makes deposits D_t and consumes C_t. Π_t^K denotes profits transferred from capital producing firms, and tr_t denotes startup funds paid to new firms and revenues remitted from exiting firms.

The consumption Euler equation and labor supply equation are:

$$1 = R_t E_t(\Lambda_{t,t+1}), \tag{3}$$

$$w_t = \chi L_t^{\varphi} U_{Ct}^{-1}, \tag{4}$$

where $\Lambda_{t,t+1} = \beta U_{Ct+1}/U_{Ct}$ and $U_{Ct} = (C_t - hC_{t-1})^{-1} - \beta h E_t(C_{t+1} - hC_t)^{-1}$.

3.2. Investors

Investors collect deposits from households and lend to firms. They observe the credit quality of each firm and issue debt to them. Investors require a risk-free return R_t in every

[9] We include habit persistence to better match the standard deviation of output and investment in the data. It does not affect the cyclicality of secured and unsecured debt qualitatively.

state of the world for both secured and unsecured debt. Investors do not play a meaningful role in the model other than making sure that households hold a diversified loan portfolio across firms.

3.3. Capital goods producers

A representative capital goods producer buys previously installed capital and combines it with investment good I_t to produce new capital. Newly produced capital is sold back to the firms within the same period. Production of new capital is subject to convex investment adjustment cost $Adj_t = 0.5\Psi^I \left(I_t/I_{t-1} - 1\right)^2$. The evolution of aggregate capital K_t is given by:

$$K_t = (1-\delta)K_{t-1} + (1 - Adj_t)I_t. \tag{5}$$

Capital goods producers maximize the sum of discounted expected future profits, $E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\Pi^K_{t+s}$, where $\Pi^K_t = Q_t[K_t - (1-\delta)K_{t-1}] - I_t$. The first order condition for the optimal investment choice is:

$$1 = Q_t\left[1 - Adj_t - \Psi^I \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + E_t\left[\Lambda_{t,t+1}Q_{t+1}\Psi^I\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right]. \tag{6}$$

3.4. Firms

In the firm sector, there is a unit measure of firms $j \in [0,1]$. Each firm carries a publicly observed label $i \in \{G, B\}$ which denotes high and low credit quality respectively.[10] The label may change over time and we discuss how the label determines a firm's borrowing options later. Firms produce with the following Cobb-Douglas production function:

$$Y^i_{jt} = A_t(\omega_{jt}K^i_{jt-1})^{\alpha}(L^i_{jt})^{1-\alpha}, \tag{7}$$

where A_t denotes the total factor of productivity (TFP), and ω_{jt} denotes a privately observable idiosyncratic shock to the firm's capital quality. The variable ω_{jt} follows an exogenous log-normal distribution with mean 1 and variance σ^2_{t-1}, i.e., $\log(\omega_{jt}) \sim N(-\frac{1}{2}\sigma^2_{t-1}, \sigma^2_{t-1})$. The cumulative distribution function and probability density function are $F(\omega_{jt}; \sigma_{t-1})$ and $f(\omega_{jt}; \sigma_{t-1})$ respectively. The idiosyncratic shock is independently distributed across firms and time, and is orthogonal to aggregate shocks.

In period $t-1$, a firm with label i purchases capital K^i_{jt-1} at the price Q_{t-1}. At the beginning of period t, the firm faces an idiosyncratic productivity shock, and effective capital becomes $\omega_{jt}K^i_{jt-1}$. The firm then hires labor, produces and sells depreciated capital to capital

[10]Note that there is no intrinsic difference between firms with different credit quality, however. This assumption is relaxed in Section 8.3.

producing firms. The firm chooses labor to maximize $(Y^i_{jt} - w_t L^i_{jt})$, and optimal choice of labor satisfies $w_t L^i_{jt} = (1-\alpha)Y^i_{jt}$, which implies that all firms have the same labor to output ratio. The marginal product of capital is given by $r^K_t = \alpha Y^i_{jt}/(\omega_{jt} K^i_{jt-1})$.[11] It is helpful to define the average return on capital of the firm sector as:

$$R^K_t \equiv \frac{r^K_t + (1-\delta)Q_t}{Q_{t-1}}. \tag{8}$$

The return on capital of firm j is given by $\omega_{jt} R^K_t$.

A firm with label i has net worth N^i_{jt-1} in period $t-1$. To finance its purchase of capital, it borrows B^i_{jt-1} from lenders with one-period risky debt contracts. The credit contracts available to a firm depends on the firm's label. Firms with label G are eligible for secured and unsecured debt contracts, whereas firms with label B are eligible for secured debt contract only.[12] We will show later that in equilibrium G firms issue unsecured debt only.

Secured debt contracts: In the secured debt contract, the lender seizes the firm's asset in case of default. To borrow secured debt, the firm is subject to an initialization cost, which equals a proportion κ of the firm's net worth.[13] Therefore, a firm needs to borrow $B^B_{jt-1} = Q_{t-1}K^B_{jt-1} - (1-\kappa)N^B_{jt-1}$ to finance its purchase of capital. The contract specifies the amount borrowed B^B_{jt-1} and a gross non-default loan rate, Z^B_{jt}. It is helpful to define a default threshold, $\bar{\omega}^B_{jt}$, where

$$\bar{\omega}^B_{jt} R^K_t Q_{t-1} K^B_{jt-1} = Z^B_{jt} B^B_{jt-1}. \tag{9}$$

When $\omega_{jt} \geq \bar{\omega}^B_{jt}$, the firm repays the promised amount $Z^B_{jt} B^B_{jt-1}$. If $\omega_{jt} < \bar{\omega}^B_{jt}$, the firm defaults and shuts down.[14] In this situation the lender monitors the firm and gets to keep the net receipt $(1-\mu)\omega_{jt} R^K_t Q_{t-1} K^B_{jt-1}$, where μ is a proportional default cost.[15] The payoff structure of secured debt is summarized in Table 3.

Unsecured debt contracts: In the unsecured debt contract, the loan is not backed up by an underlying asset and lenders will not be paid in case of default. A firm borrows $B^G_{jt-1} = Q_{t-1}K^G_{jt-1} - N^G_{jt-1}$. The firm promises a gross non-default loan rate Z^G_{jt}. We similarly

[11] This means that $r^K_t \equiv \alpha A_t[(1-\alpha)A_t/w_t]^{(1-\alpha)/\alpha}$.

[12] In Section 8.4, we relax this assumption and B firms can issue both types of debt.

[13] Footnote 3 provides a discussion why it is more costly to raise secured debt than unsecured debt. We will show that within a certain range of values, the cost κ ensures that G firms have no incentives to raise secured debt, and we will focus on this case.

[14] We also consider a model which relaxes this assumption by allowing B firms to keep some assets in the event of default. Details are presented in Section 8.4. If the fraction of assets B firms can keep is small, our main results go through.

[15] Following Bernanke, Gertler and Gilchrist (1999), we assume a cost of bankruptcy. It can be interpreted as the auditing and legal cost, as well as losses associated with liquidation.

Table 3
Payoff structure of secured debt.

	Defaults: ($\omega_{jt} < \bar{\omega}^B_{jt}$)	Does not default: ($\omega_{jt} \geq \bar{\omega}^B_{jt}$)
B firm	Gets nothing.	Repays debt and keeps profit.
Lender	Gets liquidation value of the firm.	Receives repayment.

define a cutoff threshold $\bar{\omega}^G_{jt}$, where

$$\bar{\omega}^G_{jt} R^K_t Q_{t-1} K^G_{jt-1} = Z^G_{jt} B^G_{jt-1}. \tag{10}$$

When $\omega_{jt} < \bar{\omega}^G_{jt}$, the firm is insolvent and declares defaults. When $\omega_{jt} \geq \bar{\omega}^G_{jt}$, the firm is financially capable to make the repayment. In this situation the firm may choose to honor its debt contract. But it can also engage in a strategic default.[16] By doing so the firm refuses to pay off the debt but then is unable to borrow on an unsecured basis in the future, i.e., losing the G label.[17] After it defaults, the firm undergoes debt restructuring. With probability ζ, debt restructuring is successful and the firm retains its operating profit $\omega_{jt} R^K_t Q_{t-1} K^G_{jt-1}$. With probability $1-\zeta$, debt restructuring is unsuccessful and the firm shuts down and exits the market. Without loss of generality, assuming that the G firm chooses to default when $\omega_{jt} < \tilde{\omega}^G_{jt}$, where $\tilde{\omega}^G_{jt} \geq \bar{\omega}^G_{jt}$, we get the following summarized payoff of unsecured debt in Table 4.[18]

Table 4
Payoff structure of unsecured debt.

	Defaults: ($\omega_{jt} < \tilde{\omega}^G_{jt}$)	Does not default: ($\omega_{jt} \geq \tilde{\omega}^G_{jt}$)
G firm	With Prob= ζ, keeps assets and becomes B firm; With Prob $= 1-\zeta$, gets nothing.	Repays debt and keeps profit.
Lender	Gets nothing.	Receives repayment.

At this point, let us drop the firm subscript j on all variables. This minimizes notation, and reflects the fact (see below) that the equilibrium values of these variables in any period only depend on the type $i \in \{G, B\}$ of the firm in that period but not on j. For this reason, in the discussion below, we treat firms of the same type in a given period as a single agent.

[16] We call the decision by a borrower to stop making payments (i.e., to default) on a debt, despite having the financial ability to make the payments, as strategic default.

[17] In the data, high credit quality is positively correlated with a firm's historical productivity. This is reflected in our model because the precedence of a credit downgrade implies that B firms on average have lower historical productivity.

[18] To see why $\tilde{\omega}^G_{jt} \geq \bar{\omega}^G_{jt}$, suppose a firm draws a shock such that $\lim \omega_{jt} \to (\bar{\omega}^G_{jt})^+$. After the repayment, the firm's net worth is $(\omega_{jt} - \bar{\omega}^G_{jt}) R^K_t Q_{t-1} K_{jt-1} \to 0^+$. This firm is better off defaulting.

Perfectly-competitive investors lend in both secured and unsecured debt markets, and they require a non-state-continent risk-free return R_t.[19] For type i firms, lenders offer a menu of debt and cutoff values which satisfies the lenders' participation constraint. The participation constraint of the secured lender is given by:

$$R_t^K Q_{t-1} K_{t-1}^B \left[\int_{\bar{\omega}_t^B} \bar{\omega}_t^B dF_{t-1} + (1-\mu) \int^{\bar{\omega}_t^B} \omega dF_{t-1} \right] \geq R_{t-1} B_{t-1}^B, \tag{11}$$

where the left hand side is the expected gross return on the loan and the right hand side is the opportunity cost of lending measured by the risk-free return.

The participation constraint of the unsecured lender is:

$$R_t^K Q_{t-1} K_{t-1}^G \left(\int_{\tilde{\omega}_t^G} \bar{\omega}_t^G dF_{t-1} \right) \geq R_{t-1} B_{t-1}^G. \tag{12}$$

An unsecured lender takes strategic defaults into account. The left hand side represents the gross expected return from borrowers who experience $\omega \geq \tilde{\omega}_t^G$, and the borrower does not engage in strategic default. For borrowers who experience $\omega < \tilde{\omega}_t^G$, they either are incapable of making the repayment or choose to deliberately default on their debt. In both situations the return to the lender will be zero. The right hand side measures the opportunity cost of lending.[20]

We are ready to spell out the borrowers' problems. Let $V_t^i(N_t^i)$ denote the firms' valuation function. We then have the following Bellman equations. First, for B firms:

$$V_t^B(N_t^B) = \max_{K_t^B, \bar{\omega}_{t+1}^B} E_t \Lambda_{t,t+1} \int_{\bar{\omega}_{t+1}^B} \left\{ \begin{array}{c} \theta V_{t+1}^B[(\omega - \bar{\omega}_{t+1}^B) R_{t+1}^K Q_t K_t^B] \\ +(1-\theta)(\omega - \bar{\omega}_{t+1}^B) R_{t+1}^K Q_t K_t^B \end{array} \right\} dF_t. \tag{13}$$

where the parameter θ is the exogenous survival probability common across firm types.[21] With probability $(1-\theta)$ the firm exits and transfer the net worth to the households.

The Bellman equation of G firms is

$$V_t^G(N_t^G) = \max_{K_t^G, \bar{\omega}_{t+1}^G} E_t \Lambda_{t,t+1} \int \max \left\{ V_{t+1}^{G,ND}, V_{t+1}^{G,D} \right\} dF_t, \tag{14}$$

[19] Lenders get a certain return by investing in a lot of firms.

[20] We assume that in every state of the world, there are at least some borrowers who make the repayment such that unsecured lenders always break even. In this paper we do not consider a potential bad equilibrium in which all G firms are expected to default and the unsecured debt market shuts down. This equilibrium is analyzed by Cui and Kaas (forthcoming), Azariadis, Kaas and Wen (2016) and Gu, Mattesini, Monnet and Wright (2013).

[21] Following Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999) and Gertler and Karadi (2011), this assumption prevents firms from growing out of their financial constraints.

where $V^{G,ND}$ denotes the firm value when it repays and $V^{G,D}$ denotes the firm value when it defaults. They are given by:

$$V_t^{G,ND} \quad = \quad \theta V_t^G[(\omega - \bar{\omega}_t^G)R_t^K Q_{t-1} K_{t-1}^G] + (1-\theta)(\omega - \bar{\omega}_t^G)R_t^K Q_{t-1}K_{t-1}^G, \tag{15}$$

and

$$V_t^{G,D} \quad = \quad \theta\zeta V_t^B[\omega R_t^K Q_{t-1}K_{t-1}^G] + (1-\theta)\zeta\omega R_t^K Q_{t-1}K_{t-1}^G. \tag{16}$$

To summarize, B firms maximize their value (13) subject to the participation constraint (11) in the secured debt market, whereas G firms maximize their value (14) subject to the participation constraint (12) in the unsecured debt market. They take the prices R_t^K, R_t and Q_t as given.

Since both objectives and participation constraints are constant returns to scale, it follows that all firms of the same type select the same cutoff value and the same capital to net worth ratio. We guess the value functions are given by $V_t^i(N_t^i) = \lambda_t^i N_t^i$ for $i \in \{B, G\}$, where λ_t^B and λ_t^G are the marginal values of net worth of B firms and G firms respectively. We define the leverage ratio of B firms as $\phi_t^B \equiv Q_t K_t^B/[(1-\kappa)N_t^B]$, and the leverage ratio of G firms as $\phi_t^G \equiv Q_t K_t^G/N_t^G$. Let us also define $\xi_t \equiv \bar{\omega}_t^G/\tilde{\omega}_t^G$ as strategic default decision of G firms, so a larger ξ_t means fewer strategic defaults.

In Appendix B, we show that when the initialization cost for secured debt satisfies $\kappa \in (\kappa_0, \kappa_1)$ (the lower bound makes sure that unsecured debt is preferable to secured debt, or $\lambda_t^G > \lambda_t^B$, and the upper bound makes sure that B firms prefer keeping their business to liquidation, or $\lambda_t^B > 1$), there is a separating equilibrium in which G firms only borrow unsecured debt.[22] Due to the existence of financial frictions, the firms' return on capital is higher than the cost of borrowing, so all firms will borrow up to the limit, which means that the participation constraints for both types of firms hold with equality. Moreover, the G firms choose a default strategy given by:

$$\xi_t = 1 - \zeta\frac{\Omega_t^B}{\Omega_t^G} \leq 1, \tag{17}$$

where $\Omega_t^i \equiv \theta\lambda_t^i + 1 - \theta$ for $i \in \{B, G\}$ are the probability weighted average of the marginal values of net worth of continuing and exiting firms at $t+1$. Since $\lambda_t^G > \lambda_t^B$, (17) implies that $\xi \in (0,1)$, *i.e.* some G firms default strategically. One can think of the ratio Ω_t^G/Ω_t^B as

[22]The parametric expressions for κ_0 and κ_1 are given in Appendix B. In our numerical exercise, our calibrated initialization cost κ is 0.017, which is in between $\kappa_0 = 0.013, \kappa_1 = 0.041$. In a similar setting, De Fiore and Uhlig (2011) use a screening cost of 2.8% of firms' net worth for European banks.

the reputation of being a G firm. When the ratio is large, it is costly to default strategically so we expect fewer strategic defaults. Indeed, (17) shows that ξ_t is increasing in Ω_t^G/Ω_t^B.

The marginal values of net worth for B firms and G firms are given by:

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1}\Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t, \tag{18}$$

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1}\Omega_{t+1}^G R_{t+1}^K \left[(1-\xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \bar{\omega}_{t+1}^G) dF_t \right]. \tag{19}$$

Consider (18) first. Conditional on realized ω, a unit of net worth in a B firm is leveraged up by $(1-\kappa)\phi_t^B$ times, yielding a discounted aggregate return $\Lambda_{t,t+1}\Omega_{t+1}^B R_{t+1}^K$. If $\omega \geq \bar{\omega}_{t+1}^B$ the firm receives a share $(\omega - \bar{\omega}_{t+1}^B)$ of the revenue after debt repayment. Equation (19) can be understood similarly.

The firms' optimal demand for secured and unsecured debt respectively are given by the following two conditions:

$$\lambda_t^B = \frac{(1-\kappa)E_t \Lambda_{t+1}\Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}^B) - \mu \bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]}. \tag{20}$$

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G)]}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]}. \tag{21}$$

Once $\bar{\omega}_{t+1}^B$ and $\tilde{\omega}_{t+1}^G$ are solved for, one can use the participation constraints to back out the quantity of each type of debt.

3.5. Aggregation and accumulation of net worth

Since each type of firms has the same capital to labor ratio and leverage ratio, we only need to keep track of sector-level quantities. For $X \in \{Y, K, L, N, B\}$, we define $X_t^i \equiv \int_i X_{jt}^i dj$, where $i \in \{G, B\}$. We also define economy-wide variables $X_t \equiv X_t^G + X_t^B$. We have:

$$N_t^G \phi_t^G = Q_t K_t^G, \tag{22}$$

$$(1-\kappa) N_t^B \phi_t^B = Q_t K_t^B. \tag{23}$$

It is helpful to define the aggregate leverage ratio of the economy as $\phi_t \equiv Q_t K_t / N_t$.

We write down the evolution of net worth for G and B firms. We assume that in each period, new firms enter to keep the number of firms of each type constant. We assume households transfer to a new firm a small fraction τ of the net worth of the average firm with the same credit rating.[23] These initial funds are one-time lump-sum transfer. G firms' net worth evolves as follow:

$$N_t^G = \theta \int_{\tilde{\omega}_t^G} (\omega - \bar{\omega}_t^G) R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} + \tau N_{t-1}^G, \tag{24}$$

where the first term represents the net worth of firms which are G firms in period $t-1$ and remain G firms in period t. The second term denotes the transfer received by new entrants.

Net worth of B firms evolves as follow:

$$\begin{aligned} N_t^B &= \theta \int^{\tilde{\omega}_t^G} \zeta \omega R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} \\ &\quad + \theta \int_{\bar{\omega}_t^B} (\omega - \bar{\omega}_t^B) R_t^K Q_{t-1} K_{t-1}^B dF_{t-1} + \tau N_{t-1}^B. \end{aligned} \tag{25}$$

The first term represents the net worth of G firms who default in the last period and becomes B firms in period t. The second term represents the net worth of B firms in period $t-1$ who remain B firms in period t. The last term is the transfer received by new entrants.[24]

The goods market clearing condition is given by:

$$\begin{aligned} Y_t &= C_t + I_t + (1-\zeta) \int^{\tilde{\omega}_t^G} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^G \\ &\quad + \mu \int^{\bar{\omega}_t^B} \omega dF_{t-1} R_t^K Q_{t-1} K_{t-1}^B + \kappa N_t^B. \end{aligned} \tag{26}$$

Goods are consumed, invested, wasted due to default by G firms and B firms, and spent as initialization costs for B firms. Finally, the debt market clears:

$$D_t = B_t. \tag{27}$$

3.6. Shocks

There are two shocks in the economy, namely a TFP shock and a shock to σ_t, the cross-sectional variance of ω, which we call a risk shock, following Christiano, Motto and Rostagno

[23] The parameter τ is small in the calibration, and it has little impact to the dynamics of the system.

[24] The net transfer from households is given by:

$$\begin{aligned} tr_t &= \tau N_{t-1} - (1-\theta) \int_{\tilde{\omega}_t^G} (\omega - \bar{\omega}_t^G) R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} \\ &\quad -(1-\theta) \int^{\tilde{\omega}_t^G} \zeta \omega R_t^K Q_{t-1} K_{t-1}^G dF_{t-1} - (1-\theta) \int_{\bar{\omega}_t^B} (\omega - \bar{\omega}_t^B) R_t^K Q_{t-1} K_{t-1}^B dF_{t-1}. \end{aligned}$$

(2014). These shocks follow exogenous AR(1) processes as follows:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{At}, \qquad \epsilon_{At} \sim N(0, s_A^2) \tag{28}$$

$$\ln \sigma_t = (1-\rho_\sigma) \ln \sigma + \rho_\sigma \ln \sigma_{t-1} + \epsilon_{\sigma t}, \qquad \epsilon_{\sigma t} \sim N(0, s_\sigma^2). \tag{29}$$

This completes the description of the model.[25]

4. Model properties

In this section, we explore the key properties of the credit contracts. Due to the different payoff structures for the borrowers and lenders in secured and unsecured contracts, the resulting leverage ratios for G firms and B firms as well as their dynamics are different. We first discuss the leverage ratios in levels, and then turn to their dynamics over the business cycle.

4.1. Level of leverage ratios

We first study the borrowers. Their credit demand satisfies the first order conditions (20) and (21). Let us define $E_t(R^K_{t+1})/R_t$ as the external finance premium. By rearranging the first order conditions, we relate the external finance premium to the cutoff values for B firms and G firms respectively:[26]

$$E_t\left(\frac{R^K_{t+1}}{R_t}\right) = E_t \rho^B(\bar{\omega}^B_{t+1}; \sigma_t) \geq 1, \tag{30}$$

$$E_t\left(\frac{R^K_{t+1}}{R_t}\right) = E_t \rho^G(\tilde{\omega}^G_{t+1}, \xi_{t+1}; \sigma_t) \geq 1. \tag{31}$$

We discuss the key properties of the ρ^B and ρ^G functions. First, $\partial\rho^B/\partial\bar{\omega}^B > 0$ and $\partial\rho^G/\partial\tilde{\omega}^G > 0$. For a given risk-free rate, a higher expected return on capital induces firms to take on more debt and risk higher chance of default, so the cutoff values rise. Second, ρ^B and ρ^G, and so the external finance premium, are weakly greater than unity in equilibrium due to financial frictions. Resources lost in the event of default have to be compensated by the wedge between the risk-free rate and the expected return on capital. Third, $\partial\rho^B/\partial\sigma$ and $\partial\rho^G/\partial\sigma > 0$ because, given the same external finance premium, a more spread-out distribution of idiosyncratic shock increases the chance of default, which induces firms to borrow less.

[25] Appendix A shows the full system.

[26] The functional forms of ρ^B and ρ^G and proofs regarding their mathematical properties are provided in Appendix B.

Most important, for any given cutoff value $\bar{\omega}$, the slope of ρ^G with respective to the cutoff value is always steeper than the slope of ρ^B. The difference in slopes is due to the different payoff structures to the borrowers of secured and unsecured debt. For secured borrowers, when the realization of the idiosyncratic shock ω is below a certain default threshold $\bar{\omega}$, the borrower goes bankrupt and payoff is zero; when ω is above the default threshold, secured borrowers make the promised repayment and keep the profit, which is increasing linearly with ω. For unsecured borrowers, when $\omega < \bar{\omega}$, default occurs but the borrowers avoid bankruptcy with positive probability, so the payoff is positive and increasing in ω over this range; and when $\omega \geq \bar{\omega}$, unsecured borrowers make the promised repayment and the payoff is increasing in ω (at a faster rate). The piecewise linear payoff structure for both types of borrowers means that they enjoy the upside risk above the face value of their debt, and they have an incentive to borrow excessively and shift the downside risk to the lenders. This risk-shifting incentive is more prominent for secured borrowers, because the marginal value with respect to ω for secured borrowers is more 'convex' than for unsecured borrowers.

Therefore, suppose that a secured borrower and an unsecured borrower choose the same cutoff value at some given expected return. The above argument implies that a marginal increase in expected return would induce the secured borrower to demand more credit relative to the unsecured borrower, and the cutoff value for the secured borrower rises more than for the unsecured borrower. In other words, the slope of the function ρ^B with respect to the cutoff value is less steep than that of ρ^G.

The top panel of Figure 2 plots the ρ^B function (blue solid line) and the ρ^G function (red dashed line) against the cutoff value $\bar{\omega}$, fixing other variables at their steady-state values (with the benchmark calibration). As discussed above, these two lines are upward sloping, and the slope of ρ^G is always steeper than the slope of ρ^B for any $\bar{\omega}$.

We now turn to the lenders' side. It is useful to write the lenders' participation constraint in each market as a relationship between the cutoff threshold and the leverage ratio specific to the borrower type:

$$\phi_{t-1}^B = PC^B\left(\bar{\omega}_t^B, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right) \equiv \left\{1 - \frac{R_t^K}{R_{t-1}}\left[\int_{\bar{\omega}_t^B} \bar{\omega}_t^B dF_{t-1} + (1-\mu)\int^{\bar{\omega}_t^B} \omega dF_{t-1}\right]\right\}^{-1} \quad (32)$$

$$\phi_{t-1}^G = PC^G\left(\tilde{\omega}_t^G, \xi_t, \frac{R_t^K}{R_{t-1}}; \sigma_{t-1}\right) \equiv \left[1 - \frac{R_t^K}{R_{t-1}}\left(\xi_t \int_{\tilde{\omega}_t^G} \tilde{\omega}_t^G dF_{t-1}\right)\right]^{-1}. \quad (33)$$

For both participation constraints, the leverage ratio is increasing in the respective cutoff value. The reason is that, other things unchanged, when a firm borrows more, it has a higher chance of default, and the lender has to be compensated by a higher contractual interest rate, which implies a higher cutoff value.

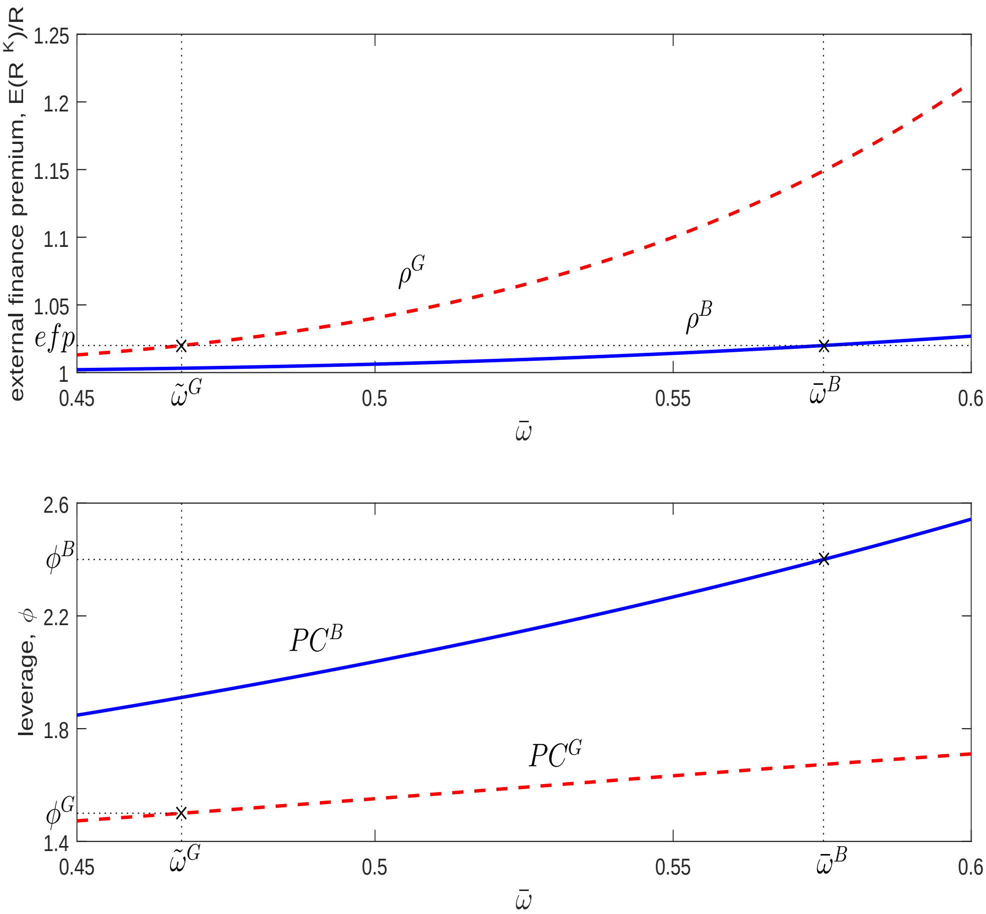

Fig. 2. Comparative static analysis of credit demand functions (ρ^G, ρ^B), and participation constraints (PC^G, PC^B) based on steady-state calibrations. The top panel plots the external finance premium with respect to cutoff value, and the bottom panel plots the leverage with respect to cutoff value. $\times$ denotes the steady state values of the external finance premium and leverage obtained by benchmark calibration in Section 5.

The two participation constraints have different slopes. Suppose a secured borrower and an unsecured borrower choose credit contracts that imply the same cutoff value. What happens if both borrowers ask for an additional unit of debt? The lender will require a larger increase in the cutoff value of the unsecured debt contract than in the secured debt contract. This is because the lender receives nothing when the unsecured borrower defaults; on the other hand, the lender receives the remaining value of the firm when the secured borrower defaults. In other words, for any given cutoff value, the slope of the participation constraint is steeper for secured lenders than unsecured lenders.

The bottom panel of Figure 2 plots the PC^B function (blue solid line) and the PC^G function (red dashed line) against the cutoff value $\bar{\omega}$, fixing other variables at their steady-state values. It is observed that the two lines are upward-sloping. Furthermore, PC^B is always above PC^G and the slope always steeper.

The differences in the relative slopes of the ρ functions and PC functions imply that, in equilibrium, the leverage ratio of G firms is always lower than the leverage ratio of B firms. Figure 2 illustrates the intuition. As explained above, in the top panel, ρ^G is steeper than ρ^B. Facing the same external finance premium, unsecured borrowers choose a lower cutoff value than secured borrowers, *i.e.*, $\tilde{\omega}^G_{t+1} < \bar{\omega}^B_{t+1}$. In the bottom panel, PC^B is steeper than PC^G, so we must have $\phi^B_t > \phi^G_t$. Our result that G firms have lower leverage than B firms is consistent with stylized fact 2.

4.2. Dynamics of leverage ratios

This subsection explains why our model is consistent with stylized fact 3, *i.e.* that unsecured debt has a higher correlation with output than secured debt. We start with a comparative static analysis to show the key intuition.

Figure 3 illustrates what happens when there is a negative TFP shock.[27] The top panels represent the credit demand functions (ρ^B, ρ^G), and the bottom panels show the participation constraints (PC^B, PC^G). The left and right panels refer to the secured and unsecured debt contracts respectively. The shock leads to a fall in the stock of capital, and a rise in expected return of capital, which drives up the external finance premium. From the figure, both secured and unsecured borrowers demand more credit per unit net worth, so cutoff values increase. The bottom panels show that a rise in the external finance premium, ceteris paribus, increases lenders' revenue for any $\bar{\omega}$, so the participation constraints shift up (from blue lines to red lines). As a result, the leverage ratios in both debt markets increase.

[27]The numerical values in Figure 3 correspond to the benchmark calibration to be discussed in the next section. We assume that the system is in the steady state initially. The magnitude of the rise in the external finance premium is the initial jump in the corresponding impulse response function. In this comparative statics exercise, we assume ξ and σ are fixed at the steady state.

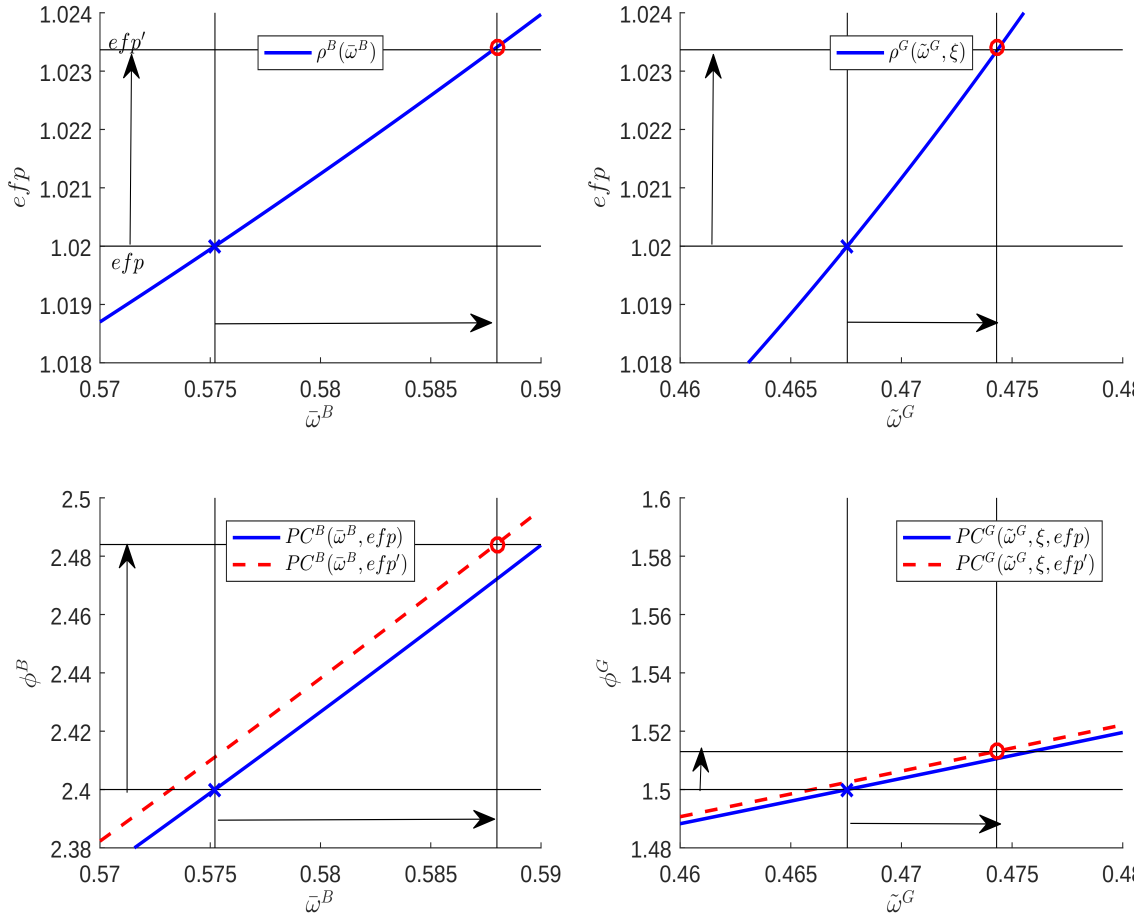

Fig. 3. This figure illustrates the relationships among cutoff values, the external finance premium and leverage. The top left and right panels plot the credit demand functions in secured and unsecured debt contracts. The bottom left and right panels plot the participation constraints in secured and unsecured debt contracts. All plots use the calibrated parameters in the benchmark calibration. Blue lines represent the steady-state relationships. Red lines show the relationships after the external finance premium increases by the initial jump in response to a one standard deviation negative TFP shock. The reputation value ξ and cross-sectional dispersion of idiosyncratic productivity σ are fixed at the steady state values throughout. $\times$ denotes the partial equilibrium before shock. o denotes the partial equilibrium after shock.

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Moreover, the rise in the external finance premium has different effects on secured and unsecured debt contracts. As ρ^G is steeper than ρ^B, so $\tilde{\omega}^G_{t+1}$ shifts to the right by less than $\bar{\omega}^B_{t+1}$. In addition, the participation constraint PC^G is less steep than PC^B, which further moderates the rise in ϕ^G, relative to ϕ^B.[28] In summary, the contractual features are such that in a downturn borrowers of secured debt are relative more willing to borrow and lenders of secured debt more willing to lend. As a result, the leverage ratio of secured borrowers rises more than the leverage ratio of secured borrowers.

How does this account for the cyclicality of secured and unsecured debt? To answer this question, we rewrite B^G_t and B^B_t in terms of their net worth and leverage ratios as follows:

$$B^G_t = (\phi^G_t - 1)N^G_t, \qquad B^B_t = (1-\kappa)(\phi^B_t - 1)N^B_t.$$

These equations state that the level of unsecured and secured debt is increasing in net worth and the leverage ratio. A negative shock reduces both N^G_t and N^B_t. But, as explained above, the leverage ratio of unsecured borrowers rises less than the leverage ratio of secured borrowers, unsecured debt falls more than secured debt. As a result, unsecured debt is more procyclical than secured debt.

In the rest of the paper, we study whether this channel can quantitatively match the cyclical pattern of secured and unsecured debt in the data.

5. Calibration

We solve and simulate the model numerically by log-linearizing the system around its non-stochastic steady state. This section discusses our calibration strategy.

Each period is a year. The parameters in production and household sectors are relatively standard in the macroeconomic literature and are given in Table 5. We set $\beta = 0.96$, which corresponds to around 4% steady-state interest rate. We set $\chi = 5$, so households devote 41 percent of their time to work. The parameter that governs the Frisch elasticity of labor supply is set to $\varphi = 1$. For production, the capital share is $\alpha = 0.33$, and the depreciation rate to $\delta = 0.08$. The curvature of investment adjustment costs Ψ^I and the consumption habit parameter h are set to match the output and investment volatility in the US data.

Our calibration strategy for financial parameters is as follows.[29] The survival rate of firms is $\theta = 0.87$ so that an average firm exits in 7.7 years.[30] Following Davydenko, Strebulaev and

[28] Of course, there is an effect coming from the change in reputation value ξ. It turns out that this effect is quantitatively small and dominated by the relative slopes of the credit demand functions and participation constraints.

[29] Appendix C shows the details of our calibration.

[30] Morris (2009) estimates that US firms have average life expectancies of 7 to 11 years.

Table 5
Calibrated parameters.

Parameter	Value	Meaning
β	0.96	Subjective discount factor
α	0.33	Capital share in production
δ	0.08	Capital depreciation rate
χ	5	Labor disutility
φ	1	Inverse of Frisch labor elasticity
Ψ^I	0.85	Convexity of investment adjustment costs
h	0.49	Consumption habit
θ	0.87	Firm survival probability
κ	0.017	Initialization cost for secured debt
μ	0.2	Default costs
ζ	0.31	Debt restructuring success rate
σ	0.257	Steady-state std. dev. of idiosyncratic shock
τ	0.068	Firm initial transfer
ρ_A	0.58	Persistence of TFP shock
ρ_σ	0.87	Persistence of risk shock
s_A	0.024	Std. dev. of TFP shock innovation
s_σ	0.023	Std. dev. of risk shock innovation

Zhao (2012), we set the default cost to $\mu = 0.2$, which is between the value used in BGG and Carlstrom and Fuerst (1997). We calibrate the remaining parameters to match four targets. First, the external finance premium R^K/R is 2% following Gilchrist and Zakrajsek (2012). Second, we target an unsecured debt to total debt ratio $B^G/B = 0.75$, consistent with the Compustat data in Figure 1. Third, we target a steady-state leverage ratio of B firms to $\phi^B = 2.4$. Fourth, we target a steady-state leverage ratio of G firms to $\phi^G = 1.5$. These leverage ratios are in line with the leverage ratios of firms with credit quality 'AA and above' and 'CC or below' in our dataset as well as the findings of Rauh and Sufi (2010). They imply that the aggregate leverage of the firm sector is 1.59, which is in between 2 used in BGG and 1.43 found in De Fiore and Uhlig (2011) for the period 1999-2007. These conditions pin down $\{\sigma, \zeta, \kappa, \tau\}$.

The shock parameters are calibrated as follows. We calibrate the persistence and standard deviation of the risk shock using annual industry-level TFP data in 1982-2011 by the National Bureau of Economic Research and the Center for Economic Studies. We linearly detrend each industry-level TFP series and compute the cross-sectional variance at each point in time. We fit an AR(1) process and obtain $\rho_\sigma = 0.87$ and $s_\sigma = 0.023$. This procedure follows Nuno and Thomas (2017). For the TFP shock we use the annual TFP series in 1982-2011 constructed by the CSIP at the Federal Reserve Bank of San Francisco. The log-TFP series is HP-filtered (smoothing parameter =100) fitted with an AR(1) process. We get $\rho_A = 0.58$ and $s_A = 0.024$.

6. Model results

6.1. Impulse responses

Figures 4 and 5 show the response of macroeconomic and financial variables to a one standard deviation fall in TFP and a one standard deviation increase in the cross-sectional dispersion of idiosyncratic productivity respectively. All variables are presented as percentage deviation from their steady-state value. For sectoral variables, the blue solid lines denote G firms and the red dashed lines denote B firms.

In Figure 4, a negative TFP shock reduces the realized return on capital. This reduces the net worth of all firms in the economy and limits their ability to borrow in subsequent periods. As a result, investment demand drops, the price of capital Q falls, and the external finance premium rises. A fall in the price of capital further reduces the realized return on capital, increasing the break-even contractual interest rate. Therefore, the cutoff values rise. This increases the initial fall in the net worth of the firms through the financial accelerator effect discussed in BGG. This effect leads to a large and persistent fall in output and investment.

We are interested in the dynamics of secured and unsecured debt, which is driven by the net worth effect and the leverage effect. Figure 4 shows that, as a negative TFP shock hits, N^B falls by more than 8% whereas N^G falls by around 5%. Since B firms borrow with a higher steady-state leverage ratio than G firms, N^B is more volatile. Meanwhile, ϕ^B rises by about 3.5%, which is more than four times the rise ϕ^G (0.8%). The mechanism behind the different responses in leverage ratio is explained in Section 4. A fall in net worth combined with a dampened response in the leverage ratio of G firms implies that unsecured debt falls strongly. By contrast, a sharp increase in the leverage ratio of B firms partially offsets the fall in secured debt, resulting in a small fall in secured debt.

Figure 5 shows the response to a rise in the cross-sectional dispersion of idiosyncratic firm productivity. This shock increases the default probability of the firms. Ceteris paribus, lenders require higher cutoff values to break even. This reduces firms' net worth and the price of capital, triggering the financial accelerator mechanism.

The effects of a risk shock on unsecured and secured debt borrowing can be understood similarly. Following the shock, both the price of capital and net worth fall. Since a risk shock is mean-preserving, its effect on the price of capital and firms' net worth is smaller than a TFP shock. The shock affects the leverage ratios through multiple channels. First, a risk shock increases the external finance premium. The secured and unsecured debt markets respond differently as the credit demand functions (ρ^G, ρ^B) and the participation constraints

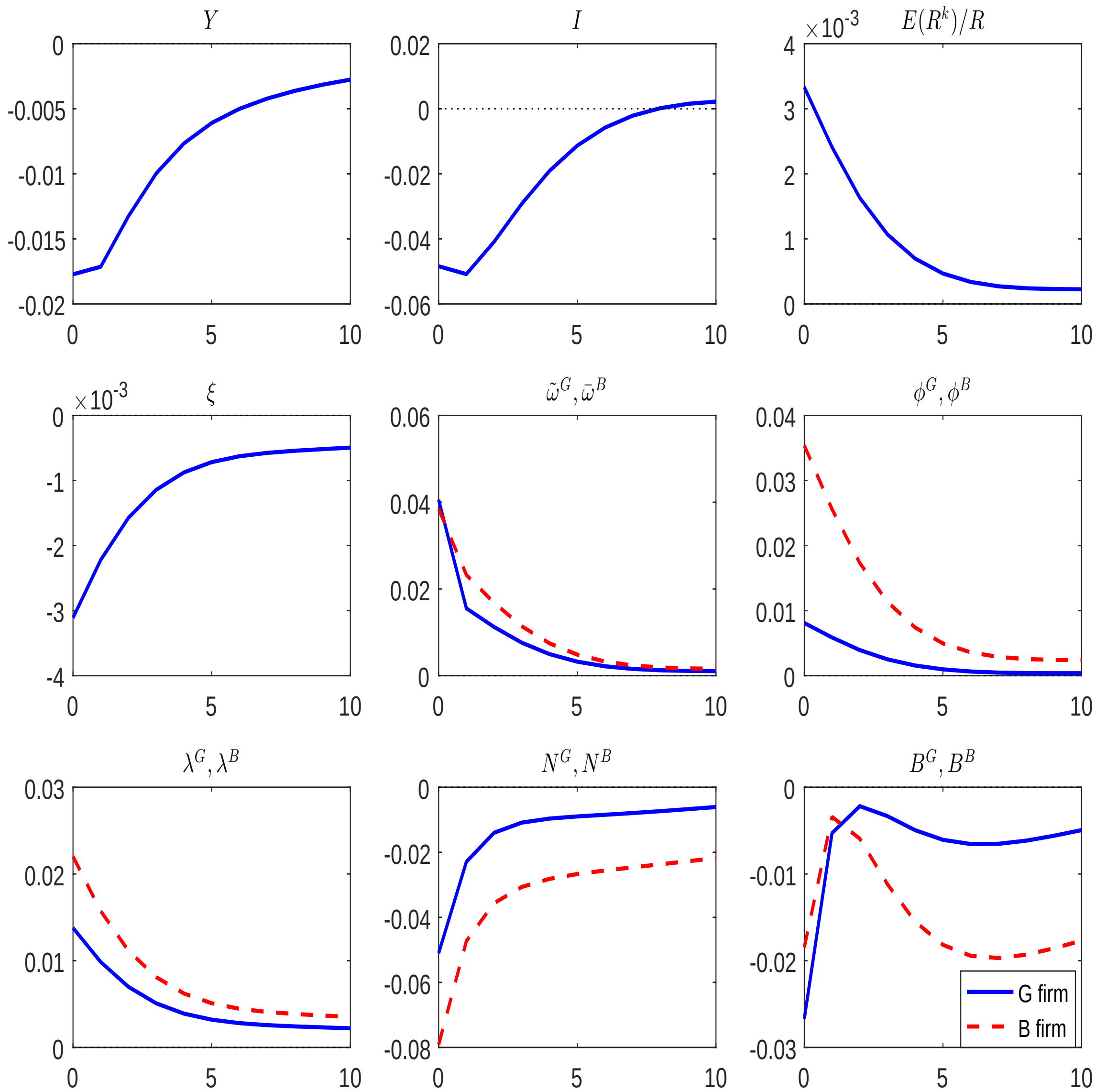

Fig. 4. Impulse response to a negative TFP shock. *Note*: The impulse response functions measure the response to a one standard deviation negative shock to the innovations in TFP as percent deviation from the steady state.

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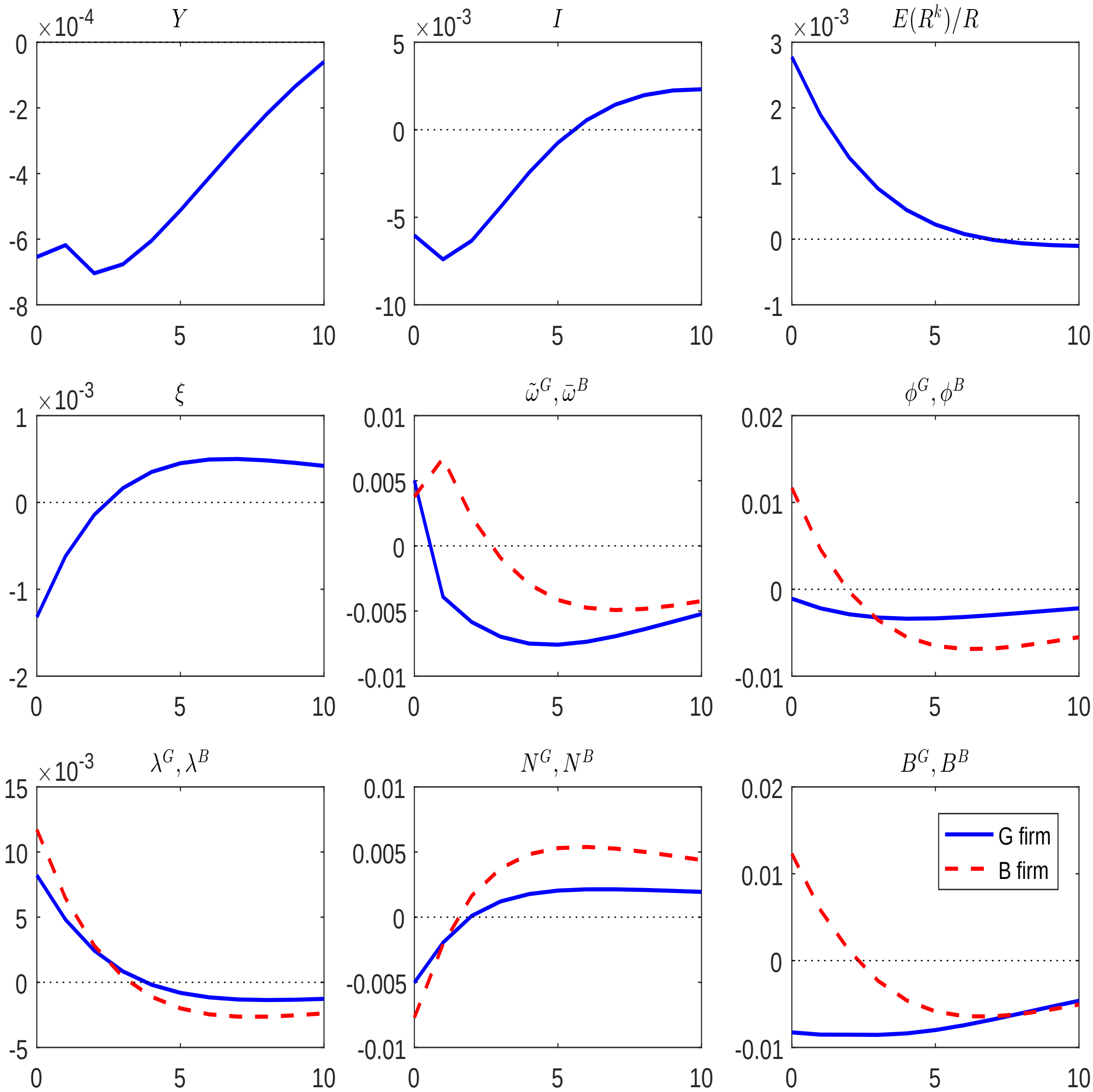

Fig. 5. Impulse response to a positive risk shock. *Note*: The impulse response functions measure the response to a one standard deviation increase in the innovations in the cross-sectional variance of the idiosyncratic shock as percent deviation from the steady state.

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(PC^G, PC^B) have different slopes. Second, a risk shock shifts up ρ^G and ρ^B. Intuitively, when there is more cross-sectional risk, firms borrow less for a given external finance premium. In equilibrium, the upward shift of ρ^G and ρ^B reduces the initial jump of the default thresholds $\tilde{\omega}^G_{t+1}$ and $\bar{\omega}^B_{t+1}$. Third, a risk shock shifts PC^G and PC^B downwards because, ceteris paribus, lenders have to cut lending to break even. So both ϕ^G_t and ϕ^B_t jump up by less. Figure 5 shows that, in response to the shock, the leverage ratio of B firms increases whereas the leverage ratio of G firms falls on impact. As a result, unsecured debt falls by about 1% and secured debt rises by more than 1%.

6.2. Comparing model with data

We now take the model to the U.S. data. We use the observed TFP and cross-sectional dispersion series to simulate the predicted output, investment, aggregate leverage, and the secured/unsecured debt ratio. Figure 6 plots the model predicted time series with the U.S. data for the period 1982-2011. As shown, the model is able to replicate the movements in output and investment over the sample period. Importantly, the model does a great job in matching the dynamics of the secured and unsecured debt ratio, including the long-lasting fall in the ratio between mid 1990s and mid 2000s as well as the sharp rise in the ratio during the Great Recession. Regarding the aggregate leverage, the model-generated series is highly countercyclical due to the financial accelerator effect. It captures relatively well the dynamics of leverage after 2000, though it fails to match the highly volatile and procyclical movement in leverage in the first half of the sample period.

Table 6 presents the model's performance along with the empirical moments. Panel A shows the standard deviation of output produced by the model, while Panel B and C report the relative standard deviation and correlation of other variables with output. The most important result that emanates from Table 6 is that the model is able to reproduce the cyclicality of secured and unsecured debt. Unsecured debt is highly procyclical with $Corr(B^G, Y) = 0.62$, whereas secured debt is only slightly procyclical $Corr(B^B, Y) = 0.22$. They are close to the corresponding empirical moments for rated firms (and all firms): 0.48(0.50) for unsecured debt and 0.06(0.15) for secured debt. The model performs well in terms of matching other moments characterizing the business cycle. Output and investment volatility perfectly match the US data as they are calibration targets. Consumption in the model is less volatile than output, although a bit less than its empirical counterpart. The correlations of consumption and investment with output in the model is also consistent with the data. Lastly, the model is able to reproduce the correlation of total debt with output.

The model underestimates the volatility of secured, unsecured, and total debt compared to the data. This result is not unexpected as a BGG type model is not able to generate large

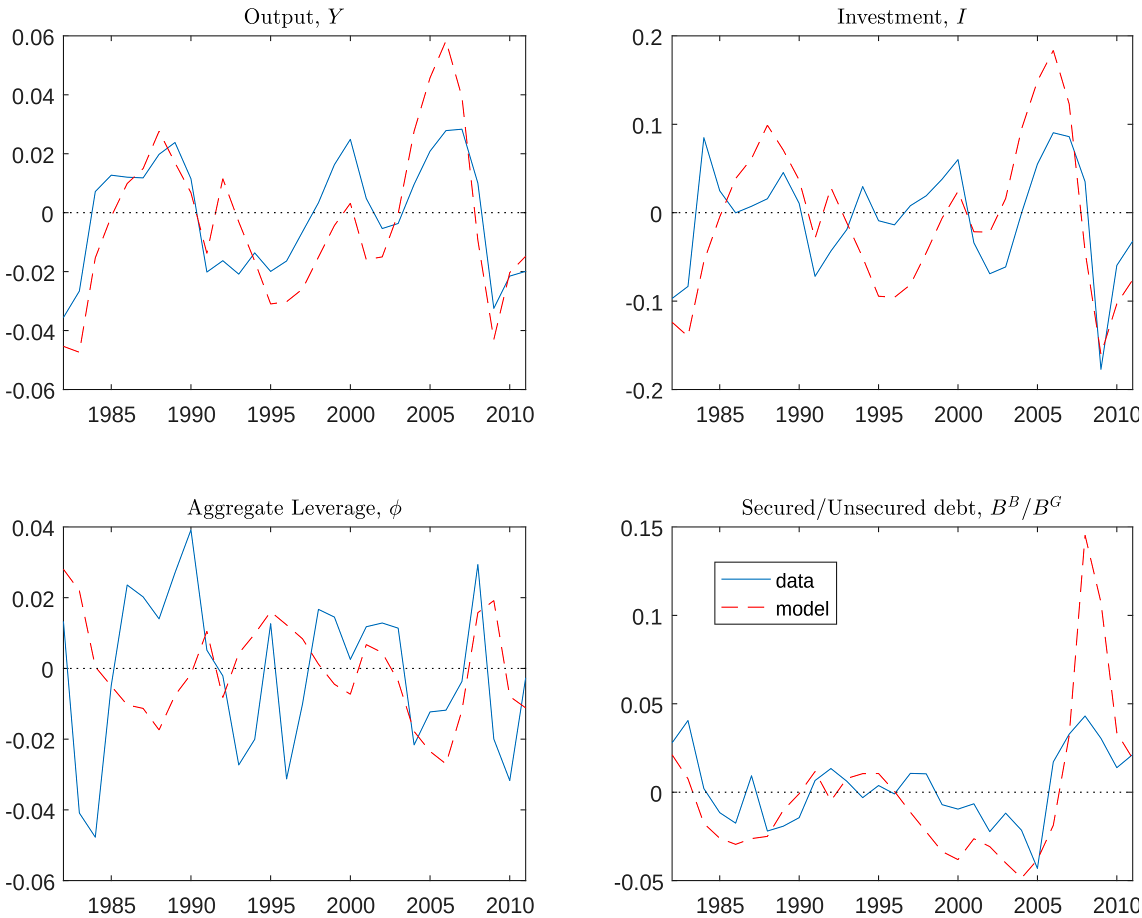

Fig. 6. Historical and model-generated series, all shocks. The historical series are in logs and HP-filtered with smoothing parameter 100. Model-generated series are log-deviations from steady state with smoothing parameter 100. Source: NBER, CES and Compustat.

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Table 6
Moments.

	U.S. Data	Benchmark Model
Panel A: Standard Deviation		
Output (Y)	1.81	1.81
Panel B: Standard Deviation/ std.(Y)		
Consumption (C)	0.90	0.60
Investment (I)	3.18	3.18
Unsecured Debt (B^G)	7.68	1.35
Secured Debt (B^B)	5.60	1.18
Total Debt (B)	4.39	1.18
Panel C: Correlation with Output		
Consumption (C)	0.94	0.97
Investment (I)	0.87	0.98
Unsecured Debt (B^G)	**0.48**	**0.62**
Secured Debt (B^B)	**0.06**	**0.22**
Total Debt (B)	0.53	0.58

Moments of U.S. data are computed by using annual data from 1981 to 2016. The numbers from the model are theoretical moments based on the benchmark calibration. Panel A reports the standard deviation of output. Panel B reports the relative standard deviations with respect to output. Panel C reports the contemporaneous correlations with output. Model-generated series are HP-filtered with smoothing parameter 100.

fluctuations in debt.[31] Rannenberg (2016) compares moments generated by different types of models with financial frictions and shows that a Gertler and Karadi (2011) type model with financial frictions in the banking sector can better match the standard deviation of debt to output. Nevertheless, our model is able to capture the relative size of the volatility of secured, unsecured, and total debt, with unsecured debt the most volatile and total debt the least.

7. Comparison with one-sector financial accelerator model

What are the macroeconomic implications of using a model with both secured and unsecured debt? To answer this question, we compare our model with a standard one-sector BGG model and a real business cycle (RBC) model without financial frictions.[32]

To make the comparison fair, we calibrate the one-sector BGG model to have the same steady-state aggregate leverage ratio and external finance premium as the benchmark model.

[31] Using our calibrated parameters, a standard BGG model yields $std(B)/std(Y) = 1.18$, much smaller than US data.

[32] We describe the details of the one-sector BGG system and the RBC model in the Appendix.

We assume that the monitoring technology μ is available to the one sector BGG model. Finally, we assume that in the one-sector BGG model the initialization cost is given by $\tilde{\kappa} = \kappa \bar{N}^B / \bar{N}$, where $\bar{N}^B$ are $\bar{N}$ are the steady-state value of N_t^B and N_t in the benchmark model. This means that the initialization costs are now shared evenly by every firm. Other parameters are common across all three models. With these assumptions the steady state of the one-sector BGG model is very close to the benchmark model.

Figure 7 and 8 show the impulse responses to a negative TFP shock and a positive risk shock.[33] The financial mechanism embedded in the benchmark model and one-sector BGG model leads to more volatile fluctuations in macroeconomic variables, relative to the RBC model. Furthermore, our model has stronger amplification than the one-sector BGG model, as shown by the larger fall in aggregate net worth and debt, and also by the sharper increase in the external finance premium. For instance, in response to a risk shock, the fall in output and investment nearly doubles that in the one-sector BGG model; whereas the fall in aggregate debt, asset prices and the external finance premium more than double.

To understand these results, notice that the financial accelerator mechanism has two effects, which affect macroeconomic volatility in opposite directions. In response to a negative shock, firms' net worth falls, but firm leverage increases. In the one-sector BGG model, the fall in firms' net worth dominates, so debt falls at a time when firms' net worth is low. This financial accelerator mechanism magnifies the fall in investment and the price of capital. In our model, both secured and unsecured debt are subject to the financial accelerator mechanism. Moreover, for unsecured borrowers, the rise in the leverage ratio is dampened in a downturn, so their borrowing capacity is even lower. Overall, there is even less borrowing in the benchmark economy relative to the one-sector BGG model, and hence larger volatility.

8. Model extensions

In this section we discuss four model extensions. The purpose is to show that our key mechanism holds under a more general environment. We outline each of the extensions below and report their key moments (i.e. the correlations of secured and unsecured debt with output) in Table 7.[34] Note that in all extensions, unsecured debt remains highly procyclical whereas secured debt is much less procyclical.

[33] Using the way we calibrate the one-sector BGG model, $\bar{\sigma}^{BGG} = 0.465$, which is bigger than $\bar{\sigma}$ in the benchmark model. In Figure 8 we show the impulse reponses to a 1 s.d. innovation o cross-sectional dispersion in the benchmark model, together with a (0.257/0.465) s.d. innovation of cross-sectional dispersion in the one-sector BGG model.

[34] Detailed descriptions of each of these extensions are discussed in an additional appendix available from the authors.

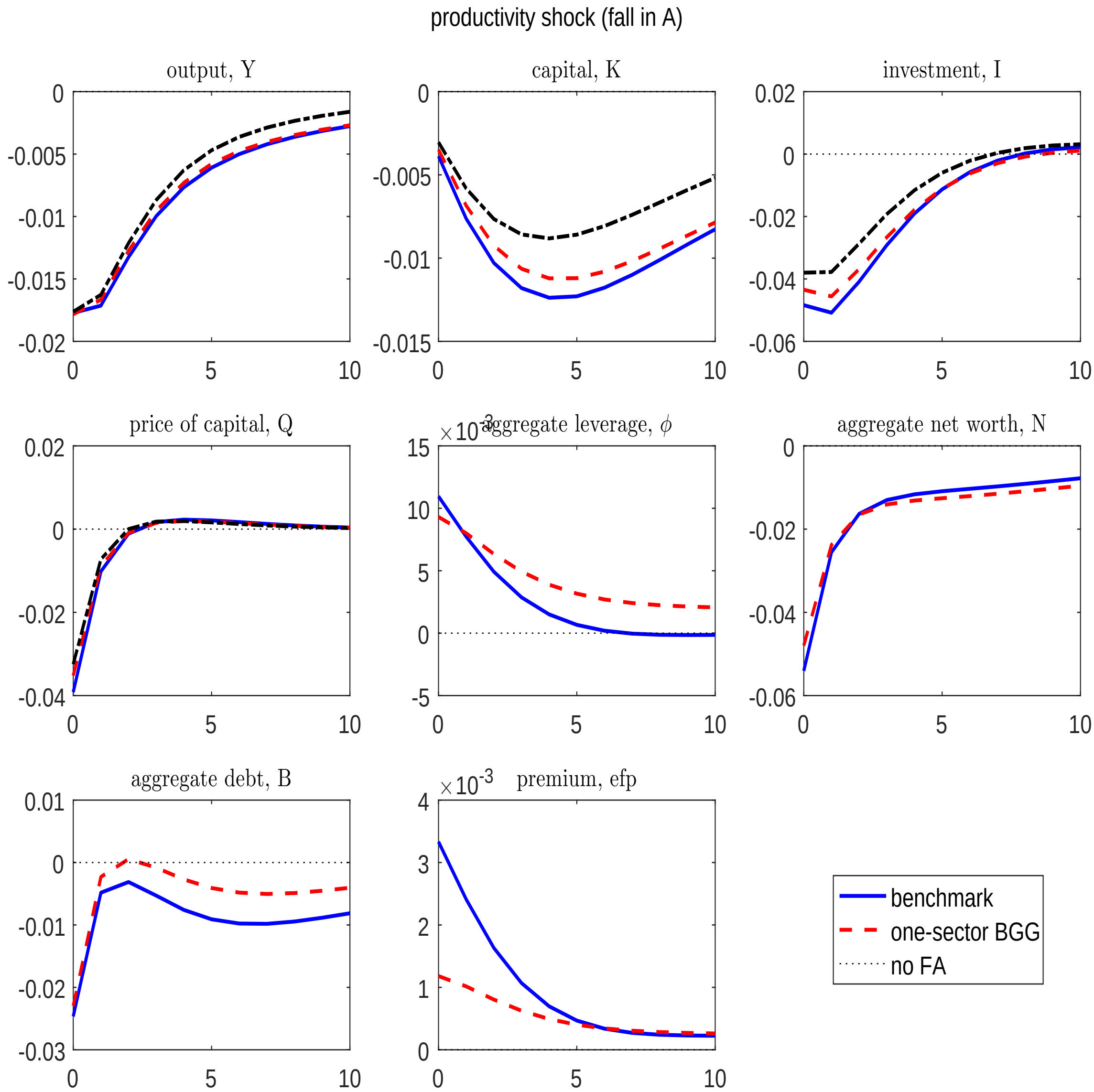

Fig. 7. Impulse response to a negative TFP shock. *Note*: The impulse response functions measure the response to a one standard deviation negative shock to the innovations in TFP as percent deviation from the steady state.

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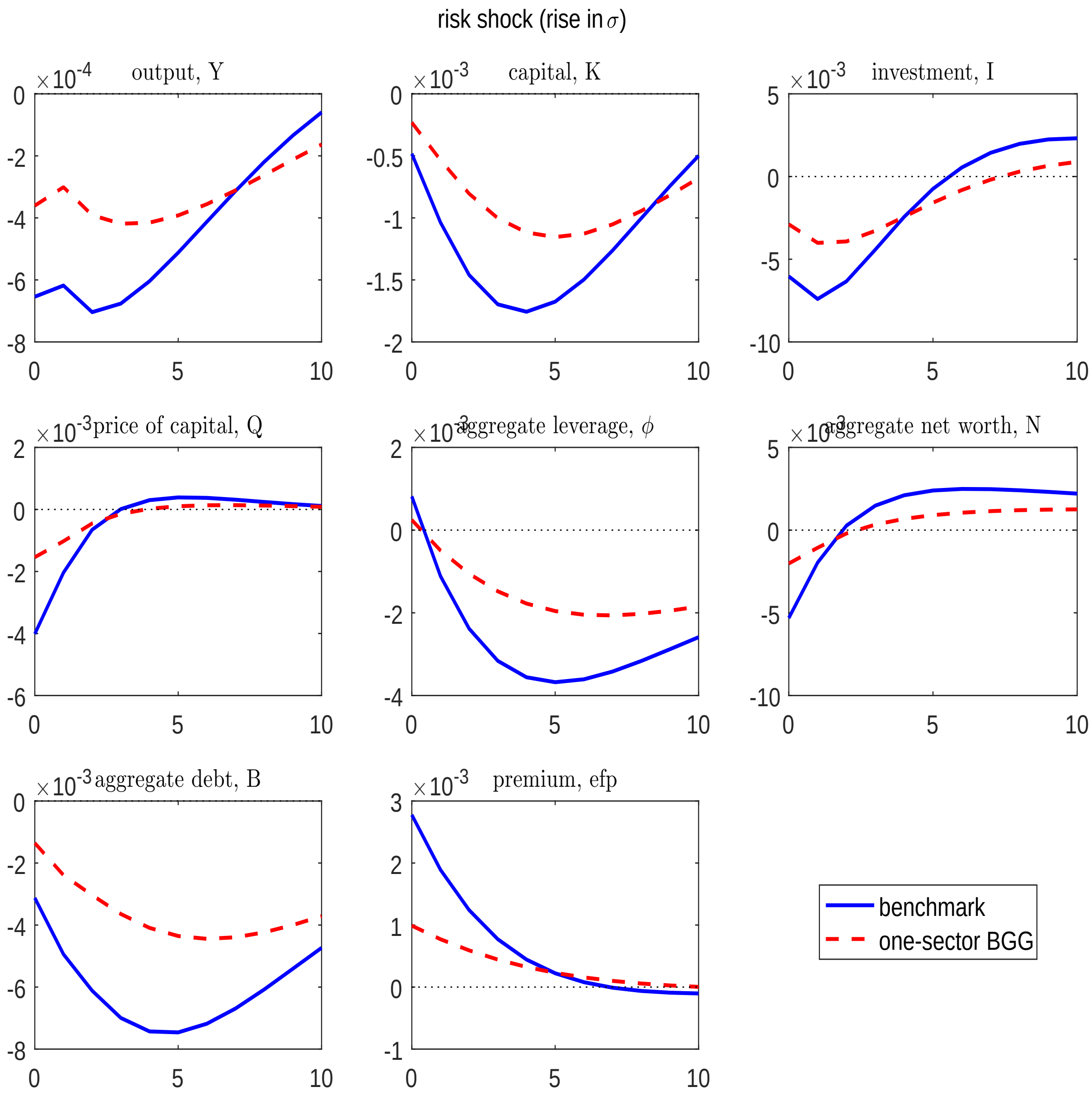

Fig. 8. Impulse response to a positive risk shock. *Note*: The impulse response functions measure the responses to a one standard deviation increase in the innovations in the cross-sectional variance of the idiosyncratic shock in the benchmark model as percent deviation from the steady state. The shock in the BGG model has the same size.

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Table 7
Model extensions.

	Correlations with output	Unsecured Debt	Secured Debt
Data	Rated firms	0.48	0.06
	All firms	0.50	0.15
Model	Benchmark	0.62	0.22
	Credit upgrade	0.62	0.23
	Positive recovery ratio	0.64	0.30
	Different avg. productivity	0.58	0.28
	Mixed debt	0.76	0.36

Note: This table reports the contemporaneous correlations with output.

8.1. Credit upgrade

In the benchmark model, firms that are downgraded to B firms will not become G firms in any future periods. In reality some firms do regain high credit ratings and favorable terms with lenders. We allow for this in the current extension. Following Cui and Kaas (forthcoming), we assume there is an exogenous probability γ^{up} that a B firm becomes a G firm in a given period. We also assume an exogenous probability γ^{down} that a G firm becomes a B firm in the next period. To implement this, the future marginal values of net worth are modified to:

$$\Omega_t^G = \theta[(1-\gamma^{down})\lambda_t^G + \gamma^{down}\lambda_t^B] + 1 - \theta, \tag{34}$$

$$\Omega_t^B = \theta[(1-\gamma^{up})\lambda_t^B + \gamma^{up}\lambda_t^G] + 1 - \theta. \tag{35}$$

The rest of the credit contract equations remain unchanged. For small values of γ^{up} and γ^{down}, all of our analytical results remain valid. To simulate this model, we set the credit upgrade parameter to $\gamma^{up} = 0.1$ to corresponds to a firm staying at a B rating for 10 years on average.[35] The exogenous downgrade is set to $\gamma^{down} = 0.013$ which keeps the steady-state ratio of net worth in G and B firms roughly the same as in the benchmark model.

8.2. Positive recovery ratio in unsecured debt

We assume in the benchmark model that lenders of unsecured debt do not have access to a firm's asset when it defaults. In reality there is usually a secondary market for distressed unsecured debt, so the recovery rate is not zero. In this extension we allow for a positive recovery rate for unsecured debt. Specifically, we assume that if a unsecured borrower defaults, lenders get a fraction $\varrho < 1$ of the remaining value of the firm. The borrower has a

[35] This calibration corresponds to the bankruptcy flag for sole proprietors filing for bankruptcy under Chapter 7 of US Bankruptcy Code.

probability ζ of retaining the remaining $(1-\varrho)$ fraction of net worth and becomes a B firm. With probability $(1-\zeta)$ the borrower gets nothing.[36] We set $\varrho = 0.3$ in our simulations, matching the mean recovery ratio of unsecured debt in the Moody's Default Risk Service (DRS) dataset for the period 1970-2008.

8.3. Different average productivity

The benchmark model does not allow for *ex ante* productivity to differ across firms. As a result, all firms face the same expected return on capital $E_t R^K_{t+1}$. In this extension we relax this assumption. Specifically, in each period a fraction π of firms have high productivity A^H, and the remaining $(1-\pi)$ fraction has low productivity A^L such that $A_H > A_L$. For simplicity, productivity in each period is uncorrelated. Firms produce with the following Cobb-Douglas production function:

$$Y^{m,i}_{jt} = A_t A^m (\omega_{jt} K^{m,i}_{jt-1})^{\alpha} (L^{m,i}_{jt})^{1-\alpha}, \tag{36}$$

where A_t denotes the TFP of the economy, A^m where $m \in \{H, L\}$ is the firm's productivity type such that $A^H > A^L$, and ω_{jt} is an idiosyncratic shock to a firms' capital quality. A^m has an i.i.d two point distribution with $Pr(A^H) = \pi$ and mean 1. Its realization is observed by lenders when the loan contracts are decided.

Now, the average return on capital of the firm whose current productivity is A^m is given by:

$$R^{m,K}_t \equiv \frac{r^{m,K}_t + (1-\delta)Q_t}{Q_{t-1}}. \tag{37}$$

where $r^{m,K}_t \equiv \alpha A_t A^m \left(\frac{(1-\alpha)A_t A^m}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$. Clearly $R^{H,K}_t > R^{L,K}_t$.

For each average productivity $\{A^H, A^L\}$, there are G and B firms. More importantly, all of our analytical results hold for G and B firms with the same average productivity. But firms with average productivity A^H face a higher expected return and external finance premium than firms with low average productivity A^L.

In the simulation exercise, we set the fraction of productive firms to be $\pi = 20\%$, which is common in the literature. We choose $A^H = (1.15)^{\alpha}$, and $A^L = (1-\pi A^H)/(1-\pi)$ so that the unconditional productivity is 1.

8.4. Mixed debt in low credit quality firms

In the data, low credit quality firms usually have a multi-tier debt structure, borrowing both secured and unsecured debt (Rauh and Sufi (2010)). In this extension, we assume that

[36] The benchmark model is a special case in which $\varrho = 0$.

B firms borrow a fixed fraction $(1-\nu)$ of unsecured debt and the remaining fraction ν of secured debt. For simplicity, assume that a firm either repays or defaults all its debt obligations. In the case of default, the secured lender is entitled to $\nu(1-\mu)\omega_{jt+1}R^K_{t+1}Q_tK^B_{jt}$ fraction of assets after monitoring. The default B firm undergoes debt restructuring. With probability ζ, debt restructuring is successful and the firm retains $(1-\mu)(1-\nu)\omega_{jt+1}R^K_{t+1}Q_tK^B_{jt}$, but it loses its B label and is excluded from *any* loans in future. With probability $(1-\zeta)$, debt restructuring is unsuccessful, the firm shuts down and has nothing left.

We show that it is optimal for a B firm to choose to default when $\omega < \tilde{\omega}^B_t$, where $\tilde{\omega}^B_t = (\xi^B_t)^{-1}\bar{\omega}^B_t$, and $\xi^B_t \leq 1$ is the reputation value of being a B firm. Furthermore, the value of a firm is still given by $V^i_t(N^i_{jt}) = \lambda^i_t N^i_{jt}$ for $i \in \{G, B, X\}$, where $\lambda^G_t > \lambda^B_t > \lambda^X_t > 1$ for all t, and 'X' is the label for a firm which is excluded from the financial market.

We use the same calibration strategy for financial parameters, targeting a 75% unsecured debt to total debt ratio $[B^G + (1-\nu)B^B]/(B^G + B^B)$, and a $\nu = 80\%$ secured debt share in B firms.

9. Conclusion

In this paper, we study the important features of firms' debt structure. We find that firms with a high-credit-rating rely almost exclusively on unsecured debt, while those with a low credit quality use a multi-tiered debt structure often consisting of a large share of secured debt. We show that debt heterogeneity is a first-order aspect of firms' capital structure, and is essential to the understanding of debt dynamics and cyclical fluctuations.

We embed secured and unsecured debt in a dynamic stochastic general equilibrium model featuring costly state verification. In our model, unsecured borrowers may default and still keep their assets, which allows them to strategically default on their borrowing and run the risk of losing their high credit rating. Under this contractual arrangement, market participants of unsecured debt are relatively cautious, relative to participants in the secured debt market. This accounts for low leverage ratios in high-credit-rating firms. This effect implies that lenders cut lending disproportionately on unsecured debt in a recession, thus leading to a higher correlation between output and unsecured debt than for secured debt.

A calibrated version of our economy matches well with the observed volatility and correlations of output, firm credit, and investment. We find that the amplification effect of an economic shock in our model is larger than that generated by a model featuring secured debt only. We conclude that unsecured debt and its dynamics are important to a better understanding of fluctuations in business cycles.

10. Acknowledgments

The views expressed in this paper are those of the authors and do not necessarily reflect those of the Hong Kong Monetary Authority, the Hong Kong Institute of Monetary and Financial Research, or the International Monetary Fund. We would like to thank two anonymous referees, Wei Cui, Nan Hu, John Kuong, Sang Seok Lee, Marco del Negro, John Taylor, Zhiwei Xu and participants in Economic Fluctuation and Growth Research Workshop (Spring 2018), 2nd Research Conference of the CEPR Network on MMCN and 14th Dynare conference for their helpful comments and suggestions. All errors are our own.

Appendix A. Full system

The full system has a macroeconomic part and a credit contract part. The macroeconomic part is given by:

$$\Lambda_{t-1,t} = \beta \frac{C_{t-1}}{C_t} \tag{A.1}$$

$$1 = R_t E_t(\Lambda_{t,t+1}) \tag{A.2}$$

$$w_t = \chi L_t^{\varphi} U_{Ct}^{-1} \tag{A.3}$$

$$w_t L_t = (1-\alpha) Y_t \tag{A.4}$$

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{A.5}$$

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\psi^I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{A.6}$$

$$Y_t = C_t + I_t + (1-\zeta) G(\tilde{\omega}_t^G) R_t^K Q_{t-1} K_{t-1}^G + \mu G(\bar{\omega}_t^B) R_t^K Q_{t-1} K_{t-1}^B + \kappa N_t^B \tag{A.7}$$

$$\begin{aligned} 1 = {} & Q_t \left[1 - \frac{\psi^I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \psi^I \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] \\ & + E_t \left[\Lambda_{t,t+1} Q_{t+1} \psi^I \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right] \end{aligned} \tag{A.8}$$

$$R_t^K = \frac{\alpha \frac{Y_t}{K_{t-1}} + (1-\delta) Q_t}{Q_{t-1}} \tag{A.9}$$

The credit contract part:

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left\{1 - \xi_{t+1}[G(\tilde{\omega}_{t+1}^G) + \tilde{\omega}_{t+1}^G(1 - F(\tilde{\omega}_{t+1}^G))]\right\} \quad (A.10)$$

$$1 - \frac{1}{\phi_{t-1}^G} = \frac{R_t^K}{R_{t-1}} \xi_t \tilde{\omega}_t^G [1 - F(\tilde{\omega}_t^G)] \quad (A.11)$$

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1}(1 - F(\tilde{\omega}_{t+1}^G))}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1}[1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]} \quad (A.12)$$

$$\bar{\omega}_t^G = \xi_t \tilde{\omega}_t^G \quad (A.13)$$

$$\xi_t = 1 - \frac{\zeta(\theta\lambda_t^B + 1 - \theta)}{\Omega_t^G} \quad (A.14)$$

$$\lambda_t^B = (1 - \kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K [1 - G(\bar{\omega}_{t+1}^B) - \bar{\omega}_{t+1}^B(1 - F(\bar{\omega}_{t+1}^B))] \quad (A.15)$$

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}} \left\{\bar{\omega}_t^B [1 - F(\bar{\omega}_t^B)] + (1 - \mu)G(\bar{\omega}_t^B)\right\} \quad (A.16)$$

$$\lambda_t^B = \frac{(1 - \kappa)E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t}[1 - F(\bar{\omega}_{t+1}^B) - \mu\bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]} \quad (A.17)$$

$$K_t = K_t^G + K_t^B \quad (A.18)$$

$$Q_t K_t^G = N_t^G \phi_t^G \quad (A.19)$$

$$Q_t K_t^B = (1 - \kappa) N_t^B \phi_t^B \quad (A.20)$$

$$N_t^G = \left(\theta R_t^K \phi_{t-1}^G \{1 - G(\tilde{\omega}_t^G) - \bar{\omega}_t^G [1 - F(\tilde{\omega}_t^G)]\} + \tau\right) N_{t-1}^G \quad (A.21)$$

$$N_t^B = \zeta G(\tilde{\omega}_t^G) \theta R_t^K \phi_{t-1}^G N_{t-1}^G + (1 - \kappa)\theta\{1 - G(\bar{\omega}_t^B) - \bar{\omega}_t^B [1 - F(\bar{\omega}_t^B)]\} R_t^K \phi_{t-1}^B N_{t-1}^B + \tau N_{t-1}^B \quad (A.22)$$

$$\Omega_t^B = \theta\lambda_t^B + 1 - \theta \quad (A.23)$$

$$\Omega_t^G = \theta\lambda_t^G + 1 - \theta \quad (A.24)$$

where $f(\bar{\omega}_t; \sigma_{t-1}) \equiv \frac{\partial}{\partial \bar{\omega}_t} F(\bar{\omega}_t; \sigma_{t-1})$ is the probability density function of $\bar{\omega}_t$, and $G(\bar{\omega}_t; \sigma_{t-1}) \equiv \int^{\bar{\omega}_t} \omega dF(\omega, \sigma_{t-1})$. The above 24 equations solve the following 24 variables

$$\{\Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R_t^K, R_t, \lambda_t^G, \phi_t^G, \tilde{\omega}_t^G, \bar{\omega}_t^G, \xi_t, N_t^G, K_t^G, \Omega_t^G, \lambda_t^B, \phi_t^B, \bar{\omega}_t^B, N_t^B, K_t^B, \Omega_t^B\}.$$

Appendix A.1. BGG system

This appendix presents the BGG system. The macroeconomic part is identical to our benchmark model, except that the goods market clearing condition is now given by:

$$Y_t = C_t + I_t + \mu G(\bar{\omega}_t) R_t^K Q_{t-1} K_{t-1} + \tilde{\kappa} N_t \quad (A.25)$$

The credit contract part is:

$$\lambda_t = (1-\tilde{\kappa})\phi_t E_t \Lambda_{t,t+1}\Omega_{t+1}R^K_{t+1}[1-G(\bar{\omega}_{t+1})-\bar{\omega}_{t+1}(1-F(\bar{\omega}_{t+1}))] \quad \text{(A.26)}$$

$$1-\frac{1}{\phi_{t-1}} = \frac{R^K_t}{R_{t-1}}\{\bar{\omega}_t[1-F(\bar{\omega}_t)]+(1-\mu)G(\bar{\omega}_t)\} \quad \text{(A.27)}$$

$$\lambda_t = \frac{(1-\tilde{\kappa})E_t\Lambda_{t+1}\Omega_{t+1}R^K_{t+1}[1-F(\bar{\omega}_{t+1})]}{E_t\frac{R^K_{t+1}}{R_t}[1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}f(\bar{\omega}_{t+1})]} \quad \text{(A.28)}$$

$$Q_tK_t = (1-\tilde{\kappa})N_t\phi_t \quad \text{(A.29)}$$

$$N_t = (1-\tilde{\kappa})\theta\{1-G(\bar{\omega}_t)-\bar{\omega}_t[1-F(\bar{\omega}_t)]\}R^K_t\phi_{t-1}N_{t-1}+\tau N_{t-1} \quad \text{(A.30)}$$

$$\Omega_t = \theta\lambda_t+1-\theta \quad \text{(A.31)}$$

The 15-equation system solves the following 15 variables:

$$\{\Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R^K_t, R_t, \lambda_t, \phi_t, \bar{\omega}_t, N_t, \Omega_t\}.$$

Appendix A.2. The simple RBC system

The simple RBC system has a macroeconomic system similar to our benchmark model, except that the goods market clearing condition is now given by:

$$Y_t = C_t + I_t, \quad \text{(A.32)}$$

and the return on capital is equal to the risk-free rate:

$$1 = E_t\left[\Lambda_{t,t+1}\frac{\alpha\frac{Y_{t+1}}{K_t}+(1-\delta)Q_{t+1}}{Q_t}\right]. \quad \text{(A.33)}$$

The system solves the following 9 variables:

$$\{\Lambda_{t-1,t}, C_t, w_t, L_t, Y_t, K_t, I_t, Q_t, R_t\}.$$

Appendix B. Proofs

This appendix provides the mathematical proofs. The first proposition proves the optimality of the solution to the optimal financial contracting problem stated in Section 3. The remaining five propositions establish the properties of the ρ^G, ρ^B and PC^G, PC^B functions claimed in Section 4.

Proposition 1. *Suppose that initialization cost satisfies some $\kappa \in (\kappa_0, \kappa_1)$. The solution to the firms' problem is characterized by the following features:*

1. *G firms only borrow unsecured debt.*
2. *The default strategy of G firms is given by:*

$$\xi_t = 1 - \zeta \frac{\Omega_t^B}{\Omega_t^G} \leq 1, \tag{B.1}$$

   where $\Omega_t^i \equiv \theta \lambda_t^i + 1 - \theta$ for $i \in \{B, G\}$.

3. *The marginal values of net worth for G firms and B firms evolve as follows:*

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t, \tag{B.2}$$

$$\lambda_t^G = \phi_t^G E_t \Lambda_{t,t+1} \Omega_{t+1}^G R_{t+1}^K \left[(1-\xi_{t+1}) \int^{\tilde{\omega}_{t+1}^G} \omega dF_t + \int_{\tilde{\omega}_{t+1}^G} (\omega - \tilde{\omega}_{t+1}^G) dF_t \right]. \tag{B.3}$$

4. *The default threshold for secured debt contract, $\bar{\omega}_t^B$, satisfies:*

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K [1 - F(\bar{\omega}_{t+1}^B)]}{E_t \frac{R_{t+1}^K}{R_t} [1 - F(\bar{\omega}_{t+1}^B) - \mu \bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]}. \tag{B.4}$$

5. *The default threshold for unsecured debt contract, $\tilde{\omega}_t^G$, satisfies:*

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G)]}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1} [1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]}. \tag{B.5}$$

6. *The participation constraints hold with equality.*

Proof of proposition 1: The proof has two parts. The first part assumes that G firms only borrows unsecured debt and derives the optimal decisions for the borrowers. The second part proves that for κ bigger than some κ_0 defined below, even if G firms can use secured debt, they will choose not to.

Part 1: With perfect competition, the participation constraints hold with equality. We begin by solving the problem for the B firms. We substitute the guess into the objective function. The objective function is rewritten as:

$$V_t^B(N_{jt}^B) = \max E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t, \tag{B.6}$$

where $\Omega_t^B \equiv \theta \lambda_t^B + 1 - \theta$.

We write down the Lagrangian as

$$\begin{aligned} V_t^B(N_{jt}^B) &= \max E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t \\ &+ lm_{jt}^B \left[\frac{R_{t+1}^K}{R_t} \frac{Q_t K_{jt}^B}{(1-\kappa)} \left(\int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{Q_t K_{jt}^B}{(1-\kappa)} + N_{jt}^B \right], \end{aligned}$$

where lm_{jt}^B is the Lagrange multiplier. The envelope condition says that $\lambda_t^B = lm_{jt}^B$. The first order condition for K_{jt}^B is:

$$\begin{aligned} K_{jt}^B \ &: \ 0 = E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \left(\int_{\bar{\omega}_{jt+1}^B} (\omega - \bar{\omega}_{jt+1}^B) dF_t \right) \\ &+ \lambda_t^B \left[\frac{R_{t+1}^K}{R_t} \frac{1}{(1-\kappa)} \left(\int_{\bar{\omega}_{jt+1}^B} \bar{\omega}_{jt+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{jt+1}^B} \omega dF_t \right) - \frac{1}{(1-\kappa)} \right]. \end{aligned} \tag{B.7}$$

In this equation, $\bar{\omega}_{jt}^B$ is the only firm-specific variable. This implies that every firm chooses the same cutoff value $\bar{\omega}_t^B$. The participation constraint implies every firm chooses the same leverage ratio:

$$\frac{R_{t+1}^K}{R_t} \left(\int_{\bar{\omega}_{t+1}^B} \bar{\omega}_{t+1}^B dF_t + (1-\mu) \int^{\bar{\omega}_{t+1}^B} \omega dF_t \right) = 1 - \frac{1}{\phi_t^B}, \tag{B.8}$$

where $\phi_t^B \equiv Q_t K_{jt}^B / [(1-\kappa) N_{jt}^B]$. Rearranging the first order condition for K_{jt}^B, we obtain:

$$\lambda_t^B = (1-\kappa) \phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t. \tag{B.9}$$

Using the results that $V_t^B(N_{jt}^B) = \lambda_t^B N_{jt}^B$ and $\phi_t^B = Q_t K_{jt}^B / [(1-\kappa) N_{jt}^B]$, the objective function is expressed as:

$$V_t^B(N_{jt}^B) = E_t\Lambda_{t,t+1}\Omega_{t+1}^B R_{t+1}^K Q_t K_{jt}^B \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t$$

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t\Lambda_{t,t+1}\Omega_{t+1}^B R_{t+1}^K \int_{\bar{\omega}_{t+1}^B} (\omega - \bar{\omega}_{t+1}^B) dF_t. \quad \text{(B.10)}$$

This is the same as the first order condition for K_{jt}^B. Our guess is verified.

The first order condition for $\bar{\omega}_{t+1}^B$ is given by:

$$\lambda_t^B = \frac{(1-\kappa)E_t\Lambda_{t+1}\Omega_{t+1}^B R_{t+1}^K[1-F(\bar{\omega}_{t+1}^B)]}{E_t\frac{R_{t+1}^K}{R_t}[1-F(\bar{\omega}_{t+1}^B)-\mu\bar{\omega}_{t+1}^B f(\bar{\omega}_{t+1}^B)]}. \quad \text{(B.11)}$$

In the steady state

$$\frac{\lambda^B}{\theta\lambda^B+1-\theta} = \frac{(1-\kappa)[1-F(\bar{\omega}^B)]}{[1-F(\bar{\omega}^B)-\mu\bar{\omega}^B f(\bar{\omega}^B)]}. \quad \text{(B.12)}$$

We need $\lambda^B > 1$ in the steady state, which requires that $\kappa < \kappa_1$ where

$$\kappa_1 \equiv 1 - \frac{[1-F(\bar{\omega}^B)-\mu\bar{\omega}^B f(\bar{\omega}^B)]}{[1-F(\bar{\omega}^B)]}.$$

We turn to the problem of G firms. We substitute $V_t^G(N_{jt}^G) = \lambda_t^G N_{jt}^G, V_t^B(N_{jt}^B) = \lambda_t^B N_{jt}^B$ into the objective function. The maximization problem in the integral becomes:

$$\max\{\Omega_{t+1}^G(\omega - \bar{\omega}_{jt+1}^G), \zeta\Omega_{t+1}^B\omega\}, \quad \text{(B.13)}$$

where $\Omega_t^i \equiv \theta\lambda_t^i + 1 - \theta$, for $i \in \{B, G\}$. This means that default is chosen when $\omega < \tilde{\omega}_{jt}^G$, where $\tilde{\omega}_{jt}^G \in [\bar{\omega}_{jt}^G, \infty)$ (because we rule out the case that all G firms default) and is given by:

$$\tilde{\omega}_t^G = \xi_t^{-1}\bar{\omega}_t^G, \qquad \xi_t \equiv 1 - \frac{\zeta\Omega_t^B}{\Omega_t^G}. \quad \text{(B.14)}$$

These mean that we can rewrite the objective function as:

$$V_t^G(N_{jt}^G) = \max E_t\Lambda_{t,t+1}\Omega_{t+1}^G R_{t+1}^K Q_t K_{jt}^G \left((1-\xi_{t+1}) \int^{\tilde{\omega}_{jt+1}^G} \omega dF_t + \int_{\tilde{\omega}_{jt+1}^G} (\omega - \bar{\omega}_{jt+1}^G) dF_t \right),$$

and the participation constraint as:

$$R^K_{t+1}Q_tK^G_{jt}\left(\int_{\tilde{\omega}^G_{jt+1}}\bar{\omega}^G_{jt+1}dF_t\right) = R_t(Q_tK^G_{jt}-N^G_{jt}). \tag{B.15}$$

We write down the Lagrangian as

$$\begin{aligned} V^G_t(N^G_{jt}) &= \max E_t\Lambda_{t,t+1}\Omega^G_{t+1}R^K_{t+1}Q_tK^G_{jt}\left((1-\xi_{t+1})\int^{\tilde{\omega}^G_{jt+1}}\omega dF_t+\int_{\tilde{\omega}^G_{jt+1}}(\omega-\bar{\omega}^G_{jt+1})dF_t\right)\\ &\quad +lm^G_{jt}\left[\frac{R^K_{t+1}}{R_t}Q_tK^G_{jt}\left(\int_{\tilde{\omega}^G_{jt+1}}\bar{\omega}^G_{jt+1}dF_t\right)-Q_tK^G_{jt}+N^G_{jt}\right], \end{aligned} \tag{B.16}$$

where lm^G_{jt} is the Lagrange multiplier. The envelope conditions says that $\lambda^G_t = lm^G_{jt}$. The first order condition for K^G_{jt} is:

$$\begin{aligned} K^G_{jt} \;:\; 0 &= E_t\Lambda_{t,t+1}\Omega^G_{t+1}R^K_{t+1}\left((1-\xi_{t+1})\int^{\tilde{\omega}^G_{jt+1}}\omega dF_t+\int_{\tilde{\omega}^G_{jt+1}}(\omega-\bar{\omega}^G_{jt+1})dF_t\right)\\ &\quad +\lambda^G_t\left[\frac{R^K_{t+1}}{R_t}\left(\int_{\tilde{\omega}^G_{jt+1}}\bar{\omega}^G_{jt+1}dF_t\right)-1\right]. \end{aligned} \tag{B.17}$$

In this equation, $\bar{\omega}^G_{jt}$ is the only firm-specific variable. This implies that every firm chooses the same cutoff value $\bar{\omega}^G_t$. Then the participation constraint implies every firm chooses the same leverage:

$$1-\frac{1}{\phi^G_t} = \frac{R^K_{t+1}}{R_t}\left(\int_{\tilde{\omega}^G_{t+1}}\bar{\omega}^G_{t+1}dF_t\right), \tag{B.18}$$

where $\phi^G_t \equiv Q_tK^G_{jt}/N^G_{jt}$. Rearranging the first order condition for K^G_{jt}, we obtain:

$$\lambda^G_t = \phi^G_tE_t\Lambda_{t,t+1}\Omega^G_{t+1}R^K_{t+1}\left((1-\xi_{t+1})\int^{\tilde{\omega}^G_{t+1}}\omega dF_t+\int_{\tilde{\omega}^G_{t+1}}(\omega-\bar{\omega}^G_{t+1})dF_t\right). \tag{B.19}$$

We substitute these results back to the objective function to verify the guess $V^G_t(N^G_{jt}) = \lambda^G_tN^G_{jt}$ is indeed correct:

$$\begin{aligned} V^G_t(N^G_{jt}) &= E_t\Lambda_{t,t+1}\Omega^G_{t+1}R^K_{t+1}Q_tK^G_{jt}\left[(1-\xi_{t+1})\int^{\tilde{\omega}^G_{t+1}}\omega dF_t+\int_{\tilde{\omega}^G_{t+1}}(\omega-\bar{\omega}^G_{t+1})dF_t\right]\\ \lambda^G_t &= \phi^G_tE_t\Lambda_{t,t+1}\Omega^G_{t+1}R^K_{t+1}\left[(1-\xi_{t+1})\int^{\tilde{\omega}^G_{t+1}}\omega dF_t+\int_{\tilde{\omega}^G_{t+1}}(\omega-\bar{\omega}^G_{t+1})dF_t\right]. \end{aligned} \tag{B.20}$$

This is the same as the first order condition for K_{jt}^G. Our guess is verified.

The first order condition for $\tilde{\omega}_{t+1}^G$ is given by:

$$\lambda_t^G = \frac{E_t \Lambda_{t+1} R_{t+1}^K \Omega_{t+1}^G \xi_{t+1}[1 - F(\tilde{\omega}_{t+1}^G)]}{E_t \frac{R_{t+1}^K}{R_t} \xi_{t+1}[1 - F(\tilde{\omega}_{t+1}^G) - \tilde{\omega}_{t+1}^G f(\tilde{\omega}_{t+1}^G)]}. \tag{B.21}$$

In the steady state, this implies:

$$\frac{\lambda^G}{\theta\lambda^G + 1 - \theta} = \frac{1 - F(\tilde{\omega}^G)}{1 - F(\tilde{\omega}^G) - \tilde{\omega}^G f(\tilde{\omega}^G)} > 1. \tag{B.22}$$

Part 2: This part of the proof is by construction. Suppose some G firms j deviate from the above in period t and choose to borrow secured debt. Because lenders observe the G labels of these firms, they may agree on a contract with different cutoff values and amounts borrowed. In order for secured lenders to participate, the following participation constraint has to hold:

$$R_t^K Q_{t-1} K_{jt-1}^G \left(\int_{\hat{\omega}_{jt}^G} \hat{\omega}_{jt}^G dF_t + (1-\mu) \int^{\hat{\omega}_{jt}^G} \omega dF_t \right) = R_{t-1}[Q_{t-1}\hat{K}_{jt-1}^G - (1-\kappa) N_{jt-1}^G], \tag{B.23}$$

where $\hat{\omega}_{jt}^G$ and $\hat{K}_{jt-1}^G$ denote the cutoff value and capital a deviating G firm would choose. Note that since the firm borrows secured debt, the firm is subject to the initialization cost κ.

The deviating firms' value function, called $\hat{V}_t^G(N_{jt}^G)$ is given by:

$$\hat{V}_t^G(N_{jt}^G) = \max_{\hat{\omega}_{jt+1}^G, \hat{K}_{jt}^G} E_t \Lambda_{t,t+1} \int_{\hat{\omega}_{jt+1}^G} \left\{ \theta V_{t+1}^G[(\omega - \hat{\omega}_{jt+1}^G) R_{t+1}^K Q_t K_{jt}^G] + (1-\theta)(\omega - \hat{\omega}_{jt+1}^G) R_{t+1}^K Q_t K_{jt}^G \right\} dF_t \tag{B.24}$$

Notice that the firm only deviates in period t, so in period $t+1$, if the firm does not default or exit, the firm keeps the G label, and so we have V_{t+1}^G on the right hand side.

We conjecture that $\hat{V}_t^G(N_{jt}^G) = \hat{\lambda}_t^G N_{jt}^G$ where $\hat{\lambda}_t^G$ is the marginal value of the firm if it deviates. We maximize (B.24) subject to (B.23). Following similar steps as part 1 of the proof above, we get that $\hat{\omega}^G_{jt+1}, \hat{K}^G_{jt}, \hat{\lambda}^G_t$ satisfies the following first order conditions:

$$\hat{\lambda}^G_t = \frac{Q_t \hat{K}^G_{jt}}{N^G_{jt}} E_t \Lambda_{t,t+1} \Omega^G_{t+1} R^K_{t+1} \int_{\hat{\omega}^G_{jt+1}} (\omega - \hat{\omega}^G_{jt+1}) dF_t \tag{B.25}$$

$$\hat{\lambda}^G_t = \frac{(1-\kappa) E_t \Lambda_{t,t+1} \Omega^G_{t+1} R^K_{t+1} [1 - F(\hat{\omega}^G_{jt+1})]}{E_t \frac{R^K_{t+1}}{R_t} [1 - F(\hat{\omega}^G_{jt+1}) - \mu \hat{\omega}^G_{jt+1} f(\hat{\omega}^G_{jt+1})]} \tag{B.26}$$

and the participation constraint (B.23). After some algebra, one can show that $\hat{\omega}^G_{jt+1} = \bar{\omega}^B_{t+1}$.

Clearly, a large κ discourages G firms from deviating. By construction, we can find κ_0 such that for $\kappa > \kappa_0$, we have $\lambda^G_t > \hat{\lambda}^G_t$, in which case no G firm will find borrowing secured debt rather than unsecured debt profitable. In the steady state, the last equation implies:

$$\kappa_0 \equiv \frac{\lambda^G [1 - F(\bar{\omega}^B) - \mu \bar{\omega}^B f(\bar{\omega}^B)]}{\Omega^G [1 - F(\bar{\omega}^B)]}.$$

In our numerical exercise, $\kappa_0 = 0.013, \kappa_1 = 0.041$. We calibrate $\kappa = 0.017$, which is within the bounds.

Proposition 2. *The cutoff value for secured debt contract, $\bar{\omega}^B_t$, satisfies:*

$$E_t \left(\frac{R^K_{t+1}}{R_t} \right) = E_t \rho^B(\bar{\omega}^B_{t+1}; \sigma_t) \geq 1, \tag{B.27}$$

where the function $\rho^B(\bar{\omega}^B_{t+1}; \sigma_t)$ is increasing in the cutoff value $\bar{\omega}^B_{t+1}$, and increasing in the cross-sectional dispersion of idiosyncratic productivity σ_t. Furthermore $\lim_{\bar{\omega}^B_{t+1} \to 0} \rho^B(\bar{\omega}^B_{t+1}; \sigma_t) = 1$.

Proposition 3. *The cutoff value for the unsecured debt contract, $\tilde{\omega}^G_t$, satisfies:*

$$E_t \left(\frac{R^K_{t+1}}{R_t} \right) = E_t \rho^G(\tilde{\omega}^G_{t+1}, \xi_{t+1}; \sigma_t) \geq 1, \tag{B.28}$$

where the function $\rho^G(\tilde{\omega}^G_{t+1}, \xi_{t+1}; \sigma_t)$ is increasing in the cutoff value $\tilde{\omega}^G_{t+1}$, decreasing in ξ_{t+1}, and increasing in the cross-sectional dispersion of idiosyncratic productivity σ_t. Furthermore, $\lim_{\tilde{\omega}^G_{t+1} \to 0} \rho^G(\tilde{\omega}^G_{t+1}, \xi_{t+1}; \sigma_t) = 1$.

Proof of proposition 2 and 3: We first derive the functions $\rho^B(\bar{\omega}^B; \sigma)$ and $\rho^G(\tilde{\omega}^G, \xi; \sigma)$

and then prove some important properties. It is helpful to define:

$$\begin{aligned} G(\bar{\omega}_t;\sigma_{t-1}) &\equiv \int^{\bar{\omega}_t} \omega dF(\omega,\sigma_{t-1}), \\ \Gamma(\bar{\omega}_t;\sigma_{t-1}) &\equiv G(\bar{\omega}_t;\sigma_{t-1}) + \bar{\omega}[1 - F(\bar{\omega}_t;\sigma_{t-1})]. \end{aligned}$$

The function G denotes the mean of the idiosyncratic shock conditional on the shock below a given threshold $\bar{\omega}$. The function Γ adds the function G and a constant return $\bar{\omega}$ if the realization of idiosyncratic shock is above the threshold. This function is the share of revenue transferred to lenders (before monitoring) in the secured debt contract. We denote G_ω, Γ_ω the first derivatives of G and Γ with respect to $\bar{\omega}$, and denote G_σ, Γ_σ the first derivatives of G and Γ with respect to σ, and so on. In the following, we suppress the arguments of the functions when this does not cause any confusions.

To derive the function ρ^B, we first note that the evolution of λ_t^B, the optimal threshold $\bar{\omega}_{t+1}^B$ and the participation constraint can be written as:

$$\lambda_t^B = (1-\kappa)\phi_t^B E_t \Lambda_{t,t+1} \Omega_{t+1}^B R_{t+1}^K [1 - \Gamma(\bar{\omega}_{t+1}^B)], \tag{B.29}$$

$$\lambda_t^B = \frac{(1-\kappa) E_t \Lambda_{t+1} \Omega_{t+1}^B R_{t+1}^K \Gamma_\omega(\bar{\omega}_{t+1}^B)}{E_t \frac{R_{t+1}^K}{R_t}[\Gamma_\omega(\bar{\omega}_{t+1}^B) - \mu G_\omega(\bar{\omega}_{t+1}^B)]}, \tag{B.30}$$

$$1 - \frac{1}{\phi_{t-1}^B} = \frac{R_t^K}{R_{t-1}}[\Gamma(\bar{\omega}_t^B) - \mu G(\bar{\omega}_t^B)]. \tag{B.31}$$

We roll the participation constraint one period forward, rearrange these three equations to eliminate the leverage ratio and the marginal value λ_t^B to get, up to a first order approximation:

$$E_t\left(\frac{R_{t+1}^K}{R_t}\right) = E_t \rho^B(\bar{\omega}_{t+1}^B;\sigma_t), \tag{B.32}$$

where

$$\rho^B(\bar{\omega}_{t+1}^B) \equiv \frac{\Gamma_\omega(\bar{\omega}_{t+1}^B)}{[1-\Gamma(\bar{\omega}_{t+1}^B)][\Gamma_\omega(\bar{\omega}_{t+1}^B) - \mu G_\omega(\bar{\omega}_{t+1}^B)] + [\Gamma(\bar{\omega}_{t+1}^B) - \mu G(\bar{\omega}_{t+1}^B)]\Gamma_\omega(\bar{\omega}_{t+1}^B)}. \tag{B.33}$$

Following the same procedures, we show that for the unsecured debt contract, we have,

$$E_t\left(\frac{R_{t+1}^K}{R_t}\right) = E_t \rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1};\sigma_t), \tag{B.34}$$

where

$$\rho^G(\tilde{\omega}^G_{t+1}, \xi_{t+1}) \equiv \frac{\Gamma_\omega(\tilde{\omega}^G_{t+1})}{[1-\xi_{t+1}\Gamma(\tilde{\omega}^G_{t+1})][\Gamma_\omega(\tilde{\omega}^G_{t+1}) - G_\omega(\tilde{\omega}^G_{t+1})] + \xi_{t+1}[\Gamma(\tilde{\omega}^G_{t+1}) - G(\tilde{\omega}^G_{t+1})]\Gamma_\omega(\tilde{\omega}^G_{t+1})}. \tag{B.35}$$

We now analyze the properties of ρ^B, ρ^G. First, it is straightforward to show that:

$$F(\bar{\omega};\sigma) = \Phi\left(\frac{\log\bar{\omega} + 0.5\sigma^2}{\sigma}\right) > 0, \quad G(\bar{\omega};\sigma) = \Phi\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right) > 0.$$

where we define $\Phi(.), \phi(.)$ as the cdf and pdf of a standard normal distribution.

The first derivatives are:

$$\begin{aligned}
F_\omega &= \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{\log\bar{\omega} + 0.5\sigma^2}{\sigma}\right) > 0, \\
F_\sigma &= -\frac{1}{\sigma}\phi\left(\frac{\log\bar{\omega} + 0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right) > 0, \\
G_\omega &= \frac{1}{\sigma\bar{\omega}}\phi\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right) > 0, \\
G_\sigma &= -\frac{1}{\sigma}\phi\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega} + 0.5\sigma^2}{\sigma}\right) > 0, \\
\Gamma_\omega &= 1 - F > 0, \\
\Gamma_\sigma &= G_\sigma - \bar{\omega}F_\sigma = -\phi\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right) < 0,
\end{aligned}$$

where $\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) < \left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) < 0$ because the default probability is small in economically relevant cases.[37]

[37]To derive the expression for Γ_σ we note that:

$$\begin{aligned}
\bar{\omega}F_\sigma &= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right)\phi\left(\frac{\log\bar{\omega} + 0.5\sigma^2}{\sigma}\right), \\
&= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{[(\log\bar{\omega} - 0.5\sigma^2) + \sigma^2]^2}{\sigma^2}\right\}, \\
&= -\frac{\bar{\omega}}{\sigma}\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(\log\bar{\omega} - 0.5\sigma^2)^2}{\sigma^2}\right\}\exp(-\log\bar{\omega}), \\
&= -\frac{1}{\sigma}\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right)\phi\left(\frac{\log\bar{\omega} - 0.5\sigma^2}{\sigma}\right).
\end{aligned}$$

The following second derivatives are useful:

$$
\begin{aligned}
G_{\omega\omega} &= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) - \frac{1}{\sigma^2\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\\
&= -\frac{1}{\sigma\bar{\omega}^2}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right) > 0,\\
G_{\omega\sigma} &= -\frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right) - \frac{1}{\sigma^2\bar{\omega}}\phi'\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right)\\
&= \frac{1}{\sigma^2\bar{\omega}}\phi\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left[\left(\frac{\log\bar{\omega}-0.5\sigma^2}{\sigma}\right)\left(\frac{\log\bar{\omega}+0.5\sigma^2}{\sigma}\right)-1\right] > 0,\\
\Gamma_{\omega\omega} &= -F_\omega < 0,\\
\Gamma_{\omega\sigma} &= -F_\sigma < 0,
\end{aligned}
$$

where we have used $\phi'(x) = -x\phi(x)$.

Using the above relations it is easy to show that $\rho^G, \rho^B \geq 1$, and

$$
\rho^B_\omega = \frac{\mu(1-\Gamma)}{[(1-\Gamma)(\Gamma_\omega-\mu G_\omega)+(\Gamma-\mu G)\Gamma_\omega]^2}(\Gamma_\omega G_{\omega\omega}-\Gamma_{\omega\omega}G_\omega) > 0. \quad \text{(B.36)}
$$

$$
\rho^G_\omega = \frac{(1-\xi\Gamma)}{[(1-\xi\Gamma)(\Gamma_\omega-G_\omega)+\xi(\Gamma-G)\Gamma_\omega]^2}(\Gamma_\omega G_{\omega\omega}-\Gamma_{\omega\omega}G_\omega) > 0. \quad \text{(B.37)}
$$

$$
\rho^B_\sigma = \frac{(1-\Gamma)\mu[\Gamma_\omega G_{\omega\sigma}-G_\omega\Gamma_{\omega\sigma}]+\mu\Gamma_\omega[\Gamma_\omega G_\sigma-\Gamma_\sigma G_\omega]}{[(1-\Gamma)(\Gamma_\omega-\mu G_\omega)+(\Gamma-\mu G)\Gamma_\omega]^2} > 0. \quad \text{(B.38)}
$$

$$
\rho^G_\sigma = \frac{(1-\xi\Gamma)[\Gamma_\omega G_{\omega\sigma}-G_\omega\Gamma_{\omega\sigma}]+\xi\Gamma_\omega[\Gamma_\omega G_\sigma-\Gamma_\sigma G_\omega]}{[(1-\xi\Gamma)(\Gamma_\omega-G_\omega)+\xi(\Gamma-G)\Gamma_\omega]^2} > 0. \quad \text{(B.39)}
$$

Next, we show that $\rho^G_\xi < 0$. Clearly,

$$
\rho^G_\xi = -\frac{\Gamma_\omega}{[(1-\xi\Gamma)(\Gamma_\omega-G_\omega)+\xi(\Gamma-G)\Gamma_\omega]^2}(\Gamma G_\omega - G\Gamma_\omega). \quad \text{(B.40)}
$$

Notice that

$$
\begin{aligned}
\Gamma G_\omega - G\Gamma_\omega &= [G+\bar{\omega}(1-F)]G_\omega - G(1-F),\\
&= GG_\omega + (1-F)(\bar{\omega}G_\omega - G).
\end{aligned}
$$

The first term is clearly positive. We show that the second term is also positive by studying the function G_ω:

$$
G_\omega = \frac{1}{\sigma\tilde{\omega}^G}\phi\left(\frac{\log\tilde{\omega}^G-0.5\sigma^2}{\sigma}\right) = \frac{1}{\sigma}\phi\left(\frac{\log\tilde{\omega}^G+0.5\sigma^2}{\sigma}\right).
$$

This means that $\lim_{\tilde{\omega}^G \to 0} G_\omega(\tilde{\omega}^G) = 0$. Furthermore, $G_{\omega\omega} > 0$ for $\omega \in [0, \tilde{\omega}^G]$. These mean that

$$\tilde{\omega}^G G_\omega(\tilde{\omega}^G) > \int_0^{\tilde{\omega}^G} G_\omega d\omega = G(\tilde{\omega}^G) - \lim_{\tilde{\omega}^G \to 0} G(\tilde{\omega}^G) = G(\tilde{\omega}^G).$$

Therefore, $\tilde{\omega}^G G_\omega > G$, so $\Gamma G_\omega - G\Gamma_\omega > 0$, which means that $\rho^G_\xi < 0$.

Finally, since $\lim_{\bar{\omega} \to 0} G_\omega(\bar{\omega}) = 0$ and $\lim_{\bar{\omega} \to 0} G(\bar{\omega}) = 0$, we substitute these results into ρ^G, ρ^B to get $\lim_{\bar{\omega} \to 0} \rho^G(\bar{\omega}) = \lim_{\bar{\omega} \to 0} \rho^B(\bar{\omega}) = 1$.

Proposition 4. *For any $\xi_t \in (\mu, 1)$, $\sigma_{t-1} > 0$ and $\bar{\omega}_t > 0$, we have*

$$\frac{\partial \rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1})}{\partial \bar{\omega}_t} > \frac{\partial \rho^B(\bar{\omega}_t; \sigma_{t-1})}{\partial \bar{\omega}_t}, \tag{B.41}$$

and $\rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1}) > \rho^B(\bar{\omega}_t; \sigma_{t-1})$.

Proof of proposition 4: Consider $\rho^B_\omega, \rho^G_\omega$ in (B.36) and (B.37). We evaluate these functions at a given $\bar{\omega} > 0$. Clearly, the numerator of ρ^B_ω is smaller than the numerator of ρ^G_ω. Furthermore, the denominator of ρ^B_ω is larger than the denominator of ρ^G_ω. To see this, notice that

$$\begin{aligned} & [(1-\Gamma)(\Gamma_\omega - \mu G_\omega) + (\Gamma - \mu G)\Gamma_\omega] - [(1-\xi\Gamma)(\Gamma_\omega - G_\omega) + \xi(\Gamma - G)\Gamma_\omega], \\ = \; & G_\omega[1 - \mu + \Gamma(\mu - \xi)] + (\xi - \mu)G\Gamma_\omega, \\ > \; & G_\omega[\Gamma(1-\mu) + \Gamma(\mu - \xi)] + (\xi - \mu)G\Gamma_\omega, \\ > \; & G_\omega \Gamma(1-\xi), \\ > \; & 0, \end{aligned}$$

for $\xi > \mu$. Therefore, $\frac{\partial \rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1})}{\partial \bar{\omega}_t} > \frac{\partial \rho^B(\bar{\omega}_t; \sigma_{t-1})}{\partial \bar{\omega}_t}$. Together with the fact that ρ^G and ρ^B are continuous in $\bar{\omega}$, and $\lim_{\bar{\omega} \to 0} \rho^G(\bar{\omega}) = \lim_{\bar{\omega} \to 0} \rho^B(\bar{\omega}) = 1$, it follows that $\rho^G(\bar{\omega}_t, \xi_t; \sigma_{t-1}) > \rho^B(\bar{\omega}_t; \sigma_{t-1})$.

Proposition 5. *For any $\xi_t \in (\mu, 1)$, $R^K_t / R_{t-1} > 1$, $\sigma_{t-1} > 0$ and $\bar{\omega}_t > 0$, we have*

$$\frac{\partial PC^B\left(\bar{\omega}_t, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right)}{\partial \bar{\omega}_t} > \frac{\partial PC^G\left(\bar{\omega}_t, \xi_t, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right)}{\partial \bar{\omega}_t} > 0, \tag{B.42}$$

and $PC^B\left(\bar{\omega}_t, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right) > PC^G\left(\bar{\omega}_t, \xi_t, \frac{R^K_t}{R_{t-1}}; \sigma_{t-1}\right)$.

Proof of proposition 5: We consider the two participation constraints:

$$1-\frac{1}{\phi_{t-1}^{B}} = \frac{R_t^K}{R_{t-1}}[\Gamma(\bar{\omega}_t^B)-\mu G(\bar{\omega}_t^B)], \tag{B.43}$$

$$1-\frac{1}{\phi_{t-1}^{G}} = \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma(\tilde{\omega}_t^G)-G(\tilde{\omega}_t^G)]. \tag{B.44}$$

When $\bar{\omega}_t=\bar{\omega}_t^B=\tilde{\omega}_t^G$,

$$1-\frac{1}{\phi_{t-1}^{B}} = \frac{R_t^K}{R_{t-1}}[\Gamma(\bar{\omega}_t)-\mu G(\bar{\omega}_t)] > \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma(\bar{\omega}_t)-G(\bar{\omega}_t)] = 1-\frac{1}{\phi_{t-1}^{G}}. \tag{B.45}$$

Therefore, $\phi_{t-1}^B > \phi_{t-1}^G$.

Furthermore,

$$\frac{\partial\left(1-\frac{1}{\phi_{t-1}^{B}}\right)}{\partial\bar{\omega}_t} = \frac{R_t^K}{R_{t-1}}[\Gamma_\omega(\bar{\omega}_t)-\mu G_\omega(\bar{\omega}_t)] > \frac{R_t^K}{R_{t-1}}\xi_t[\Gamma_\omega(\bar{\omega}_t)-G_\omega(\bar{\omega}_t)] = \frac{\partial\left(1-\frac{1}{\phi_{t-1}^{G}}\right)}{\partial\bar{\omega}_t}.$$

Therefore,

$$\frac{\partial\phi_{t-1}^B}{\partial\bar{\omega}_t} > \left(\frac{\phi_{t-1}^B}{\phi_{t-1}^G}\right)^2\frac{\partial\phi_{t-1}^G}{\partial\bar{\omega}_t} > \frac{\partial\phi_{t-1}^G}{\partial\bar{\omega}_t}. \tag{B.46}$$

Next, we prove that the two participation constraints are upward-sloping. We consider the function $\Psi(\bar{\omega})\equiv\Gamma(\bar{\omega})-G(\bar{\omega})$ and show that $\Psi_\omega>0$ for a relevant range of ω. To see this we write:

$$\begin{aligned}
G_\omega(\bar{\omega}) &= \bar{\omega}f(\bar{\omega}) = \bar{\omega}h(\bar{\omega})(1-F(\bar{\omega}))>0,\\
\Gamma_\omega(\bar{\omega}) &= G_\omega(\bar{\omega})+(1-F(\bar{\omega}))-\bar{\omega}f(\bar{\omega}) = 1-F(\bar{\omega})>0,\\
\Psi_\omega(\bar{\omega}) &= \Gamma_\omega(\bar{\omega})-G_\omega(\bar{\omega}) = (1-F(\bar{\omega}))(1-\bar{\omega}h(\bar{\omega})),
\end{aligned}$$

where $h(\bar{\omega})=f(\bar{\omega})/(1-F(\bar{\omega}))$ is the hazard rate. For the log-normal distribution, $\bar{\omega}h(\bar{\omega})=0$ when $\bar{\omega}=0$, $\lim_{\bar{\omega}\to\infty}\bar{\omega}h(\bar{\omega})=\infty$, and $\bar{\omega}h(\bar{\omega})$ is increasing in $\bar{\omega}$. Hence, there exists an $\bar{\omega}^*$ such that $\Psi_\omega(\bar{\omega})>0$ for $\bar{\omega}<\bar{\omega}^*$ and $\Psi_\omega(\bar{\omega})<0$ for $\bar{\omega}>\bar{\omega}^*$. For any $\bar{\omega}_1$ such that $\bar{\omega}_1>\bar{\omega}^*$, there exist a $\bar{\omega}_2$ such that $\bar{\omega}_2<\bar{\omega}^*<\bar{\omega}_1$ and $\Psi(\bar{\omega}_2)=\Psi(\bar{\omega}_1)$. Since the smaller $\bar{\omega}_2$ implies a smaller bankruptcy rate for the borrower than $\bar{\omega}_1$ while keeping the lenders' share of profit unchanged, any $\bar{\omega}_1>\bar{\omega}^*$ will never be chosen. Hence, $\bar{\omega}$ has an interior solution and in the optimal contract $\Psi_\omega(\bar{\omega}) > 0$. This means that:

$$\frac{\partial PC^B\left(\bar{\omega}_t, \frac{R_t^K}{R_{t-1}}\right)}{\partial \bar{\omega}_t} > \frac{\partial PC^G\left(\bar{\omega}_t, \xi_t, \frac{R_t^K}{R_{t-1}}\right)}{\partial \bar{\omega}_t} > 0. \tag{B.47}$$

Finally, as PC^B and PC^G are continuous in $\bar{\omega}$, and $PC^B\left(0, \frac{R_t^K}{R_{t-1}}\right) = PC^G\left(0, \xi_t, \frac{R_t^K}{R_{t-1}}\right) = 1$, the previous result implies that $PC^B\left(\bar{\omega}_t, \frac{R_t^K}{R_{t-1}}\right) > PC^G\left(\bar{\omega}_t, \xi_t, \frac{R_t^K}{R_{t-1}}\right)$ for $\bar{\omega}_t > 0$.

Proposition 6. *In equilibrium, the leverage ratio of G firms is always lower than the leverage ratio of B firms. That is $\phi_t^B > \phi_t^G$.*

Proof of proposition 6: We know from Proposition 1 that $\lim_{\bar{\omega}_t \to 0} \rho^B(\bar{\omega}_t) = \lim_{\bar{\omega}_t \to 0} \rho^G(\bar{\omega}_t, \xi_t) = 1$, and ρ^B, ρ^G are increasing in $\bar{\omega}_t$. Moreover, Proposition 2 show that $\rho_\omega^G(\bar{\omega}_t, \xi_t) > \rho_\omega^B(\bar{\omega}_t)$. These mean that, for any external finance premium such that $E_t(R_{t+1}^K)/R_t = E_t\rho^G(\tilde{\omega}_{t+1}^G, \xi_{t+1}) = E_t\rho^B(\bar{\omega}_{t+1}^B)$, we must have $\tilde{\omega}_{t+1}^G < \bar{\omega}_{t+1}^B$.

Then

$$\phi_t^G = PC^G\left(\tilde{\omega}_{t+1}^G, \xi_{t+1}, \frac{R_{t+1}^K}{R_t}\right) < PC^B\left(\tilde{\omega}_{t+1}^G, \frac{R_{t+1}^K}{R_t}\right) < PC^B\left(\bar{\omega}_{t+1}^B, \frac{R_{t+1}^K}{R_t}\right) = \phi_t^B, \tag{B.48}$$

where the first inequality is proved in Proposition 3, and the second inequality makes use of the fact that PC^B is increasing in $\bar{\omega}$ and that $\tilde{\omega}_{t+1}^G < \bar{\omega}_{t+1}^B$.

Appendix C. Details of calibration

We discuss our calibration strategy of the benchmark model. We first use the following equations for the secured debt contracts:

$$\begin{aligned}
\lambda^B &= (1-\kappa)\phi^B\beta\Omega^B R^K[1-G(\bar{\omega}^B)-\bar{\omega}^B(1-F(\bar{\omega}^B))] && \text{(C.1)}\\
1-\frac{1}{\phi^B} &= \beta R^K\left\{\bar{\omega}^B[1-F(\bar{\omega}^B)]+(1-\mu)G(\bar{\omega}^B)\right\} && \text{(C.2)}\\
\lambda^B &= \frac{(1-\kappa)\Omega^B[1-F(\bar{\omega}^B)]}{[1-F(\bar{\omega}^B)-\mu\bar{\omega}^B f(\bar{\omega}^B)]} && \text{(C.3)}\\
\Omega^B &= \theta\lambda^B+1-\theta && \text{(C.4)}
\end{aligned}$$

We use the steady-state conditions for the unsecured debt contracts:

$$\begin{aligned}
\lambda^G &= \phi^G\beta\Omega^G R^K\left\{1-\xi[G(\tilde{\omega}^G)+\tilde{\omega}^G(1-F(\tilde{\omega}^G))]\right\} && \text{(C.5)}\\
1-\frac{1}{\phi^G} &= \beta R^K\xi\tilde{\omega}^G[1-F(\tilde{\omega}^G)] && \text{(C.6)}\\
\lambda^G &= \frac{\Omega^G\xi(1-F(\tilde{\omega}^G))}{\xi[1-F(\tilde{\omega}^G)-\tilde{\omega}^G f(\tilde{\omega}^G)]} && \text{(C.7)}\\
\xi &= 1-\frac{\zeta(\theta\lambda^B+1-\theta)}{\Omega^G} && \text{(C.8)}\\
\Omega^G &= \theta\lambda^G+1-\theta && \text{(C.9)}
\end{aligned}$$

Furthermore, the steady-state ratio of secured and unsecured debt is given by:

$$\frac{B^G}{B^B}=\frac{K^G-N^G}{K^B-(1-\kappa)N^B}=\frac{\frac{K^G}{N^G}-1}{\frac{K^B}{N^B}\frac{N^B}{N^G}-(1-\kappa)\frac{N^B}{N^G}}=\frac{N^G}{N^B}\times\frac{\phi^G-1}{(\phi^B-1)(1-\kappa)} \tag{C.10}$$

where the evolution of net worth of B firms in the steady state gives the following relation:

$$\frac{N^G}{N^B}=\frac{1-\tau-(1-\kappa)\theta\{1-G(\bar{\omega}^B)-\bar{\omega}^B[1-F(\bar{\omega}^B)]\}R^K\phi^B}{\zeta G(\tilde{\omega}^G)\theta R^K\phi^G}.$$

The evolution of net worth of G firms in the steady state gives the following relation:

$$\tau=1-\theta R^K\phi^G\{1-G(\tilde{\omega}^G)-\xi\tilde{\omega}^G[1-F(\tilde{\omega}^G)]\} \tag{C.11}$$

The four steady-state conditions pin down $R^K/R, \phi^B, \phi^G, B^G/B^B$. The above eleven equations solve for the remaining steady-state values of $\{\bar{\omega}^B, \tilde{\omega}^G, \lambda^B, \lambda^G, \Omega^B, \Omega^G, \xi\}$ and the parameters $\{\sigma, \kappa, \zeta, \tau\}$.

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