Trend Inflation and the New Keynesian Phillips Curve

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Abstract

We investigate the nature of structural breaks in the dynamics of U.S. inflation, in the presence of a stochastic trend in inflation. This is done by deriving and estimating unobserved component trend-cycle models of inflation that are consistent with the New Keynesian Phillips Curve. Our derivation suggests that the transitory component of inflation is a function of real economic activity and a potentially serially correlated component, with the latter signifying the importance of the backward-looking component. Our empirical results suggest that a forward-looking Phillips curve can provide a good fit to postwar U.S. inflation data, except for the period of the Great Inflation in the 1970s. These results imply that the relatively poor performance of recursive and rolling AR forecasts of inflation in the last decades results from a disappearance of the backward-looking component in the Phillips curve in the early 1980s. Furthermore, the models presented in this paper display good in-sample and out-of-sample forecasting abilities relative to benchmark models in the literature that forecast inflation well.

Keywords: Inflation, Inflation forecasting, New Keynesian Phillips Curve, Trend inflation, Unobserved components model.
JEL Classification: E31, E32, E37.

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This version: 30 October 2011

1. Introduction

The New Keynesian Phillips Curves (NKPC) are generally estimated under the assumption that inflation is a stationary process. However, one cannot rule out the possibility that inflation has a unit root. For example, dating back to at least Nelson and Schwert (1977), there is substantial empirical evidence that U.S. inflation has a unit root (Ireland, 1999; Bai and Ng, 2004; Henry and Shields, 2004).

Recently, authors have incorporated the possibility of a stochastic trend in their models of inflation. For example, Cogley and Sbordone (2008) derive a version of the NKPC that incorporates a time-varying inflation trend and examine whether it explains the dynamics of inflation. Within a VAR framework with drifting coefficients, Cogley and Sargent (2005) investigate the nature of inflation dynamics and the time-varying long-run trend component of inflation. Based on unobserved component trend-cycle models of inflation with heteroscedastic shocks, Stock and Watson (2007) and Kang et al. (2009) investigate the evolving nature of inflation dynamics. Harvey (2008) specifies a bivariate unobserved components model for inflation and real output based on a reduced form Phillips curve, with the lagged inflation terms replaced by a random walk. Lee and Nelson (2007), in particular, derive an unobserved components trend-cycle model for inflation and unemployment, as implied by the NKPC. They show that the slope of the implied Phillips curve will depend critically on the horizon of the forward-looking inflation expectation.

In this paper, we investigate the nature of structural breaks in the dy-

namics of U.S. inflation, in the presence of a unit root. This is done by using the unobserved components trend-cycle models of inflation as implied by the NKPC. We first derive the transitory component of inflation to be a function of real economic activity and a potentially serially correlated unobserved component, which signifies the importance of the backward-looking component in the NKPC. We then incorporate the possibility that the autoregressive coefficients in the transitory component as well as the variances of the shocks to inflation may evolve over time. Within the framework outlined above, we hope to re-evaluate the significance or the temporal significance of the backward-looking component in the NKPC characterizing the dynamics of U.S. inflation. The forecasting performances of the proposed models are also evaluated.

The contents of this paper are organized as follows. Section 2 outlines the model specifications. Section 3 presents and discusses the estimation of the proposed models. The forecasting performances of the proposed models are presented in Section 4. Section 5 provides a summary and conclusion.

2. Model Specification

Consider the following New Keynesian Phillips curve:

$$\pi_t = (1-\alpha)E_t(\pi_{t+1}) + kx_t + \alpha\pi_{t-1}, \tag{1}$$

$$0 \leq \alpha \leq 1$$

where $E_t(.)$ refers to expectation formed conditional on information up to t, and x_t is the output gap. By rewriting equation (1), we have:

$$\pi_t = E_t(\pi_{t+1}) + kx_t + z_t^*, \tag{2}$$

where $z_t^* = \alpha(\pi_{t-1} - E_t(\pi_{t+1}))$. By iterating equation (2) in the forward direction, we have:

$$\pi_t = E_t(\pi_\infty) + k\sum_{j=0}^{\infty} E_t(x_{t+j}) + \tilde{z}_t \tag{3}$$

where $\tilde{z}_t = \sum_{j=0}^{\infty} E_t(z_{t+j}^*)$. Note that, with $\alpha = 0$, we have a purely forward-looking Phillips curve. With $\alpha \neq 0$, however, we have a hybrid Phillips curve and the $\tilde{z}_t$ term in equation (3) would be serially correlated[1].

Lee and Nelson (2007) estimate a version of the Phillips curve based on equation (3) with $\alpha = 0$, and they show that the slope of the Phillips curve critically depends on the horizon of the forward-looking expectation given that the dynamics of x_t is persistent. For example, by assuming that x_t follows an AR(1) process with the persistence parameter ϕ, they consider the following expectation of π_{t+J} conditional on information up to t:

$$E_t(\pi_{t+J}) = E_t(\pi_\infty) + k^* x_t, \tag{4}$$

[1]See for examples, Fuhrer and Moore (1995) and Gali and Gertler (1999) for analyses and discussions related to the forward-looking and hybrid versions of the NKPC.

where $k^* = k \times \frac{\phi^J}{1-\phi}$. Equation (4), combined with equation (3) results in the following Phillips curve which depends upon the J-step-ahead forecast of inflation:

$$\pi_t = E_t(\pi_{t+J}) + \tilde{k}x_t + \tilde{z}_t, \tag{5}$$

where the slope of the Phillips curve $\tilde{k} = k\frac{1-\phi^J}{1-\phi}$ depends upon the forecasting horizon J.

In this paper, we focus on the nature of structural breaks in the New Keynesian Phillips curve that characterizes the U.S. inflation dynamics. Our focus is on the stability of the α coefficient that describes the importance of the backward-looking component in equation (1). We note again that the serial correlation in the $\tilde{z}_t$ term in equation (3) implies a non-zero value for the α coefficient. We thus allow for the possibility that the dynamics of the $\tilde{z}_t$ term may be time-dependent[2]. Furthermore, we consider the heteroscedastic nature of the shocks to the permanent and transitory components of inflation.

In the presence of a unit root in inflation, we note that the $E_t(\pi_\infty)$ term, the limiting forecast of inflation as the forecast horizon goes to infinity, can be interpreted as a the random walk stochastic trend component of inflation. This is in the spirit of Beveridge and Nelson (1981). Furthermore, for an

[2]In the presence of a stochastic trend rate of inflation, we may not be able to estimate the α coefficient. This is why we propose and estimate unobserved components models that are consistent with the NKPC and investigate the importance of the α coefficient by allowing the persistence of z_t to vary over time.

empirical estimation of the model given above, we replace the $E_t(x_{t+j})$ term in equation (3) by $E_{t-1}(x_{t+j})$ to obtain the following representation of the basic NKPC in the absence of structural breaks:

$$\pi_t = \pi_t^* + k \sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) + z_t, \tag{6}$$

$$\pi_t^* = \pi_{t-1}^* + v_t, \tag{7}$$

where $z_t = k(\sum_{j=0}^{\infty} E_t(x_{t+j}) - \sum_{j=0}^{\infty} E_{t-1}(x_{t+j})) + \tilde{z}_t$. Note that the first element of z_t is a function of economic agents' revision on the present value of future output gaps and thus may be correlated with x_t. Furthermore, the z_t term can be potentially serially correlated. However, if $\tilde{z}_t$ in equation (3) is serially uncorrelated, the z_t term in equation (6) is also serially uncorrelated, which implies a purely forward-looking NKPC.

Depending upon whether we assume the output gap (x_t) to be observed or not, we have different model specifications. In what follows, we first consider a model specification in which the output gap is assumed to be observed. We then extend the model to the case of an unobserved output gap, which can be extracted from an unobserved components model of real output.

2.1. A Benchmark Model with Observed Output Gap: Model 1

$$\pi_t = \pi_t^* + k \sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) + z_t, \tag{8}$$

$$\pi_t^* = \pi_{t-1}^* + v_t, \quad v_t \sim i.i.d.N(0, \sigma_v^2), \tag{9}$$

$$z_t = \psi z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0, \sigma_\epsilon^2), \tag{10}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, \quad u_t \sim i.i.d.N(0, \sigma_u^2). \tag{11}$$

In the specification above, we allow z_t to be correlated with the output gap x_t with correlation coefficient $\rho_{\epsilon,u}$.

A state-space representation of the model is given by:

Measurement equation

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ z_t \\ x_t^* \\ x_{t-1}^* \end{bmatrix} + \begin{bmatrix} k \sum_{j=0}^{\infty} E_{t-1}(x_{t+j}^*) \\ 0 \end{bmatrix} \tag{12}$$

$$\left(\Pi_t = H\beta_t + \tilde{Z}_t\right)$$

Transition equation

$$\begin{bmatrix} \pi_t^* \\ z_t \\ x_t^* \\ x_{t-1}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^* \\ z_{t-1} \\ x_{t-1}^* \\ x_{t-2}^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_t \\ \epsilon_t \\ u_t \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} v_t \\ \epsilon_t \\ u_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_v^2 & 0 & 0 \\ 0 & \sigma_\epsilon^2 & \rho_{\epsilon,u}\sigma_\epsilon\sigma_u \\ 0 & \rho_{\epsilon,u}\sigma_\epsilon\sigma_u & \sigma_u^2 \end{bmatrix} \right)$$

$$(\beta_t = F\beta_{t-1} + U_t, \quad U_t \sim i.i.d.N(0, Q))$$

Note that the $\sum_{j=0}^{\infty} E_{t-1}(x_{t+j})$ term in the measurement equation can be calculated as:

$$\sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) = e_3'(I_4 - F)^{-1}\beta_{t|t-1}, \tag{14}$$

where $e_3' = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ and $\beta_{t|t-1} = E(\beta_t|I_{t-1})$.

2.2. A Benchmark Model with Unobserved Output Gap: Model 2

When the output gap x_t is unobserved, we can extract it from the following unobserved components model of real output:

$$y_t = \tau_t + x_t, \tag{15}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, \quad u_t \sim i.i.d.N(0, \sigma_u^2), \tag{16}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t, \quad \eta_t \sim i.i.d.N(0, \sigma_\eta^2), \tag{17}$$

where real output y_t is decomposed into a stochastic trend component τ_t and a cyclical component x_t. This unobserved components model for real output is combined with that for inflation, given below:

$$\pi_t = \pi_t^* + k \sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) + z_t, \tag{18}$$

$$\pi_t^* = \pi_{t-1}^* + v_t, \quad v_t \sim i.i.d.N(0, \sigma_v^2), \tag{19}$$

$$z_t = \psi z_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0, \sigma_\epsilon^2). \tag{20}$$

As before, we allow the shocks to z_t and the output gap x_t to be correlated through a non-zero correlation coefficient $\rho_{\epsilon,u}$.

A state-space representation of the model is given by:

Measurement equation

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ z_t \\ x_t \\ x_{t-1} \\ \tau_t \end{bmatrix} + \begin{bmatrix} k \sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) \\ 0 \end{bmatrix} \tag{21}$$

$$\left(\Pi_t = H\beta_t + \tilde{Z}_t\right)$$

Transition equation

$$\begin{bmatrix} \pi_t^* \\ z_t \\ x_t \\ x_{t-1} \\ \tau_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \psi & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^* \\ z_{t-1} \\ x_{t-1} \\ x_{t-2} \\ \tau_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_t \\ \epsilon_t \\ u_t \\ \eta_t \end{bmatrix}, \tag{22}$$

$$\begin{bmatrix} v_t \\ \epsilon_t \\ u_t \\ \eta_t \end{bmatrix} \sim i.i.d.N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 \\ 0 & \sigma_\epsilon^2 & \rho_{\epsilon,u}\sigma_\epsilon\sigma_u & 0 \\ 0 & \rho_{\epsilon,u}\sigma_\epsilon\sigma_u & \sigma_u^2 & 0 \\ 0 & 0 & 0 & \sigma_\eta \end{bmatrix} \right)$$

$$(\beta_t = \tilde{\mu} + F\beta_{t-1} + U_t, \quad U_t \sim i.i.d.N(0, Q))$$

Note that the $\sum_{j=0}^{\infty} E_{t-1}(x_{t+j})$ term in the measurement equation can be calculated as:

$$\sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) = \tilde{e}_3'(I_4 - F)^{-1}\beta_{t|t-1}, \tag{23}$$

where $\tilde{e}_3' = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ and $\beta_{t|t-1} = E(\beta_t|I_{t-1})$.

2.3. Incorporating Structural Breaks in the Model

We assume that we know the nature of structural breaks in the output dynamics. That is, we take the productivity slowdown in 1973Q1 and the

Great Moderation in 1984Q2 as stylized facts.

(i) Productivity slowdown in 1973Q1 for real output: structural break in μ for the model with unobserved output gap

$$\mu_{D_{1t}} = (1 - D_{1t})\mu_0 + D_{1t}\mu_1, \quad \mu_1 < \mu_0, \quad D_{1t} = \{0, 1\} \tag{24}$$

$$D_{1t} = \begin{cases} 0, & \text{if } t \leq 1973Q1, \\ 1, & \text{otherwise} \end{cases}$$

(ii) Great Moderation in 1984Q2 for real output: reduction in the variances of the shocks to real output

$$\sigma^2_{\eta,D_{2t}} = (1 - D_{2t})\sigma^2_{\eta,0} + D_{2t}\sigma^2_{\eta,1}, \quad \sigma^2_{\eta,1} < \sigma^2_{\eta,0}, \quad D_{2t} = \{0, 1\} \tag{25}$$

$$\sigma^2_{u,D_{2t}} = (1 - D_{2t})\sigma^2_{u,0} + D_{2t}\sigma^2_{u,1}, \quad \sigma^2_{u,1} < \sigma^2_{u,0}, \quad D_{2t} = \{0, 1\} \tag{26}$$

where

$$D_{2t} = \begin{cases} 0, & \text{if } t \leq 1984Q2, \\ 1, & \text{otherwise} \end{cases}$$

On the contrary, we allow for structural breaks with unknown break dates for the dynamics of the inflation equation. In particular, as suggested by the existing literature, we allow for two endogenous break dates in the inflation dynamics. The following summarizes how we

incorporate structural breaks in the inflation equation:

(iii) Structural breaks for the inflation equation: unknown break dates

$$\theta_{S_t} = S_{1t}\theta_1 + S_{2t}\theta_2 + S_{3t}\theta_3, \tag{27}$$

where

$$S_{jt} = \begin{cases} 1, & \text{if } S_t = j, \\ 0, & \text{otherwise} \end{cases}$$

where $\theta_{S_t} = \{\psi_{S_t}, \sigma^2_{\epsilon,S_t}, \sigma^2_{v,S_t}\}$. Unlike D_{1t} and D_{2t}, S_t is a latent variable. In order to allow for two unknown structural breaks, we assume that S_t follows a first-order Markov-switching process with absorbing states, with the following matrix of transition probabilities:

$$\tilde{P} = \begin{bmatrix} p_{11} & 1-p_{11} & 0 \\ 0 & p_{22} & 1-p_{22} \\ 0 & 0 & 1 \end{bmatrix}, \tag{28}$$

the $(j,i) - th$ element of which is defined as $Pr[S_t = j | S_{t-1} = i]$.

By incorporating the structural breaks outlined above, the complete empirical models can be specified as:

Model 1 with Structural Breaks

$$\pi_t = \pi_t^* + k\sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) + z_t, \tag{29}$$

$$\pi_t^* = \pi_{t-1}^* + v_t, \quad v_t|S_t \sim i.i.d.N(0, \sigma^2_{v,S_t}), \tag{30}$$

$$z_t = \psi_{S_t} z_{t-1} + \epsilon_t, \quad \epsilon_t|S_t \sim i.i.d.N(0, \sigma^2_{\epsilon,S_t}), \tag{31}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, \quad u_t|D_{2t} \sim i.i.d.N(0, \sigma^2_{u,D_{2t}}), \tag{32}$$

where $cov(\epsilon_t, u_t|D_{2t}, S_t) = \rho_{\epsilon,u}\sigma_{u,D_{2t}}\sigma_{\epsilon,S_t}$. All the other covariance terms are assumed to be zero.

Model 2 with Structural Breaks

$$\pi_t = \pi_t^* + k\sum_{j=0}^{\infty} E_{t-1}(x_{t+j}) + z_t, \tag{33}$$

$$\pi_t^* = \pi_{t-1}^* + v_t, \quad v_t|S_t \sim i.i.d.N(0, \sigma^2_{v,S_t}) \tag{34}$$

$$z_t = \psi_{S_t} z_{t-1} + \epsilon_t, \quad \epsilon_t|S_t \sim i.i.d.N(0, \sigma^2_{\epsilon,S_t}), \tag{35}$$

$$y_t = \tau_t + x_t, \tag{36}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, \quad u_t|D_{2t} \sim i.i.d.N(0, \sigma^2_{u,D_{2t}}), \tag{37}$$

$$\tau_t = \mu_{D_{1t}} + \tau_{t-1} + \eta_t, \quad \eta_t|D_{2t} \sim i.i.d.N(0, \sigma^2_{\eta,D_{2t}}), \tag{38}$$

where $cov(\epsilon_t, u_t|D_{2t}, S_t) = \rho_{\epsilon,u}\sigma_{u,D_{2t}}\sigma_{\epsilon,S_t}$. All the other covariance terms are assumed to be zero.

Once cast into state-space form, the two unobserved components models with structural breaks are estimated with the Kim filter (Kim, 1994) by the method of maximum likelihood.

3. Empirical Results

The data set that we use to estimate the unobserved components models contain quarterly U.S. time series data that span 1952Q1 to 2007Q3. The beginning of the sample is chosen to avoid large swings in inflation resulting from the Korean war, and the end of the sample marks the quarter prior to the 2007 recession. For the inflation series, we use the annualized log-difference of the quarterly PCE chain-type price index. As measures of the output gap in Model 1, we use the CBO's output gap measure. In Model 2, we extract measures of the output gap from the annualized log RGDP series. All data is taken primarily from the Federal Reserve Economic Database (FRED).

3.1. Parameter Estimates and Interpretations

The empirical models that we estimate contain structural breaks in the variances of the shocks to both the permanent and transitory components of inflation. We find that the variance of the permanent shock to inflation is constant throughout the postwar period, and therefore we also estimated models with constant trend inflation variance. Based on likelihood tests and diagnostic checks, we select the model in which the variance of the permanent shock to inflation is constant. The results are discussed below.

Table 1 contains the estimated parameters for Model 1 with structural breaks. The parameter estimates have magnitudes that fall within the range reported in the literature, and are estimated with reasonable accuracy. In-

terpretations of the results are as follows. First, the standard deviation of the shock to trend inflation (σ_v) is 0.34, suggesting that permanent shocks to realized inflation are important and trend inflation exhibits significant time variation over the postwar period.

Next, we find that the sum of the AR coefficients that describe the CBO output gap process is 0.92, suggesting that the CBO gap measure is highly persistent. It is also volatile across periods, as its standard deviation of shocks (σ_u) is 1.03 prior to the Great Moderation and 0.46 thereafter. Nevertheless, the output gap turns out to be a small driving force of overall inflation dynamics as the estimate of the Phillip curve's slope parameter k is 0.02, which implies a flat-sloped Phillips curve. As is well known in the literature, forward-looking specifications often produce slope estimates that are statistically insignificant or of the wrong sign, especially when the CBO gap is used as a measure of real economic activity. However, that is not the case here.

The transition probabilities p_{11} and p_{22} suggest that postwar U.S. inflation dynamics underwent two structural changes over the postwar period. The first structural break is dated 1971Q1, which occurred approximately around the collapse of the Bretton Woods system and the beginning of the Great Inflation. The second structural break occurred in 1980Q4, which approximately marks the end of the Great Inflation that followed the Volcker disinflation. By estimating a univariate unobserved components model for inflation with structural breaks, Kang et al. (2009) also find similar break

dates. These estimated break dates lend support to the traditional view that the behavior of inflation is closely tied to the monetary policy regime in place.

In each regime, the dynamics of z_t, which contains the backward-looking component of the NKPC, varies substantially. In the first and third regimes, estimates of its AR coefficient ψ, are close to zero and statistically insignificant, implying no serial correlation in z_t. This finding suggests that in these periods, the purely forward-looking NKPC with stochastic trend inflation can provide a good description of U.S. inflation dynamics. In contrast, z_t is highly persistent and volatile throughout the second regime that corresponds to the Great Inflation. However, the finding of a persistent z_t process during the Great Inflation period may be due to the occurence of supply shocks. Therefore, to isolate the effects of persistent supply shocks, we also estimated the model with core PCE inflation, yet z_t remains highly persistent during the Great Inflation. This finding suggests that the backward-looking component in the NKPC is important during the Great Inflation as there remains considerable persistence that the purely forward-looking Phillips curve fails to explain. Our empirical results therefore, stand in contrast to those of Cogley and Sbordone (2008). By estimating a version of the NKPC with time-varying trend inflation and NKPC parameters, these authors find that backward-looking elements are redundant in explaining the dynamics of U.S. inflation throughout the postwar period once time variation in trend inflation is taken into account.

Next, Table 2 presents the estimation results for Model 2 with structural

breaks. The estimates of the parameters that describe the dynamics of inflation are similar to those of Model 1, thus confirming the results described above.

Model 2 is an unobserved components model for inflation and real output, which allows us to examine the characteristics of U.S. output as implied by the NKPC with stochastic trend inflation. According to our estimation results, the sum of the output gap's AR coefficients are as high as 0.94, implying that the unobserved output gap measure consistent with a NKPC is highly persistent[3]. By estimating a bivariate unobserved components model for inflation and real output that uses inflation expectations survey data to measure the forward-looking element in the NKPC, Basistha and Nelson (2007) also find that the implied output gap series consistent with a NKPC is highly persistent.

Turning to examine the output trend growth rates μ_0 and μ_1, the annual growth rate of U.S. real output was 3.72% before the productivity slowdown in 1973Q1, and 2.99% thereafter. These estimates are roughly in line with others reported in the literature, such as those obtained from the univariate unobserved components model for output of Perron and Wada (2009). In addition, similar to these authors, we find that the standard deviation of shocks to the output trend function (σ_η) is not significantly different from

[3]According to Morley et al. (2003), persistent output cycles may arise due to the zero correlation restriction for output trend and cycle components. Therefore, we also estimate Model 2 allowing for correlation between output trend and cycle components. However, this unrestricted model delivered similar empirical results to the restricted model.

zero, implying a deterministic trend for equilbrium real output[4].

Last, we check the validity of our empirical results by testing for serial correlation in the unobserved components models' standardized residuals and their squares for the inflation series. Serial correlation in the residuals often signals model misspecification, and serial correlation in their squares implies remaining ARCH effects that are left unexplained by the model. The p-values associated with the Q-statistics under the null hypothesis of no serial correlation up to lag 8 are reported in Table 3 for both unobserved components models. The results suggest that there is no evidence of remaining serial correlation in the models' standardized residuals and their squares at the 5% significance level, implying that the unobserved components models are generally well specified.

3.2. Estimates of Unobserved Components

Both unobserved components models with structural breaks deliver similar estimates for the latent state variables. Due to space considerations, we only present and discuss the results from Model 2, in which the output gap is treated as an unobserved process.

Figure 1 shows a plot of the smoothed estimates of trend inflation, $\pi^*_{t|T}$,

[4]Perron and Wada (2009) show that their deterministic trend result follows from allowing for a one-time break in the slope function of U.S. trend output to account for the productivity slowdown that occured in 1973Q1. To check the robustness of our results, we estimate Model 2 with two alternative specifications for the output trend function. However, we find that whether the slope term is specificied as a constant or a random walk drift process, U.S. trend output is deterministic.

alongside actual U.S. inflation. It is evident that trend inflation is highly persistent, and is an important determinant of actual inflation over the post-war period. As the figure illustrates, trend inflation varies and has the same general movement as realized inflation. That is, trend inflation was low and steady in the early 1960s, started to rise in the mid 1960s, fell after the Volcker disinflation of the early 1980s, and remained low and stable around 2% since the early 1990s.

This general trajectory of trend inflation is similar to those reported elsewhere, although the exact estimates differ in some details. For example, Cogley and Sbordone (2008)'s reduced form VAR with drifting coefficients and stochastic volatility produces a slightly smoother trend, with trend inflation peaking just above 4% in the mid 1970s and trend inflation well anchored at 2% since the 1990s. However, the confidence bands associated with their estimates are relatively wide, and contains our estimates of trend inflation. By estimating a dynamic stochastic general equilibrium model with a time-varying inflation target, Ireland (2007) finds that trend inflation during most of the 1970s was higher, peaking at around 6%. Compared to Figure 1, estimates of trend inflation produced by the unobserved components model of Lee and Nelson (2007) are much more volatile, peaking as high as the inflation series itself during the Great Inflation.

Figure 2 is a plot of the inflation cycle along with the inflation gap. We define the inflation cycle as the second term in equation (6), which can be interpreted as the transitory component of inflation that is driven by agents'

forecasts of current and future output gaps. As for the inflation gap, we follow the recent literature and define it as the deviation of actual inflation from its latent trend, thus being the sum of the inflation cycle and the z_t component. The inflation gap is a widely used measure of short-run inflation dynamics, and we can observe from Figure 2 that the U.S. inflation gap was significantly more volatile and persistent during the Great inflation. By estimating VARs with drifting coefficients and stochastic volatility, Cogley et al. (2010) arrives at a similar conclusion about U.S. inflation gap dynamics. As for the implications that we can draw from our estimates of the inflation cycle, we observe from Figure 2 that the inflation cycle generally moves in the same direction as the inflation gap, suggesting that the output gap is an important driver of inflation dynamics in the short run.

3.3. A Comparison of the Output Gaps

The two output gap measures that enter the two unobserved components models are obtained from quite disparate methods. The CBO output gap in Model 1 is estimated from a large-scale multisector growth model, whereas the unobserved output gap in Model 2 is a byproduct of estimating a bivariate unobserved components model in inflation and real output that is consistent with the NKPC. Furthermore, the CBO output gap is a two-sided estimate, as it is constantly revised after new information about the macroeconomy becomes available in the data. On the other hand, estimates of the output gap obtained from Model 2 is a one-sided measure that only relies on (revised)

data up until date t. Yet, from the empirical results reported in Tables 1 and 2, the sum of the AR coefficients and the standard deviation estimates of the shocks to both output gap measures are remarkably similar.

Just how similar are these two output gap measures? Figure 3 presents a plot of the two output gaps series. Upon a quick glance, both output gaps contain movements that are generally similar. To better quantify the differences between the two, in Figure 4 we plot the CBO output gap alongside the 95% confidence interval associated with Model 2's estimates of the output gap. As shown, the CBO output gap is well contained within the 95% confidence set. In other words, the differences between the two output gap series are not statistically significant at the 5% significance level. Furthermore, we observe that the output gap associated with the bivariate unobserved components model has relatively narrow confidence bands when compared to other measures in the literature. These findings strengthens Kuttner (1994)'s case of using the bivariate unobserved components modeling approach as an effective shortcut method to obtain estimates of the output gap in place of a comprehensive supply-side analysis.

4. Inflation Forecasting

The forecasting performance of a model is often viewed as a useful metric for evaluating its empirical relevance. In this section, we conduct in-sample and out-of-sample inflation forecasting exercises with our proposed models to evaluate their ability to explain the data, as well as to evalu-

ate the role of the output gap in producing inflation forecasts. We focus on forecasting $h-$period average inflation (at an annual rate), defined as $\pi_t^h = h^{-1} \sum_{i=0}^{h-1} \pi_{t-i}$. We use the notation $\pi_{t+h|t}^h$ to denote $h-$period-ahead forecasts of π_t^h made using data through time t.

We compare the forecasting performances of Models 1 and 2 with structural breaks against three univariate inflation forecasting models as described below.

Model 3: Atkeson and Ohanian (2001)'s inflation forecasting model:

$$\pi_{t+4|t}^4 = \pi_t^4 = \frac{1}{4}(\pi_t + ... + \pi_{t-3}). \tag{39}$$

The specification above, often referred to as the AO model, is well known in the literature for its ability to forecast inflation. It is a simple model that predicts the average four-quarter rate of inflation to be the same as the average rate of inflation over the previous four quarters[5]. Atkeson and Ohanian (2001) were the first to formally point out that since the mid- 1980s, four-quarter-ahead out-of-sample inflation forecasts obtained from such a specification has been more accurate than those implied by Phillips curve models. Based on more comprehensive analyses, Fisher et al. (2002), Stock and Watson (2003, 2007), and others confirm the AO result.

The AO finding implies that since the mid- 1980s, information contained

[5]In Atkeson and Ohanian (2001)'s paper, they define π_t^4 as the percentage change in the inflation rate between quarters $t-4$ and t.

in measures of real economic activity has limited predictive content for inflation. Therefore, we also consider the following univariate unobserved components model, which is a restricted version of our proposed models with $k = 0$.

Model 4: A univariate unobserved components model without output gap ($k = 0$):

$$\pi_t = \pi_t^* + z_t, \tag{40}$$

$$\pi_t^* = \pi_{t-1}^* + \upsilon_t, \tag{41}$$

$$z_t = \psi_{S_t} z_{t-1} + \epsilon_t, \tag{42}$$

$$\begin{bmatrix} \upsilon_t \\ \epsilon_t \end{bmatrix} \sim i.i.d.N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\upsilon^2 & 0 \\ 0 & \sigma_{\epsilon,S_t}^2 \end{bmatrix} \right). \tag{43}$$

Note that with $k = 0$ in Model 1 or Model 2, inflation is no longer influenced by movements in the output gap. Thus, by comparing the performance of the above specification to Models 1 and 2, we will be able to evaluate the role of the output gap in our inflation forecasts.

In our forecasting exercise, all four competing models are used to forecast three inflation series: the chain-weighted GDP deflator, CPI inflation, and the chain-weighted PCE inflation. We compute both one-quarter-ahead in-sample inflation forecasts based on the full sample period 1952Q1-2007Q3, and four-quarter-ahead out-of-sample inflation forecasts for the sample 1994Q1-2007Q3 and the most recent period 2001Q1-2007Q3. For out-of-sample fore-

casting, a recursive procedure is followed. This means that for a forecast made at date t for date $t+4$, all estimation is made with data beginning in 1952Q1 through date t. Then, the procedure is repeated again with the same starting date but an expanding data window.

Table 4 reports the root mean squared errors (RMSEs) associated with the in-sample and out-of-sample inflation forecasts calculated from all four models. For the in-sample inflation forecasts, our proposed models, Models 1 and 2, outperform Models 3 and 4 for all inflation measures. Similarly, for the out-of-sample case, our proposed models have lower RMSEs than the two competing univariate models.

To assess whether the differences in the out-of-sample predictive accuracies of our proposed models and the two univariate models are statistically significant, we analyze the out-of-sample inflation forecasting results using the modified Diebold-Mariano test statistic. The original Diebold-Mariano test statistic is a t-statistic associated with the null hypothesis that the mean squared errors of the two forecasts being compared is zero (Diebold and Mariano, 1995). The modified version as derived by Harvey et al. (1997) attempts to correct for the poor size property of the original test statistic in small samples.

The modified Diebold-Mariano test statistic are reported in Table 5 with their corresponding p-values in parentheses. From the evidence shown in the first three rows of Table 5, neither our proposed models nor the competing univariate models have superior forecasting power for the out-of-sample

period 1994Q1-2007Q3. Two exceptions are that the RMSEs from our proposed models are lower than the AO model at the 10% level of significance when forecasting the PCE and CPI inflation series. Focusing on the more recent period 2001Q1-2007Q3, however, our proposed models outperforms the competing univariate models.

In sum, the results presented in this section provide evidence that leading inflation forecasting model in the literature, namely the AO model, may be inferior to the models we propose in this paper in terms of both in-sample and out-of-sample forecasting. Note again that the main differences between our models and theirs is that our proposed models are based on a NKPC, thus allowing innovations of the output gap to influence inflation. Thus, in contrast to the generally accepted view in the literature, we provide evidence that from the mid-1980s onwards, real economic activity still contains predictive content for inflation, and Phillips curve models remain useful for forecasting inflation.

5. Summary and Conclusion

We estimate unobserved components models of inflation that are consistent with the New Keynesian Phillips curve in the presence of a stochastic trend in inflation. The models we present can also be considered as extensions of Stock and Watson (2007), who show that the univariate inflation process is well described by an unobserved component trend-cycle model with stochastic volatility. While Stock and Watson (2007) assume a serially uncorrelated

transitory component of inflation, we derive the transitory component to be a function of real economic activity and a potentially serially correlated component, as implied by the hybrid New Keynesian Phillips curve.

The empirical results suggest that a forward-looking Phillips curve can provide a good fit to postwar U.S. inflation data, except for the period of the Great Inflation in the 1970s. The backward-looking component plays an important role in explaining the dynamics of inflation in the 1970s, as captured by the serially correlated transitory component in our model. Our results stand in contrast to those of Cogley and Sbordone (2008), who show that the backward-looking component redundant in explaining the dynamics of U.S. inflation throughout the postwar period once time variation in trend inflation is taken into account.

In explaining why inflation has become harder to forecast since the mid-1980s, Stock and Watson (2007) suggest that the changing auto-regressive (AR) coefficients and the deterioration of the low-order AR approximation accounts for the relatively poor performance of recursive and rolling AR forecasts. Our models and empirical results suggest that the relatively poor performance of recursive and rolling AR forecasts of inflation results from a disappearance of the backward-looking component in the Phillips curve in the early 1980s.

The model we present also exhibit better in-sample and out-of-sample inflation forecasts relative to benchmark models in the literature that are generally known for their good forecasting performance. Furthermore, we

find that the estimates of the output gap from our model display relatively narrow confidence bands, and the dynamics of the estimated output gap are quite close to those of the CBO output gap.

Acknowledgements

Manopimoke gratefully acknowledges the support of the Buechel Memorial Fellowship, the Grover and Creta Ensley Fellowship in Economic Policy, the CSWEP Summer Economics Fellows Program, and the Research Fellowship Program at Federal Reserve Bank of Dallas. Kim acknowledges financial support from the National Research Foundation of Korea (KRF-2008-342-B00006) and the Bryan C. Cressey Professorship at the University of Washington. Nelson acknowledges financial support from the Van Voorhis Professorship at the University of Washington.

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Table 1: **Estimation results for Model 1 with structural breaks [1952Q1-2007Q3]**

Parameters	Estimates (Standard error)		
Phillips curve slope and AR coefficients of the CBO output gap			
k	0.019 (0.008)		
ϕ_1	1.191 (0.063)		
ϕ_2	-0.267 (0.062)		
Standard deviations, correlations, and persistence of shocks			
σ_v	0.340 (0.060)		
$\sigma_{u,0}$	1.030 (0.067)		
$\sigma_{u,1}$	0.460 (0.034)		
$\rho_{\epsilon,u}$	-0.077 (0.084)		
	Regime 1	*Regime 2*	*Regime 3*
σ_ϵ	0.740 (0.089)	1.554 (0.194)	0.843 (0.075)
ψ	-0.072 (0.169)	0.902 (0.086)	-0.002 (0.102)
Transition probabilities			
p_{11}	0.987 (0.013) $\rightarrow$ *break date: 1971Q1*		
p_{22}	0.974 (0.026) $\rightarrow$ *break date: 1980Q4*		
Log-likelihood value -362.954			

Table 2: **Estimation results for Model 2 with structural breaks [1952Q1-2007Q3]**

Parameters	Estimates (Standard error)		
Phillips curve slope, output trend drifts, and AR coefficients of the unobserved gap			
k	0.017 (0.008)		
μ_0	0.931 (0.041)		
μ_1	0.747 (0.018)		
ϕ_1	1.208 (0.063)		
ϕ_2	-0.271 (0.063)		
Standard deviations, correlations, and persistence of shocks			
σ_v	0.340 (0.060)		
$\sigma_{\eta,0}$	0.000 (0.021)		
$\sigma_{\eta,1}$	0.000 (0.062)		
$\sigma_{u,0}$	1.025 (0.066)		
$\sigma_{u,1}$	0.468 (0.035)		
$\rho_{\epsilon,u}$	-0.074 (0.090)		
	Regime 1	*Regime 2*	*Regime 3*
σ_ϵ	0.748 (0.088)	1.556 (0.193)	0.844 (0.073)
ψ	-0.056 (0.146)	0.901 (0.102)	0.001(0.044)
Transition probabilities			
p_{11}	0.987 (0.013) → *break date: 1971Q1*		
p_{22}	0.974 (0.026) → *break date: 1980Q4*		
Log-likelihood value: -366.043			

Table 3: **Tests for serial correlation in the standardized residuals and their squares [1952Q1-2007Q3]**

Lag	Model 1	Model 2
	Standardized Residuals	
1	0.659	0.645
2	0.824	0.817
3	0.417	0.403
4	0.469	0.453
5	0.383	0.367
6	0.153	0.142
7	0.185	0.173
8	0.052	0.050
	Squares of Standardized Residuals	
1	0.812	0.931
2	0.966	0.990
3	0.564	0.571
4	0.680	0.679
5	0.783	0.778
6	0.835	0.836
7	0.904	0.904
8	0.947	0.947

Note: Reported are p-values associated with the Q-statistic under the null of no serial correlation.

Table 4: **RMSEs associated with in-sample and out-of-sample inflation forecasts**

	Forecasting Models			
Inflation Measure	Model 1	Model 2	Model 3	Model 4
One-step ahead in-sample forecasts, 1952Q1-2007Q3				
GDP Deflator	1.017	1.021	1.173	1.039
CPI	1.558	1.559	1.752	1.588
PCE	1.155	1.157	1.353	1.182
Four-step-ahead out-of-sample forecasts, 1994Q1-2007Q3				
GDP Deflator	0.477	0.476	0.488	0.491
CPI	0.858	0.891	0.922	0.910
PCE	0.665	0.669	0.690	0.679
Four-step-ahead out-of-sample forecasts, 2001Q1-2007Q3				
GDP Deflator	0.536	0.547	0.561	0.583
CPI	0.981	0.988	1.086	1.109
PCE	0.657	0.673	0.700	0.719

Note 1: Reported are the RMSEs from inflation forecasts. The RMSE statistic for the time period t_1 to t_2 is calculated as $RMSE_{t_1,t_2} = \sqrt{\frac{1}{T}\sum_{t=t_1}^{t_2}(\pi_{t+h}^h - \pi_{t+h|t}^h)^2}$, with $T = t_2 - t_1 - 1$. Note that $h = 1$ for in-sample one-quarter-ahead inflation forecasts, and $h = 4$ for out-of-sample four-quarter-ahead inflation forecasts.

Note 2: Model 1 is the unobserved components model with observed CBO output gap; Model 2 is the unobserved components model with unobserved output gap; Model 3 is Atkeson and Ohanian (2001)'s inflation forecasting model; Model 4 is a univariate unobserved components model similar to Models 1 and 2 but with $k = 0$.

Table 5: **Evaluation of out-of-sample inflation forecasting performances**

	Competing Models			
Inflation Measure	Model 1 vs. Model 3	Model 2 vs. Model 3	Model 1 vs. Model 4	Model 2 vs. Model 4
Four-step-ahead out-of-sample forecasts, 1994Q1-2007Q3				
GDP Deflator	-0.387	-0.418	-0.276	-0.298
	(0.350)	(0.339)	(0.392)	(0.384)
CPI	-1.581	-0.603	-0.657	-0.236
	(0.059)	(0.274)	(0.257)	(0.407)
PCE	-1.314	-0.979	-0.223	-0.162
	(0.097)	(0.166)	(0.412)	(0.436)
Four-step-ahead out-of-sample forecasts, 2001Q1-2007Q3				
GDP Deflator	-0.583	-0.362	-1.487	-1.267
	(0.283)	(0.360)	(0.074)	(0.108)
CPI	-1.797	-1.339	-3.302	-2.357
	(0.042)	(0.096)	(0.001)	(0.013)
PCE	-2.072	-1.133	-1.732	-1.349
	(0.024)	(0.134)	(0.048)	(0.094)

Note 1: Reported are the modified Diebold-Mariano test statistics. In parentheses are their corresponding p-values under the null of equal predictive accuracy.

Note 2: Model 1 is the unobserved components model with observed CBO output gap; Model 2 is the unobserved components model with unobserved output gap; Model 3 is Atkeson and Ohanian (2001)'s inflation forecasting model; Model 4 is a univariate unobserved components model similar to Models 1 and 2 but with $k = 0$.

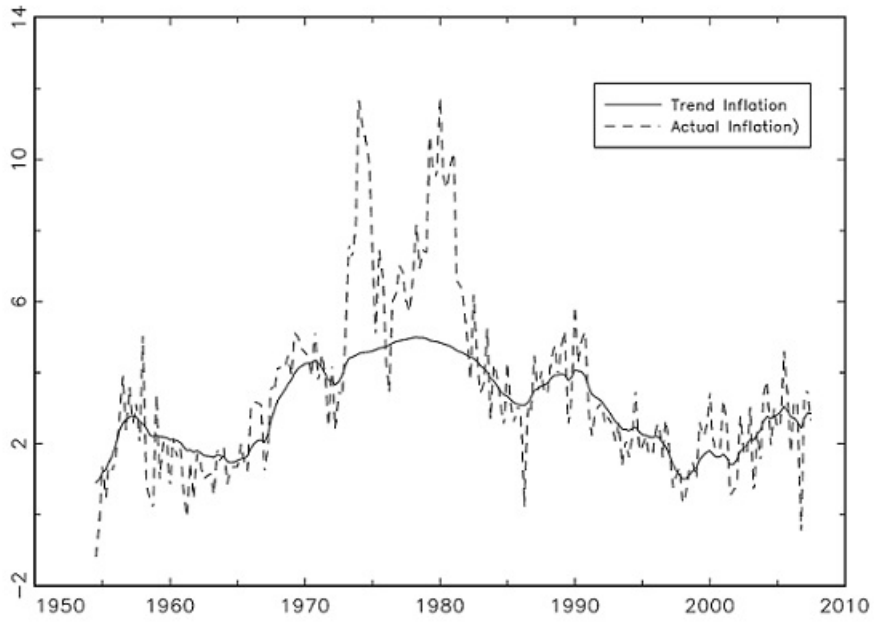

Figure 1: **Actual Inflation and Smoothed Estimates of Trend Inflation**

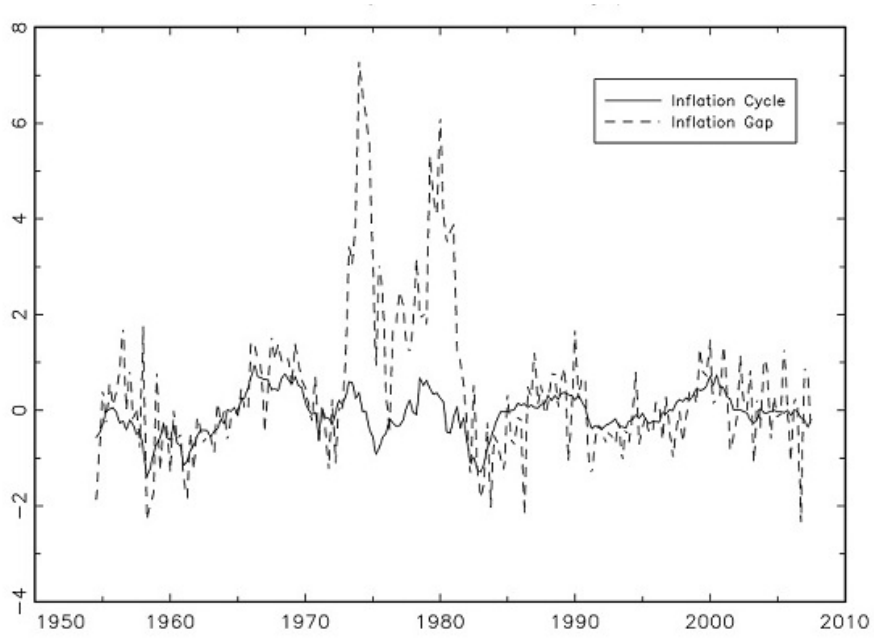

Figure 2: **Inflation Cycle and Inflation Gap**

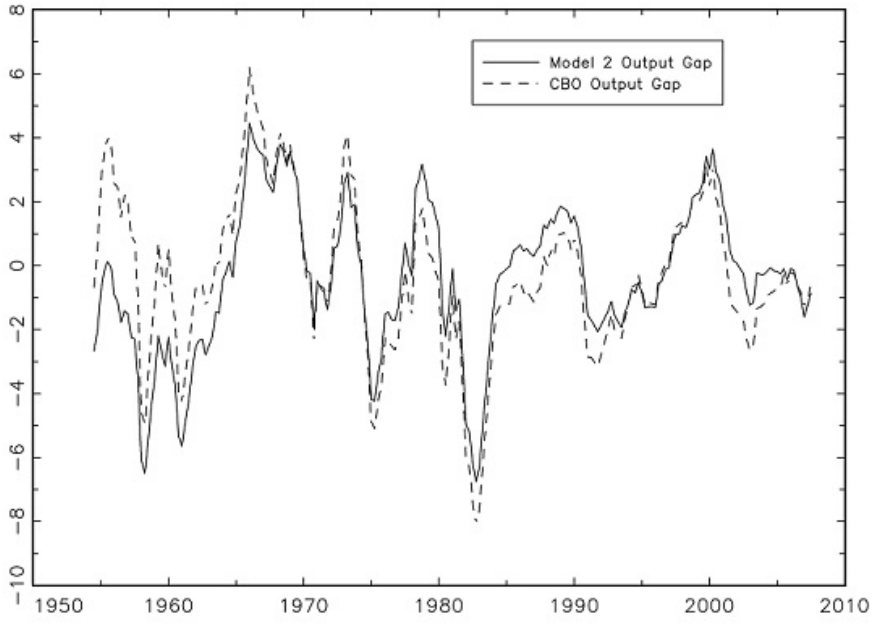

Figure 3: **Model 2 Output Gap Estimates and the CBO Output Gap**

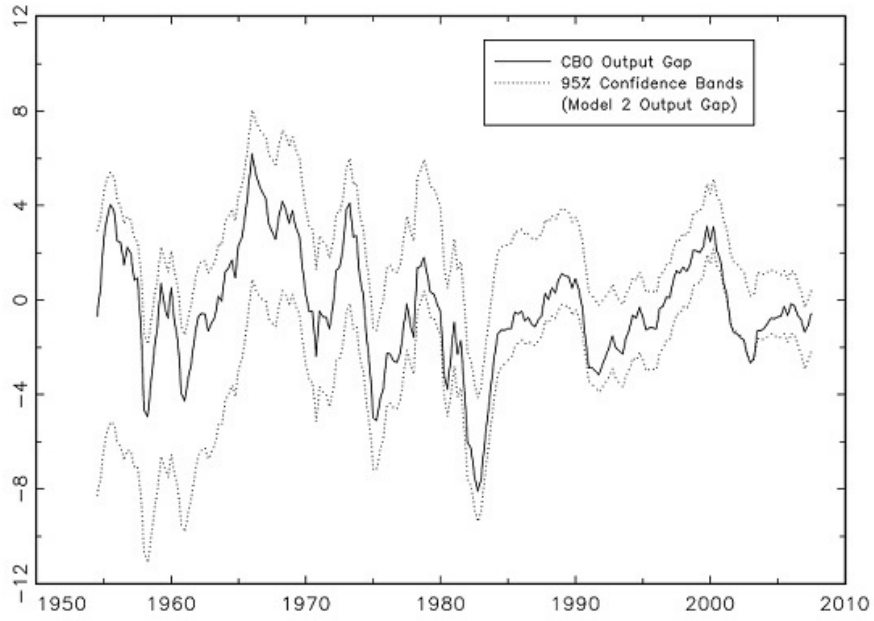

Figure 4: **CBO Output Gap and 95 Percent Confidence Bands for Model 2 Output Gap Estimates**