Mortgage Market Innovations and Housing Investment A Theoretical and Empirical Investigation *

Quoc Hung Nguyen †

This version: September, 2009

Abstract

This paper addresses the impact of recent mortgage market innovations on the housing market, particularly on housing investment. The paper begins by documenting two stylized facts for OECD countries that models with perfect credit markets fail to explain: (i) housing investment is about five times as volatile as GDP, and (ii) housing investment becomes more volatile in economies with more recent mortgage market innovations. The paper then develops a DSGE model where there is a borrowing constraint and housing is used as collateral. In this model, the housing collateral value, which defines households' borrowing capacity, is endogenously determined by housing prices and stock. Consequently, the collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to shocks and becomes stronger in economies with deeper mortgage markets. As a result, the model with a housing collateral constraint can explain the high volatility of housing investment and the fact that mortgage market innovations may destabilize the housing market. The paper calibrates the collateral constraint model to the U.K economy.

JEL Classification: E22, E32, F34, F41

Keywords: Housing Investment, Collateral Constraint, Mortgage Market

*This paper is previously circulated under the title "Housing Investment: What makes it so volatile? Theory and Evidence from OECD countries". I am especially grateful to Michael B.Devereux for continual academic support and encouragement. I would like to thank Paul Beaudry, Viktoria Hnatkovska, Amartya Lahiri, and Henry Siu for their useful comments and feedback. I thank the Bank of International Settlements for providing data. I also derived benefits from talking with colleagues at UBC, particularly David Freeman and Subrata Sarker. All errors are solely mine.

†University of British Columbia and Hong Kong Institute for Monetary Research, Hong Kong Monetary Authority. Email: quochung@interchange.ubc.ca

1 Introduction

"Why is housing investment so volatile and have recent mortgage market innovations introduced greater volatility in the housing markets?" The questions are of interest because of several reasons. First, the current global financial meltdown has generated wide interest in the impact of recent mortgage market innovations on the housing sector and the overall business cycle, particularly the concern that these innovations may destabilize the housing market. Second, housing investment shocks account for a large share of variance in GDP in many economies and housing investment offers the best early warnings of an oncoming recession among GDP components. [1] Therefore, it is important to understand the dynamics of housing investment in order to control business cycles. Third, in the US, housing investment has been documented to be both pro-cyclical and highly volatile and while the pro-cyclicality has obtained satisfactory explanations [2] the highly volatile behavior has not. [3]

Data from OECD countries first indicate that the highly volatile behavior of housing investment is not a distinguishing feature of the U.S economy. Across 17 OECD countries, housing investment is on average about five times as volatile as GDP and significantly more volatile than non-housing investment.[4] Housing investment also tends to be more volatile in economies with more recent mortgage market innovations like Australia, the U.K, and the U.S. In these economies, the standard deviation of housing investment is about two times as large as its non-housing counterpart and is about six to seven times as large as GDP. This positive association is interesting.[5] If housing is just a durable consumption good and consumers tend to smooth consumptions then the more developed mortgage market, which implies a broader access to credit markets, should allow households to smooth more efficiently against fluctuations. Nonetheless,

[1] For example, housing demand shocks account for 20-25% of variance in GDP in the U.S and Japan (IMF, 2008) and in the past 60 years, eight out of ten recessions in the US were preceded by substantial problems in housing (Leamer, 2007).

[2] The regularity that housing investment co-moves with other investments and is pro-cyclical with GDP. The co-movement in multi-sector models is not as straightforward as it might appear, since there is a strong incentive to switch labor/production between sectors in response to sector-specific productivity shocks. See Charles Leung (2004) for further literature review and explanations.

[3] The existing literature has limited success in explaining the volatility. The exception is Davis and Heathcote (2005), which explains the high volatility from the supply side, but does not address the mortgage market. The paper will review this later.

[4] The volatility of these non-housing durable goods is already very high from the business cycle perspective; it is about four times that of GDP.

[5] The positive correlation is not limited by cross-country evidence but is also reflected by time series data. The volatility of housing investment relative to GDP has significantly risen along dramatic innovations in the mortgage market. The paper discusses more in the empirical part

consumption smoothing for housing is not supported by empirical evidence.

Standard business cycle models with perfect credit markets are at odds with these empirical findings. First, these models are unable to explain the positive correlation between housing investment volatility and mortgage market depth since the degree of mortgage market development should be immaterial under a perfect credit market assumption. Second, standard models are also at odds in reconciling the high volatility of housing investment. I shows in this paper that a quantitative two-sector model with free borrowing fails generating a realistic volatility of housing investment.

To explain the aforementioned stylized facts, I develop a Dynamic Stochastic General Equilibrium (DSGE) model with a borrowing constraint. Specifically, I consider a limited obligation environment in which borrowers do not repay unless debts are secured by collateral and housing plays the collateral role for household debt.[6] The collateral constraint is inspired by the evidence that the major part of household borrowing has been in the form of collateralized debt. For example, the shares of mortgage debt in total outstanding household debt are about 80% in the US and 70% in Canada. There is also evidence of borrowers' limited obligations. For instance, when the subprime mortgage market worsened, many borrowers just walked away from their housing collateral without any further obligations. Housing collateral is rationalized by the fact that housing is a very good store of value and an important component of wealth for most households.[7]

The mechanism through which a housing collateral constraint affects the dynamics of housing investment goes as follows. The value of housing collateral is endogenously determined by the housing stock and prices, which in turn define households' borrowing capacity. As a result, the housing collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to shocks and explains the high volatility of housing investment. Intuitively, increased demand for housing in good times drives up both the housing stock and housing prices. These increases in turn raise the collateral value, enabling households to borrow more from capital gains to consume and further invest in housing, thereby creating a borrowing-consuming spiral. In other words, a boom in the housing market increases the collateral value, allowing households to borrow more to consume more. However, increased consumption including housing purchases in turn fuels the housing

[6]This includes, but is not limited, to mortgage debt.

[7]The value of housing structures excluding land is similar to the combined value of private non-housing structures and equipment, similar to annual GDP, and three times as large as the total stock of all other consumer durables. Moreover, the median value of a house is often much higher than the annual income of a typical household even in advanced countries, therefore, the owner usually has to access mortgage credit to purchase a house. In mortgage lending, housing naturally becomes collateral.

boom further, making housing investment highly volatile. Moreover, by anticipating the value of collateralizable housing in relaxing the borrowing constraint, credit constrained households rationally purchase a greater amount of housing in good times, which also accounts for the high volatility of housing investment.

The housing collateral constraint can also account for the positive correlation between housing investment volatility and the degree of mortgage market development. In other words, mortgage market innovations may introduce instability in the housing market. The underlying reason is that in economies with more flexible and developed mortgage markets, credit-constrained households can borrow a higher amount for the same value of collateral and easily withdraw equity from increased collateral for consumption.[8] As a result, more developed mortgage markets intensify the collateral role of housing, thereby encouraging credit constrained households to purchase more houses in good times. Besides, more developed markets also strengthen the link between the housing market and the consumption decisions, hence creating a stronger borrowing-consuming spiral.

This work is related to the business cycle literature that incorporates the housing sector. This literature documents regularities, distinguishes housing investment from its non-housing counterpart, and attempts to explain the co-movement between the two types of investment.[9] These authors, however, often have difficulty in accounting for the relatively high volatility of housing investment. For example, Baxter (1996) finds that consumption of durables that include housing investment is less volatile than business investment; Fisher (1997) is unable to generate household investment more volatile than business investment for all specifications. Davis and Heathcote (2005) explain the co-movement and the high volatility by building a model where housing and the other sectors all use three intermediate goods, albeit in different proportions. The high volatility mainly results from their calibration that the housing construction sector uses a relatively higher proportion of intermediate goods which are relatively more volatile. It is, however, unclear whether their estimate of the Solow residual of housing construction production is due to productivity shocks or the mixed equilibrium outcome of supply and demand in the housing sector. By contrast, this paper explains the high volatility from the

[8] It is applied even under the case that households don't directly acquire debt or withdraw equity for consumption since by accessing more mortgage debt there would be more credit available for general consumption.

[9] Greenwood and Hercowitz (1991) and Baxter (1996) assume reversibility between residential and business capital and also assume/calibrate the same or highly correlated productivity shocks between two sectors. Fisher (1997) assumes complementarity between the household and business capital in goods production. Chang (2000) argues that if there are adjustment costs in capital accumulation and substitutability between leisure time and durable goods in home production, then when households work more in periods of high productivity they also demand more durables.

demand side, particularly from the imperfect credit aspect of the housing sector.

The housing collateral constraint, the key ingredient of this paper, originates from the seminal work of Kiyotaki and Moore (KM) (1997) and Kocherlakota (2000). These authors show that collateral effects can be a powerful propagation mechanism by which relatively small, temporary shocks can generate large, persistent fluctuations in output and asset prices. Campbell and Hercowitz (CH) (2005) develop a one-sector real business cycle model to address the impact of credit market innovations on macroeconomic volatility. Their mechanism is through the labor supply: less tight collateral constraints weaken the connection between constrained households' housing investment and their hours worked. Iacoviello (2005) incorporates the New Keynesian monetary policy framework into the work of KM. Collateral effects enable his model to match the positive response of spending to a housing price shock. Calza et al. (2007) extend Iacoviello's work to allow production of new housing and endogenous asset price movement. They also model institutional features of the mortgage market and argue that the correlation between consumption and house prices increases with the degree of mortgage market development, and the transmission of monetary policy shocks to consumption and to housing prices is stronger in countries with more developed mortgage markets. More recently, Monacelli (2008) argues that introducing a collateral constraint into the New Keynesian framework can reconcile the co-movement of durable and non-durable spending in response to monetary shocks.

My work differs from these in many key aspects. Unlike the CH work, it develops a two-sector model and incorporates asset price movement to explore the amplification mechanism of collateral effects. In contrast with the others, which are New Keynesian models with nominal sticky prices and nominal debt, this paper is based on an RBC model with flexible prices and real debt to study the impact of the productivity shock. Moreover, the existing literature considers a closed economy model with heterogeneous agents where patient savers lend to impatient borrowers; this paper considers an open economy model in which domestic agents can access international credit markets, which captures the increasingly global credit market.[10] The paper also incorporates capital to better characterize the dynamics of the current account. Particularly, it is shown in the quantitative section that collateral effects improve the performance of the model in terms of generating the counter-cyclicality of the current account compared to models in the existing open economy literature such as Backus et al. (1992) and Mendoza (1991). Finally, the small open economy model allows the paper to have a representative agent,

[10]This is also rationalized by the fact that this paper studies 17 OECD countries, most of which can be regarded as small open economies in the global economy. Even for the U.S economy, thanks to recent dramatic financial deregulation, the major part of mortgage debts has been held by international investors.

which makes the model simple.[11]

This paper is organized as follows. Section 2 describes data, particularly two mortgage market depth indicators, and documents stylized facts about housing investment and its association with mortgage market depth. Section 3 explains the empirical findings using a basic model with a borrowing constrained representative household. Section 4 extends the basic model to include heterogeneous households, discusses the model's dynamics, and calibrates it for the U.K. Section 5 concludes.

2 Stylized Facts

This section documents major stylized facts about housing with emphasis on housing investment and the mortgage market in 17 advanced OECD countries from Q1-1980 to Q3-2007.[12]

2.1 Data

All time series data are quarterly, except Germany's annual and Italy's half-year house prices. House prices are mainly provided by the Bank of International Settlements, and other missing values are filled and updated via Datastream. Real house prices are then obtained by deflating nominal house prices with the consumer price index (CPI).

Housing investment or residential investment, non-housing investment, total investment, and GDP are in real values, i.e., in constant or chained prices, and obtained via Datastream and OECD Stat.[13]

I utilize two specific indicators to measure the degree of mortgage market development in these OECD countries. The first one is a synthetic mortgage market index constructed by the IMF.[14] The second measure is the ratio of total outstanding amount of mortgage debt over GDP, the mortgage-debt-to-GDP ratio or the mortgage depth, which is often used in literature.[15]

[11] In an extended model, I also consider an economy with heterogeneous households.

[12] The choice of 17 OECD countries is mainly based on the availability of data. They are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Sweden, the U.K, and the U.S. I choose post 1980s period since most of innovations in mortgage markets of these countries began in the early 1980s.

[13] For more details about the code of each specific variable, see the Data Appendix.

[14] They are taken from Table 3.1 of Chapter 3 of IMF World Economic Outlook (WEO) April 2008: "The changing housing cycle and the implications for monetary policy".

[15] For example: Warnock and Warnock (2008) use this ratio or maximum possible of this ratio to measure mortgage market depth or market size. Some other series of OECD working papers also use this particular measure. The data for mortgage-debt-to-GDP ratio (2001-2006 average) for all countries except New Zealand

In particular, although most of advanced OECD countries have moved toward more competitive and developed housing finance markets thanks to recent deregulation and innovations in the mortgage market, there are still significant cross-country differences in the level of mortgage market development in terms of market liberalization, legal procedures, and regulatory structures. The cross-country differences in mortgage market development are reflected through: (1) The typical ratio of a mortgage loan to property's value or loan-to-value (LTV) ratio and the standard length of mortgage loans; (2) The ability to make home equity withdrawals and to prepay mortgages without a fee; (3) Developments of secondary markets for mortgage loans. These differences then imply different households' access to housing-related financing in each country. To summarize cross-country differences in mortgage market development, a synthetic mortgage market development index is constructed.[16] The index lies between 0 and 1, with higher values indicating easier household access to mortgage credit. The IMF's mortgage market index (henceforth MMI) and the mortgage-debt-to-GDP ratio or the mortgage depth (henceforth MD) are closely positively correlated, i.e., the economies with a higher mortgage market index often have a bigger or deeper mortgage market size (Figure 3). Figures 1 and 2 show evidence that there are significant differences in the degree of mortgage market development and mortgage size, even among advanced OECD countries.

Since the IMF's index is a one-period time indicator, which may be able to capture precisely only the current degree of mortgage market development, I extend data for the second indicator, the mortgage-debt-to-GDP ratio, to the last 10 years in order to examine the development of the mortgage market over time.[17] Figure 4 suggests that the degree of mortgage market depth has been increasing for most of these countries but the rank remains the same, i.e., those countries that currently have deeper mortgage markets also possessed deeper ones in the 1990s. Therefore, I conclude that the IMF's index reflects the comparative degree of mortgage market development, at least from the 1990s.

2.2 Stylized Facts

The first stylized fact about housing in OECD countries is that its real prices are significantly pro-cyclical with real GDP, which is contrary to the counter-cyclicality of non-housing invest-

is taken from the IMF. Data for New Zealand are taken from Warnock (2008).

[16] For more detail about the construction method, see Chapter 3 of WEO 2008

[17] Sources: European Mortgage Federation, IMF, FRB release, Reserve Bank of New Zealand, OECD, and Keen (2007). Although some countries like the U.S, U.K and Australia have data before 1997, I could not find longer data for some European countries.

Figure 1: Mortgage Market Index

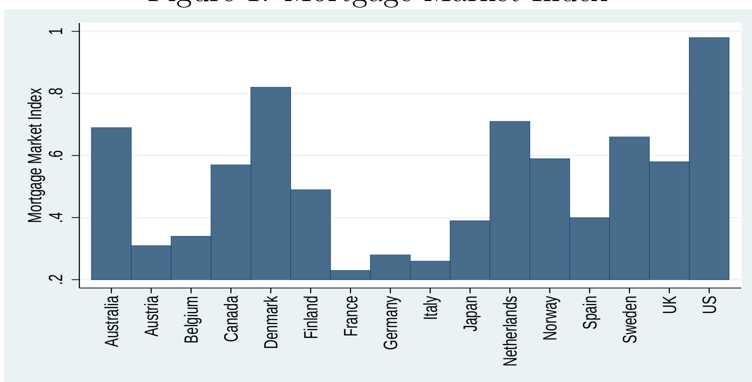


Figure 2: Mortgage Depth

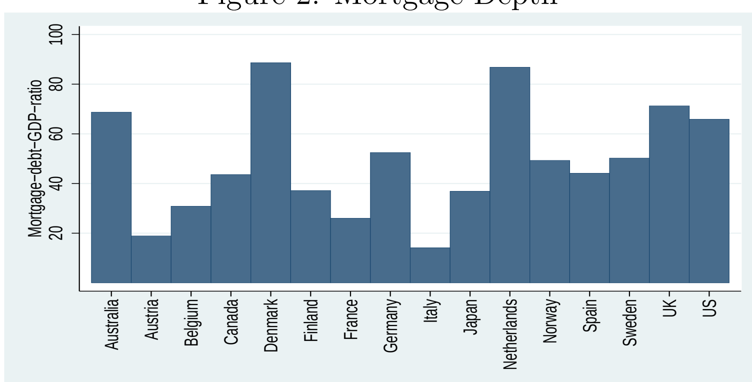


Figure 3: Mortgage Market Index and Mortgage Depth

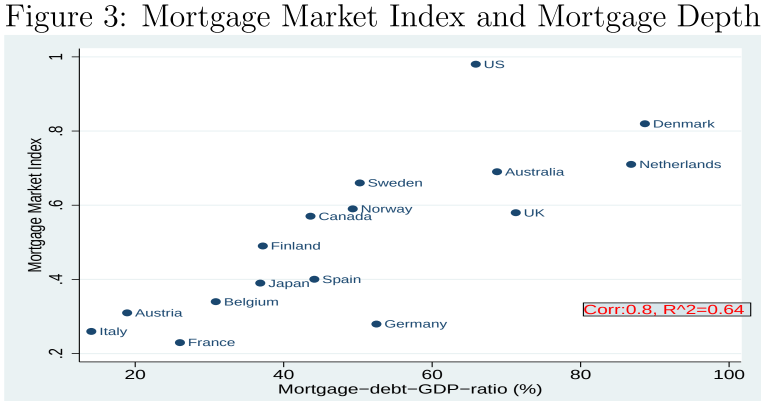
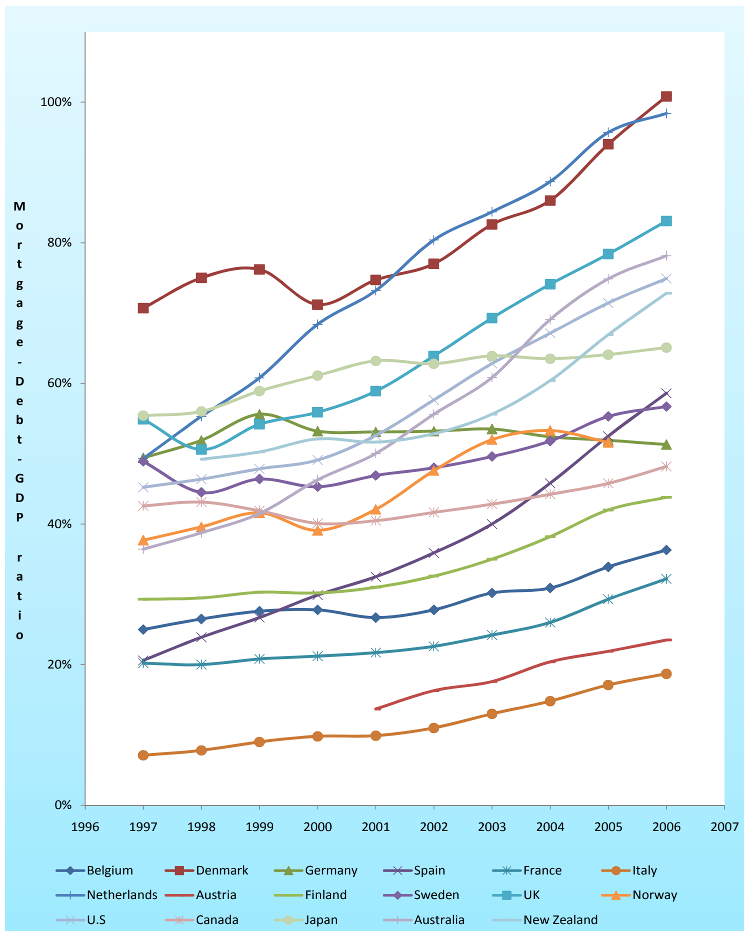


Figure 4: Mortgage Depth Development



Sources: DataStream, European Mortgage Federation, Federal Reserve Release (2008), Federal Reserve of New Zealand (2008, 2004), IMF, Keen (2007), OECD Outlook ment's real prices[18] (The 2nd Column of Table 1).

Table 1: **Statistic I**

	Correlation with GDP				Std.dev relative to GDP		
Country	HP	RES	NRES	INV	RES	NRES	INV
Australia	0.41	0.65	0.66	0.79	6.50	5.47	4.87
Austria	0.23	-0.12	0.81	0.85	4.00	3.13	2.00
Belgium	0.38	0.58	0.40	0.58	3.89	2.89	2.78
Canada	0.52	0.48	0.66	0.73	4.09	3.57	3.07
Denmark	0.53	0.39	0.58	0.65	5.08	4.50	3.83
Finland	0.74	0.62	0.70	0.78	3.11	3.37	2.84
France	0.49	0.71	0.86	0.88	2.89	3.22	3.00
Germany	0.23	0.55	0.75	0.76	3.33	3.78	3.22
Italy	0.26	0.20	0.76	0.96	2.56	4.11	3.22
Japan	0.64	0.63	0.80	0.87	4.73	2.18	2.18
Netherlands	0.68	0.57	0.63	0.75	4.00	4.00	3.30
New Zealand	0.30	0.72	0.78	0.84	5.86	4.43	4.29
Norway	0.49	0.23	-0.08	-0.01	4.85	5.08	4.31
Spain	0.33	0.11	0.51	0.54	4.50	4.33	3.33
Sweden	0.77	-0.44	0.78	0.60	6.15	2.85	2.23
U.K	0.58	0.57	0.48	0.72	6.44	3.60	3.16
U.S	0.40	0.64	0.78	0.91	6.67	3.57	3.40
Average	0.47	0.42	0.64	0.72	4.63	3.77	3.24

Notes: HP is real house prices, RES is real housing investment, NRES is real non-housing investment, INV is real aggregate investment, GDP is real GDP. Correlations are correlation with GDP. RES/GDP, NRES/GDP, and INV/GDP denote the relative volatility of RES, NRES, and INV to that of real GDP, respectively. All series are in logs and Hodrick-Prescott filtered

The 3rd and 4th Column of Table 1 present evidence that housing investment co-moves with non-housing investment and is pro-cyclical with GDP. The co-movement property is prevalent in these advanced OECD countries and has an important implication for theoretical models that this paper will address later. Note that the pro-cyclicality of both real housing prices and housing investment makes it challenging for those models that try to explain the high volatility

[18]As documented by Greenwood et al. (1997) and Fisher (2006), the real non-housing investment price measured by the business equipment deflator divided by consumption deflator is significantly counter-cyclical with GDP: The unconditional correlation for the U.S economy is -0.54

of housing investment from supply side, particularly the housing sector specific productivity shocks.

Compared to its non-housing counterpart, housing investment is also different in terms of volatility and cross-country dispersion. According to Table 1 (Column 10 and 11), the standard deviation of housing investment relative to GDP is not only significantly higher than that of non-housing investment but also varies widely across countries. The former ratio ranges from 2.56 in Italy to 6.67 in the U.S, whereas the latter ratio is stable at 3.8. The F-test for variances of the two groups is rejected with significant level (p-value is 4%) and the t-test for equality of the two ratios is strongly rejected (p-value=0.2%). I obtain the same conclusions when comparing the housing investment with aggregate investment: housing investment is, on average, much more volatile and varies widely across countries than aggregate investment.

With regard to the mortgage market, Figures 5 and 6 first show significant positive correlations between the volatility of GDP and the two mortgage market indicators. More interestingly, Figures 7 and 8 present evidence that the volatility of housing investment relative to GDP is higher in economies with more developed mortgage markets, i.e., economies with higher mortgage market indices and larger mortgage market size, while there is no significant correlation between the volatility of non-housing investment and degree of mortgage market development (Figure 9 and 10). In other words, these figures show that while GDP tends to be more volatile in economies with deeper mortgage markets, housing investment is still more volatile. Therefore the volatility of housing investment to GDP significantly increases in these countries.

Finally, I explore housing investment from a historical perspective. Since most deregulation and innovation in the housing finance system in advanced OECD countries just began in the early 1980s, and it is evident that the current system has been much developed compared to that in the early stage of deregulation and innovation, I divide samples into 2 periods: prior and post Q1-1995.[19] Table 2, first, presents evidence of the so-called *Great Moderation* in the last decade. Particularly, the volatility of output has dropped dramatically over time across advanced OECD countries: post 1995, the average standard deviation of GDP is about two times as low as that prior to 1995. However, the volatility of housing investment has not fallen by that much so that the volatility relative to GDP has risen significantly. [20] In short, housing investment has become relatively more volatile along with dramatic innovations in the mortgage market in these OECD countries.

[19] I use 10 out of 17 countries that have relatively long enough observations before Q1-1995.

[20] The volatility of housing still varies widely among countries. The t-test for the equality of the two ratios of relative volatility is rejected with 10%.

Figure 5: GDP Volatility and MMI

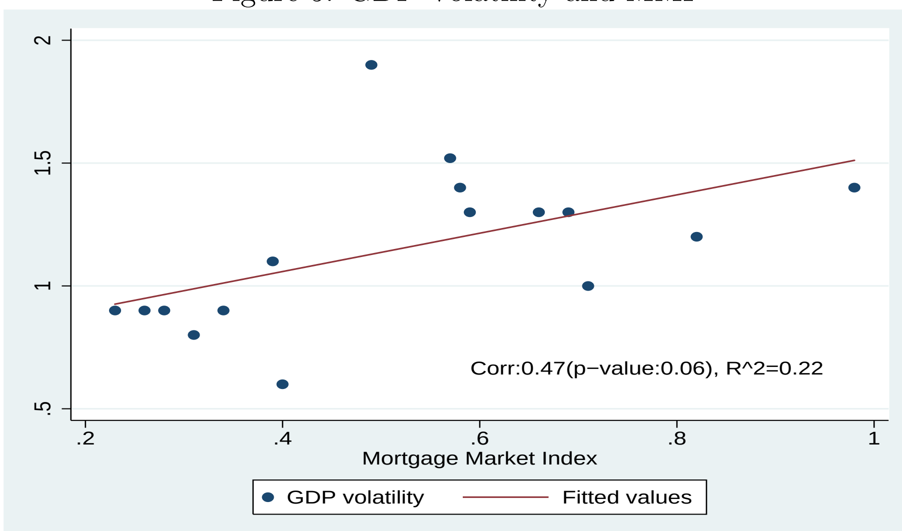


Figure 6: GDP Volatility and MD

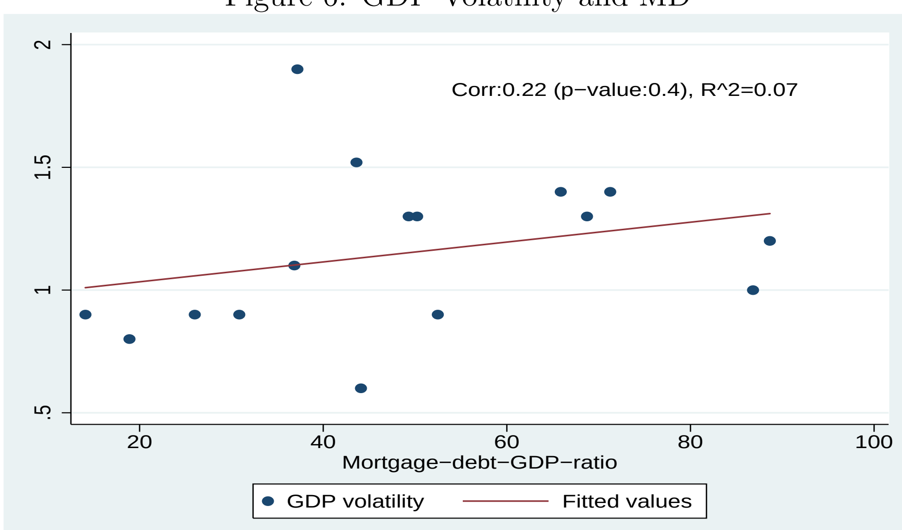


Figure 7: Housing Investment Volatility and MMI

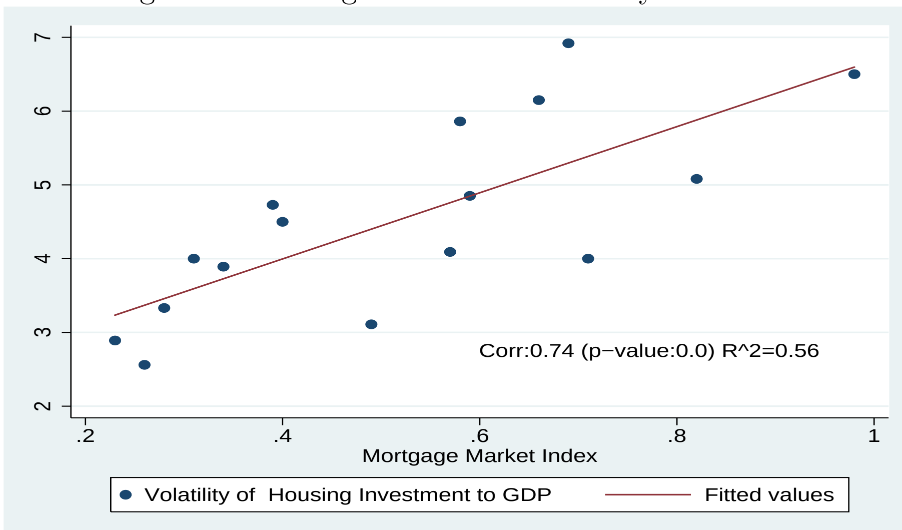
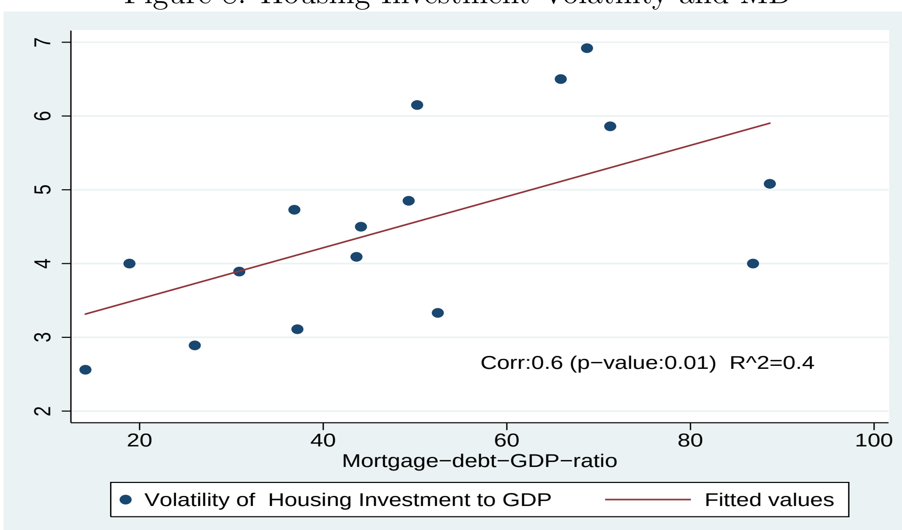


Figure 8: Housing Investment Volatility and MD



3 Basic Model

To explain the aforementioned stylized facts, I construct a two-sector RBC model in which a representative household faces a borrowing constraint and housing plays the collateral role. A two-sector model is necessary to analyze housing which is a durable and non-tradable good.

3.1 Household

The representative household maximizes its expected lifetime utility defined over random sequences of non-durable consumption goods (c_t), housing services from the housing stock (h_t),

Figure 9: Non-Housing Investment Volatility and MMI

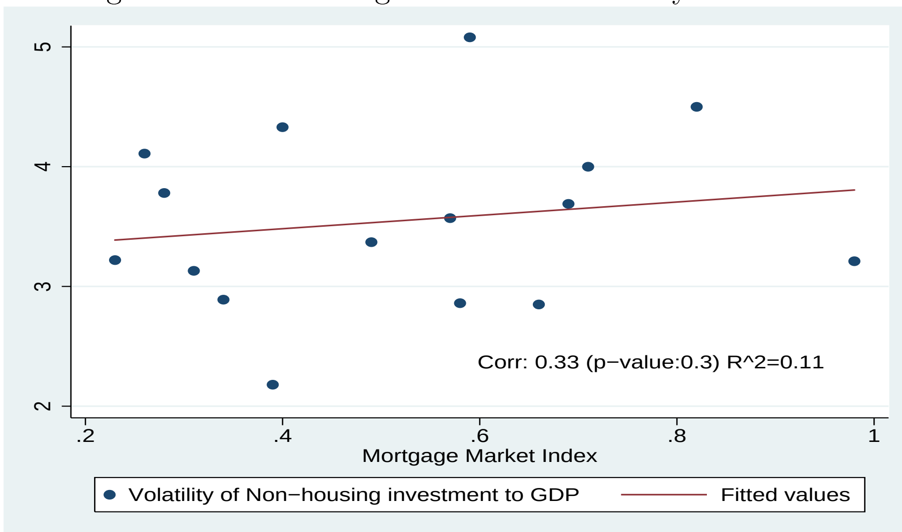
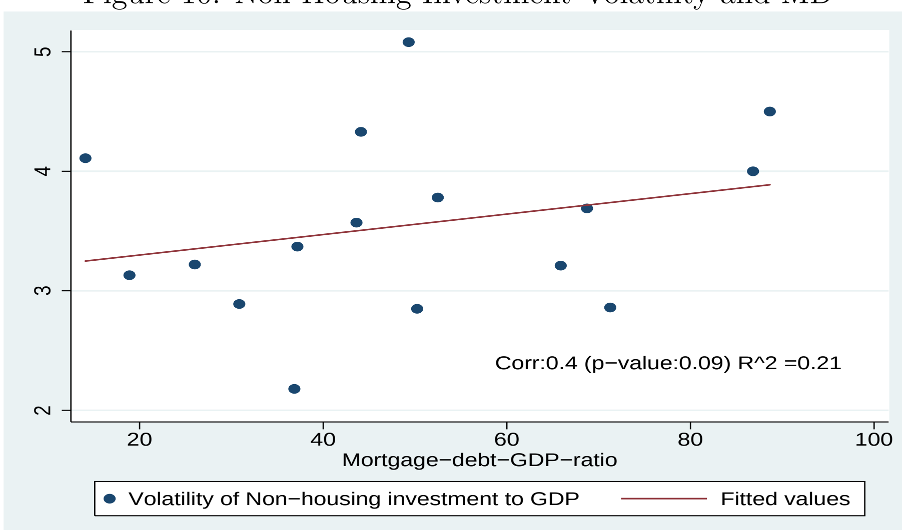


Figure 10: Non-Housing Investment Volatility and MD



and labor disutility (l_t):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, h_t, l_t\big) \tag{3.1}$$

The budget constraint of the representative household is given by:

$$\begin{aligned} c_t + q_t[h_t - (1-\delta_h)h_{t-1}] + \frac{\phi_h}{2}\frac{(h_t - h_{t-1})^2}{h_{t-1}} + i_t^c + i_t^h + (1+r_{t-1})d_{t-1} \\ \leq w_t l_t + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + d_t \end{aligned} \tag{3.2}$$

Table 2: **Statistic II**

	Prior 95			Post 95		
Country	RES	GDP	RES/GDP	RES	GDP	RES/GDP
Australia	9.12	1.78	5.12	8.59	0.61	14.08
Austria	3.99	0.78	5.12	2.51	0.80	3.14
Canada	7.63	1.94	3.93	4.19	0.87	4.82
Finland	5.70	2.50	2.28	6.00	1.05	5.71
France	3.15	0.97	3.25	1.99	0.74	2.69
Italy	2.39	1.04	2.30	2.16	0.76	2.84
New Zealand	8.07	1.77	4.56	8.40	1.05	8.00
Norway	7.60	1.53	4.97	5.00	0.98	5.10
U.K	9.67	1.51	6.40	3.90	0.38	10.26
U.S	10.42	1.53	6.81	5.01	0.87	5.76
Average	6.77	1.54	4.47	4.78	0.81	6.24

Notes: RES is real housing investment, GDP is real GDP. RES/GDP denotes the relative volatility of RES to that of real GDP. All series are in logs and Hodrick-Prescott filtered

$$i_t^c = k_t^c - (1-\delta_k)k_{t-1}^c + \frac{\phi_k}{2}\frac{(k_t^c - k_{t-1}^c)^2}{k_{t-1}^c} \tag{3.3}$$

$$i_t^h = k_t^h - (1-\delta_k)k_{t-1}^h + \frac{\phi_k}{2}\frac{(k_t^h - k_{t-1}^h)^2}{k_{t-1}^h} \tag{3.4}$$

Each period, the household can borrow internationally traded debt,[21] d_t, subject to a constraint described later, at an exogenous real interest rate, r_t. It supplies labor, l_t, at the real wage rate, w_t, and lends sector specific capital, k_{t-1}^c, k_{t-1}^h, to capital markets at prices r_t^c, r_t^h, where k_{t-1}^c, k_{t-1}^h are capital for non-durable and durable production, respectively. The household then spreads its income on non-durable consumption goods, c_t, debt repayment, $(1+r_{t-1})d_{t-1}$, investments on two types of non-housing capitals i_t^c, i_t^h, housing investment, $q_t(h_t-(1-\delta_h)h_{t-1})$, and its adjustment costs, $\frac{\phi_h}{2}\frac{(h_t-h_{t-1})^2}{h_{t-1}}$, where q_t is real housing prices and δ_h is the depreciation rate of housing stock.

In addition to the budget constraint, the representative household faces the following collateral borrowing constraint:

$$(1+r_t)d_t \leq \phi E_t(q_{t+1}h_t) \tag{3.5}$$

[21]This includes, but is not limited to, mortgage debt.

which means that at any time the amount the household can borrow, $(1+r_t)d_t$, is limited by the expected future value of his property. As in Kiyotaki and Moore (1997) and Kocherlakota (2000), this borrowing constraint is rationalized by the borrower's limited obligations. If the household repudiates its debt obligations, the lenders can foreclose the property after paying the transaction costs, $(1-\phi)E_t(q_{t+1}h_t)$. The parameter ϕ, which presents the fraction of collateral value a household can use for borrowing, reflects market liberalization, legal procedures, and regulatory structures or institutional features prevailing in the mortgage market, therefore indicating the degree of the mortgage market flexibility and development. A higher ϕ corresponds to a higher mortgage market index and indicates a more developed and flexible mortgage market in the model.

In this paper, I specialize preferences as below:

$$U(c_t,h_t,l_t)=\frac{(x_t-\kappa\frac{l_t^{\omega}}{\omega})^{1-\sigma}-1}{1-\sigma} \tag{3.6}$$

$$x_t=[(1-\gamma)^{\frac{1}{\eta}}c_t^{\frac{\eta-1}{\eta}}+\gamma^{\frac{1}{\eta}}h_t^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{3.7}$$

This is the GHH preference function introduced by Greenwood, Hercowitz and Huffman (1988) and is widely used in small open economy literature.[22] x_t is the composite consumption, the CES function of nondurable consumption, c_t, and housing services from the housing stock h_t. $\gamma>0$ is the share of housing services in the composite consumption index. $\eta\geq 0$ is the elasticity of substitution between non-durables and housing services. σ denotes the inverse elasticity of intertemporal substitution, ω determines the elasticity of labor supply, and κ determines the amount of leisure in the steady state.

Let's denote the multiplier on the borrowing constraint at time t by λ_t then the first order conditions for the representative household read:

$$U_{ct}[1+\phi_k(\frac{k_t^c-k_{t-1}^c}{k_{t-1}^c})]=\beta E_t\{U_{ct+1}[1-\delta_k+r_{t+1}^c+\frac{\phi_k}{2}((\frac{k_{t+1}^c}{k_t^c})^2-1)]\}$$

$$U_{ct}[1+\phi_k(\frac{k_t^h-k_{t-1}^h}{k_{t-1}^h})]=\beta E_t\{U_{ct+1}[1-\delta_k+r_{t+1}^h+\frac{\phi_k}{2}((\frac{k_{t+1}^h}{k_t^h})^2-1)]\}$$

$$w_t=-\frac{U_{lt}}{U_{ct}}$$

$$U_{ct}-\lambda_t=\beta E_t\{U_{ct+1}(1+r_t)\} \tag{3.8}$$

[22]GHH preferences have the property that the marginal rate of substitution between consumption and leisure is independent of the consumption level within the period or there is no wealth effect on labor supply. GHH preferences provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)).

$$U_{ct}(q_t + \phi_h \frac{h_t - h_{t-1}}{h_{t-1}}) = U_{ht} + \phi \lambda_t E_t\{q_{t+1}\}$$
$$+ \beta E_t\{U_{ct+1}[q_{t+1}(1-\delta_h) + \frac{\phi_h}{2}((\frac{h_{t+1}}{h_t})^2 - 1)]\} \tag{3.9}$$

The first two equations are standard optimality conditions for capital with adjustment costs while the third one is a standard labor supply equation. The last two equations present distinguishing features of the borrowing constraint model. Equation (3.8) is a modified Euler equation and is reduced to a standard Euler equation in case of a non-binding constraint, i.e., $\lambda_t = 0$. When the constraint binds, the shadow value of borrowing is positive, $\lambda_t > 0$, so there is an intertemporal distortion in non-durable goods consumption between two different times. In other words, when $\lambda_t > 0$, this equation implies that $U_{ct} > \beta E_t\{U_{ct+1}(1+r_t)\}$, which means the marginal utility of current non-durable consumption is higher than the marginal gain of shifting one unit of non-durables to the next period. A higher λ_t implies a tighter constraint, hence encouraging the household to purchase more collateralizable housing to relax the borrowing constraint, enabling it to increase current consumption.

Equation (3.9) is the efficiency condition for the intratemporal choice of durable housing that requires the household to equate the marginal utility of non-durable consumption, weighted by the relative housing prices and adjustment costs, to the marginal utility of housing services. The marginal utility of housing service consists of three components: (i) the direct utility gain of an additional unit of housing; (ii) the marginal gain from relaxing the collateral constraint; (iii) the expected utility derived from expanding future consumption by means of re-selling the amount of housing invested in the previous period. When the constraint doesn't bind, $\lambda_t = 0$, the distortion component $\phi \lambda_t E_t\{q_{t+1}\}$ vanishes, hence the marginal benefit of housing consists of only terms (i) and (iii), which is the standard intratemporal optimality condition.

For the sake of exposition at the moment, let's assume away adjustment costs. After integrating (3.9) forward, I obtain the following demand function for housing:

$$q_t U_{ct} = E_t\Big\{\sum_{j=0}^{\infty}[(1-\delta_h)\beta]^j U_{ht+j}\Big\} + E_t\Big\{\sum_{j=0}^{\infty}[(1-\delta_h)\beta]^j \phi \lambda_{t+j} q_{t+1+j}\Big\} \tag{3.10}$$

The first term in the RHS of (3.10) is the discounted stream of utility from housing services.[23] The second term is the current and expected benefits from the opportunity to increase consumption by the additional borrowing enabled by increased collateral value. This term depends on the degree of mortgage market development represented by parameter ϕ, the expected prices of housing, and the tightness of credit constraint λ_{t+j}. When the constraint doesn't bind,

[23]This term is set to current marginal utility of housing U_{ht} when $\delta_h = 1$.

$\lambda_t = 0$ for all t, this term is equated to zero, hence, the weighted marginal utility of non-durable consumption in the LHS equates to the discounted stream of utility from housing services.

To explore further, I follow Manacelli (2008) to express the equation as a condition where the marginal rate of substitution between housing and non-durable goods consumption $\frac{U_{ht}}{U_{ct}}$ is equal to the user cost (Z_t) of housing, which in this case can be expressed as:

$$Z_t \equiv q_t - \frac{\phi\lambda_t}{U_{ct}}E_t\{q_{t+1}\} - (1-\delta_h)\beta E_t\{\frac{U_{ct+1}}{U_{ct}}q_{t+1}\} \tag{3.11}$$

When the constraint binds, $\lambda_t > 0$, the user cost of housing is determined not only by current and expected real housing prices but also by ϕ and the movement of the shadow price of borrowing, λ_t. This is one of the distinguishing features of the model. For example, suppose that λ_t rises, that is the constraint becomes tighter, then the household has more incentives to purchase more collateralizable housing to relax the borrowing constraint and increase non-durable goods consumption. However, when λ_t rises U_{ct} also tends to increase, thereby raising the (opportunity) cost of acquiring an additional unit of durable housing. Moreover, increased housing demand also often drives up housing prices, q_t, hence raising the user cost as well.

3.2 Firms

At time t, representative firms in the tradable non-durable sector rent previously installed capital, k^c_{t-1}, and labor, l^c_t, from the household to produce goods with the production function:

$$y_t = A_t(k^c_{t-1})^{\alpha_c}(l^c_t)^{1-\alpha_c} \tag{3.12}$$

Output from the tradable non-durable sector can be used as non-durable consumption c_t or investments in either type of capital goods k^c_t, k^h_t or can be exported with tb_t. Firms in the construction sector combine capital, k^h_{t-1}, with labor, l^h_t, to construct buildings (structures) for non-tradable durable housing with the following technology:

$$b_t = A_t(k^h_{t-1})^{\alpha_h}(l^h_t)^{1-\alpha_h} \tag{3.13}$$

A_t is an aggregate exogenous stochastic productivity shock with law of motion:[24]

$$log(A_{t+1}) = \rho_A log(A_t) + \epsilon_{t+1} \tag{3.14}$$

[24] I assume that an exogenous productivity shock has the same effect on both production sectors and will consider an asymmetric case later. Notice that the symmetric productivity shock implies a perfect correlated productivity shock between two sectors, as in Greenwood and Hercowitz (1991), I do not assume reversibility between housing and business capital and housing is produced separately.

Optimality conditions for tradable goods firms imply:

$$w_t = (1-\alpha_c)\frac{y_t}{l_t^c} = (1-\alpha_c)A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c} \tag{3.15}$$

$$r_t^c = \alpha_c\frac{y_t}{k_{t-1}^c} = \alpha_c A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c-1} \tag{3.16}$$

Optimality conditions for the construction sector imply:

$$w_t = q_t(1-\alpha_h)\frac{b_t}{l_t^h} = q_t(1-\alpha_h)A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h} \tag{3.17}$$

$$r_t^h = q_t\alpha_h\frac{b_t}{k_{t-1}^h} = q_t\alpha_h A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h-1} \tag{3.18}$$

3.3 Equilibrium

Given the interest rate, r_t, a competitive equilibrium in this economy is characterized by a sequence of allocations $\{c_t, l_t, h_t, d_t, k_t^c, k_t^h, i_t^c, i_t^h, y_t, l_t^c, l_t^h,\}$ and a sequence of prices $\{q_t, w_t, r_t^c, r_t^h, \lambda_t\}$ that satisfy the household and firms optimality conditions, the budget constraint, the binding borrowing constraint, production functions, and the following market clearing conditions.

Labor market clearing:

$$l_t = l_t^c + l_t^h \tag{3.19}$$

Non-tradable durable housing market clearing:

$$b_t = h_t - (1-\delta_h)h_{t-1} \tag{3.20}$$

Tradable non-durable goods market:

$$c_t + i_t^c + i_t^h + \frac{\phi_h}{2}\frac{(h_t - h_{t-1})^2}{h_{t-1}} + (1+r_{t-1})d_{t-1} = y_t + d_t \tag{3.21}$$

The trade balance, housing investment, and aggregate output can be expressed as:

$$tb_t = y_t - c_t - i_t^c - i_t^h \tag{3.22}$$

$$resi_t = q_t b_t \tag{3.23}$$

$$Y_t = y_t + q_t b_t \tag{3.24}$$

3.4 Benchmark: Free Borrowing Economy

For comparison, I also consider a benchmark: a small open economy model augmented by the presence of the non-tradable durable housing sector with free borrowing. In this economy, the borrowing constraint does not bind so the multiplier $\lambda_t = 0 \; \forall \, t$. Therefore, two optimal conditions for non-durables and housing can be written as:[25]

$$U_{ct}(1 - \phi_d(d_t - \bar{d})) = \beta E_t\{U_{ct+1}(1 + r_t)\} \tag{3.25}$$

$$q_t U_{ct} = U_{ht} + (1 - \delta_h)\beta E_t\{q_{t+1} U_{ct+1}\} \tag{3.26}$$

Hence, the demand function for durable housing becomes:

$$q_t U_{ct} = E_t\Big\{\sum_{j=0}^{\infty}[(1 - \delta_h)\beta]^j U_{ht+j}\Big\} \tag{3.27}$$

The RHS of equation (3.27) is the shadow value of durable housing. According to Barsky et al. (2007), there are two reasons that keep this value roughly constant against moderate-lived shocks. First, durable housing with low depreciation rates has high stock-flow ratios,[26] which implies that even relatively large changes in the production of the housing over a moderate horizon have small effects on the total housing stock, therefore, causing only minor changes in the service flows. Second, if δ_h is sufficiently low, the shadow value will be mainly affected by the marginal utilities of service flows in the distant future. Since the effects of the shock are temporary, the future terms in this equation remain close to their steady-state values. Thus, even if there were significant changes in the first few terms of the expansion, they would have a small percentage effect on the present value as a whole. The two observations together suggest that under the benchmark, demand for durable housing displays an almost infinite elasticity of intertemporal substitution: even a small rise in housing prices today relative to tomorrow would cause people to delay their housing purchases.

3.5 Calibration

The model period is a quarter. *Preference*: Following Schmitt-Grohe and Uribe (2003), the inverse of elasticity of substitution in consumption σ and the elasticity of labor supply ω are set to 2 and 1.6, respectively, which are in range of literature. The elasticity of substitution

[25] All other conditions remain the same as before. The introduction of asset adjustment cost is to induce stationary dynamics in a small open frictionless economy but it does not affect the quantitative results of the model since ϕ_d is very small. For more details, see Schmitt-Grohe and Uribe (2003)

[26] In this model, the steady-state stock-flow ratio is $1/\delta_h$

between non-durable goods and housing service, η, is set to unity, implying the Cobb Douglas form of the composite consumption. [27] The parameter γ is set so that the ratio of private residential investment over GDP is equal to 3.5%, the average level for the U.K private residential investment over recent 20 years. Discount factor β is chosen as 0.985, which is a bit lower than the value implied by the foreign real interest rate $\frac{1}{1+0.01} = 0.9901$ to assure the binding credit constraint at steady state. The parameter κ is selected so that a fraction $\frac{40}{24\times 7}$ of household's one unit time endowment is used for working in the labor market.

Technology: The share of capital in the production of non-durables and housing construction, α_c, α_h, are both set to 0.3. These parameters together with depreciation rates will determine the investment rate, which is 20% of GDP. The depreciation rate of non-housing capital is chosen at 12% per year or $\delta_k = 0.03$ whereas δ_h is set to 0.003, which implies the depreciation rate of housing is 1.2% annually.[28] The parameter of capital adjustment costs ϕ_k is chosen such that volatility of non-housing investment matches the data and that of housing investment ϕ_h is set equal to ϕ_k.

Steady state value of the real interest rate is set at 4% per year or r=0.01. The persistence coefficient ρ_A in the motion equation of the productivity shock $log(A_{t+1}) = \rho_A log(A_t) + \epsilon_{t+1}$ is set to 0.9 and the variance of the innovation is selected to match the volatility of output.

For U.K: I set borrowing constraint parameter ϕ to 0.4 compared to 0.6 of the U.S economy. The reason for assigning 0.6 to the U.S economy is as follows. First, for the first-time homebuyers, the down-payment rate is typically less than 20%, which means these households can borrow more than 80% of the housing collateral value. [29]For existing homeowners, Mian and Sufi (2009) show that these households on average extract 30 cents for every dollar increase in home equity. I take average of these numbers, which implies a value of 0.6 for the US. Then I scale down the IMF mortgage market development index so that 0.98 is equal to 0.6 and obtain the number 0.4 for the U.K accordingly. At the same time, standard deviation of technology innovation and capital adjustment cost parameter ϕ_k are calibrated to 0.002 and 0.33, respectively in order to match the standard deviation of output and non-housing investment in the U.K over the past 30 years, 1.15% and 4.10% respectively.

[27]There is no consensus about this elasticity of substitution yet. Piazzesi et al. (2007) argue that if η is sufficiently less than unity then the equity premium puzzle is resolved while Davis and Martin (2005) show that the value should be no less than 1.25 in order to be consistent with U.S housing stock and price data. This paper take a neutral stance to set the value to unity. I got similar qualitative results when the value is assigned in the neighborhood of unity

[28]Monacelli sets it to 1% while Davis and Heathcote and others use 1.56% per year

[29]In the U.S, the Loan-to-Value ratio can reach 100% during the recent housing boom.

3.6 Housing Investment Dynamics

I first fix ϕ to explore the borrowing constraint model's impulse responses to a positive aggregate productivity shock. I focus on the dynamics of housing investment and compare the credit constraint model and the benchmark. In both cases, a favored productivity shock reduces production costs, encouraging firms to hire more labor to extend production in both durable and non-durable sectors, [30] thereby raising wage rates and capital returns. Also because of the positive productivity shock, the housing supply curve shifts down to the right.

From the demand side, aggregate consumption increases due to the income effect. Since housing is normal goods, its demand also increases, leading to an upward shift in the demand curve, which applies for both free borrowing and credit constraint cases. The differences, however, lie on the interaction between the income effect, the substitution effect, and potentially the collateral effect of each demand's structure.

Figure 11 presents impulse responses of the free borrowing model. In a free borrowing environment, housing is just a durable consumption good so the elasticity of intertemporal substitution is almost infinite for long-lived housing.[31] As a result, increased demand from the income effect is partly offset by the high intertemporal substitution effect resulting from rising house prices. Consequently, the housing investment volatility is relatively low and unable to exceed that of non-housing investment under all reasonable parameter calibration.[32] Nonetheless, it is shown that perfect/high correlated productivity shocks combined with an exogenous interest rate (small open economy framework) can produce the correct co-movement of housing investment in this type of two sector model, consistent with Greenwood and Hercowitz (1991), and Baxter (1996). Intuitively, the income effect together with an increased housing supply originating from a positive productivity shock can offset the high substitution effect of the housing demand but at the same time, they are not strong enough to generate the high volatility of housing investment documented from empirics.

By contrast, Figure 12 presents impulse responses of the collateral constraint model in which housing plays the collateral role. There are notably two main differences from the demand side of housing. First, unlike households that smooth consumption over time in the free borrowing benchmark, borrowing-constrained households are impatient, hence tend to locate consumption toward the current period. The impatience, therefore, produces a higher demand for both durable and non-durable goods compared to the benchmark. Second, since housing

[30] The amount of capital was already determined from the previous period

[31] This also implies a very flat housing demand curve at any given time.

[32] Recall in the empirical section shows that housing investment is on average significantly more volatile than non-housing investment in the majority of OECD countries.

Figure 11: IRs to Productivity Shocks: Benchmark

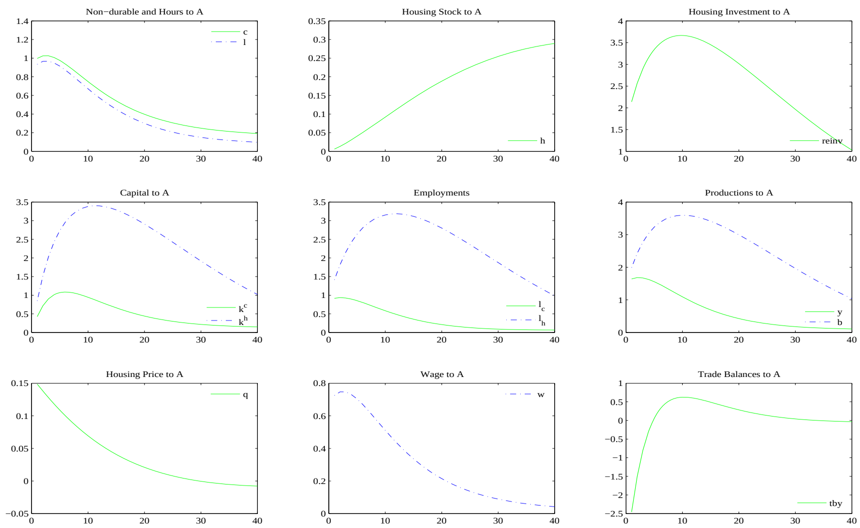
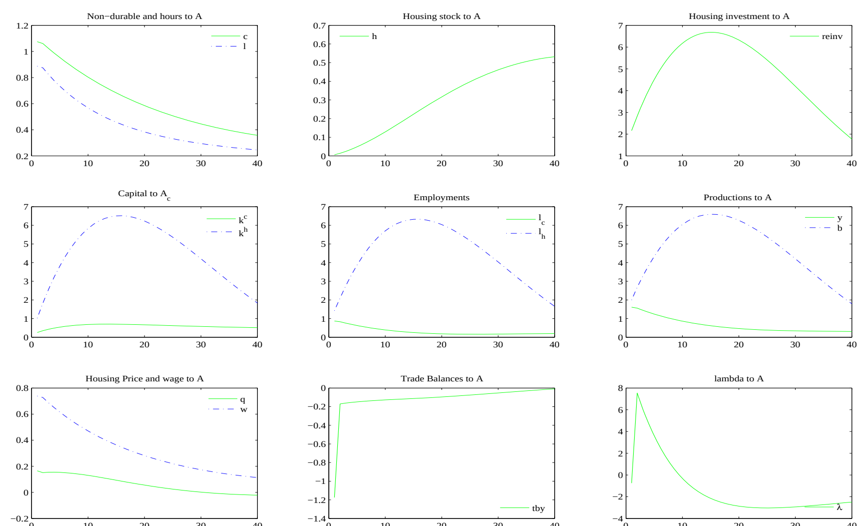
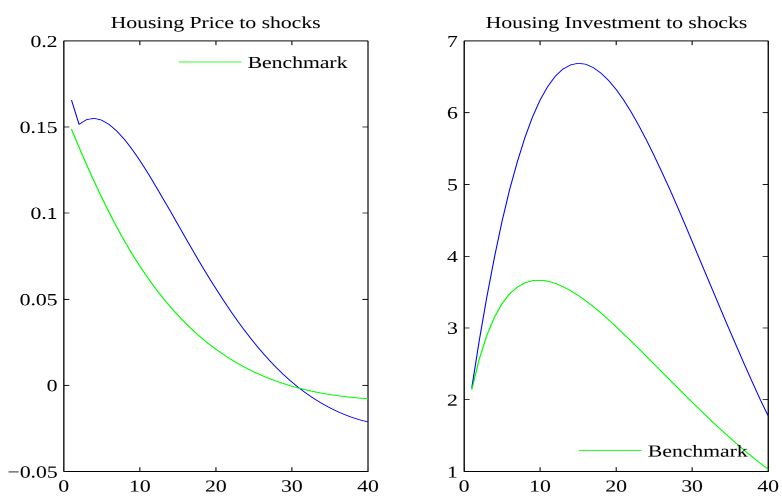


Figure 12: IRs to Productivity Shocks: Borrowing Constraint



plays an additional role as a collateralizable asset and rising collateral value will enable credit constrained households to expand consumption through further borrowing, households will have more incentives to invest in housing. In other words, besides the direct utility gain from housing services, households will also benefit from relaxing the borrowing constraint by an additional housing purchase. This is shown in the RHS of the demand equation (3.10): In addition to the standard discounted stream of utility from housing services, there is the second term presenting the current and expected benefits from the opportunity to expand consumption thanks to rising collateral value. The collateral effect implies a steeper demand curve and will shift the curve upward to a greater extent, therefore, leading to a greater response for housing investment on impact of productivity shock compared to the free borrowing benchmark. More importantly, the collateral constraint creates a borrowing-consuming spiral: initial increases in housing prices and stock raise the collateral value, enabling credit-constrained households to borrow more for consumption, which in turn will reinforce rising housing prices and housing stock. As shown in Figure 12, a higher demand on impact of the productivity shock and the amplification from the collateral effect can account for highly volatile housing investment. In other words, the income effect, increased housing supply, and the decisive collateral effect combined can dominate the substitution effect, hence reconciling the realistic volatility of housing investment. (Figure 13)

Figure 13: IRs of Housing Investment and Prices



3.7 Comparative Analysis of Mortgage Market Development

Second, I impose different values of parameter ϕ to study the impact of mortgage market development on housing investment in the borrowing constrained economy.

Table 3: **Statistics: Basic Model**

	Data	Standard Model	Basic Model
Standard deviation			
output	1.15	1.14	1.12
consumption	1.18	0.83	0.88
nres	4.1	4.15	2.3
tb/y	0.54	0.73	0.16
res	7.57	3.51	6.55
sd(res)/sd(y)	6.58	3.1	5.85
hp	5.13	0.075	0.11
Correlation w/ ouput			
consumption	0.73	0.94	0.98
nres	0.48	0.55	0.85
tb/y	-0.31	-0.07	-0.63
hp	0.58	0.93	0.79
res	0.57	0.85	0.7

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Standard model is free borrowing model. Std() is standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

In deeper mortgage markets, i.e., higher ϕ, households can borrow a higher amount of debt for the same value of collateral and withdraw more equity from increased collateral value for consumption and investment.[33] This is the *collateral effect* from mortgage market development. The collateral effect increases the volatility of housing investment because a higher ϕ intensifies the collateral role of housing, thereby encouraging credit constrained households to purchase more houses in good times. A higher ϕ also strengthens the link between the housing market and consumption decisions, therefore creating a stronger borrowing-consuming spiral.

On the other hand, mortgage market innovations, by raising ϕ, also offer the prospect of increased credit supply and a relaxation of borrowing constraints, therefore creating the *credit effect* in the borrowing constrained economy. In contrast to the collateral effect, the credit effect lowers the volatility of housing investment because higher ϕ, by providing more credits to the economy, reduces the incentive to invest in collateralizable housing for the purpose of relaxing the collateral constraint. In other words, the credit effect relatively increases the user cost of housing, hence inducing the household to substitute housing with non-durable consumption. The two effects are partly reflected through the second term in the housing demand equation (3.10): A higher value of ϕ directly increases the value of this term but at the same time eases the tightness of the borrowing constraint, thereby endogenously decreasing the current and future shadow value of borrowing λ_{t+j}. Moreover, a higher ϕ also leads to changes in the housing demand, hence affecting future expected housing prices q_{t+1+j}, which then in turn have impacts on the second term of RHS of equation (3.10) as well. Therefore, the aggregate effect of a higher ϕ on housing demand is ambiguous. It turns out that at low and medium levels of mortgage market development, when the household's credit constraint is relatively tight, the collateral effect prevails. Consequently, an improvement in the mortgage market development leads to a relatively larger increase in housing demand, causing a higher housing investment volatility. By contrast, when the mortgage market is highly developed, households are much less credit constrained, so the credit effect takes over from the collateral effect, and the household starts to substitute collateralizable housing by non-durable consumption; therefore, housing investment volatility tends to decline. Figure 14 presents an inverse U-shape in the relative volatility of housing investment with respect to the degree of mortgage market development.

[33] It is applied even under the case where households don't directly acquire debt or withdraw equity for consumption, since by accessing more mortgage debt there would be more credit available for consumption.

Figure 14: Housing Investment Volatility and ϕ

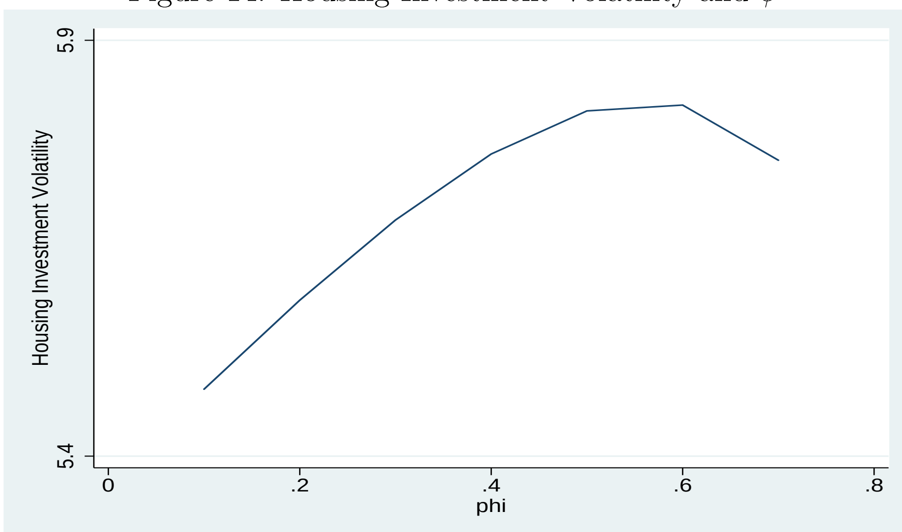
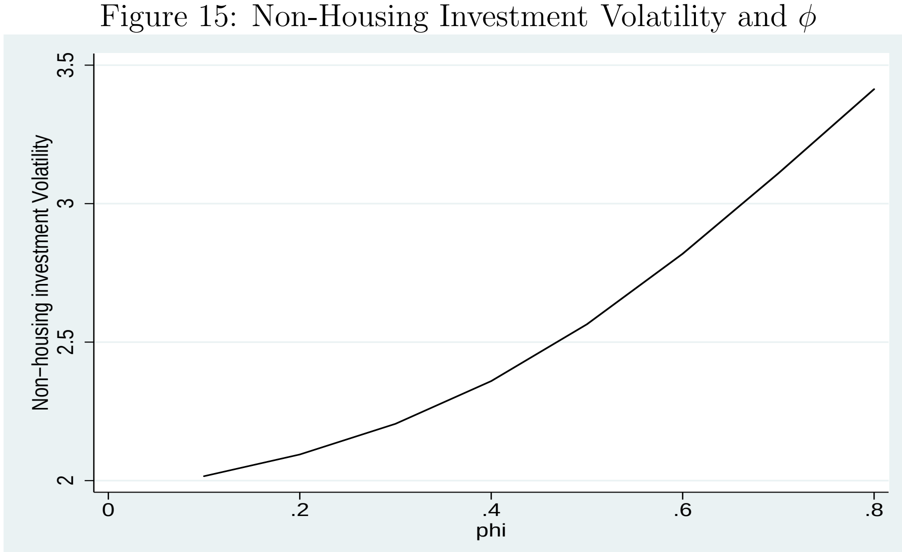


Figure 15: Non-Housing Investment Volatility and ϕ



3.8 Asymmetric Productivity Shock

I have assumed that the aggregate productivity shock A_t has a symmetric impact on both durable and non-durable sector production as in (3.12) and (3.13). In this section, I consider an asymmetric case where the aggregate productivity shock does not have any impact on the durable housing production. By assuming the asymmetric shock, I attempt to exclude the effect of increased housing supply from a positive productivity shock and therefore be able to focus on the demand side, particularly the collateral effect. The production function of non-durable goods remains the same as in (3.12) but that of durable housing has the form:

$$b_t = (k_{t-1}^h)^{\alpha_h}(l_t^h)^{1-\alpha_h} \tag{3.28}$$

When a favored aggregate productivity shock hits the economy, since there is complete productivity spillover to nondurable production, firms in this sector will take advantage of favored productivity to hire more labor and extend production, therefore raising wage rates and the capital returns. However, since labor is freely mobile, rising wage rates will hurt the housing construction sector that does not benefit from the increased productivity. Consequently, housing production costs will increase and the housing supply curve will shift upward. This is the difference in the supply side compared to the symmetric productivity case.[34]

With a negative impact from the supply side, we now witness a significant difference between the free borrowing benchmark and the borrowing constraint model with the collateral effect. In the benchmark, because of the high elasticity of intertemporal substitution, households substitute housing for relatively cheap non-durable goods. As a result, housing flow/investment falls on impact of the productivity shock and keeps falling for a while before gradually increasing. Intuitively, the positive income effect on housing demand helps to mitigate the negative substitution effect at the beginning but it weakens rapidly against the latter, causing a deeper fall in housing flow/investment. However, a fall in housing investment amid rising output and non-housing investment, i.e., the so-called co-movement problem, is at odds with empirical facts. Hence, it has been shown that under an asymmetric productivity spillover to housing production sector, a standard small open economy model is unable to correct the co-movement problem. (Figure 16)

By contrast, Figure 17 presents the dynamics of the economy with the collateral constraint. It is shown that the collateral effect can help to produce a correct co-movement even without highly correlated productivity shocks. Anticipating the collateral role of housing and speculating on rising property prices, households rationally increase investing in housing in good times

[34] Recall that the housing supply curve shifts downward in this case the complete spillover to housing production in (3.13).

Figure 16: IRs: Model without Borrowing Constraint: Asymmetric Shocks

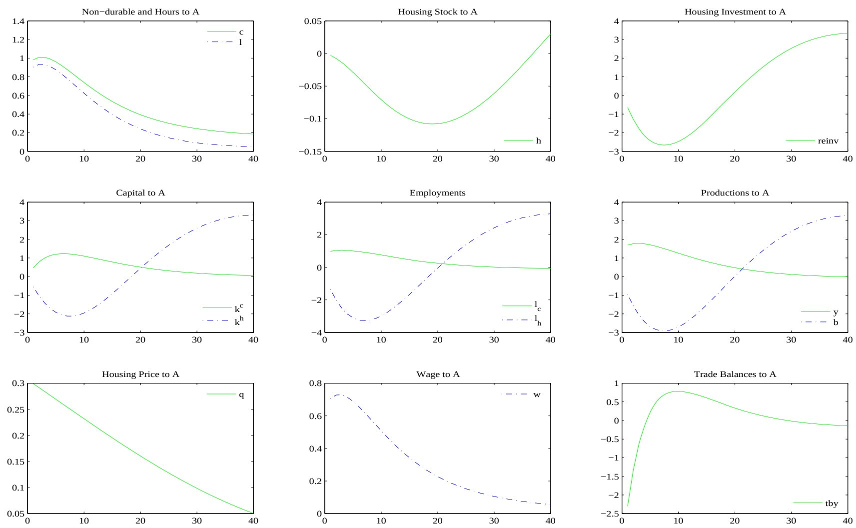
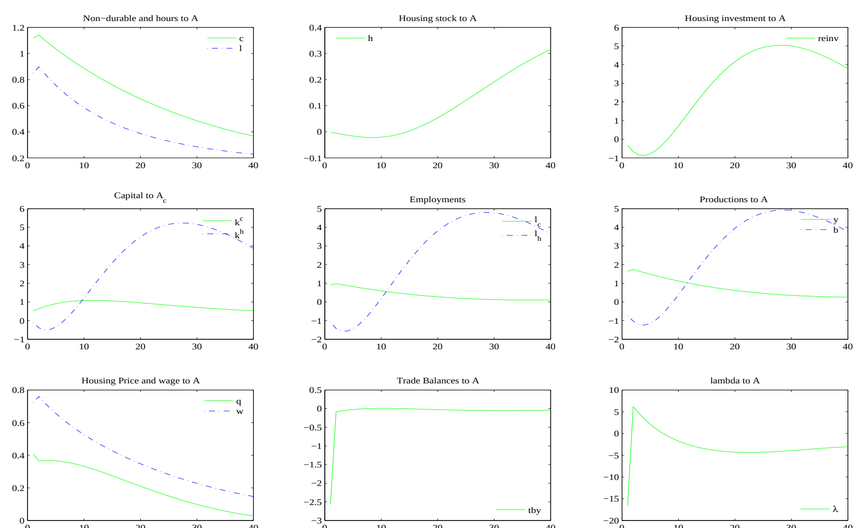
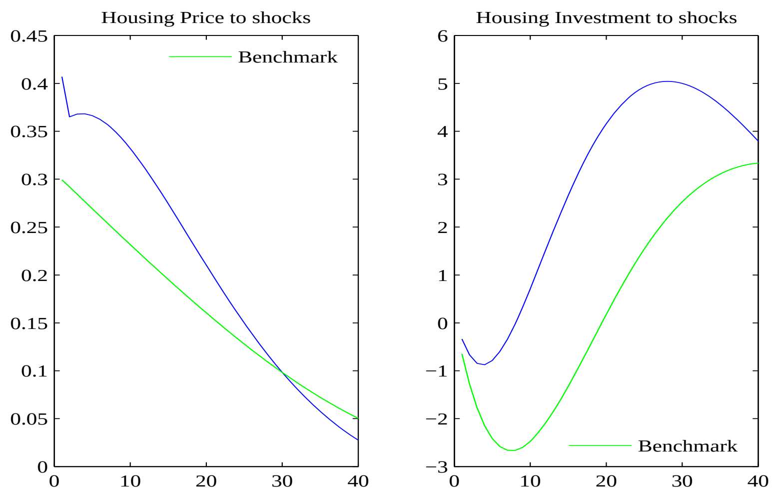


Figure 17: IRs: Model with Borrowing Constraint: Asymmetric Shocks



despite its high elasticity of intertemporal substitution. Initial increases in turn fuel the spiral, making housing investment pro-cyclical. In other words, the collateral effect together with the income effect can offset the intertemporal substitution effect, hence producing pro-cyclical housing investment as documented in the empirical work. (Figure 18)

Figure 18: IRs: Housing Investment and Prices



4 Extended Model

The basic representative household model is simple but sufficient to explain the impact of the collateral effect on housing investment dynamics. This model, however, has a weakness. It implies that the volatility of non-housing/business investment increases in economies with a more developed mortgage market, at odds with empirical evidence. (Figure 15) The underlying reason is that in the representative agent model, business capital is also owned by those who face the borrowing constraint, hence, it is affected by their borrowing capacity. In particular, because of the credit constraint, the rate of capital return is always kept higher than the borrowing interest rate, which induces credit-constrained agents increasingly to invest in business capital when the credit constraint is relaxed and the access to credit becomes broader.

In reality, business capital is often owned by corporations or *capitalists* who have much more freedom to access financial markets than a typical credit constrained household.[35] Therefore, to separate business investment decisions from credit-constrained households, I consider an

[35]For example, using data from the 1998 Survey of Consumer Finance, Diaz and Luengo-Prado document that in the U.S, households in the top 20% of the wealth distribution hold 98.9% of all financial assets while housing wealth represents 96.3% of total wealth for those in the bottom 80% of the wealth distribution.

extended model in which there are two types of households, named capitalist and borrower with measure ϵ and $1-\epsilon$, respectively. The former groups own business capital and have free access to both domestic and international financial markets. In contrast, the latter groups don't own business capital and face the collateral borrowing constraint as in the basic model. A necessary condition for this type of heterogeneous household model is that capitalists are more patient than borrowers and at equilibrium borrowers will borrow from capitalists.[36]

4.1 Capitalist

The representative capitalist maximizes his expected life-time utility defined over random sequences of non-durable consumption goods (c_{1t}), housing services from housing stock (h_{1t}), and labor dis-utility (l_{1t}):

$$U = E_0 \sum_{t=0}^{\infty} \beta_1^t U\big(c_{1t}, h_{1t}, l_{1t}\big) \tag{4.29}$$

The budget constraint of the capitalist is given by:

$$\begin{aligned} c_{1t} + q_t[h_{1t} - (1-\delta_h)h_{1t-1}] + \frac{\phi_h}{2}\frac{(h_{1t}-h_{1t-1})^2}{h_{1t-1}} + i_t^c + i_t^h + (1+r_{t-1})d_{ft-1} \\ + (1+r_{t-1}^b)d_{1t-1} + \frac{\phi_d}{2}(d_{ft}-\bar{d})^2 \le w_t l_{1t} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + d_{1t} + d_{ft} \end{aligned} \tag{4.30}$$

$$\begin{aligned} i_t^c &= k_t^c - (1-\delta_k)k_{t-1}^c + \frac{\phi_k}{2}\frac{(k_t^c - k_{t-1}^c)^2}{k_{t-1}^c} \\ i_t^h &= k_t^h - (1-\delta_k)k_{t-1}^h + \frac{\phi_k}{2}\frac{(k_t^h - k_{t-1}^h)^2}{k_{t-1}^h} \end{aligned}$$

Each period, the capitalist can either pay adjustment cost, $\frac{\phi_d}{2}(d_{ft}-\bar{d})^2$,[37] to borrow internationally traded foreign debt at an interest rate, r_t, which is exogenous, or access the domestic bond market, d_{1t}, at an interest rate, r_t^d. He supplies labor l_{1t} at the real wage rate, w_t, and lends capital, k_{t-1}^c, k_{t-1}^h to capital markets at prices r_t^c, r_t^h. The capitalist then spreads his income on non-durable tradable consumption goods, c_{1t}, debt payment $(1+r_{t-1})d_{ft-1}, (1+r_{t-1}^d)d_{1t-1}$, investments of two types of non-housing capital i_t^c, i_t^h, housing investment, $q_t[h_{1t}-(1-\delta_h)h_{1t-1}]$, and its adjustment costs, $\frac{\phi_h}{2}\frac{(h_{1t}-h_{1t-1})^2}{h_{1t-1}}$.

[36] When $\epsilon = 1$, this extended model is reduced to a standard representative model under free borrowing. However, when $\epsilon = 0$, this model is not the same as the basic model.

[37] The introduction of adjustment costs in a small open economy framework is to induce stationarity.

The first order conditions for the capitalist, which are standard, read:

$$U_{1ct}[1+\phi_k(\frac{k^c_t-k^c_{t-1}}{k^c_{t-1}})]=\beta_1 E_t\{U_{1ct+1}[1-\delta_k+r^c_{t+1}+\frac{\phi_k}{2}((\frac{k^c_{t+1}}{k^c_t})^2-1)]\}$$

$$U_{1ct}[1+\phi_k(\frac{k^h_t-k^h_{t-1}}{k^h_{t-1}})]=\beta_1 E_t\{U_{1ct+1}[1-\delta_k+r^h_{t+1}+\frac{\phi_k}{2}((\frac{k^h_{t+1}}{k^h_t})^2-1)]\}$$

$$w_t=-\frac{U_{1lt}}{U_{1ct}}$$

$$U_{1ct}(1-\phi_d(d_{ft}-\bar{d}))=\beta_1 E_t\{U_{1ct+1}(1+r_t)\} \tag{4.31}$$

$$U_{1ct}=\beta_1 E_t\{U_{1ct+1}(1+r^d_t)\} \tag{4.32}$$

$$U_{1ct}(q_t+\phi_h\frac{h_{1t}-h_{1t-1}}{h_{1t-1}})=U_{1ht}+\beta E_t\{U_{1ct+1}[q_{t+1}(1-\delta_h)+\frac{\phi_h}{2}((\frac{h_{1t+1}}{h_{1t}})^2-1)]\} \tag{4.33}$$

4.2 Borrower

The representative borrower maximizes his expected life-time utility defined over random sequences of non-durable consumption goods(c_{2t}), housing services from housing stock (h_{2t}), and labor dis-utility (l_{2t}):

$$U=E_0\sum_{t=0}^{\infty}\beta_2^t U\big(c_{2t},h_{2t},l_{2t}\big) \tag{4.34}$$

The budget constraint of the borrower is given by:

$$c_{2t}+q_t(h_{2t}-(1-\delta_h)h_{2t-1})+\frac{\phi_h}{2}\frac{(h_{2t}-h_{2t-1})^2}{h_{2t-1}} + (1+r^b_{t-1})d_{2t-1}\leq w_t l_{2t}+d_{2t} \tag{4.35}$$

I assume that the borrower is more impatient than the capitalist or $\beta_2<\beta_1$. The borrower does not hold capital.[38] Each period, he supplies labor l_{2t} at the real wage rate, w_t, borrows from the domestic bond market, d_{2t}, at the interest rate, r^d_t, but is subject to a borrowing constraint mentioned below. The borrower then spreads his income on non-durable tradable consumption goods, c_{2t}, debt payment $(1+r^d_{t-1})d_{2t-1}$, and housing investment, $q_t[h_{2t}-(1-\delta_h)h_{2t-1}]$. I also assume that the borrower is not able to access to the international foreign debt.

The borrower is also subject to the following collateral borrowing constraint:

$$(1+r^d_t)d_{2t}\leq\phi E_t(q_{t+1}h_{2t}) \tag{4.36}$$

[38]It can be shown that because of being relatively impatient, the borrower will not hold capital.

Let's denote the multiplier on the borrowing constraint by λ_t, the first-order conditions for the borrower read:

$$w_t = -\frac{U_{2lt}}{U_{2ct}} \tag{4.37}$$

$$U_{2ct} - \lambda_t = \beta_2 E_t\{U_{2ct+1}(1 + r_t^d)\} \tag{4.38}$$

$$\begin{aligned} U_{2ct}(q_t + \phi_h \frac{h_{2t} - h_{2t-1}}{h_{2t-1}}) &= U_{2ht} + \lambda_t \phi E_t\{q_{t+1}\} \\ &+ \beta E_t\{U_{2ct+1}[q_{t+1}(1 - \delta_h) + \frac{\phi_h}{2}((\frac{h_{2t+1}}{h_{2t}})^2 - 1)]\} \end{aligned} \tag{4.39}$$

4.3 Firms

Representative firms in the non-durable sector produce goods with the following technology:

$$y_t = A_t (k_{t-1}^c)^{\alpha_c} (l_t^c)^{1-\alpha_c} \tag{4.40}$$

Structures of the non-tradable durable housing are produced with following technology:

$$b_t = A_t (k_{t-1}^h)^{\alpha_h} (l_t^h)^{1-\alpha_h} \tag{4.41}$$

Optimality conditions of non-durable goods firms imply:

$$w_t = (1 - \alpha_c)\frac{y_t}{l_t^c} = (1 - \alpha_c) A_t \Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c} \tag{4.42}$$

$$r_t^c = \alpha_c \frac{y_t}{k_{t-1}^c} = \alpha_c A_t \Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c - 1} \tag{4.43}$$

Optimality conditions of construction firms imply:

$$w_t = q_t(1 - \alpha_h)\frac{b_t}{l_t^h} = q_t(1 - \alpha_h) A_t \Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h} \tag{4.44}$$

$$r_t^h = q_t \alpha_h \frac{b_t}{k_{t-1}^h} = q_t \alpha_h A_t \Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h - 1} \tag{4.45}$$

4.4 Equilibrium

Given the interest rate, r_t, a competitive equilibrium in this small open economy is characterized by a sequence of allocations $\{c_{1t}, c_{2t}, l_{1t}, l_{2t}, h_{1t}, h_{2t} d_{1t}, d_{2t}, d_{ft}, k_t^c, k_t^h, i_t^c, i_t^h, y_t, l_t^c, l_t^h, l_{1t}, l_{2t}\}$, and a sequence of prices $\{q_t, w_t, r_t^c, r_t^h, r_t^d, \lambda_t\}$ that satisfy the household and firms optimality conditions, the borrower's budget constraint, the binding borrowing constraint, production functions, and following market clearing conditions.

Labor market clearing:

$$\epsilon l_{1t} + (1-\epsilon) l_{2t} = l_t^c + l_t^h \tag{4.46}$$

Non-tradable durable housing market clearing:

$$b_t = \epsilon(h_{1t} - (1-\delta_h)h_{1t-1}) + (1-\epsilon)(h_{2t} - (1-\delta_h)h_{2t-1}) \tag{4.47}$$

Domestic bond market:

$$\epsilon d_{1t} + (1-\epsilon) d_{2t} = 0 \tag{4.48}$$

Tradable goods market:

$$\begin{aligned} \epsilon c_{1t} + (1-\epsilon)c_{2t} + \frac{\phi_h}{2}\frac{(h_{1t} - h_{1t-1})^2}{h_{1t-1}} + \frac{\phi_h}{2}\frac{(h_{2t} - h_{2t-1})^2}{h_{2t-1}} \\ + i_t^c + i_t^h + (1+r_{t-1})d_{ft-1} = y_t + d_{ft} \end{aligned} \tag{4.49}$$

4.5 Calibration

Preference: Basic parameters like σ, ω, and η, are chosen the same as those in the basic model. Both capitalists and borrowers have the same share of housing services in the composition consumption and γ_1, γ_2 are set so that the ratio of total housing investment over GDP is equal to 5%. Capitalists' discount factor β_1 is pinned down by the steady state value of the exogenous interest rate, $\frac{1}{1+r}$. Borrowers are more impatient or β_2 is set to be 0.985. κ_1, κ_2 is selected such that in steady state both capitalists and borrowers supply a fraction $\frac{40}{24\times 7}$ of household's one unit time endowment for working in the market. I set the fraction of capitalists in total population ϵ equal to 0.2, which implies that about the top 20 percent of the wealthy population in the economy own capital, have free access to both domestic and international finance markets, which is consistent with the results of Diaz and Luengo-Prado that the top 20% in the wealth distribution holds 98.9% of total financial assets.

Technology: All parameters pertaining to technology and productivity side of the model are kept the same as those in the basic model.

I calibrate the model such that steady state trade-balance-to-GDP ratio is equal to 1%, which then pins down the level of foreign debt at steady state $\bar{d}$.[39] I also follow Schmitt-Grohe et al. (2003) to set the portfolio adjustment cost ϕ_d to 0.0007.

4.6 Model Dynamics

When a favored productivity shock hits the economy, firms in both sectors hire more labor to extend production, driving up the wage rate and capital returns. Due to the positive income

[39] Although varying this ratio does not have much effect on our results.

effect, the capitalist and the borrower both increase their aggregate consumption, hence raising non-durable consumption. However, there is a contrast in the housing demand between the capitalist and the borrower. For the capitalist, since housing is just a durable good, his elasticity of intertemporal substitution for long-lived housing is almost infinite so even a small rise in price relative to future will lead him to delay current purchase. Facing an increase in the relative housing prices, the capitalist optimally substitutes his durable consumption with non-durable goods, therefore reducing his housing stock in the early stage after shock and then gradually accumulates it back later on. By contrast, to the borrower, housing is not just a durable good but also plays a collateral role for future borrowing, which therefore makes him increasingly invest in housing in good times. As shown in the simulation, the increase in the borrower's housing demand not only is able to absorb the sale of the capitalist's housing but also drives up the overall economy housing investment. Figure 19

Moreover, since business capital is owned by capitalists who are not subject to borrowing constraints, its dynamics are not affected by the development of the mortgage market. As a result, unlike housing investment whose volatility relative to GDP increases in economies with a higher mortgage market index, the volatility of non-housing investment remains almost unchanged, consistent with the empirical evidence. Figure 21

Finally, I calibrate the extended model for the U.K and Table 4.6 presents the result. For comparison, I also calibrate a standard two-sector RBC model with free borrowing and the basic model. Despite its simplicity, calibrated models' second moments match data relatively well.[40] In particular, the implied volatility of housing investment in the credit constraint model can match data quite well, whereas the volatility of housing investment in a standard free model is two times lower than the data. Although the implied volatility of housing prices from the credit constraint model is about 2-3 times higher than that in the free borrowing model, it is far below that of the data, which reflects the difficulty of business cycle models in accounting for the high volatility of asset prices. Furthermore, unlike the free borrowing benchmark, the credit constraint models also can account for the significant counter-cyclicality of the current account. The reason is that since households are credit-constrained they tend to borrow more (from foreigners) to consume in good times and the borrowing-consuming spiral also reinforces borrowing as explained above.

[40]It is not that surprising since Mendoza (1991) uses a standard model without housing and can match data for Canada quite well.

Figure 19: IRs: Extended Model with Borrowing Constraint

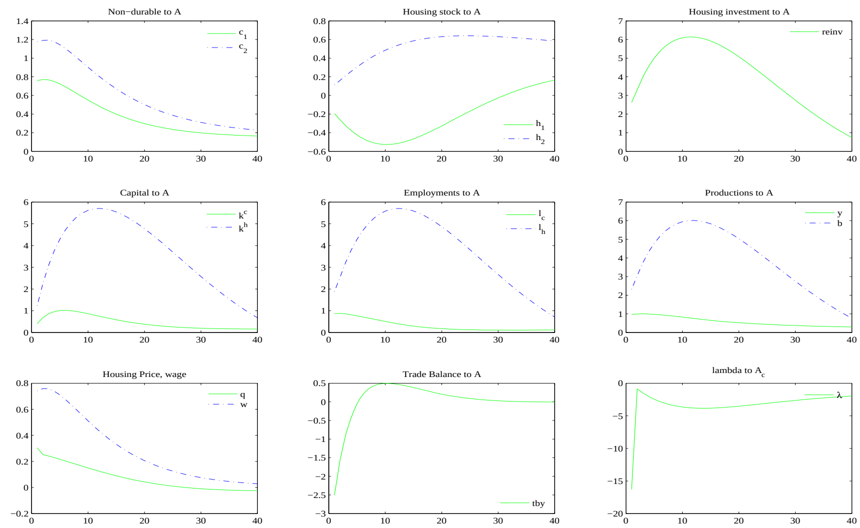


Figure 20: IRs of Housing Investment and Prices

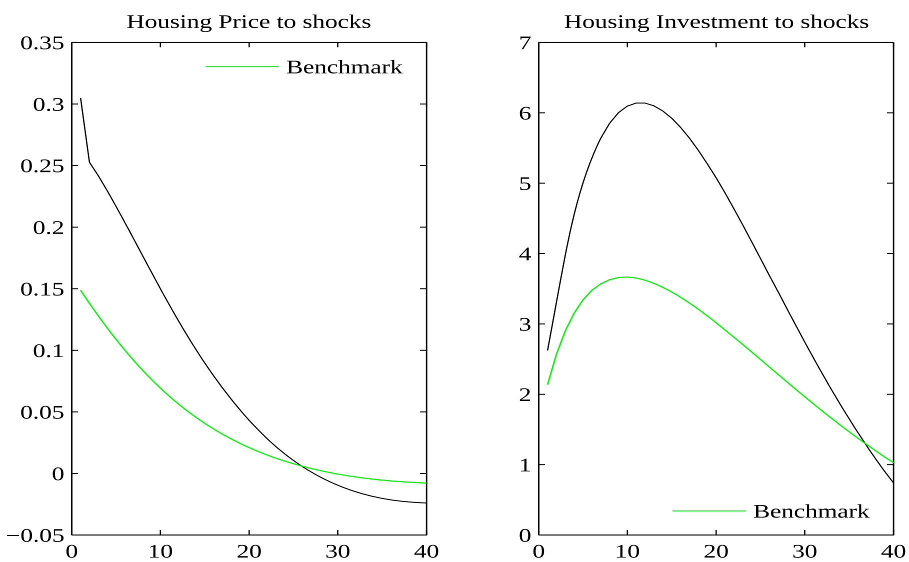
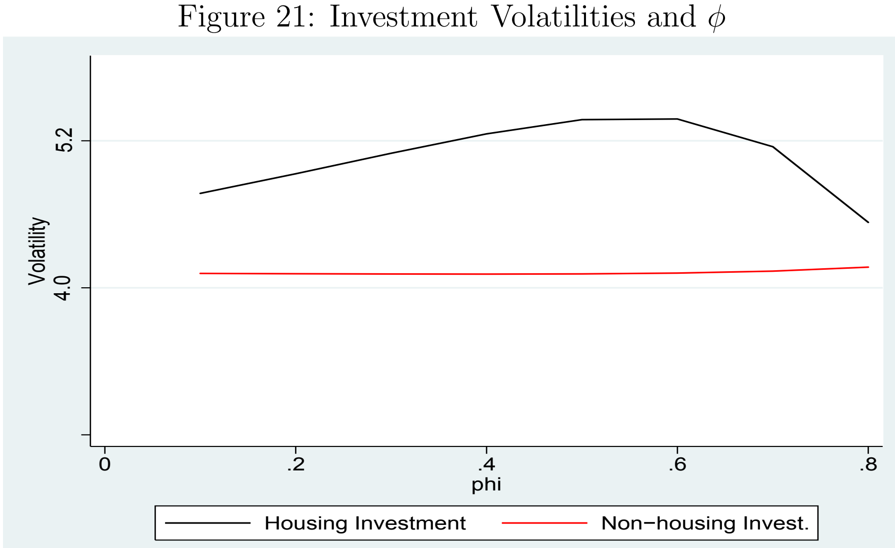


Table 4: **Statistics: Extended Model**

	Data	Std. Model	Basic Model	Extended Model
Standard deviation				
output	1.15	1.14	1.12	1.15
consumption	1.18	0.83	0.88	0.86
nres	4.1	4.15	2.3	4.13
tb/y	0.54	0.73	0.16	0.71
res	7.57	3.51	6.55	6.03
sd(res)/sd(y)	6.58	3.1	5.85	5.3
hp	5.13	0.075	0.11	0.18
Correlation w/ ouput				
consumption	0.73	0.94	0.98	0.97
nres	0.48	0.55	0.85	0.53
tb/y	-0.31	-0.07	-0.63	-0.2
hp	0.58	0.93	0.79	0.9
res	0.57	0.85	0.7	0.8

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Extended model means heterogeneous household model with credit constraint, standard model is free borrowing model. Std() is standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

Figure 21: Investment Volatilities and ϕ



5 Conclusions

This paper begins by documenting stylized facts regarding housing investment and mortgage market depth in OECD countries. Housing investment is highly volatile, especially in economies with more developed mortgage markets. The paper demonstrates that standard RBC models with a perfect credit market assumption are at odds with these empirical facts but the introduction of a housing collateral constraint can help reconcile the models with the facts. Collateral effects also enable the models to produce significant counter-cyclicality of the current account and the co-movement of different types of investments even without highly correlated productivity shocks. The paper predicts a non-monotonic impact of mortgage market depth on the volatility of housing investment. In the quantitative section, calibrated models with a housing collateral constraint can match the data in the U.K quite well.

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Appendix A: Theoretical Model

1 Solution Method

I solve the models by the perturbation method.[41] Particularly the set of optimality conditions of the economy can be expressed as follows:

$$E_t\{F(Y_{t+1}, Y_t, X_{t+1}, X_t)\} = 0 \tag{1.1}$$

E_t is the mathematical expectation operator conditional on information available at time t, Y_t is the vector of non-predetermined variables, and $X_t = [x_t^1, x_t^2]'$ is the state variable vector, x_t^1 is endogenous predetermined state variables while x_t^2 is exogenous state variables. Particularly, x_t^2 follows exogenous process given as:

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}\bar{\sigma}\epsilon_{t+1} \tag{1.2}$$

where $\tilde{\eta}, \bar{\sigma}$ are given parameter. The solution of the optimal plan is of the form:

$$Y_t = g(X_t, \bar{\sigma}) \tag{1.3}$$

$$X_{t+1} = h(X_t, \bar{\sigma}) + \bar{\eta}\bar{\sigma}\epsilon_{t+1} \tag{1.4}$$

where $\bar{\eta} = [\emptyset, \tilde{\eta}]'$, these equations describe the policy and transition functions respectively. I compute a first order expansion of the two functions around the deterministic steady state.

2 Basic Model Calibration

2.1 Deterministic Steady State

First, notice that the modified Euler equation at steady state which can be written as

$$U_c - \lambda = \beta U_c(1+r) \Rightarrow \lambda = U_c(1 - \beta(1+r)) \tag{2.5}$$

Condition for binding borrowing constraint at steady state or $\lambda > 0$ requires that $\beta < 1/(1+r)$, where r is the steady state of real world interest rate.

[41] For more details, see Schmitt-Grohe and Uribe (2004)

2.1.1 Benchmark: Free Borrowing

$$1 = \beta(1-\delta_k+r^c) = \beta(1-\delta_k+r^h); \quad 1 = \beta(1+r) \Rightarrow r^c = r^h = r+\delta_k \tag{2.6}$$

$$\frac{k^c}{l^c} = (\frac{r+\delta_k}{\alpha_c})^{\frac{1}{\alpha_c-1}}; \quad \frac{k^h}{l^h} = (\frac{r+\delta}{q\alpha_h})^{\frac{1}{\alpha_h-1}} \tag{2.7}$$

$$w = (1-\alpha_c)(\frac{r+\delta_k}{\alpha_c})^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)(\frac{r+\delta}{q\alpha_h})^{\frac{\alpha_h}{\alpha_h-1}} \tag{2.8}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(r+\delta_k)^{\frac{\alpha_c}{\alpha_c-1}-\frac{\alpha_h}{\alpha_h-1}}\alpha_c^{-\frac{\alpha_c}{\alpha_c-1}}\alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{2.9}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h}l^h}{(\frac{k^c}{l^c})^{\alpha_c}l^c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l_h}{l_c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{2.10}$$

$$l^c + l^h = l = \frac{40}{24*7} \Rightarrow l^c,\ l^h \Rightarrow k^c,\ k^h \Rightarrow y,\ h;\ \frac{i^c}{y} = \delta_k\frac{k^c}{y} = \delta_k\frac{\alpha_c}{r^c} = \delta_k\frac{\alpha_c}{r+\delta_k} \tag{2.11}$$

$$\frac{i^h}{y} = \delta_k\frac{k^h}{y} = \delta_k\frac{k^h}{qb}\frac{qb}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{rein}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{q\delta_h h}{y} \tag{2.12}$$

$$\frac{c}{y} = 1 - \frac{i^c}{y} - \frac{i^h}{y} - \frac{tb}{y} \Rightarrow c \tag{2.13}$$

$$q[1-\frac{(1-\delta_h)}{1+r}] = \frac{U_h}{U_c} = \Big[\frac{\gamma c}{(1-\gamma)h}\Big]^{\frac{1}{\eta}} \Rightarrow \gamma = \Big[1+\frac{c}{h}(\frac{1+r}{q(r+\delta_h)})^{\eta}\Big]^{-1} \tag{2.14}$$

2.1.2 Borrowing constraint

$$1 = \beta(1-\delta_k+r^c) = \beta(1-\delta_k+r^h); \quad r^c = r^h = \frac{1}{\beta} - 1 + \delta_k \equiv \bar{r} + \delta_k \tag{2.15}$$

$$\frac{k^c}{l^c} = (\frac{\bar{r}+\delta_k}{\alpha_c})^{\frac{1}{\alpha_c-1}}; \quad \frac{k^h}{l^h} = (\frac{\bar{r}+\delta_k}{q\alpha_h})^{\frac{1}{\alpha_h-1}} \tag{2.16}$$

$$w = (1-\alpha_c)(\frac{\bar{r}+\delta_k}{\alpha_c})^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)(\frac{\bar{r}+\delta_k}{q\alpha_h})^{\frac{\alpha_h}{\alpha_h-1}} \tag{2.17}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(\bar{r}+\delta_k)^{\frac{\alpha_c}{\alpha_c-1}-\frac{\alpha_h}{\alpha_h-1}}\alpha_c^{-\frac{\alpha_c}{\alpha_c-1}}\alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{2.18}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h}l^h}{(\frac{k^c}{l^c})^{\alpha_c}l_c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l^h}{l^c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{2.19}$$

$$l_c + l_h = l = \frac{40}{24*7} \Rightarrow l_c\ l_h \Rightarrow k^c,\ k^h \Rightarrow y,\ h;\ \frac{i^c}{y} = \delta_k\frac{k^c}{y} = \delta_k\frac{\alpha_c}{r_c} = \delta_k\frac{\alpha_c}{\bar{r}+\delta_k} \tag{2.20}$$

$$\frac{i^h}{y} = \delta_k\frac{k^h}{y} = \delta_k\frac{k^h}{qb}\frac{qb}{y} = \delta_k\frac{\alpha_h}{\bar{r}+\delta_k}\frac{rein}{y} = \delta_k\frac{\alpha_h}{\bar{r}+\delta_k}\frac{q\delta_h h}{y} \tag{2.21}$$

$$\frac{c}{y} = 1 - \frac{i^c}{y} - \frac{i^h}{y} - \frac{tb}{y} \Rightarrow c; \quad \frac{qh}{y} = \frac{rein}{\delta_h} \Rightarrow \phi = \frac{\frac{d}{y}}{q\frac{h}{y}} \tag{2.22}$$

$$U_c - \lambda = \beta U_c(1+r) \Rightarrow \lambda = U_c(1 - \beta(1+r)) \tag{2.23}$$

$$qU_c - \lambda\phi q = U_h + \beta U_c[q(1-\delta_h)] \tag{2.24}$$

$$\Rightarrow q[1 - \beta(1-\delta_h) - \phi(1-\beta(1+r))] = \frac{U_h}{U_c} = \Big[\frac{\gamma c}{(1-\gamma)h}\Big]^{\frac{1}{\eta}} \tag{2.25}$$

$$\gamma = \Big[1 + \frac{c}{h}\big(\frac{1}{q[1-\beta(1-\delta_h) - \phi(1-\beta(1+r))]}\big)^{\eta}\Big]^{-1}; \quad w = \kappa\frac{l^{\omega-1}}{[(1-\gamma)\frac{x}{c}]^{\frac{1}{\eta}}} \Rightarrow \kappa \tag{2.26}$$

3 Extended Model Calibration

3.1 Deterministic Steady State

$$1 = \beta_1(1 - \delta_k + r^c) = \beta_1(1 - \delta_k + r^h); \quad 1 = \beta_1(1+r) \Rightarrow r^c = r^h = r + \delta_k \tag{3.27}$$

$$\frac{k^c}{l^c} = \big(\frac{r+\delta_k}{\alpha_c}\big)^{\frac{1}{\alpha_c - 1}}; \quad \frac{k^h}{l^h} = \big(\frac{r+\delta}{q\alpha_h}\big)^{\frac{1}{\alpha_h - 1}} \tag{3.28}$$

$$w = (1-\alpha_c)\big(\frac{r+\delta_k}{\alpha_c}\big)^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)\big(\frac{r+\delta}{q\alpha_h}\big)^{\frac{\alpha_h}{\alpha_h-1}} \tag{3.29}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(r+\delta_k)^{\frac{\alpha_c}{\alpha_c-1} - \frac{\alpha_h}{\alpha_h-1}}\alpha_c^{-\frac{\alpha_c}{\alpha_c-1}}\alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{3.30}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h}l^h}{(\frac{k^c}{l^c})^{\alpha_c}l^c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l_h}{l_c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{3.31}$$

$$l^c + l^h = l = l_1 = l_2 = \frac{40}{24*7} \Rightarrow l^c,\ l^h \Rightarrow k^c,\ k^h \Rightarrow y,\ h \tag{3.32}$$

$$\frac{i^c}{y} = \delta_k\frac{k^c}{y} = \delta_k\frac{\alpha_c}{r^c} = \delta_k\frac{\alpha_c}{r+\delta_k} \tag{3.33}$$

$$\frac{i^h}{y} = \delta_k\frac{k^h}{y} = \delta_k\frac{k^h}{qb}\frac{qb}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{rein}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{q\delta_h h}{y} \tag{3.34}$$

$$\frac{c}{y} = 1 - \frac{i^c}{y} - \frac{i^h}{y} - \frac{tb}{y} \Rightarrow c \tag{3.35}$$

$$\epsilon c_1 + (1-\epsilon)c_2 = c;\ \epsilon h_1 + (1-\epsilon)h_2 = h;\ c_2 + (q\delta_h + r\phi q)h_2 = wl_2 \tag{3.36}$$

$$\gamma_1 = \gamma_2 \Rightarrow \frac{c_1}{h_1} = \alpha\frac{c_2}{h_2} \Rightarrow h_2, c_2, h_1, c_1 \Rightarrow \gamma_1 = \gamma_2 = \gamma = \Big[1 + \frac{c_1}{h_1}\big(\frac{1+r}{q(r+\delta_h)}\big)^{\eta}\Big]^{-1} \tag{3.37}$$

Appendix B: Data

House Prices: Bank of International Settlements via Mr. Markus Kramer [42]. In particular, (1) File *Residential_property_prices.csv* is used for most countries from "National sources" as per detailed documentation, (2) Residential Prop prices IT.xls for Italy from Nomisma. Japanese house prices, however, are taken from Datastream with Code name JPLANDPIF.

Consumer Price Index (CPI): Seasonally Adjusted (SA). GDP: constant or chained prices, SA. Sources: Datastream, OECD Stat. (http://www.oecd.org/home/)

Housing investment, non-residential investment, aggregate investment: real values, SA. Sources: Datastream, OECD Stats. Code means Datastream Code.

- Australia: Datastream. Housing investment: Private dwelling, code AUFXCPDWD. Aggregate Investment: Private Gross Fixed Capital Formation (GFCF), code AUPRFXCPD. Non-residential investment is obtained by subtracting housing investment from aggregate investment. Q1 1980-Q1 2008.

- Austria: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1988-Q1 2008.

- Belgium: Datastream. Housing investment: GFCF-Housing, chained prices, code BGHOUINVD. Aggregate investment: GFCF, chained prices, code BGGFCFD. Non-residential investment: GFCF-Companies, code BGBUSINVD. Q1 1995-Q1-2008

- Canada: Datastream. Housing investment: Business GFCF Residential Structures (CN100112). Aggregate Investment: Business GFCF, code CNGFCFD. Non-residential investment: Business GFCF Non-Residential Structures, code CNINRSEQD. Q1 1981-Q3 2007.

- Denmark: Datastream. Housing investment: GFCF-Housing, chained prices, code DKGFCFHOD. Aggregate investment: GFCF, code DKGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q1 1992-Q1-2008.

- Finland: Datastream. Housing investment: GFCF Construction-residential building, code FNGFCRESD. Aggregate investment: GFCF, constant prices, code FNGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q1 1990-Q1-2008.

[42] Email: markus.kramer@bis.org

- France: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1980-Q1 2008.

- Germany: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1991-Q1 2008.

- Italy: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1981-Q1 2008.

- Japan: Datastream. Housing investment: Gross Domestic Fixed Capital Formation (GDFCF) by type: DWELLINGS, constant prices, code JPGCFBLDD. Aggregate investment: GDFCF, code JPGFCFD. Non-residential investment: GDFCF subtract GDFCF Dwellings. Q1 1994-Q1-2008.

- Netherland: Datastream. Housing investment: GFCF DWELLINGS, constant prices, code NLGFRBCVD. Aggregate investment: GFCF, constant prices, code NLGFCFD. Non-residential investment: GFCF subtract GFCF Dwellings. Q1 1993-Q1-2008.

- New Zealand: Datastream. Housing investment: GFCF Residential building constant prices, code NZGFCFRBD. Aggregate investment: GFCF, constant prices, code NZGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q2 1987-Q1-2008.

- Norway: Datastream. Housing investment: GFCF Housing investment, constant prices, code NWGFCHSID. Aggre. investment: GFCF, NWGFCFD. Non-housing investment: GFCF subtract GFCF Housing. Q1 1980-Q1-2008.

- Spain: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1995-Q1 2008.

- Sweden: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1993-Q1 2008.

- U.K: Office of National Statistics through Datastream. Housing investment: Private sector New Dwellings excl. Land, constant prices, code UKDFEAD. Aggregate Investment: GFCF, code UKNPQTD. Non-residential investment: Fixed Capital Formation, Non-dwellings, code UKTONDWLD. Output: constant prices GDP, code UKABMID. Non-durable goods is the household final consumption excluding durable goods, constant price, code UKJSRVD). Trade balance is equal to net export of goods, constant prices, code UKBALGSVD. House price index, UK DCLG HOUSE PRICE INDEX (MIX ADJ.), code UKNSAQHPF. Q1 1980-Q3 2007.

- U.S: Datastream. Housing investment: Residential Private Domestic Investment, constant prices, code USGPDRESD. Aggregate Investment: Private Domestic Fixed Investment, code USGFCFD. Non-residential investment: Non-residential Private Domestic Fixed Investment, code USNRSINVD. Q1 1980-Q3 2007.