

A Long-Swing Model of the Inflation-Deflation Cycle in Hong Kong^{*}

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Abstract

This paper examines inflation in Hong Kong. Unit root tests suggest that the inflation series is non-stationary. However, the unconditional distribution of inflation appears to change culminating in the current deflation. We employ a two-regime Markov-Switching model to decompose price dynamics into a sequence of stochastic segmented time trends. The non-linear specification appears to provide a superior conditional characterisation of the data over the more usual random walk. There is strong evidence of asymmetric persistence as the inflationary regime is relatively more volatile and persistent than the deflationary regime.

Keywords: Unit root, Segmented Trends, Regime Switching, Asymmetric Persistence

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1. Introduction

Arguably the most noteworthy achievement of macroeconomic policy in recent times has been the reduction of inflation rates in industrialised countries. For example, average inflation in the G7 economies has declined from 10% during 1974-1983 to levels below 4% since 1996. For Hong Kong the reduction of inflation has culminated in the current severe and persistent deflation. While there is a deep literature studying the effect of rapid inflation, little is known about the dynamics of deflation.

Following Nelson and Plosser (1982) the consensus view is that the majority of macroeconomic variables including inflation have a univariate time series representation with a unit root. While Rose (1988) rejects the unit root in US inflation, Evans and Lewis (1995), Crowder and Hoffman (1996), Crowder and Wohar (1998), and Ng and Perron (2001), *inter alia*, find that US inflation is non-stationary, while Edwards (1998) reports extreme persistence in Latin American inflation. Henry and Shields (2002) present similar results for the USA, UK and Japan.

Recently a literature has evolved documenting structural breaks and non-linearity in inflation, see Garcia and Perron (1996), Evans and Wachtel (1993) *inter alia*. It is well known that standard unit root tests are biased towards the null hypothesis in the face of one-off changes in regime see, Zivot and Andrews (1992) and Perron (1990), (1997), *inter alia*. In the case of neglected non-linearity unit root tests also suffer from bias, see Pippenger and Goering (1993), Caner and Hansen (2001), and Henry and Shields (2002) *inter alia*. Henry and Shields (2002) employ bootstrap methods based on threshold autoregressive models to distinguish between non-linearity and/or non-stationarity in US, UK and Japanese inflation.

Much of this existing literature on non-stationarity and non-linearity concentrates on the great inflation of the 1970's and 1980's. The majority of studies conclude that inflationary shocks are extremely persistent. In contrast this paper focuses on the relatively deep and persistent deflation suffered by Hong Kong since 1998. In particular we focus on a simple question: how persistent are deflationary shocks?

The paper is made up of seven sections. Section two of this paper discusses the problems associated with tests of the unit root hypothesis in inflation. In section three we describe the data and present the results of the unit root tests. The fourth section describes the Markov model of

segmented trends. The fifth section presents the estimates of the model and discusses the implications for Hong Kong. The penultimate section reconciles the results presented in sections 3 and 5, focussing on the likely pitfalls associated with unit root and stationarity tests. The final section provides a summary and some concluding comments.

2. Testing for a Unit Root in inflation

For ease of exposition, consider the AR(1) model for inflation, x_t ,

$$x_t = \mathbf{r}x_{t-1} + u_t \quad (1)$$

If $|\mathbf{r}| = 1$ then x_t is said to contain a unit root and shocks to inflation are infinitely persistent.

On the other hand if $|\mathbf{r}| < 1$ then shocks to inflation have finite lives. In principle testing for a unit root is relatively straightforward, however in reality distinguishing between stationarity and infinite persistence in inflation is a non-trivial task.

The null hypothesis of the Dickey-Fuller (1979) unit root test is $H_0 : x_t \sim I(1)$. Allowing for a drift, λ and a trend t , these tests are usually performed as a test of $H_0 : \tilde{\alpha} = 0$ in

$$\ddot{x}_t = \mathbf{m} + \mathbf{g}x_{t-1} + \mathbf{d}t + \sum_{i=1}^k \mathbf{a}_i \ddot{x}_{t-i} + u_t \quad (2)$$

The lag order k is chosen to ensure whiteness of the residuals, u_t .

Dejong, Nankervis, Savin and Whiteman (1992) argue that many tests including the ADF type test have low power when the autoregressive parameter is close to unity. To address this loss of power, Elliot, Rothenberg and Stock (1996), hereafter ERS, present an asymptotically efficient test of the unit root hypothesis based on the regression

$$\ddot{\tilde{x}}_t = \mathbf{g}\tilde{x}_{t-1} + \sum_{i=1}^k \mathbf{a}_i \ddot{\tilde{x}}_{t-i} + u_t \quad (3)$$

where \tilde{x}_t represents the quasi-differenced data obtained from the GLS regression

$$\tilde{x}_t = x_t + \tilde{\alpha}_t \mathbf{x}(\bar{c}) \quad (4)$$

This class of test requires the choice of, \bar{c} , the local-to-unity parameter, which following ERS is selected as

$$\bar{c} = \begin{cases} 1-7/T & \text{if } \mathcal{Z}_t = \{1\} \\ 1-13.5/T & \text{if } \mathcal{Z}_t = \{1, t\} \end{cases} \quad (5)$$

Since \tilde{x}_t has already been detrended the elements of \mathcal{Z}_t need not be included in (3). The DFGLS test is based on $H_0: \tilde{a}=0$ in (3). The results presented in ERS (1996) suggest that GLS local detrending yields substantial power gains over the standard ADF unit root test constructed as (2).

On the other hand, in the presence of a large negative moving average root in the residuals, the majority of unit root tests display significant size distortions resulting in over rejection of the unit root null hypothesis (Schwert 1990, Ng and Perron 1996 *inter alia*). Inflation rates often display large negative MA roots. In constructing the ADF and ERS tests it is necessary to select k , the autoregressive truncation lag. Ng and Perron (2001) use the GLS detrended data, \tilde{x}_t , to construct four further test statistics

$$MZ_{\mathbf{a}} = (T^{-1}\tilde{x}_t^2 - f_0) / 2\mathbf{k} \quad (6)$$

$$MSB = (\mathbf{k} / f_0)^{1/2} \quad (7)$$

$$MZ_t = MZ_{\mathbf{a}} \times MSB \quad (8)$$

and

$$MPT = \begin{cases} (\bar{c}^2\mathbf{k} - \bar{c}T^{-1}\tilde{x}_t^2) / f_0 & \text{if } \mathcal{Z}_t = \{1\} \\ (\bar{c}^2\mathbf{k} + (1-\bar{c})T^{-1}\tilde{x}_t^2) / f_0 & \text{if } \mathcal{Z}_t = \{1, t\} \end{cases} \quad (9)$$

where $\mathbf{k} = \sum_{t=2}^T (x_{t-1})^2 / T^2$ and f_0 is a estimate of the residual spectral density at the zero frequency. Again the choice of the autoregressive truncation lag, k , is critical for correct calculation of f_0 . Here k is chosen using the $MIC(k)$ of Ng and Perron (2001) as $k = k_{MIC} = \arg \min_k MIC(k)$ where

$$MIC(k) = \ln \hat{\mathbf{S}}_k^2 + \frac{C_T(\mathbf{t}\Gamma(k) + k)}{T - k_{\max}} \quad (10)$$

Where $\mathbf{t}_T(k) = (\hat{\mathbf{s}}_k^2)^{-1} \hat{\mathbf{g}}^2 \sum_{l=k_{\max}+1}^T \tilde{x}_{t-l}^2$ and $\hat{\mathbf{s}}_k^2 = (T - k_{\max})^{-1} \sum_{l=k_{\max}+1}^T \hat{u}_l^2$. Ng and Perron (2001) argue that the use of $MIC(k)$ in conjunction with GLS de-trending results in a battery of tests (6) – (9) with superior size and power properties.

Perron (1990, 1997) and Zivot and Andrews (1992), *inter alia*, argue that unit root tests are biased towards the null in the presence of structural breaks. The performance of the Ng and Perron and ERS tests in the face of such a break is, as yet, unknown. However, Perron (1997) presents tests for a unit root allowing for a break under both the null and alternative hypotheses. The first model considered by Perron, usually referred to as the innovational outlier or $IO(1)$ model allows for a gradual change in the intercept. The test is based on the regression

$$x_t = \mathbf{m} + \mathbf{q}DU_t(\mathbf{y}) + \mathbf{b} + \mathbf{f}D_t(\mathbf{y}) + \mathbf{r}x_{t-1} + \sum_{i=1}^k \mathbf{a}_i \ddot{A}x_{t-i} + u_t \quad (11)$$

Here $DU_t(\mathbf{y}) = 1$ if $t < T\mathbf{y}$ and 0 otherwise; and $D_t(\mathbf{y}) = 1$ if $t = T\mathbf{y} + 1$ and 0 otherwise. Under the second model, $IO(2)$, a change in the intercept and slope are allowed for at time $T\mathbf{y}$. The test is based on the regression

$$x_t = \mathbf{m} + \mathbf{q}DU_t(\mathbf{y}) + \mathbf{b} + \mathbf{f}D_t(\mathbf{y}) + \mathbf{n}DT_t(\mathbf{y}) + \mathbf{r}x_{t-1} + \sum_{i=1}^k \mathbf{a}_i \ddot{A}x_{t-i} + u_t \quad (12)$$

with $DU_t(\mathbf{y})$ and $D_t(\mathbf{y})$ as before and $DT_t(\mathbf{y}) = t - T\mathbf{y}$ if $t > T\mathbf{y}$ and 0 otherwise. The third test, often referred to as the additive outlier or AO test, the break is in the slope of x_t and is assumed to occur rapidly. The AO test is performed in two steps. The first step de-trends the data using

$$x_t = \mathbf{m} + \mathbf{b} + \mathbf{n}DT_t(\mathbf{y}) + \tilde{x}_t \quad (13)$$

The AO test is obtained from

$$\tilde{x}_t = \mathbf{r}\tilde{x}_{t-1} + \sum_{i=1}^k \mathbf{a}_i \tilde{x}_{t-i} + u_t \quad (14)$$

All of the tests (12) – (14) are based on the t -statistic for $H_0: \tilde{\eta} = 1$ in the various regressions.

Rejection of the null hypothesis $H_0: x_t \sim I(1)$ does not imply that $x_t \sim I(0)$. Kwiatkowski, Phillips, Schmidt and Shin (1992), hereafter KPSS, present a tests for the null of stationarity, $H_0: x_t \sim I(0)$, The KPSS test involves regressing x_t against a constant λ . The test is based upon

$$\mathbf{h}_m = T^{-2} \sum_t s_t^2 / \left(T^2 f_0 \right) \quad (15)$$

Where $S_t = \sum_{r=1}^t \hat{u}_r$, the partial sum of the residuals and again f_0 represents an estimate of the spectral density of the residuals at the zero frequency. A similar test for the null of stationarity about a linear trend, \mathbf{h}_t , involves regressing x_t against a constant, λ , and a linear trend, t .

3. Data Description and test results

Monthly observations on the CPI for Hong Kong were collected for the period 1985:1 to 2002:8. The price data were transformed into annual inflation rates using

$$x_t = 100 \times \log(P_t / P_{t-12}) \quad (16)$$

-Figure 1 Here-

-Figure 2 Here-

Figure 1 presents a time series plot of the CPI index while Figure 2 displays the associated inflation data. Visual inspection of the time series plots suggests that there are two distinct regimes in the data. Between 1985:1 – 1998:9 inflation was positive in Hong Kong, but after late 1998 there has been a deflation. Were the data normally distributed, the empirical distribution would be

$$g(x_t | \mathbf{ms}) = \frac{1}{\sqrt{2\mathbf{ps}}} \exp\left(-\frac{1}{2\mathbf{s}^2}(x_t - \mathbf{m})^2\right) \quad (17)$$

We estimate the parameters of $g(x_t | \mathbf{ms})$ using maximum likelihood methods for the full sample and for the pre-and post September 1998 sub-samples. The results of this procedure are displayed in table 1.

-Table 1 Here-

For the full sample the average level of inflation is approximately 4.93% on an annual basis with an estimated standard deviation of roughly 4.67%. In the first sub-sample the estimated mean of inflation was approximately 7.26% on an annual basis with a standard deviation of 2.39%. In the second sub-sample the average rate of deflation was -2.67% on an annual basis with an estimated standard deviation of 1.44%. Clearly using the full sample underestimates the level of inflation for the first period and drastically overestimates the standard deviation of inflation. The full sample estimate misses the deflation in the second sub-sample and again overestimates the standard deviation of inflation relative to the sub-samples.

-Table 2 Here-

Table 2 reports ADF, DFGLS, Ng-Perron and KPSS tests for the inflation data. The results suggest that the inflation data are $I(1)$ series and thus shocks to inflation are infinitely persistent. Clearly some caution needs to be exercised in interpreting these results given the potential for these tests to be biased towards the null of non-stationarity in the face of a structural break. However, table 2 also reports the $IO(1)$, $IO(2)$ and AO tests of Perron (1997) which allow for a one-off break under both the null and alternative hypotheses. Again we are unable to reject the null of non-stationarity. There is however conflict in the estimated break dates from the three tests. The $IO(1)$ model predicts that a gradual change in the intercept took place after May 1998. This contrasts with the prediction of the $IO(2)$ model that a gradual change in the intercept and slope occurred after July 1990. The AO model dates a rapid break in slope to February 1993. It is not easy to resolve this conflicting evidence. However the possibility arises that there are multiple breaks in the series which could lead to multiple dates if the breaks were of differing types. Alternatively it may be the case that one or more of the models is incorrectly specifying the break, leading to unreliable inference. Similarly, it may be that the linear DGP underlying the $IO(1)$, $IO(2)$ and AO models is inappropriate and that a non-linear functional form should be considered¹. We investigate this third possibility in the next section.

4. The Markov Switching Model of Segmented Trends

The model is motivated by the apparent change in the underlying data described in Table 1. The long swing model is a special case of the Hamilton (1989) approach and was first used by Engel and Hamilton (1991) to describe long swings in nominal exchange rates². Let S_t be a variable taking the value $S_t=1$ when the inflation rate is drawn from an $N(\mathbf{m}_1, \mathbf{s}_1^2)$ distribution.

On the other hand $S_t=2$ when the data are drawn from an $N(\mathbf{m}_2, \mathbf{s}_2^2)$ distribution. When $S_t=1$

¹ Pippenger and Goering (1993), Caner and Hansen (2001), and Henry and Shields (2002) *inter alia* document the poor performance of commonly used unit root tests in the face of a neglected threshold. Similarly, Nelson, Zivot and Piger (2001) find that Markov switching can affect unit root tests and also break dating. A Monte-Carlo study of this point is beyond the scope of the current paper.

² Henry and Shields (2002) argue that UK and Japanese inflation are threshold non-linear processes. The Caner-Hansen (2001) Wald test for a threshold was not significant for the Hong Kong data. Tests for a threshold unit root could not reject the null of a unit root in the data. Further details are available from the author upon request.

the trend in prices is \mathbf{m}_1 , while when $S_t=2$ the trend is \mathbf{m}_2 . In practice S_t , the variable that identifies the regime is unobserved. Engel and Hamilton (1991) use a Markov rule to describe S_t :

$$\begin{aligned}
 p(S_t = 1 | S_{t-1} = 1) &= p_{11} \\
 p(S_t = 2 | S_{t-1} = 1) &= 1 - p_{11} \\
 p(S_t = 1 | S_{t-1} = 2) &= 1 - p_{22} \\
 p(S_t = 2 | S_{t-1} = 2) &= p_{22}
 \end{aligned}
 \tag{18}$$

Implicitly S_t depends on past realisations of x and S only through S_{t-1} .

The long swing hypothesis is not imposed on the data but rather occurs when \mathbf{m}_1 and \mathbf{m}_2 are opposite in sign and when the Markov probabilities p_{11} and p_{22} are large. Other alternatives are possible. For example, consider when \mathbf{m}_1 is large and positive and p_{11} is small while \mathbf{m}_2 is negative and small in magnitude and p_{22} is large. Here the model captures short sharp bursts of inflation but deflationary episodes are slow and persistent. Thus the model is capable of capturing asymmetry in persistence of the two regimes. In the case of the random walk the inflation rate this period is completely independent of the inflation rate in the last period. This case occurs if $p_{11} = 1 - p_{22}$.

The model represents a mixture of distributions. As a further motivation for the Markov switching model as a potential conditional characterisation of Hong Kong inflation we note the comment of Engel and Hamilton (1991, p692) that a ‘‘histogram of data drawn from such a distribution would represent the sum of two overlapping bell-shaped curves’’. Figure 3 presents a kernel based estimate of the unconditional distribution of the data, which appears remarkably similar to ‘‘a superposition of two [...] simple normal distributions’’, Engel and Hamilton (1991, p692). The central difference between the Markov model and the mixture of normals is that the draws of x_t are not independent in the Markov model.

5. Estimation and Inference

Table 3 presents maximum likelihood estimates of the Markov switching model obtained using Hamilton's (1989) approach. Clearly $\hat{\mathbf{m}}_1 > 0$ and $\hat{\mathbf{m}}_2 < 0$, while \hat{p}_{11} and \hat{p}_{22} are large. This is consistent with a long swing in Hong Kong inflation. The estimates associate state 1 with a 7.26% rate of increase in prices on an annual basis. On the other hand, in state 2 the annual rate of inflation is -2.66% , that is state 2 is a low mean or deflationary environment. These estimates are almost identical to the sub-sample estimates of \mathbf{m} reported in Table 1. The estimates of \acute{o}_1 and \acute{o}_2 do not differ from the sub-sample estimates of \acute{o} .

-Table 3 Here-

Does the Markov switching model provide a superior conditional characterisation of the data to the random walk model of inflation implied by the results in Table 2? Testing the null of no switching in the data is a non-trivial task. Under such a null hypothesis $\mathbf{m}_1 = \mathbf{m}_2$ and $\mathbf{s}_1 = \mathbf{s}_2$. However any test will have a non-standard distribution since the transition probabilities p_{11} and p_{22} are unidentified under the null, a feature known as the Davies Problem (see Davies 1987, Hansen 1993, Andrews and Ploberger 1994 *inter alia*). Furthermore there are problems associated with the Maximum Likelihood Estimator under the restrictions since the derivative of the likelihood function with respect to \mathbf{m}_1 and \mathbf{s}_1 is identically equal to zero. Garcia and Perron (1996) present a Likelihood Ratio test adopting the Davies (1987) upper bound test. Defining L_0 as the value of the log-likelihood under the null and L_1 as the same measure under the alternative we obtain $LR = 2(L_1 - L_0)$. On the assumption that the likelihood ratio has a single peak, an upper bound for the significance of LR is given as $\Pr\left[\mathbf{c}_D^2 > LR\right] + 2(LR/2)^{D/2} \exp(-LR/2) / \tilde{\Gamma}(D/2)$. Note that there are $D=2$ parameters appearing only under the alternative and that $\Gamma(\cdot)$ represents the gamma function. Obviously the upper bound is greater than $\Pr\left[\mathbf{c}_D^2 > LR\right]$, the usual marginal significance level associated with the LR test. The 5% upper bound requires a value of LR of 10.95, rather than the usual \mathbf{c}_2^2 value of 5.99.

Table 4 presents maximum likelihood estimates of the mixture of normals model. The parameter estimates are consistent with the results in Tables 1 and 3. The log-likelihood value for the long-swings model was -257.8467 . On the other hand the estimation of the mixture of distributions model implied by the restriction $p_{22} = (1 - p_{11})$ yielded a log likelihood of -472.9275 . The resulting LR statistic is 430.1616, which clearly implies rejection of the null of no switching at the 5% level.

As an alternative response to the Davies problem, Engel and Nelson (1991) test the more general hypothesis

$$H_0 : p_{11} = 1 - p_{22}; \mathbf{m}_1 \neq \mathbf{m}_2; \mathbf{s}_1 \neq \mathbf{s}_2 \quad (19)$$

Under H_0 the changes in the price level correspond to an *i.i.d.* sequence with individual densities given by a mixture of two normals. Standard distribution theory can be used to test $H_0 : p_{11} = 1 - p_{22}$ because under the null the remaining parameters are identified.

Obtaining estimates of the asymptotic variance of \hat{p}_{jj} , $\text{var}(\hat{p}_{jj})$ and $\text{cov}(\hat{p}_{11}, \hat{p}_{22})$ from the inverse of the matrix of second derivatives of the likelihood function, we may construct a Wald test of $H_0 : p_{11} = 1 - p_{22}$, distributed as χ_1^2 from

$$\frac{[\hat{p}_{11} - (1 - \hat{p}_{22})]^2}{[\text{var}(\hat{p}_{11}) + \text{var}(\hat{p}_{22}) + 2\text{cov}(\hat{p}_{11}, \hat{p}_{22})]} \quad (20)$$

The Wald statistic is highly significant at any level of confidence (Wald = 244.0854, marginal significance level = 0.0000). We note that this test statistic also exceeds the 5% Davies upper bound implying that the more restrictive version of (19) namely $H_0 : p_{11} = 1 - p_{22}$ is also rejected.

Using a similar approach we test whether the inflation rates are significantly different across regimes. If $\mathbf{m}_1 = \mathbf{m}_2$ and $\mathbf{s}_1 \neq \mathbf{s}_2$ then the states have the same rates of inflation, but differing variances. We designate this null $H_0^1 : \mathbf{m}_1 = \mathbf{m}_2$, the Wald statistic corresponding to H_0^1 , distributed as χ_1^2 is

$$\frac{(\hat{\mathbf{m}}_1 - \hat{\mathbf{m}}_2)^2}{[\hat{\text{var}}(\hat{\mathbf{m}}_1) + \hat{\text{var}}(\hat{\mathbf{m}}_2) - 2 \hat{\text{cov}}(\hat{\mathbf{m}}_1, \hat{\mathbf{m}}_2)]} \quad (21)$$

Here the Wald statistic is again highly significant at any level of confidence. (Wald = 247.0854, marginal significance level = 0.0000). Clearly the mean inflation rate differs across the regimes in a statistically significant fashion.

Finally given the significant difference in mean inflation rates across the regimes, we test whether the apparent differences in variance are significant. Again, this is a Wald type test constructed as in (21) and distributed as χ_1^2 with the null hypothesis $H_0^2 : \mathbf{s}_1^2 = \mathbf{s}_2^2$. Under H_0^2 the expected rate of inflation differs across regimes, but uncertainty about inflation is constant. The null hypothesis is clearly not satisfied for the data (Wald = 14.9133, marginal significance level = 0.0001).

Given the evidence from the LR test and Wald tests (19) – (21) it appears that movements in Hong Kong inflation are well described by a long swing. Inflation enters into regimes where it increases or decreases and it remains in these regimes for substantial periods of time. These differences in means and variances across regimes, coupled with the rejection of $H_0 : p_{11} = 1 - p_{22}$ imply that the long swing model provides a superior conditional data characterisation to the more typically used random walk model.

Figure 4 plots the smoothed regime probability for regime 2. Clearly regime 2 captures the low mean inflation regime which coincides with the deflation for Hong Kong. Taking $p_{22} > 0.5$ as evidence for the second regime our estimates imply that inflation moved into the deflationary regime in September 1998. The *IO(1)* model dates a break to May 1998, which is not inconsistent with the prediction of the long swing model. We leave the investigation of the effect of neglected non-linearity on break dating as a matter for further research.

The expected length of state i is $1/(1-p_{ii})$. This suggests that the expected length of the inflationary regime is 238 months, while the deflationary regime would on average be expected to last 130 months. At the time of writing, the deflation is in its fourth year and has not ended. However it is unlikely that the current regime will last another six years. This of course highlights the difficulties associated with measuring the duration of the inflation cycle with less than one complete cycle.

What are the advantages of the long swing model over the sub-sample approach followed in Table 1? First, the long swing approach does not impose the break date, transitions across regimes are estimated rather than imposed. Secondly the long swing model admits multiple changes of regime, although only a single transition is detected. Third, the long swing model uses the entire sample and is likely to provide more efficient estimates of the parameters of the model. Fourth, one may obtain an estimate of the expected duration of each regime from the long swing model.

6. Reconciling the results

Nelson, Piger and Zivot (2001) study the properties of unit root tests in the presence of Markov switching. The Markov switching model can be summarized as

$$\begin{aligned}
\mathbf{p}_t &= \mathbf{m}_t + v_t \\
\mathbf{m}_t &= \mathbf{m}_1 \times S_t + \mathbf{m}_2 (1 - S_t) \\
v_t &\sim N(0, \mathbf{s}_t^2) \\
\mathbf{s}_t^2 &= \mathbf{s}_1^2 \times S_t + \mathbf{s}_2^2 (1 - S_t)
\end{aligned} \tag{22}$$

This can easily be written as a model with a constant growth rate of prices, and a serially correlated heteroscedastic error term

$$\begin{aligned}
\mathbf{p}_t &= \mathbf{m} + e_t \\
e_t &= (\mathbf{m}_1 - \mathbf{m}) S_t + (\mathbf{m}_2 - \mathbf{m}) (1 - S_t) + v_t
\end{aligned} \tag{23}$$

The effect (if any) of the switching drift and variance on the performance of commonly used unit root and stationarity tests must be due to e_t . Nelson, Piger and Zivot (2001) show that the autocovariance function of e_t is

$$Cov(e_t, e_{t-k}) = (\mathbf{m}_1 - \mathbf{m}_2)^2 (E(S_t S_{t-k}) - p^2) \tag{24}$$

In the limit, under the assumption of a first order Markov process underlying, S_t this autocovariance goes to zero. However, e_t is heteroscedastic due to the switching variance in (22). Consider the lag k autocorrelation for e_t

$$\frac{(\mathbf{m}_1 - \mathbf{m}_2)^2 (p \times P(S_t = 1 | S_{t-k} = 1) - p^2)}{(\mathbf{m}_1 - \mathbf{m}_2)^2 (p - p^2) + \mathbf{s}_t^2} \tag{25}$$

If all terms that depend on the transition probabilities are ignored in (25) the ratio $(\mathbf{m}_1 - \mathbf{m}_2)^2 / \mathbf{s}_{it}^2$ determines the size of the autocorrelations. This may be interpreted as a signal to noise ratio. The larger the variance of the error term, the larger the noise masking the serial correlation induced by the drift terms. The larger (smaller) the ratio, the larger (smaller) the autocorrelations. The average value of this signal-to-noise ratio is 4.785 in our data. Evaluating the heteroscedasticity for the cases where $S_i = 1$ and $S_i = 2$ yields the ratio

$$\frac{(\mathbf{m}_1 - \mathbf{m}_2)^2 (p - p^2) + \mathbf{s}_1^2}{(\mathbf{m}_1 - \mathbf{m}_2)^2 (p - p^2) + \mathbf{s}_2^2} \quad (26)$$

The value of (26) for our data was 1.1373.

The values of (25) and (26) for our data correspond almost exactly to the high correlation, low heteroscedasticity case studied by Nelson, Piger and Zivot (2001), who demonstrate that the ADF test is subject to size distortions depending on how the lag truncation is chosen. In the high correlation, low heteroscedasticity case where $\hat{p}_{11} = \hat{p}_{22} \cong 0.99$ the Dickey Fuller test is severely undersized, rejecting less than 1% of the time, while the ADF test also suffers.

Using the parameters and transition probabilities from the estimated long-swing model we generated 20000 samples, each containing 200 observations. The lag truncation parameter was chosen using the Schwarz criterion. In only 24.85% of cases could the ADF test reject the null hypothesis of a unit root. More interestingly, the KPSS test is unable to distinguish the Markov Switching drift. The rejection rate of the null hypothesis of the KPSS test was 100%. Nelson, Piger and Zivot (2001) conclude that the failure of the ADF and DF tests in the presence of Markov switching drift is a function of the severity of the serial correlation caused by the probabilistic trend breaks. This is very likely to be the source of the apparent unit root in inflation in our Data. In the case of the KPSS test there is little distinction between the null and alternative hypotheses as $x_t \sim I(0)$ in both cases. The uniform rejection is like to be as a result of very low ‘‘power’’ in the sense that the alternative hypothesis is consistent with the stationary null hypothesis and does not really constitute an alternative after all.

7. Summary and Conclusion

Using annual rates of inflation calculated from monthly CPI data we are unable to reject the null hypothesis of a unit root in the inflation rate. However this result must be regarded as tenuous as there is strong evidence of a break in the unconditional distribution of inflation. It is well known that unit root tests are biased towards the null hypothesis of non-stationarity in the face of a structural break. However, using the Perron (1997) approach, which is robust to a one-off break under the null and alternative hypothesis, we are again unable to reject the null of a unit root. Interestingly the Perron (1997) approach presents conflicting evidence as to the timing of any such break.

We employ an alternative model for inflation due to Engel and Hamilton (1990) that characterises movements in prices as following long swings. Inflation appears to display two regimes. One regime features increasing prices and relatively high inflation uncertainty. In the other regime Hong Kong prices are falling, although the uncertainty about deflation is relatively low. The model dates a single transition across regimes that occurred in September 1998. The data display asymmetric persistence; the estimated duration of an inflation is approximately twice that of a deflation, although some caution must be associated with this conclusion. However, it is clear that deflationary periods can be highly persistent. The results are consistent with a segmented stochastic trend underlying prices. In such a scenario inflation, as the change in prices, is stationary. Using a battery of tests that allow for unidentified parameters under the null hypothesis, the data strongly reject the null of no switching. A consequence of the long swing is that unit root stationarity tests are likely to be unreliable. A direct implication of this is the tendency of linear models to dramatically over-estimate the persistence of an inflation shock.

Given the current deflation in Hong Kong the results have important implications for the construction of economic models and also for the implementation of policy. One implication of these results is that inference based on linear models of inflation is likely to be highly misleading as such models are inherently misspecified. A similar criticism applies to VAR based simulation experiments designed to investigate the effects of policy initiatives to combat the deflation. Given the pernicious effects of deflation and stagnation, this is clearly an area that merits further research.

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Tables and Figures

Table 1: Maximum Likelihood Estimates of the Parameters of the Empirical Density of x_t

$$g(x_t | \mathbf{ms}) = \frac{1}{\sqrt{2\pi}\mathbf{s}} \exp\left(-\frac{1}{2\mathbf{s}^2}(x_t - \mathbf{m})^2\right)$$

Parameter	Full sample 1985:1 – 2002:8	First sub-sample 1985:1 – 1999:9	Second sub-sample 1999:10 – 2002:8
\mathbf{m}	4.9321 (0.3369)	7.2678 (0.1934)	-2.6716 (0.2111)
\mathbf{s}	4.7649 (0.2388)	2.3920 (0.1372)	1.4471 (0.1509)

Table 2: Unit root and stationarity tests

ADF(\mathbf{t})	ADF(\mathbf{m})	ADF	KPSS(\mathbf{t})	KPSS(\mathbf{m})
-2.2384 <-3.4340>	-0.3831 <-2.8768>	-0.8489 <-1.9425>	0.3724 <0.1460>	1.0417 <0.4630>
Ng-Perron	MZ \mathbf{a}	MZ T	MSB	MP T
$\tilde{x}_t = \{1, t\}$	-2.2715 <-17.3000>	-0.9456 <-2.9100>	0.4163 <0.1680>	34.6558 <5.4800>
Ng-Perron	MZ \mathbf{a}	MZ T	MSB	MP T
$\tilde{x}_t = \{1\}$	-1.6296 <-8.1000>	-0.7411 <-1.9800>	0.4548 <0.2330>	12.0404 <3.1700>
DFGLS(\mathbf{t})	DFGLS(\mathbf{m})	IO(1)	IO(2)	AO
0.8791 <-2.9430>	-0.7339 <-1.9425>	-3.0756 <-5.1000>	-3.2435 <-5.5500>	-3.2543 <-4.6500>
		$T_{\mathbf{y}} : 1998:5$	$T_{\mathbf{y}} : 1990:7$	$T_{\mathbf{y}} : 1993:2$

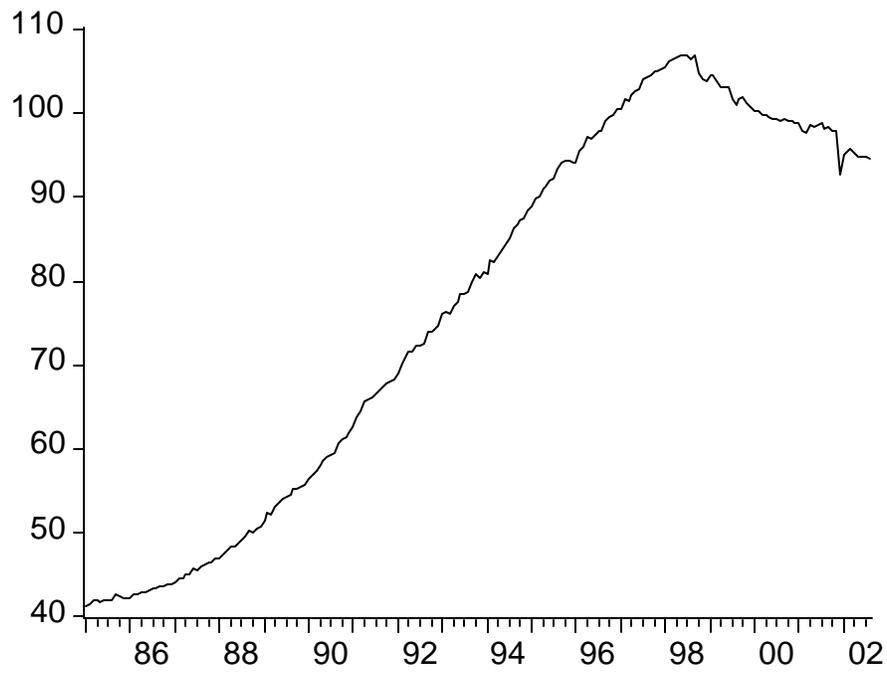


Figure1: The CPI Data

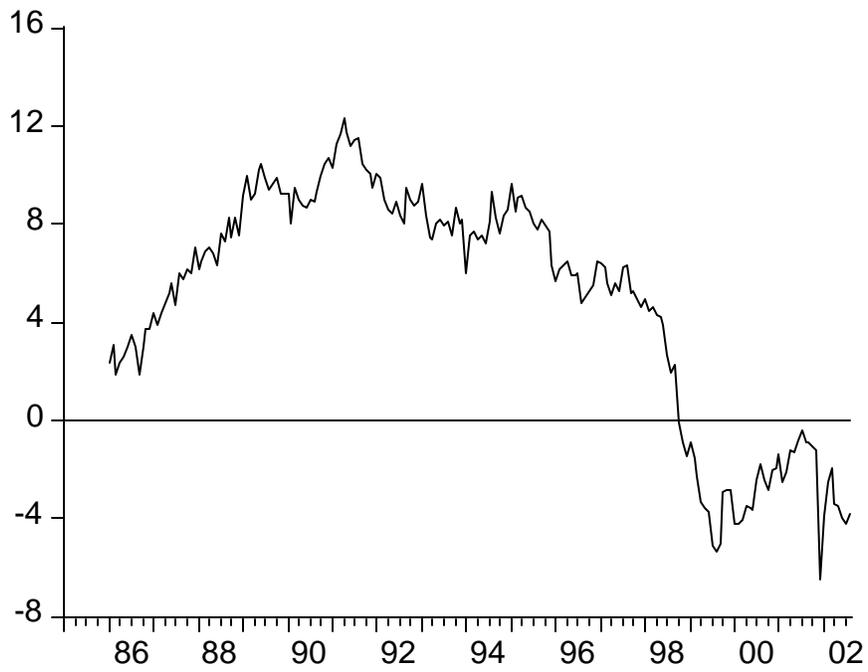


Figure 2: The Inflation Data

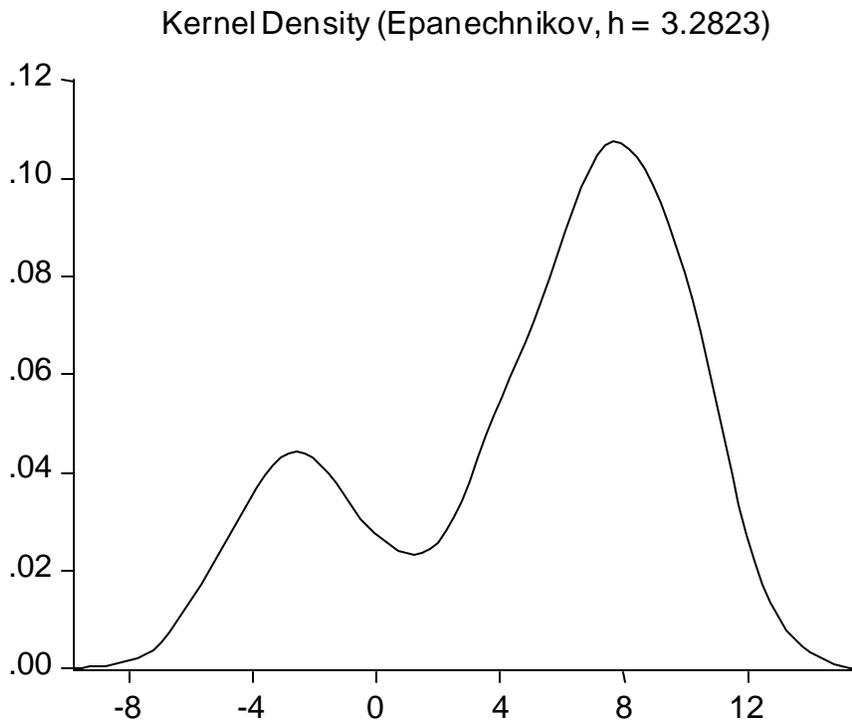


Figure 3: The estimated unconditional density of Hong Kong inflation

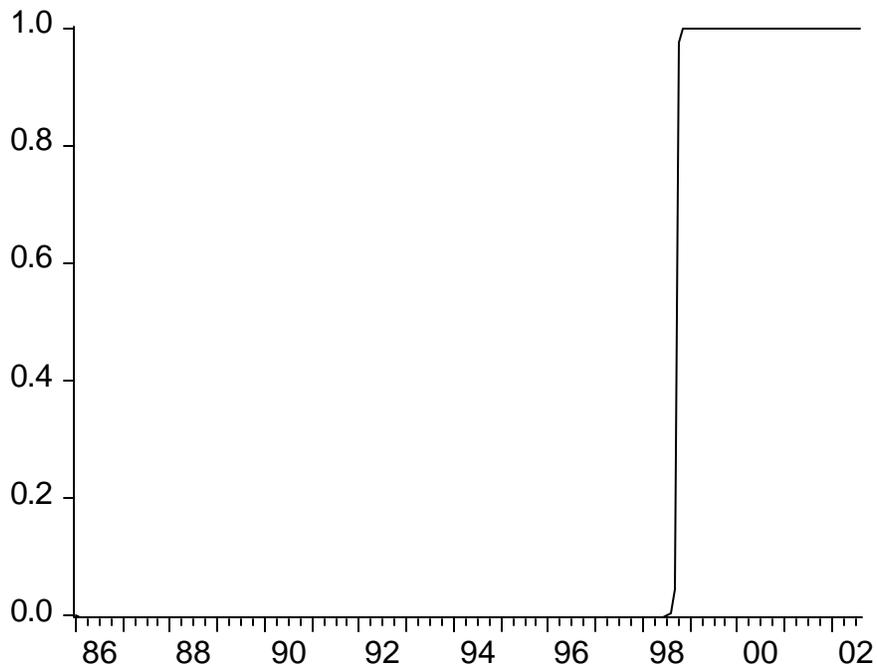


Figure 4: The smoothed regime probability for the deflationary regime $P[S_t = 2]$.