

# **How Big Should the State Pension Be?**

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## 1. Introduction

How big should the state pension be? Should it be provided to everyone at the same level? Should it be clawed back for the better off, and if so, how quickly? Or should it be positively related to earnings? And how should state pensions be financed?

This paper is an attempt to answer these questions in the simplest possible framework that respects three key principles: one, that earning abilities differ across the population; two, that first-best (ability) taxation is precluded by incentive compatibility conditions; and three, that a period of retirement follows a period of work for everyone. The first two principles lead to the optimum distortionary taxation of wage earnings, as in Mirrlees (1971). The third implies the existence of a crucial time dimension. So the analysis parts company with standard optimum income taxation models.

The final question – who should finance pensions – has obvious intragenerational implications. It may also have intergenerational ones, too. Should pensions be fully funded, or, as in much of continental Europe, largely financed by transfers from the current young? This issue is now a pressing one for many EU countries, and has provoked much excellent applied work recently (such as Boldrin et al (1999) and Miles and Timmermann (1999) and other papers discussed therein).

Flemming (1977) is the only major paper known to the author that attempts to place optimum income taxation issues squarely in an overlapping generations model, but the emphasis there is largely on the transition towards a fully funded optimal system, not on the

details of redistribution within generations. Diamond and Mirrlees (1978) offer a very penetrating study of the public policy implications of a random, unobservable collapse of earning ability due to ill health, in a variety of different time settings, but there is no continuum of abilities. Diamond (1977) provides a broad survey of relevant arguments and U.S. institutional arrangements, but no formal model as such.

Section 2 sets out the model's assumptions, while the simplest universal pension, financed by a proportional tax on earnings, is examined in section 3. Section 4 explores a pension supplement proportional to earnings and financed by an additional tax on those earnings, while a pension opt-out is investigated in section 5. Section 6 extends this to a general, non-linear pension-tax setup. The impact pension systems may have on steady state factor prices and other variables is the subject of section 7, and the key difference between fully funded and pay as you go systems is highlighted there. Some qualifications are presented, and conclusions gathered, in section 8.

## **2. Assumptions**

Providing an answer to our questions requires assumptions to be made about eleven sets of issues or variables. These are: (a) the extent of altruism; (b) the size of any exogenous government spending commitment; (c) the size of any government transfers to the young; (d) the availability and character of private-sector pensions; (e) demographic variables; (f) the dispersion of earning abilities; (g) agents' ability to borrow; (h) factor prices; (i) individuals' utility functions; (j) the social welfare function; and (k) the nature of state pensions and the taxes levied to finance it.

It is simplest to assume as follows. For (a), (b) and (c), assume none. For (d), allow everyone access to private pensions at competitive rates; as for (e), assume that there is no growth in population, and that everyone works when young and not in the second and final period of life. On (f), it is easiest to assume that the ability to earn is uniform on the interval (0,1), and, on (g), that this is completely unrestricted, including the opportunity for the young to borrow against their future pension if they wish. [These assumptions on (d) and (g) incidentally remove by fiat some of the reasons traditionally urged for state pensions, as for example in Diamond (1977), but they offer a very useful starting point for handling other issues]. On (h), let us assume that the rate of interest  $r$ , and that the wage rate for one unit of top-ability labour,  $w$ , are given, and the latter case unitary. Our assumptions on (d) and (g)

For (i) let the utility function of individual  $i$  be represented by

$$U_i = \ln c_i + \beta \ln C_i - h_i \quad (1)$$

where  $c_i$  and  $C_i$  denote  $i$ 's levels of consumption in youth and old age,  $h_i$  is his or her hours of work in youth, and  $\beta$  is a positive parameter reflecting patience. On (j), the natural starting point is the Roberts (1980) generalization

$$SW = \xi M + (1 - \xi)A \quad (2)$$

where  $A$  and  $M$  represent the steady-state values of average and minimum utility. In (2),  $\xi$  is a parameter on the interval (0,1) that captures society's (or the planner's) inequality aversion: it is unitary if social welfare is Rawlsian, and zero if Benthamite utilitarian.

Lastly, on (k). Let us begin by assuming that the state pension is a universal benefit paid to all, and financed by a proportional tax on earnings. This assumption will be relaxed later.

### **3. Universal State Pension, Proportional Labour Earnings Tax**

Given perfect capital markets, earnings-only taxation at a marginal rate  $t$ , no borrowing restriction and a universal state pension,  $b$ , every agent's intertemporal budget constraint will be

$$C - b - (1+r)(vh(1-t) - c) = 0 \quad (3)$$

Each will also face a constraint prohibiting a negative supply of labour

$$h \geq 0 \quad (4)$$

Maximizing (1) with respect to  $h$ ,  $c$  and  $C$ , subject to (3) and (4), implies (5) and (6):

$$\frac{h}{1+\beta} = \text{Max}[0, 1 - u/v] \quad (5)$$

$$\frac{c}{1-t} = \text{Max}[u, v] = C/\beta(1+r)(1-t) \quad (6)$$

where  $u = b/(1+r)(1-t)(1+\beta)$ ;  $1 - u$  is the labour force participation rate among youths, given the uniform (0,1) distribution of earning ability  $v$  and the assumption that the market price of top-ability labour is unitary.

Substitution of (5) and (6) into (1) gives indirect utility

$$U(1+\beta)^{-1} = \ln(1-t) + \beta(1+\beta)^{-1} \ln \beta(1+r) + \text{Max}[\ln u, \ln v - 1 + u/v] \quad (7)$$

Those with ability less than or equal to  $u$  supply no labour in youth, and finance their consumption in both periods by the old-age state pension,  $b$  (against which some borrowing in youth is incurred).

Those with ability above  $u$  work for some fraction of their time-endowment in youth, and consume at higher levels in both periods of life. The higher the wage rate, the greater the labour supply  $h$ , and the greater both  $c$  and  $C$  will be. Those with a wage rate of  $u(1+\beta)/\beta$

will neither save nor dissave; those with ability above (below) this level will save (dissave). So the population of youths consists of three groups: borrowing non-participants in the labour force, dissaving workers and workers who save.

The financing constraint confronting government, under fully-funded arrangements (and recalling that direct government spending and transfers to youths are both assumed to be zero) will be

$$\frac{b}{1+r} = t \int v h(v) \phi(v) dv \quad (8)$$

where  $\phi(v)$  is the p.d.f. for ability. Given that  $v$  is uniform on  $(0,1)$  and that labour supply is given by (5), (8) reduces to

$$\frac{b}{1+r} = \frac{t}{2} (1+\beta)(1-u)^2 \quad (9)$$

Furthermore, the unemployment-pension-tax rate relationship implied by (5) and (6),  $u = b/(1+r)(1-t)(1+\beta)$ , allows us to solve for the tax rate  $t$  as a function of  $u$  alone:

$$t = 2u(1+u^2)^{-1} \quad (10)$$

Under these conditions, the Roberts Social Welfare Function may be written

$$\frac{\psi - \beta \ln \beta (1+r)}{1+\beta} = \ln \frac{(1-u)^2}{1+u^2} + (\xi - u(1-\xi)) \ln u - 2(1-u)(1-\xi) \quad (11)$$

The optimal value of  $u$  may now be found as an implicit function of  $\xi$ ;

$$\xi = \frac{1 + \ln u + 2 \frac{1+u}{(1-u)(1+u^2)}}{1 + \ln u + \frac{1}{u}} \quad (12)$$

For example, when  $\xi = 1$  so that  $\psi$  is Rawlsian,  $u$  satisfies

$$(1-u)^3 - 4u^2 = 0 \quad (13)$$

Here,  $u \sim .2956$ , with  $t$  (the wages tax) 54.4%, and the size of the state pension is about  $0.1349/(1+r)(1+\beta)$ . If  $\xi = 0$ , on the other hand,  $\psi$  is Benthamite, and  $u$  obeys

$$2(1+u) - (1-u)(1+u^2)(1+\ln u) = 0 \quad (14)$$

which solves uniquely at  $u \sim .173$ . Here the tax rate is nearly 33.6% and the universal state pension is somewhat lower, at about  $.1149/(1+r)(1+\beta)$ .

#### 4. Adding a Proportion of Earnings to the State Pension

Let us now amend the setup in the previous section by introducing an earnings-related element in the state pension. Up to now everyone received the basic pension  $b$  in old age. Now, the State gives a pensioner  $b + \varepsilon vh$ , let us say, if she earned  $vh$  in her youth. We are now in a position to contrast the Continental European system (large  $\varepsilon$  with little or no  $b$ ) with the ‘‘Anglo-Saxon’’ pension system (where  $\varepsilon$  is zero or at least inapplicable to most workers). The Continental system is inherited from Bismark’s social reforms in late nineteenth century Germany, and relies, essentially, on pensions and pension contributions that are both proportional to labour earnings. The Anglo-Saxon system derives from the welfare state proposals of William Beveridge in the early 1940s, and the Lloyd George system that preceded it by thirty five years, which (in the simplest form) made both pensions and pension contributions lump sums common to all. This implies a modified budget constraint for the individual:

$$C - b - \varepsilon vh - (1+r)(vh(1-t) - c) = 0 \quad (3')$$

Maximizing (1) subject to (3') and (4) leaves the value of  $h$  unchanged. But  $c$  now changes to

$$c = w \left( 1 - t + \frac{\varepsilon}{1+r} \right) \quad (4')$$

The government’s financing constraint becomes

$$t - \frac{\varepsilon}{1+r} = \frac{2u}{1+u^2} \quad (10')$$

in place of (10). The LHS of (10') represents the true marginal rate of tax on earnings, discounted to present value. It follows immediately that  $\varepsilon$  has no role to play in determining the optimum basic pension, or the labour-force participation rate  $1 - u$ , no matter how minimum and mean utilities are weighted in social welfare. The term  $\varepsilon$  here is wholly ornamental, given that the apparent rate of earnings taxation,  $t$ , is raised to cover the element  $\frac{\varepsilon}{1+r}$ . In this event, it has no bearing upon labour supply, or consumption in either period of life. Essentially, whatever the State's earnings-related pension seeks to achieve is undone, one for one, by offsetting changes in private pension contributions (or borrowing). Moreover, this result is quite general, and not confined to the particular utility function (1).

The case for an earnings-related element in pensions, if there is one, can only be based on other considerations which our model has excluded. Examples could include non-optimizing, tax-illusion, or imperfections in the private pension system (higher administrative costs than in the state system, or monopolistic overpricing).

In the present context, an earnings-related element in the state pension is not a source of inefficiency. It is an irrelevance. It exerts no behavioural effects if agents perceive their true discounted marginal rate of taxation on earnings; and it has no redistributive effect, either. Essentially, an (additional) pension proportional to earnings and financed by a proportional (additional) tax on earnings merely replicates what agents would (some of) want to do by way of providing for their old age themselves. It is neutral in its distributive effects.

## 5. A Basic State Pension, with a Private Opt-out



If section 4 has established that it is pointless to give workers an earnings-related supplement on top of the basic state pension, because they can do this for themselves, what about allowing the better off to opt out of the basic state pension in return for a lower tax rate? This is the question addressed in the present section.

Let us imagine that a worker can opt between (i) the basic state pension,  $b$ , with a tax rate  $t$  on labour earnings, and (ii) no state pension, but a lower income tax rate,  $s$ .

Those choosing option (ii) set  $c$  and  $h$  to maximize

$$\ln c + \beta \ln(1+r)[vh(1-s)-c] - h \quad (15)$$

yielding labour supply of  $1 + \beta$ , and consumption in youth of  $v(1-s)$ . The indirect utility derived from this choice,  $V$  call it, will be

$$V = (1 + \beta) \ln v(1-s) + \beta \ln \beta (1+r) - 1 - \beta \quad (16)$$

A comparison of (16) with (7), which is what the agent will achieve under option (i), implies the following condition of indifference

$$\ln \frac{1-s}{1-t} = \frac{u}{v^*} \quad (17)$$

where  $v^*$  is the critical ability level for someone indifferent between these two choices. All for whom  $v > v^*$  will opt for (iii); all those for whom  $v^* > v$  will choose (i). Since  $1 > t > s > 0$ ,  $v^* > u$ . Private saving for those opting out will be  $\beta v(1-s)$ .

The finance constraint, under full funding, now becomes

$$\begin{aligned} \frac{bv^*}{1+r} &= s(1+\beta)\int_{v^*}^1 vdv + t(1+\beta)\int_u^{v^*} (v-u)dv \\ &= \frac{(1+\beta)}{2} [s(1-v^{*2}) + t(v^*-u)^2] \end{aligned} \quad (18)$$

The crucial question is, will the introduction of an opt-out prove beneficial?

When social welfare is Rawlsian, answering this question reduces to asking whether

$$u(1-t) = u \frac{[1+u^2 - 2uv^*]}{[1+u^2 + (1-v^{*2}) (e^{u/v^*} - 1)]} \quad (19)$$

reaches its maximum where  $v^* < 1$ , or not. The answer is that minimum utility is highest when  $v^* \sim .7$  and  $u \sim .308$ .

The Rawlsian optimum prescribes a tax rate of about 25.2% for those who opt out, and 51.8% for those remain in the pension system. The top 30% of earners elect for the low tax/no pension combination. This represents a Pareto-improvement over the Rawlsian universal-pension optimum discussed in section 3 (where everyone faced a tax rate of 54.4%).

In the Benthamite optimum, it is again better for some to opt out, than none. Benthamite social welfare is maximized with tax rates of 33.8% for pension qualifiers and 15.6% for those opting out (who constitute 26% of the population of youths and 31.7% of those employed). The unemployment rate is 18%. Everyone is better off in this equilibrium than in the Benthamite optimum with a universal pension.

The means testing of pensions, and of other social security benefits, is of course widespread in practice, and its careful use is often warmly recommended (e.g. by Coulter et al. (1997)).

In the setting above, the means testing is essentially devolved to the individual, who herself decides whether it suits her. This ensures there is no violation of truth telling constraints.

## 6. The Non-Linear Pension

In section 5 there was a basic flat-rate pension from which high earners would prefer to exempt themselves if this gave them an appropriate cut in their rate of tax on earnings. There are two levels of state pension, basic or zero, and two marginal income tax rates, a high one for those who opt for the pension and a lower one for those who do not.

This prompts the question: why stop at two? Why not extend the principle of a state pension, and a marginal rate of tax on earnings, that both fall continuously as earnings rise? The present section is devoted to exploring this issue.

Accordingly, let  $t(v)$  be the marginal tax rate for which a person with ability  $v$  will opt, and  $x(v)$  denote the pre-tax discounted present value of the pension. People will be free to select from the range of possible  $(t, x)$  bundles, and will, we assume, choose the  $(t, x)$  combination that suits them best.

For example, consider a working individual with ability  $v$ . If he or she chooses the  $t(v), x(v)$  bundle, indirect utility will be

$$U = \beta \ln \beta (1+r) + (1+\beta) \left[ \ln v + \ln(1-t(v)) - 1 + \frac{x(v)}{v(1+\beta)} \right] \quad (20)$$

If he or she were to choose the  $t, x$  combinations ideal for those of ability  $v+\varepsilon$  or  $v-\varepsilon$ , indirect utility would be, respectively:

$$U = \beta \ln \beta (1+r) + (1+\beta) \left[ \ln v + \ln(1-t(v+\varepsilon)) - 1 + \frac{x(v+\varepsilon)}{v(1+\beta)} \right] \quad (21)$$

$$\beta \ln \beta (1+r) + (1+\beta) \left[ \ln v + \ln(1-t(v-\varepsilon)) - 1 + \frac{x(v-\varepsilon)}{v(1+\beta)} \right] \quad (22)$$

Let us assume that  $\varepsilon$  is small enough that our individual is effectively indifferent between the  $t(v), x(v)$  and  $t(v+\varepsilon), x(v+\varepsilon)$  combinations. In that case, the right hand sides of (20) and (21) are vanishingly close, and

$$\ln(1-t(v+\varepsilon)) \sim \ln(1-t(v)) + \frac{x(v) - x(v+\varepsilon)}{v(1+\beta)} \quad (23)$$

Let us further assume that the schedule of marginal tax rates displays the functional form

$$[1-u] \ln(1-t(v)) = (1-v) \ln(1-t(u)) + (v-u) \ln(1-t(1)) \quad (24)$$

This last equation stipulates that the marginal income retention rate,  $1-t(v)$ , is a geometric mean of the income retention rates for top and bottom earners,  $(1-t(1))$  and  $(1-t(u))$  respectively. It allows for the possibility of a single tax rate (so that  $t(1)=t(u)$ ) as one extreme, but generalizes beyond it to a wide range of other possibilities. The functional form of (24) has the further merit of conforming to the character of (23) and yielding the following simple relationship between  $x(v)$  and  $x(u)$ :

$$x(v) \sim x(u) + (1+\beta) \frac{1-\delta}{1-u} \frac{v^2 - u^2}{2} \ln(1-t(u)) \quad (25)$$

where  $\delta \equiv \ln(1-t(u)) / \ln(1-t(1))$ .

The finance constraint for fully funded pensions, in the absence of direct government spending or transfers to youths, implies

$$u(1-t(u))x(u) + \int_u^v x(v)dv = (1+\beta) \int_u^v vt(v)dv \quad (26)$$

Given (25), this reduces to the condition

$$\frac{[1-ut(u)]x(u)}{1+\beta} + \frac{1+2u}{6}(1-u)^2 D = \frac{1-u^2}{2} + \frac{[1-t(u)]^\delta (1+D) - (1-t(u))[1+uD]}{D^2} \quad (27)$$

where  $D \equiv \frac{1-\delta}{1-u} \ln(1-t(u))$ .

If social welfare is Rawlsian, the task is to maximize  $u(1-t(u))$ , since minimum utility equals  $\beta \ln \beta (1+r) + (1+\beta) \ln u(1-t(u))$ , subject to (27). The instruments here are three:  $\delta$ , the elasticity of the bottom to the top marginal retention rate; the labour force non-participation rate,  $u$ ; and  $t(u)$ , the bottom tax rate. There is a single optimum for this problem, with  $t(u)$  at 52.5%,  $u$  at nearly 30.35%, and  $\delta$  at 0.28. The top tax rate is just over 18.8%. and all agents' utilities are higher in this equilibrium than in the universal flat-rate pension Rawlsian optimum in section 3. The base pension is nearly 6.9% higher under the non-linear Rawlsian optimum; top earners receive a slightly negative pension (so that  $x(1)$  is nearly -32% of  $x(u)$ ).

With Benthamite social welfare, the top rate of tax,  $t(1)$ , is zero, so that  $\delta = 0$ . Social welfare reduces to

$$\beta \ln \beta (1+r) + (1+\beta) \left[ -2(1-u) + \frac{3}{4}(1+u) \ln(1-t(u)) - u \ln u + \frac{u^2}{2} \frac{\ln u}{1-u} \ln(1-t(u)) \right] \quad (28)$$

which is to be maximized, with respect to  $u$  and  $t(u)$ , subject to the finance constraint (27).

Once  $\delta = 0$ , (27) simplifies to

$$\frac{[1-ut(u)]x(u)}{1+\beta} + \frac{1+2u}{6}(1-u) \ln(1-t(u)) = \frac{1-u^2}{2} + \frac{t(u) + D(1-u(1-t(u)))}{D^2} \quad (29)$$

with  $D \equiv \frac{\ln(1-t(u))}{1-u}$ .

The solution to this problem yields a bottom tax rate,  $t(u)$ , of 52.9%, with a labour force non-participation rate,  $u$ , of 30.28%. The maximum state pension is nearly 99% as large as in the Rawlsian case. The expression in square brackets in (28), the core term for Benthamite social welfare, rises to  $-1.709$  (as against  $-1.717$  in the opt-out Benthamite optimum in section 5).

Although top earners, with ability level 1, benefit from the absence of a marginal tax rate, they pay a considerably bigger lump-sum than in the corresponding Rawlsian optimum. Their lump sum tax is 131% of the pension paid to bottom earners (and labour force non-participants). For top earners,  $\frac{U - \beta \ln \beta (1+r)}{1+\beta}$  is  $-1.188$ , as against  $-1.236$  under the universal pension and  $-1.17$  under the opt-out Benthamite optima respectively.

The really striking feature of our Benthamite optimum results here is how close the maximum pension  $x(u)$  and the unemployment rate  $u$  are to their values in the Rawlsian optimum. That is in sharp contrast with the standard static optimum taxation framework, where optimum tax schedules, and any resulting optimum unemployment, are highly sensitive to the shape of the social welfare function. Our example, at least, suggests that political controversy about the weightings between mean and minimum utility in the Roberts Social Welfare function should have little bearing upon some aspects of the pension redistribution system.

## **7 Steady State Factor Prices, and the Difference Between Fully Funded and Pay As You Go Pension Systems**

So far, factor prices (the rate of interest, and the marginal product of labour of unit ability) have been assumed given. Does redistribution of the various types examined above affect the steady state value of these prices? This is an important question: If  $r$  and  $w$  were to respond

to the pension-taxation system, our analysis would only have been partial, and the responses of those variables would have to be taken into account.

Flemming (1977) was the first to ask and answer this question. He provides conditions under which  $w$  and  $r$  are, in fact, invariant to fiscal redistribution through pensions. In the context of the model of this paper, with exogenous fertility, and where there is no government spending and pensions are fully funded, agents work for only the first period of life, have common preferences, and never inherit anything, and the production function displays constant returns to scale and other usual properties, Fleming's condition for redistribution – invariance of  $w$  and  $r$  in the steady state – is that lifetime utility may be written in the separable  $U$  form  $(F(c,C), h)$  with  $F$  homothetic. In this case the ratio of  $C$  to  $c$  is independent of  $h$  and varies only with the rate of interest. Now the mean value of  $C$ , will equal  $\bar{k}_{+1}(1+r_{+1})(1+n)$ , where  $\bar{k}_{+1}$  is the mean value of capital per youth in the succeeding period. From this it follows that  $\bar{c}$ , the mean value of consumption in youth, is proportional to  $\bar{k}_{+1}$  and varies only with  $r_{+1}$ , and the same will also be true of  $\overline{wh} - \bar{c}$ , the gross-of-tax level of total saving (public and private) per youth. Our utility function (1) satisfies the additive separability and  $F$  – homotheticity properties.

Factor-price-invariance to redistribution does not hold, however, if the pension system is less than fully funded. In that event, some part, at least, of the pension(s) paid to the current old are financed by levies on the current working young. Total saving per youth is now less than  $\overline{wh} - \bar{c}$ , and redistribution does affect the steady-state value of the aggregate capital-labour ratio on which factor prices depend.

The key difference between fully funded (FF) and pay as you go (PAYG) pension systems may be illustrated thus. Consider a simplified representative – agent version of our model where pensions are fully funded and completely Bismarckian (so that pensions and pension contributions are proportional to labour income in youth). Let the contribution rate be  $\theta$ , and let the pension paid in old age equal

$$p = \theta wh(1 + r_{+1}); \text{ agents maximising lifetime utility (1) will set } h = 1 + \beta, c = w \text{ and}$$

$C = \beta w(1 + r_{+1})$ . State pensions of this kind will be anticipated and fully offset by private saving.

Let there be an Cobb-Douglas, constant returns to scale aggregate production function, where  $T$  denotes the (stationary) level of technology and  $\gamma$  is the share of income accruing, given perfect competition, to the owners of capital.

The resulting steady state value of capital per youth,  $k^*$ , in the FF system will be

$$k^* (1 + \beta) [\beta T \frac{1 - \gamma}{1 + n}]^{\frac{1}{1 - \gamma}} \quad (30)$$

The associated values of  $w$  and  $r$  are

$$\left. \begin{aligned} w^* &= (1 - \gamma) T [\beta T \frac{1 - \gamma}{1 + n}]^{\frac{\gamma}{1 - \gamma}} \\ r^* &= \frac{\gamma}{\beta} \frac{1 + n}{1 - \gamma} \end{aligned} \right\} \quad (31)$$

By contrast, a fully Bismarckian PAYG system operates like this. The earnings of the current young,  $wh$  per head, are taxed at rate  $\theta$  to pay the pensions of the current old.

So

$$p = \theta w_{+1} h_{+1} (1 + n) \quad (32)$$



Since there are  $1+n$  old people per youth. Let us assume, however, that the youth believes that her old age pension will be

$$p = \theta wh(1+r_{+1})(1-\delta) \quad (33)$$

Here,  $\delta$  is a “seepage” parameter that reconciles (32) and (33). In the steady state, (32) and (33) together imply

$$1-\delta = \frac{1+n}{1+r_{+1}} \quad (34)$$

So that  $\delta$  vanishes if and only if the Golden Rule condition for maximum steady-state consumption is satisfied – the “biological interest rate rule”,  $n = r$ . When the economy is dynamically efficient,  $r \geq n$  and  $\delta \geq 0$ .

The youth still chooses a labour supply of  $1+\beta$  under this PAYG system, but consumption in youth falls to  $w(1-\delta\theta)$ . Steady state capital per youth falls to

$$\tilde{k} = (1+\beta) \left[ \beta T \frac{1-\gamma}{1+n} (1-\delta\theta) \right]^{\frac{1}{1-\gamma}} \quad (35)$$

and the associated factor prices will be

$$\tilde{w} = (1-\gamma) T \left[ \beta T \frac{1-\gamma}{1+n} (1-\delta\theta) \right]^{\frac{\gamma}{1-\gamma}} \quad \} \quad (36)$$

$$\tilde{r} = \frac{\gamma}{\beta} \frac{1+n}{(1-\gamma)(1-\delta\theta)} \quad \}$$

Meanwhile, (34) and (36) allow us to express  $\theta$  as an explicit function of  $\delta$  :

$$\theta = \frac{1}{\delta} \left[ 1 - \frac{1-\delta}{\delta+n} \frac{\gamma}{\beta} \frac{1+n}{1-\gamma} \right] \quad (37)$$

It is clear from (36) that a PAYG pension system will affect steady state factor prices. Given dynamic efficiency ( $r \geq n$ ), the lowering of  $\tilde{k}$  can only lower  $\tilde{w}$  and increase  $\tilde{r}$  whenever  $\theta \geq 0$ .

Steady state utility, it can easily be shown, is lower in a PAYG pension system than a FF one. All that matters for this result is that  $r$  should equal  $n$ . A PAYG system is a fine method of removing unwanted surplus capital, but dynamic efficiency will imply that there is none. This does not mean, however, that the transition from a PAYG to a FF system is painless. In fact, as Boldrin et al (1999) and Miles and Timmermann (1999) both emphasize, at least one generation has to suffer if this transition is made.

Several points arise here. One, if the planner's intertemporal social welfare maximand is Rawlsian between the generations, any inherited PAYG system should in fact be retained in full. Lexicographic preferences of this kind make any burden on a transition generation that suffers quite intolerable, because the gains of a better off posterity can never compensate. Second, if the planner's function is less than infinitely averse to inequality aversion across the generations, and displays no discounting of utilities, the transition to a FF system (granted dynamic efficiency) should always be done, in full and starting at once. Third, pure time discounting of utilities by the planner will typically imply an optimum steady state interest rate above the rate of population growth, and hence, possibly, that an incompletely funded pension system may be preferable to a fully funded one. Fourth, the optimum transition speed rises, the lower the social rate of relative inequality aversion across the generations.

The Roberts social welfare function, (2), is a normative construction that is somewhat distanced from the reality of public choice. One way of extending it (as in Sinclair (1997)) is

to incorporate a term reflecting the utility of the median individual (whose earning ability will be  $\frac{1}{2}$  under our assumptions), alongside mean and minimum utilities. In any referendum, the median individual could become a dictator; with the symmetric ability distribution assumed, she will not favour any redistribution, and the greater the planner's weight on her utility, the lower the pensions, tax schedules and resulting unemployment we might expect. In the present context of a transition from a PAYG to a FF pension system, the relevance of voting is particularly important. Posterity must wait to vote; a referendum is limited, presumably, to the young and old alive at the time. Unfortunately, unless a constitution specifically prohibits this (and how could there be democratic approval for it?), any referendum would be expected to vote to delay the implementation of the transition until those current alive had died. So the transition might never actually come about.

## **8 Qualifications and Conclusions**

The model presented in this paper has abstracted from numerous real world complications. Capital market imperfections (preventing individuals from saving for themselves on competitive terms) and deficient foresight are advanced as reasons for a public pension system (Diamond (1977)), and yet they play no role here. There is no uncertainty, no direct government spending, no international capital transactions, no intergenerational altruism, and no allowance, in the simple 2 period lives setup, for government transfers to the very young to finance their education. As Boldrin et al (1999) stress, early educational expenses financed by taxes on current labour earnings go some way to offset the effects of PAYG pensions to the current old financed from the same source, reducing the steady state damage done to capital and welfare in a dynamically efficient economy. With demographics now giving European Union pensioners an average span of over fifteen years in retirement, however,

with a span now rising by some 28 days each year, and state pensions costing considerably more than schooling in most countries, the magnitude of this offset may be rather modest.

Despite these and other reservations about the assumptions underlying the model, the paper has provided some clear cut answers to the key questions posed at the outset. If redistribution within generations is warranted, by diminishing marginal utilities of consumption, inequality aversion or both, progressivity in the pension-taxation system is desirable. Ideally a non-linear setup is best, with pensions and marginal earnings tax rates falling over the range of income, and a surprising degree of uniformity between Benthamite and Rawlsian optima. So long as the system is fully funded, our assumptions have ensured that there is no impact on long run factor prices. A fully funded system dominates a pay as you go one, granted dynamic efficiency and finite intergenerational inequality aversion. But political considerations make the transition fraught with difficulties. It may never even get off the ground.

An ideal pension system should attempt to strike the best balance between five desiderata:

- (i) allocative efficiency in the trade-off between leisure or labour and consumption within each generation;
- (ii) justice, or at least distributive efficiency, in the distribution of consumption between different individuals within each generation;
- (iii) intertemporal efficiency in consumption allocation between different periods of life within each generation;
- (iv) efficiency in consumption across generations;
- (v) justice in consumption across generations.

A purely Bismarckian or purely Beveridge pension system, with pensions and pension contributions proportional to earnings in the first case, and lump sums in the latter, achieve (i) but violate (ii). An optimum pension-contributions system, that achieves the best feasible balance between them while recognizing the impossibility of ability taxes, has been explored in some detail. This system respects (iii), by not taxing interest income for pensioners (although in other settings, such as Diamond and Mirrlees' (1978) random ill health model, this may be undesirable). And there has been some initial scrutiny of the issues posed by (iv) and (v) in the context of PAYG and FF systems.

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