Are Speed Limit Policies Robust?

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Abstract

Both price level targeting and speed limit policies have been suggested as alternatives to inflation targeting that may confer benefits when a central bank operates under discretion, even if society's loss function is specified in terms of inflation (instead of price level) volatility. Here we show that price level targeting dominates a speed limit policy under perfect credibility and rational expectations. However, a speed limit policy is more robust than a price level target. Even for small deviations from either rational expectations or perfect credibility, a speed limit policy dominates a price level target.

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1. Introduction

When a central bank cannot commit to following a monetary policy rule, monetary policy is confined to being discretionary. This generally imposes costs on the central bank, and also the public. A number of possible solutions have been proposed elsewhere that might serve to move the optimal discretionary solution towards that which can be obtained under commitment. For example, several papers have focused on the potential benefits of the central bank targeting the price level, even if society's loss function is specified in terms of inflation variability. Svensson (1999), using a Neo Classical Phillips curve, argued that there is an advantage to doing so provided the output gap is sufficiently persistent, and labeled this a 'free lunch.' Others have found even stronger support for price level targeting when agents are forward looking. These papers utilize variants of the New Keynesian framework, outlined in Roberts (1995) and Clarida, Gali and Gertler (1999). This framework assumes that changing prices is costly, so prices that are set today reflect future expectations of inflation. For example, Dittmar and Gavin (2000) find that the inflation-output variability trade-off is better with a price level target than an inflation target. They argue that adding a price level target with a small weight has little cost in terms of the real side of the economy yet is beneficial in reducing inflation volatility. Vestin (2000), using a similar framework, finds that price level targeting under discretion outperforms inflation targeting under discretion. In some cases, he finds that price level targeting under discretion can result in the same outcome as inflation targeting under commitment, provided the parameters of the loss function are suitably adjusted.

Walsh (2003) considers an alternative solution, by focusing on what he refers to as 'speed limit' policies. A speed limit policy is one where the output gap is replaced by the change in the output gap in the loss function delegated to the central bank. Walsh demonstrates that this moves the discretionary policy in the direction of the optimal commitment solution and argues that, in contrast to price level targeting, it is consistent with the language used by members of the Federal Open Market Committee to describe the policy process.

However, it is not in general clear how robust these alternative rules are. As Yetman (2003) argues, the 'free lunch' results from price level targeting have been based on a model in which the central bank enjoys perfect credibility, and agents' expectations are fully rational. Yet while these assumptions provide reasonable starting points for considering monetary policy, any desirable policy should also be robust to alternative assumptions, such as when a portion of agents use rules-of-thumb to form expectations, or the central bank enjoys less than perfect credibility. He shows that in general, the free lunch from price level targeting is sensitive to such perturbations. While the presence of rule-of-thumb forecasters or imperfect credibility is typically costly with either an inflation or a price level target, it is especially so with a price level target.

In this paper, we carry out a similar exercise to Yetman (2003) on Walsh's (2003) proposal of a speed limit policy. We find that, in contrast to price level targets, a speed limit policy is generally robust to rule-of-thumb expectations and imperfect credibility.

The basic model, demonstrating the potential role for a speed limit policy, follows. Section 3 considers a case of rule-of-thumb forecasting, while section 4 considers imperfect credibility. Conclusions then follow.

2. A Simple New-Keynesian Model

Here I describe a simple linear-quadratic, forward-looking model similar to that in Vestin (2000) and Walsh (2003) in which price level targeting and speed limit policies can play a role in improving the trade-off between output and inflation variability faced by the central bank when monetary policy is set under discretion. The economy is assumed to consist of a Phillips curve given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \tag{1}$$

where π_t is inflation, x_t is the output gap and u_t is an exogenous shock term that is known by the central bank. Expectations of future inflation are assumed rational, and all variables are expressed in logs. For simplicity, x_t can be considered the policy instrument of the central bank.

An inflation-targeting central bank seeks to minimize a standard, quadratic loss function given by

$$L = \sum_{i=0}^{\infty}[(\pi_{t+i} - \pi^*)^2 + \lambda x_{t+i}^2], \tag{2}$$

subject to (1), for some inflation target π^*. Under discretionary policy, the central bank is assumed to lack the means to commit to future policy actions. Optimal monetary policy will therefore minimize the period loss function, taking the form

$$x_t = \frac{\kappa(1-\beta)}{\kappa^2 + \lambda(1-\beta)}\pi^* - \frac{\kappa}{\kappa^2 + \lambda}u_t, \tag{3}$$

while inflation evolves according to

$$\pi_t = \frac{\kappa^2}{\kappa^2 + \lambda(1-\beta)}\pi^* + \frac{\lambda}{\kappa^2 + \lambda}u_t. \tag{4}$$

To examine monetary policy with a price level target, note that (1) may be rewritten as

$$E_t(p_{t+1} - p^*_{t+1}) = \frac{1+\beta}{\beta}(p_t - p^*_t) - \frac{1}{\beta}(p_{t-1} - p^*_{t-1}) - \frac{\kappa}{\beta}x_t - \frac{1}{\beta}u_t + \frac{1-\beta}{\beta}\pi^*, \qquad (5)$$

where $p^*_t = p^*_{t-1} + \pi^*$. The appropriate quadratic period loss function is then given by

$$L_t = (p_t - p^*_t)^2 + \lambda x_t^2, \qquad (6)$$

where λ is appropriately scaled to reflect the difference between the magnitude of price level and inflation rate volatility. In contrast to inflation targeting, today's policy affects losses in future periods, implying the presence of state variables in the model. Following the methodology of Currie and Levine (1993), the paths of output and inflation under optimal discretionary monetary policy may be defined by

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2, \qquad (7)$$

$$(p_t - p^*_t) = \phi_1[(p_{t-1} - p^*_{t-1}) + u_t] + \phi_2, \qquad (8)$$

where the solution values of the coefficients are given in Appendix 1.

To examine monetary policy with a speed limit policy, the appropriate quadratic period loss function is then given by

$$L_t = (\pi_t - \pi^*)^2 + \lambda(x_t - x_{t-1})^2, \qquad (9)$$

where λ should be scaled to reflect the difference between the magnitude of output gap volatility and the volatility of the change in the output gap. As in Walsh (2003), the paths of output and inflation under optimal discretionary monetary policy may be defined by

$$x_t = \xi_0 + \xi_1 x_{t-1} + \xi_2 u_t, \qquad (10)$$

$$\pi_t = \zeta_0 + \zeta_1 x_{t-1} + \zeta_2 u_t, \tag{11}$$

for ξ_0, ξ_1, ξ_2, ζ_0, ζ_1, ζ_2 given in Appendix 2.

We can now consider the policy frontiers obtained under either a price level or an inflation target. The parameters considered here are the same as in Vestin (2000) and Yetman (2003): $\kappa = \frac{1}{3}$; $\beta = 1$; $\pi^* = 0$; and $0 \leq \lambda \leq \infty$. To construct the policy frontier, the economy is simulated 1000 times for 100 periods for different levels of λ, and the average level of output and inflation volatility in the final period is computed. 100 periods is sufficient to ensure that the results are independent of the starting point, while averaging across 1000 simulations results in policy frontiers that are virtually indistinguishable from those that may be obtained analytically. While analytical solutions may be easily obtained for this base case, they are difficult to obtain for some of the cases that follow; hence the reliance on numerical results.

Figure 1 contains the policy frontiers for the base model. Over the entire range of possible values of λ, either a price level target or a speed limit policy results in a better trade-off between output volatility and inflation volatility than an inflation target. Note however, that for these parameter values, a speed limit policy is dominated by a price level target. Further, this result is robust to the choice of the volatility of cost-push shocks, as well as κ. However, it is sensitive to the discount rate. Figure 2 plots the trade-offs for different values of β. As the discount rate declines, the free lunch from pursuing a price level target diminishes at a faster rate than that from pursuing a speed limit policy. But for realistic discount rates, a price level target dominates both an inflation target and a speed limit policy.

Both price level targeting and speed limit policies introduce history dependence into the conduct of monetary policy, and thereby drive the economy towards outcomes that would be obtained under commitment. However, the mechanisms by which they do this are very different. A price level target obliges the central bank to fully correct for past deviations of the inflation rate from its desired level, while a speed limit policy induces the central bank to produce persistent deviations of the inflation rate from its desired level.

3. Rule-Of-Thumb Forecasting

The base result outlined above assumes that agents form rational expectations of inflation. This assumes complete knowledge of the structure of the economy, the loss function of the central bank, and the size of the shock term. Suppose that gathering this information were costly. Then it is possible that optimizing agents may follow simple rules-of-thumb in forming expectations.

When inflation expectations are biased towards long-run average

Consider the case where some agents' expectations of future inflation are biased away from their rational expectation towards the long-run average inflation rate, while others form inflation expectations that take into account the current policy target. This may result if agents are not sophisticated enough to form rational expectations for example, or do not know the model or size of the shock term.

In this model, which is linear and entails symmetric shock terms and additive uncertainty, as $\beta \to 1$ the average long-run rate of inflation is π^* with any of the three policies being considered. Replacing the Phillips Curve in (1) with

$$\pi_t = \omega E_t(\pi_{t+1}) + (1-\omega)\pi^* + \kappa x_t + u_t \tag{12}$$

where $(1-\omega)$ is the portion of unsophisticated agents, we can solve the model with an inflation target and obtain identical results to those in the preceding section, since $E_t(\pi_{t+1}) = \pi^*$. Hence inflation expectations being biased towards the long-run average inflation rate does not introduce any distortion with an inflation target.

To examine monetary policy with a price level target, note that (12) may be rewritten as

$$E_t(p_{t+1} - p^*_{t+1}) = \frac{1+\omega}{\omega}(p_t - p^*_t) - \frac{1}{\omega}(p_{t-1} - p^*_{t-1}) - \frac{\kappa}{\omega}x_t - \frac{1}{\omega}u_t. \qquad (13)$$

The price level path may then be solved in a manner consistent with that outlined in Appendix 1 (see Yetman (2003)); similarly with a speed limit policy (see Appendix 3).

The policy frontiers for different values of ω are given in Figure 3. We see here that as the portion of agents who take account of the target when forming expectations declines, the 'free lunch' resulting from following a price level target diminishes at a much faster rate than that from following a speed limit policy. Indeed if more than approximately 30% of agents forms expectations using this simple rule-of-thumb, then a speed limit policy dominates a price level target.

4. Imperfect Credibility

The base result outlined in section 2 assumes that agents form rational expectations that condition on the true central bank target. That is, the central bank has perfect credibility. While there is some evidence that the credibility enjoyed by many central banks has improved in recent years, few would believe that credibility is perfect. In reality, if inflation were to remain away from the target for a period of time or were to move far from a constant target due to a particular realization of shocks, some agents may interpret this as a change in the target. One can think of credibility in a number of different ways, two of which will be explored here. In each case, agents do not believe the target price path of the central bank, and base their expectations on an alternative price path, in terms of either the starting point or the growth rate.

When agents do not believe the starting point

One form of credibility that is important with a price level target but of little importance with either an inflation target or a speed limit policy is the starting point of the price path. Because bygones are bygones with the latter types of policy, the starting point (in terms of the price level) is relevant for one period only, while with a price level target it is relevant for all periods. I consider the situation where agents form expectations based on the belief that the starting point of the price path differs from the true one by an amount δ. Therefore expectations are based on a price level target of $\hat{p}_t^*$ where

$$\hat{p}_t^* = p_t^* + \delta, \tag{14}$$

and agents' expectations consistently exceed the true target.

With an inflation target or a speed limit policy, the results are as in Section 2 above beyond the first period. With a price level target, expectations are formed based on an incorrect target, and policy is set optimally, conditional on the forecasts (see Yetman (2003)).

Figure 4 shows the policy frontiers for $\delta = 0.001$ (0.1%), $\delta = 0.01$ (1%), and $\delta = 0.1$ (10%) respectively. Even a small bias in the perceived anchor for the desired price level path has a large impact on the policy frontier with a price level target, but has no impact with either of the other forms of monetary policy. The behavior of the economy in the United Kingdom during the return to the Gold Standard after World War I may be thought of as an example of the economic cost of the cost of the public not believing the starting point of the target price path (see Smith 1998 for a discussion).

When agents do not believe the price level growth rate

An alternative form of credibility that is important with all three types of policy relates to the desired growth rate in the price level. In the case of an inflation target or a speed limit policy, this will have a negative impact on the policy frontier that is uniform (in expectation) over time. With a price level target, the impact of imperfect credibility on output and inflation volatility will increase over time, as the perceived target and the actual target diverge. Clearly a price level target will then generally be costly relative to alternative policies.

A less extreme form of imperfect credibility may result from agents instead making non-systematic errors in predicting the target price level growth rate (π^*). This might occur with a time-varying inflation target, for example. To be precise, suppose

$$E_t(\pi^*_{t+1}) = \pi^*_{t+1} + \delta_t, \quad \delta_t \sim N(0, \sigma^2_\delta). \tag{15}$$

With an inflation target or a speed limit policy, δ_t will be equivalent from the central bank's point of view to an additional inflation shock:

$$\bar{u}_t = u_t + \delta_t. \tag{16}$$

With a price level target, the price level the price and output paths may be solved as outlined in Appendix 4.

Figure 5 shows the policy frontiers for $\sigma_\delta = 0.0001$ (0.01%), $\sigma_\delta = 0.001$ (0.1%), and $\sigma_\delta = 0.01$ (1%). Once again, the results illustrate the fragility of the monetary policy free lunch with a price level target.

5. Conclusions

Price level targeting and speed limit policies have been suggested as alternatives to inflation targeting that may confer benefits where a central bank operates under discretion, even if society's loss function is specified in terms of inflation (instead of price level) volatility. Both introduce history dependence into the conduct of monetary policy, driving the economy towards outcomes that would be obtained under commitment. However, the mechanisms by which they do this are very different. A price level target obliges the central bank to fully correct for past deviations of the inflation rate from its desired level, while a speed limit policy induces the central bank to propagate deviations of the inflation rate from its desired level into future periods. Here we have shown that under perfect credibility, and completely rational expectations, both price level targeting and speed limit policies confer benefits on society relative to inflation targeting, with price level targeting dominating speed limit policies.

However, with imperfect credibility or less than completely rational expectations, a price level target may become costly because correcting for past deviations of inflation from target requires that the central bank increase output volatility. In contrast, with a speed limit policy, past deviations of inflation from target are not corrected (but instead propagated). Even for small deviations from either rational expectations or perfect credibility, a speed limit policy dominates a price level target.

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Appendix 1

To solve the price level targeting model under optimal discretionary policy, optimal discretionary monetary policy will satisfy

$$V_t = \min E_t[L_t + \beta V_{t+1}], \tag{A1}$$

where the relevent terms of V_t are

$$V_t = \gamma_1(p_{t-1} - p^*_{t-1})^2 + 2\gamma_2(p_{t-1} - p^*_{t-1}) + ... \tag{A2}$$

Following Currie and Levine (1993), the solution price-path may be written as

$$E_t(p_{t+1} - p^*_{t+1}) = \phi_1(p_t - p^*_t) + \phi_2. \tag{A3}$$

Equations (5) and (A3) imply that

$$(p_t - p^*_t) = \frac{1}{1+\beta-\beta\phi_1}\left((p_{t-1} - p^*_{t-1}) + \kappa x_t + u_t + [\beta\phi_2 - (1-\beta)\pi^*]\right). \tag{A4}$$

Substituting (A2) (iterated forward) and (A4) into (A1) and differentiating with respect to x_t yields the optimal discretionary policy rule

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2, \tag{A5}$$

$$\theta_1 = \frac{-\kappa(1+\beta\gamma_1)}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2},$$
$$\theta_2 = -\frac{\kappa(1+\beta\gamma_1)[\beta\phi_2 - (1-\beta)\pi^*] + \kappa\beta\gamma_2(1+\beta-\beta\phi_1)}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2}.$$

Substituting (A5) back into (A4) yields

$$(p_t - p^*_t) = \frac{\lambda(1+\beta-\beta\phi_1)}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2}\left((p_{t-1} - p^*_{t-1}) + u_t\right) + \frac{\lambda(1+\beta-\beta\phi_1)[\beta\phi_2 - (1-\beta)\pi^*] - \kappa^2\beta\gamma_2}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2}. \tag{A6}$$

Interating forward one period, taking expectations, and combining with (A3) yields

$$\phi_1 = \frac{\lambda(1+\beta-\beta\phi_1)}{\kappa^2(1+\beta\gamma_1)+\lambda(1+\beta-\beta\phi_1)^2}, \tag{A7}$$

$$\phi_2 = \frac{\lambda(1+\beta-\beta\phi_1)[\beta\phi_2-(1-\beta)\pi^*]-\kappa^2\beta\gamma_2}{\kappa^2(1+\beta\gamma_1)+\lambda(1+\beta-\beta\phi_1)^2}. \tag{A8}$$

Finally, substituting (A2) iterated forward, (A5), and (A6) back into (A1) and equating the relevent coefficients with (A2) yields

$$\gamma_1 = (1+\beta\gamma_1)\phi_1^2+\lambda\theta_1^2, \tag{A9}$$

$$\gamma_2 = (1+\beta\gamma_1)\phi_1\phi_2+\lambda\theta_1\theta_2+\beta\gamma_2\phi_1. \tag{A10}$$

Solving these numerically, we then have the law of motion for both prices and output and can calculate output and inflation volatility.

Appendix 2

To solve the speed limit model, optimal discretionary monetary policy will satisfy

$$V_t = \min E_t[L_t+\beta V_{t+1}], \tag{A11}$$

where the relevent terms of V_t are

$$V_t = \gamma_1 x_{t-1}^2 + 2\gamma_2 x_{t-1} + ... \tag{A12}$$

Equations (1) and (11) imply that

$$\pi_t = \beta\zeta_0 + (\beta\zeta_1+\kappa)x_t + u_t. \tag{A13}$$

Substituting (A12) (iterated forward) and (A13) into (A11) and differentiating with respect to x_t yields the optimal discretionary policy rule

$$x_t = \xi_0 + \xi_1 x_{t-1} + \xi_2 u_t, \tag{A14}$$

$$\xi_0 = \frac{(\beta\zeta_1 + \kappa)(\pi^* - \beta\zeta_0) - \beta\gamma_2}{(\beta\zeta_1 + \kappa)^2 + \lambda + \beta\gamma_1},$$

$$\xi_1 = \frac{\lambda}{(\beta\zeta_1 + \kappa)^2 + \lambda + \beta\gamma_1},$$

$$\xi_2 = \frac{-(\beta\zeta_1 + \kappa)}{(\beta\zeta_1 + \kappa)^2 + \lambda + \beta\gamma_1}.$$

Substituting (A14) and (11) into (1) yields

$$\pi_t = \zeta_0 + \zeta_1 x_{t-1} + \zeta_2 u_t, \tag{A15}$$

$$\zeta_0 = \beta\zeta_0 + (\beta\zeta_1 + \kappa)\xi_0,$$

$$\zeta_1 = (\beta\zeta_1 + \kappa)\xi_1,$$

$$\zeta_2 = (\beta\zeta_1 + \kappa)\xi_2 + 1.$$

Finally, substituting (A12) iterated forward, (A14) and (A15) into (A11) and equating coefficients with (A12) yields

$$\gamma_1 = \frac{\zeta_1^2 + \lambda(\xi_1 - 1)^2}{1 - \beta\xi_1^2}, \tag{A16}$$

$$\gamma_2 = \frac{\zeta_1(\zeta_0 - \pi^*) + \lambda\xi_0(\xi_1 - 1) + \beta\gamma_1\xi_0\xi_1}{1 - \beta\xi_1}. \tag{A17}$$

Solving these numerically, we then have the law of motion for both inflation and output and can calculate output and inflation volatility.

Appendix 3

With inflation expectations biased towards their long-run trend, the model can be solved in an analygous fashion to Appendix 2. For $\beta = 1$, the results are

$$\xi_0 = \frac{\omega(\omega\zeta_1 + \kappa)(\pi^* - \zeta_0) - \gamma_2}{(\omega\zeta_1 + \kappa)^2 + \lambda + \gamma_1},$$

$$\xi_1 = \frac{\lambda}{(\omega\zeta_1 + \kappa)^2 + \lambda + \gamma_1},$$

$$\xi_2 = \frac{-(\omega\zeta_1 + \kappa)}{(\omega\zeta_1 + \kappa)^2 + \lambda + \gamma_1},$$

$$\zeta_0 = \omega\zeta_0 + (1-\omega)\pi^* + (\omega\zeta_1 + \kappa)\xi_0,$$

$$\zeta_1 = (\omega\zeta_1 + \kappa)\xi_1,$$

$$\zeta_2 = (\omega\zeta_1 + \kappa)\xi_2 + 1,$$

and γ_1 and γ_2 are defined as before.

Appendix 4

Agent's inflation expectations are formed under the assumption that the actual target is equal to the perceived target:

$$E_t(p_{t+1} - \hat{p}^*_{t+1}) = \phi_1(p_t - \hat{p}^*_t) + \phi_2, \tag{A18}$$

for ϕ_1, ϕ_2 outlined in Appendix 1. In contrast, prices in fact follow

$$E_t(p_{t+1} - p^*_{t+1}) = \phi_1(p_t - p^*_t) + \phi_2 + \phi_1\delta_t + (1-\phi_1)\sum_{i=0}^{\infty}\delta_{t-i}. \tag{A19}$$

The relevent terms of the the value function are

$$V_t = \gamma_1(p_{t-1} - p^*_{t-1})^2 + 2\gamma_2(p_{t-1} - p^*_{t-1}) + 2\gamma_3(p_{t-1} - p^*_{t-1})\sum_{i=0}^{\infty}\delta_{t-i}..., \tag{A20}$$

implying that

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2 + \theta_3\delta_t + \theta_4\sum_{i=0}^{\infty}\delta_{t-i}, \tag{A21}$$

$$\theta_3 = \frac{-\beta\kappa(1+\beta\gamma_1)\phi_1}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2},$$

$$\theta_4 = \frac{-\beta\kappa[(1+\beta\gamma_1)(1-\phi_1) + \gamma_3(1+\beta-\beta\phi_1)]}{\kappa^2(1+\beta\gamma_1) + \lambda(1+\beta-\beta\phi_1)^2},$$

$$\gamma_3 = (1+\beta\gamma_1)\phi_1(1-\phi_1) + \lambda\theta_1\theta_4 + \beta\gamma_3\phi_1.$$

The solutions to ϕ_1, ϕ_2, θ_1, θ_2, γ_1 and γ_2 coincide with those in Appendix 1.

Figure 1. Base Model

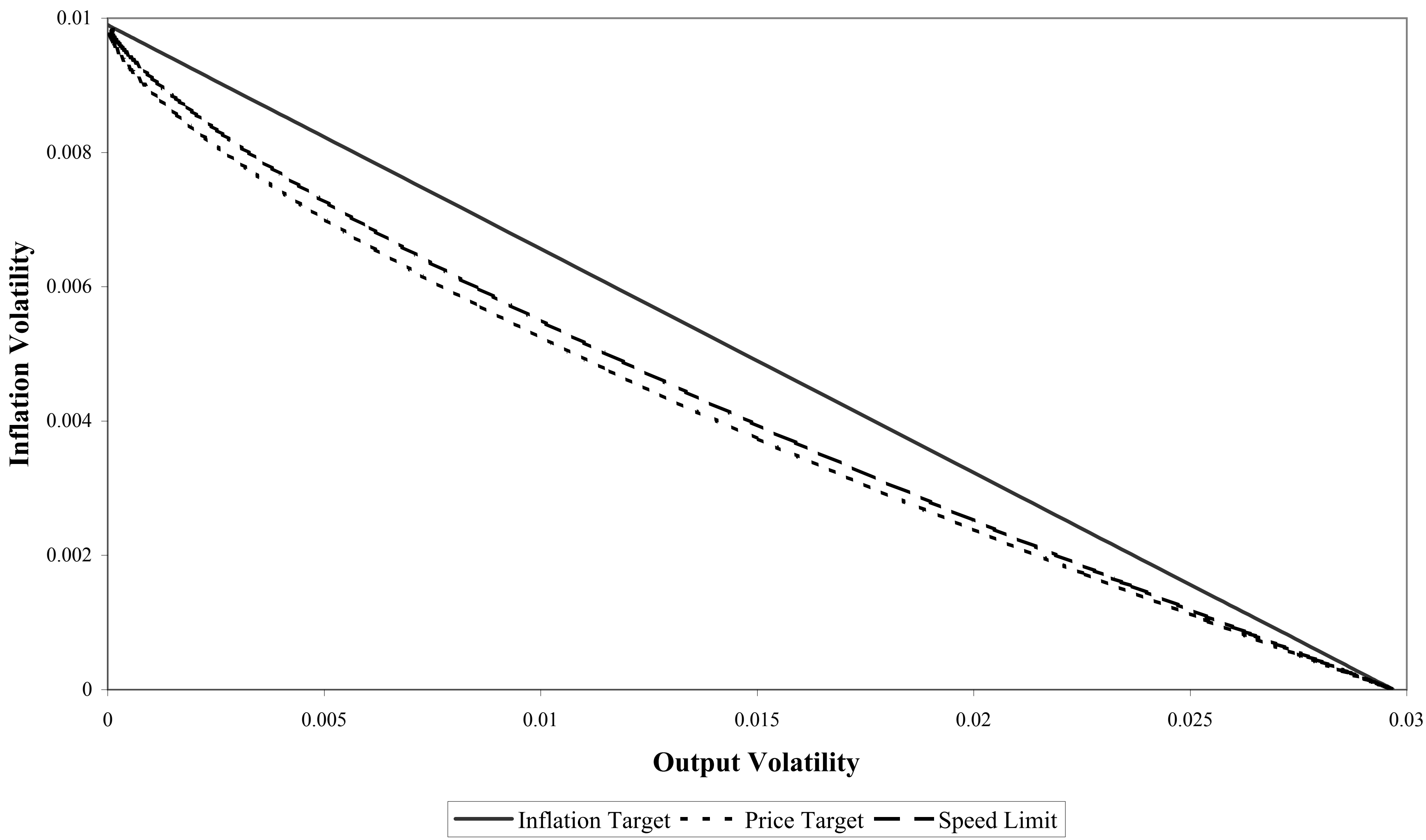


Figure 2. Different Discount Rates

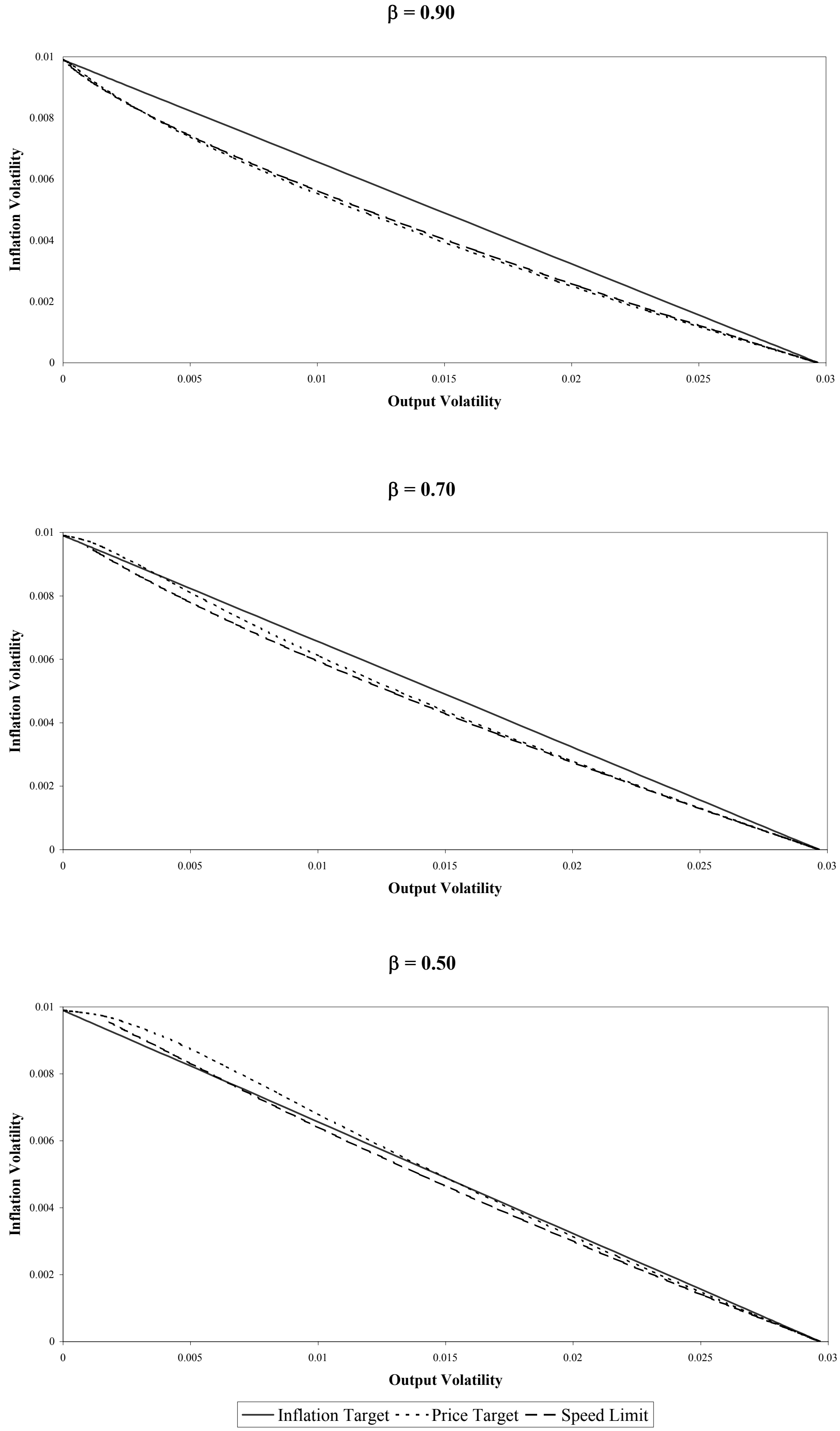
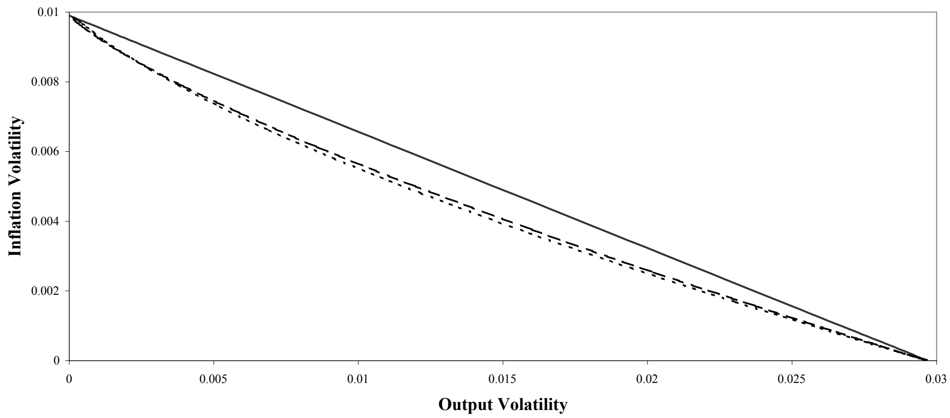
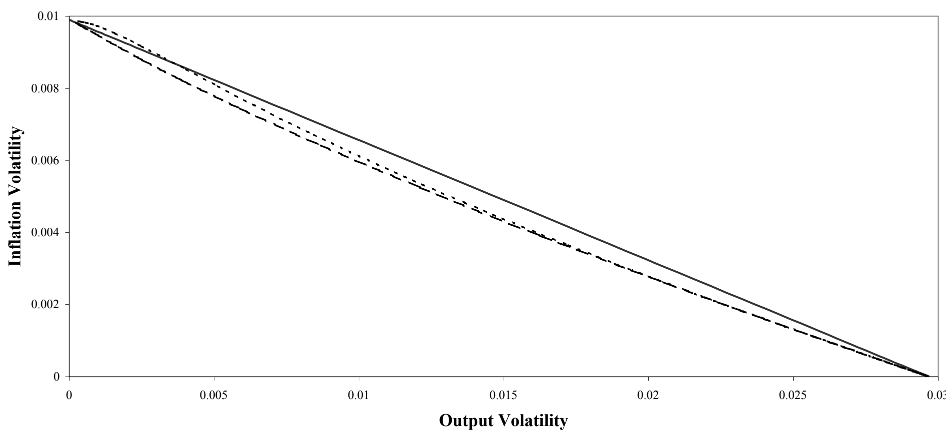


Figure 3. Expectations Biased Towards Long-Run Average

ω = 0.90



ω = 0.70



ω = 0.50

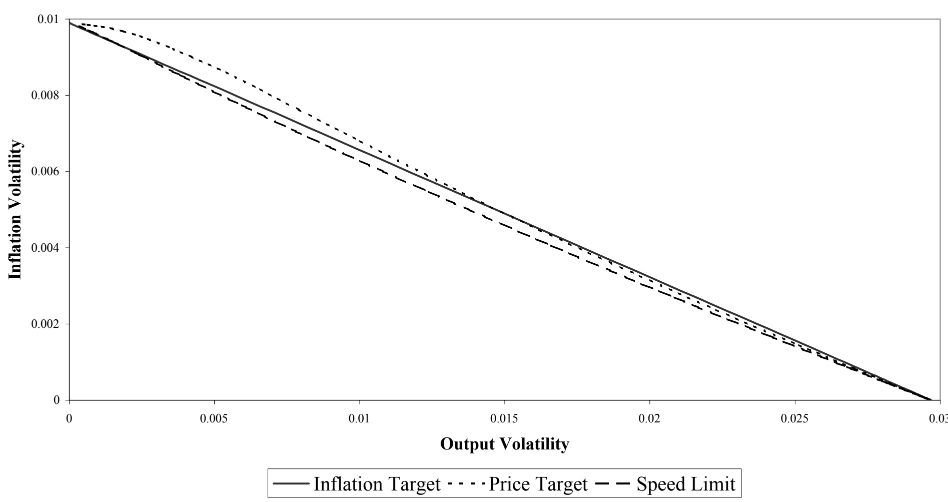


Figure 4. Agents Do Not Believe Starting Point

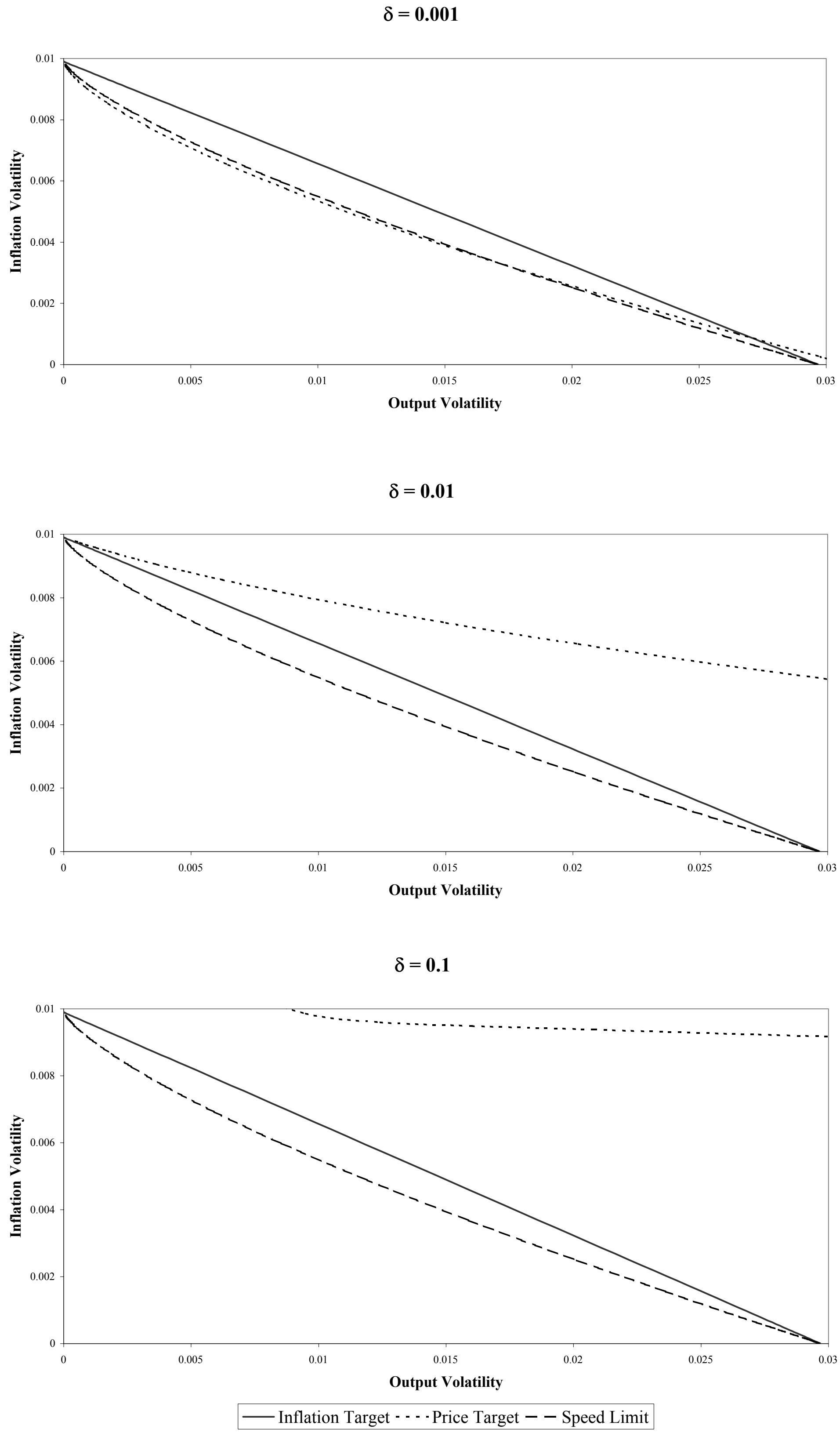
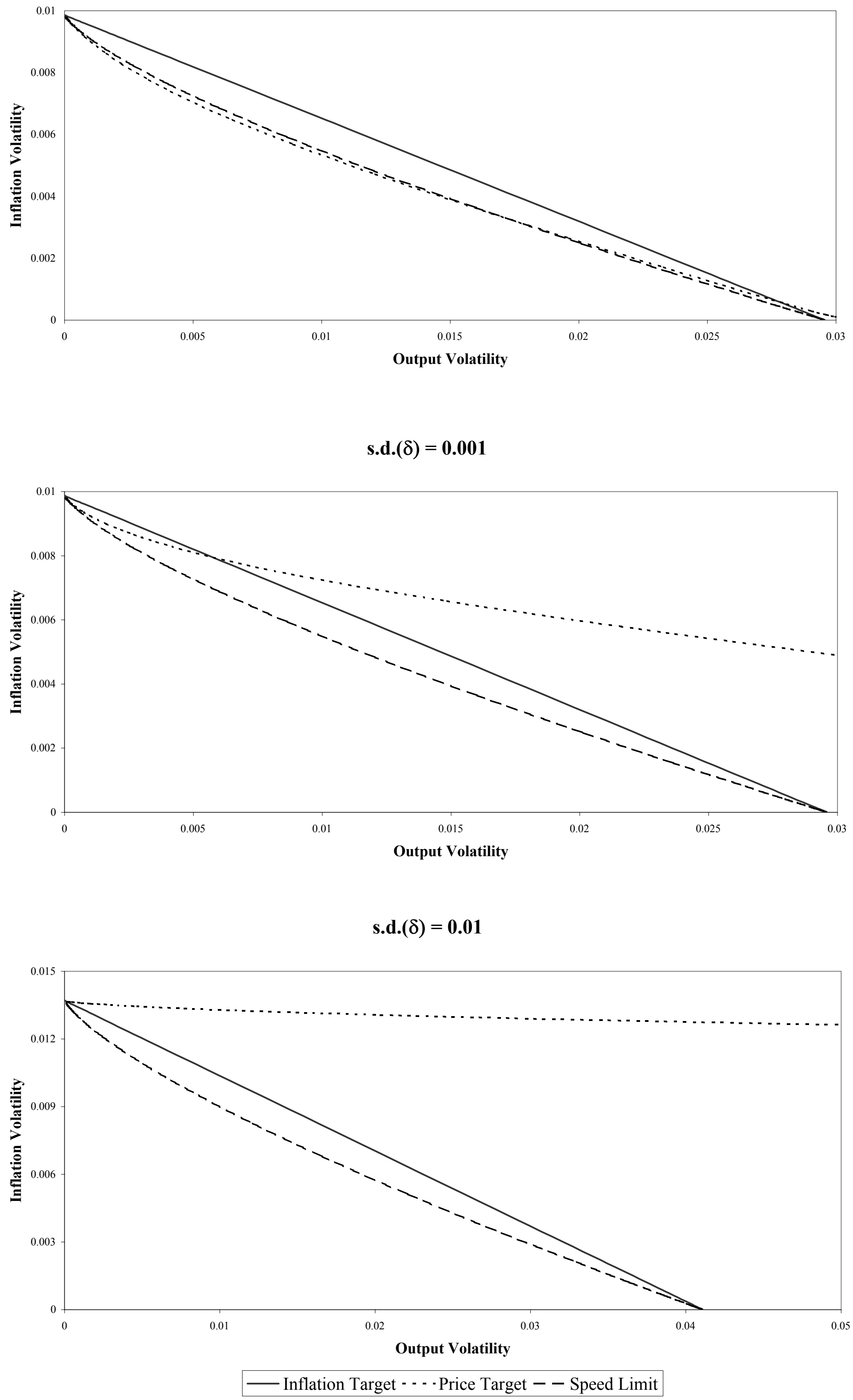


Figure 5. Agents Do Not Believe Growth Rate

s.d.(δ) = 0.0001



The Credibility of the Monetary Policy "Free Lunch"

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Abstract

Price level targeting has been proposed as an alternative to inflation targeting that may confer benefits if a central bank sets policy under discretion, even if society's loss function is specified in terms of inflation (instead of price level) volatility. This paper demonstrates the sensitivity of this argument. If even a small portion of agents use a rule-of-thumb to form inflation expectations, or does not fully understand the nature of the target, price level targeting may in fact impose costs on society rather than benefits.

Keywords: Price Level Targeting, Inflation Targeting, Credibility, Free Lunch, Discretion

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1. Introduction

In recent times, a growing number of central banks have commenced explicitly targeting inflation as a means to improving economic performance. Associated with this has been growing central bank credibility, as explicit targets have been achieved, and deviations from the targets have been clearly articulated (see Bernanke, Laubach, Mishkin, and Posen (1999) and Johnson (1998) for evidence of this).

Some have argued that central banks should go further. One characteristic of an inflation target is that past mistakes are ignored: if inflation lies above or below the target one period, the inflation target for the following period does not change. In a world in which there are nominal rigidities, optimal policy will generally include some degree of history dependence, so that current policy should condition on past mistakes.

One special case of a history dependent monetary policy that has received particular attention is price level targeting. Several recent papers have focused on the potential benefits of the central bank targeting the price level when the central bank operates under discretion, even if society's loss function is specified in terms of inflation variability. Svensson (1999), using a Neo Classical Phillips curve, argued that there is an advantage to doing so provided the output gap is sufficiently persistent, and labeled this a "free lunch." Others have found even stronger support for price level targeting when agents are forward looking. These papers utilize variants of the New Keynesian framework, outlined in Roberts (1995) and Clarida, Gali and Gertler (1999). This framework assumes that changing prices is costly, so prices that are set today reflect future expectations of inflation. For example, Dittmar and Gavin (2000) find that the inflation-output variability trade-off is better with a price level target than an inflation target. They argue that adding a price level target with a small weight has little cost in terms of the real side of the economy yet is beneficial in reducing inflation volatility.

Vestin (2000), using a similar framework, finds that price level targeting under discretion outperforms inflation targeting under discretion. In some cases, he finds that price level targeting under discretion can result in the same outcome as inflation targeting under commitment, provided the parameters of the loss function are suitably adjusted. In a related paper, Barnett and Engineer (2001) provide a summary of the literature and consider a hybrid New Keynesian – Neo Classical Phillips curve. They argue that optimal monetary policy precludes long-run price-level drift if inflation expectations are sufficiently forward looking, among other contexts.

The existing literature has assumed that agents are fully rational and that the central bank enjoys perfect credibility. Therefore whether a central bank has an inflation target or a price level target, agents are assumed to know the target, and form expectations that condition on it. Here the robustness of these results is considered. Suppose, for example, that constructing rational expectations of inflation requires substantial costs. Then a portion of economic agents may use rules-of-thumb to form expectations instead. Alternatively, suppose that agents do not know or believe the stated target of the central bank. They would then base their inflation expectations on a policy target that differs from the true target of the central bank.

A number of examples of rule-of-thumb forecasting and imperfect credibility are considered below. I find that in nearly all cases considered, the existing results are sensitive. As the assumptions in the existing literature are relaxed, the "free lunch" rapidly loses its culinary value, especially if inflation volatility has a high weight in society's loss function. While the presence of rule-of-thumb forecasters or imperfect credibility is typically costly with either an inflation or a price level target, it is especially so with a price level target.

The basic model, demonstrating the potential role for price level targets, follows. Section 3 discusses history-dependent monetary policy, section 4 considers the impact of rule-of-thumb forecasting, while section 5 considers imperfect credibility. Conclusions then follow.

2. A Simple New-Keynesian Model

Here I describe a simple linear-quadratic, forward-looking model similar to that in Vestin (2000) in which price level targeting can play a role in improving the trade-off between output and inflation variability faced by the central bank when monetary policy is set under discretion. The economy is assumed to consist of a Phillips curve given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \tag{1}$$

where π_t is inflation, x_t is the output gap and u_t is an exogenous shock term that is known by the central bank. Expectations of future inflation are assumed rational, and all variables are expressed in logs. For simplicity, x_t can be considered the policy instrument of the central bank.

An inflation-targeting central bank seeks to minimize a standard, quadratic loss function given by

$$L = (1-\beta)E_t \sum_{i=0}^{\infty} \beta^i L_{t+i},$$

$$L_{t+i} = (\pi_{t+i} - \pi^*)^2 + \lambda x_{t+i}^2, \tag{2}$$

subject to (1), for some inflation target π^*. Note that I am assuming, as in Svensson (1999) and Vestin (2000), that inflation targeting or price level targeting central banks continue to pay attention to real variables in the economy, except in the special case where $\lambda = 0$.

Under discretionary policy, the central bank is assumed to lack the means to commit to future policy actions. Given the absence of state variables with inflation targeting, optimal monetary policy will therefore minimize the period loss function, taking the form

$$x_t = \frac{\kappa(1-\beta)}{\kappa^2 + \lambda(1-\beta)}\pi^* - \frac{\kappa}{\kappa^2 + \lambda}u_t, \tag{3}$$

while inflation evolves according to

$$\pi_t = \frac{\kappa^2}{\kappa^2 + \lambda(1-\beta)}\pi^* + \frac{\lambda}{\kappa^2 + \lambda}u_t. \tag{4}$$

To examine monetary policy with a price level target, note that (1) may be rewritten as

$$E_t(p_{t+1} - p^*_{t+1}) = \frac{1+\beta}{\beta}(p_t - p^*_t) - \frac{1}{\beta}(p_{t-1} - p^*_{t-1}) - \frac{\kappa}{\beta}x_t - \frac{1}{\beta}u_t + \frac{1-\beta}{\beta}\pi^*, \tag{5}$$

where $p^*_t = p^*_{t-1} + \pi^*$. The appropriate quadratic period loss function is then given by

$$L_{t+i} = (p_{t+i} - p^*_{t+i})^2 + \lambda x^2_{t+i}, \tag{6}$$

where λ is appropriately scaled to reflect the difference between the magnitude of price level and inflation rate volatility. In contrast to inflation targeting today's policy affects losses in future periods even without commitment technology, implying the presence of state variables in the model. We follow the methodology of Currie and Levine (1993), and find that under optimal discretionary monetary policy, the paths of output and inflation are defined by

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2, \tag{7}$$

$$p_t = p^*_t + \phi_1[(p_{t-1} - p^*_{t-1}) + u_t] + \phi_2, \tag{8}$$

for θ_1, θ_2, ϕ_1, ϕ_2 given in Appendix 1.

We can now consider the policy frontiers obtained under either a price level or an inflation target. The parameters considered here are the same as in Vestin (2000): $\kappa = 1/3$; $\beta = 1$; $\pi^* = 0$; and $0 \leq \lambda \leq \infty$. Note that in the limit, as $\beta \to 1$, $E(L_t) =\text{Var}(\pi_t - \pi^*)+\text{Var}(x_t)$ with an inflation target, and $E(L_t) =\text{Var}(p_t - p^*_t)+\text{Var}(x_t)$ with a price level target. To construct the policy frontier, the economy is simulated 1000 times for 100 periods for different levels of λ, and the average level of output and inflation volatility in the final period is computed. 100 periods is sufficient to ensure that the results are independent of the starting point, while averaging across 1000 simulations results in policy frontiers that are virtually indistinguishable from those that may be obtained analytically. While analytical solutions may be easily obtained for this base case, they are difficult to obtain for some of the cases that follow; hence the reliance on numerical results.

Figure 1 contains the policy frontiers for the base model. Over the entire range of possible values of λ, a price level target results in a better trade-off between output volatility and inflation volatility than an inflation target. Note that the graphs given here are in terms of the standard deviations of output and inflation rather than the variances, to aid interpretation. Hence the trade-off between output and inflation volatility with an inflation target obtained here is a straight line, whereas Vestin (2000) obtained a convex relation.

Note also that the outcome with strict inflation targeting coincides with that when there is strict price level targeting- in either case, inflation volatility equals zero, while output volatility is at its highest level. Finally, for intermediate points, the desired level of λ with a price level target will differ systematically from that with an inflation target. As Vestin (2000) argues, this is as a result of both differences between the magnitudes of price level volatility and inflation volatility and differences in the desired degree of conservatism. Figure 2 plots the optimal choice of λ with a price level target against that with an inflation target for each level of λ in society's loss function. If society is relatively concerned about inflation volatility, a price level targeting central bank should have a smaller λ than an inflation targeting central bank (that is, be more conservative); the reverse is true if society is relatively concerned about output volatility.

3. History-Dependent Monetary Policy

To illustrate the history dependence of a price level target, note that (8) above can be rewritten as

$$\pi_t = (\phi_1 - 1)(p_{t-1} - p^*_{t-1}) + \phi_1 u_t + (\phi_2 + \pi^*). \tag{9}$$

In comparison with the inflation path under inflation targeting given by (4), there is an additional argument representing the difference between the price level target and the realized price level in the previous period.

Also, with inflation targets, if agents discount the future and the central bank cares about both inflation and output ($\beta < 1$ and $\lambda > 0$), the weight on π^* in (4) is less than one, and the expected long-run average inflation rate is less than π^*. By contrast, with price level targeting the expected long-run average inflation rate is equal to π^*. To see this, (8) may be rewritten as

$$\pi_t = \pi^* + \phi_1 u_t + (\phi_1 - 1) \sum_{i=1}^{\infty} \phi_1^i u_{t-i}, \tag{10}$$

which has an unconditional expectation of π^*. With price level targeting, the average inflation rate may deviate substantially from π^* in the short run, while the long run average inflation rate will tend to π^*. This is because the further the price level deviates from the target path, the greater will be the loss to the central bank, and therefore the size of the monetary policy response to bring the price level back to the desired price path (and the inflation rate back to π^*).

One important point to note is that price level targeting is just one example of a history-dependent monetary policy, and not all history-dependent monetary policies are equal. Suppose, for example, that the central bank were to seek to attain a time-varying inflation target where the time-variation is sufficient to correct for past mistakes, so that

$$\pi_t^* = \pi_{t-1}^* - \pi_{t-1} + \pi^*. \tag{11}$$

One could think of this as a price level target, specified in inflation terms. Under these circumstances, the optimal policy under discretion would result in the policy frontier in Figure 3. Clearly the outcome is worse than with either an inflation target or a price level target, over the full range of possible values of λ. The intuition behind this result is that the expectations term in the Phillips curve is given by

$$E_t(\pi_{t+1}) = E_t(p_{t+1}) - p_t. \tag{12}$$

With a target expressed in inflationary terms and discretionary monetary policy, this entire term is taken to be exogenous by the central bank. In contrast, if the target is expressed in price level terms, optimal discretionary policy takes account of the impact of the current price level on future expectations (for example, from (8) with $p_t^* = 0$, $E_t(\pi_{t+1}) = (\phi_1 - 1)p_t$). Therefore price level targeting incorporates more information than inflation targeting, and results in a better trade-off. Note that this effect would be obtained whenever expectations are forward looking. If expectations were backward looking, then pursuing a price level target in inflation terms would result in no loss to the central bank.

4. Rule-Of-Thumb Forecasting

The base result outlined above assumes that agents form rational expectations of inflation. This assumes complete knowledge of the structure of the economy, the loss function of the central bank, and the size of the shock term. Suppose that gathering this information were costly. Then it is possible that optimizing agents may follow simple rules-of-thumb in forming expectations. Two such cases are considered below, for $\beta = 1$.

When inflation expectations are biased towards current inflation

I first consider the case where a portion $(1 - \omega)$ of agents expect future inflation to be equal to its current level. Then aggregate inflation expectations are biased away from their rational expectation towards the current inflation rate. This may result if some agents have short memory, or perceive changes in the inflation rate to reflect a change in the inflation target, for example.

In the New Keynesian framework, agents set prices today based on what they expect inflation to be in the future. If some agents expect future inflation to be equal to current inflation, they will set prices accordingly, and inflation expectations will appear to be less forward looking. In what follows I will assume that the agents who are forward looking (including the central bank) are sophisticated in that they take account of the effect of less sophisticated agents when forming their expectations. Replacing (1) above with the following

$$\pi_t = [\omega E_t(\pi_{t+1}) + (1-\omega)\pi_t] + \kappa x_t + u_t, \tag{13}$$

and solving the model for different values of ω captures differing levels of credibility. The policy frontiers are very qualitatively similar to those in Figure 1. Rewriting (13) as

$$\pi_t = E_t(\pi_{t+1}) + \frac{\kappa}{\omega} x_t + \frac{u_t}{\omega}, \tag{14}$$

one can see why. A reduction in credibility here increases the slope of the Phillips curve and also the potency of inflation shocks. But these changes have equal effects on the policy frontier with either a price level or an inflation target. Therefore the "free lunch" remains. Figure 4 illustrates these results for differing values of ω.

When inflation expectations are biased towards long-run average

Now consider the case where some agents' expectations of future inflation are biased away from their rational expectation towards the long-run average inflation rate, while others form inflation expectations that take into account the current policy target. This may result if agents are not sophisticated enough to form rational expectations, or do not know the model or size of the shock term.

In this model, which is linear and entails symmetric shock terms and additive uncertainty, as $\beta \to 1$ the average long-run rate of inflation is π^* with either a price-level or an inflation target. Replacing the Phillips Curve in (1) with

$$\pi_t = \omega E_t(\pi_{t+1}) + (1-\omega)\pi^* + \kappa x_t + u_t \tag{15}$$

where $(1-\omega)$ is the portion of unsophisticated agents, we can solve the model with an inflation target and obtain

$$x_t = -\frac{\kappa}{\kappa^2+\lambda}u_t, \tag{16}$$

$$\pi_t = \pi^* + \frac{\lambda}{\kappa^2+\lambda}u_t, \tag{17}$$

which are identical to the case of $\omega = 1$. This is because inflation targeting implies that agents' rational expectation of next period's inflation rate is simply the inflation target, which is also the long-run average inflation rate. Hence inflation expectations being biased towards the long-run average inflation rate does not introduce any distortion into the model.

To examine monetary policy with a price level target, note that (15) may be rewritten as

$$E_t(p_{t+1} - p^*_{t+1}) = \frac{1+\omega}{\omega}(p_t - p^*_t) - \frac{1}{\omega}(p_{t-1} - p^*_{t-1}) - \frac{\kappa}{\omega}x_t - \frac{1}{\omega}u_t. \tag{18}$$

The price level path may then be solved using the method outlined in the appendix.

The policy frontiers for different values of ω are given in Figure 5. We see here that as the portion of agents who focus only on the target when forming expectations increases, the "free lunch" resulting from following a price level target diminishes.

5. Imperfect Credibility

The base result outlined in section 2 assumes that agents form rational expectations that condition on the true central bank target. That is, the central bank has perfect credibility. While there is some evidence that the credibility enjoyed by many central banks has improved in recent years, few would believe that credibility is perfect. In reality, if inflation were to remain away from the target for a period of time or were to move far from a constant target due to a particular realization of shocks, some agents may interpret this as a change in the target. Also, if a central bank were to introduce a price level target, even if it had previously enjoyed high credibility for an inflation target, it would not necessarily enjoy immediate credibility for its new target. To the extent that credibility must be attained for the new target, the expectations channel by which price level targeting delivers superior output and inflation variability to inflation targeting may be weakened.

One can think of credibility in a number of different ways, two of which will be explored here. In each case, agents do not believe the target price path of the central bank, and base their expectations on an alternative price path, in terms of either the starting point or the growth rate.

When agents do not believe the starting point

With an inflation target, bygones are bygones, and so the starting point (in terms of the price level) of the target price path is relevant for one period only. In contrast, with a price level target, the starting point is relevant for all future periods. I consider the situation where agents form expectations based on the belief that the starting point of the price path differs from the true one by an amount δ. Therefore expectations are based on a price level target of $\hat{p}_t^*$ where

$$\hat{p}_t^* = p_t^* + \delta, \tag{19}$$

and agents' expectations consistently exceed the true target.

With an inflation target, the results are as in (3) and (4) above beyond the first period. With a price level target, expectations are formed based on an incorrect target, and policy is set optimally, conditional on the forecasts. The solution to the policy problem is outlined in Appendix 2.

Figure 6 shows the policy frontiers for $\delta = 0.001$ (0.1%), $\delta = 0.01$ (1%), and $\delta = 0.1$ (10%) respectively. A small bias in the perceived anchor for the desired price level path results in a rapid loss of the "free lunch" from pursuing a price level target, but has no impact with an inflation target. This contrast points to the importance for a central bank to credibly communicate the desired price path under a price level target.

Historically, examples that are consistent with this argument may be found from the Gold Standard era. A gold standard may be viewed as equivalent to a price level target in the sense that deviations from the standard must in principle be corrected by future policy. As outlined in Capie and Collins (1983) and discussed in Smith (1998), the United Kingdom left the Gold Standard during World War I, and sought to return to it shortly after the end of the war. The process of returning was associated with a large and persistent increase in unemployment (from below 5% to a peak of more than 20%), and a large decline in industrial production. The implication of my analysis here is that to the extent that credibility played a role in this recession, it could have been prevented by simply locking into a new Gold Standard at the post-war rate rather then returning to the old gold standard.

When agents do not believe the price level growth rate

An alternative form of credibility that is important with either a price level target or an inflation target is the growth rate. In the case of an inflation target, this will have a negative impact on the policy frontier that is uniform (in expectation) over time. With a price level target, the impact of imperfect credibility on output and inflation volatility will increase over time, as the perceived target and the actual target diverge. Clearly a price level target will then generally be costly relative to an inflation target.

In the two examples above, imperfect credibility results in agents making systematic errors in forecasting inflation. While this may be realistic in the short-run (for example, immediately after adopting an inflation target), this is likely to be unrealistic in the long run, since agents may be expected to learn about the inflation target over time. A less extreme form of imperfect credibility, with agents making non-systematic errors in predicting the target price level growth rate (π^*), may occur with a time-varying inflation target. To be precise, suppose

$$E_t(\pi^*_{t+1}) = \pi^*_{t+1} + \delta_t, \quad \delta_t \sim N(0, \sigma^2_\delta). \tag{20}$$

Many central banks allow for the idea of such a time-varying target in their inflation objective. For example, the objective of the Australian central bank is to maintain "consumer price inflation between 2 and 3 per cent, on average, over the cycle" (Reserve Bank of Australia 2003), and the New Zealand central bank objective is "to keep future CPI inflation outcomes between 1 per cent and 3 per cent on average over the medium term" (Reserve Bank of New Zealand 2002). Not only do these objectives leave some discretion with the central bank in choosing the average inflation objective, they also allow for discretion in the choice of the exact target at any given point in time.

With an inflation target, δ_t will be equivalent from the central bank's point of view to an additional inflation shock:

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda}(u_t + \delta_t), \tag{21}$$

$$\pi_t = \pi^* + \frac{\lambda}{\kappa^2 + \lambda}(u_t + \delta_t). \tag{22}$$

With a price level target, the price level the price and output paths may be solved in an analogous fashion to Appendix 2; see Appendix 3 for details.

Figure 7 shows the policy frontiers for $\sigma_\delta = 0.0001$ (0.01%), $\sigma_\delta = 0.001$ (0.1%), and $\sigma_\delta = 0.01$ (1%). Once again, the results illustrate the fragility of the monetary policy free lunch.

6. Conclusions

Price level targeting has been proposed as an alternative to inflation targeting that may confer benefits where a central bank operates under discretion, even if society's loss function is specified in terms of inflation (instead of price level) volatility. This paper demonstrates the sensitivity of this argument to the expectations formation process of agents. If even a small portion of agents believes that inflation will continue in the future at current levels, or does not fully understand the nature of a price level target, price level targeting may in fact impose costs on society rather than benefits, especially if inflation volatility has a high weight in society's loss function. While rational expectations and perfect credibility are generally beneficial with either a price level or an inflation target, an inflation target is more robust to alternative assumptions.

These results suggest that caution should be exercised in considering a price level target as the basis for monetary policy, unless society has preferences specified in terms of price level (rather than inflation) volatility.

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Notation

π_t inflation rate

β discount rate

x_t output gap

u_t cost-push shock

π^* inflation target

λ weight on output objective in central bank loss function

κ slope of Phillips curve

Appendix 1

To solve the price level targeting model under optimal discretionary policy, optimal discretionary monetary policy will satisfy

$$V_t = \min E_t[L_t + \beta V_{t+1}], \tag{A1}$$

where the relevant terms of V_t are

$$V_t = \gamma_1(p_{t-1} - p^*_{t-1})^2 + 2\gamma_2(p_{t-1} - p^*_{t-1}) + ... \tag{A2}$$

Following Currie and Levine (1993), the solution price-path may be written as

$$E_t(p_{t+1} - p^*_{t+1}) = \phi_1(p_t - p^*_t) + \phi_2. \tag{A3}$$

Equations (5) and (A3) imply that

$$(p_t - p^*_t) = \frac{1}{1 + \beta - \beta\phi_1}\big((p_{t-1} - p^*_{t-1}) + \kappa x_t + u_t + [\beta\phi_2 - (1-\beta)\pi^*]\big). \tag{A4}$$

Substituting (A2) (iterated forward) and (A4) into (A1) and differentiating with respect to x_t yields the optimal discretionary policy rule

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2, \tag{A5}$$

$$\theta_1 = \frac{-\kappa(1 + \beta\gamma_1)}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}, \tag{A6}$$

$$\theta_2 = -\frac{\kappa(1 + \beta\gamma_1)[\beta\phi_2 - (1-\beta)\pi^*] + \kappa\beta\gamma_2(1 + \beta - \beta\phi_1)}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}. \tag{A7}$$

Substituting (A5) back into (A4) yields

$$\begin{aligned}(p_t - p^*_t) =& \frac{\lambda(1 + \beta - \beta\phi_1)}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}\big((p_{t-1} - p^*_{t-1}) + u_t\big) \\ &+ \frac{\lambda(1 + \beta - \beta\phi_1)[\beta\phi_2 - (1-\beta)\pi^*] - \kappa^2\beta\gamma_2}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}.\end{aligned} \tag{A8}$$

Iterating forward one period, taking expectations, and combining with (A3) yields

$$\phi_1 = \frac{\lambda(1+\beta-\beta\phi_1)}{\kappa^2(1+\beta\gamma_1)+\lambda(1+\beta-\beta\phi_1)^2}, \tag{A9}$$

$$\phi_2 = \frac{\lambda(1+\beta-\beta\phi_1)[\beta\phi_2-(1-\beta)\pi^*]-\kappa^2\beta\gamma_2}{\kappa^2(1+\beta\gamma_1)+\lambda(1+\beta-\beta\phi_1)^2}. \tag{A10}$$

Finally, substituting (A2) iterated forward, (A5), and (A8) back into (A1) and equating the relevant coefficients with (A2) yields

$$\gamma_1 = (1+\beta\gamma_1)\phi_1^2 + \lambda\theta_1^2, \tag{A11}$$

$$\gamma_2 = (1+\beta\gamma_1)\phi_1\phi_2 + \lambda\theta_1\theta_2 + \beta\gamma_2\phi_1. \tag{A12}$$

Solving these numerically, we then have the law of motion for both prices and output and can calculate output and inflation volatility.

Appendix 2

Agent's inflation expectations are formed under the assumption that the actual target is equal to the perceived target. From (A8),

$$E_t(p_{t+1}-\hat{p}^*_{t+1}) = \phi_1(p_t-\hat{p}^*_t)+\phi_2, \tag{A13}$$

for ϕ_1, ϕ_2 outlined in appendix 1. Combining (A13) with the Phillips curve (5) yields

$$(p_t-p^*_t) = \frac{1}{1+\beta-\beta\phi_1}\big((p_{t-1}-p^*_{t-1})+\kappa x_t+u_t+[\beta(\phi_2+\delta(1-\phi_1))-(1-\beta)\pi^*]\big). \tag{A14}$$

On the other hand, the central bank takes expectations as given and optimizes monetary policy conditional on them. Their expectations will satisfy

$$E_t(p_{t+1}-p^*_{t+1}) = \phi_1(p_t-p^*_t)+\hat{\phi}_2, \tag{A15}$$

for some $\hat{\phi}_2$. Combining (A15) with (5) and equating coefficients with (A14) implies

$$\hat{\phi}_2 = \phi_2 + \delta(1 - \phi_1), \tag{A16}$$

with corresponding new values implied for the other constant coefficients ($\hat{\theta}_2$ and $\hat{\gamma}_2$) as well. The paths of output and inflation are then functions of ϕ_1, θ_1, $\hat{\phi}_2$, and $\hat{\theta}_2$.

Appendix 3

In this case, (A13) implies that prices follow

$$E_t(p_{t+1} - p^*_{t+1}) = \phi_1(p_t - p^*_t) + \phi_2 + \phi_1\delta_t + (1 - \phi_1)\sum_{i=0}^{\infty}\delta_{t-i}. \tag{A17}$$

The relevant terms of the value function are

$$V_t = \gamma_1(p_{t-1} - p^*_{t-1})^2 + 2\gamma_2(p_{t-1} - p^*_{t-1}) + 2\gamma_3(p_{t-1} - p^*_{t-1})\sum_{i=0}^{\infty}\delta_{t-i}..., \tag{A18}$$

implying that

$$x_t = \theta_1[(p_{t-1} - p^*_{t-1}) + u_t] + \theta_2 + \theta_3\delta_t + \theta_4\sum_{i=0}^{\infty}\delta_{t-i}, \tag{A19}$$

$$\theta_3 = \frac{-\beta\kappa(1 + \beta\gamma_1)\phi_1}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}, \tag{A20}$$

$$\theta_4 = \frac{-\beta\kappa[(1 + \beta\gamma_1)(1 - \phi_1) + \gamma_3(1 + \beta - \beta\phi_1)]}{\kappa^2(1 + \beta\gamma_1) + \lambda(1 + \beta - \beta\phi_1)^2}, \tag{A21}$$

$$\gamma_3 = (1 + \beta\gamma_1)\phi_1(1 - \phi_1) + \lambda\theta_1\theta_4 + \beta\gamma_3\phi_1. \tag{A22}$$

The solutions to ϕ_1, ϕ_2, θ_1, θ_2, γ_1 and γ_2 coincide with those in Appendix 1.

Figure 1. Base Model

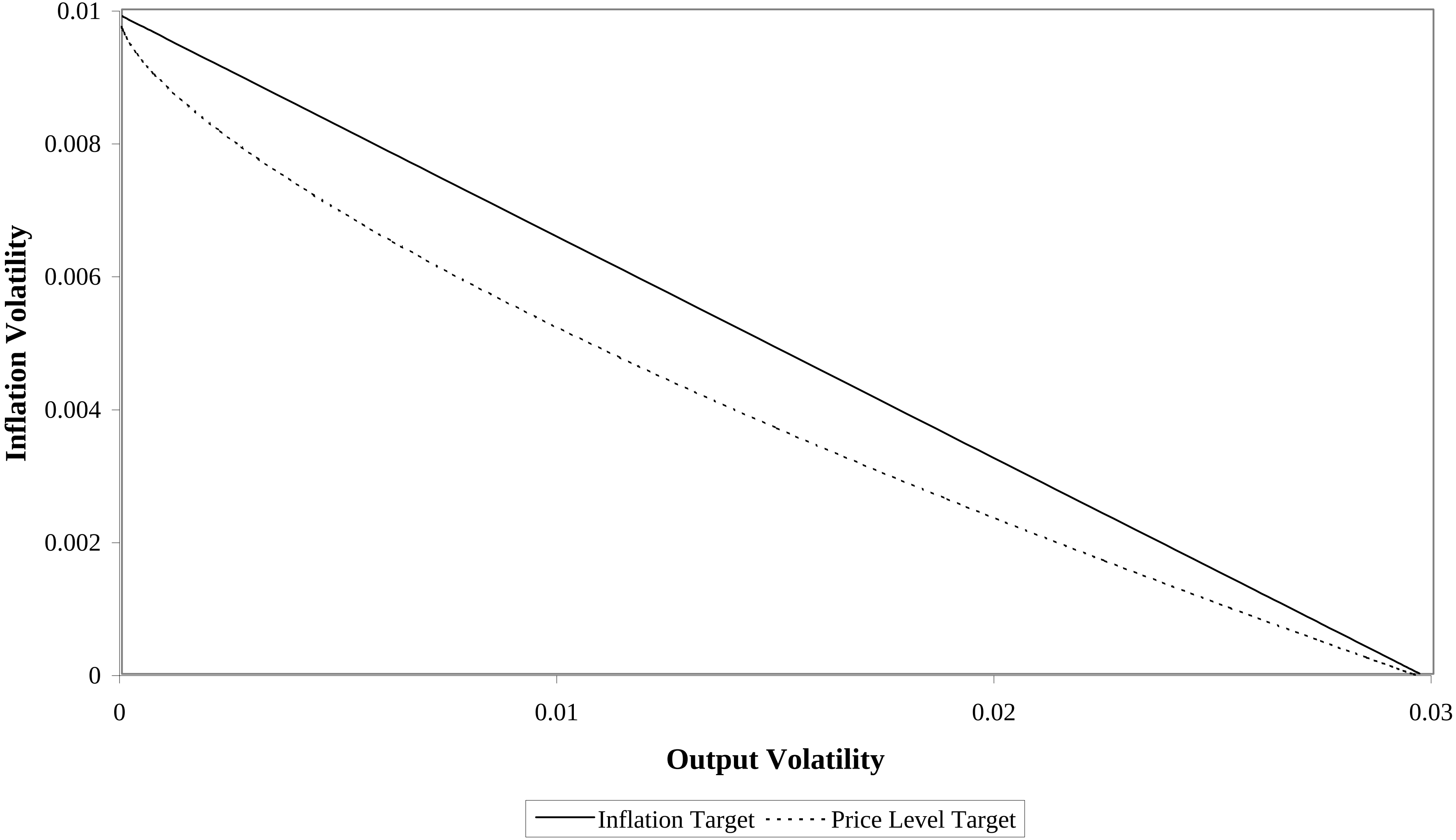


Figure 2. Optimal l

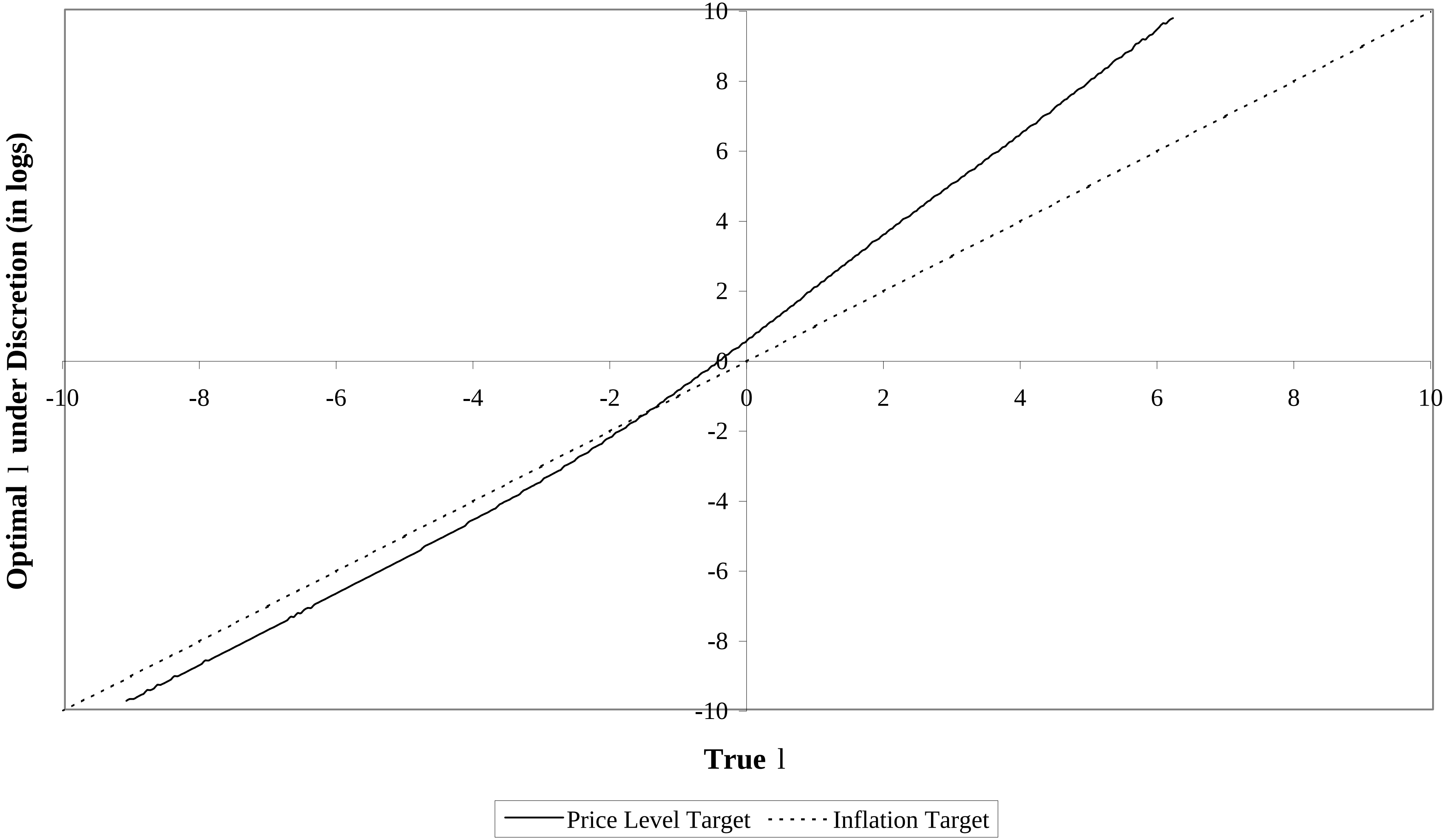


Figure 3. Price Level Targeting in Inflation Terms

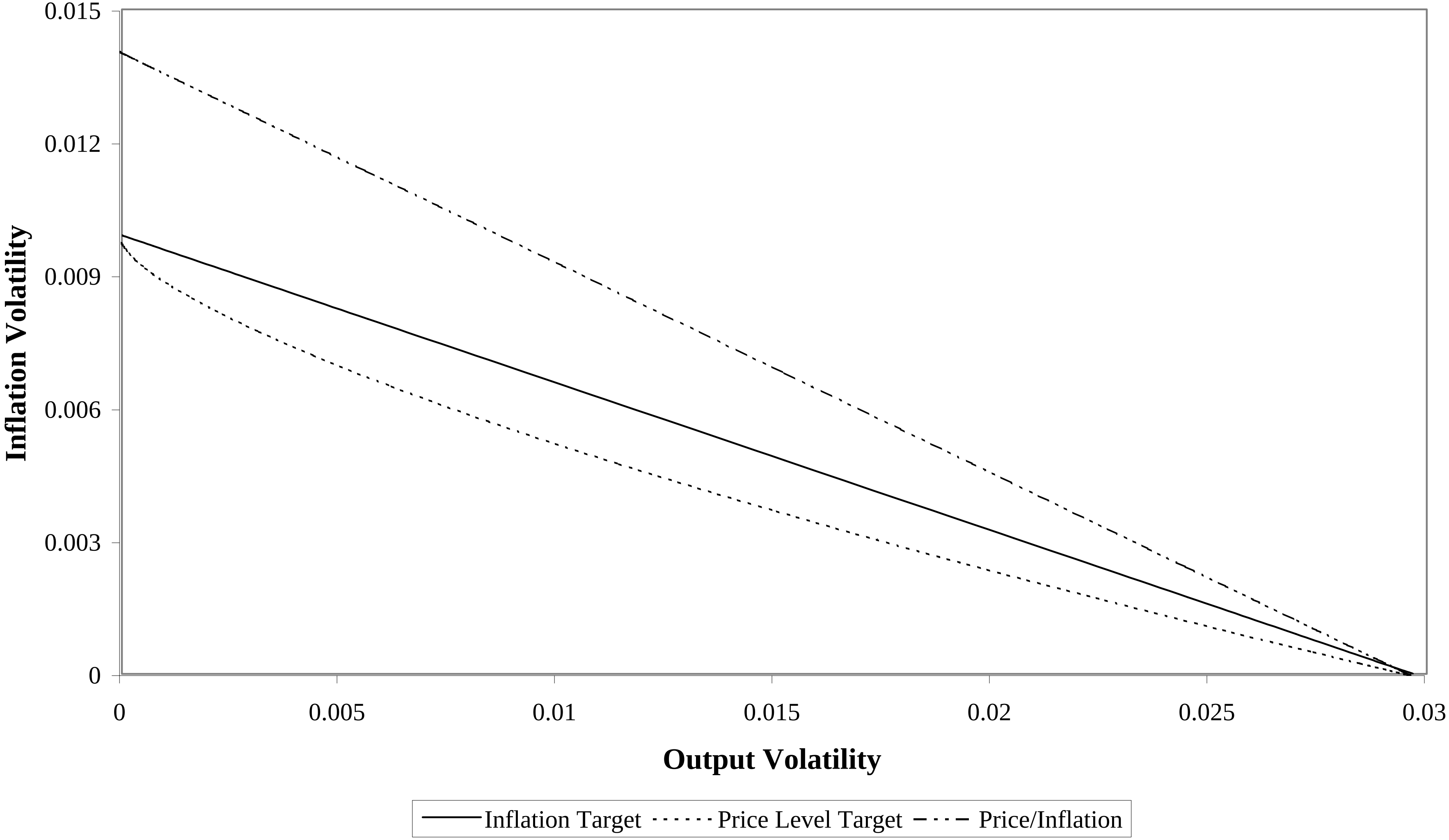
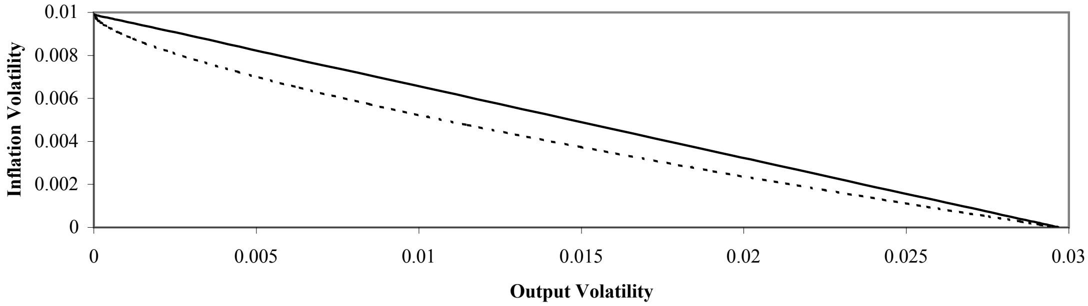
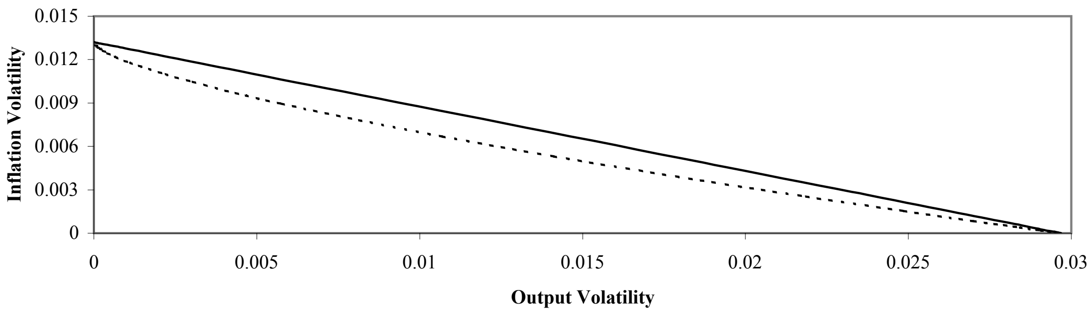


Figure 4. Expectations Biased Towards Current Inflation

ω=1.0



ω=0.75



ω=0.50

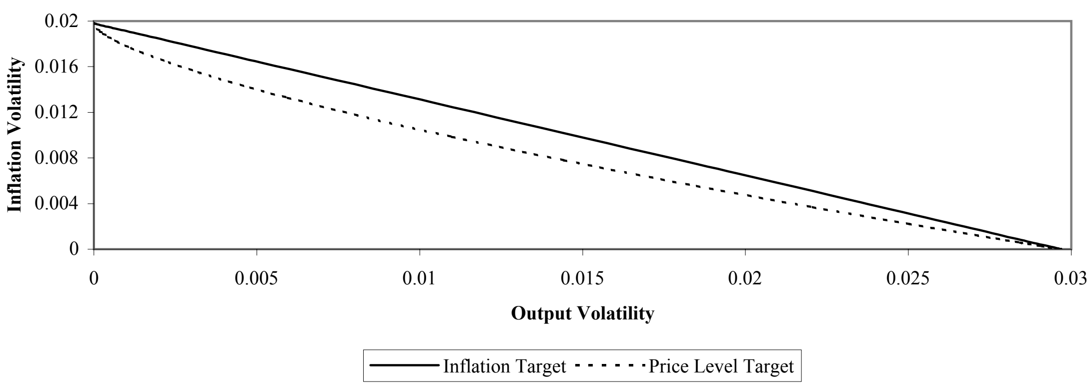


Figure 5. Expectations Biased Towards Long-Run Average

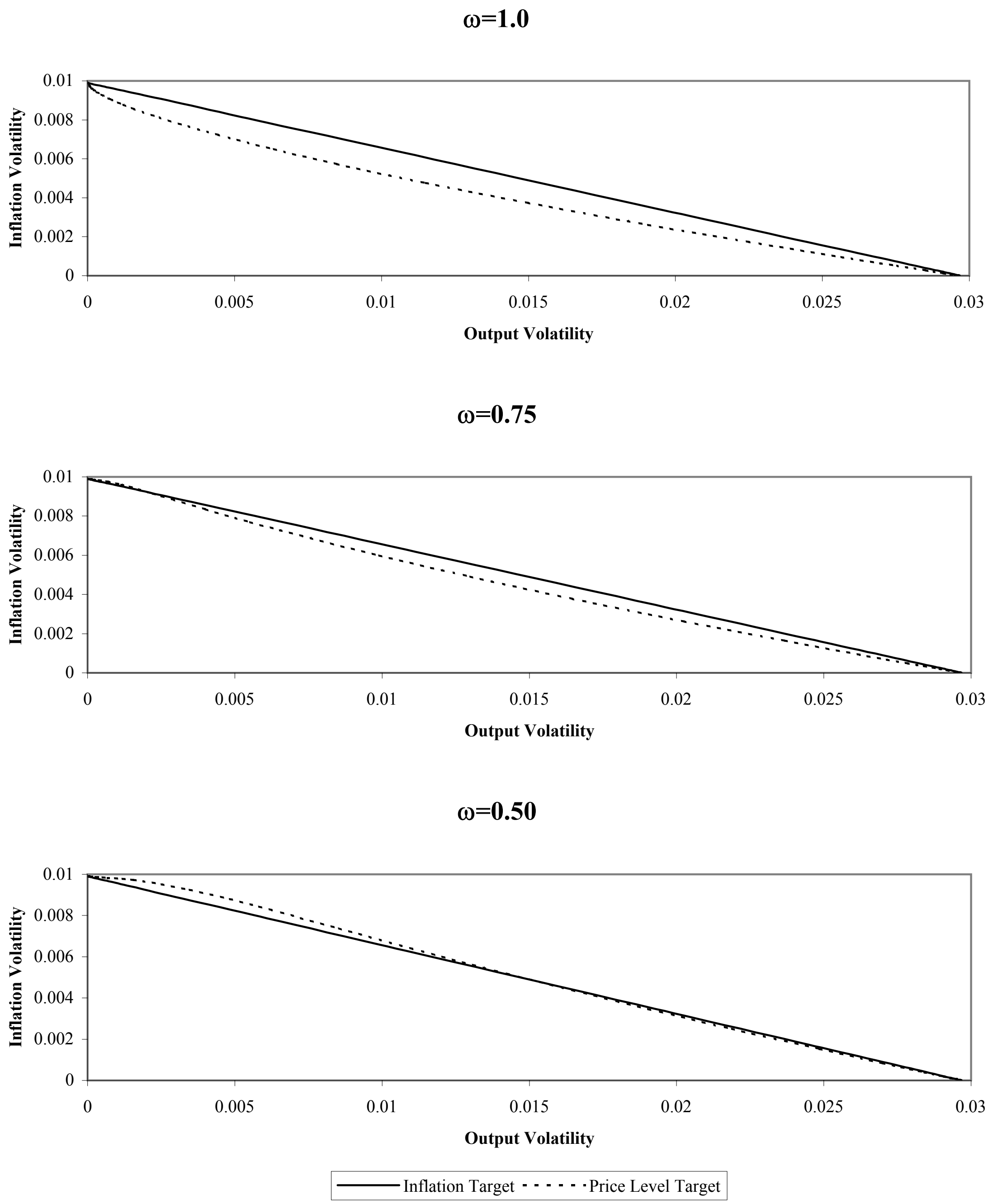
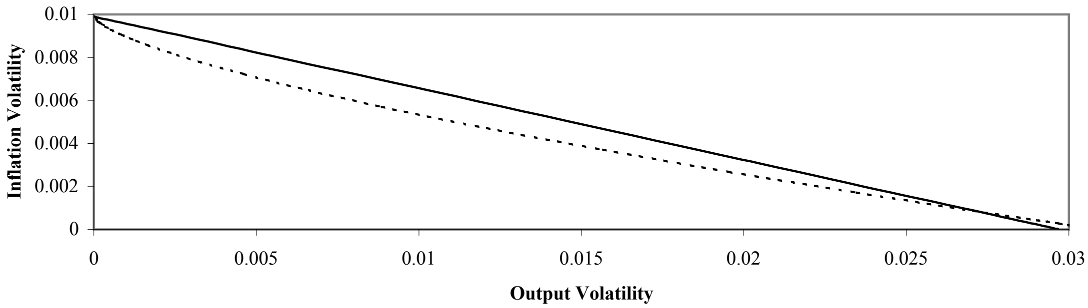
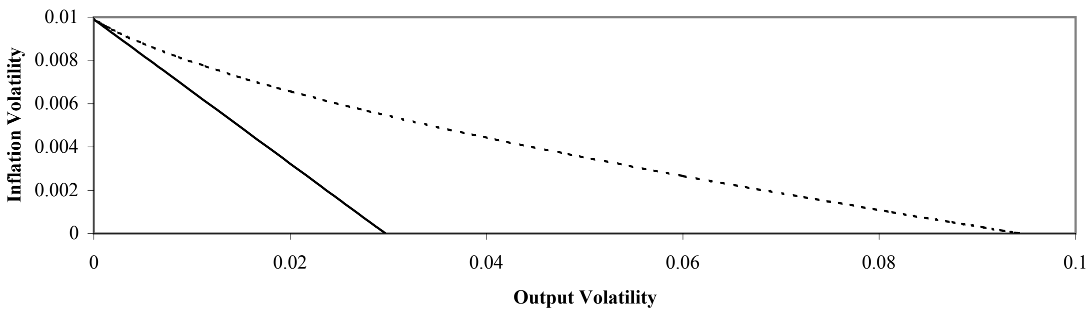


Figure 6. Agents Do Not Believe Starting Point

δ=0.001



δ=0.01



δ=0.1

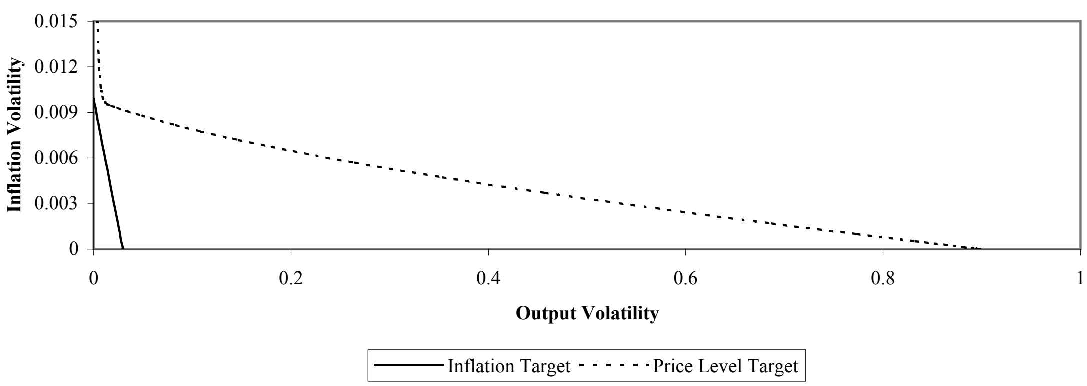
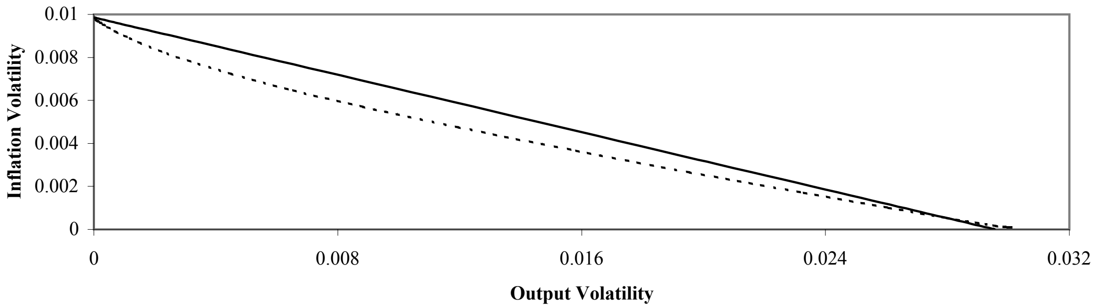
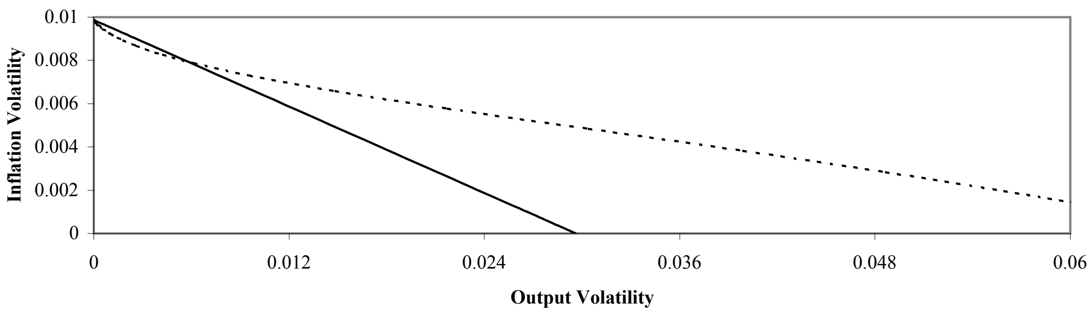


Figure 7. Agents Do Not Believe Growth Rate (Random δ)

s.d.(δ)=0.0001



s.d.(δ)=0.001



s.d.(δ)=0.01