Endogenous Exchange Rate Pass-through

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Abstract

This paper develops a model of endogenous exchange rate pass-through an open economy, where both pass-through and the exchange rate are simultaneously determined, and interact with one another. Pass-through is endogenous because firms have the choice of which currency in which they set their export prices. There is a unique equilibrium rate of pass-through under the condition that exchange rate volatility rises as the degree of pass-through falls. We show that the relationship between exchange rate volatility and economic structure may be substantially affected by the presence of endogenous pass-through. Our key results show that pass-through is related to the *relative* stability of monetary policy. Countries with relatively low volatility of money growth will have relatively low rates of exchange rate pass-through, while countries with relatively high volatility of money growth will have relatively high pass-through rates.

Introduction

A large body of empirical evidence has found that pass-through of exchange rate changes to import prices is less than complete.[1] However, the degree of pass-through is not uniform across countries or industries.[2] Exchange rate pass-through matters for many questions; e.g. the predicted volatility of the real exchange rate, the international transmission of macroeconomic shocks, and the welfare benefits of international policy coordination[3]. It is therefore important to understand the underlying determinants of pass-through. While there is a large literature that has examined long-run pass-through – the optimal pricing choice of firms when markets are segmented and competition is imperfect – considerably less study has been undertaken of pass-through in the short run when there may be some nominal price stickiness.

In this paper, we analyze the determinants of an exporting firm's choice of currency in which to pre-set prices. Since in an environment of nominal price stickiness, the degree of exchange rate pass-through is determined by the choice of currency in which to pre-set prices, our paper therefore develops a model of endogenous exchange rate pass-through. Moreover, we do this in a model where the exchange rate is endogenously determined. We show that there is a two-way interaction between exchange rate pass-through and exchange rate volatility. The volatility of the exchange rate determines the price-setting choices of a firm, and therefore, given the choices of all

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[1] A short list of citations includes Knetter (1989, 1993), Feenstra (1989), Feenstra, Gagnon, and Knetter (1996), Goldberg and Knetter (1997, 1998), Goldberg (1995), and Goldberg and Verboven (2001) for studies of pass-through to import prices. Engel (1993), Engel and Rogers (1996) and Parsley and Wei (2001), among many others, have studies pass-through to consumer prices.

[2] This point was emphasized in the survey of Goldberg and Knetter (1997).

[3] See for example, Betts and Devereux (1996, 2000), Devereux and Engel (2000), Tille (2000), and Lane (2001).

firms, the degree of aggregate exchange rate pass-through. But the degree of exchange rate pass-through itself determines the volatility of the exchange rate.

The starting point of our analysis is the assumption that prices are sticky in the short run. There is a long tradition of nominal price stickiness in models of macroeconomics. But in an open economy, the question of price stickiness is more problematic. Clearly, the exchange rate is not sticky. As a result, when a good is traded between countries with flexible exchange rates, the currency in which the price of the good is fixed becomes an important factor in determining the effect of exchange rate changes. If prices are sticky in the currency of the exporter (we denote this as PCP, or `producer currency pricing'), then pass-through from exchange rate changes to final consumers will be complete, and imported goods will display considerable price flexibility. On the other hand, if goods prices are fixed in consumer's currency (LCP, or `local currency pricing'), there is no pass-through at all, and imported goods prices are unaffected by exchange rate changes.

We therefore examine the choice that firms make over their currency of price setting. When a firm sells abroad, would it prefer to follow PCP or LCP? This question has been addressed before, but mostly in partial equilibrium settings, which take as exogenous key variables that are influenced by the price-setting configuration itself. For instance, in general equilibrium, the behavior of exchange rates, labor costs, and demand may themselves depend on how prices are set.

Our analysis proceeds in three stages. In the first stage, we examine the choice of currency of price setting for a firm that has local market power in a stochastic environment, taking as given the distribution of exchanges rates, market demand, and prices of other firms. We establish a very simple rule for the choice of price-setting currency. If a firm is choosing its prices optimally, then, up to a second order approximation, its decision depends only on the variance of the (log) exchange rate and the covariance of the exchange rate with marginal costs. The higher is the variance of the exchange rate, the more incentive the firm has to set prices in its own currency. The higher is the covariance of the exchange rate and marginal costs, the more the firm would wish to set its price in foreign currency. A remarkable aspect of the result is that if the firm is setting its price optimally, then the currency of pricing decision is independent of the variance of market demand and the prices of all other firms.

We go on to place the firm in a two-country intertemporal general equilibrium environment in which both the exchange rate and marginal costs are determined by stochastic money shocks in each country. Each country has a continuum of firms that export goods to the other country. The degree of aggregate exchange rate pass-through is determined by the measure of firms that choose to follow PCP, basing on their decision on the behavior of exchange rates and marginal costs.

The key property is that while firms' decisions with respect to currency of pricing depend on the distribution of exchange rates and marginal costs, these distributions themselves depend on the degree of aggregate exchange rate pass-through in each country, which itself depends on the pricing decision made by firms. Thus, there is a two way inter-relationship between exchange rate volatility and exchange rate pass-through.

Is there a unique equilibrium degree of exchange rate pass-through? If pass-through depends on exchange rate volatility, and exchange rate volatility depends on pass-through, there arises the possibility of multiple equilibria[4]. Roughly speaking, the condition for a unique equilibrium is that exchange rate volatility is higher in an economy where exchange rate pass-through is lower. On the other hand, if declining pass-through is associated with a decline in exchange rate volatility, then multiple equilibria may exist. But in our model, this is not likely to occur.

The overall degree of exchange rate pass-through depends on various structural features of the economy. Pass-through is higher the more stable are marginal costs in each country, and the higher is the elasticity of substitution between domestic and foreign goods. But when the volatility of money shocks is the same in the two countries, pass-through does *not* depend on the volatility of money.

In an environment of endogenous exchange rate pass-through, conventional results on the determinants of exchange rate volatility must be applied with caution. For instance, in many recent papers, it is shown that a high volatility of the real exchange rate may be determined by a combination of a low consumption elasticity of demand for money and a low degree of exchange rate pass-through[5]. But our results indicate that this combination is unlikely to happen. Precisely because exchange rate variance is high with a low elasticity of consumption demand, firms will tend to follow PCP, and the degree of exchange rate pass-through will be high.

In the third stage of our analysis, we examine the relationship between monetary policy and pass-through. Our key results relate to the impact of differential monetary shocks on the degree of exchange pass-through. When countries have differences in the

[4] This was pointed out by Devereux and Engel (2001). A slightly different perspective on multiple equilibria in the decision over invoicing currency is presented by Bacchetta and Van Wincoop (2001). We discuss Bachetta and Van Wincoop (2001) more fully below.
[5] See for instance, Chari, Kehoe and McGrattan (2000), Devereux (1997), and Kollman (2002).

volatility of money growth, our model predicts that exporting firms in *both* countries will tend to pre-set their prices in the country that has the more stable money growth. This leads to an important link between monetary policy and price stability. A country that follows a successful credible policy of inflation targeting, reducing the mean and variance of its money growth, will experience a price-stability `bonus'. This is because foreign exporters will begin more and more to fix their prices in that country's currency, thereby reducing the impact of exchange rate changes on the country's CPI. But the flip side of this is that the foreign country experiences a price-stability `penalty', since exporters in the inflation targeting country will also begin to pre-set their prices in domestic currency. Thus, there is a `beggar-thy-neighbor' aspect to inflation targeting policies in an environment of endogenous pass-through.

This paper is part of a wider literature on sticky price open economy macroeconomic models[6]. More particularly, a number of other papers have looked at the determination of the degree of exchange rate pass-through in general equilibrium models with endogenous exchange rates. Devereux and Engel (2001) and Storgaard (2001) present a very similar analysis of the decision with respect to PCP versus LCP, in separate works that have been combined to form the present paper. Bacchetta and Van Wincoop (2000b) present numerical results on equilibrium pass-through in a static environment. They find a positive connection between risk-aversion and local currency pricing. In some cases they find that there are no pure strategy equilibria for firms pricing decisions, a theme we take up below. Bacchetta and Van Wincoop (2001) focus on the choice of invoicing currency (or currency of price setting) in a static general

[6] See Obstfeld and Rogoff (1995, 1998, 2000), Lane (2001), Bacchetta and Van Wincoop (2000a), Devereux and Engel (2001), and many others.

equilibrium framework, providing analytical results. Their partial equilibrium results take on much of the flavor of theoretical conclusions of Feenstra, Gagnon, and Knetter's (1996) – that pass-through is greater when exporting firms have a high degree of market power. They emphasize the possibility of multiple equilibria that arise because of strategic complementarities between the price setting decisions of firms. They also explore the role of multiple countries, and the impact of a monetary union on the equilibrium invoicing currency in international trade. In their paper, multiple equilibria arise due to diminishing returns to scale in a manner that is absent in our work. But they do not focus on the two-way interaction between exchange rate pass-through and exchange rate volatility, nor do they examine the implication of differences in monetary policies across countries.

The paper is organized as follows. The next section sets out the problem of a single firm in a stochastic environment, and establishes a simple rule for the determination of the currency of pricing. Section 2 sets out the general equilibrium model. Section 3 combines section 1 and section 2 to determine the degree of exchange rate pass-through. Section 4 explores the implications of differences in the variance of money growth among countries.

Section 1. The Decision of a Firm in a Stochastic Environment

Take a firm *i* in the home country selling a differentiated good to a foreign market. Assume that the firm faces the CES demand curve

$$Y(P(i)) = \left(\frac{P(i)}{P}\right)^{-\lambda} \left(\frac{P}{P^*}\right)^{-\theta} Y^*, \quad \lambda > 1. \tag{1.1}$$

$P(i)$ is the price the foreign consumer pays for good *i*. P is the price index for all home goods purchased by the foreign consumer, and P^* is the foreign country consumer price index. Without loss of generality let $P(i), P,$ and P^* be denominated in foreign-currency. Y^* is a demand shift variable. λ is the price elasticity of demand facing the domestic firm i. This must exceed unity if the second order condition for profit maximization is to be satisfied. θ is the foreign price elasticity of demand for domestic goods. We assume that firm *i* is a small enough supplier that it ignores the impact of its pricing decision on the price index of home goods in the foreign market.

Equation (1.1) imposes a particular functional form on the firm's demand schedule. This is done so as to be consistent with the general equilibrium model developed later on. But we make no specific assumptions about the distribution of P, P^*, and Y^*. We allow these variables to be stochastic, and have an arbitrary cross-correlation structure, as well as being possibility correlated with the exchange rate.

Assume that the firm has a constant returns to scale production function, and faces the (possibly stochastic) marginal cost W. We also assume that the firm evaluates profits using the (stochastic) discount factor d. In the general equilibrium model, we determine the exact form of this discount factor.

PCP versus LCP

The firm has to decide whether to set its price in domestic or foreign currency. Whatever currency it chooses, it must set the price before the state of the world is known. If the firm bears no additional cost when setting prices in foreign currency, it will simply set prices in the currency which gives the highest discounted expected profits.

If firm *i* sets its price in its own currency, (PCP), then expected profit is

$$E\Pi^{PCP} = E\left[d(P^{PCP}(i) - W)\left(\frac{P^{PCP}(i)}{SP}\right)^{-\lambda}\left(\frac{P}{P^*}\right)^{-\theta} Y^*\right], \qquad (1.2)$$

where S is the exchange rate (domestic-currency price of foreign currency).

If the firm sets its price in the foreign currency (LCP), then expected profit is

$$E\Pi^{LCP} = E\left[d(SP^{LCP}(i) - W)\left(\frac{P^{LCP}(i)}{P}\right)^{-\lambda}\left(\frac{P}{P^*}\right)^{-\theta} Y^*\right]. \qquad (1.3)$$

The profit-maximizing price for the firm, under PCP and LCP, respectively, may easily be derived as follows:

$$P^{PCP}(i) = \frac{\lambda}{\lambda - 1}\frac{E(WS^{\lambda}Z)}{E(S^{\lambda}Z)}, \qquad P^{LCP}(i) = \frac{\lambda}{\lambda - 1}\frac{E(WZ)}{E(SZ)},$$

where $Z = dP^{\lambda-\theta}P^{*\theta}Y^*$.

Now, using these solutions, we can derive the expressions for expected profits, conditional on prices being set optimally:

$$E\Pi^{PCP} = \tilde{\lambda}E(S^{\lambda}Z)^{\lambda}E(S^{\lambda}ZW)^{1-\lambda} \qquad (1.4)$$

$$E\Pi^{LCP} = \tilde{\lambda}E(SZ)^{\lambda}E(ZW)^{1-\lambda} \qquad (1.5)$$

where $\tilde{\lambda} = \frac{1}{\lambda - 1}\left(\frac{\lambda}{\lambda - 1}\right)^{-\lambda}$.

From expressions (1.4) and (1.5), we may establish the following proposition:

Proposition 1

The firm sets its price for the foreign market in home (foreign) currency if

$$\left[\frac{\mathrm{var}(s)}{2} - \mathrm{cov}(w, s)\right] > 0,\ (< 0),$$

where $s = \ln(S)$, and $w = \ln(W)$. Proof: see appendix.

This condition says that (log) exchange rate variance leads the firm to set its price in terms of home currency. But a positive covariance between (the log of) the exchange rate and (the log of) marginal costs leads the firm to set its price in foreign currency. In net, independently of what other home and foreign firms do, the firm will choose to set its price in terms of the home currency when the variance of the exchange rate exceeds two times the covariance of the exchange rate and marginal costs.

To explain this condition, take expressions (1.2) and (1.3) again. In any given state of the world, under either pricing policy, profits are increasing in the exchange rate. Under PCP, a rise in the exchange rate will increase demand for the firm's good, holding other firm's prices constant in foreign currency. Under LCP, a rise in the exchange rate will increase the home currency value of sales. But note that under PCP, the profit function in any state of the world is strictly convex in the exchange rate, for $\lambda > 1$. But under LCP the profit function is linear in the exchange rate. This means that, holding other variables constant, an increase in exchange rate variance increases profits under PCP pricing relative to LCP. If this were the only consideration, the firm would follow PCP pricing if there is any exchange rate uncertainty.

But there is a secondary channel, arising from the uncertainty of marginal costs. Holding expected marginal cost constant, then if the covariance between the exchange rate and marginal cost is positive, this tends to increase expected total costs under PCP, since the firms demand is higher precisely when the cost of production is higher. Under LCP however, demand is independent of the exchange rate (holding other variables constant), so that expected total costs do not depend on the covariance between the exchange rate and marginal cost. Therefore, a positive covariance between the exchange rate and marginal cost increases the incentive to follow LCP pricing.

When we add both of these channels together, we arrive at exactly the condition described in the proposition.

Note a striking feature of Proposition 1. The condition does not depend on the variance of Z (which itself depends on total demand, the prices of other home firms, the foreign CPI, and the stochastic discount factor), or the covariance of Z with S or W. It follows that Proposition 1 holds in any environment in which the firm's demand schedule can be described by (1.1). In particular, it will apply in the same form for the general equilibrium model that we construct below. Thus, given $\text{var}(s)$ and $\text{cov}(w,s)$, the firm's optimal currency of pricing is independent of the pricing policies of other firms, the assumptions about international financial markets, or the characteristics of any other macro variables in the domestic or foreign economies.

Why does the condition in proposition 1 not depend on the distribution of Z? The reason is that the covariance between Z and the exchange rate and marginal cost is already taken into account in the optimal pricing decision. This means that, up to a second order approximation, the impact of Z on profits is equalized across the two pricing schemes. To see this rewrite the profit expressions (1.4) and (1.5) in the following way:

$$E\Pi_i^{PCP} = \frac{1}{\lambda}\left[E(S^{\lambda}Z)\right]\left[P^{PCP}(i)\right]^{1-\lambda} \tag{1.6}$$

$$E\Pi_i^{LCP} = \frac{1}{\lambda}\left[E(ZS)\right]\left[P^{LCP}(i)\right]^{1-\lambda} \tag{1.7}$$

At first glance, it would seem that the covariance between the exchange rate and Z will affect a comparison of (1.6) and (1.7). Holding the firm's price constant, since $\lambda > 1$, positive covariance between Z and S would raise (1.6) relative to (1.7). But in fact, an increase in the covariance between Z and S will reduce the LCP price relative to the PCP price, because it raises expected marginal revenue under LCP (the exchange rate is high when foreign demand is high, increasing the expected value of the foreign currency earnings under LCP.) The endogenous reduction in $P^{LCP}(i)$ is such that, at the level of second order approximation, the increase in the covariance between the exchange rate and Z has no bearing for a comparison of profits between LCP and PCP.[7]

The situation of firm in a foreign exporting to the domestic market is entirely analogous, so long as demand can be described as in equation (1). Thus we may state:

Corollary to Proposition 1.

The foreign firm sets its price for the home market in foreign (home) currency if

$$\left[\frac{\text{var}(s)}{2} + \text{cov}(w^*, s)\right] > 0, \ (< 0).$$

[7] How does the condition of proposition 1 relate to the partial equilibrium models of Giovannini (1988) (see also Friberg (1998))? In Giovannini (1988), it is assumed that the exchange rate is the only source of uncertainty in the firms pricing problem. He then shows that if profits under PCP are concave (convex) in the exchange rate, then LCP (PCP) is preferred to PCP (LCP) by the firm. Profits are concave (convex) in the exchange rate if the market demand curve is concave (convex). In our analysis, holding marginal cost constant, profits must be *convex* in the exchange rate, because we use a CES demand system in which the demand schedule is convex by construction. Therefore, were the exchange rate the only source of uncertainty, all firms would wish to follow PCP (as we have shown). But our interest is in analyzing the two-way *interaction* between exchange rate pass-through *and* exchange rate determination. Since the exchange rate and marginal costs are both driven by the underlying aggregate shocks to the economy, we cannot assume that marginal costs are constant. Hence the condition underlying proposition 1.

Section 2. The General Equilibrium Model

We now move to a general equilibrium model, where the distribution of the exchange rate is endogenous. There are two countries, home and foreign, with consumers, firms and governments in each country. There are *n* households and firms in the home country, and *1-n* in the foreign country. Prices are chosen in advance, by monopolistically competitive firms. Wages are chosen by monopoly suppliers of labor, as in Obstfeld and Rogoff (2000). We also assume that a certain fraction of wages in each country must be chosen in advance. The structure of the model has been developed in other studies, so only a brief sketch of the main elements is given in the text. The full details of the model are given in the appendix.

Preferences and Market Structure

Each consumer *k* in the home country maximizes expected lifetime utility

$$U_t(k) = E_t\left(\sum_{s=t}^{\infty} \beta^{s-t} u_s(k)\right), \qquad 0 < \beta < 1$$

where $u_s(k) = \frac{1}{1-\rho} C_s^{1-\rho}(k) + \chi \ln\left(\frac{M_s(k)}{P_s}\right) - \frac{\eta}{1+\psi} L_s^{1+\psi}(k), \quad \rho > 0$.

C(k) is a consumption index, $M(k)/P$ are domestic real balances, and *L(k)* is the labor supply of the representative home agent.. Consumption is decomposed into the consumption of home and foreign sub-aggregates. Thus

$C_t = \left(n^{\frac{1}{\theta}} C_{ht}^{1-\frac{1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_{ft}^{1-\frac{1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$. In turn, home and foreign consumption is defined over the consumption of a continuum of goods so that $C_{ht} = \left(n^{-\frac{1}{\lambda}} \int_0^n C_{ht}(i)^{1-\frac{1}{\lambda}} di \right)^{\frac{1}{1-\frac{1}{\lambda}}}$

and $C_{ft} = \left((1-n)^{-\frac{1}{\lambda}} \int_n^1 C_{ft}(i)^{1-\frac{1}{\lambda}} di \right)^{\frac{1}{1-\frac{1}{\lambda}}}$.

The elasticity of substitution between goods produced within a country is λ (>1), and the elasticity of substitution between home goods and foreign goods is θ. The consumer price index may be written as $P_t = \left(nP_{hht}^{1-\theta} + (1-n)P_{fht}^{1-\theta} \right)^{\frac{1}{1-\theta}}$, where P_{ij} represents the price of country *i's* good for sale in country *j*. Prices set in foreign currency are denoted with an asterisk. Prices for each period are set before all information about the period is known. All goods sold by local firms are priced in local currency, but when exporting, firms have the option of setting prices either in their own currency (PCP), or in the currency of their customers (LCP). Let the fraction of home (foreign) firms that engage in LCP be z (z^*). For now we take these values as given.

Using this notation, the home country price index of foreign goods is

$$P_{fht} = \left[\frac{1}{1-n} \int_n^{n+(1-z^*)(1-n)} (S_t P_{fht}^*(i))^{1-\lambda} di + \frac{1}{1-n} \int_{n+(1-z^*)(1-n)}^1 P_{fht}(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}},$$

where S_t represents the exchange rate. This expression shows that (holding goods prices fixed) the degree of pass-through from exchange rate changes to home prices depends on the fraction of foreign firms who engage in LCP. As $z^* \to 1$, pass-through is zero.

We assume that international financial markets are imperfect. Consumers can trade internationally only in non-contingent nominal bonds. Thus, there is incomplete international risk sharing. Within the domestic economy however, we assume that there is full risk sharing. This eliminates the individual uncertainty in wage income, so that workers have equal consumption, whether or not they adjust wages ex-post.

Finally, firms produce using labor only, with constant returns to scale. Labor is differentiated, however. The production function for firm *i* in the home country is

$$y(i) = \left[\frac{1}{n} \int_0^n L(k)^{1-\frac{1}{\omega}} dk \right]^{\frac{1}{1-\frac{1}{\omega}}}.$$

Thus, the elasticity of substitution between types of labor is ω. Each worker then faces a specific labor demand curve with wage elasticity of demand ω.

Equilibrium Conditions

Table 1 outlines the main equations of the model. Table 1a describes the optimality conditions for the consumer and the firm. The consumer chooses a stock of domestic currency denominated bonds to maximize utility, given the nominal interest rate r_{t+1}.[8] Money demand depends positively on consumption and negatively on the nominal interest rate. Each consumer-worker sets the wage as a markup over the marginal rate of substitution between consumption and hours. A fraction *v* of the total *n* workers set wages ex-post, after the state of the world is realized, while the fraction 1-*v* set wages in advance. All prices are set in advance, as described in the previous section.   The nominal discount factor used by firms in their evaluation of expected profits is now defined as $d_{t-1} = \beta \frac{C_{t-1}^{\rho} P_{t-1}}{C_t^{\rho} P_t}$. That is, firms evaluate nominal profits using the same discount factor of the home consumer (home firms are owned by home consumers).

[8] We abstract from foreign bonds, since in our linear approximate solution, they are identical to domestic bonds.

The wage and price indices are described in Table 1b. The wage index faced by the firm will adjust to shocks only through changes in the wages set by the *v* flexible-wage workers. The consumer price index will depend on the exchange rate to the extent that some foreign firms follow PCP pricing.

Table 1c describes the market clearing relationships. Employment of fixed wage and flexible-wage workers will in general differ (although the income effects of this are diversified away). The home country current account (per capita) is equal to total income per capita less consumption. All home consumers receive the same income, where income comes from sales to domestic consumers, foreign consumers, through both PCP and LCP firms[9].

Table 1(a) Optimal conditions for consumer and firm			
Euler equation	$\frac{C_t^{-\rho}}{P_t} = \beta(1+r_{t+1})E_t\frac{C_{t+1}^{-\rho}}{P_{t+1}}$	Home Price	$P_{ht} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}(d_{t-1}C_{ht}W_t)}{E_{t-1}(d_{t-1}C_{ht})}$
Money demand	$\frac{M_t}{P_t} = \chi C_t^{\rho}\frac{1+r_{t+1}}{r_{t+1}}$	PCP Price	$P_{hft} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}(Z_tS_t^{\lambda}W_t)}{E_{t-1}(Z_tS_t^{\lambda})}$
Flexible wage	$W_t^a = \frac{\omega\eta}{\omega-1}P_tC_t^{\rho}L_t^{a\psi}$	LCP Price	$P_{hft}^* = \frac{\lambda}{\lambda-1}\frac{E_{t-1}(Z_tW_t)}{E_{t-1}(Z_tS_t)}$
Fixed wage	$W_t^f = \frac{\omega\eta}{\omega-1}\frac{E_{t-1}(L_t^{f(1+\psi)})}{E_{t-1}\left(\frac{L_t^f}{P_tC_t^{\rho}}\right)}$	Definition	$Z_t = d_{t-1}P_{hft}^{\lambda-\theta}P_t^{*\theta}C_t^*$

[9] Because there is full intra-country risk sharing, the share of total home income received by each home consumer is equal. Thus, the profits of the two types are firms are evenly distributed across home consumers.

Table 1 (b) Price and Wage Index			
Wage index	$W_t = \left(v(W_t^a)^{1-\omega} + (1-v)(W_t^f)^{1-\omega}\right)^{\frac{1}{1-\omega}}$	CPI	$P_t = \left(nP_{ht}^{1-\theta} + (1-n)P_{ft}^{1-\theta}\right)^{\frac{1}{1-\theta}}$
Import Price Index	$P_{ft} = \left[(1-z^*)(SP_{fht}^*)^{1-\lambda} + z^* P_{fht}^{1-\lambda}\right]^{\frac{1}{1-\lambda}}$		

Table1 (c) Market Equilibrium	
Employment (flex wage)	$L_t^a = v\left(\frac{W_t^a}{W_t}\right)^{-\omega}\left[Y_{ht} + (1-z)Y_{hft} + zY_{hft}^*\right]$
Employment (fix wage)	$L_t^f = (1-v)\left(\frac{W_t^f}{W_t}\right)^{-\omega}\left[Y_{ht} + (1-z)Y_{hft} + zY_{hft}^*\right]$
Home sales	$Y_{ht} = n\left(\frac{P_{ht}}{P_t}\right)^{-\theta} C_t$
Foreign sales (PCP)	$Y_{hft} = (1-n)\left(\frac{P_{hft}}{S_t P_{ht}^*}\right)^{-\lambda}\left(\frac{P_{ht}^*}{P_t^*}\right)^{-\theta} C_t^*$
Foreign sales (LCP)	$Y_{hft}^* = (1-n)\left(\frac{P_{hft}^*}{P_{ht}^*}\right)^{-\lambda}\left(\frac{P_{ht}^*}{P_t^*}\right)^{-\theta} C_t^*$
Balance of payments	$P_t C_t + B_{t+1} = P_{ht}Y_{ht} + (1-z)P_{hft}Y_{hft} + zS_t P_{hft}^* Y_{hft}^* + (1+r_t)B_t$

Model solution

Using Table 1a-c, we solve the model for the exchange rate and marginal costs. Let the money stock in each country follow a random walk in logs. For the home country, we have[10]

$$\ln M_{t+1} = \ln M_t + u_{t+1} \qquad E_t(u_{t+1}) = 0 .$$

While there is no exact solution for the model, for given z and z^*, we may solve for the equilibrium sequence by linear approximation around an initial non-stochastic equilibrium. We know from section 1 that in order to determine the currency in which firms wish to set their prices, the only information we need is the second moment properties of the log of the exchange rate and wages. But this is exactly what is obtained from the linear approximation.

Let $x_{t+j} = \ln X_{t+j} - E_{t-1} \ln X_{t+j}$ represent the log deviation from *time t-1* expectation for any variable X_{t+j}, $j \geq 0$. A very convenient property of the money demand specification, in combination with the assumption about the money supply process, is that the nominal interest rate is constant. This is because, given that the money stock follows a random walk, so does the term $P_t C_t^{\rho}$. Using this fact, taking the money market equilibrium for the home country from Table 1a, and the analogous conditions for the foreign country, linearizing, and taking differences, gives

$$c_t - c_t^* = \frac{m_t - m_t^*}{\rho} - \frac{(1 - zn - z^*(1-n))s_t}{\rho} . \qquad (2.1)$$

This says that, when there is full pass-through of exchange rates into prices in both countries, i.e. $z = z^* = 0$, purchasing power parity holds at all times, and (2.1) represents

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[10] To keep the notation simple, we abstract from a trend growth rate of money.

a simple `monetary model' of the exchange rate. Alternatively, with zero pass-through, (2.1) indicates that shocks to relative consumption are determined by shocks to relative money supplies alone, since the CPI's in both countries are constant.

From time t+1 onwards, in expectation, there is full money neutrality (in the linear approximate model). Then, using the time $t+1$ balance of payments condition, labor market and product market clearing, we may establish that

$$E_t(c_{t+1} - c^*_{t+1}) = \frac{r}{\sigma}\frac{dB_{t+1}}{(1-n)\overline{P}\overline{C}}\,, \tag{2.2}$$

where $\sigma = \dfrac{\left(1 + \dfrac{(1-\rho)}{\psi\theta} + \dfrac{\rho}{\psi}\right)}{(1+\dfrac{1}{\psi\theta})}$, r represents the steady state nominal interest rate, and $\overline{P}\overline{C}$ describes the initial steady state value of nominal consumption. This condition says that, if the home country is expected to have an increase in net foreign assets, beginning in time $t+1$, then it is also expected to have an increase in its relative consumption.

Using the balance of payments condition for time t, the expressions for foreign and domestic sales, and the price indices from Table 1b), we obtain the following

$$c_t - c^*_t + \frac{dB_{t+1}}{(1-n)\overline{P}\overline{C}} = \left[(\theta-1)(1-z(1-n)-z^*n)+(1-n)z^*+nz\right]s_t\,. \tag{2.3}$$

This equation says that shocks to the exchange rate, by affecting the relative income of home and foreign country, affect the path of relative consumption and the current account. Then, putting (2.2) and (2.3) together, we obtain

$$c_t - c^*_t + \frac{\sigma}{r}E_t(c_{t+1} - c^*_{t+1}) = \left[(\theta-1)(1-z(1-n)-z^*n)+(1-n)z^*+nz\right]s_t\,. \tag{2.4}$$

Equation (2.4) says that the income effects of exchange rate changes are spread over current and expected future relative consumption.

Finally, from the home and foreign Euler equations (Table 1a), we may obtain the following condition relating consumption growth across the two countries:

$$E_t(c_{t+1} - c_{t+1}^*) = c_t - c_t^* - \frac{(zn + z^*(1-n))}{\rho} s_t \,. \tag{2.5}$$

Equation (2.5) says that an unanticipated exchange rate depreciation in period *t*, by causing a real exchange rate depreciation for the home country (when $z, z^* \neq 0$), reduces the relative home country interest rate, and causes a fall in expected consumption growth in the home country, relative to the foreign country.

We may put (2.1)-(2.5) together to obtain a solution for the impact of money shocks on the current exchange rate. This is given by:

$$s_t = \frac{(1+\frac{\sigma}{r})(m_t - m_t^*)}{\Delta} \quad , \tag{2.6}$$

where $\Delta = \left[(1+\frac{\sigma}{r}) + \left(zn + z^*(1-n)\right)(\rho - 1) + \rho(\theta - 1)(1 - z(1-n) - z^* n) \right]$.

The response of the exchange rate to unanticipated money shocks depends on the elasticity of demand for home goods, the inter-temporal elasticity of substitution, and the measure of LCP firms in the home and foreign countries. Two special cases of (2.6) are reported in Table 2 are of particular interest.

Table 2 Exchange Rate Solutions: special cases			
z=z=0*	$s_t = \frac{\left(m_t - m_t^*\right)(1+\frac{\sigma}{r})}{\left(\rho(\theta-1) + (1+\frac{\sigma}{r})\right)}$	*z=z*=1*	$s_t = \frac{\left(m_t - m_t^*\right)(1+\frac{\sigma}{r})}{\left(\rho + \frac{\sigma}{r}\right)}$

With full pass-through from exchange rates to price, the response of the exchange rate is lower the greater is the elasticity of demand between home and foreign goods. On the other hand, when pass-through is zero, the exchange rate response is negatively related to ρ, the consumption elasticity of money demand.

From Table 1 we may derive the response of marginal cost to a money shock as

$$w_t = v(m_t + \psi l_t^a)\,. \qquad (2.7)$$

Marginal cost depends directly on unanticipated home money shocks, and indirectly on home and foreign money shocks, through the movements in employment of the flexible wage setters. We may solve for this employment response to a money shock as:

$$l_t^a = -\omega(1-v)w_t^a + nc_t + (1-n)c_t^* + (1-n)\theta(1-nz^* - z(1-n))s_t\,. \qquad (2.8)$$

Employment depends negatively upon the wage of the flexible wage setters, positively on the movement in aggregate world consumption, and, through `expenditure switching' effects, positively on the nominal exchange rate, so long as there is some pass-through of exchange rates into prices (i.e. when $z, z^* < 1$).

From the money market equilibrium conditions in both countries, we may obtain the movement in world consumption as

$$nc_t + (1-n)c_t^* = \frac{nm_t + (1-n)m_t^* - (1-n)n(z-z^*)s_t}{\rho}\,. \qquad (2.9)$$

An unanticipated increase in home or foreign money raises world consumption. But when $z \neq z^*$, an exchange rate depreciation has a compositional impact on total consumption. For instance, when $z > z^*$, a depreciation raises the home CPI more than it reduces the foreign CPI. Ceteris paribus, this implies that a weighted sum of home and foreign consumption will fall.

We now put the components of section 1 and section 2 together, examining the interaction between the determination of exchange rates and exchange rate pass-through.

Section 3. Equilibrium Pass-through with Identical Monetary Policies

We evaluate the conditions underlying proposition 1 and its corollary, using the results from (2.6), (2.7), and (2.8). First, define the function $\Phi(z, z^{*}, \sigma_u^2, \sigma_{u*}^2)$ as the relative benefit to the firm of pricing by LCP as opposed to PCP. That is:

$$\Phi(z, z^{*}, \sigma_u^2, \sigma_{u*}^2) = \left[\operatorname{cov}_{t-1}(w_t, s_t) - \frac{\operatorname{var}_{t-1}(s_t)}{2}\right].$$

Because exchange rate variance and the covariance of marginal cost and the exchange rate depend on the underlying monetary shocks, as well as on the measure of firms in each country following LCP, we may write the function in this way. In the same way, we define $\Phi^{*}(z, z^{*}, \sigma_u^2, \sigma_{u*}^2)$ as:

$$\Phi^{*}(z, z^{*}, \sigma_u^2, \sigma_{u*}^2) = -\left[\operatorname{cov}_{t-1}(w_t^{*}, s_t) + \frac{\operatorname{var}_{t-1}(s_t)}{2}\right].$$

Table 3 uses the results from section 2 to define the conditional variance of the exchange rate and the conditional covariance of exchange rate and marginal costs.

All our results may be obtained from the combination of the expressions in Table 3 with the definitions of the Φ and Φ^{*} functions. We first focus on symmetric equilibria, where $n = 0.5$, $\sigma_u^2 = \sigma_{u*}^2$, $\Phi = \Phi*$, and $z = z*$. Countries are therefore identical in all respects, and firms in the home and foreign country follow the same pricing policy.

There are three candidate symmetric equilibria, described as follows:

A) Symmetric PCP, $z = z* = 0$. This requires $\Phi(0, 0, \sigma_u^2, \sigma_u^2) < 0$,

B) Symmetric LCP, $z = z^* = 1$. This requires $\Phi(1,1,\sigma_u^2,\sigma_u^2) > 0$,

C) Symmetric mixed, $0 \le z = z^* \le 1$. This requires $\Phi(z,z,\sigma_u^2,\sigma_u^2) = 0$.

In the third case, at the equilibrium value of *z*, firms are indifferent between pricing in the home and foreign currency.

Table 3	
$\mathrm{var}_{t-1}(s_t)$	$\dfrac{(1+\frac{\sigma}{r})^2(\sigma_u^2+\sigma_{u*}^2-2\sigma_{uu*})}{\Delta^2}$
$\mathrm{cov}_{t-1}(w_t,s_t)$	$v\,\mathrm{cov}_{t-1}(w_t^a,s_t)$
$\mathrm{cov}_{t-1}(w_t^a,s_t)$	$=\Theta\left[\mathrm{cov}_{t-1}(u_t,s_t)\left(1+\dfrac{\psi n}{\rho}\right)+\psi\left(\dfrac{1-n}{\rho}\right)\mathrm{cov}_{t-1}(u_t^*,s_t) +\psi(1-n)\left[\theta(1-nz^*-z(1-n))-\dfrac{n}{\rho}(z-z^*)\right]\mathrm{var}_{t-1}(s_t)\right]$, $\quad \Theta=\dfrac{1}{1+\psi\omega(1-v)}$
$\mathrm{cov}_{t-1}(w_t^*,s_t)$	$v\,\mathrm{cov}_{t-1}(w_t^{*a},s_t)$
$\mathrm{cov}_{t-1}(w_t^{*a},s_t)$	$=\Theta\left[\mathrm{cov}_{t-1}(u_t^*,s_t)\left(1+\dfrac{\psi(1-n)}{\rho}\right)+\psi\left(\dfrac{n}{\rho}\right)\mathrm{cov}_{t-1}(u_t,s_t) -\psi n\left[\theta(1-nz^*-z(1-n))+\dfrac{(1-n)}{\rho}(z-z^*)\right]\mathrm{var}_{t-1}(s_t)\right]$

To establish the existence of equilibrium we need to evaluate the Φ function at each value of *z*. It is easy to establish that in the symmetric case:

$$\Phi(z,z,\sigma_u^2,\sigma_u^2) \propto \tilde{v}\left[(\rho(\theta-1)+\psi\theta(1+\frac{\sigma}{r}))(1-z)+(\rho-1)z\right]-(1+\frac{\sigma}{r})(1-\tilde{v}), \tag{3.1}$$

where the term `$\propto$' denotes `proportional to', and $\tilde{v} = \dfrac{v}{1+\psi\omega(1-v)} < 1$.

Using expression (3.1), we may establish the following proposition

Proposition 2: Symmetric equilibrium

a) If $\theta > \dfrac{1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\rho\tilde{v}}}{1+\frac{\psi}{\rho}(1+\frac{\sigma}{r})}$, and $\rho > 1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\tilde{v}}$, LCP is the unique equilibrium.

b) If $\theta < \dfrac{1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\rho\tilde{v}}}{1+\frac{\psi}{\rho}(1+\frac{\sigma}{r})}$, and $\rho < 1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\tilde{v}}$, PCP is the unique equilibrium.

c) If $\theta > \dfrac{1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\rho\tilde{v}}}{1+\frac{\psi}{\rho}(1+\frac{\sigma}{r})}$ and $\rho < 1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\tilde{v}}$ the unique equilibrium is $\bar{z} \in (0,1)$,

such that $\Phi(\bar{z}, \bar{z}, \sigma_u^2, \sigma_u^2) = 0$.

d) If $\theta < \dfrac{1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\rho\tilde{v}}}{1+\frac{\psi}{\rho}(1+\frac{\sigma}{r})}$ and $\rho > 1+(1+\frac{\sigma}{r})\frac{(1-\tilde{v})}{\tilde{v}}$, there are three equilibria: PCP,

LCP, and an interior (unstable) equilibrium $\hat{z} \in (0,1)$ such that $\Phi(\hat{z}, \hat{z}, \sigma_u^2, \sigma_u^2) = 0$.

Proof: Part a) If the two inequalities hold, each firm has an incentive to set export prices in terms of the local currency, whatever other firms do. Moreover, since the $\Phi(z, z, \sigma_u^2, \sigma_u^2)$ function is monotonic in z, there is no interior equilibrium where $0 < z < 1$. Thus, the only equilibrium can be one where all firms follow LCP. For LCP to be a unique symmetric equilibrium, the consumption elasticity of money demand must be at least unity, and the elasticity of substitution between home and foreign goods must be sufficiently high. As is clear from section 1, the results are very sensitive to the degree of ex-post flexibility in marginal cost. If most wages are pre-set, then LCP cannot be an equilibrium. But if v=1, so that all wages are adjusted ex-post, then LCP is the unique equilibrium when $\rho > 1$ and $\theta > 1$.

Part b) If the two inequalities of part b hold, then each firm will follow PCP pricing, no matter what other firms do. This outcome is more likely, the lower is v, and the lower are θ and ρ. In the first case, the calculation the firm makes is dominated by the variance of the exchange rate, since the covariance of marginal costs and the exchange rate is small when most wages are pre-set. In the second case, the lower are θ and ρ, the higher is the volatility of the exchange rate, relative to the $\text{cov}_{t-1}(w_t, s_t)$ term, whatever pricing policy is chosen. As a result the optimal policy for all firms is to choose PCP pricing.

Part c) In part c, the incentives for pricing will depend on what other firms do. If all firms follow PCP pricing, then any one firm would have an incentive to deviate and choose LCP. But if all firms follow LCP, then again, any one firm would have an incentive to deviate and choose PCP. Thus, there is no equilibrium where all firms follow the same pricing policy. By continuity, an intermediate equilibrium exists in which some firms follow PCP and some firms LCP. For a given value of v, this outcome is more likely, the higher is θ and the lower is ρ. In that case, exchange rate volatility is quite low under PCP, relative to $\text{cov}_{t-1}(w_t, s_t)$, giving firms an incentive to engage in LCP pricing. But if all firms follow LCP pricing, then with a low value of ρ, exchange rate volatility is high, relative to $\text{cov}_{t-1}(w_t, s_t)$. This means that LCP is not an equilibrium. In the intermediate equilibrium $(1-\bar{z})$ firms follow PCP and $\bar{z}$ firms follow PCP. No firm has an incentive to deviate from this outcome.

Note that uniqueness arises because of the negative relationship between exchange rate pass-through and exchange rate volatility. With all firms following PCP, exchange rate volatility is relatively low, giving any one firm an incentive to follow LCP. As more and more firms engage in LCP, there is a fall in exchange rate pass-through. The fall in pass-through increases the volatility of the exchange rate. But the rise in exchange rate volatility tends to limit the incentive to follow an LCP pricing strategy, therefore limiting the degree to which pass-through falls.

Part d). Part d is the opposite of part c. In this case, all firms have an incentive to follow PCP if all other firms do also. Conversely, all firms have an incentive to follow LCP if all other firms do also. For a given value of *v*, this outcome is more likely, the lower is θ and the higher is ρ. Exchange rate volatility is then quite *high* under PCP, relative to $\mathrm{cov}_{t-1}(w_t, s_t)$, giving firms and incentive to engage in PCP pricing, when $z=0$. But if all firms follow LCP pricing, then with a high value of ρ, exchange rate volatility is reduced. This encourages firms to engage in LCP pricing when $z=1$. Thus, both $z=0$ and $z=1$ are equilibria[11]. Thus, a *positive* relationship between exchange rate pass-through and exchange rate volatility gives rise to the possibility of non-uniqueness.

Figure 1 describes the four possible equilibrium configurations, in terms of the value of the $\Phi(z, z, \sigma_u^2, \sigma_u^2)$ function over the range of *z*. Figure 1a describes the unique LCP outcome. Figure 1b shows the unique PCP outcome. Figure 1c shows the unique intermediate outcome, while Figure 1d illustrates the possibility of multiple equilibria.

We note from the Figures that multiple equilibria are possible only when the $\Phi(z,z,\sigma_u^2,\sigma_u^2)$ function is upward sloping. The slope of the function is proportional to

$$-\tilde{v}\left[\rho(\theta-1)+\psi\theta(1+\frac{\sigma}{r})+1-\rho\right] \tag{3.2}$$

As suggested by the Proposition, this is more likely, the higher is ρ, and the lower is θ. More informally, multiple equilibria are more likely when LCP tends to be associated with low exchange rate volatility, while PCP associated with high exchange rate volatility. Empirically, however, multiple equilibria are not very likely in our model. The empirically relevant range θ exceeds unity, and should certainly exceed the consumption elasticity of money demand ρ. Even if there is a very high elasticity of labor supply (low ψ), the expression (3.2) is therefore negative for reasonable parameter values.

It is also worth noting that the equilibrium pass-through in all cases of Proposition 2 is independent of the distribution of the money supplies. This is because both the variance of the exchange rate and the covariance of exchange rates and marginal costs are affected equally by monetary variability in the symmetric case.

How would identical firms coordinate on which pricing policy to follow to achieve an outcome where there is ex-post heterogeneity? If we think of firms engaged in a coordination game in their price setting decisions, we can think of these interior outcomes as mixed strategy equilibria of the game, where firms choose in advance a probability of the currency in which to set prices[12]. Alternatively, as shown in the appendix, we can rationalize this equilibrium as one in which firms incur differential

[11] There is also an internal equilibrium. This equilibrium is unstable, however, since if a small measure of

`menu costs' of setting prices in foreign currency relative to home currency, and the costs are heterogeneous across firms. Ranking the firms by menu costs of foreign price setting, the firms with the lowest costs will engage in LCP first, and then firms with progressively higher costs. An interior equilibrium is by the firm whose net benefit from LCP pricing just equals the menu cost. For very small menu costs relative to expected profits, this mechanism is approximately equivalent to the mixed strategy equilibrium of the pricing game, with the equivalence being exact in the limit.

In case c) of the proposition, the equilibrium $\overline{z}$ is given by

$$\overline{z} = \frac{\rho(\theta-1)+(1+\frac{\sigma}{r})(\psi\theta-\frac{(1-\tilde{v})}{\tilde{v}})}{\rho(\theta-1)+\psi\theta(1+\frac{\sigma}{r})+1-\rho}. \tag{3.3}$$

Inspection of (3.3) indicates that $\overline{z}$ is increasing in $\tilde{v}$. As a greater fraction of wages are set ex-post, the equilibrium degree of pass-through declines. Similarly, $\overline{z}$ is increasing in ρ and θ. Since both parameters tend to reduce exchange rate volatility for any given *z*, the result is to increase the number of firms who engage in LCP, and reduce pass-through.

This illustrates one of the key points of the paper; the relationship between exchange rate volatility and economic structure may be substantially altered by taking account of the endogeneity of exchange rate pass-through. To illustrate this, let's take a special case where $\psi = 0$ and $\tilde{v} = 1$. Taking pass-through as given, we may re-write the expression for exchange rate volatility as

firms deviate by increasing (decreasing) *z*, then all others will wish to follow, so that *z* goes to 1 (0).

[12] Bacchetta and Wincoop (2000b) give this interpretation in their simulations.

$$\frac{(1+\frac{\sigma}{r})^2(\sigma_u^2+\sigma_{u*}^2-2\sigma_{uu*})}{\left[(1+\frac{\sigma}{r})^2+z(\rho-1)+\rho(\theta-1)(1-z)\right]^2}.$$

If we begin in a situation where $\theta>1$, $\rho>1$, so that $z=1$ holds (part a of the proposition), then exchange rate volatility is $\left(\frac{1+\frac{\sigma}{r}}{\frac{\sigma}{r}+\rho}\right)^2(\sigma_u^2+\sigma_{u*}^2-2\sigma_{uu*})$. Here, exchange rate volatility is less than the variance of monetary fundamentals, $(\sigma_u^2+\sigma_{u*}^2-2\sigma_{uu*})$. Now let ρ fall below unity. Ignoring the response of pass-through, we would predict that this would increase exchange rate volatility, so that volatility exceeded the variance of monetary fundamentals. A number of papers have argued that it is possible to explain high variability of real exchange rates with a combination of low pass-through and a low consumption elasticity of labor demand (e.g. Chari et al, 2000). But in our model, this will not happen. When ρ falls below unity, part c of the proposition (or Figure 1c) applies. Now we have pass through falling, so that $\bar{z}=\frac{\rho(\theta-1)}{\rho(\theta-1)+1-\rho}$. Exchange rate volatility is now given by $(\sigma_u^2+\sigma_{u*}^2-2\sigma_{uu*})$. Exchange rate volatility will in this example never exceed the monetary fundamentals.

Section 4. Pass-through with Differential Monetary Policies

We now allow for differences in money growth volatility across countries. Without loss of generality, assume that the home country has a lower monetary growth volatility than the foreign country. As discussed previously, the motivation for this might be based on the experience of a country that institutes a successful program of inflation targeting, reducing the growth rate and the volatility of the money supply.

As discussed at the end of the previous section, we focus on equilibria where firms employ mixed strategies. Thus, if $\Phi(z, z^*, \sigma_u^2, \sigma_{u*}^2) = 0$ and $\Phi^*(z, z^*, \sigma_u^2, \sigma_{u*}^2) = 0$ we say $\{z, z^*\}$ is an equilibrium where each home (foreign) firm chooses a probability *z* (*z**), ex ante, of setting its export price in foreign (home) currency, and *1-z*, (*1-z**) of setting its price in home (foreign) currency.

To simplify the presentation of results, we first make the additional assumption that preferences display linearity in labor supply, so that $\psi = 0$. This assumption is commonly used in the literature on exchange rates and price stickiness (Devereux and Engel 2001, Corsetti and Pesenti 2001). Qualitatively, none of the results are affected by the assumption. The general case where $\psi > 0$ is used in the simulations below. In addition, further to our discussion of the last section, we focus only on the cases of unique equilibrium. Thus, we restrict attention to the set of equilibria where the Φ and Φ^* functions are downward sloping[13].

Using Table 3, it may be established that

$$\Phi \propto \tilde{v}\left[\rho(\theta-1)(1-z^* n - z(1-n)) + (\rho-1)(zn + z^*(1-n)) + (1+\frac{\sigma}{r})\right] - (1+\frac{\sigma}{r})\frac{(\sigma_u^2 + \sigma_{u*}^2)}{2\sigma_u^2}$$

$$\Phi^* \propto \tilde{v}\left[\rho(\theta-1)(1-z^* n - z(1-n)) + (\rho-1)(zn + z^*(1-n)) + (1+\frac{\sigma}{r})\right] - (1+\frac{\sigma}{r})\frac{(\sigma_u^2 + \sigma_{u*}^2)}{2\sigma_{u*}^2}$$

Using these two expressions, we may establish the following proposition.

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[13] In the specific case where $\psi = 0$, this requires that $\rho(\theta-1)(1-n) - (\rho-1)n > 0$ and $\rho(\theta-1)n - (\rho-1)(1-n) > 0$.

Proposition 3

Let $\Omega = (1+\frac{\sigma}{r})\frac{(\sigma_u^2+\sigma_{u*}^2)}{2\sigma_u^2}$, $\quad \Omega^* = (1+\frac{\sigma}{r})\frac{(\sigma_u^2+\sigma_{u*}^2)}{2\sigma_{u*}^2}$, and

$\Gamma(z,z^*) = \tilde{v}\left[\rho(\theta-1)(1-z^*n-z(1-n))+(\rho-1)(zn+z^*(1-n))+(1+\frac{\sigma}{r})\right]$. Note that from our assumption that $\sigma_{u*}^2 > \sigma_u^2$, we have $\Omega > \Omega^*$. The equilibrium is described by the set $\mu = \{z, z^*\}$. The equilibrium has the following properties:

a) If $\Gamma(1,1) = \tilde{v}((\rho-1)+(1+\frac{\sigma}{r})) > \Omega$, then $\mu = \{1,1\}$.

b) If $\Gamma(\hat{z},1) = \tilde{v}(\rho(\theta-1)(1-n)(1-\hat{z})+(\rho-1)(1-n+\hat{z}n)+(1+\frac{\sigma}{r})) = \Omega$, $\Gamma(\hat{z},1) > \Omega^*$, and $0 < \hat{z} < 1$, then $\mu = \{\hat{z},1\}$.

c) If $\Gamma(0,1) = \tilde{v}(\rho(\theta-1)(1-n)+(\rho-1)(1-n)+(1+\frac{\sigma}{r})) < \Omega$ and $\Gamma(0,1) > \Omega^*$, then $\mu = \{0,1\}$.

d) If $\Gamma(0,\hat{z}^*) = \tilde{v}(\rho(\theta-1)(1-n\hat{z}^*)+(\rho-1)\hat{z}^*(1-n)+(1+\frac{\sigma}{r})) < \Omega$, $\Gamma(\hat{z}^*,1) = \Omega^*$, and $0 < \hat{z}^* < 1$, then $\mu = \{0, \hat{z}^*\}$.

e) If $\Gamma(0,0) = \tilde{v}(\rho(\theta-1)+(1+\frac{\sigma}{r})) < \Omega$, and $\Gamma(0,0) < \Omega^*$, then $\mu = \{0,0\}$.

Proof: For each part, the proof follows by direct construction. In case a), if $\Gamma(1,1)$, exceeds Ω, then full LCP is an equilibrium for both the home and foreign firms. Moreover, because we assume that $\Gamma(z,z^*)$ is decreasing in both variables (i.e. because we rule out multiple equilibria), this is the only equilibrium outcome. In case b), a measure $\hat{z}$ of home country firms follow LCP, while all foreign firms follow LCP. Note that $\hat{z}$ is implicitly defined by the equality $\Gamma(\hat{z},1)=\Omega$. In case c), all home country firms follow PCP, whereas foreign firms all follow LCP. In case d), all home country firms follow PCP, while a measure $\hat{z}^*$ of foreign firms follow LCP. Finally, in case e), all firms, both home and foreign, follow PCP.

A notable feature of Proposition 3 is that the exchange rate pass-through into the home economy is always less than or equal to that into the foreign economy. By reducing the volatility of home money growth, the home country will either leave exchange rate pass-through into the home economy unchanged, or decrease it. Conversely, exchange rate pass-through into the foreign country either remains unchanged, or increases. Relative monetary growth stability tends to be associated with a fall in exchange rate pass-through to the stabilizing country, and a rise in pass-through for the other country. Thus, firms tend to set their export prices in the currency which is associated with the more stable monetary growth.

Which of the five categories of Proposition 3 will come about depends on parameter values, and the relative size of money growth variances. As in section 3, the smaller is v, the fraction of wage contracts that are subject to ex-post adjustment, the more likely that firms in both countries will follow PCP, since marginal costs will tend to have a smaller covariance with exchange rate movements. For a given ρ, the greater is the elasticity of substitution between home and foreign products, θ, the more likely is LCP, since exchange rate variance will, ceteris paribus, be smaller. One clear result is that as a country progressively eliminates its monetary variability, its firms will always follow PCP. That is, when the variance of money growth, relative to foreign money growth, falls to zero, exchange rate pass-through becomes complete. The reason is that reducing money growth variance to zero tends to fully stabilize marginal cost for the home country. With a positive exchange rate variance determined by foreign monetary instability, it is always optimal for home country firms to set prices in their domestic currency. For foreign firms on the other hand, the variance of the exchange rate tends to fall, relative to the covariance of the exchange rate and marginal cost, since more and more of exchange rate volatility is driven by their own monetary shocks; the same shocks that are driving marginal costs.

Take a particular example of the impact of changes in the variance of monetary growth and focus on it more closely. We do this by way of the following Proposition.

Proposition 4.

Begin in an initial symmetric equilibrium $\mu = \{\tilde{z}, \tilde{z}\}$, where $\Gamma(\tilde{z}, \tilde{z}) = (1 + \sigma / r)$, with $0 < \tilde{z} < 1$. Then a fall in the variance of home country monetary growth will reduce pass-through into the home country, and increase pass-through into the foreign country. The new equilibrium will be either a) $\mu = \{z', 1\}$, b) $\mu = \{0, 1\}$, or c) $\mu = \{0, z^{*}{}'\}$, where $z' < \tilde{z}$, and $z^{*}{}' > \tilde{z}$.

Proof: Using the same arguments as Proposition 3, it is easy to show that the impact of the fall in the variance of home country monetary growth must lead to one of cases b), c) or d) of Proposition 3. In particular, given that the function $\Gamma(z, z^{*})$ is common to both countries, the impact of a fall in monetary growth in the home country is either to fully eliminate exchange rate pass-through into the domestic economy, or to increase pass-through to 100 percent in the foreign economy. With differences in monetary growth variance, it is no longer possible to have partial pass-through in both economies.

These results provide a theoretical rationale for the conjecture that low and stable inflation rates may lead to a reduction in the pass-through from exchange rate movements into the CPI. In our model, a fall in one country's monetary instability will reduce exchange rate pass-through into that country, and hence stabilize its price level from the effects of exchange rate movements. But note a critical aspect of results; the rate of pass-through depends on *relative* variances of monetary growth rates, not on the absolute variances. Thus, pass-through is unaffected by a parallel reduction in monetary growth stability in all countries. Moreover, a country achieves a low degree of pass-through only by increasing exchange rate pass-through into its trading partner.

From a slightly different perspective, the model gives an additional link between inflation targeting and price stability. By pursuing a policy of low and stable inflation, a country can achieve price stability. But with endogenous pass-through, it gets an additional `bonus' with respect to price stability. By stabilizing its money growth rate, it encourages foreign exporters to set prices in terms of its currency. In doing so, it stabilizes the imported goods component of its CPI, thereby further achieving price stability. But the flip side of this is that the policy also encourages domestic exporters to favor the home currency for price-setting of goods to be sold in foreign markets. As a result the price-stability bonus achieved for the home economy comes at the expense of a price stability `penalty' imposed on the foreign economy, as its price level becomes more unstable. In this respect, there is a type of `beggar-thy-neighbor' feature in the determination of exchange rate pass-through, and more generally in the effect of monetary policy on price stability in an open economy with endogenous pass-through.

Figure 2 and 3 illustrate the results with the more general model, without the assumption of linearity in labor supply in the utility function. Since the Φ and Φ* functions are no longer linear dependent (in *z* and *z**), it is now possible that there are simultaneous interior equilibrium values for pass-through in both countries. The parameter values used are reported in Table 4, and are mostly quite standard.

Table 4. Parameter Values			
θ	8	r	0.1
ρ	1.25	σ	1
v	.75	ω	1.5
ψ	1	n	0.5

In Figure 2, we show the impact in the volatility of monetary growth in the home country, starting from the point where home and foreign money growth volatility is the same. At the initial point, *z=z*=0.35*, so there is pass-through from exchange rates to imported goods prices equal to 65 percent. As σ_u^2 falls however, *z* falls sharply, and *z** increases, so that pass-through into the foreign economy increases to 100 percent when σ_{u*}^2 rises 4 percent above σ_u^2, and pass-through into the home economy falls to zero when σ_{u*}^2 rises 7 percent above σ_u^2.

Figure 3 illustrates the case where only 10 percent of wage contracts are adjusted ex post (*v=0.1*). In this case, the initial symmetric equilibrium is one where both firms follow PCP, and pass-through is complete in both economies. But as $(\sigma_{u*}^2/\sigma_u^2)$ rises to 1.1, foreign firms switch quite quickly to LCP, and *z** rises to unity, so that pass-through into the home economy falls to zero. This example points quite dramatically to the importance of relative volatility of money growth in determining exchange rate pass-through in our environment. The symmetric equilibrium indicates a very strong preference for PCP, given that marginal cost shows little ex-post responsiveness to the exchange rate. But the rise in $(\sigma_{u*}^2/\sigma_u^2)$ still increases the importance of marginal costs for foreign firms so much that they will switch over to pricing in home currency.

Conclusions

This paper develops a general framework for analyzing the determinants of exchange rate pass-through in a open economy macroeconomic model. The key implications of the model are essentially equivalent to the Lucas critique – changes in economic policy may lead to changes in equilibrium decision rules. We have given one example where changes in relative monetary stability have very strong implications equilibrium exchange rate pass-through in all countries. But more generally, we conjecture that allowing for endogenous exchange rate pass-through may have significant implications for the international transmission of shocks, for optimal monetary policy, and for the gains from international coordination of monetary policies.

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Appendix A

Proof of proposition 1

From (1.4), profits under PCP are given as

$$E\Pi^{PCP} = \tilde{\lambda} E(S^{\lambda} Z)^{\lambda} E(S^{\lambda} ZW)^{1-\lambda}.$$

This expression may be rewritten as

$$\tilde{\lambda}(E\exp(\ln Z)\exp(\lambda \ln S))^{\lambda}(E\exp(\ln Z)\exp(\lambda \ln S)\exp(\ln W))^{1-\lambda}. \qquad \text{(A1)}$$

Now use the second order approximation:

$$E\exp(\ln Z)\exp(\lambda \ln S) \approx \exp(E\ln Z)\exp(\lambda E\ln S)\times \left(1+\frac{1}{2}\mathrm{var}(\ln Z)+\frac{\lambda^2}{2}\mathrm{var}(\ln S)+\lambda\,\mathrm{cov}(\ln Z,\ln S)\right). \qquad \text{(A2)}$$

Using the same approximation for the expression $E\exp(\ln Z)\exp(\lambda \ln S)\exp(\ln W)$, we get an approximation for profits equal to

$$\Sigma\left(1+\frac{1}{2}\mathrm{var}(\ln Z)+\frac{\lambda^2}{2}\mathrm{var}(\ln S)+\lambda\,\mathrm{cov}(\ln Z,\ln S)\right)^{\lambda} \times \begin{pmatrix} 1+\frac{1}{2}\mathrm{var}(\ln Z)+\frac{\lambda^2}{2}\mathrm{var}(\ln S)+\frac{1}{2}\mathrm{var}(\ln W) \\ +\lambda\,\mathrm{cov}(\ln S,\ln Z)+\mathrm{cov}(\ln Z,\ln W)+\lambda\,\mathrm{cov}(\ln S,\ln W) \end{pmatrix}^{1-\lambda},$$

where $\Sigma = \tilde{\lambda}\exp(E\ln Z)\exp(\lambda E\ln S)\exp((1-\lambda)E\ln W)$

Taking logs, we get expected profits equal to

$$\ln\Sigma+\left(\frac{1}{2}\mathrm{var}(\ln Z)+\frac{\lambda^2}{2}\mathrm{var}(\ln S)+\frac{(1-\lambda)}{2}\mathrm{var}(\ln W)\right)+ \left(\lambda\,\mathrm{cov}(\ln Z,\ln S)+\lambda(1-\lambda)\,\mathrm{cov}(\ln W,\ln S)+(1-\lambda)\,\mathrm{cov}(\ln Z,\ln W)\right). \qquad \text{(A3)}$$

Now, expected profits under LCP are written as

$$E\Pi^{LCP} = \tilde{\lambda}\left[EZS\right]^{\lambda}\left[EZW\right]^{1-\lambda}$$

Using the same approximation, they may be written as

$$\ln\Sigma+\left(\frac{1}{2}\mathrm{var}(\ln Z)+\frac{\lambda}{2}\mathrm{var}(\ln S)+\frac{(1-\lambda)}{2}\mathrm{var}(\ln W)\right)+ \left(\lambda\,\mathrm{cov}(\ln Z,\ln S)+(1-\lambda)\,\mathrm{cov}(\ln Z,\ln W)\right). \qquad \text{(A4)}$$

Now comparing (A3), and (A4), we can immediately establish part Proposition 1.

Appendix B

Here we derive the results that are obtained in section 2 of the paper. The full model is described by the equations of Table 1. This gives 15 equations for the home country, plus the balance of payments equations. Adding on 15 corresponding equations for the foreign country (the foreign country's balance payments equation is dropped, by Walras' Law), we arrive at 31 equations in the 31 endogenous variables listed as follows: $C_t, C_t^*, W_t, W_t^*, W_t^a, W_t^{*a}, W_t^f, W_t^{*f} L_t^a, L_t^{*a}, L_t^f, L_t^{*f}\ Y_{ht}, Y_{hft}, Y_{hft}^*, Y_{ft},$
$Y_{fht}, Y_{fht}^*, S_t, B_t, r_t\ P_t, P_t^*, P_{ht}, P_{ft}^*, P_{ht}^*, P_{ft}, P_{hft}, P_{hft}^*, P_{fht}^*, P_{fht}$.

Solution technique

To solve this system, we take a linear approximation around an initial symmetric steady state, where net foreign assets are zero, all prices are equal, and the exchange rate is initially unity. The solution procedure is as follows. First, we take the linear approximation around an initial steady state equilibrium. We define $\hat{x}_t = \ln X_t - \ln \overline{X}$, as a log deviation from the initial steady state. Define $x_{t+j} = \hat{x}_{t+j} - E_{t-1}\hat{x}_{t+j}$ as the unexpected component of the deviation from the initial steady state. Using this, we may compute the conditional variance and covariance of the exchange rate and marginal costs.

Then the linearized versions of the pricing equations of Table 1 are

$$\hat{p}_t = n\hat{p}_{ht} + (1-n)\hat{p}_{ft} = n\hat{p}_{ht} + (1-n)((1-z^*)(\hat{s}_t + \hat{p}_{fht}^*) + z^*\hat{p}_{fht}) \qquad \text{(B1)}$$

$$\hat{p}_t^* = n\hat{p}_{ht}^* + (1-n)\hat{p}_{ft}^* = n(z\hat{p}_{hft}^* + (1-z)(\hat{p}_{hft} - \hat{s}_t)) + (1-n)\hat{p}_{ft}^* \qquad \text{(B2)}$$

$$\hat{p}_{ht} = E_{t-1}\hat{w}_t \qquad \hat{p}_{ft}^* = E_{t-1}\hat{w}_t^* \qquad \text{(B3)}$$

$$\hat{p}_{hft}^* = E_{t-1}(\hat{w}_t - \hat{s}_t) \qquad \hat{p}_{hft} = E_{t-1}\hat{w}_t \qquad \text{(B4)}$$

$$\hat{p}_{fht} = E_{t-1}(\hat{w}_t^* + \hat{s}_t) \qquad \hat{p}_{fht}^* = E_{t-1}(\hat{w}_t^*) \qquad \text{(B5)}$$

This implies that the CPI prices may be written as:

$$\hat{p}_t = nE_{t-1}\hat{w}_t + (1-n)((1-z^*)(\hat{s}_t + E_{t-1}\hat{w}_t^*) + z^*E_{t-1}(\hat{w}_t^* + \hat{s}_t)) \qquad \text{(B6)}$$

$$\hat{p}_t^* = n((1-z)(E_{t-1}\hat{w}_t - \hat{s}_t) + zE_{t-1}(\hat{w}_t - \hat{s}_t)) + (1-n)E_{t-1}\hat{w}_t^*. \qquad \text{(B7)}$$

Linearizing the balance of payments condition including the home and foreign demand schedules from Table 1 gives

(B8)

$$\hat{c}_t + \frac{d\hat{B}_{ft+1}}{\overline{P}\overline{C}} = n\hat{c}_t + n(1-\theta)(\hat{p}_{ht} - \hat{p}_t) + (1-n)z \times$$
$$\left[(1-\lambda)(\hat{p}^*_{hft} - \hat{p}^*_{ht}) + (1-\theta)(\hat{p}^*_{ht} - \hat{p}^*_t) + (\hat{p}^*_t + \hat{s}_t - \hat{p}_t) + \hat{c}^*_t\right]$$
$$+(1-n)(1-z)\left[(1-\lambda)(\hat{p}_{hft} - \hat{s}_t - \hat{p}^*_{ht}) + (1-\theta)(\hat{p}^*_{ht} - \hat{p}^*_t) + (\hat{p}^*_t + \hat{s}_t - \hat{p}_t) + \hat{c}^*_t\right]$$
$$+(1+r)\frac{d\hat{B}_{ft}}{\overline{P}\overline{C}}$$

Linearizing the employment conditions for the fixed and flexible wage sectors gives

$$\hat{l}^a_t = -\omega(\hat{w}^a_t - \hat{w}_t) + n\left[-\theta(\hat{p}_{ht} - \hat{p}_t) + \hat{c}_t\right] +$$
$$(1-n)\left[z(-\lambda(\hat{p}^*_{hft} - \hat{p}^*_{ht}) - \theta(\hat{p}^*_{ht} - \hat{p}^*_t)) + (1-z)(-\lambda(\hat{p}_{hft} - \hat{s}_t - \hat{p}^*_{ht}) - \theta(\hat{p}^*_{ht} - \hat{p}^*_t)) + \hat{c}^*_t\right] \tag{B9}$$

$$\hat{l}^f_t = -\omega(\hat{w}^f_t - \hat{w}_t) + n\left[-\theta(\hat{p}_{ht} - \hat{p}_t) + \hat{c}_t\right] +$$
$$(1-n)\left[z(-\lambda(\hat{p}^*_{hft} - \hat{p}^*_{ht}) - \theta(\hat{p}^*_{ht} - \hat{p}^*_t)) + (1-z)(-\lambda(\hat{p}_{hft} - \hat{s}_t - \hat{p}^*_{ht}) - \theta(\hat{p}^*_{ht} - \hat{p}^*_t)) + \hat{c}^*_t\right] \tag{B10}$$

$$\hat{l}^{*a}_t = -\omega(\hat{w}^{*a}_t - \hat{w}^*_t) + (1-n)\left[-\theta(\hat{p}^*_{ft} - \hat{p}^*_t) + \hat{c}^*_t\right] +$$
$$(1-n)\left[z^*(-\lambda(\hat{p}_{fht} - \hat{p}_{ft}) - \theta(\hat{p}_{ft} - \hat{p}_t)) + (1-z^*)(-\lambda(\hat{p}^*_{fht} + \hat{s}_t - \hat{p}_{ft}) - \theta(\hat{p}_{ft} - \hat{p}_t)) + \hat{c}_t\right] \tag{B11}$$

$$\hat{l}^{*f}_t = -\omega(\hat{w}^{*f}_t - \hat{w}^*_t) + (1-n)\left[-\theta(\hat{p}^*_{ft} - \hat{p}^*_t) + \hat{c}^*_t\right] +$$
$$(1-n)\left[z^*(-\lambda(\hat{p}_{fht} - \hat{p}_{ft}) - \theta(\hat{p}_{ft} - \hat{p}_t)) + (1-z^*)(-\lambda(\hat{p}^*_{fht} + \hat{s}_t - \hat{p}_{ft}) - \theta(\hat{p}_{ft} - \hat{p}_t)) + \hat{c}_t\right] \tag{B12}$$

Linearizing the implicit labor supply schedules for the fixed and flexible wage setters gives

$$\hat{w}^a_t = \hat{p}_t + \rho\hat{c}_t + \psi\hat{l}^a_t \tag{B13}$$

$$\hat{w}^f_t = E_{t-1}(\hat{p}_t + \rho\hat{c}_t + \psi\hat{l}^f_t) \tag{B14}$$

$$\hat{w}^{*a}_t = \hat{p}^*_t + \rho\hat{c}^*_t + \psi\hat{l}^{*a}_t \tag{B15}$$

$$\hat{w}^{*f}_t = E_{t-1}(\hat{p}^*_t + \rho\hat{c}^*_t + \psi\hat{l}^{*f}_t) \tag{B16}$$

Finally, the linearization of the Euler equation and the money market clearing conditions (using the fact that the nominal interest rate is constant in equilibrium) gives:

$$\hat{p}_t + \rho\hat{c}_t = E_t(\hat{p}_{t+1} + \rho\hat{c}_{t+1}) \tag{B17}$$

$$\hat{p}^*_t + \rho\hat{c}^*_t = E_t(\hat{p}^*_{t+1} + \rho\hat{c}^*_{t+1}) \tag{B18}$$

$$\hat{m}_t - \hat{p}_t = \rho\hat{c}_t \tag{B19}$$

$$\hat{m}^*_t - \hat{p}^*_t = \rho\hat{c}^*_t \tag{B20}$$

To get (2.1) of the text, use equations (B19) and (B20), together with (B6) and (B7), using the definition, $x_t = \hat{x}_t - E_{t-1}\hat{x}_t$, noting that for all prices, this variable will be zero, given that prices are set in period *t-1*.

To get (2.2) of the text, use the balance of payments condition (B8), substituting in the pricing equations, and taking expectations dated *t-1*, gives

$$E_{t-1}(\hat{c}_t - \hat{c}^*_t) = (1-\theta)E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) + \frac{rd\hat{B}_{ft}}{(1-n)\bar{P}\bar{C}} \tag{B21}$$

Doing the same for the employment equations, noting that in expected terms (in the linear approximation), employment and wages of both groups will be the same, gives us

$$E_{t-1}(\hat{l}_t - \hat{l}^*_t) = -\theta E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) \tag{B22}$$

Finally, from the labor supply equations, we have

$$E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) = \rho E_{t-1}(\hat{c}_t - \hat{c}^*_t) + \psi E_{t-1}(\hat{l}_t - \hat{l}^*_t) \tag{B23}$$

From (B22) and (B23),

$$E_{t-1}(\hat{w}_t - \hat{w}^*_t - \hat{s}_t) = \frac{\rho}{(1+\psi\theta)} E_{t-1}(\hat{c}_t - \hat{c}^*_t), \tag{B24}$$

Combining (B21) and (B24), and updating to period *t* gives (2.2) of the text.

Now take the balance of payments equation (B8) again, substituting in for prices, and take away date *t-1* expectations (i.e. use the definition $x_t = \hat{x}_t - E_{t-1}\hat{x}_t$ again), gives (2.3) of the text.

To get (2.5) of the text, use equations (B17) and (B18), substitute in for the price definitions, and take away date *t-1* expectations.

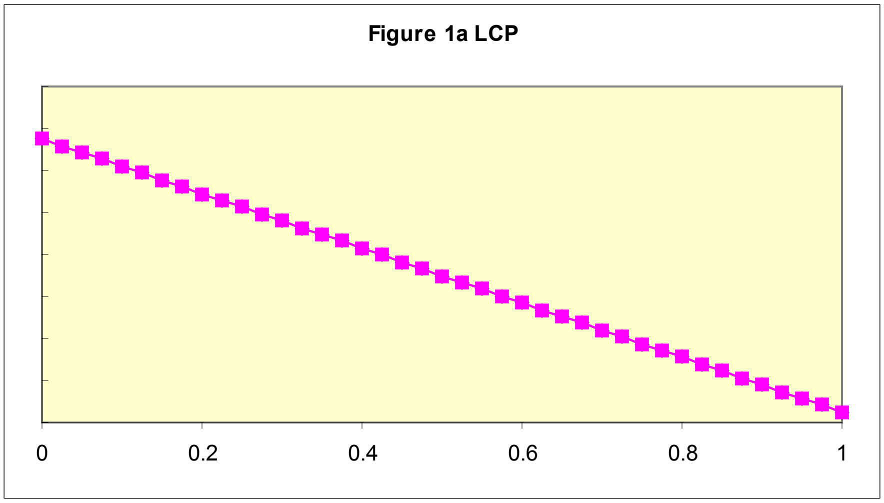
Figure 1a LCP
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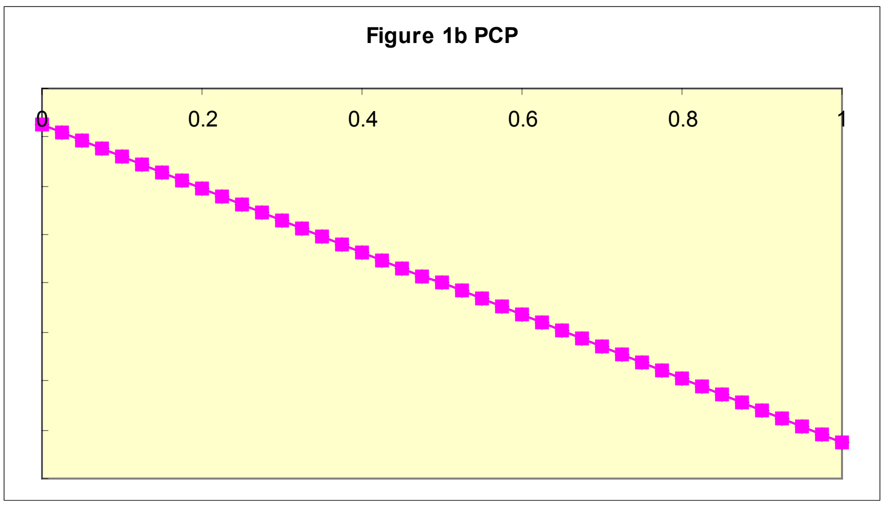
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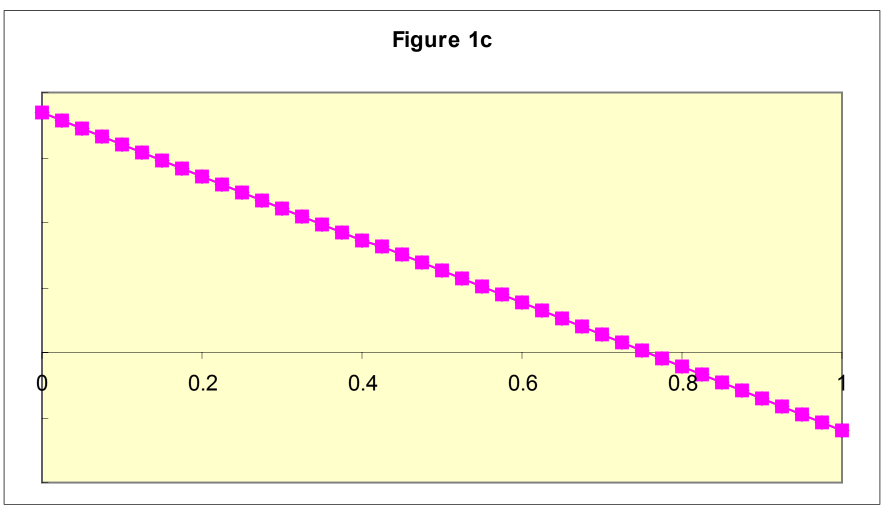
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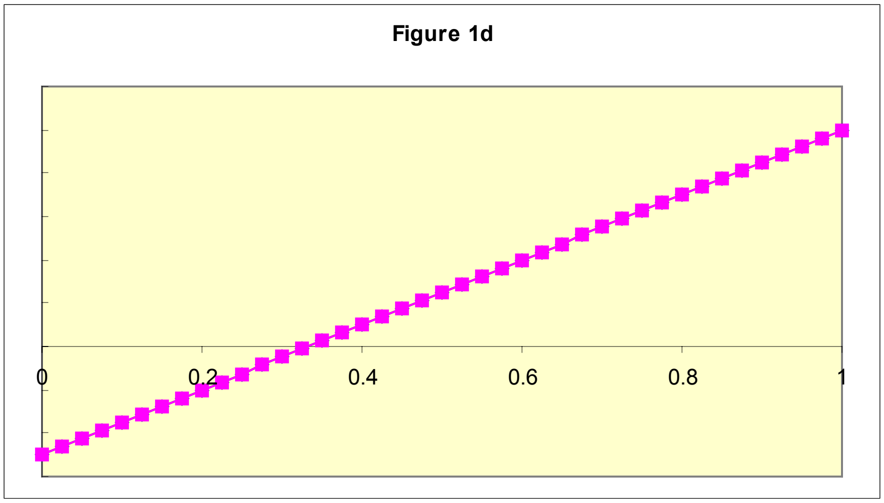
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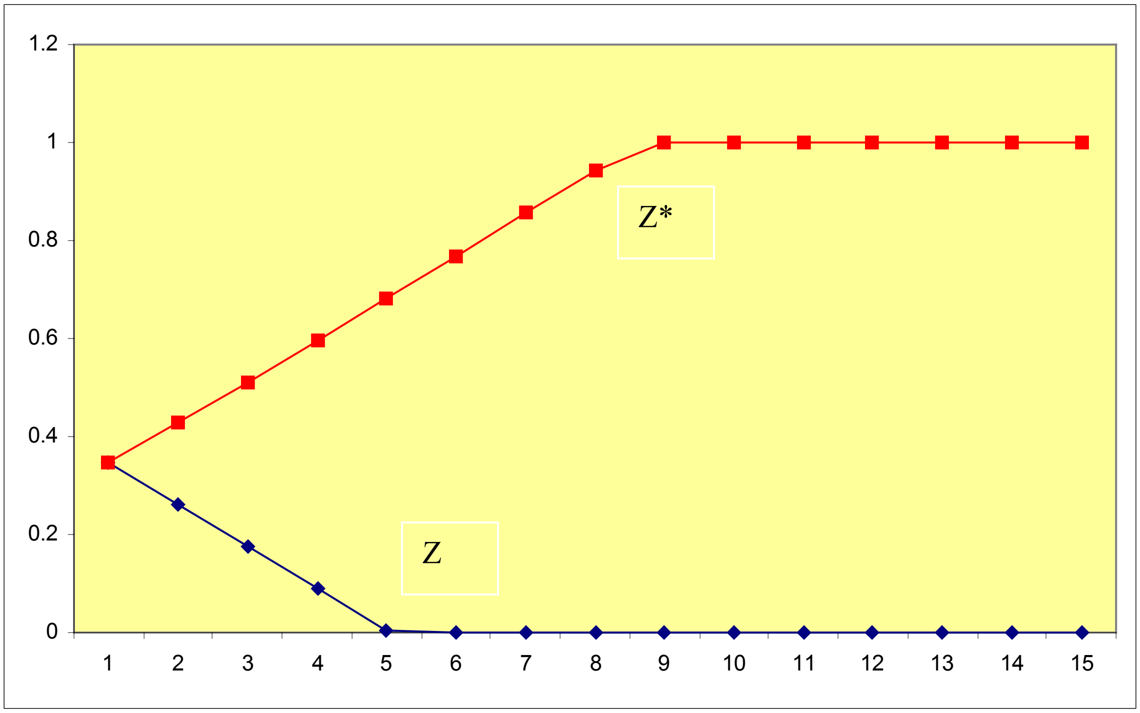

Figure 2

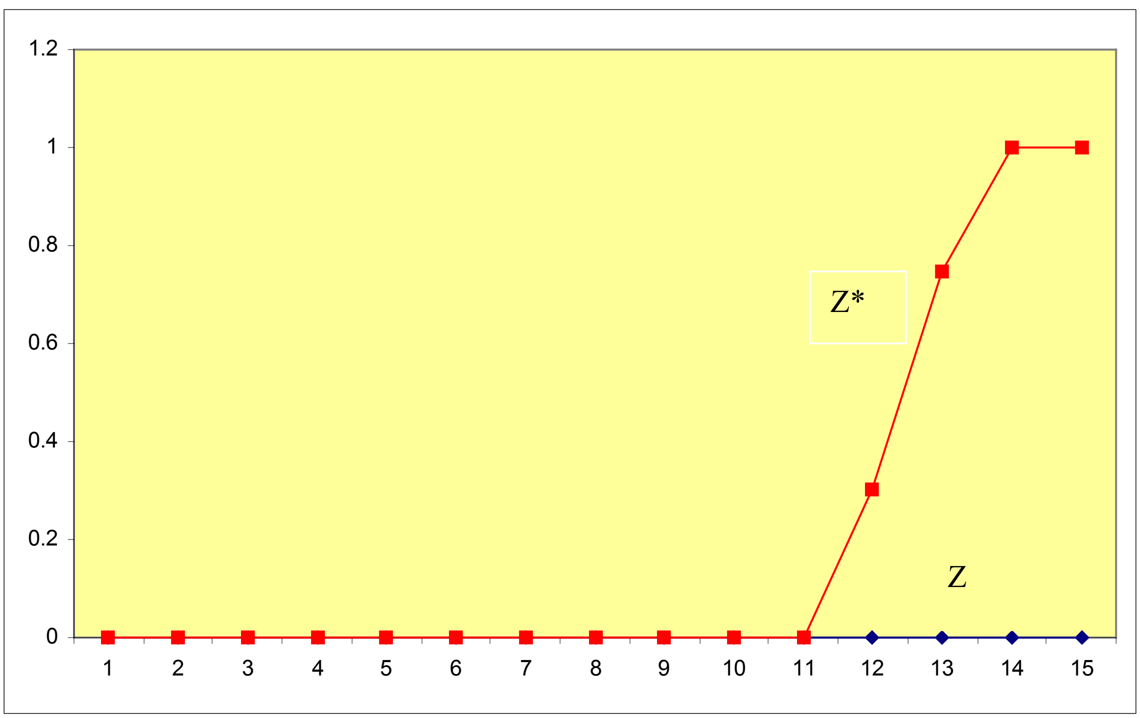

Figure 3