

# The Instability of the Banking Sector and Macrodynamics: Theory and Empirics\*

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## Abstract

This paper studies the issue of local instability of the banking sector and how it may spillover to the macroeconomy. The banking sector is considered here as representing a wealth fund that accumulates capital assets, can heavily borrow and pays bonuses. We presume that the banking system faces not only loan losses but is also exposed to a deterioration of its balances sheets due to adverse movements in asset prices. In contrast to previous studies that use the financial accelerator – which is locally amplifying but globally stable and mean reverting – our model shows local instability and globally multiple regimes. Whereas the financial accelerator leads, in terms of econometrics, to a one-regime VAR we demonstrate the usefulness of a multi-regime VAR (MRVAR). We estimate our model for the US with a MRVAR using a constructed financial stress index and industrial production. We also undertake an impulse-response study with an MRVAR which allows us to explore regime dependent shocks. We show that the shocks have asymmetric effects depending on the regime the economy is in and the size of the shocks. As to the recently discussed unconventional monetary policy of quantitative easing, we demonstrate that the effects of monetary shocks are also dependent on the size of the shocks.

JEL classifications: E2, E6 and C13

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# 1 Introduction

As many of the historical financial crises have shown, the crises may have originated in adverse shocks to firms, households, foreign exchange, stock market or sovereign debt. Yet, as Reinhard and Rogoff (2009) and Gorton (2009, 2010) have demonstrated, the banking sector could seldom escape the crises. In fact, most of crises ended up as a meltdown of the banking sector, and the banking sector has usually exacerbated and amplified the crisis whatever origin it had. As Gorton (2010) shows, in the past loan losses and bank runs were the conventional mechanisms by which the crises were triggered, but more recently, banking crises seem to be strongly related to adverse shocks in asset prices and financial stress. We want to study how this channel may have some exacerbating or even destabilizing effects on the macroeconomy.

The issue is do we have proper models to explain this? Do we have models that help to understand this central aspect of sudden financial meltdowns? There are the earlier non-conventional studies by Kindleberger and Aliber (2005) and Minsky (1976, 1982) that view the role of credit as significantly amplifying forces. In Kindleberger it is the instability of credit, and in Minsky it is the way financing becomes de-linked from collaterals that contributes to a downward spiral once large real or financial shocks occur. This is surely an important tradition that captured many of the aspects of the boom-bust scenarios that we have seen historically.

Recently, there has been significant work by Greenwald and Stiglitz (1998), Bernanke, Gertler and Gilchrist (1999) that shows that finance has amplifying effects. In the DSGE tradition there is only a locally magnifying effect, through collaterals.<sup>1</sup> Collaterals rise at high level of economic activity, making credit available and cheap, and the reverse happens at low level of economic activity. The debt to asset value ratio is predicted to fall in the

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<sup>1</sup>Theoretical literature has studied the amplifying effect of shocks near the steady state, for example, see Carlstrom, Fuerst (2009), and Curdia and Woodford (2009a,b).

boom and rise in recessions.<sup>2</sup> Yet, in most models of this type, there is no tracking of the debt dynamics and the fragility of the debt dynamics does not come into play. The models are solved through local linearizations about a unique and stable steady state, and the amplifying effects occur only with respect to deviations from the steady state. The departure from the steady state is eventually mean reverting. Although the economy is accelerating, it will revert back to the steady state. Empirically, this is often shown in a one-regime VAR, see Gilchrist et al. (2009, 2010), Christensen and Dib (2008), and Del Negro et al. (2010).

Many students of the great depression developed the perception that locally destabilizing effects, arising from the banking sector, are missing in modern macroeconomic modeling. So far the financial accelerator theory has mainly been applied to firms and households. Yet, as the recent meltdown of the years 2007-9 has demonstrated, the financial accelerator applied to the balance sheets of banks seems to be destabilizing rather than mean reverting. There is this important work by Adrian and Shin (2009, 2010) and Brunnermeier and Pederson (2009), where financial intermediaries<sup>3</sup> are not able to fulfill their functions as intermediaries. Often they have to liquidate their capital, when asset prices get depressed and margin requirements in the money market rise, which forces the financial intermediaries to take a hair cut and to delever further, with another subsequent fall of asset prices reinforcing the downward trajectory. The depressed asset prices, generated by firesales of assets by some intermediaries, have external effects on the industry, and bank runs can exacerbate this effect, see Gorton (2010). There was a large body of literature that has shown that there might be a downward

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<sup>2</sup>This is for example empirically stated in Gilchrist et al (2010). Yet, as Geanakoplos (2010) mentions, the empirical measure is distorted through the way the debt asset ratio is measured, namely as total assets over equity. Equity value rises in the boom and falls in a recession.

<sup>3</sup>We also include here what has been called by Gorton (2010) the shadow banking system, such as investment firms, brokers and money market dealers. Those have been growing rapidly in the US in the last 15 to 20 years.

spiral through interconnectedness, interlinkages and contagion. Such studies have started with Greenwald and Stiglitz (1996) and continued with Adrian and Shin (2009, 2010), Gorton (2010), Geanakoplos (2010), Geanakoplos and Farmer (2009), and Brunnermeier and Sannikov (2010).

Following this line of research, we want to show how this process works through the balance sheets of banks. Banks in the first instance usually have extensive loan losses. This may be arising from default of the firm or household sector, the foreign sector or resulting from sovereign debt. The shocks to asset prices will affect the banks balance sheets through first loan losses but also and substantially through the asset and liability side of the balance sheets of banks.<sup>4</sup> This in turn affects the banks' availability of credit in the interbank credit market and the price of credit, thus the actual interest the banks have to pay.

Usually, with deteriorating balance sheets of the financial intermediaries due to loan losses and falling asset prices, the risk premium they are asked to pay in the interbank loan market rises rapidly. Frequently, banks have to liquidate more assets to keep liquidity and payment obligation afloat. With the value and of their capital basis shrinking, banks have to sell assets, and this might trigger a firesale of assets by some intermediaries, making their capital basis even weaker. This has effects on other intermediaries (as well as firms and households). The repo rate, the Ted spread, and credit spreads in general, indicating financial conditions, will rise. Thus one would expect low financial stress and narrow credit spreads in a period of high economic activity, and high financial stress and widened credit spreads in a period of low economic activity. Hence the dissipating liquidity, the credit constraints and credit spreads might be locally destabilizing rather than locally mean reverting.

The remainder of the paper is organized as follows. Section 2 builds up a model that reflects those features. Section 3 discusses some extensions. Sec-

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<sup>4</sup>As Gorton (2010) shows, this is often magnified through bank runs.

tion 4 solves numerically some model variants using dynamic programming. Section 5 employs a financial stress variable – that captures the financial conditions of banks – and industrial production to empirically estimate the model using a Multi-Regime Var (MRVAR). Section 6 concludes the paper.

## 2 The Basic Model

Next, let us present the above developed ideas in a more formal model. The best way to explain the model is to refer to the balance sheets of the financial intermediaries

Assets	Liabilities
$p_t k_t$	$d_t$
	$n_t = p_t k_t - d_t$
total assets	$\alpha(p_t k_t - d_t) + (1 - \alpha)(p_t k_t - d_t)$

Table 1: The Balance Sheet of Banks

On the left hand side there are assets, valued at current asset prices. On the right hand side there is debt  $d_t$  and net worth  $n_t = p_t k_t - d_t$ . Moreover, the equity might be divided up into inside equity  $\alpha(p_t k_t - d_t)$  and outside equity  $(1 - \alpha)(p_t k_t - d_t)$ . The latter may be state dependent.

Next, let us introduce the dynamics of the variables. The asset price, the capital stock and the debt may evolve as follows

$$dp_t = \mu_i p_t dt + \sigma_t p_t dZ_t \quad (1)$$

$$dk_t = (\varphi(i_t/k_t) - \delta)k_t dt + \sigma_t k_t dZ_t \quad (2)$$

$$dd_t = (rd_t - (ak_t - i_t))dt \quad (3)$$

The growth rate of asset prices follows a geometric Brownian motion. In fact, since it is formulated here as social planning or monopoly problem, prices will be implicit in the solution, given by the preferences and the equs. (2)-(3), see Brunnermeier and Sannikov (2010). But actual price movements can affect the dynamics when we consider the role of asset price shocks to financial intermediaries. The asset price shocks will reduce the collateral value of the financial intermediaries, and the fast depreciation of asset prices— possibly triggered by a firesale of assets— will have extensive externality effects on other intermediaries, leading to a general loss of net worth. This may magnify the downward movement of the spiral. Though, at first sight, the asset prices do not have a special role in the above equation, they will come into play below.

The assets of the financial intermediaries will increase with investment,  $i_t/k_t$ , the function  $\varphi(i_t/k_t)$  includes some adjustment cost which is concave in the argument, and  $\delta$  is a depreciation rate. The actual gross capital of the bank increases at the rate  $i_t/k_t$ . The debt evolves at a rate that is essentially determined by the excess spending of investment over capital income, which is defined here as  $ak_t$ . Investment in the second equation will generate a greater stock of assets for financial intermediaries, but the high rate of purchase of assets will increase their debt, once the investment spending exceeds their income. We have taken here the interest rate,  $r$ , to be paid on debt, as a constant, it may later be made endogenous depending on net worth. Note that only the first and second equations are stochastic.

So far we have neglected the bonuses of the executives which can be viewed to serve the consumption stream of the executives.<sup>5</sup> We can define the executives bonuses as an optimal consumption stream, to be derived optimally through some intertemporal decision making process. We can also have the investment being computed as optimal, with  $g_t = i_t/k_t$ . Then we

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<sup>5</sup>In Semmler and Bernard (2010) bonus payments of the six largest US investment banks are computed. Bonus payment, as a percent of revenues, went up from roughly 10 percent in 2000 to 35 percent in 2007, see figure 17 in appendix 2.

have the dynamic decision problem:

$$V(k, d) = \max_{c_t, g_t} \int_0^{\infty} e^{-rt} U(c_t) dt \quad (4)$$

s.t.

$$dp_t = \mu_t p_t dt + \sigma_t p_t dZ_t \quad (5)$$

$$dk_t = (\varphi(i_t/k_t) - \delta)k_t dt + \sigma_t k_t dZ_t \quad (6)$$

$$dd_t = (rd_t - (ak_t - i_t - c_t))dt \quad (7)$$

The latter model includes now bonus payments of the executives,  $c_t$ , which is used for a consumption stream.<sup>6</sup> Note that we have here  $g_t = i_t/k_t$ . Note also that in equ. (7) if the excess of spending for new assets and bonus payments exceeds the income generated, then the debt of the financial intermediary will rise. As mentioned before, for the problem of a social planner, which is equivalent to a monopoly problem<sup>7</sup> of the financial intermediary, the prices are endogenous and do not play a role at first.

We want to remark that the above is a standard model of wealth management as it is now commonly used for wealth management of financial intermediaries, see He and Krishnamurthy (2008). If we replace the constant income for a unit of wealth,  $a$  in  $ak_t$ , by a weighted average of risky and risk free returns of a wealth fund  $k_t$ , then the remaining parts of the equations above are reasonably familiar from the wealth management literature, see also Semmler et al. (2009). Yet the explicit equation for the evolution of debt of the financial intermediary, as represented in equ. (7), is missing. This reflects the innovative part of the model by Brunnermeier and Sannikov

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<sup>6</sup>In recent attempts of financial market reforms in Europe the cash payment of bonus payments is planned to be restricted to 20 percent of total bonus payments, the remaining part is only allowed to be paid out in subsequent years via common stocks. In our model we leave aside those complications.

<sup>7</sup>See Brunnermeier and Sannikov (2010).

(2010) and other recent literature:<sup>8</sup> Here then, the financial intermediaries are encouraged by more risk taking through transfer of risk to outside investors, consequently the financial intermediaries will build up their debt and thus their default risk.

Now let us derive a dynamic equation for the debt-asset ratio.<sup>9</sup> Let us take as the debt-asset ratio:  $d_t/k_t$ : We can rewrite this, for convenience, as  $\omega = -(d_t/k_t)$ .<sup>10</sup> Taking log and time derivative of this, we can write the asset accumulation and debt dynamics with the previous objective function of the financial intermediaries as:<sup>11</sup>

$$V(\omega_t) = \max_{\tilde{c}_t, g_t} \int_0^\infty e^{-rt} U(\tilde{c}_t) dt \quad (8)$$

$$d\omega_t = ((g_t - r + \sigma^2)\omega_t + a - \tau(g_t))dt - \tilde{c}_t + \sigma_t \omega_t dZ_t \quad (9)$$

Hereby  $\tilde{c}_t$  is the new control variable.<sup>12</sup> Term  $\tilde{c}_t$  is the consumption wealth ratio,  $\frac{c}{k}$ . The expression  $\tau(g_t)$  represents a convex adjustment cost which is affecting the size of borrowing to achieve a growth rate  $g_t$ . This is modeled by following the capital adjustment cost literature. Yet, of course only the growth of wealth  $g_t$  appears in the equation for the evolution of assets  $k_t$ . The other expressions in the latter equation are straight forward derivations from the negative of the growth rate of the debt-asset ratio as stated above.

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<sup>8</sup>See for example Hall (2010) who also includes an equation for the evolution of debt.

<sup>9</sup>Note that we use stocks of assets and debt, in contrast to Geaokoplos (2010) who uses flows as leverage measure, hereby then leveraging is highly positively correlated with booms.

<sup>10</sup>See Brunnermeier and Sannikov (2010)

<sup>11</sup>For a similar approach, see Brunnermeier and Sannikov (2010) and Hall (2010).

<sup>12</sup>A derivation of a dynamic equation in the stochastic case, using Itoh's lemma, is given in Brunnermeier and Sannikov (2010). The term  $\sigma^2$  comes in through Itoh's lemma.

### 3 Extensions of the Basic Model

In the next section we want to treat several extensions that amplify some of the mechanisms studied above in the basic model. We might think about four types of extensions.

First, in the context of the above model, the effect of asset price changes can easily be discussed. Note that fundamental asset price movements are implicitly contained in the above model. So far we have presented a model where asset prices are endogenous. Brunnermeier and Sannikov (2010) provide basic proofs of the probability of instability with endogenous asset prices. But what one could be interested in, as indicated above, are potential significant deviations from the fundamental asset price movements. Those can result from market price movements (driven by trading strategies of agents with different expectations, sentiments and opinion dynamics) which we could model as

$$dp_t = (\mu_t + z_t)p_t dt + \sigma_t p_t dZ_t \quad (10)$$

Hereby  $z_t$  could represent such market price movements.<sup>13</sup>

When we consider the role of asset price shocks for financial intermediaries, we might posit that the asset price movements are likely to affect the balance sheet dynamics of the financial intermediaries. Considering net worth, such as  $p_t k_t - d_t$  on the balance sheets of banks, an adverse asset price shock reduces the collateral value of the financial intermediaries, its equity, and thus since the bank has to offer less collateral, it will face a greater haircut and higher repo rate or greater default premium. It will thus demand

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<sup>13</sup>Such market price movements, such as  $z_t$ , resulting from sentiments are, for example, studied in Lux (2009). A market sentiment is also at play in the theory of Geanakoplos (2010), where leveraging drives asset prices. The role of heterogeneous expectations and trading strategies for market price movements are explored in Chiarella et al. (2009: chs 6-9). Yet, there are more general effects that can make the market price of the asset deviate from its fundamental price, as present value of future cash flow, for example liquidity problems, fire sales of assets and market dysfunctions, see Geneva Report (2009).

less capital, and with demand for capital falling asset prices will fall further. This has contagion effects: classes of similar types of assets will fall in price too, and a fast depreciation of asset prices— possible triggered by a firesale of assets— will have extensive externality effects for other intermediaries in the form of increased financial stress and credit spreads.<sup>14</sup> It might also affect the asset holdings and activities of firms and households that subsequently will have to sell assets to meet liquidity or payment requirements. The distinct contagion and externality effects are that a general loss of net worth could occur that may magnify the downward spiral. Though, at first sight, the asset price did not have a special role in the model above, it is easy to see how it might magnify the downward spiral.<sup>15</sup> Furthermore, it is very likely that positive and negative asset price shocks may have asymmetric effects.<sup>16</sup>

The second type of extension pertains to the bonus payments. We could assume if the net worth, as a ratio of net worth to total assets, falls below a certain safe threshold, then the bonus payments are reduced. Equivalently we could postulate that if the debt to asset ratio rises above some threshold, lets say  $\omega = -(d_t/k_t) \leq \omega^*$ , then the bonus payments are cut or reduced to zero. It could hold that bonus payments are used to give the managers an incentive to reduce leverage, so when the leverage is lower, a higher bonus payments could be allowed.<sup>17</sup> This might be considered as a type of penalty on risk taking and high indebtedness – the latter resulting from leveraged asset purchases. The dynamics of the debt-wealth ratio, once those policies are introduced, might be of interest. This modification will also be studied in our numerical section.

One could consider a third extension that takes into account the avail-

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<sup>14</sup>Those positive feedback effects are extensively studied in Geanakoplos (2010) and Gorton (2010).

<sup>15</sup>For further details and a number of other effects that falling asset prices may have, see Brunnermeier and Sannikov (2010)

<sup>16</sup>This is also discussed in Basel III.

<sup>17</sup>This is for example planned by Basil III, where it refers to “linkages of the total variable compensation pool to the need ...to maintain a sound capital base“

ability of funds for the financial intermediaries. There might be a fraction of households that accumulate risky assets<sup>18</sup>, which will provide funds for the financial intermediaries. A fraction of funds could also come from capital inflows, see Caballero and Krishnamurthy (2009). To be more formal, with  $\psi$  the fraction of assets being held by financial intermediaries,<sup>19</sup> the evolution of their capital assets and debt would be formulated as follows:

$$dk_t = (\psi(i_t/k_t) - (1 - \psi)\sigma)k_t dt + \sigma_t k_t dZ_t \quad (11)$$

$$dd_t = (rd_t - (ak_t - \psi i_t - c_t))dt \quad (12)$$

In this context, the inflow of funds from the Central Bank could be considered, which for example took place in the US in the years 2008 and 2010 when the Fed employed an unconventional monetary policy, called quantitative easing, buying bad — and rapidly declining — assets from the financial intermediaries. The latter would have more of a mitigating effect on the unstable forces generated by the banking system. An estimation of this effect will be presented in section 5. On the other hand, the precautionary motives of households (and firms), the “run into high quality assets”,<sup>20</sup> would lead to a reduction of financial funds for the financial intermediaries.

A fourth type of extension could relate to the interest rate paid by the financial intermediaries. So far, in our basic model, the interest rate paid is a constant,  $r$ , but one could assume, as in Brunnermeier and Sannikov (2010), that there is a cost of state verification which will depend on net worth of the financial intermediary. In fact, it is likely that adverse asset price shocks, as discussed in our basic model, will affect borrowing cost by banks through the LIBOR, Ted spread, or margin requirements, as discussed in the first

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<sup>18</sup>See He and Krishnamurthy (2008) and Brunnermeier and Sannikov (2010).

<sup>19</sup>For further details of this effect on the stability of the banking system, see Brunnermeier and Sannikov (2010, sect. 3)

<sup>20</sup>Gorton (2010) calls this the run into “information insensitive assets”, since one does not need acquire much information when one wants to hold them like treasury bonds.

type of extension above. Therefore, when there is a shock to asset prices and a magnification of a downward spiral, the credit spread and thus borrowing cost for financial intermediaries will rise. We thus will have:

$$d\omega_t = ((g_t - r(\omega) + \sigma^2)\omega_t dt + a - \tau(g_t))dt - \tilde{c}_t + \sigma_t \omega_t dZ_t \quad (13)$$

Brunnermeier and Sannikov (2010) have also included the effect of a rising volatility  $\sigma_t$  on the spread.<sup>21</sup> The above variant, with  $r(\omega)$  and  $r_\omega < 0$ <sup>22</sup> can also be numerically solved and it might be very important to study its effect on the overall stability of the banking system. On the other hand, one might argue that the financial intermediaries have in fact transferred risk to outside investors through securitization, i.e. through pooling and tranching of mortgage debt or other kind of liabilities, through MBSs or CDOs. Successfully undertaking the transfer of risk encourages them to take on more risk, but passes the verification cost on to someone else. The verification cost usually defines the amount that financial intermediaries have to pay, but if it is passed on, they can generally borrow at a lower risk premium, and their evolution of debt is determined by an almost constant interest rate as defined in our basic model.<sup>23</sup> A model with state or time depending spread can also be solved by our numerical procedure.

Overall, as we can see from the above considerations, some of the extensions may have further destabilizing effects, and some may have more stabilizing effects.

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<sup>21</sup>The importance of the effect of a rising volatility has also been indicated by the financial stress index developed by the Fed of Kansas City. It will be relevant in a distance to default model where it is shown that the distance to default shrinks with rising volatility.

<sup>22</sup>Note that we have the derivative  $r_\omega < 0$ , since we have the negative of the debt asset ratio as argument.

<sup>23</sup>See Brunnermeier and Sannikov (2010: sect. 4) for details of such considerations.

## 4 Solution Method and Numerical Results

As Brunnermeier and Sannikov (2010) correctly state, the dynamics for a model such as represented by eqs. (8)-(9) should not be studied by common linearization techniques. The first or even second order Taylor approximations to solve for the local dynamics of a model such as (4)-(7) or (8)-(9) will not properly capture the global instabilities of the model in particular in some regions of the state space. We have used the dynamic programming method by Gruene and Semmler (2004) to study the dynamics of the stochastic version of the basic model (8)-(9) and some extensions. Here, the debt to asset ratio is the state variable, and the control variables are the growth rate of assets and consumption, which can be interpreted as bonus payments.

The dynamic programming method can explore the local and global dynamics by using a coarse grid for a larger region of the state space, and then employing grid refinement for smaller region. Here we use dynamic programming, which can provide us with the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker *et al.*, 2007). In contrast, local linearization, as has been argued there, and also in Brunnermeier and Sannikov (2010), does not give sufficient information on the global dynamics. A more detailed description of the method is given in appendix 1. We want to study two major cases, a model with large bonuses and a model with small bonuses, and explore the stability properties of each variant.

Note that in both cases prices are implicitly given by the solution of the dynamic decision problem, in our case by the derivative of the value function.<sup>24</sup> Here, their effects are not separately considered, an issue we will make comments on further below. We also do not consider specifically the inflow of funds from households, the public (for example, through TARP) or from abroad (as, for example in the case of Citigroup in 2008, after the

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<sup>24</sup>Note that the derivative of the value function is equivalent to the co-state variable using the Hamiltonian, or the Lagrangian multiplier, using the Lagrangian approach. The latter two are usually used in asset pricing theories as the shadow price for capital.

insolvency of Lehman Brothers). At first we also neglect counter cyclical credit spreads for the financial intermediaries, since we first abstract from large non-fundamental asset price movements, and also the cost of state verification resulting in cyclically varying credit spreads. The latter will be added later.

## 4.1 Solution with Large Bonuses

In the first variant of our model we allow for negative and positive growth rates of the assets purchased by the financial intermediaries. We constrain the growth of the assets to  $-0.1 < g_t < 0.1$ , and the consumption capital ratio by  $0.01 < \tilde{c}_t < 0.8$ . The latter is always positive but is allowed to be rather large.<sup>25</sup> The growth rates of assets and consumption can be chosen optimally, and the latter is permitted to be rather large.

Brunnermeier and Sannikov (2010) conjecture that when the bonus payouts are chosen endogenously “the system is relatively stable near its “steady state” ... but becomes unstable below the steady state... “(BS, 2010:17). Moreover they state: “Papers such as BGG (Bernanke, Gertler and Gilchrist) and KM (Kiyotaki and Moore) do not capture the distinction between relative stable dynamics near the steady state, and much stronger amplification loops below the steady state... “(BS, 2010:18).

The reason for the different result is “With endogenous payout, the steady state naturally falls in the relative unconstrained region where amplification is low, and amplification below the steady state is high” (BS, 2010:18). Brunnermeier and Sannikov make this statement with respect to the ratio of net worth to assets. Since we take the negative of the debt to asset ratio, the statements can be immediately translated into the properties of our model

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<sup>25</sup>Note that the low bonus payments reflect roughly the time period until 2002 and then large bonus payments set in, see figure 17, appendix 2. In the solution method using DP we have allowed for a larger maximum bonus payment in order to make the change of the subsequent dynamics visible. Note also that we could also allow for dividend payments, in fact as our model is constructed the bonus payments can encompass dividend payments.

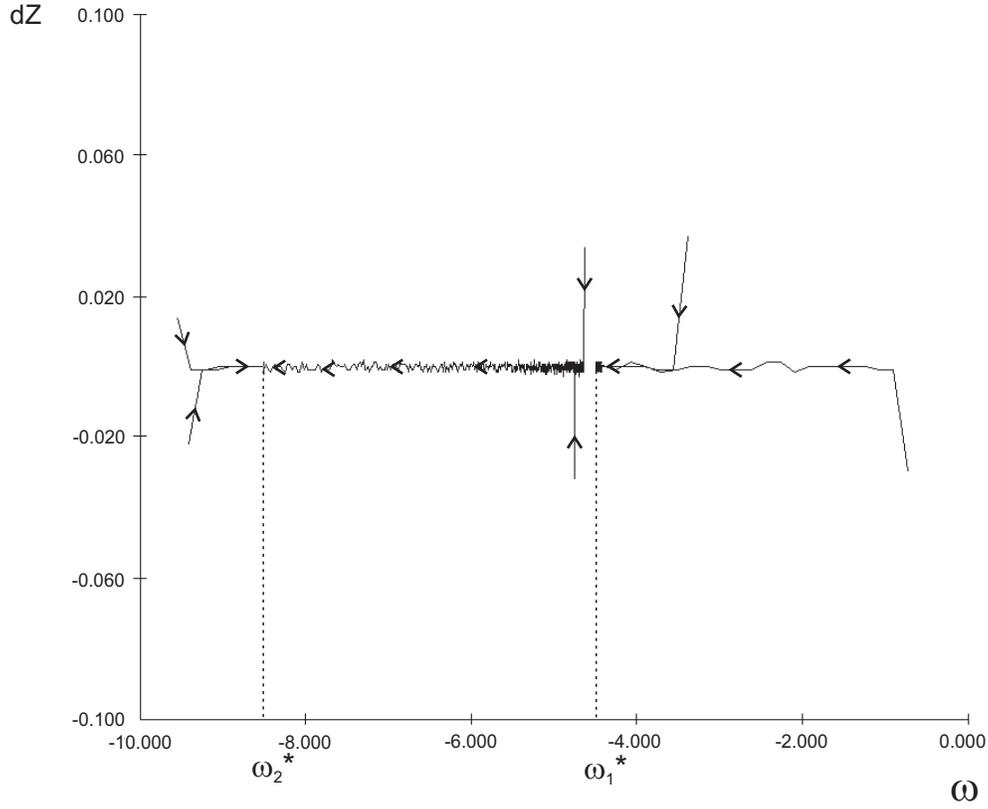


Figure 1: Trajectories for large bonuses

using the debt to asset ratio.

As to the parametrization of our model we take:  $a = 0.5$ ,  $\alpha = 0.3$ ,  $\sigma = 0.008$ ,  $\gamma = 0.03$ , and  $r = 0.03$ .

The figure 1 shows on the horizontal axis the state variable  $\omega$  and on the vertical axis the stochastic path for state variable  $\omega$ . Since we have stochastic shocks, with pre-defined standard deviation  $\sigma = 0.008$ , the path of  $\omega$  varies in the state space, and thus there is no unidirectional vector field, i.e. the path of  $\omega_t$  is not a straight line. In our numerical procedure the shocks are drawn from a distribution having a pre-defined standard deviations  $\sigma = 0.008$ . As visible from the numerical solution path in figure 1, for different initial conditions there is a dynamic path from a low level of debt to asset ratio

( $\omega$  close to zero) to roughly  $\omega = -4.5$ , see the first steady state  $\omega_1^*$ . This indicates that large bonuses will make the debt to asset ratio rise, moving the  $\omega_t$  toward the first steady state of  $\omega_1^* = -4.5$ .<sup>26</sup>

To the left of this first steady state, the debt to asset ratio rises, moving roughly to  $\omega = -8.5$ . This is a second, high debt to asset value steady state  $\omega_2^* = -8.5$ , which is stable and thus attracting, but it might be considered much too high. For the possibly high bonuses the first steady state  $\omega_1^*$  of about -4.5, is attracting only from the right and repelling starting from the left of -4.5. That means that with large bonuses in the interval  $0.01 < \tilde{c}_t < 0.8$ , as optimal choices, the debt to asset ratio will rise even if the debt to asset ratio is low.

As figure 2 shows, the value function, computed through our numerical solution procedure, increases once the bonuses start rising, see the upward slope of the value function to the right of about  $\omega = -4.5$ . The rise of the value function is reasonable, since it is the welfare from the rising bonuses that make the value function increase. At the same time, in the region to the right of  $\omega = -4.5$ , higher (optimal) bonus payment are allowed. Yet this eventually gives rise to a higher debt to asset ratio.<sup>27</sup>

## 4.2 Solution with Small Bonuses

Brunnermeier and Sannikov (2010:32) state further that allowing the debt to asset ratio rise too much, driven by the incentives of the intermediaries to take on too much risk for the sake of short term profits, paying out high

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<sup>26</sup>Note that we do not pursue the issue here at what leverage ratio bankruptcy would occur. This depends on the distance to default, which is defined by the KMV model by the distance of the asset value of the bank to the debt, divided by the standard deviation (volatility) of the asset value. We are not pursuing this question here, since we do not explicitly compute the asset value of the financial firm. This issue is pursued in Grüne and Semmler (2005).

<sup>27</sup>Note that the shape of the value function is roughly the same as shown in Brunnermeier and Sannikov (2010) in their figure 7, though we have negative values on the vertical axis, since we are taking  $\log \tilde{c}_t$ , not  $\tilde{c}_t$ , in the preferences.

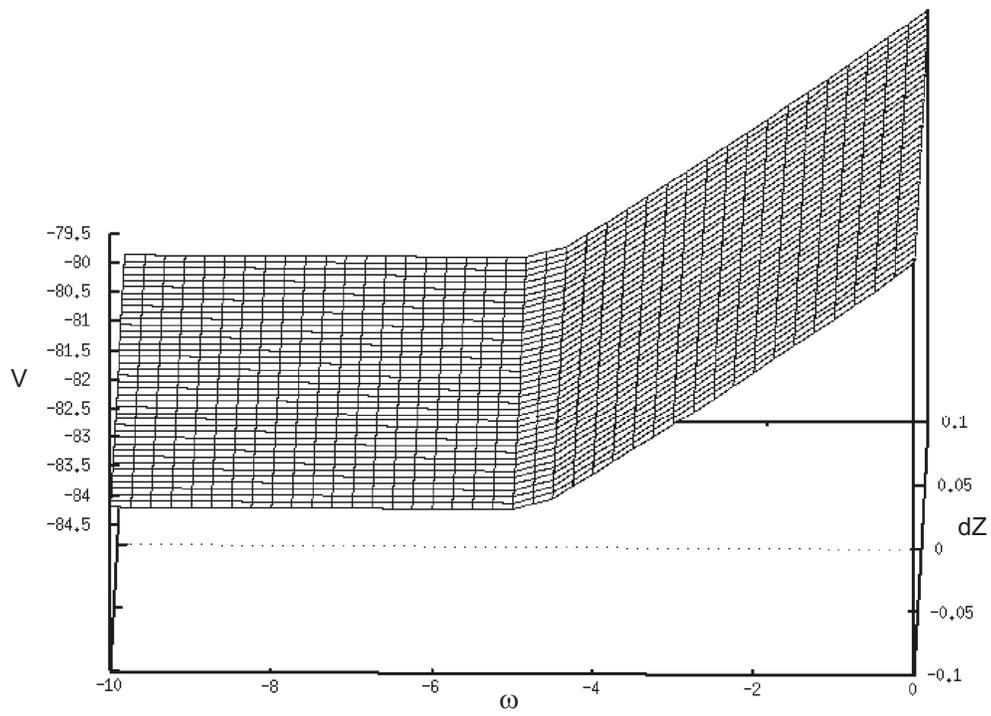


Figure 2: Value function for large bonuses

bonuses, and neglecting externalities may lead to damages and downturns. In their view the triggering of the downturn in the financial, product and labor markets results from not taking into account the full extent of the externalities, and they argue that a competitive financial sector is likely to trigger such events even more frequently.

They thus state that limiting bonuses should be welfare improving. More explicitly they say: “We would like to argue that a regulator can improve social welfare by a policy that limits bonus payments within the financial sector. Specifically, suppose that experts are not allowed to pay themselves as long financial intermediaries are not sufficiently capitalized” (BS, 2010: 32). This type of regulatory effort would keep sufficient capital within the financial system and make it more stable.<sup>28</sup> Actually, this conjecture can also be shown to hold using our DP solution algorithm.

In order to explore this variant we, as before, allow for negative and positive growth rates of the assets purchased by the financial intermediaries to be in the range  $-0.1 < g_t < 0.1$ ,<sup>29</sup> but we constrain the consumption to capital ratio by  $0.01 < \tilde{c}_t < 0.1$ . Again, the latter is always positive but it is constrained not to be too large. Under the condition that the growth rate of assets and the consumption rate can be chosen optimally, consumption will be constrained to be low. Figure 3 shows that now the first steady state,  $\omega_1^* = -4.5$  becomes a complete repeller: with lower debt and low bonus payouts the debt to asset ratio will go to zero. No dangers of large externalities, financial stress and meltdowns will appear. Again to the left of  $\omega_1^* = -4.5$ <sup>30</sup> the debt to asset ratio will rise, possibly going up to roughly -9, see  $\omega_2^* = -9$ . Thus, as before, the second high level debt to asset ratio is still there, but a lower initial debt to asset ratio, with low payouts, will produce

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<sup>28</sup>A similar view is present in the Geneva Report (2009, sect. 6.2 ) and Basel III.

<sup>29</sup>In the subsequent use of our DP algorithm we have used a maximum bonus payment of 0.1, reflecting roughly the time period before 2003, see figure 17, appendix 2.

<sup>30</sup>Actually the rise of the debt to asset ratio will start immediately to the left of  $\omega_1^* = -4.5$ ; the trajectories with initial conditions immediately to the left of  $\omega_1^* = -4.5$ , not shown here, would also go to  $\omega_2^* = -9$ .

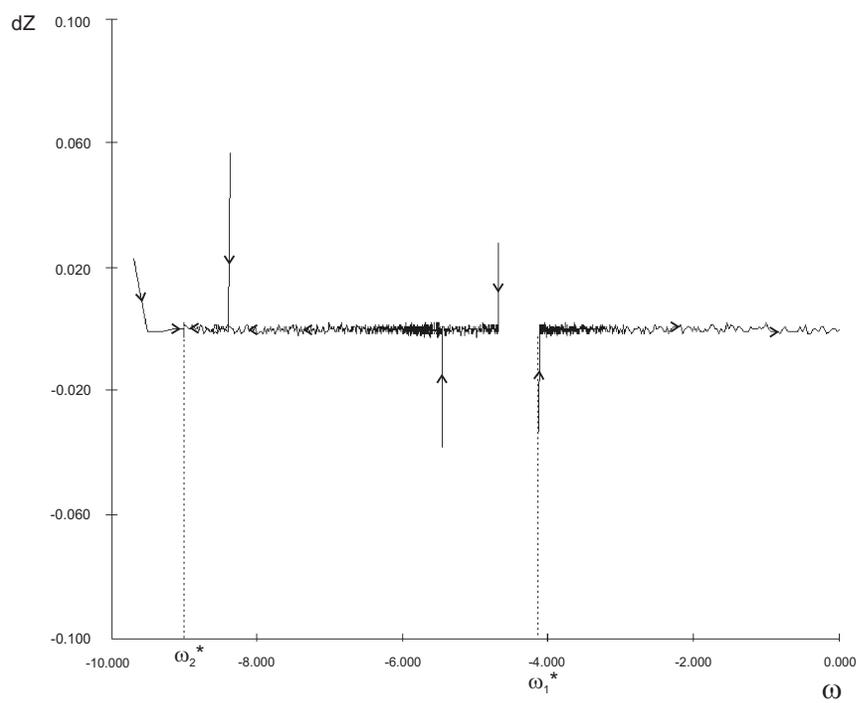


Figure 3: Trajectories for small bonuses

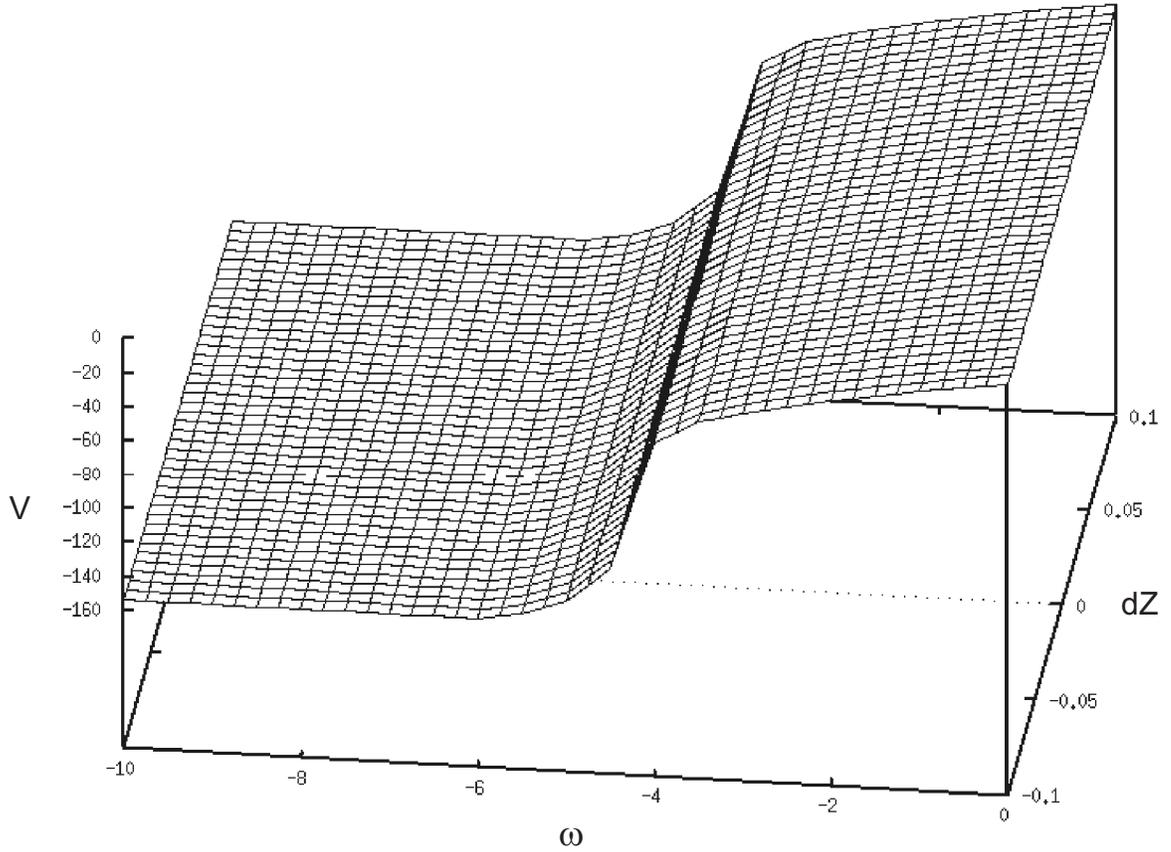


Figure 4: Value function for small bonuses

stability.

Figure 4 shows the corresponding value function, revealing again that total welfare (for the financial intermediaries) is rising with lower debt to asset ratio, but it is, in terms of level, also higher as compared to the variant with a weak constraint on the payouts.

### 4.3 Solution with a Varying Risk Premium

Since the publication of the financial accelerator principle by Bernanke et al (1999) the economists have been greatly concerned with the fact that

borrowing cost moves counter-cyclically, and the ease of lending standards cyclically. Accordingly, in sect. 3, in equ. (13) we have proposed that we might have not a constant interest rate for the debt dynamics but a state dependent interest rate  $r(\omega_t)$  with  $r_\omega < 0$ . Thus the interest payment could be made state dependent, such as<sup>31</sup>

$$r(\omega_t) = \frac{\alpha_1}{(\alpha_2 + (1 + \omega_t))^\mu} r d_t \quad (14)$$

The risk premium, and thus the credit spread, is hereby made state dependent thus  $r(\omega_t)$  rises with the leveraging. The parameters  $\alpha_1$ ,  $\alpha_2$  are positive constants. If they are appropriately chosen, the risk premium goes to zero and a constant (risk free) interest rate will re-emerge with no leverage. The constant interest rate, as assumed in the previous version of the model, see equ. (9), is a limit case of the above scenario.<sup>32</sup>

Using information economics and the theory of costly state verification, we can say that equ. (14) reflects the standard case of the financial accelerator, according to which the risk premium rises with leverage, since a greater cost of state verification is needed with higher leverage. If there are no possible losses and no verification cost, the constant (risk free) interest rate will be charged. A case close to this may emerge, according to the argument by Brunnermeier and Sannikov (2010, sect. 4), if the financial intermediaries can transfer risk through the securitization of loans and selling them as CDOs to a secondary risk market. This will not only reduce their risk exposure, but also give them less incentives for monitoring loans and increase leveraging and thus increase systemic risk: if idiosyncratic shocks are fully hedged out through securitization, the financial intermediaries then “face the cost of borrowing of only  $r$  ... Lower cost of borrowing leads to higher leverage and quicker payouts. As a result the system becomes less stable”. (Brunnermeier

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<sup>31</sup>For further details, see Gruene et al (2004, 2007)

<sup>32</sup>See Gruene et al (2004, 2007). For a similar formulation, but in terms of net worth, see Christensen and Dib (2008)

and Sannikov, 2010:39).<sup>33</sup>

The cases of a constant interest rate, with larger and smaller bonus payments have already been numerically solved in sects 4.1 and 4.2. Now it may be of interest to solve the case of a state dependent interest rate  $r(\omega_t)$ . Yet, the results of this case of a debt dynamics of eqs. (14) with  $r(\omega_t)$  are easy to anticipate. The dynamics to the left of  $\omega_1^*$  in figure 3 will be more unstable, debt and the leverage ratio will increase faster to the left of  $\omega_1^*$  and the leverage will decrease faster to the right of  $\omega_1^*$ . Since the results are rather obvious we do not explore this case numerically here.

On the other hand, one might argue that risk premia embodied in credit spreads cannot solely be measured by leverage ratios. There are other factors affecting risk premia, such as financial stress being built up through externalities and contagion effects generated by financial intermediaries, as well as resulting from macro economic risk. Adrian and Shin (2010) have defined such a risk premium as a macro economic risk premium. They summarize the macro risk in one indicator using principle component analysis. So, we might argue that, in fact, we should have a risk premium varying with leverage, as well as other risk factors such as externality and contagion effects and asset price volatility. We might expect then a varying risk premium that is impacted by several factors and exhibits some periodic movements.

We can estimate such periodic movements in risk premia by the estimation of harmonic oscillations in the data using Fast Fourier Transform. This has been done in Semmler and Hsiao (2009) to estimate time varying asset returns. One of the most important measure for macro risk is the BAA/AAA spread or the BAA/T-Bill spread. Many studies have worked with the former measure.<sup>34</sup> We employ here time series data from 1983.1 to 2009.4 to estimate periodic components in such a macro risk premium.<sup>35</sup>

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<sup>33</sup>They further argue that though in principle securitization may be good, since it allows for sharing of idiosyncratic risk, it also leads to the creation of severe leverage and the amplification of systemic risk.

<sup>34</sup>See for example, Gilchrist and Zagrajsek (2010).

<sup>35</sup>We want to note that one might also take the periodic components of the BAA/AAA

The periodic component of our risk measure, which is estimated from empirical data, can be employed in our dynamic programming algorithm as presented in appendix 1. Doing so we have the following extended system to be solved on risk premia. We can write

$$V(\omega_t) = \max_{\tilde{c}_t, g_t} \int_0^\infty e^{-rt} U(\tilde{c}_t) dt \quad (15)$$

$$d\omega_t = ((g_t - r(x_t) + \sigma^2)\omega_t + a - \tau(g_t))dt - \tilde{c}_t + \sigma_t \omega_t dZ_t \quad (16)$$

$$dx_t = 1dt \quad (17)$$

The latter dynamic equation above creates a time index  $x_t$  through which the actual periodic components in the credit cost, including a risk premium,<sup>36</sup> can be read into the DP algorithm. If we use the notation  $r(x_t)$  in equ. (16), this indicates that there are risk factors at work that make the credit cost  $r(x_t)$  time varying.<sup>37</sup> Formally our stochastic dynamic decision problem will have two decision variables and three state variables, the leverage ratio  $\omega_t$ , the time index  $x_t = t$  and the stochastic term  $dZ_t$ .

As shown in detail in appendix 2, the estimated time varying credit cost represented by the BAA bond yield, which includes a premium, takes on the form:

$$x_t = 0.0862 - 0.0022(t - t_0) + \sum_{i=1}^n \left( a_i \sin\left(\frac{2\pi}{\tau_i}(t - t_0)\right) + b_i \cos\left(\frac{2\pi}{\tau_i}(t - t_0)\right) \right). \quad (18)$$

Note that the first two terms in the above equation represent the time trend of credit cost, the next terms the periodic variations. Appendix 2 also

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spread as measure of the time variation of financial stress. Actually our measure of financial stress is highly correlated with other measures, see sect. 5.

<sup>36</sup>Note that in equ. (16) we have, with  $r(x_t)$ , the actual credit cost modeled that includes a risk premium.

<sup>37</sup>Of course, the interest rate set by the Central Bank also affects  $r(x_t)$ .

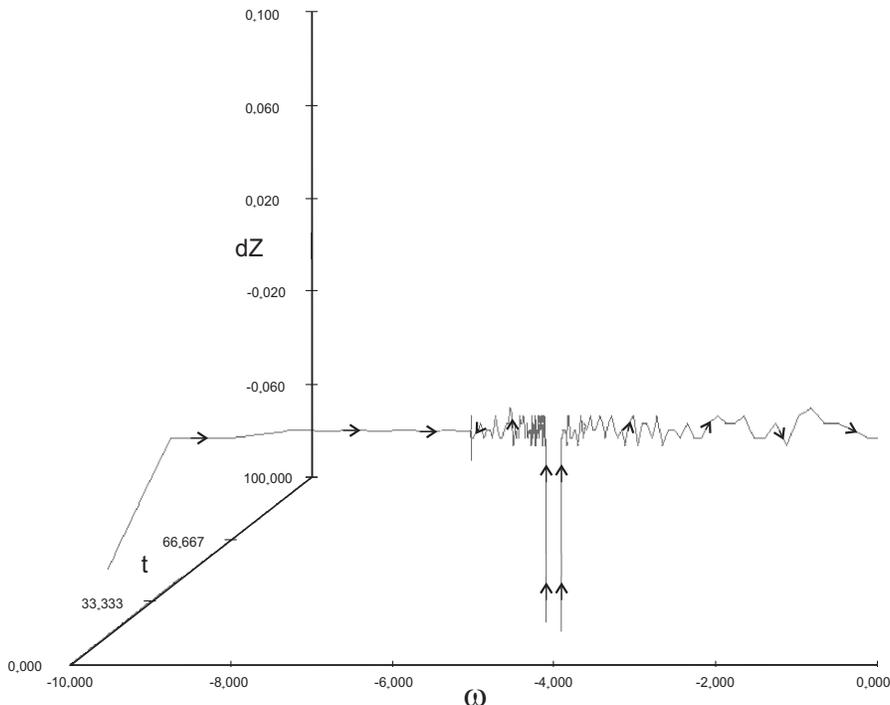


Figure 5: Trajectories with varying risk premium

discusses how many periodic components are needed to properly replicate the actual time series of the credit cost that includes a risk component. Since we are only interested in low frequency components, it turns out that in our case we need three oscillatory components.<sup>38</sup>

Using (18) and solving the problem (15) -(17) through DP generates the trajectories as shown in figure 5.

Using our set up of low bonus payments of sect. 4.2, there are again two steady states, a stable one around  $\omega_2^*$ , close to  $-5$ , and a repelling steady state, close to  $-4$ . Yet, compared to the fluctuations seen in figure 3, the

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<sup>38</sup>From the estimation procedure, as discussed in Appendix 2, we get the following parameters for equ (18):  $a_{11} = 0.006$ ,  $a_{12} = 0.0062$ ,  $a_{13} = 0.0063$ ,  $b_{11} = 0.0049$ ,  $b_{12} = 0.006$ ,  $b_{13} = 0.0016$ ,  $c_{11} = -0.002$ ,  $c_{12} = 0.0862$ ,  $\tau_{11} = 305$ ,  $\tau_{12} = 152.5$ ,  $\tau_{13} = 101.5$ .

periodic fluctuations of the credit cost (including a risk premium) increase the volatility of the state variable  $\omega_t$ . The shock  $dZ_t$ , as before, moves the trajectories, along the vertical axis, whereas the periodic fluctuations of the credit spread moves the trajectories along the time axis  $t$ . Note that here again we have the shock drawn from the range  $-0.1 < dZ_t < 0.1$ , and we have used  $-10 < \omega_t < 0$ . For our third dimension, our time index, the range is defined as  $0 < t < 100$ .<sup>39</sup>

As one can observe from figure 5, to the right of the middle steady state,  $-4$ , the inclusion of the time varying risk premium in the credit cost, the use of  $r(\omega)$ , not only amplifies the volatility of  $\omega_t$ , but moves the leverage ratio to zero faster as compared to a constant credit cost. On the other hand it will also built up financial stress faster, once the middle steady state is,  $-4$ , is surpassed.<sup>40</sup>

## 5 Financial Intermediaries and Financial Stress Measures

In the previous section we have postulated that the financial intermediaries are not only exposed to the tides of the financial market, but also amplify them. As we saw, a variety of constellations are feasible, depending on potential restrictions on bonus payments, and whether there is a constant or time varying credit spread. Yet the different variants show that a shock to the banks' balance sheets can entail a considerable instability.

Thus, shocks to asset prices, and therefore to capital assets,  $p_t k_t$ , and net worth,  $n_t = p_t k_t - d_t$ , (increasing debt) will be amplifying, particularly in the

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<sup>39</sup>We do not pursue here to compute and graph the value function, since the 3-dim problem (15)-(18) makes it more cumbersome to present the value function.

<sup>40</sup>A varying risk premium is might thus prevent the agents to go to a too high leverage, but on the other hand if the middle steady state is surpassed it creates a vicious cycle of credit spreads and higher indebtedness, as often has been observed for companies as well as sovereign debt.

case of large bonus payments, represented by figure 1, where initial leverage ratios to the right of  $\omega_1^*$  will be increasing, and even a very low or zero leverage will create higher leverage and financial stress. The trajectories will be attracted to a high steady state leverage ratio for high bonus payments. Yet, the leverage may fall in the case of the low bonus payment scenario, see figure 3. So we may observe some superior stability properties of the leverage ratio for small bonus payments. The strength of the local instabilities will be the empirical issue to be explored next.

The problem is, however, what measures can one utilize to empirically evaluate the predictions of the model and undertake empirical estimates. What actual measurements should one take to evaluate how financial stress of banks is interlinked with the financial market, or more specifically, to asset prices? In the context of our model in sects. 2-4, one could take leverage ratios stemming from the balance sheets of the financial intermediaries as measuring this linkage: high leverage implying high financial stress and low leverage the reverse.

However, there is an issue whether the ratio of net worth to capital assets, or the reverse measure, the leveraging  $\omega$  can be good measures of financial stress. First, both are greatly affected by the market valuation of assets as well as liabilities, which is not easy to undertake. In particular, asset valuation is heavily impacted by the confidence and estimate of income streams the asset generate, as well as presumed discount rates, and the liabilities such as bonds or short and long term loans are strongly affected by their corresponding risk premia.<sup>41</sup> Moreover, credit constraints, for example, as measured by the Fed index of changes in credit standards to determine the ease and tightness of obtaining credit as well as credit spreads and short term liquidity, are also important financial stress factors for financial intermediaries. All this will affect credit demand and supply of financial intermediaries. We thus need more extensive measures than only leverage to evaluate financial stress.

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<sup>41</sup>This is implicit in Merton's risk structure of interest rates, see Merton (1974)

The Federal Reserve Bank of Kansas City and the Fed St. Louis have thus developed a general financial stress index, called KCFSI and STLFSI respectively. The KCFSI and the STLFSI,<sup>42</sup> take into account the various factors generating financial stress. They can be taken as substitutes for the net worth or leverage ratios as measuring financial stress of financial intermediaries. Other financial stress indices have been developed before, for example the Bank of Canada index<sup>43</sup> for Canada, and the IMF (2008) index. Both of them include a number of variables that are included in the KCFSI and STLFSI but are less broad. Both the KCFSI and STLFSI assert that the times of stress: 1) increase the uncertainty of the fundamental value of the assets, often resulting in higher volatility of the asset prices, 2) increase uncertainty about the behavior of the other investors, 3) increase the asymmetry in information, 4) increase the flight to quality, 5) decrease the willingness to hold risky assets, and 6) decrease the willingness to hold illiquid assets.<sup>44</sup>

Following this characterization of the period of financial stress, the above mentioned FSIs take the following variables: The TED spread (spread between the 3 month LIBOR/T-bill), the 2 year swap spread, the AAA/10-year Treasury spread, the BAA/AAA spread, the high yield bond/BAA spread, Consumer ABS/5 year Treasury spread, the correlation between returns on stocks and Treasury bonds (a measure for the flight to quality), the VIX (implied volatility of bank stocks) and the cross dispersion of banks stocks. As one can see here, spreads, volatility and dispersion measures are taken as variables for a financial stress index. The principle component analysis is used to obtain the FSI<sup>45</sup>. We want to note that most of the above variables

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<sup>42</sup>The KC index is a monthly index, the STL index a weekly index, to capture more short run movements. Another recent work on financial stress stress indexes can be in Hatzius et al (2010).

<sup>43</sup>See Illing and Lui (2006).

<sup>44</sup>The latter tendencies have described by Gorton (2010) as a flight from information sensitive to information insensitive assets.

<sup>45</sup>This is done as follows. Linear OLS coefficients are normalized through their standard deviations and their relative weights computed to explain the index. A similar procedure

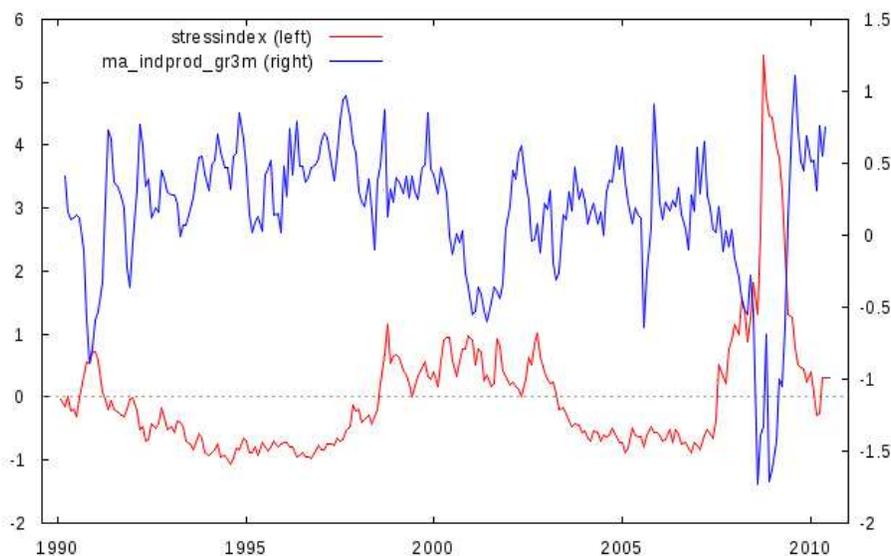


Figure 6: Financial stress index (KCFSI, lower graph) plotted against growth rates of industrial production (3 month moving average, upper graph)

are highly correlated and the leading variables are the spread variables.<sup>46</sup>

Combining all the variables with appropriate weight in a stress index produces a clearly counter-cyclical behavior. This is illustrated in figure 6.

As the comparison of the smoothed growth rate of the production index and the stress index in figure 6 shows there is less financial stress in good times, but more in bad times. Although we are not using balance sheet variables directly, nevertheless we can safely presume that financial intermediaries are clearly doing better in economic booms than in recessions<sup>47</sup>. Given the linkages between the financial stress index and economic activity, we would also expect a strong linkage between net worth, or leveraging, of financial intermediaries and economic activity, since the financial stress is

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is used by Adrian and Shin (2010) to compute a macro economic risk premium.

<sup>46</sup>In the sense that they have the highest weight in the index, for details see Hakkio and Keeton (2009, tables 2-3.)

<sup>47</sup>This coincides also with the empirical study by Gorton (2010) that there is more insolvency of financial institutions in bad times, see also figure 15 in appendix 2.

affecting the balance sheets of financial intermediaries.<sup>48</sup> We want to note that the financial stress index is also highly linked to some broader index of economic activity.<sup>49</sup>

A “one-regime VAR” has been used many times to study the financial accelerator.<sup>50</sup> Yet those “one-regime VAR” studies presume only local instability, symmetry effects of shocks and mean reversion after the shocks. What we will pursue here is an MRVAR. Our MRVAR<sup>51</sup> takes the aforementioned stress index KCFSI as empirical measure of financial stress, and the growth rate of the monthly production index as a threshold variable to define regimes.

## 6 Empirical Analysis Using a MRVAR

To empirically investigate of how strong the local instabilities are – and whether one finds sufficient empirical evidence for instabilities at all – requires an empirical approach that can accommodate varying dynamic patterns across alternative states of the economy. For this reason, we adopt a multi-regime modeling strategy, which allows us to explore regime-dependencies of responses to shocks to the system. A related question is whether there might be local stability for small disturbances, but not for larger shocks. Thus, we will also explore whether the size of shocks matter.<sup>52</sup>

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<sup>48</sup>The fact that the leverage ratio is rising in recessions and falling in booms, is documented in Gilchrist et al. (2009).

<sup>49</sup>See Hakkio and Keeton (2009).

<sup>50</sup>Estimating the financial accelerator for the macroeconomy with a “one regime VAR”, see Christensen and Dib (2008). and for the application of the financial accelerator to study financial intermediaries in a “one regime VAR”, see Hakkio and Keeton (2009) and Adrian and Shin (2009, 2010).

<sup>51</sup>For using a MRVAR, see Mittnik and Semmler (2009) and Ernst, Mittnik and Semmler (2010).

<sup>52</sup>Since we work with historical data since the 1990s it is probably realistic to assume a historical period of large bonus payments, for data and computation on this, see Semmler and Lucas (2009)..

## 6.1 Methodology

To assess the dependence of the responses to shocks to the stress index, we employ a multi-regime VAR (MRVAR) approach. A major limitation of conventional linear VAR models is that shock responses are independent of the economy's state at the time a shock occurs. Also, VAR response profiles are invariant with respect to the sign and size of a shock. That is, responses to positive and negative shocks are sign-symmetric; and the response to shocks of different sizes are simply scaled versions of the response to a shock of size one. To capture state dependencies and asymmetries of shock responses, a nonlinear model or a linear model with state dependencies needs to be specified. The mildest form of generalizing a linear, constant-parameter VAR is to adopt a piecewise linear VAR, such as Markov switching autoregressions (Hamilton, 1989) or threshold autoregressions (Tong, 1978, 1983). A characteristic of Markov switching autoregressions is that the states are unobservable and, hence, do not necessarily have a clear interpretation. Also, a given observation cannot directly be associated with any particular regime. Only conditional probabilistic assignments are possible via statistical inference based on past information.

For our purposes, namely, state-dependent response analysis, states are associated with specific stages of the business cycle as measured, for example, in terms of output growth. Multi-regime vector autoregression (MRVAR) models in the form of threshold autoregression models of Tong (1978, 1983) or, in a vector setting, of multivariate threshold autoregressions (Tsay, 1998) are obvious candidates. In contrast to Markov switching autoregressions or standard multivariate threshold autoregressions, our approach assumes that we can, based on some observable variable, define upfront a meaningful set of regimes, which are not a result of some estimation procedure, but rather motivated by the objective of the empirical analysis. This is preferable in our setting, where we are interested in evaluating the potential effectiveness of policy measures for a particular state of the economy.

The MRVAR specification adopted here is given by

$$y_t = c_i + \sum_{j=1}^{p_i} A_{ij} y_{t-j} + \varepsilon_{it}, \text{ if } \tau_{i-1} < r_{t-d} \leq \tau_i, \varepsilon_{it} \sim NID(0, \Sigma_i), i = 1, \dots, M, \quad (19)$$

where  $r_{t-d}$  is the value of the threshold variable observed at time  $t - d$ ; and regimes are defined by the (prespecified) threshold levels  $-\infty = \tau_0 < \tau_1 < \dots < \tau_M = \infty$ . In the following analysis we estimate a two-regime VAR, with the output-growth rate as the threshold variable, and the average growth rate delineating the threshold for the sample.

In addition to the more straightforward regime interpretation, MRVAR models are also more appealing than Markov switching autoregressions as far as estimation is concerned. Rather than EM-estimation, MRVARs with predefined threshold levels resemble conventional VARs and can be estimated regime by regime, using standard common least-squares—provided the regime-specific sample sizes permit this, or using Bayesian techniques.

Response analysis for linear VAR models is straightforward. Point estimates and asymptotic distributions of shock response can be derived analytically from the estimated VAR parameters (cf. Mittnik and Zadrozny, 1993). In nonlinear settings, this is, in general, not possible, and one has to resort to Monte Carlo simulations. Following Koop et al. (1996), the so-called generalized impulse responses, which depend on the overall state,  $z_t$ , type of shock,  $v_t$ , and the response horizon,  $h$ , are defined by

$$GIR_h(z_t, v_t) = E(y_{t+h} | z_t, u_t + v_t) - E(y_{t+h} | z_t, u_t), \quad (20)$$

where the overall state,  $z_t$ , reflects the relevant information set. For a Markov-switching VAR process,  $z_t$  comprises information about the past realizations of  $y_t$  and the states; for an MRVAR process with known threshold levels, only information about past realizations  $y_{t-1}, \dots, y_{t-p_{\max}}$ , with  $p_{\max} = \max(p_1, \dots, p_M)$ , is required.

To understand the differences in the dynamic characteristics between the different regimes, regime-specific response analysis as in Ehrmann et al. (2003) is helpful. Regime-specific responses of MRVAR models assume that the process remains within a specific regime during the next  $h$  periods. This is particularly reasonable when regimes tend to persist or when we are interested in short-term analysis, and helps to understand regime-specific dynamics.

## 6.2 Estimation

For our bivariate analysis, we use monthly data on U.S. industrial production and the KCFSI stress index covering the period February 1990 to June 2010.<sup>53</sup>

We estimate a standard VAR and an MRVAR model for the IP growth rate and absolute changes in the stress index, and define  $y_t = (100\Delta \log IP_t, \Delta KCFSI_t)'$ . We use the AIC for model selection. For MRVAR model (19), the AIC is given by

$$AIC(M, p_1, \dots, p_M) = \sum_{j=1}^M \left[ T_j \ln |\hat{\Sigma}_j| + 2n \left( np_j + \frac{n+3}{2} \right) \right], \quad (21)$$

where  $M$  is the number of regimes;  $p_j$  is the autoregressive order of Regime  $j$ ;  $T_j$  reflects the number of observations associated with Regime  $j$ ;  $\hat{\Sigma}_j$  is the estimated residual covariance matrix for Regime  $j$ ; and  $n$  denotes the number of variables in vector  $y_t$ . Formulation (21) differs from that in Chan et al. (2004) in that we account for possible heterogeneity in the constant terms,  $c_j$ , and residual covariance,  $\Sigma_j$ , across regimes.<sup>54</sup>

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<sup>53</sup>Seasonally-adjusted industrial-production (IP) data (Series ID INDPRO) come from Board of Governors of the Federal Reserve System; the KCFSI stress-index data were obtained from the Federal Reserve Bank of Kansas City.

<sup>54</sup>When employing (21) to discriminate between an MRVAR and a standard VAR specification (i.e., a one-regime MRVAR), we need to include the  $n$  parameters in the intercept vector,  $c$ , and the  $n(n+1)/2$  parameters in the residual covariance matrix for an equivalent parameter count.

Based on the AIC, a VAR of order  $p = 4$  is suggested. Specifying a two-regime MRVAR with the threshold,  $\tau$ , set to the sample mean of the IP-growth rate given by 0.165, we assign observations associated with below-mean (above-mean) growth rates to Regime 1 (Regime 2). Then, the AIC suggests an autoregressive order of four for Regime 1 and order of three for Regime two. Although the MVAR has quite a few more free parameters than the fitted VAR (35 vs. 21 parameters), the AIC favors the two-regime MRVAR with AIC ( $M = 2, p_1 = 4, p_2 = 3$ ) = -1084.9 (and regime-specific sample sizes  $T_1 = 112$  and  $T_2 = 126$ ) over a standard VAR with AIC( $M = 1, p = 4$ ) = -842.1.

### 6.3 Response analysis

To assess the effects of linear versus nonlinear model specification, we first look at the estimates of the cumulative unit-shock responses for the VAR model and the regime-specific responses for the MRVAR model. To derive structural responses, we assume that a shock to IP simultaneously affects the stress index, whereas IP reacts with a one-period delay to a stress shock. As figures 15 and 16 in appendix 2 show, when compared to figure 6, a positive stress shock reveals a high co-movement with a rise in the overall leverage ratio and the rise of banking failures. We can thus view the positive stress shock as measuring the deterioration of the balance sheets of banks and a rise of insolvency risk of banks.

The cumulative responses to unit shocks are shown in Figure 7.

In our analysis we will focus on the responses of IP due to shocks in the financial stress index, since we want to evaluate the impact of the change of financial stress on the banking system and the macroeconomy. In our case, the latter is measured by output growth.

The results for the conventional VAR model (Figure 7) suggests that a positive one-standard-deviation stress-index shock has an increasingly negative cumulative IP growth effect which settles at -2% after about 10 months.

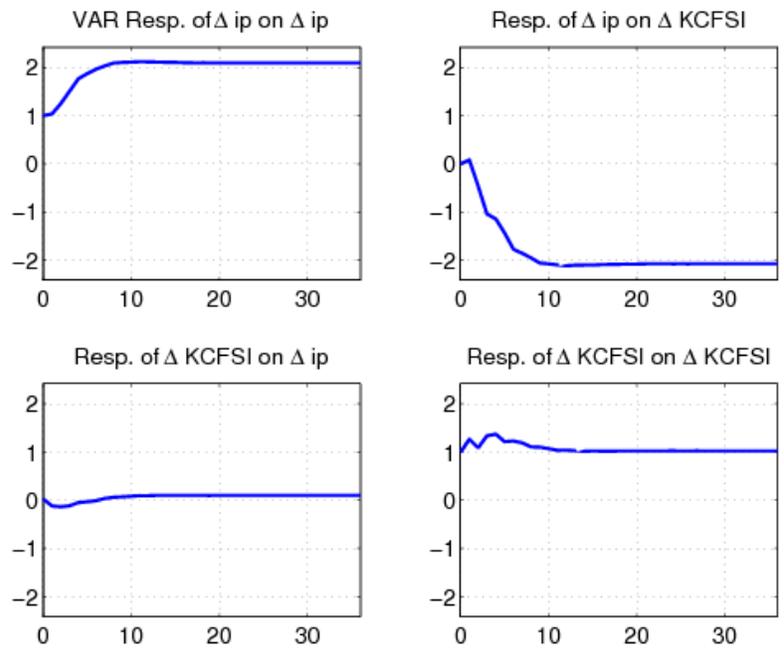


Figure 7: Cumulative Responses from a one-regime linear VAR Model

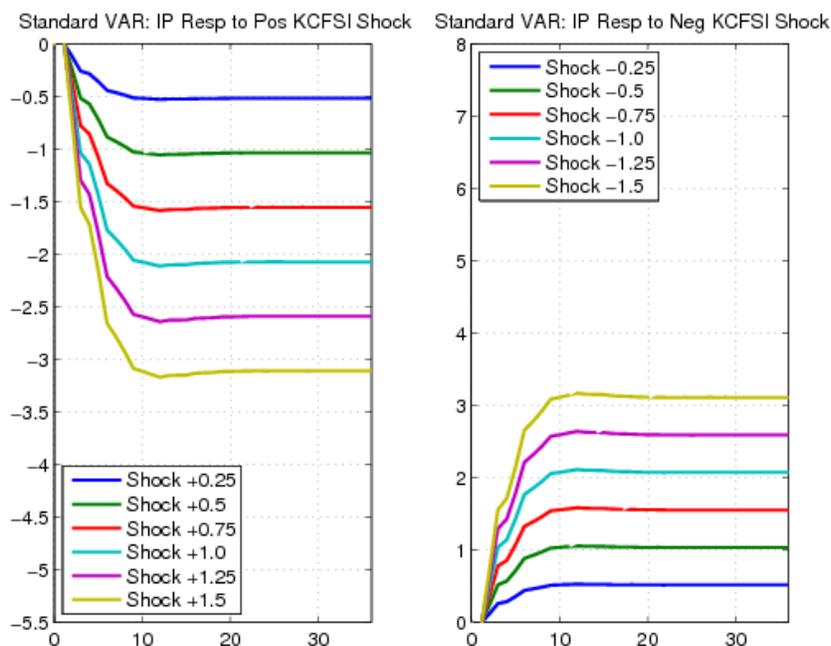


Figure 8: Cumulative responses from a one-Regime linear VAR with negative and positive shocks of different size

Additionally, the output responds strongly to output, but the stress index responds weakly to the output change and the stress shock itself. These are all responses that one would expect from macroeconomic one-regime VARs.

The symmetry and size independence of responses of the shock in the one-regime linear VAR is shown in figure 8. As one can observe, the positive and negative stress shocks have symmetric effects and the effects are linearly dependent on the size of the shocks. There is no difference whether there is a positive or negative shock that its the economy.

Next we want to explore regime-specific responses that may help to understand the dynamic properties of the estimated regimes. Assuming that

one stays within one regime after a shock will not be very realistic. Results of such an exercise are not convincing. Such an exercise will be of limited use when trying to assess the overall impact of a shock for two reasons. First, the process is not expected to stay within a given regime for an extended period of time; it will rather switch between regimes. Second, by looking at the within-regime dynamics, we would solely focus on the regime-specific autoregressive parameters and ignore the level effects induced by the differences in the regime intercepts. They will induce additional variation in the dynamics as the process switches between regimes.

In order to investigate the economy's overall reaction to shocks, we simulate generalized cumulative response functions to unit-impulse shocks. We do this for specific states at which the shock is assumed to occur. The two states we select are given by the sample averages we observe for the two regimes and are, thus, representative for low- and high-growth states of the economy. In the mean in low-growth regime is  $\bar{y}_{low} = (-0.3463, -0.0294)'$ , and that for the high-growth one is  $\bar{y}_{high} = (0.6137, 0.0296)'$ . For each case we simulate two shocks to the stress-index: a positive and a negative unit-shock. The mean cumulative responses to output and one-standard deviation confidence bands are shown in Figure 9.<sup>55</sup>

The responses strongly suggest that the impact of a stress-shock on output varies with the state of the economy. A *positive* unit-shock in the average high-growth state (top left plot in Figure 9) causes IP to drop by about -4% within about 36 months. The same shock applied in the average low-growth state (bottom left plot), results in a less severe output contraction, namely, -2% after three years. Thus, in a boom period an increase in financial stress curbs growth more strongly than in a recession. This presumably comes from the fact that an increase of financial stress, and thus a deterioration of the balance sheets of banks and a rising insolvency risk in the high growth

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<sup>55</sup>The generalized cumulative responses were simulated based on 100 replications, which were repeated 200 times to approximate the standard errors of the responses.

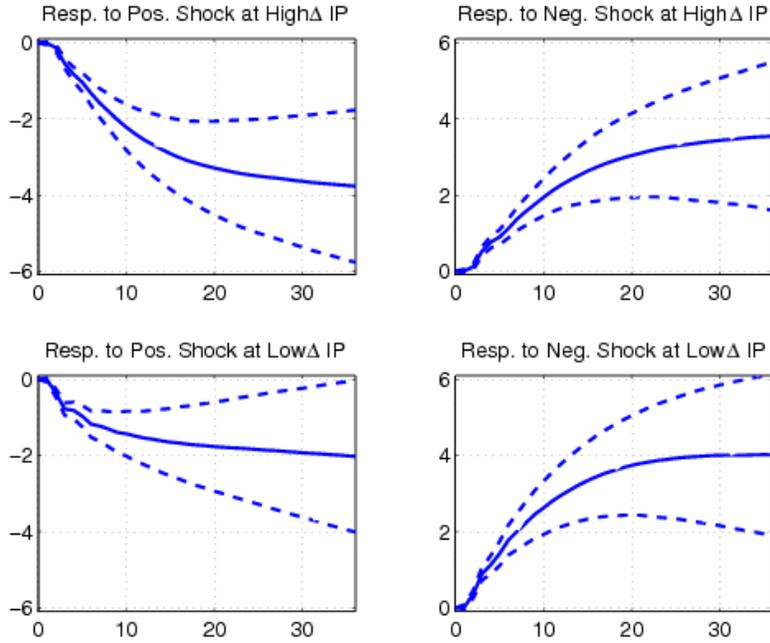


Figure 9: Cumulative MRVAR Responses to Positive (left panel) and Negative Stress-index Shocks (right panel) in High- (upper panel) and Low-growth States (lower panel)

regime, can trigger sharp downturns. This might come from the tendency that even our risk measure (and any risk measure) declines during the boom, even though risk might build up in the background. This often leads to the paradox that while the risk is rising in terms of higher leveraging in high growth regimes, the financial risk measure (for example due to falling credit spreads) shows a decline, see figure 6.

On the other hand, a reversion of the sign of the stress-shock, a negative stress-shock, also indicates some asymmetries in the IP response. A *negative* unit stress-shock in the high-growth state (top right plot) produces a 3.5% IP increase in the long-run. In contrast, in the depressed state (bottom right plot) the negative shock boosts IP by 4%. Thus, a negative stress shock in the low growth regime seems to be more effective in boosting the economy. We will come back to this issue when we consider larger shocks.

Moreover, the state-dependent response analysis indicates that in a neg-

ative growth period the IP responds very differently with the sign of the stress–shock: A positive unit–shock reduces IP by about -2%, whereas a stress reduction by a unit induces an IP increase of 4%. This cannot be observed for the linear one regime VAR as shown in figure 8. There is virtually no such asymmetry: The size (not the sign) of the IP response is virtually identical for positive and negative shocks to financial stress.

Next, we investigate to what extent the *size* of the shock to financial stress matters. Instead of a *unit* shock to the stress index we simulate the cumulative IP responses to stress shocks with different sizes. Specifically, we impose positive and negative shocks of sizes 0.25, 0.5, 0.75, 1.0, 1.25, and 1.5, always measured in terms of standard deviations of the stress index.

The consequences of positive shocks after 36 months differ quite dramatically with the magnitude of the shocks. Figure 10 compares the response profiles. Whereas the responses to a small (+0.25 std. dev.) shock are similar in both regimes, namely, -0.8% in the average high– and -0.7% in the average low–growth regime, this does not hold for larger shocks. For larger shocks, 0.5 standard deviations and more, IP drops roughly about twice as much in the high– compared to the low–growth state. One can see here not only that large positive shocks have quite a different final impact than small shocks, but that the effects of the large shocks are quite different in a high growth regime as compared to low growth regime. As already argued above, a given the fragility of the finance sector likely to be built up during the boom, a sudden large financial stress shock in the high growth regime will likely trigger a significant deterioration of balance sheets through externality effects in the interconnectedness of the financial firms and a rise of a credit risk spread (and possibly a cascade of insolvencies), generating strong chain effects of shocks in the positive regime.

As Figure 11 shows, we find an analogous but somewhat less extreme divergence for negative shock scenarios. For small negative shocks (-0.25 and -0.5) IP responds more or less identically. Larger stress reductions, however,

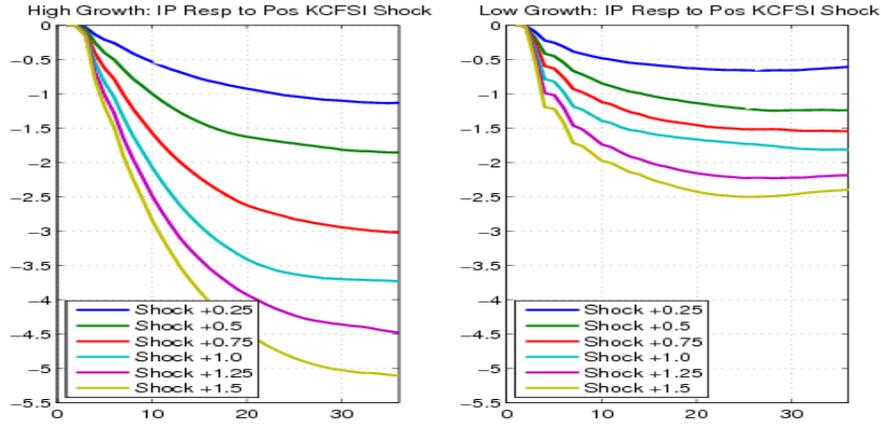


Figure 10: Cumulative MRVAR Responses to Positive Stress-index Shocks in Average High- (left) and Low-growth States (right)

have a much stronger positive effect on IP growth when the economy is in a recessionary rather than a boom period, a phenomenon observed earlier for the unit shock. In case of larger shocks (-1.25 and -1.5), the impact in low-growth is about 50% larger than in the high-growth.

A comparison of the left plots in Figures 10 and 11 reveals that—as in the unit-shock experiment—there is also not much asymmetry in the IP responses when the shock size varies. In a low-growth state, however, there is a strong asymmetry. This holds especially, for large shocks. A 1.5 standard-deviation reduction in the stress index raises IP by about three times as much as a stress-increase of the same size would lower IP. This seems to us a very relevant observation concerning the asymmetric impact of monetary policy on the economy. Monetary policy shocks—in particular what has recently been called unconventional monetary policy<sup>56</sup>—is likely to have large effects if the shocks are large.

<sup>56</sup>For example of quantitative easing, as pursued by the Fed since 2008.

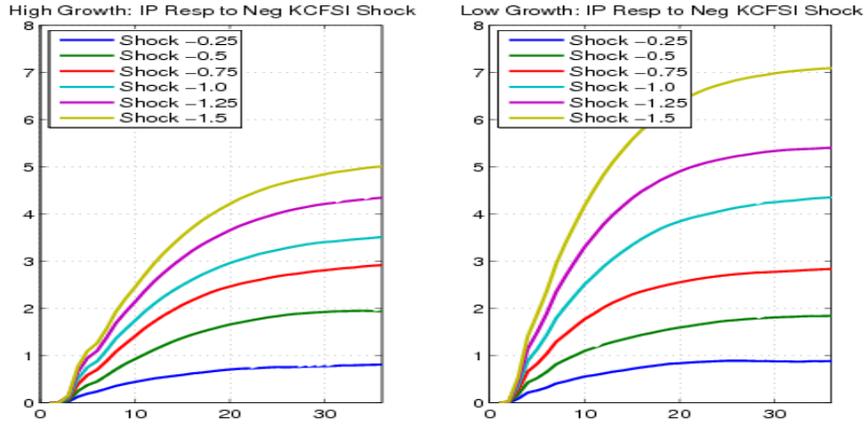


Figure 11: Cumulative MRVAR Responses to Negative Stress-index Shocks in Average High- (left) and Low-growth States (right)

We can stress that our empirical results suggest that the timing of policy actions affecting financial stress can be crucial to the success of such measures. The findings are compatible with recent studies which argue that unconventional monetary policy is needed in a depressed economy that is accompanied by a sharp rise in credit spreads, which, more so than asset-price volatility, constitute the dominant component of the stress index.<sup>57</sup> The results suggest that not only a decrease in the interest rate but also in credit spreads are required to induce significant expansionary effects.

## 7 Conclusions

Though most of the historical economic crises ended up as a meltdown of the banking sector, the banking sector has usually exacerbated and amplified the crisis whatever origin it had. To investigate those feedback effects, we have studied the linkage of asset prices and the financial intermediaries' balance sheets. We have modeled the financial intermediaries as they are affected by adverse asset price shocks, but we also considered the reverse effect from the instability of the banking system to the macroeconomy, which is here

<sup>57</sup>See for example Curdia and Woodford (2009a 2009b) and Gertler and Karadi (2009).

represented by the growth rate of output. In particular we studied the issue of local instability of the banking sector that is exposed to asset price shocks, credit spread shocks and financial stress.<sup>58</sup> We modeled financial intermediaries as wealth fund that accumulates capital assets, can heavily borrow and pays bonuses. When the banking sector is exposed to a deterioration of its balances sheets, it turns out that the size of the bonus payments plays an important role for the dynamics of the leverage ratio, the financial stress and the local instability.

In contrast to previous studies that use the financial accelerator – which is locally amplifying but globally stable and mean reverting – our model admits local instability and globally multiple regimes. Whereas the financial accelerator leads, in terms of econometrics, to a one-regime VAR, the multi-regime dynamics studied here requires to use a multi-regime VAR (MRVAR). Using a financial stress index as evaluating the financial intermediaries’ stress and output growth, measuring the state of the macroeconomy, our method of a MRVAR estimate permits us to undertake an impulse-response study which lets us explore regime dependent shocks.

We show that the shocks have asymmetric effects depending on the regime the economy is in, but we also show that the effects of the shocks are dependent on the size of the shocks.<sup>59</sup> Large positive financial stress shocks in booms seem to have a stronger contractionary effect than in a recessions, but large negative stress shocks in recessions appear to have a stronger expansionary effect in recessions than in booms. The latter result seems to us very important for the evaluation of an unconventional monetary policy, since frequently not only the timing, but also the strength of policy actions matter.

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<sup>58</sup>Note that most of the components of the financial stress index are some measure of credit spread.

<sup>59</sup>In earlier literature on Keynesian macrodynamics this has been called “corridor stability”: small shocks have only small effects but larger shocks could be destabilizing and have large effects.

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## Appendix 1: The Numerical Solution of the Model

We have used the dynamic programming method by Grüne and Semmler (2004) to study the dynamics of the stochastic version of debt-asset ratio with consumption and growth rate of assets as controls. The dynamic programming method can explore the local and global dynamics by using a coarse grid for a larger region and then employing grid refinement for a smaller region. As Brunnermeier and Sannikov (2010) correctly state, the dynamics should not be studied by first or second order Taylor approximations to solve for the local dynamics, since this will not capture the global instabilities of the model, in particular below the steady state. Instead we use dynamic programming, which can provide us with the truly global dynamics in a larger region of the state space without losing much accuracy (see Becker *et al.*, 2007). In contrast, local linearization, as has been argued there, does not give sufficient information on the global dynamics.

Hence, before going into the model discussion, we start by briefly describing this dynamic programming algorithm and the mechanism by which it enables us to numerically solve our dynamic model variants. The adaptive discretization of the state space feature of the dynamic programming algorithm leads to high numerical accuracy with moderate use of memory. In particular, the algorithm is applied to discounted infinite horizon dynamic decision problems of the type introduced for the study of our search and matching models. In our model variants we have to numerically compute  $V(x)$ :

$$\begin{aligned}
V(x) &= \max_u \int_0^\infty e^{-rt} f(x, u) dt \\
\text{s.t.} \quad & \dot{x} = g(x, u), x(0) = x_0 \in \mathbb{R}^1
\end{aligned}$$

where  $u$  represents the decision variable, and  $x$  a vector of state variables. Note that the time index  $t$ , as used in sect 4.3 of the paper, could be one of the state variables.

In the first step, the continuous time optimal decision problem has to be replaced by a first order discrete time approximation given by

$$V_h(x) = \max_{u \in U} J_h(x, u)$$

where  $J_h(x, u) = h \sum_{i=0}^{\infty} (1 - \theta h) f(x_h(i), u_i)$ , and  $x_h$  is defined by the discrete dynamics

$$x_h(0) = x, \quad x_h(i+1) = x_h(i) + hg(x_h(i), u_i)$$

and  $h > 0$  is the discretization time step. Note that  $U$  denotes the set of discrete control sequences  $u = (u_1, u_2, \dots)$  for  $u_i \in U$ .

The value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

$$V_h(x) = \max_{u \in U} \{hf(x, u) + (1 - \theta h)V_h(x_h(1))\} \quad (22)$$

where  $x_h(1) = x + hg(x, u)$  denotes the discrete solution corresponding to the control and initial value  $x$  after one time step  $h$ . Using the operator

$$T_h(V_h)(x) = \max_{u \in U} \{hf(x, u) + (1 - \theta h)V_h(x_h(1))\}$$

the second step of the algorithm now approximates the solution on a grid  $\Gamma$  covering a compact subset of the state space, i.e. a compact interval  $[0, K]$  in our setup. Denoting the nodes of  $\Gamma$  by  $x^i$  with  $i = 1, \dots, P$ , we are now

looking for an approximation  $V_h^\Gamma$  satisfying

$$V_h^\Gamma(x^i) = T_h(V_h^\Gamma)(x^i) \quad (23)$$

for each node  $x^i$  of the grid, where the value of  $V_h^\Gamma$  for points  $x$  which are not grid points (these are needed for the evaluation of  $T_h$ ) is determined by linear interpolation. We refer to Gr̈unke and Semmler (2004) for the description of iterative methods for the solution of (23). This procedure allows then the numerical computation of approximately optimal trajectories.

In order to distribute the nodes of the grid efficiently, we make use of an *a posteriori* error estimation. For each cell  $C_l$  of the grid  $\Gamma$  we compute

$$\eta_l := \max_{k \in C_l} | T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) |$$

More precisely, we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators  $\eta_l$  give upper and lower bounds for the real error (i.e., the difference between  $V_j$  and  $V_h^\Gamma$ ) and hence serve as an indicator for a possible local refinement of the grid  $\Gamma$ . It should be noted that this adaptive refinement of the grid is particularly effective for computing step value functions, non-differential value functions and models with multiple equilibria, see Gr̈unke *et al.* (2004) and Gr̈unke and Semmler (2004). These are all cases where local linearizations are not sufficiently informative.

## Appendix 2: Periodic Components in Credit Spreads, Bank Failures, Leveraging, and Bonus Payments

We take the BAA bond yield as a proxy of time varying credit cost that includes a risk premium. We apply the *Fast Fourier Transformation* to Moody's Seasoned BAA Corporate Bond Yield. The time period is from

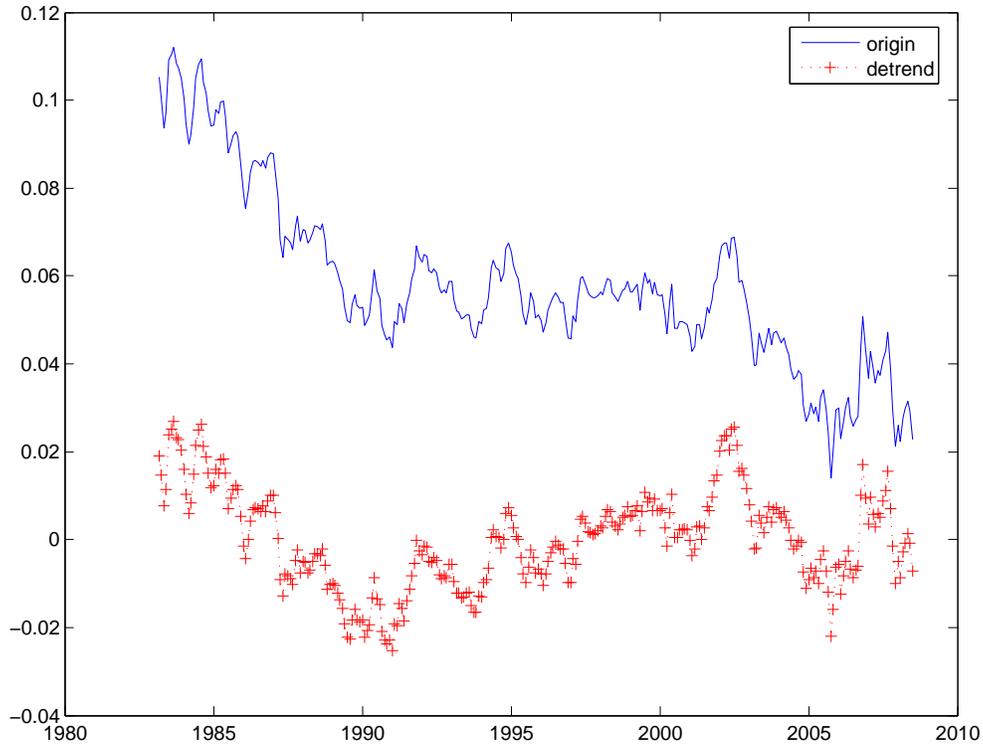


Figure 12: Original and de-trended real Baa Yields

February 1983 to June 2008 at monthly frequency (305 data points). Data are: A) Moody's BAA corporate bond yield from St Louis Fed,<sup>60</sup> and the inflation rate: B) the CPI (seasonal adjusted) consumer price index of all urban areas from Bureau of Labor Statistics of U.S. Department of Labor.<sup>61</sup> The realized real bond yield is then: A)-B).

First we de-trend the real BAA yields

$$\text{Detrend rb} = \text{Original rb} - (-0.0022(t - t_0) + 0.0862) \quad (24)$$

and illustrate it in Figure 12.

<sup>60</sup>See <http://research.stlouisfed.org/fred2/series/BAA>

<sup>61</sup>(See <http://www.bls.gov/cpi/home.htm>.)

Next we apply the FFT on the de-trended real Baa Yields and obtain the loading/power of periods, which helps us to select the first few harmonic components of the fit. The harmonic fit is implemented, the coefficients estimated (reported in the table below), and the results are then illustrated in Figures 13-14. The empirical estimates are based on linear regressions based on the trigonometric functions, which means we fit the time series  $x_t$  using the sin/cos functions of the given period. The linear regression formula is given by

$$x_t = \sum_{i=1}^n \left( a_i \sin \left( \frac{2\pi}{\tau_i} (t - t_0) \right) + b_i \cos \left( \frac{2\pi}{\tau_i} (t - t_0) \right) \right). \quad (25)$$

$i =$	1	2	3	4	5	6
$\tau_i(\text{month})$	305.0	152.5	101.7	30.5	43.6	27.7
$a_i$	-0.0066	0.0062	0.0063	0.0007	0.00222	0.0025
$b_i$	0.0049	0.0031	0.0016	-0.0033	-0.0026	-.0004

Table 1: Coefficients of the harmonic fit of the real bond yield in the equation (25)

Figure 13 gives the sum of squared errors of the harmonic fit for the Baa corporate bond yields, table 1 the estimated coefficients of the harmonic fit, and figure 14 the periodic components.

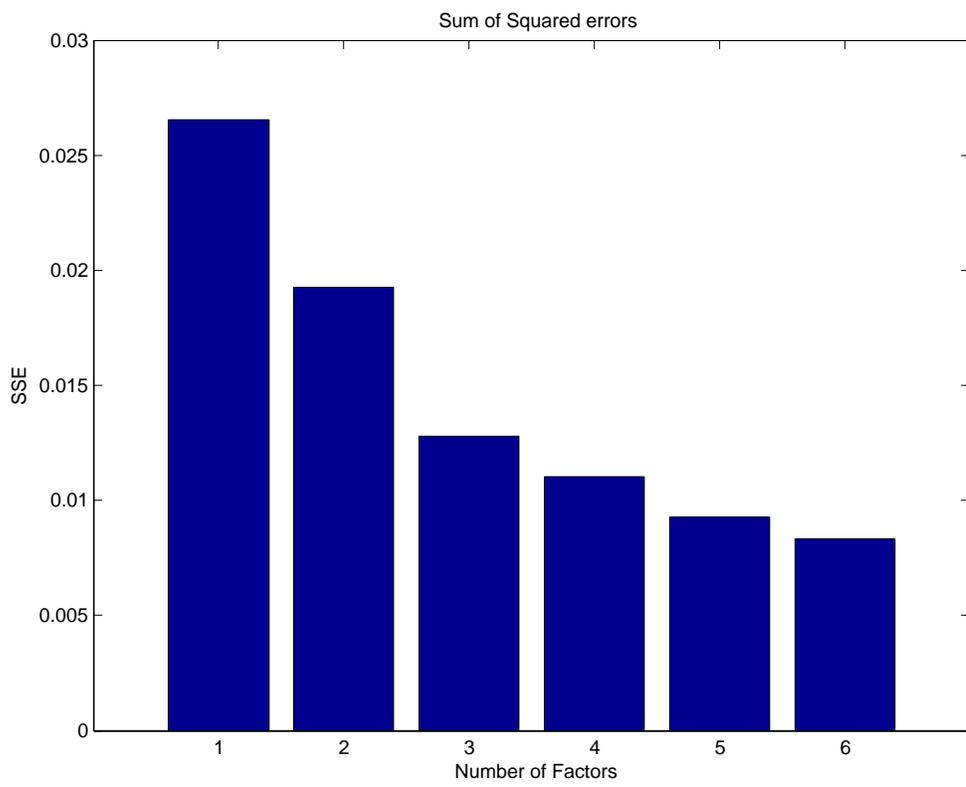


Figure 13: Sum of the squared errors

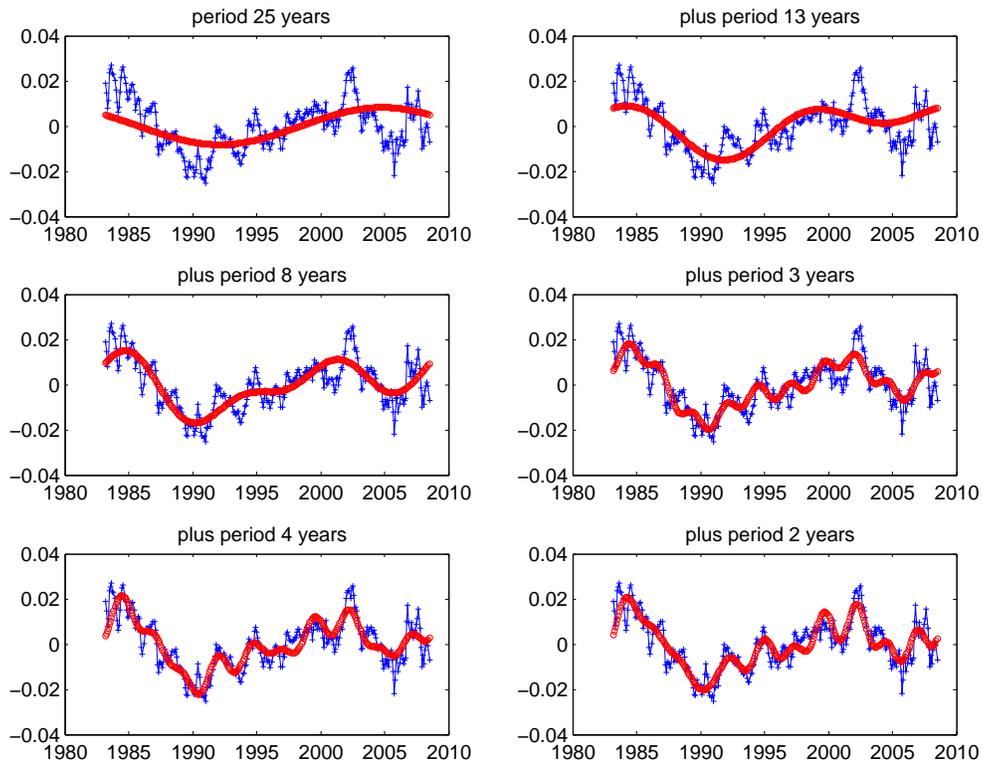


Figure 14: Harmonic fit of the Baa yields

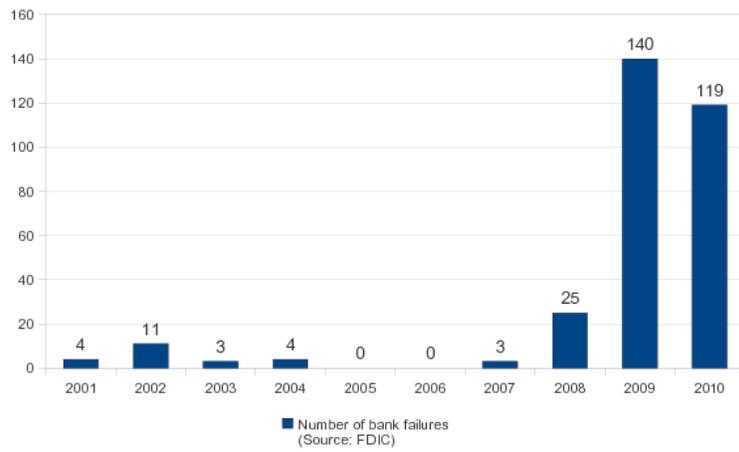


Figure 15: Bank failures

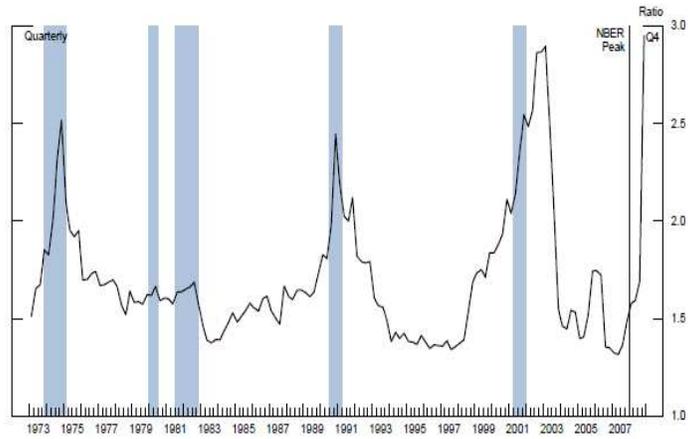


Figure 16: Leveraging (source: Gilchrist et al., 2009)

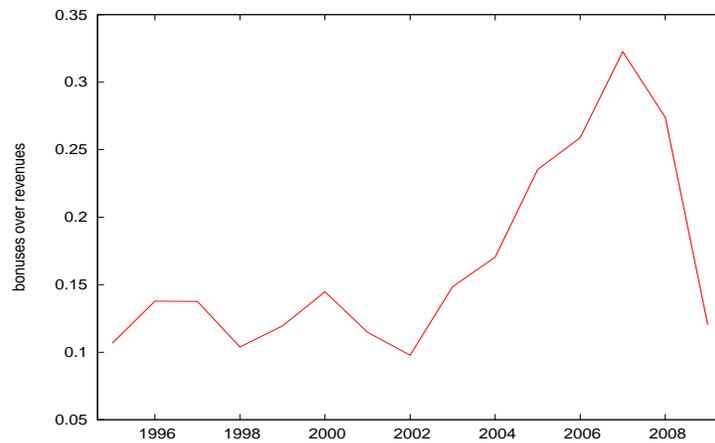


Figure 17: Ratio of bonus payments to revenue (source: DiNapoli and Bleiwas, 2010)