A Model of the Lender of Last Resort[1]

Charles A. E. Goodhart
Financial Markets Group, London School of Economics

Haizhou Huang
Monetary and Exchange Affairs Department, International Monetary Fund

First Draft: September 1998; Revised: December 2000

Abstract

This paper attempts to develop a model of the lender of last resort (LOLR) from a Central Bank (CB) viewpoint. In a simple static setting, the CB should only rescue banks which are above a threshold size, thus providing an analytical basis for "too big to fail". In a dynamic setting, whereby both the probability of a (further) failure and the likelihood of a bank requiring LOLR being insolvent in each period are a function of CB's prior actions, which then influence the actions of banks and depositors, CB's optimal policy in liquidity support depends on the trade off between contagion and moral hazard effects. The optimal policy may be non-monotonic in bank size and is time varying, and it is also contingent on the probability of a failure, the likelihood of a bank requiring LOLR being insolvent, and random shocks. Our results show that contagion is the key factor affecting CB's incentives in providing LOLR and they also provide a rationalization for "constructive ambiguity".

JEL Classification Number: E50, E58, G21, G28

Keywords: lender of last resort, banking crises, financial contagion, moral hazard

[1] We would like to thank William Alexander, Peter Clark, Tito Cordella, Fabrizio Coricelli, Gianni de Nicolo, Carsten Detken, Stanley Fischer, Xavier Freixas, Frank Heinemann, Heinz Herrmann, Gerhard Illing, Barry Johnston, Allan Meltzer, Hyun Song Shin, Charles Siegman, Mark Spiegel, Jukka Vesala, Geoffrey Wood, and participants in seminars at IMF and ECB and conferences in Stanford, LSE, Siena, and Frankfurt for helpful comments and conversations, Satvinder Singh for research assistance. All remaining errors are our own. The usual disclaimer applies.

1. INTRODUCTION

There have been few formal models seeking to explain and to analyze how and why central banks have provided lender of last resort (LOLR) services to individual commercial banks, even though such acts have been a regular, albeit often contentious, part of a central bank's armory since Bagehot (1873). One reason why there have been few formal models of LOLR is that many economists in this field believe that providing LOLR to individual banks, rather than to the market as a whole, is fundamentally misguided.

All economists accept the lessons of history that banking panics can occur, with depositors seeking to switch out of the deposits of banks perceived as riskier into currency, gold, foreign exchange or those banks perceived as safer. But many economists argue that the central bank's role in such cases should be limited to open market operations (OMO), injecting extra cash into the system as a whole, in order to maintain the aggregate money stock at its desired level.[2] As a generality, they believe central banks should not lend to individual banks, e.g. through a discount window; the market is as well or better informed than the central bank (CB) about the relative solvency of a bank short of liquidity. Given an aggregate sufficiency of high-powered money, illiquid (but solvent) banks will be able to borrow, e.g. in the interbank market, whereas potentially insolvent banks will be driven out of the system. Moreover, the monetary authorities will have incentives to exercise forbearance (Kane, 1992) and rescue banks that should have been closed; and the pursuit of financial stability by direct intervention may divert the CB from achieving its primary goal of controlling the monetary aggregates so as to achieve price stability.[3]

There are two ripostes to this position. The first, though not the subject of this

[2]See Bordo (1990), Goodfriend and King (1988), Humphrey (1989), Kaufman (1991), Schwartz (1988), amongst others.

[3]It is presumably on grounds such as these that the European Central Bank was prevented from assuming any LOLR role, (except by unanimous agreement of the participating members) in the Maastricht Treaty, Article 105.6.

paper, is the potential for "market failure". For example, when the Bank of New York computer malfunctioned in 1985 and would not accept incoming payments for bond market dealings, the resultant illiquidity position soon ballooned to a point where no one counterparty bank could take on the risk of making a sufficiently large loan. It would have required a coordinated syndicate, but such syndicates take time to organize, and time was scarce. Next, in the aftermath of the BCCI failure, there were considerable deposit withdrawals from a string of small banks, run by Asians and serving the Asian community in the UK. They were (unjustly) tainted by association. They had relied, almost entirely, on deposits from the local community, and their names were not known in the wholesale banking market. Although illiquid, rather than insolvent, they were not getting help from the market, so the Bank of England assisted them (Bank of England, 1992, 1993, 1996).

Most central banks would argue that their supervisory role - or their ready access to supervisory information - should give them access to additional information, not available in the market. Moreover, as in the case of the Bank of New York, when there is any large-scale need to redirect reserves, there must be a coordination problem. No one commercial-counterparty can single-handedly assume the credit risk, and there is no incentive for a single commercial bank to take on the time, effort and cost of coordinating the exercise of sorting out the problem. The Bank of England would tend to argue that most of its historical LOLR actions have primarily involved the provision of additional information combined with a coordinating role to encourage private sector financial institutions to resolve the problem, primarily by themselves.[4]

Focusing on the micro aspects of central banks' intervention in dealing with market failure, Freixas, Parigi and Rochet (1998) build one of the rare formal models of LOLR. Using the framework of Diamond and Dybvig (1983), they analyze the

[4]The problem in the case of Barings was that there was insufficient information on the potential close-out cost of Leeson's derivative position. With the Bank of England being unwilling to provide a guarantee to limit any such loss, no commercial institution was willing to buy Barings over the key weekend.

moral hazard problem caused by bank managers' incentive to choose an inefficient technology that gives them some private benefit. This moral hazard problem, as in Holmstrom and Tirole (1998), sets an upper limit to the finance that would be provided at interim dates by outside investors. When liquid shocks cannot be disentangled from solvency shocks, moral hazard on the commercial bank's investment creates a market failure. In the absence of CB intervention there is excessive liquidation of banks: the optimal continuation threshold is above the solvency threshold. Then "The role of the central banks is to mutualize the solvency shocks: lucky banks will be taxed and unlucky banks subsidized." This is a stimulating model, but it does not address the macroeconomic policy concerns of central banks, nor does it deal with contagious risks in the banking system. But their work did provide an incentive to our attempt to model the main macro-policy considerations lying behind LOLR action and to examine both contagious and moral hazard risks in the macro framework.

We thus wish to focus here on a second concern that is at the macro level and affected by both contagious risks and moral hazard. We believe this concern has been important in the minds of central bankers in their application of LOLR. At the macro level, even if the CB is aware of particular examples of "market failure" at micro level, there is still an important judgement to be made about the appropriate balance between "market failure" on the one hand and "official intervention failure" on the other hand. See, for example, Bernanke (1983), Goodhart and Schoenmaker (1995), Gorton (1985), among others for related discussions.[5]

Moreover, contagious risks provide a strong and compelling call for CB to play the role of LOLR. Since the seminal contribution by Diamond and Dybvig (1983) and the recent banking crises in Latin America and Asia, a growing literature on banking crises and financial contagion has emerged. This literature shows that the damage can spill over from the original bank to many other banks, which are either directly

[5] It would be an interesting historical exercise to go over a central bank's LOLR record to see how far such actions could be explained as arising from particular examples of "market failure".

connected with the original bank or merely trading with those banks which are directly connected to the original troubled bank, and as a result the total damage may be compounded in a nonlinear fashion. For example, Allen and Gale (1998, 2000) analyze financial contagion as arising from a combination of incomplete markets and interconnectedness between a failing institution and other institutions. Incomplete markets are important because diversification is limited and the failure of one institution may worsen the position of another through a variety of channels. The literature identifies several channels for transmitting financial contagion, including payments/settlements systems (e.g. Bankhaus Herstatt), interbank deposit and correspondent systems (e.g. Continental Illinois), and effects via asset/liquidity prices.[6] While contagious risks provide a compelling reason for CB to play the role of LOLR, moral hazard effects and uncertainty at the macro level pose important challenges to CB in conducting LOLR.

Our aim is to provide a model of how central bankers behave and why they do so. To our best knowledge, our model of LOLR is the first attempt to address both effects of contagious risks and moral hazard at the macro level. Our main claim is that most studies of this topic implicitly involve a certainty equivalent postulate and thus ignore financial contagious risks. That is, the CB is just as confident and knowledgeable about the optimal level of open market operations, high-powered money and aggregate money stock after the onset of bank failures and panic, as it would have been if such a panic was prevented at the outset. We argue that when failures occur, it can lead to substantial financial contagious risks, as people start to panic and their behavior is likely to become far less predictable, and financial markets become much more volatile. Policy mistakes become much more likely in such an environment.

Let us take three examples.[7] First, the bank failures in the USA in the 1930s shifted

[6]For contagious risks in payment/settlements system, see Rochet and Tirole (1996); in interbank market, see Aghion, Bolton and Dewatripont (2000) and Huang and Xu (2000); and in asset/liquidity markets, see Caballero and Krishnamuthy (1999), Diamond and Rajan (1998) and Kodres and Pritsker (1999), among others. See Huang (2000) for a survey on the growing literature.

[7]See Appendix for further empirical evidence of bank failures causing macro uncertainty.

the high-powered money (H) to aggregate money (M) ratio. Although, as Kaldor (1958) noted, the Fed's actions led to a much faster, than previous, growth in H during these years, M still fell. Second, after the 1987 stock market collapse, central banks lowered interest rates aggressively scared of a replay of 1929, only to discover a couple of years later that they had overdone such ease. Third, in Japan, some 90% of respondents to a survey in 1998 stated that they lacked confidence in their banks. Interest rates were rock bottom and H was growing fairly fast. What should the Monetary Policy Committee (MPC) of the Bank of Japan do? The published Minutes of the MPC there reveal their uncertainties.

A key feature in our static model is that we formalize the loss to a CB when it allows bank failures to occur. We assume that the CB is trying to achieve an (exogenously given) desired level of deposits in the system. When a failure occurs, due to financial contagion it causes further changes in deposits in the banking system as a whole. Although the CB can immediately take open market operations to offset the deposit change, it can still suffer a loss, as a result of getting macro-policy wrong in the uncertain macroeconomic environment with financial contagion. In a simple static setting, the CB should only rescue banks which are above a threshold size, thus providing an analytical basis for "too big to fail".

Another feature in our model is that we explicitly model both effects of financial contagion and moral hazard in dynamic setting, and our optimal LOLR policy is derived from the trade-off between the two effects. In the dynamic setting, both the probability of a further failure and the likelihood of a bank requiring LOLR being insolvent in each period are a function of CB's prior actions, which then through moral hazard effects influence the actions of banks and depositors. The optimal policy, based on the trade-off between contagion and moral hazard effects, may be non-monotonic in bank size and is time varying, and it is also contingent on the probability of a failure, the likelihood of a bank requiring LOLR being insolvent, and random shocks. Our results thus provide a rationalization for "constructive ambiguity".

The remaining of the paper is organized as follows. Section 2 sets up and analyzes the static model, which is followed by the dynamic model in Section 3. Concluding remarks are in Section 4, and some further empirical evidence of bank failures causing macro uncertainty in the Appendix.

2. THE STATIC MODEL

2.1 The Basic Setup

We start with a commercial banking system with many banks of varying sizes, and assume all of which choose a similar risk profile (h). We shall try to justify this assumption as a reasonable approximation to reality shortly. Given this preferred risk profile there is a probability (p) of a bank holding j of the system's deposits coming to the CB asking for LOLR assistance in any period. We set the initial volume of deposits at D^*, a pre-determined desired level, $0 < j < D^*$.

If no bank seeks LOLR, as occurs with probability $(1-p)$, CB takes no action, D remains equal to D^*, and the CB suffers no loss. If p occurs, with $j > 0$, the CB has to decide whether to say yes to the request for LOLR ($S_t = 1$), or no ($S_t = 0$). There is a probability, x, where $x = f(h)$, that the bank (or banks) coming to the CB will also be insolvent. This is not revealed to the CB at this stage, and the CB also at this stage has to decide on its OMO.[8] By OMO the CB is assumed to be able to change D by any desired amount, OMO being unlimited in size and direction, so the CB can always achieve its desired expected value of D, i.e., it can make $\mathrm{E}[D] = D^*$, on the strong assumption that the demand for money function itself remains unchanged by bank failures.[9,10]

[8] All practical descriptions of LOLR activities reveal that the CB is under tremendous time-pressure to take decisions before the market reopens and has to do so when in possession of only sketchy details of the financial position of the commercial bank needing help.

[9] This biases the analysis against LOLR, since in a panic the demand for money function is also liable to become unpredictable.

[10] Panics also act to increase uncertainty about the appropriate selection of interest rates, and the analysis can be done similarly. During panics there is a rush to safety, so interest rates on high-quality, short-term government paper frequently fall. Since risk-premia and spreads then typically

If the CB does not provide LOLR, the illiquid bank(s) will shut. This will cause the public to move out of deposits into cash, via the linear relationship $B_1 j + j\epsilon$, where B_1 is a positive coefficient known to the CB, and ϵ is a stochastic variable, with $\mathrm{E}[\epsilon] = 0$ and $\mathrm{Var}(\epsilon) = k$, where k is also known to the CB. The loss from getting macro-policy wrong is assumed to be quadratic, and for simplicity as $(D - D^*)^2$.

The identity of the illiquid bank, which has been supported by LOLR, is then made known. If it is insolvent, with probability x, the CB will face a cost Z, where $Z = n + B_2 j$ $(n > 0,\ B_2 > 0)$.

Thus, in the static model, with h, f and x all given and constant, the CB wishes to minimize the loss functions,

$$\min \left[\mathrm{E}(D - D^*)^2,\ \mathrm{E}Z\right] \tag{1}$$

The sequence of events is as in Figure 1.

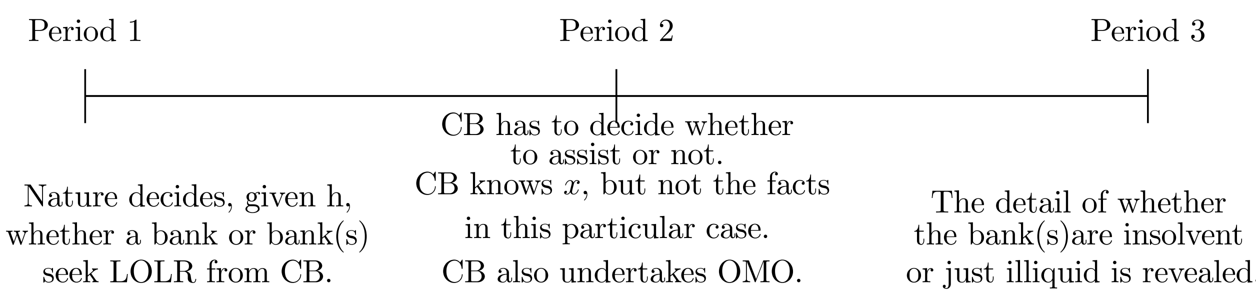

FIG. 1. Time Line

2.2 The Analysis

By using OMO to off set the expected amount that the public move out of deposits into cash, $B_1 j$, the CB can always achieve its desired expected value of deposits, i.e., $\mathrm{E}[D] = D^*$. But this is the best the CB can achieve by using OMO, as the CB cannot eliminate the variance item in $\mathrm{E}(D - D^*)^2$, which is kj^2, thus $\min[\mathrm{E}(D - D^*)^2] = kj^2$.

widen, low quality debt yields may simultaneously increase.

Comparing kj^2 with $(n + B_2 j)x$, LOLR is preferable if and only if:

$$\mathrm{E}Z = (n + B_2 j)x \leq kj^2 = \min[\mathrm{E}(D - D^*)^2].$$

That is[11]

$$j \geq \overline{j} \equiv \frac{B_2 x + \sqrt{B_2^2 x^2 + 4knx}}{2k}. \tag{2}$$

Thus we reach our first result regarding the comparison between LOLR and OMO in a static setting.

Proposition 1 *In a static setting, LOLR is preferable to OMO if and only if the size of the bank seeking assistance is above a threshold level $\overline{j}$; otherwise OMO is preferable.*

Clearly the crucial feature of this model is that the costs of allowing a bank to fail (quadratic in j) rise at a faster rate with respect to the size of bank than the costs of rescuing a bank that may turn out to be insolvent (proportional to j). But so long as the costs of failure rise consistently faster than the costs of rescue with respect to size, the same qualitative results will hold.

There are several reasons for this asymmetry. Let us start with the quadratic loss function from policy error, kj^2. In virtually all the CB independence literature, policy errors, e.g. deviations of inflation from target, are taken to be quadratic. But the justification for quadratic loss functions is rarely profound, and often just based on mathematical convenience. More convincingly, the literature of banking crises and financial contagion as described in the introductory section shows that the damage can spill over from the original bank to many other banks, which can further spill over to many more banks. The total damage is thus a nonlinear function of the size of the original troubled bank. What we claim is that the risks arising from such contagion is a nonlinear convex function of the size of the original troubled bank, and the exact quadratic form is just a mathematical convenience.

[11] The technical condition for $\overline{j} < D^*$ is assumed to be satisfied.

Moreover, the costs of rescue rise less fast. Here there are two costs, reputational and financial costs, when the rescued bank is insolvent, with the latter falling on some combination of surviving banks, CB[12] and taxpayers. There is a significant fixed element in reputational costs. The CB may, or may not, make an obvious, publicly observable, error of judgement. Again, there is a fixed cost in making the taxpayer, or other surviving banks, face the reality of having to contribute to an ex post bailout at all. That fixed cost may be large, perhaps even so large that no banks, even the largest, will actually be supported (e.g. see the problem in Japan of sharing out the costs of rescue). But, once taxpayers have come to accept that they must bear the costs of rescue, then our assertion is that the disutility is just proportional to the cash burden.

According to the above proposition, $\overline{j}$ only depends on x, k, n and B_2, but not on D^* or B_1. Some comparative statics on $\overline{j}$ further reveal that $\overline{j}$ increases as x increases, k decreases, n increases, or B_2 increases. These results have the following intuition: The CB should raise the threshold level of bank size and thus only rescues bigger size banks, when the probability of insolvency x increases, when the risk of deposits moving out of the banking system k decreases, or when either the fixed cost (n) or variable cost (B_2) of rescuing banks that turn out to be insolvent increases.

Moreover, if the risk of deposits moving out of banking system k is very small, then $\overline{j}$ can be very large; thus OMO becomes optimal even for the largest bank; if

[12] CB losses may be minimized by an appropriate requirement for collateral. But as Goodfriend and King (1988, p.12) note, "Fully collateralizing a loan with prime paper such as U.S. Treasury bills would make the value of a central bank's line of credit minimal, since a bank could acquire the funds by simply selling the bills on the private market."

Even if a commercial bank seeking help from a CB will usually have used up its best collateral already, (to borrow on finer terms from the market), the CB may be able to extract such tough terms for its LOLR lending that its own resources are largely protected in the case of an insolvency. But some (junior) creditors would then be hit all the harder, and there would still be a reputational loss to the CB, perhaps the more severe if its was perceived as refusing to take its "share" of the losses – especially if it was also responsible for bank supervision.

the probability of insolvency x is very small, then $\bar{j}$ can be very small; thus LOLR becomes optimal even for very small banks.

We have now formally explained how a policy of "too big to fail" minimizes the CB's loss function in a static setting. But if decisions whether to provide LOLR are a function of the size of the commercial bank in difficulties, one may wonder whether these decisions will make individual choice of risk profile (h) a function of size, and thus contradicting our initial simplifying assumption. Not necessarily so. Because the benefits that a manager obtains, both pecuniary and non-pecuniary, from his (continued) position are a positive function of the size of his bank as well as a function of its profitability, the larger the bank, the less that the manager would want to put his status at risk. So greater size may on the one hand be predicted to lead to more probable CB intervention, but on the other hand to make managers more unwilling to put their own position at risk. Moreover, large size banks may be subject to a tighter monitoring and regulation by CB.

Of course, if the value of $\bar{j}$ was public knowledge, there would be a discontinuity. Banks just larger than the cut-off point would have an incentive to increase risk, whereas banks just below the cut-off point would become very risk averse. But $\bar{j}$ is not generally publicly observable.[13] This is partly because individual banks have idiosyncratic features making the CB more, or less likely, to intervene on their behalf,[14] partly because $\bar{j}$ varies over time (and in a somewhat unpredictable fashion, as we will show in subsequent sections), and partly because the monetary authorities maintain a policy in this respect of "constructive ambiguity". Indeed our model enables us to interpret this latter as a rational response by the CB precisely to prevent

[13] There are some exceptions to this dictum. The Comptroller of the Currency, at the time of the Continental Illinois crisis, stated that all larger banks would also be automatically rescued. The Japanese monetary authorities in recent years have made it publicly known that the large City banks are ring-fenced against failure.

[14] For example Johnson Matthey's involvement in the gold market in London. Again, BCCI and Drexel Lambert could be let go, because their interconnectedness was low. *Per contra*, the interconnectedness of LTCM was high.

commercial banks' chosen risk profiles becoming a function of size.[15]

In summary, we believe that even though the CB response to a commercial bank in difficulties will be a function of the size of that bank, a bank's chosen risk profile need not necessarily be a function of size. "Too big to fail" is widely accepted to be an almost universal phenomenon, yet we are not aware of any empirical finding that risk preference among bankers is a positive function of size. This is not to suggest that commercial banks' risk profile choices do not respond to the actions and signals of the CB. Indeed the next Section focuses directly on that. Rather we claim that our model simplification, whereby all banks react similarly to such CB signals, despite being of differing initial sizes, is sufficiently close to reality to make our model results interesting.

3. THE DYNAMIC MODEL

In the static model above, the probability of a commercial bank needing LOLR (p), the probability of it then also being insolvent (x) and its risk profile (h) were all taken as given. In this Section we consider the dynamic equilibrium in which, p, x, and h will become time-varying in response of the CB's actions and signals.

There are two main channels of intertemporal interactions that we identify here. The first is contagion, whereby failure now, when the CB refuses the request of LOLR ($S_t = 0$), is likely to lead to more failures subsequently. The second is moral hazard, whereby a rescue ($S_t = 1$), is likely to cause commercial banks to increase their risk profiles, thereby raising both p and x. In reality, of course, moral hazard will be much worse if the CB provides LOLR assistance to an insolvent bank than to a solvent, but illiquid, bank, (when there might be no moral hazard). We have justified the above simplified assumption with the observation that generally commercial banks, and the general public, will not be able to observe for some time whether a LOLR support exercise involves an insolvent bank, or not, and so will have to condition on the LOLR

[15] See, for example, Enoch, Stella and Khamis (1997) for empirical evidence.

action itself, rather than on the full characteristics of the banks supported.[16]

In the dynamic setting, this problem the CB faces generalizes to:

$$\min_{S_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) x_t S_t \right] \right\}, \tag{3}$$

where $0 < \delta < 1$ is discount factor, and $0 \leq S_t \leq 1$ is the CB's control variable. This is subject to the equation of motion of p_t or/and x_t. We choose to solve the dynamic programming problem by using the Lagrange method.[17]

In order to derive basic economic intuitions out of clear-cut closed form solutions, we start our analysis by allowing one of the two stochastic variables, p or x, to vary over time, holding the other fixed. We treat the analysis with p_t time-varying (x constant) as primarily about contagion and present it in subsection 3.1; in subsection 3.2 we present the analysis with x_t time-varying (p_t constant) as focussing on moral hazard. Then in subsection 3.3, we allow both contagion and moral hazard to operate, i.e., with both p_t and x_t stochastically time-varying. With linearization of the first order conditions around the steady states, we also obtain a closed form solution in this case.

3.1 Contagion in Dynamic Model

To focus on the contagion problem, we can set $h_t + \zeta_t = x$ as constant. Thus, the objective function is:

$$\min_{S_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) x S_t \right] \right\}. \tag{4}$$

[16] It takes a long time, often measured in years rather than months, to evaluate the final balance sheet position of banks that are liquidated, or taken over by authorities with a view to subsequent resale.

[17] Our problem in the case of contagion, or of moral hazard alone, is a standard dynamic programming problem and can be solved by using the standard dynamic programming approach. We decided to choose the Lagrange approach because it appears more efficient and useful in dealing with non-quadratic objective functions (the joint case of contagion and moral hazard in subsection 3.3). For consistency in approach and easy exposition, we use the Lagrange approach for the whole paper. See Chow (1997) for more discussions of this method.

This is subject to the equation of motion for p_t, which will depend on the CB's actions, that is either to grant or refuse LOLR ($S_t = 1$ or 0), so that

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j)(1 - S_t) - (\alpha_2 + \beta_2 j)S_t + \alpha_3 p_t + \varepsilon_{t+1}, \tag{5}$$

where ε_{t+1} is a stochastic term with mean zero and a constant variance. Note that allowing a bank to fail will raise the probability of future failures, i.e., contagion, so that $\alpha_1 > 0$ and $\beta_1 > 0$, so providing LOLR assistance can reduce the probability of failure by containing the contagious risk. With any CB action, in response to a call for assistance from a commercial bank lending, which leads to further difficulties (i.e., p_t rising), there is a question whether the banking system is globally stable, and past history suggests that such worries could have some foundation. We show the necessary stability conditions and discuss their economic intuition below.

The Lagrange function of this question is[18]

$$\begin{aligned} \mathfrak{L} \;=\; & E_0\{\sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k (1 - S_t) + (n + B_2 j)x S_t\right] \\ & -\delta^{t+1}\lambda_{t+1}\left[p_{t+1} - \alpha_0 - (\alpha_1 + \beta_1 j)(1 - S_t) + (\alpha_2 + \beta_2 j)S_t - \alpha_3 p_t - \varepsilon_{t+1}\right]\}. \end{aligned}$$

The first order conditions (FOC) of this Lagrange with respect to S_t and p_t yield:

$$p_t\left[j^2 k - (n + B_2 j)x\right] + \delta\left(\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j\right) E_t \lambda_{t+1} = 0, \tag{6}$$

$$j^2 k (1 - S_t) + (n + B_2 j)x S_t + \delta \alpha_3 E_t \lambda_{t+1} = \lambda_t. \tag{7}$$

For a quadratic objective function like ours, we can conjecture λ_t as a linear function:

$$\lambda_t = \rho_0 + \rho_1 p_t, \tag{8}$$

thus,

$$E_t\lambda_{t+1} = \rho_0 + \rho_1 E_t p_{t+1} = \rho_0 + \rho_1\left[\alpha_0 + (\alpha_1 + \beta_1 j)(1 - S_t) - (\alpha_2 + \beta_2 j)S_t + \alpha_3 p_t\right].$$

[18] Technically, we should have a further Lagrange multiplier, τ_t, and include $\tau_t(1 - p_t)$ in the Lagrange function to take fully into consideration the case of $0 \leq p_t \leq 1$. For simplicity, we assume $0 < p_t < 1$ (and $0 < h_t < 1$) thus $\tau_t = 0$, and thus ignore it in the Lagrange function here (and below). We also assume that the technical condition for $0 \leq S_t \leq 1$ is satisfied.

Substituting this $E_t\lambda_{t+1}$ in (6) and (7) respectively, solving for ρ_0 and ρ_1, we finally arrive at the following proposition regarding the solution for S_t in the case of contagion, denoted as S_t^c.[19]

Proposition 2 *In dynamic setting, the optimal monetary policy dealing with contagion case is:*

$$S_t^c = \gamma_0 + \gamma_1 p_t, \tag{9}$$

whereby:

$$\gamma_0 = 1 - \frac{\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta}}{2 - \delta\alpha_3}\frac{(n + B_2 j)x}{j^2 k - (n + B_2 j)x} + \frac{1}{2 - \delta\alpha_3}\frac{\alpha_0 - \alpha_2 - \beta_2 j}{\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j},$$

$$\gamma_1 = \frac{\sqrt{\delta^2\alpha_3^2 - \delta}}{\delta(\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j)}.$$

This result has the following implications. Notice first that $\gamma_1 > 0$ holds for all the given parameter settings, as long as $\delta\alpha_3^2 > 1$, i.e. $\alpha_3 > 1/\sqrt{\delta} > 1$, which we will further discuss in connection with the stability condition below. Therefore, if p_t rises, i.e., there is a structural shift making the banking system less stable, then the CB always wants to accommodate banks' calls for assistance more frequently, i.e. $\partial S_t^c/\partial p_t > 0$, regardless whether their sizes are above the threshold level or not. Thus it will accommodate banks' calls for assistance more frequently than in the static case and rescue "smaller" as well as all big banks. This should not be surprising when contagion is the main concern of the CB.

Moreover, the system will tend to a long-run equilibrium value for p, set as p^c, where p^c is given by the following equation

$$\begin{aligned} p^c \quad = \quad & \frac{\delta}{(2 - \delta\alpha_3)(\sqrt{\delta^2\alpha_3^2 - \delta} - \delta\alpha_3 - \delta)}\{(1 - \delta\alpha_3)(\alpha_0 - \alpha_2 - \beta_2 j) \\ & + \left(\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta}\right)(\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j)\frac{(n + B_2 j)x}{j^2 k - (n + B_2 j)x}\}. \end{aligned}$$

Since $\sqrt{\delta^2\alpha_3^2 - \delta} - \delta\alpha_3 - \delta > 0$ and $\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta} > 0$, then if $1 - \delta\alpha_3 > 0$ (that is $1/\sqrt{\delta} < \alpha_3 < 1/\delta$) and hence $2 - \delta\alpha_3 > 0$, the value of p^c critically depends

[19] It is easy to check that the sufficient conditions for global optimization in this case of contagion, and moral hazard below, are satisfied.

on whether $j^2k - (n + B_2j)x > 0$ or not. If $j^2k - (n + B_2j)x > 0$, that is the cost of LOLR is smaller than that of OMO for a given j in the static model, then $\frac{(n+B_2j)x}{j^2k-(n+B_2j)x} > 0$. Comparative statics analysis on p^c in this case indicates that p^c goes up if α_0, $(\alpha_1 + \beta_1 j)$, or $(n + B_2j)x$ goes up, or j^2k goes down. These results are consistent with intuition, because when $j^2k - (n + B_2j)x > 0$, the CB has more incentive to provide LOLR. A higher α_0 implies a higher constant risk level in the banking system; a higher $\alpha_1 + \beta_1 j$ implies a stronger effect of OMO on the risk level of the banking system; a higher $(n+B_2j)x$ implies higher costs of LOLR, and a lower j^2k implies lower costs of OMO. The effect of $\alpha_2 + \beta_2 j$ on p^c is more complicated, as the main instrument in this case is LOLR. The system will fluctuate around p^c as ε_t varies stochastically.

If $j^2k - (n + B_2j)x < 0$, that is the cost of LOLR is bigger than that of OMO for a given j in the static model, however, then $\frac{(n+B_2j)x}{j^2k-(n+B_2j)x} < 0$, and for $p^c > 0$ we need

$$(1-\delta\alpha_3)(\alpha_0-\alpha_2-\beta_2 j) > \left(\delta\alpha_3 - \sqrt{\delta^2\alpha_3^2 - \delta}\right)(\alpha_1+\beta_1 j+\alpha_2+\beta_2 j)\frac{(n + B_2j)x}{(n + B_2j)x - j^2k}.$$

Comparative statics analysis on p^c in this case indicates that p^c goes up if α_0, j^2k goes up, or $(\alpha_2 + \beta_2 j)$, $(n + B_2j)x$ goes down. These results are also consistent with intuition, because when $j^2k - (n + B_2j)x < 0$, the CB has more incentive to provide OMO rather than LOLR. A lower $\alpha_2+\beta_2 j$ implies a weaker effect of providing LOLR on the risk level of the banking system. The effect of $\alpha_1 + \beta_1 j$ on p^c is more complicated, as the main instrument in this case is OMO.[20]

[20]Notice further that, if failures have occurred in a series of large banks, i.e., $j^2k-(n+B_2j)x > 0$, then p^c may even be close to its upper bound, 1, for some parameter configurations. The intuition is that when the CB is only concerned with contagious risk, which is more severe for large banks, it would rescue too many banks so that the banking system risk level would then become extremely high. Similarly, if the only failures to have occurred were among small banks, i.e., $j^2k-(n+B_2j)x <$ 0, p^c may even be close to its lower bound, 0, for some parameter configurations. The intuition is that even when the CB is concerned with contagious risk, yet the troubled banks have been small, it would limit access to LOLR and rather use OMO, so that the banking system risk level would be extremely low.

Furthermore, from the equation of motion for p_t above and substituting S_t, we get:

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j)(1 - \gamma_0) - (\alpha_2 + \beta_2 j)\gamma_0 + \left[\alpha_3 - \sqrt{\alpha_3^2 - 1/\delta}\right] p_t + \varepsilon_{t+1}.$$

Thus in addition to the required stability condition from p^c, $j^2 k \neq (n + B_2 j)x$, the stability condition (on α_3 and δ) is $\alpha_3 - 1 < \sqrt{\alpha_3^2 - 1/\delta}$. Combining this condition with $1/\sqrt{\delta} < \alpha_3 < 1/\delta$, and noticing that $\frac{1+\delta}{2\delta} > \frac{1}{\sqrt{\delta}}$ for $0 < \delta < 1$, we thus have the stability condition for p_t:

$$\begin{cases} \frac{1+\delta}{2\delta} < \alpha_3 < \frac{1}{\delta}, \\ j^2 k \neq (n + B_2 j)x. \end{cases} \tag{10}$$

It is easy to check that for $0 < \delta < 1$ there is a non-empty set for α_3.

Finally, notice that the result of the above proposition can also be interpreted in terms of bank size j. Setting $S_t^c = \widehat{S}$ in (9), where $\widehat{S}$ is close to the upper bound of S_t and the CB will accommodate bank j's request for LOLR if $S \geq \widehat{S}$, we get:

$$\gamma_0(j) + \gamma_1(j) p_t = \widehat{S}. \tag{11}$$

Equation (11) becomes a cubic function of j after we multiply $[j^2 k - (n + B_2 j)x](\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j)$ on its both sides. Thus there may exist either one or three real-number roots, which are functions of all the parameters and variables including p_t. Therefore, a same size bank may be rescued by the CB for a given p_t, but may not be rescued for another value of p_{t+1}. This shifting of CB's policy reflects the time dynamics and is caused by the changes in the state-variable p_t.

Even for the same value of p_t, the CB may optimally rescue only these banks that have the "right sizes". As the above cubic equation (11) may have three real-number roots, which are non-negative yet smaller than D^*, these roots also clearly define useful bank size categories and the CB should rescue these banks within such categories, but not to do so for those banks which are not in the categories.

For a variety of possible parameter settings, some small or large banks may be rescued, while banks whose sizes are intermediate may not be. Consequently, the CB's optimal behavior may appear non-monotonic in bank size and thus ambiguous if viewed from outside, which will further enhance the constructive ambiguity which we described in the above section.

Next we turn to the case of moral hazard.

3.2 Moral Hazard in Dynamic Model

We focus on moral hazard by switching off contagion (i.e., holding p constant), but allowing universal risk preference (h) to increase alongside with the probability of a bank requiring assistance also being insolvent (i.e., x rises).

We assume that x is a linear function of h, and without loss of generality we set the coefficient equal to unity, so $x_t = h_t$. With p constant, we can drop that from the CB's objective function, which involves setting S_t so as to minimize:

$$\min_{S_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) h_t S_t \right] \right\}, \tag{12}$$

subject to the equation of motion for h, whereby:

$$h_{t+1} = a_0 - (a_1 + b_1 j)\left(1 - S_t\right) + (a_2 + b_2 j) S_t + a_3 h_t + e_{t+1}. \tag{13}$$

Where e_{t+1} is a stochastic term with mean zero and a constant variance, and $a_1 > 0$, $b_1 > 0$, $a_2 > 0$, $b_2 > 0$, with $a_1 < a_2$ and $b_1 < b_2$.

Again, the Lagrange function of this question is:

$$\begin{aligned} L \;=\; & E_0 \{ \sum_{t=0}^{\infty} \delta^t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) h_t S_t \right] \\ & - \delta^{t+1} l_{t+1} \left[h_{t+1} - a_0 + (a_1 + b_1 j)\left(1 - S_t\right) - (a_2 + b_2 j) S_t - a_3 h_t - e_{t+1} \right] \}. \end{aligned}$$

The FOC of this Lagrange with respect to S_t and h_t yield:

$$j^2 k - (n + B_2 j) h_t - \delta(a_1 + b_1 j + a_2 + b_2 j) E_t l_{t+1} = 0, \tag{14}$$

$$(n + B_2 j) S_t + \delta a_3 E_t l_{t+1} = l_t. \tag{15}$$

Again, we can conjecture l_t as a linear function:

$$l_t = r_0 + r_1 h_t,$$

thus,

$$E_t l_{t+1} = r_0 + r_1 E_t h_{t+1} = r_0 + r_1 \left[a_0 - (a_1 + b_1 j)(1 - S_t) + (a_2 + b_2 j) S_t + a_3 h_t \right].$$

Substituting this $E_t l_{t+1}$ in (14) and (15) respectively, solving for r_0 and r_1, we finally arrive at the following proposition regarding the solution for S_t in the case of moral hazard, denoted as S_t^m.

Proposition 3 *In dynamic setting, the optimal monetary policy dealing with moral hazard case is:*

$$S_t^m = g_0 + g_1 h_t, \tag{16}$$

whereby:

$$g_0 = 1 + \frac{\delta a_3 - \sqrt{\delta^2 a_3^2 - \delta}}{2 - \delta a_3} + \frac{1}{2 - \delta a_3} \frac{a_0 + a_2 + b_2 j}{a_1 + b_1 j + a_2 + b_2 j},$$

$$g_1 = -\frac{\sqrt{\delta^2 a_3^2 - \delta}}{\delta (a_1 + b_1 j + a_2 + b_2 j)}.$$

This result has the following implications. Notice first that $g_1 < 0$ holds for all given parameter settings. Therefore, if h_t rises, i.e., there is a structural shift making the risk level in the banking system higher, then the CB will accommodate less often, i.e. $\partial S_t^m / \partial h_t < 0$, regardless of the banks' sizes; and vice versa. This is obviously the opposite case to contagion, whereby $\partial S_t^c / \partial p_t > 0$, and thus the CB always wants to accommodate banks' calls for assistance more often.

The system will tend to a long-run equilibrium value for h, set as h^m, where h^m is given by the following equation

$$h^m = \frac{\delta[(3 - \delta a_3)(a_0 + a_2 + b_2 j) + (\delta a_3 - \sqrt{\delta^2 a_3^2 - \delta})(a_1 + b_1 j + a_2 + b_2 j)]}{(2 - \delta a_3)(\sqrt{\delta^2 a_3^2 - \delta} - \delta a_3 - \delta)}.$$

For $\delta a_3^2 < 1$ and $\delta a_3 < 2$, comparative statics analysis on h^m indicates that h^m goes down if a_0, $a_1 + b_1 j$, or $a_2 + b_2 j$ goes down.

The equilibrium risk level in moral hazard does not so critically depend on the sizes of failing banks as it did in the previous contagion case. Although the equilibrium risk level does depend on the average size of failing banks, in particular the higher is that average size, the higher is the equilibrium risk level for $a_1 + b_1 j > a_0$, the difference is much smaller than in contagion. These results are again consistent with intuition, because when moral hazard is the sole concern of the CB, then it should always have strong incentive to reject the request from the troubled banks for LOLR, and as a result of such an extreme policy, the equilibrium risk level in the banking system should be quite low, for small and large banks as well.

Furthermore, from the equation of motion for h_t above and substituting S_t, we get:

$$h_{t+1} = a_0 + a_2 + b_2 j - (a_1 + b_1 j + a_2 + b_2 j) g_0 + \left(a_3 - \sqrt{a_3^2 - 1/\delta} \right) h_t + e_{t+1}.$$

Thus, similar to the case of contagion alone, the stability conditions are:

$$\delta(a_3 - 1) < \sqrt{\delta^2 a_3^2 - \delta}.$$

Again combining this condition with $1/\sqrt{\delta} < a_3 < 1/\delta$, and noticing that $\frac{1+\delta}{2\delta} > \frac{1}{\sqrt{\delta}}$ for $0 < \delta < 1$, we thus have:

$$\frac{1+\delta}{2\delta} < a_3 < \frac{1}{\delta}. \tag{17}$$

And it is easy to check that there is a non-empty set for a_3.

In the case of moral hazard, bank size does not play a key role. When contagion is the main concern of the CB, it is the big banks which worry the CB most, and require the CB's prompt LOLR action. If moral hazard is the main concern of the CB, CB's rescuing policy is more uniform across banks with different sizes. The banking system can be more easily stabilized (than that in the case of contagion alone); and the stability conditions do not depend on the size of the illiquid banks.

Once again, the threshold (j) for LOLR with moral hazard is time-varying. Again setting $S_t^m = \widehat{S}$, we get:

$$g_0(j) + g_1(j) h_t = \widehat{S}. \tag{18}$$

Multiplying $(a_1 + b_1 j + a_2 + b_2 j)$ on both sides of equation (18), it becomes a linear increasing function of j. Thus there exists a real-number root, which is a function of all the parameters and variables including h_t. Therefore, a same size bank may be rescued by the CB for a low value of h_t, but may not be rescued when h_{t+1} increases. This changing of CB's policy reflects the time dynamics and is caused by the changes in time-varying state-variable, h_t.

As the left-hand-side of equation (18) is a monotonic increasing function of j, and thus a cut-off point in j, $\widehat{j}$, exists for a given h_t and a rescue threshold $\widehat{S}$, the CB will rescue any bank with size above $\widehat{j}(h_t, \widehat{S})$. Therefore, "too big too fail" holds even in the dynamic setting with moral hazard being the main concern.

3.3 Contagion and Moral Hazard in Dynamic Model

When both p and x are allowed to be time-varying, we assume (as above) that x is a linear function of h, and we set the coefficient equal to unity, so $x_t = h_t$. Thus the new problem is to set S_t optimally so as to minimize:

$$\min_{S_t} E_0 \left\{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) h_t S_t \right] \right\}, \tag{19}$$

subject to the equations of motion for h_t and p_t, whereby:

$$h_{t+1} = a_0 - (a_1 + b_1 j) \left(1 - S_t\right) + (a_2 + b_2 j) S_t + a_3 h_t + \varepsilon_{t+1}, \tag{20}$$

$$p_{t+1} = \alpha_0 + (\alpha_1 + \beta_1 j) \left(1 - S_t\right) - (\alpha_2 + \beta_2 j) S_t + \alpha_3 p_t + e_{t+1}, \tag{21}$$

where again ε_{t+1} and e_{t+1} are stochastic terms with mean zero and constant variances, and all other coefficients are positive.

Again, the Lagrange function of this question is:

$$\begin{aligned} \pounds \;\; = \;\; & E_0 \{ \sum_{t=0}^{\infty} \delta^t p_t \left[j^2 k \left(1 - S_t\right) + (n + B_2 j) h_t S_t \right] \\ & - \delta^{t+1} l_{t+1} \left[h_{t+1} - a_0 + (a_1 + b_1 j) \left(1 - S_t\right) - (a_2 + b_2 j) S_t - a_3 h_t - \varepsilon_{t+1} \right] \\ & - \delta^{t+1} \lambda_{t+1} \left[p_{t+1} - \alpha_0 - (\alpha_1 + \beta_1 j) \left(1 - S_t\right) + (\alpha_2 + \beta_2 j) S_t - \alpha_3 p_t - e_{t+1} \right] \}. \end{aligned}$$

The FOC of this Lagrange with respect to S_t, h_t and p_t yield, respectively:

$$p_t \left[j^2 k - (n + B_2 j) h_t \right] - \delta (a_1 + b_1 j + a_2 + b_2 j) E_t l_{t+1} + \delta (\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j) E_t \lambda_{t+1} = 0,$$

$$p_t(n + B_2 j)S_t + \delta a_3 E_t l_{t+1} = l_t,$$

$$j^2 k\,(1 - S_t) + (n + B_2 j)h_t S_t + \delta\alpha_3 E_t \lambda_{t+1} = \lambda_t.$$

Intuitively, the joint case of contagion and moral hazard is some form of convex combination of each case alone, with both h_t and p_t changing over time. As discussed above, the contagion case is a special case of the joint case, with h_t fixed; and similarly, the moral hazard case is a special case of the joint case, with p_t fixed.

To solve a problem with a quadratic first order conditions like ours, we shall start with linearization of the FOC around the steady state.[21] Doing so leads to:

$$\begin{aligned} 0 &= (n + B_2 j)\overline{h}\overline{p} - (n + B_2 j)\overline{p}h_t + [j^2 k - (n + B_2 j)\overline{h}]p_t \\ &\quad -\delta(a_1 + b_1 j + a_2 + b_2 j)E_t l_{t+1} + \delta(\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j)E_t \lambda_{t+1}, \end{aligned}$$

$$l_t = (n + B_2 j)(1 - \overline{S})\overline{p} - (n + B_2 j)\overline{S}p_t - (n + B_2 j)\overline{p}\,(1 - S_t) + \delta a_3 E_t l_{t+1},$$

$$\lambda_t = (n{+}B_2 j)\overline{h}\left(1 - \overline{S}\right){+}(n{+}B_2 j)\overline{S}h_t{+}[j^2 k\,(1 - S_t){-}(n{+}B_2 j)\overline{h}\,(1 - S_t){+}\delta\alpha_3 E_t \lambda_{t+1}.$$

Where $\overline{h}$ and $\overline{p}$ denote the steady state for h_t and p_t respectively, and $\overline{S}$ denotes S_t evaluated at $\overline{h}$ and $\overline{p}$.

With a linear FOCs, we can conjecture

$$\begin{pmatrix} l_t \\ \lambda_t \end{pmatrix} = \begin{pmatrix} r_0 \\ \rho_0 \end{pmatrix} + \begin{pmatrix} r_1 & 0 \\ 0 & \rho_1 \end{pmatrix} \begin{pmatrix} h_t \\ p_t \end{pmatrix}.$$

Following the same procedure as in the case of contagion, or moral hazard, we finally arrive at the following proposition regarding the solution for S_t in the joint case of contagion and moral hazard, denoted as S_t^*.

Proposition 4 *In the dynamic setting, the optimal monetary policy dealing with both contagion and moral hazard is:*

$$S_t^* = \mu_0 + \mu_h h_t + \mu_p p_t, \tag{22}$$

[21] Since both the objective function and two constraints are convex in S_t, h_t and p_t, global optimality is warranted. Consequently we can linearize the first order conditions around the steady states to solve the problem analytically.

whereby:

$$
\begin{aligned}
\mu_0 &= \overline{S} + \mu_h \overline{h} - \mu_p \overline{p}, \\
\mu_h &= -\frac{2\overline{p} + \delta a_3 \Delta_h}{\delta \Delta_h (a_1 + b_1 j + a_2 + b_2 j)}, \\
\mu_p &= \frac{2[j^2 k - (n + B_2 j)\overline{h}] + \delta \alpha_3 \Delta_p}{\delta \Delta_p (\alpha_1 + \beta_1 j + \alpha_2 + \beta_2 j)}; \\
\Delta_h &= (a_1 + b_1 j + a_2 + b_2 j)\overline{S} - 2a_3 \overline{p} - \sqrt{(a_1 + b_1 j + a_2 + b_2 j)^2 \overline{S}^2 - 4/\delta}, \\
\Delta_p &= (n + B_2 j)\overline{S} - 2\alpha_3 [j^2 k - (n + B_2 j)\overline{h}] \\
&\quad - \sqrt{\{(n + B_2 j)\overline{S} - 2\alpha_3 [j^2 k - (n + B_2 j)\overline{h}]\}^2 - 4[j^2 k - (n + B_2 j)\overline{h}]^2/\delta}.
\end{aligned}
$$

This result has the following implications. Notice first that $\mu_h < 0$ holds, except for the case in which $\Delta_h < 0$ and $2\overline{p} + \delta\alpha_3\Delta_h > 0$, which happens if $\delta a_3 \overline{p}(a_1 + b_1 j + a_2 + b_2 j) > 1$ and $2\overline{p} + \delta\alpha_3\Delta_h > 0$. In general, we shall expect $\mu_h < 0$, and thus similar to the case of moral hazard alone, if h_t rises, i.e., there is a structural shift making the risk level in the banking system higher, then the CB will accommodate less often, i.e. $\partial S_t^* / \partial h_t < 0$, regardless whether the banks' sizes are above or below some cutoff points; and vice versa. Notice that, unlike in the case of moral hazard alone, μ_h becomes dependent on many parameters, including the equilibrium states, $\overline{h}$, $\overline{S}$, and $\overline{p}$ which were not in the moral hazard case alone.

Moreover, because $\Delta_p > 0$, $\mu_p > 0$ if and only if

$$2[j^2 k - (n + B_2 j)\overline{h}] + \delta\alpha_3\Delta_p > 0.$$

A strong condition for this to be true is

$$j^2 k > (n + B_2 j)\overline{h},$$

that is the cost of LOLR is smaller than that of OMO when the risk level is in equilibrium. More precisely,

$$j \geq \overline{j}^* \equiv \frac{B_2 \overline{h} + \sqrt{B_2^2 \overline{h}^2 + 2k(n\overline{h} - \delta\alpha_3\Delta_p)}}{2k},$$

thus $\overline{j}^* < \overline{j}(x = \overline{h})$ and it is smaller than the cutoff size in the static model.

We may compare this result with that for contagion alone. In the case of contagion alone, $\gamma_1 > 0$ holds regardless of bank size, and so the CB always had an incentive to rescue banks. In the joint case, the CB will only provide LOLR for large banks. The CB is still willing to provide LOLR even if moral hazard effects are considered, but its incentives to provide LOLR are not as strong as in the case where moral hazard is excluded.

Similarly, the system will tend to a long-run equilibrium value for both h and p, set as $\overline{h}$ and $\overline{p}$, which are more complicated than in the case of moral hazard or contagion alone. But it is quite easy to see that both the equilibrium risk levels, $\overline{h}$ and $\overline{p}$, critically depend on the average size of failing banks. $\overline{p}$ critically depends on bank size, because μ_p critically depends on bank size. To see that $\overline{h}$ also critically depends on bank size, we shall take notice that the equilibrium $\overline{p}$ ultimately affect $\overline{h}$.

The dynamics of the joint case of contagion and moral hazard are more interesting and complicated. From our above analysis, we can see that the system will, as before, fluctuate with the stochastic shocks, but also both contagion and moral hazard provide an inbuilt cycling mechanism. When h_t is (temporarily) low, the CB will be induced to accommodate more, but that will signal the commercial banks to raise h_t and raise the contagion risk p_t, which will cause the CB to refuse accommodation more often, and so on. For plausible values of the coefficients, however, we would expect this cycle to be damped. But the combination of stochastic shocks to riskiness together with this inbuilt damped cycling mechanism will always leave the choice of whether to accommodate, or not, fluctuating around its long-run equilibria.

Finally, there may be multiple values of j around which CB's policy should shift, and such values will always be changing over time, more often and perhaps more strongly than in the contagion case alone, as can be seen more clearly below.

If we set $S_t^* = \widehat{S}$ in (22), we get:

$$\mu_0(j) + \mu_1(j)h_t + \mu_2(j)p_t = \widehat{S}. \tag{23}$$

This is a high-order equation in j, hence there may exist several real-number roots which are obviously functions of all the parameters and variables including h_t and p_t. Thus, when h_t and/or p_t change, CB should adjust its rescuing policy accordingly. Therefore, a bank of equal size may be rescued by the CB for a set of values for h_t and p_t, but may not be rescued when $h_{t+1}$and p_{t+1} change. Such a shift in CB's policy reflects the time dynamics and is caused by the changes in time-varying variables, h_t and p_t.

Moreover, even for the same value of h_t and p_t, the CB may optimally rescue only these banks that have the "right sizes". For the above high-order equation (23), there may be several real number roots which are non-negative yet smaller than D^*. These roots also clearly define bank size categories and the CB should rescue these banks within such categories, but not rescue banks outside these categories.

4. CONCLUSION

This paper has developed a model of the lender of last resort. In a simple static setting, the CB should only rescue banks above a threshold size. This result provides an analytical basis for the well known "too big too fail" syndrome. If that key threshold size were known to commercial banks, it would influence their risk preferences. To avoid this the regulatory authorities should, and do, use "constructive ambiguity" to make their decisions on which banks they are likely to rescue.

In a dynamic setting, wherein both the probability of a failure and the likelihood of a bank requiring LOLR being insolvent in each period are a function of CB's prior actions, which then influence the actions of banks and depositors, we focus our analysis on the effects of contagion and/or moral hazard. We show that CB's optimal rescuing policy, whether to support or not, depends not only on bank size, but also on the time-varying variables, such as the probability of a failure and the likelihood of a bank requiring LOLR being insolvent.

Unlike the static setting wherein the CB only rescues banks above a single threshold size, in dynamic setting with concern on contagion CB's optimal rescuing policy may be non-monotonic in bank size, and its optimal policy is time-varying. More importantly, we have found that if contagion is the main concern, then the CB in general would have an excessive incentive to rescue banks through LOLR, though its incentives to rescue big (small) banks are very strong (weak) and thus the equilibrium risk level is high (low). If moral hazard is the main concern, then the CB in general would have less incentive to rescue banks through LOLR than in the dynamic setting with contagion alone, and it should only rescue banks above a threshold size. When both contagion and moral hazard are included as main concerns, then the CB's incentives to rescue though LOLR are stronger than in the static setting but weaker than in the dynamic setting with contagion alone, and the CB's optimal policy in handling moral hazard is similar to the dynamic setting with moral hazard alone.

A key result coming out of our model is that contagion is the key factor affecting CB's incentive in providing LOLR, while moral hazard is not. When contagion is the main concern, the CB has a very strong incentive to provide LOLR. When moral hazard is also included, in the joint case, even though it weakens the CB's incentive to rescue in general, its effects are quite week, and the qualitative features of CB's incentive remain the same as when contagion is the main concern. This is so because moral hazard can be viewed as an unpleasant by-product of contagion. If it was not for worries about contagion, then CB's incentive to provide LOLR would be very weak, and consequently there would be very little moral hazard.

This conclusion has some implications for the ongoing debates over CB's, and in particular the IMF's, rescuing policy in financial crises. When contagion becomes a main concern, even if moral hazard is also present, LOLR becomes (perceived as) necessary and justified. Attacks on such LOLR policies, largely based on arguments from moral hazard, are insufficient and unsatisfactory unless they also address the possibility of contagion.[22]

Turning finally to further possible research, in our model we have assumed that commercial bank managers' appetite for risk remains constant as size varies. This

[22] For international perspectives on the LOLR function, see Fischer (1999), Giannini (1999), and Goodhart and Huang (2000).

was because we assumed that the incentive for risk-taking inherent in too-big-to-fail was (roughly) balanced by the greater risk aversion of managers in high status large banks. This assumption can be examined. In future work, we intend to model the incentives on commercial bank managers, in such a game, more rigorously. We also wish to examine the effects of contagion and moral hazard on the optimal decision of international lender of last resort.

Appendix: Bank Failures Cause Macroeconomic Uncertainty — Further Empirical Evidence

Almost by definition, a panic is a situation in which behavior becomes less predictable, and sensible decision-making more difficult.[23] For the purpose of this exercise we have assumed that the monetary authorities know exactly what is the socially optimal level of bank deposit, D^*. In panic conditions this is less likely to be true. In a panic, such as in Russia and Indonesia in 1998, the external and internal value of the currency is likely to be under threat, so should one keep D^* down? But at the same time there is likely to be economic dislocation and a credit crunch, which suggests a case for a higher D^*.

Such examples can be multiplied. But we shall stick to our strong assumption that D^* is known and given. So the problem in our model for a CB is to vary H, the high powered money base, by OMO, so as to hit D^*. What we shall assume here is that the CB tries to estimate the appropriate level of OMO by predicting the H/M ratio on the basis of a simplified (ARIMA) model, as follows:

$$\left(\frac{H}{M}\right)_t = d_0 + d_1 \left(\frac{H}{M}\right)_{t-1} + d_2 \left(\frac{H}{M}\right)_{t-2} + d_3 i_t + d_4 i_{t-1} + e_t. \tag{24}$$

Our hypothesis is that the residuals from this equation will be higher during, and in the immediate aftermath of, a panic. If so, we take this as evidence that it will be much harder for the CB to adjust OMO and H, so that $D = D^*$.

In the three cases that we examined, this hypothesis seemed to hold. These three cases were:

(1) USA: 1872-1914 and 1921-1940, annual data;[24]

[23] See, for example, "panic" in the Shorter Oxford English Dictionary (3rd Edition, Reprinted with Corrections, 1959): *"A condition of widespread apprehension in relation to financial and commercial matters, leading to hasty and violent measures, the tendency of which is to cause financial disaster."*

[24] For the USA, the monetary data are taken from Friedman and Schwartz (1982). Data on bank failures (deposits, post 1918, and number of suspended banks), are taken from *Historical Statistics of the United States – Colonial Times to 1970.* The monetary data for H and M are annual averages; call money rates are annual averages of monthly data.

(2) Australia: 1861-1913, annual data;[25]

(3) Mexico: May 1980 to November 1997, monthly data.[26]

In Chart 1 (USA), particular periods of instability in the H/M rates are 1873-1875, 1884-1886, 1893-1895 and 1907-1909. 1873 **was** a financial and economic panic even though the number of bank failures was not particularly high, and 1893 and 1907 were amongst the most severe banking panics of the National Banking era (see Sprague, 1910). The H/M ratio would rise (econometrically significant) when bank suspensions increased, and fall when suspensions fell back again.

The inter-war experience is even better known, largely from the work of Friedman and Schwartz (1963). The H/M ratio shot up amidst the bank closures in the early 1930s, so that even though the Fed expanded H at an unusually rapid rate (see Kaldor, 1958), M still fell. As closures declined in the mid 1930s, the H/M ratio became more predictable again, only for the residuals from our basic equation (24) to rise, alongside further bank suspensions at the end of the period.

[Insert Charts 1 and 2 here]

For Australia, we do not have accurate data on bank failures year by year. What we do know, however, is that 1893 was a year of massive bank failures, so under our hypothesis that residual from our simple, predictive equation (24) should be larger in 1893. Chart 3 below shows that this was indeed so.

[Insert Chart 3 here]

[25] For Australia, the monetary data were mainly taken from Butlin, Hall and White (1971) and the interest rate data from Mitchell (1983). For deposits, the data are for the final quarter (average) of each year. For reserves, data are reported as of December for each year until 1900. For 1900-1912, data are as of June for each year, and so were averaged over two years in order to centre on December. Data on currency in circulation (end year) are taken from Mitchell (1983). Both H and M are thus approximately end-year.

[26] The monthly data for Mexico are taken from IFS over the period May 1990 till November 1997. The interest rate used is the Treasury bill rate, reported as a monthly average.

For Mexico, under our hypothesis the residuals from our simple predictive equation (24) should increase during and immediately after the crisis at end 1994/start 1995. The data shown in Chart 4 are consistent with that hypothesis.

[Insert Chart 4 here]

REFERENCES

[1] Aghion, P., P. Bolton, and M. Dewatripont (2000), "Contagious Bank Failures," mimeo, Princeton University.

[2] Allen, F. and D. Gale (1998b), "Optimal Financial Crises", *Journal of Finance*, 53(4), 1245-1284.

[3] Allen, F. and D. Gale (2000), "Financial Contagion", *Journal of Political Economy.*

[4] Bagehot, W. (1873), *Lombard Street: A Description of the Money Market.* London: H.S. King.

[5] Bank of England (1996), *Financial Stability review*, No. 1, Autumn, 1996.

[6] Bank of England (1992, 1993), *Annual Report*, 1991-1992, and 1992-1993.

[7] Bernanke, B. (1983), "Non Monetary Effects of Financial Crisis in Propagation of the Great Depression", *American Economic Review*, 73(3), 257-276.

[8] Bordo, M.D. (1990), "The Lender of Last Resort: Alternative Views and Historical Experience", Federal Reserve Bank of Richmond, *Economic Review*, 18-29.

[9] Butlin, S.J., A.R. Hall and R.C. White (1971), "Australian Banking and Monetary Statistics, 1817 - 1945", Federal Reserve Bank of Richmond, *Occasional Paper, No. 4A*.

[10] Caballero, R. and A. Krishnamuthy (1999), "Emerging Markets Crisis: An Asset Markets Perspective", IMF WP/99/129, International Monetary Fund, Washington, DC.

[11] Chow, G.C. (1997), *Dynamic Economics.* New York: Oxford University Press.

[12] Diamond, D. and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy*, 91, 401-419.

[13] Diamond, D., and R.G. Rajan (1998), "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of banking," mimeo, Graduate School of Business, University of Chicago.

[14] Enoch, C., P. Stella and M. Khamis (1997), "Transparency and Ambiguity in Central Bank Safety Net Operations", *IMF Working Paper*, Washington, DC: International Monetary Fund.

[15] Fischer, S. (1999), "On the Need for an International Lender of Last Resort", Speech Given at the Joint Luncheon of the American Economic Association and the American Finance Association, New York, January 3, 1999.

[16] Freixas, X., B. Parigi and J.C. Rochet (1998), "The Lender of Last Resort: A Theoretical Foundation", Memo.

[17] Friedman, M. and A. Schwartz (1963), *A Monetary History of the United States, 1867-1960.* Princeton, NJ: Princeton University Press.

[18] Friedman, M. and A. Schwartz (1982), *Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices and Interest Rates, 1867 - 1975.* Chicago, IL: University of Chicago Press.

[19] Giannini, C. (1999), ""Enemy of None But a Common Friend of All?" An International Perspective on the Lender-of-Last-Resort Function", *Essays in International Finance No. 214*, Princeton University.

[20] Goodfriend, M. and R. King (1988), "Financial Deregulation, Monetary Policy, and Central Banking", *Federal Reserve Bank of Richmond Economic Review*, May/June, 3-22.

[21] Goodhart, C.A.E. (1988), *The Evolution of Central Bank.* Cambridge, MA: MIT Press.

[22] Goodhart, C.A.E. (1995), *The Central Bank and the Financial System.* Cambridge, MA: MIT Press.

[23] Goodhart, C.A.E. and H. Huang (2000), "A Simple Model of International Lender of Last Resort", WP/00/75, International Monetary Fund, Washington, DC.

[24] Goodhart, C.A.E. and D. Schoenmaker (1995), "Should the Functions of Monetary Policy and Bank Supervision be Separated?", *Oxford Economic Papers*, 47, 539-560.

[25] Gorton, G. (1985), "Bank Suspension of Convertibility", *Journal of Monetary Economics*, 177-193.

[26] Holmstrom, B. and J. Tirole (1998), "Private and Public Supply of Liquidity", *Journal of Political Economy*, 106(1), 1-40.

[27] Huang, H. (2000), "Financial Contagion: ABC Channels", *IMF Research Bulletin*, 1(2).

[28] Huang, H. and C. Xu (2000), "Financial Institutions, Financial Contagion, and Financial Crises", WP/00/92, International Monetary Fund, Washington, DC.

[29] Kaldor, N. (1958), "Monetary Policy, Economic Stability and Growth", Collected in Musella, M. and C. Panico ed., *The Money Supply in the Economic Process: A Post Keynesian Perspective.* Elgar Reference Collection. International Library of Critical Writings in Economics, vol. 60. Aldershot, U.K.: Elgar: 1995, 49-74.

[30] Kane, E.J. (1992), "The Incentive Incompatibility of Government-Sponsored Deposit-Insurance Funds", in Barth, J.R. and R.D. Brumbaugh eds. *The reform of federal deposit insurance: Disciplining the government and protecting taxpayers.* New York: Harper Collins,144-66.

[31] Kaufman,G.G. (1991), "Lender of Last Resort: A Contemporary Perspective", *Journal of Financial Services Research*, 5(2), 95-110.

[32] Kodres, L. and M. Pritsker (1998), "A Rational Expectations Model of Financial Contagion", mimeo, International Monetary Fund, Washington, DC.

[33] Rochet, J.C.. and J. Tirole (1996), "Interbank Lending and Systemic Risk", *Journal of Money, Credit and Banking*, 28:4, 733-762.

[34] Schwartz, A. (1988), "Financial Stability and the Federal Safety Net", in Haraf, W.S. and R.M. Kushmeider ed. *Restructuring banking and financial services in America,* Washington, D.C.: American Enterprise Institute for Public Policy Research distributed by UPA, Lanham, Md. and London, 34-62.

[35] Sprague, O.M.W. (1910), *History of Crisis under National Banking System.* National Monetary Commission, 61st Congress, 2nd session, Senate, Doc. No. 538.