Exchange Rates, Yield Curves, and the Macroeconomy

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Abstract. The nominal exchange rate is both a key macroeconomic variable equilibrating international goods markets and a financial asset that embodies expectations and prices risks about cross border currency-holdings. Recognizing this, we adopt a joint macro-finance strategy to model exchange rates, incorporating both macroeconomic stabilization in monetary policy setting, as well as financial expectations and systematic risks embodied in the shape of the cross-country yield curves. We extract the latent factors in the Nelson and Siegel (1987) yield curve model, and combine them with VAR dynamics for the exchange rate and monetary policy targets (output gap and inflation). Using monthly data between 1985 and 2005 for Canada, Japan, the UK and the US, we show that both the yield curve factors and the macroeconomic fundamentals explain future exchange rate dynamics, with the factors playing an especially significant role in explaining the yen and the pound movements relative to the dollar. Next, decomposing the yield curves into expected short rates and term premia, we show that both drive exchange rate dynamics and excess currency returns. These findings provide support for our parsimonious model and the view that the exchange rate dynamics is determined by both macroeconomic as well as financial forces.

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1 Introduction

This paper proposes to model nominal exchange rate by incorporating both macroeconomic determinants as well as latent financial factors, combining lessons from several strands of recent literature. First, the international macro literature has shown that models in which monetary policy follow an explicitly Taylor-type interest rate rule deliver improved empirical performance, offering a glimmer of hope over decades of negative results in the empirical exchange rate literature. This literature also emphasizes that nominal exchange rate should be viewed as an asset price, or the net present value of its expected future macroeconomic fundamentals. While recognizing the presence of risk, empirical tests in this literature largely ignore it, rendering it an "unobservable".[1] On the finance side, recent research efforts show that systematic sources of financial risk, as captured by latent factors, drive excess currency returns, especially across currency portfolios (see, for example Lustig et al 2009). These models, however, are silent on the role of macroeconomic forces, including monetary policy, in exchange rate behavior or in expectation formation and risk determination.[2] Our paper shows that the two approaches should be combined.

The joint macro-finance strategy has proven fruitful in modeling other financial assets. For example, Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006) among others, illustrate that a joint macro-finance modeling strategy provides the most comprehensive description of the term structure of interest rates (the yield curve). As stated in Diebold, Piazzesi, and Rudebusch (2005), the joint approach captures both the macroeconomic perspective that the short rate is a monetary policy instrument used to stabilize the economy, as well as the financial perspective that yields of all maturities are risk-adjusted averages of expected future short rates. The exchange rate certainly fits the same description of playing a key role in both international macroeconomic and financial markets. Nominal exchange rate links prices across borders and equilibrates purchasing parity and money markets; it is also an asset that prices expected future macro fundamentals, such monetary policy action, as well as uncertainties. Its dynamics, based on the no-arbitrage condition in the international asset markets, should be pinned down by the risk-adjusted

[1] See, Engel, Mark, and West (2007), Molodstova and Papell (2009), for example. Engel et al establish a link between exchange rates and fundamentals in a present value framework. Explicitly recognizing the possibility that risk premia may be important in explaining exchange rates, they "do not explore that avenue in this paper, but treat it as an 'unobserved fundamental." Molodstova and Papell show that Taylor rule fundamentals (interest rates, inflation rates, output gaps and the real exchange rate) forecasts better than the commonly used interest rate fundamentals, monetary fundamentals and PPP fundamentals. Again, they explain exchange rate using only observed fundamentals and do not account for risk premium.

[2] See Lustig, Roussanov and Verdelhan (2009) and Farhi et al (2009) and references therein. Inci and Lu (2004) and Krippner (2006) both aim at connecting the term structure and exchange rate, but again they do not include macroeconomic fundamentals via monetary policy.

cross-country yield differences at the corresponding maturities, relating it to both monetary policy as well as the shapes of the yield curves. As such, the exchange rate, together with the yield curves, should also be modeled from a joint macro-finance approach.

We present an open economy model where central banks follow a Taylor-type interest rate rule that stabilizes expected inflation and output gap, and show that nominal exchange rate is the net present value of expected future Taylor-rule fundamentals as well as time-varying risks. To capture these expectations and the systematic risks in the financial markets, we propose to use the latent Nelson and Siegel (1987) factors extracted from the cross-country yield curves. The yield curves are of particular interest because they offer continuous readings of market expectations and their responses to changes in macroeconomic conditions, and have become a common indicator for central banks for receiving speedy feedback to their policy actions. In addition, by looking at the *relative* yield curves between two countries, we obtain the two aspects of the asset-price attribute of the exchange rate we aim to capture: expectations and perceived risks.[3] Traditional models of the yield curve posit that the shape of the yield curve is determined by expected future paths of short rates and perceived future uncertainty, or the term premia.[4] Given that short rates are monetary policy instruments that react to macroeconomic conditions, expected long yields (averaged expected short rates) thus capture expected future macroeconomic conditions. The term premium perceived for holding bonds over the maturity duration captures market pricing of risk, of various origins, over the holding period.[5] As discussed in Diebold et al (2005), latent factors such as Nelson-Siegel summarize well the small number of sources of systematic risk that underlies the pricing of a myriad of tradable financial assets. We expect that the risk premium in the currency markets - captured by excess currency returns or deviations from uncovered interest parity - will be highly correlated with the risks captured in the two countries' relative yield curves, as it prices the same latent risks. Our framework allows us to extract these yield curve term premia and separate them from market expectation about future macro fundamentals, in order to study their relative impact on currency returns and risks.

Specifically, we look at monthly exchange rate changes for three currency pairs

[3] Throughout the paper, we assume symmetry between countries (e.g. in how economic fundamentals affect their relative currency values, in how yield curves embody local information)

[4] That is, a long yield at time t of maturity m can be decomposed into: 1) the average of the current time t one-period yield and the expected one-period yields for the upcoming $m-1$ periods, and 2) the term risk premium, perceived at t, associated with holding the long bond until $t+m$: ${}_t\rho_{t+m}$ (The Expectation Hypothesis.)

[5] Kim and Orphanides (2007) provide a good discussion of the bond market term premium, covering both systematic risks associated with macroeconomic conditions, variations in investors' risk-aversion over time, as well as liquidity considerations and geopolitical risky events.

over the period August 1985 to July 2005: the Canada dollar, the British pound, and the Japan yen relative to the US dollar.[6] For each corresponding observation, we extract three Nelson-Siegel factors from the zero-coupon yield *differences* between the three countries and the US, using yield data of 17 maturities ranging from one month to ten years. These three factors, which we refer to as the relative level, relative slope, and relative curvatures, capture movements at the long, short, and medium part of the *relative* yield curves between the two countries. We then estimate a 6-variable VAR by combining the three relative factors with 1-month exchange rate change, relative output gap and relative inflation. The model is a state-space system and can be estimated by maximum likelihood. Via the VAR structure, we obtain expected relative yields at different maturities for a country-pair and use them to construct the term premia in the relative yield curves at each time for different future maturities.[7] We demonstrate that:1) the macro-fundamental based empirical exchange rate equations miss out on two crucial elements that drive currency dynamics: expectation and risk; 2) both elements are captured in the latent factors extracted from the cross-country yield curves, which are found to be empirically very important ; 3) decomposing the yield curves into expectations of future short rates and term premia, we show that both contain information on future exchange rate changes and excess currency returns. These findings support the view that exchange rates should be modeled using a joint macro-finance framework.

In the subsequent sections, we discuss the net present value framework of exchange rate determination in the context of Taylor rule monetary policy, the Nelson-Siegel latent factors, our estimation strategy, and our empirical results.

2 Monetary Policy and the Exchange Rate

Several recent papers emphasize the importance of monetary policy rules, and in particular, the Taylor rule, in modeling exchange rates (see Engel and West (2005), Molodtsova and Papell (2008), and Wang and Wu (2009) among others). This approach assumes that central banks adjust short-term interest rates in response to target variables such as the output gap and inflation, and together with uncovered interest rate parity condition, it can also deliver a set of fundamentals relevant to our discussion.

We assume the monetary policy instruments, the home interest rate i_t and the foreign rate i_t^*, are set as follows:

[6] We present results based on the dollar cross rates, though the qualitative conclusions extend to other pair-wise combinations of currencies.

[7] The term premium at time t for maturity m is just the difference between the actual maturity-m yield and the predicted yield. See Diebold, Rudebusch and Aruoba (2006) and Cochrane and Piazzesi (2006), for more discussions.

$$i_t = \mu_t + \beta_y y_t^{gap} + \beta_\pi \pi_t^e + u_t$$

$$i_t^* = \mu_t^* + \beta_y y_t^{*,gap} + \beta_\pi \pi_t^{*e} - \delta q_t \; where \; q_t = s_t - p_t + p_t^*$$

where y_t^{gap} is the output gap, π_t^e is the expected inflation, β_y, $\beta_\pi > 0$ and μ_t contains the inflation and output targets, the equilibrium real interest rate, and other omitted terms, and u_t captures policy errors. The foreign corresponding variables are denoted with a "*", and following the literature, we assume the foreign central bank to explicitly target the real exchange rate or purchasing power parity $q_t = s_t - p_t + p_t^*$ in addition, with p denoting the overall price level. For notation simplicity, we assume the home and foreign central banks to have the same weights β_y and β_π.[8] The efficient market condition for the foreign exchange markets, under rational expectations, equates cross border interest differentials $i_t - i_t^*$ with the expected rate of home currency depreciation, adjusted for the risk premium associated with home currency holdings:

$$i_t - i_t^* = E_t \Delta s_{t+1} + \widetilde{\rho}_t^H$$

We note that $\widetilde{\rho}_t^H$ denote the "relative risk" of holding home currency over foreign currency, and it is a function of the general perceived risks within each country: $\widetilde{\rho}_t^H = function(\rho_t^H - \rho_t^F)$, again assuming symmetry across countries. In subsequent sections, will relate the county specific risk, ρ_t^H and ρ_t^F, to risks or term premia extracted from the country's yield curves about different future horizons. Combining the above equations and letting $v_t = \mu_t - \mu_t^*$, we can express exchange rates in the following differenced expectation equation:

$$s_t = \gamma f_t^{TR} + \kappa \widetilde{\rho}^H + \psi E_t s_{t+1}$$

where $f_t^{TR} = \{(p_t - p_t^*), (y_t - y_t^*), (\pi_t^e - \pi_t^{*e})\}$. Iterating the equation forward, we show that the Taylor-rule based model can deliver the net present value equation where exchange rate is determined by expected future values of cross country output, inflation, and interest rates:

$$s_t = \lambda \sum_{j=0}^{\infty} \psi^j E_t(f_{t+j}^{TR} | I_t) + \zeta \sum_{j=0}^{\infty} \psi^j \rho_t^H$$

As shown above, nominal exchange rate embodies two main elements, expected future macro dynamics as well as risks. In the next section, we discuss that the

[8]Our setup is what Papell et al (2008) term "asymmetric homogenrous" in their analysis of the performance of variations of the Taylor-rule based forecasting equations.

Taylor-rule fundamentals are exactly the macroeconomic indicators for which the yield curves appear to embody information. Empirically, nominal exchange rate is best approximated by a unit root process, so we express equation (2) in a first-differenced form. From here, rather than following the common approach in the literature and imposing additional assumptions about the statistical processes driving the fundamentals, we discuss in the next section how to use the information in the yield curves to proxy the expected discounted sum on the right-hand side of equation.[9]

3 The Yield Curve and the Nelson-Siegel Factors

We now discuss the yield curve literature and how the yield curve is connected to the macroeconomy (see Chen and Tsang (2009a) for a more detailed discussion). The yield curve or the term structure of interest rates describes the relationship between yields and their time to maturity. Traditional models of the yield curve posit that the shape of the yield curve is determined by expected future paths of interest rates and perceived future uncertainty (the risk premia). While the classic expectations hypothesis is rejected frequently, research on the term structure of interest rates has convincingly demonstrated that the yield curve contains information about expected future economic conditions, such as output growth and inflation.[10] Below we give a brief presentation on the Nelson-Siegel (1987) framework for characterizing the shape of the yield curve and summarize findings in the macro-finance literature about its predictive content. Next we discuss how term premia is estimated and interpreted in the literature.

The Nelson-Siegel (1987) factors offer a succinct approach to characterize the shape of the yield curve. To derive the factors, they first approximate the forward rate curve at a given time t with a Laguerre function that is the product between a polynomial and an exponential decay term. This forward rate is the (equal-root) solution to the second order differential equation for the spot rates. A parsimonious approximation of the yield curve can then be obtained by averaging over the forward rates, with the resulting function capable of capturing the relevant shapes of the empirically observed yield curves: monotonic, humped, or S-shaped. It takes the following form:

[9] See Chen and Tsang (2009a) for a more detailed discussions of the standard estimation techniques that impose a joint statistical process for the fundamentals.

[10] Briefly, the expectations hypothesis says that a long yield of maturity m can be written as the average of the current one-period yield and the expected one-period yields for the coming $m-1$ periods, plus a term premium. See Thornton (2006) for a recent example on the empirical failure of the expectations hypothesis.

$$i_t^m = L_t + S_t\left(\frac{1-\exp(-\lambda m)}{\lambda m}\right) + C_t\left(\frac{1-\exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)\right)$$

where i_t^m is the continuously-compounded zero-coupon nominal yield on a m-month bond. The parameter λ controls the speed of exponential decay, and instead of imposing the usual value of 0.0609 we estimate the parameter directly below. As discussed earlier, one of the main advantages of the Nelson-Siegel approach, compared to the popular no-arbitrage affine or quadratic factor models, is that the three factors, L_t, S_t, and C_t, are easy to estimate and have simple intuitive interpretations. The level factor L_t , with its loading of 1, has the same impact on the whole yield curve. The loading on the slope factor S_t starts at 1 when $m = 0$ and decreases down to zero as maturity m increases. This factor captures short-term movements that mainly affect yields on the short end of the curve, and an increase in the slope factor means the yield curve becomes flatter, holding the long end of the yield curves fixed. The curvature factor C_t is a “medium” term factor, as its loading is zero at the short end, increases in the middle maturity range, and finally decays back to zero. It captures how curvy the yield curve is at the medium maturities. These three factors typically capture most of the information in a yield curve. The R^2 of the cross-section fit is usually close to 0.99.

There is long history of using the term structure to predict output and inflation. Mishkin (1990a and 1990b) shows that the yield curve predicts inflation, and that movements in the longer end of the yield curve are mainly explained by changes in expected inflation. Barr and Campbell (1997) use data from the UK index-linked bonds market and show that long-term expected inflation explains almost 80% of the movements in the long yields. Estrella and Mishkin (1996) show that the term spread is correlated with the probability of a recession, and Hamilton and Kim (2002) find that it can forecast GDP growth.

The more recent macro-finance literature connects the observation that the short rate is a monetary policy instrument with the idea that yields of all maturities are risk-adjusted averages of expected short rates. This more structural approach offers deeper insight into the relationship between the yield curve factors and macroeconomic dynamics. This new literature can be divided into two groups. The first group does not model the macroeconomic fundamentals structural and capture their dynamics using a general VAR. Ang, Piazzesi and Wei (2006) estimate a VAR model for the US yield curve and GDP growth. By imposing non-arbitrage condition on the yields, they show that the yield curve predicts GDP growth better than a simple unconstrained OLS of GDP growth on the term spread. More specifically, they find that the term spread (the slope factor) and the short rate (the sum of level and slope factor) outperform a simple AR(1) model in forecasting GDP growth 4 to 12 quarters ahead. Diebold, Rudebusch and Aruoba (2006) is similar to Ang, Piazzesi and Wei, but they use the Nelson-Siegel curve instead of a no-arbitrage affine model. The second group pushes further and models the macroeconomic variables structurally, using with a three-equation New Keynesian model. Bekaert, Cho and Moreno (2006) demonstrate that the level factor is mainly moved by changes in the central bank's inflation target, and monetary policy shocks dominate the movements in the slope and curvature factors. Dewachter and Lyrio (2006) estimate an affine model for the yield curve with macroeconomic variables. They find that the level factor reflects agents' long run inflation expectation, the slope factor captures the business cycle, and the curvature represents the monetary stance of the central bank. Last but not least, Rudebusch and Wu (2007, 2008) contend that the level factor incorporates long-term inflation expectations, and the slope factor captures the central bank's dual mandate of stabilizing the real economy and keeping inflation close to its target. They provide macroeconomic underpinnings for the factors, and show that when agents perceive an increase in the long-run inflation target, the level factor will rise and the whole yield curve will shift up. They model the slope factor as behaving like a Taylor-rule, reacting to the output gap and inflation. When the central bank tightens monetary policy, the slope factor rises, forecasting lower growth in the future.

As we jointly estimate a model of the yield curve and a VAR system of the unobserved components and macroeconomic variables, our paper is similar to the macro-finance literature of Diebold, Rudebusch and Aruoba (2006) and Ang, Piazzesi and Wei (2006). We use the Nelson-Siegel curve without imposing no-arbitrage condition. As argued in Diebold, Rudebusch and Aruoba (2006), the Nelson-Siegel is flexible enough to avoid arbitrage opportunities in the data, and, if arbitrage opportunities do exist, our model avoids the misspecification problem. Replacing the VAR system in our model with a two-country New Keynesian model will be a challenging follow-up to this paper.

The term premium of maturity m is defined as the difference between the current m-period yield and the average of the current 1-period yield and its expected value in the coming $m-1$ periods. Different measures of the term premium come from different methods of forecasting the short rates. As the short rate is a highly persistent and predictable variable, The term premium can be understood as the compensation for bearing the risk from holding long-term instead of short-term bond. Despite a long history of interest in the term premium, there is no consensus among economists on its sources and its effects on the macroeconomy. According to the "common sense" interpretation of the term premium among practitioners, a drop in term premium, which reduces the spread between short and long rates, is expansionary and predicts an increase in real activity. Bernanke (2006) agrees with such a view. According to the canonical New Keynesian framework, the term premium has no such implication. As pointed out by Rudebusch, Sack and Swanson (2007), only the expected path of short rate matters in the dynamic output Euler equation and term premium does not predict more real activity in the future. For the purpose of this paper, we use the difference between the term premium between two countries to measure the difference in interest rate risk, and we do not attempt to explain the movements of the term premium.

4 A Dynamic Latent Factor Model

To connect the exchange rate with the term structure, we estimate a model that describes the dynamics among the exchange rate, yield curve factors and the macroeconomic fundamentals. We begin with a discussion of the latent-factor representation of the Nelson-Siegel (1987) yield curve. Given a panel of yields, we can estimate the level, slope and curvature factors as latent variables that follow a first-order vector autoregression. Next, we explain how to include exchange rate and other macroeconomic fundamentals in the model.

4.1 The Yields-Only Model

Noting that the exchange rate fundamentals discussed in previous section are in cross-country differences, we propose to measure the discounted present value with the cross country differences in their yield curves. Assuming symmetry and exploiting the linearity in the factor-loadings, we fit three Nelson-Siegel factors of the relative level (L_t^R), the relative slope (S_t^R), and the relative curvature (C_t^R). The interpretation of the relative factors is straightforward. For example, an increase in the relative level factor means the vertical difference of the home yield curve to the foreign one is more positive or less negative. We now proceed to estimate the yields-only model for relative yields. At each point of time t, we can fit the cross-section of yields i_t^m, where m denotes maturity, with the Nelson-Siegel curve:

$$i_t^m - i_t^{m*} = L_t^R + S_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m} \right) + C_t^R \left(\frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right) + \epsilon_t^m \quad (1)$$

Each yield of maturity m has a loading of 1 on the level factor, a loading of $\frac{1-\exp(-\lambda m)}{\lambda m}$ on the slope factor, and a loading of $\frac{1-\exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)$ on the curvature factor. The parameter λ, which will be estimated, controls the at which maturity the loading on the curvature is maximized. As the number of yields is larger than the number of factors, the factors cannot give a perfect fit to all the yields. As a result, an error term ϵ_t^m is appended to each yield as a measure of the goodness of fit. The typical application of the Nelson-Siegel curve involves estimating (1) period by period, and does not model how the yield curve evolves over time. We model the three factors together as a VAR(1) system:[11]

$$f_t - \mu = A(f_{t-1} - \mu) + \eta_t \tag{2}$$

where

$$f_t - \mu = \begin{pmatrix} L_t^R - \mu_L \\ S_t^R - \mu_S \\ C_t^R - \mu_C \end{pmatrix}.$$

The term η_t is a 3 by 1 vector of disturbances, and the term A is a 3 by 3 matrix of VAR coefficients describing the dynamics of the three factors. With (2), we can write the Nelson-Siegel curve (1) more succinctly as vectors:

$$i_t = \Lambda f_t + \epsilon_t \tag{3}$$

Since Equation (2) and (3) together form a state-space system, which can be estimated by the Kalman filter. For the estimation to be feasible, the two sets of error terms are uncorrelated:

$$\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim i.i.d.N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right]$$

The matrix Q is non-diagonal and the matrix H is diagonal (i.e. the error terms for the different yields in (3) are uncorrelated). As shown in Diebold, Rudebusch and Aruoba (2006), this yields-only model provides good fit to the US data (not relative). Two of their results are worth mentioning. First, persistence decreases and variance (as measured by the diagonal terms in Q) increases from L_t to S_t to C_t. Second, the null of a diagonal covariance matrix Q is strongly rejected by the data, implying strong interaction among the factors.

The data we examine consists of monthly observations from August 1985 to July 2005 for the US, Canada, Japan, and the United Kingdom. The US is defined as the home country for the rest of the paper. We look at zero-coupon bond yields for maturities 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60 72, 84, 96, 108 and 120 months, where the yields are computed using the Fama-Bliss (1987) methodology.[12] To accommodate with the macroeconomic variables, the yields are measured at the

[11] While we do not have a rigorous justification for this specification, Borak, Härdle, Mammen and Park (2007), for a high-dimensional system, it is acceptable to describe its dynamics with a VAR for a few factors.

[12] For details on the data, please see Diebold, Li and Yue (2007).

second trading day of *next month* (i.e. the yields for May 2001 are yields quoted on the second trading day of June 2001).

Through the Kalman filter, we estimate the model using maximum likelihood (See Nelson and Kim (1998) or Harvey (1981) for a discussion of estimating a state-space model by maximum likelihood). To ensure that the variances in the model are positive, we estimate log variances and obtain standard errors by the delta method. Since we have a large number of parameters, choosing the initial values for the optimization problem becomes an important issue. We try two sets of initial values. First, we set the variances to 1, λ to 0.0609 (the value commonly imposed for the Nelson-Siegel curve) and all other parameters to 0. The model takes long to converge with these initial values. Second, we use the Diebold-Li (2005) two-step method and obtain the factors with OLS. We then estimate a VAR and use the coefficient estimates to initialize the Kalman filter. The model converges faster with these initial values but the final results are almost identical to those using the first set. The Marquart algorithm is used for the optimization, and the convergence criterion is set to 10^{-6}.

Tables 1-3 show the estimated factors (smoothed) for the three countries.[13] Compare to the US yield curve (which is not relative) in Diebold, Rudebusch and Aruoba (2006), we again find the level factor to be the most persistent and the curvature factor the least, but we find more cross-factor dynamics among the relative yield factors for all three country pairs. We reject a diagonal Q matrix for all three countries. From the off-diagonal coefficients in the A matrix, for all three country pairs, we learn that a high lagged curvature predicts a high current value of any of the three factors. According to the off-diagonal coefficients in the Q matrix, the reduced-form shock to curvature is negatively correlated with the other two shocks for all three countries. To ensure that our model has a good fit to the data, we compare our smoothed factors to the factors obtained by the Diebold and Li (2006) 2-step OLS method. For each factor, the correlation between the two versions is 0.9 or above, implying that our model gives a reliable description of the data.

4.2 The Yields-ER-Macro Model

Knowing the yields-only model works well, our next goal is to add in the exchange rate and two macroeconomic variables into the VAR system and explain jointly the interaction between the relative term structure and the macroeconomy. We measure exchange rate s as the unit of foreign currency for each USD, which is measured at the end of each month. A larger number means an appreciation of the home currency,

[13] The smoothed (which uses information in the whole sample) and the filtered (which uses information up to time t through the Kalman filter) factors are essentially the same. Notice that even the filtered factors are not out sample: the parameter estimates fed into the Kalman filter are obtained using the whole sample.

the USD. For all horizons, we define exchange rate change as the change of the log exchange rate s. We denote the 1-month exchange rate, from the end of last month $t-1$ to the end of this month t, as Δs_t. We take inflation and output gap as the macroeconomic fundamentals. Using data from the IMF's International Financial Statistics database, we define relative inflation π_t^R as the difference of the 12-month percentage change of the CPI between the home and foreign countries, and relative output gap $\tilde{y}_t^R$ as the difference of the log industrial production index (detrended by fitting the original series on a quadratic trend) between the home and foreign countries.

The state-space system (2) and (3) is essentially the same, except that the vector f_t now has six variables $\tilde{y}_t^R, \pi_t^R, \Delta s_t, L_t^R, S_t^R, C_t^R$, and we call this the yields-ER-macro model. The six-variable VAR, as in the yields-only model, is in reduced form, and we can impose a recursive ordering of the variables. The relative output gap and inflation are ordered the first, followed by the 1-month exchange rate change (which is measured at the end of the month). Since the yields are measured on the second trading day of the *next* month, the three yield factors are ordered the last. Tables 4-6 show the results for the model. The 1-month exchange rate change is disconnected from both the two macroeconomic variables and the yield curve factors. The dynamics among the yield curve factors are similar as the yields-only model. Unlike DRA, we do not find a link from the macroeconomic variables to the yield curve or the other way round. Notice that the interaction found in DRA is mainly driven by the short rate in their VAR system (See their Table 3), and since we do not have the short rate in the VAR, it is not surprisingly for us to not finding the linkage. Notice that the coefficients in the VAR system only tell us the tie among the variables from the previous to the current month. Though unable to find a short-term connection from the yield curves or macroeconomic fundamentals to the exchange rate, our model is capable of extracting information from the yield curves and the macroeconomic variables for explaining longer term exchange rate movements.

4.3 Implications for Longer Horizons

Based on the estimated parameters of the state-space system, from the VAR we can calculate forecasts of exchange rate change of any horizon in each month. According to the model, (*ex ante*) exchange rate change from time t to any future period $t+m$ is a function of the time t values of the six VAR variables. In this section, we first check the performance of the model by comparing the exchange rate change as implied by the model with its *ex post* value. If the yield factors incorporate expectations of the fundamentals, they should play a larger role than the current output gap and inflation in explaining exchange rate change. We estimate a VAR with only the macroeconomic fundamentals and the exchange rate (the macro-ER model), and a model with only the yields and the exchange rate (the yields-ER model), and compare them with the yields-ER-macro model.

Our VAR model $f_t - \mu = A(f_{t-1} - \mu) + \eta_t$ allows us to calculate forecasts for any horizon. Given f_t, we can calculate the forecast for f_{t+m}as $A^m(f_t - \mu)$. In particular, the third row of the vector $A^m(f_t - \mu)$ gives you a forecast of Δs_{t+m}. We denote the third row of the vector as $A^m(f_t - \mu)_3$. To forecast exchange rate change from period t to period $t+m$, $s_{t+m} - s_t$,we simply need to calculate (we also annualize the forecast in our calculation):

$$A(f_t - \mu)_3 + A^2(f_{t-1} - \mu)_3 + ... + A^m(f_{t-1} - \mu)_3$$

With A and μ estimated using the whole sample, we can treat the above as an in-sample forecast.[14] Notice that the overlapping variable $s_{t+m} - s_t$ is not used directly in the VAR, and in the model we only have the 1-month exchange rate changeΔs_t. Given the parameter estimates, we explain future exchange rate change using only current exchange rate change Δs_t, current macroeconomic fundamentals $\tilde{y}_t^R, \pi_t^R$, and the current term structure L_t, S_t, C_t.

We compare the performance of our model (2)-(3), the same model without the yields (which is essentially just a VAR), and the same model without the macroeconomic variables. Results are shown in Figures 1-3, and the root mean squared errors (RMSE) are in Table 7.[15] Unlike the typical long-horizon regressions that put an m-period change on the LHS, we do not have exchange rate change of any horizon longer than one period in our model. The in-sample forecast we make is relies only on the yields and the six variables in the VAR, and we avoid the problems that usually plague the long-horizon regressions. From the figures we can see that both the yield curves and the macroeconomic variables are useful in explaining exchange rate change. The model's explanatory power is impressive for Canada and Japan, with the forecast tracing closely the actual exchange rate change, but we are unable to capture the large fluctuations in the UK pound during the ERM crisis. Table 7 tells us that our model fits better than the random walk forecast of "no change".

[14] Our short sample and the large number of parameters keep us from forecasting out of sample. Despite the practical attractiveness of out-sample forecasting, Engel, Mark and West (2007) argue that it is not a reliable criterion of measuring a model.

[15] For the period October 1990 - September 1992, the UK was a participant of the Exchange Rate Mechansim (ERM). During that period, the UK pound was effectively pegged within a small margin to countries in the European Community. Since the UK pound was a "semi-flexible" currency for that period, we dropped that period when doing the RMSE comparison in the previous section. Since the sample size is too small (24 observations), it is infeasible to fit our model and investigate the relationship between the UK pound and the term structure just for that period.

Also, we have done the same estimation for the non-US pairs of Canada-Japan, UK-Japan and Canada-UK and find similar forecasting results.

For Canada the model with only the macroeconomic variables fits the best, for the UK only the model with yields only fits the best, and for Japan both sets of variables are important.

5 Expectations or Term Premia?

In this section, we make use of our VAR model to break the term structure factors into an expectations part and a term premia part. We then investigate their separate role in explaining future exchange rate movements and excess returns.

In each month we can use the VAR model to forecast future relative short rates (i.e. 1-month rate) or rates of any maturity. The procedure is similar to the one in the previous section, but instead of forecasting the exchange rate we forecast the three yield curve factors in the VAR. With the forecasts of the three factors, we can compute the forecasts of the relative short rate based on the Nelson-Siegel curve. Consider some horizon m. Take the average of the time t short rate, and the $t+1, ..., t+m-1$ short rate forecasts, we can subtract it from the time t yield of m-period maturity to obtain the term premium of maturity m. The VAR approach is adopted by Diebold, Rudebusch and Aruoba (2006) and Cochrane and Piazzesi (2006), among others. While this approach may suffer from inconsistency between the yield curve at time t and forecasts of the yields, Rudebusch, Sack and Swanson (2007) shows that the VAR measure of the term premium behaves similarly as other measures that impose no-arbitrage conditions.[16]

We calculate the relative term premium using the relative yield curve factors, and we denote it as:

$$\rho_t^{(m)} \equiv i_t^m - i_t^{m*} - \frac{1}{m} \sum_{j=0}^{m-1} E_t \left[i_{t+j}^m - i_{t+j}^{m*} \right]$$

The expectation sign E_t refers to using the VAR model based on variables known at time t. The relative term premium of maturity m can be interpreted as the difference in the amount of risk in the home and foreign bond markets at horizon m. More specifically, it measures the difference of the amount of compensation required fore bearing the interest rate risk from holding long-term (maturity m) instead of short-term (maturity 1) debt between home and foreign countries. Due to the missing observations on 3-month yields, we only calculate relative premium for maturities of 6, 12, 18, 24 and 36 months. Figures 4-6 plot the relative term

[16] For example, see Rudebusch and Wu (2008) and Kim and Wright (2005). The first paper combines a no-arbitrage affine term structure model with a New Keynesian model, while the second paper estimates a three-factor no-arbitrage model without connection to macroeconomic variables.

premium for the three countries. An increase in the premium can be interpreted as an increase in the interest risk of maturity m in the home country relative to the foreign country. With the beginning of the inflation targeting regime at the end of 1995, interest rate risk in Canada is lower than that of the US at all horizons. The same applies to the UK where inflation targeting was first implemented in 1992. Term premium is lower in the UK than in the US for all horizons after that regime change, except for a brief period around the year 1997. For Japan its interest risk is lower than that of the US almost at all horizons for the whole sample. As the horizon increases, the average of the short rate forecast will approach the sample mean of the short rate, and the relative term premium of maturity m is roughly equal to the relative yield of maturity m minus a constant.

After separating the term premia from the yield curve, we propose a regression that tells us the separate role of expectations and term premia. By definition, the level factor in the relative yield curve can be separated into the sum of the average short rate forecast and a time-varying term premium:

$$L_t^R = \lim_{m\to\infty} \frac{1}{m} \sum_{j=0}^{m} E_t \left[i_{t+j}^1 - i_{t+j}^{1*} \right] + \rho_t \simeq \overline{i^1} - \overline{i^{1*}} + \rho_t \tag{4}$$

Equation (4) has no information if we do not observe at least two of the three components, but since we observe the relative level factor L_t^R we can define ρ_t as the relative level factor minus some constant. The approximation is due to the assumption in our VAR model that all yields are stationary, and that the expectations term will approach the sample mean of the short rate difference. The term premium ρ_t can be interpreted as compensation for risk in the distant future, but we can also interpret it as the perceived relative long-run inflation target (as in Kozicki and Tinsley (2001)). If people perceive the target to be higher by 1% temporarily, ρ_t will go up by 1%.

The slope factor is likewise defined as:[17]

$$S_t^R = i_t^1 - i_t^{1*} - L_t \simeq \left(i_t^1 - i_t^{1*} \right) - \left(\overline{i^1} - \overline{i^{1*}} \right) - \rho_t \tag{5}$$

Following the previous example, if people perceive the target to be higher by 1%, $i_t^1 - i_t^{1*}$will also go up by 1% and leave the slope unchanged. If the increase in ρ_t is not due to a higher perceived inflation target but due to higher long-term risk, the short rate stays the same and the yield curve gets steeper, and S_t^R goes down. All yields are linear function of the two factors plus a curvature term, as in the Nelson-Siegel curve:

[17] Strictly speaking, the slope factor should be defined as the difference between the instantaneous yield i_t^0 instead of the 1-period yield i_t^1. For exposition purpose we neglect the difference here.

$$
\begin{aligned}
i_t^m &= L_t^R + \beta_m S_t^R + \alpha_m C_t^R \\
&= \overline{i^1} - \overline{i^{1*}} + \rho_t + \beta_m \left(i_t^1 - i_t^{1*}\right) - \beta_m \left(\overline{i^1} - \overline{i^{1*}}\right) - \beta_m \rho_t + \alpha_m C_t^R \qquad (6) \\
&= (1-\beta_m)\left(\overline{i^1} - \overline{i^{1*}}\right) + \beta_m \left(i_t^1 - i_t^{1*}\right) + (1-\beta_m)\rho_t + \alpha_m C_t^R \qquad (7)
\end{aligned}
$$

The parameter β_m approaches 0 as m approaches infinity. The curvature term C_t^R takes care of nonlinearity in the middle part of the term structure. The constant term is unimportant. The second term can be interpreted as the expectations part of the yield: as the current 1-period relative yield changes, the average expected future path of the 1-period yield changes as well. The amount of change in the path depends on the horizon. The shorter is the horizon, the larger is the impact of a current change. The third term is the term premium ρ_t. Again, if there is an increase of 1% in the relative perceived inflation target both $i_t^1 - i_t^{1*}$ and ρ_t will go up by 1%, leaving i_t^m higher by 1% as well (i.e. the whole yield curve shifts up). If the long-run compensation for risk increases, the yield i_t^m will go up as well, with the amount of increase depending on β_m.

We do not have a clear interpretation for the relative curvature factor C_t^R, except that it captures movements in the middle part of the yield curve. To understand the role of the term premia in explaining exchange rate, we can break the curvature factor into the term premia. Denote the term premia we have calculated above as $\rho_t^{(6)}, \rho_t^{(12)}, \rho_t^{(18)}, \rho_t^{(24)}, \rho_t^{(36)}$. Running an OLS regression, we find that the level factor, the slope factor and the five term premia account for more than 80% of the movement of the curvature factor, which justifies a regression of the form:

$$
s_{t+m} - s_t = a_0 + a_1 \left(L_t^R + S_t^R\right) + a_2 L_t^R + a_3 \rho_t^{(6)} + a_4 \rho_t^{(12)} + a_5 \rho_t^{(18)} + a_6 \rho_t^{(24)} + a_7 \rho_t^{(36)} + e_t \qquad (8)
$$

On the RHS, the first term is the expectations part, the second term is the long-run relative term premium or inflation target, and the remaining terms take care of term premia in the middle part of the term structure. The above regression allows the following extreme cases: i) only expectations matter ($a_2 = a_3 = ... = a_7 = 0$), ii) only long-horizon term premium (above 36 months) matters ($a_1 = 0, a_3 = ... = a_7 = 0$), and iii) only shorter-horizon term premia matter (($a_1 = 0, a_3 = 0$).

Next we define excess return, the other LHS variable. We denote s_t as the price of the foreign currency for each US dollar. An investor can make an investment for m periods in two ways. The investor can buy the foreign currency and then buy the foreign m-period bond, or the investor can buy the domestic m-period bond. The difference between the two investments is the excess currency return of horizon m:

$$ex_{t+m,t} = i_t^m - i_t^{m*} - (s_{t+m} - s_t) \quad (9)$$

The notation $ex_{t+m,t}$ denotes the excess return realizes in period $t+m$ that starts in period t. The excess return is simply the deviation from the UIP, as the UIP says that the two investments should have the same expected return.

We use the smoothed yield curve factors for the regression (using the filtered factors give very similar results), and the term premia are calculated using the method mentioned at the beginning of this section. To avoid overlapping observations problem, we pick the first month of each quarter, each half-year and each year to create the quarterly, bi-annual, and annual non-overlapping samples. Using the corresponding non-overlapping samples, we look at the 3, 6, and 12 months horizons for exchange rate change, and we look at the 6 and 12 months horizons for excess return.

Results are in tables 8 and 9. For both exchange rate change and excess return, the hypothesis that the expectations part is redundant is rejected overwhelmingly. Expectations on future short rates, or the future path of monetary policy, appear to be important in explaining future exchange rate and excess return. The term premia variables are relevant too. For most horizons, we cannot reject the null hypothesis that the coefficients on either group of the term premia are zero. If the term structure of term premia is not important for explaining exchange rate and excess return, then replacing the term premia with only the curvature factor will give a similar fit. The last two columns in tables 8 and 9 show that splitting the curvature factor into the term premia terms improves the fit, and in some cases substantially.

Figures 7-9 plot the coefficients on the five term premia and the relative level factor.

6 Conclusion

This paper is the first step of combining the monetary policy approach and the finance approach to modeling exchange rate. The first strand of literature argues that macroeconomic fundamentals usually included in the Taylor rule forecast exchange rate well, but it ignores the presence of risk in the analysis. The second strand of literature explains exchange rate movements or excess return using return-based latent factors, but it does not link the factors to monetary policy directly. We connect the two by estimating a model that jointly describes the dynamics of exchange rate, yield curve factors, inflation and output gap. The model fits the data well, especially at long horizons. Based on the term premia estimated from the VAR model, we show that both the expected path of relative short rate and term premia explain future exchange rate movements and excess return. Investors' view on the future path of monetary policy (which is driven by current and future fundamentals) and their risk appetite are both factors that move future exchange rate.

While this is the first step of bridging the two approaches, this is certainly not the last. Our results call for a model that jointly accounts for forward premium and term premium, tracing both back to preference.

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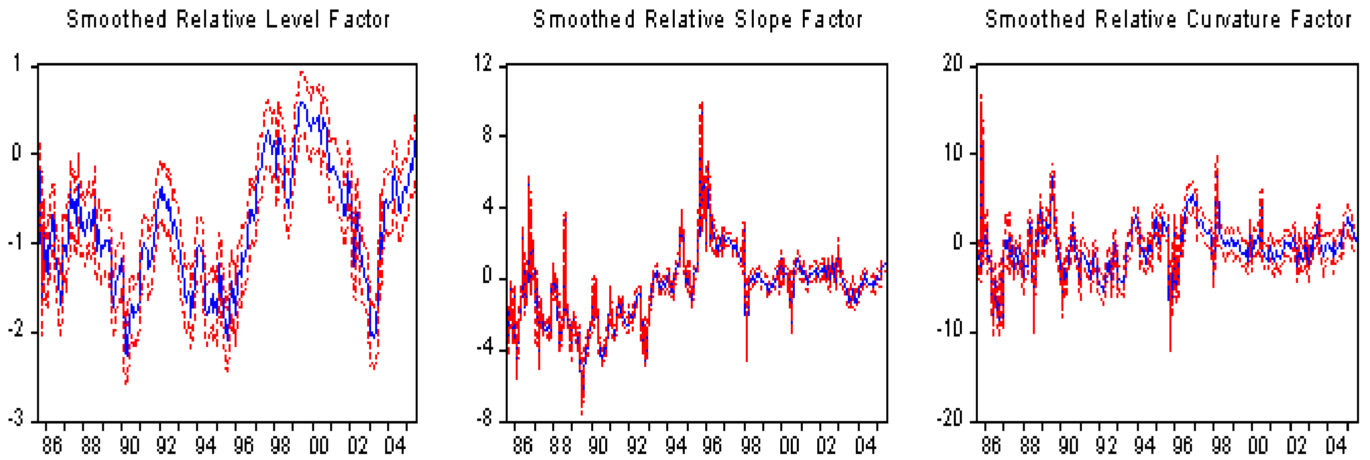
Table 1: Relative Level, Slope and Curvature Factors for Canada (Yields-Only Model)

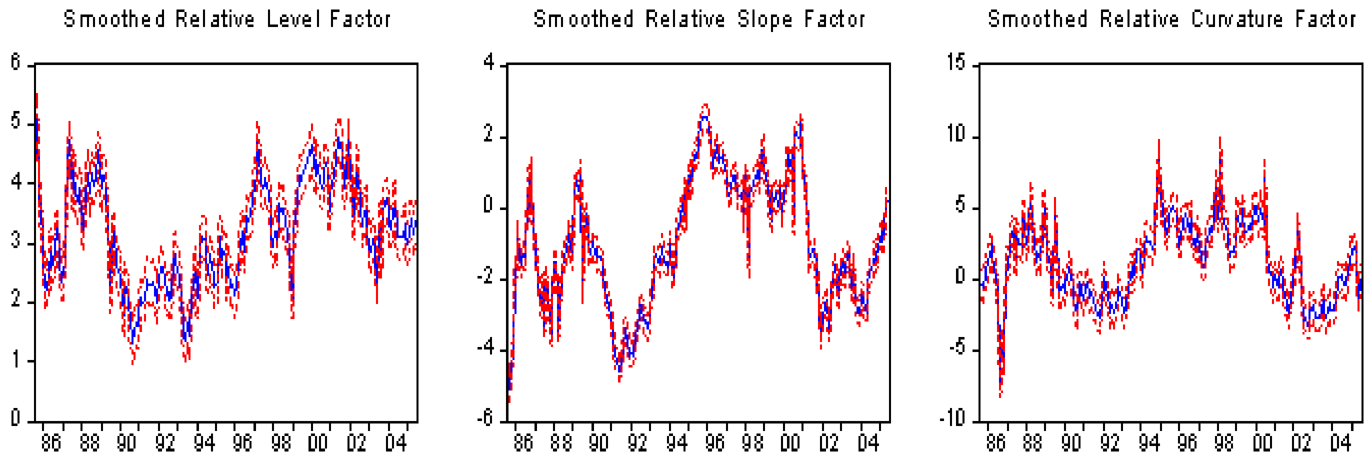
Estimated λ: 0.111 (0.004), P-value for the Null of a Diagonal Q: 0.0000



$$A = \begin{pmatrix} \mathbf{0.909} & 0.003 & \mathbf{0.037} \\ (0.039) & (0.015) & (0.009) \\ -0.074 & \mathbf{0.786} & \mathbf{0.138} \\ (0.168) & (0.047) & (0.037) \\ 0.308 & \mathbf{0.266} & \mathbf{0.543} \\ (0.376) & (0.110) & (0.065) \end{pmatrix} \quad Q = \begin{pmatrix} \mathbf{0.081} & 0.068 & \mathbf{-0.363} \\ (0.017) & (0.059) & (0.142) \\ & \mathbf{1.894} & \mathbf{-2.471} \\ & (0.164) & (0.297) \\ & & \mathbf{6.378} \\ & & (0.934) \end{pmatrix}$$

Table 2: Relative Level, Slope and Curvature Factors for Japan (Yields-Only Model)

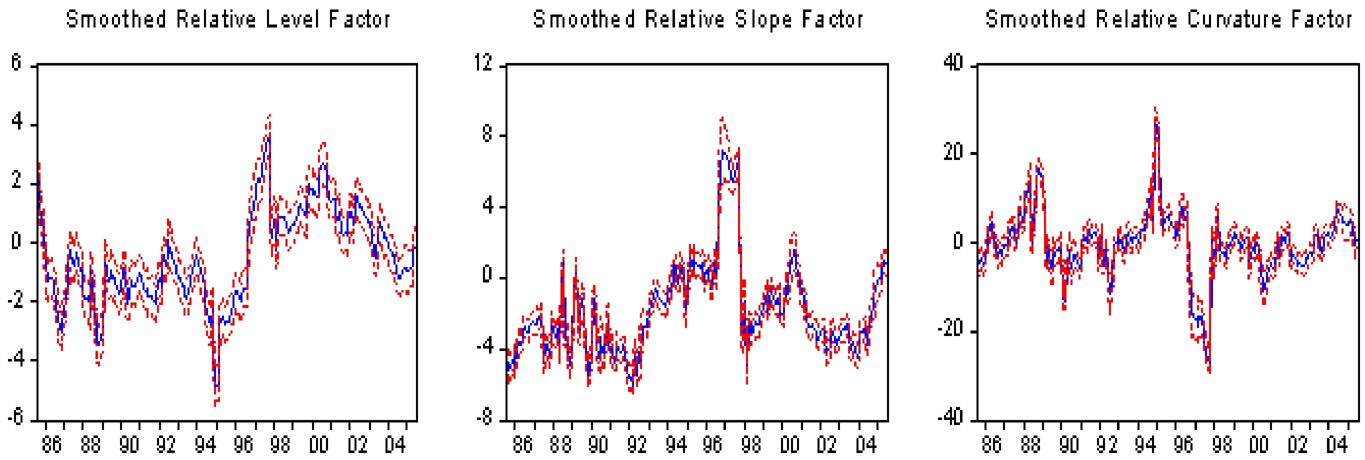
Estimated λ: 0.086 (0.003), P-value for the Null of a Diagonal Q: 0.0000



$$A = \begin{pmatrix} \mathbf{0.894} & -0.008 & \mathbf{0.026} \\ (0.036) & (0.018) & (0.013) \\ 0.028 & \mathbf{0.873} & \mathbf{0.089} \\ (0.068) & (0.031) & (0.016) \\ 0.033 & \mathbf{0.195} & \mathbf{0.797} \\ (0.143) & (0.071) & (0.036) \end{pmatrix} \quad Q = \begin{pmatrix} \mathbf{0.136} & \mathbf{-0.082} & -0.083 \\ (0.023) & (0.023) & (0.060) \\ & \mathbf{0.383} & \mathbf{-0.460} \\ & (0.046) & (0.063) \\ & & \mathbf{1.798} \\ & & (0.223) \end{pmatrix}$$

Table 3: Relative Level, Slope and Curvature Factors for the UK (Yields-Only Model)

Estimated λ: 0.073 (0.002), P-value for the Null of a Diagonal Q: 0.0000



$$A = \begin{pmatrix} \mathbf{0.988} & 0.020 & 0.015 \\ (0.045) & (0.023) & (0.010) \\ 0.048 & \mathbf{0.940} & \mathbf{0.041} \\ (0.106) & (0.035) & (0.018) \\ -0.263 & -0.050 & \mathbf{0.808} \\ (0.351) & (0.120) & (0.061) \end{pmatrix} \quad Q = \begin{pmatrix} \mathbf{0.309} & \mathbf{0.281} & \mathbf{-1.743} \\ (0.078) & (0.079) & (0.396) \\ & \mathbf{1.280} & \mathbf{-2.823} \\ & (0.163) & (0.356) \\ & & \mathbf{14.805} \\ & & (1.760) \end{pmatrix}$$

Table 4: Yields-ER-Macro Model for the Canada

Estimated λ: 0.111 (0.005), P-value for the Null of a Diagonal Q: 0.0000

P-value of No Yields to ER: 0.7167   P-value of No Macro to ER: 0.4255

P-value of No Yields to Macro: 0.9688   P-value of No Macro to Yields:0.8478

$$A = \begin{pmatrix} \mathbf{0.748} & -0.059 & -0.006 & 0.046 & -0.008 & 0.018 \\ (0.065) & (0.131) & (0.043) & (0.096) & (0.028) & (0.026) \\ 0.005 & \mathbf{0.945} & -0.005 & 0.001 & 0.003 & -0.001 \\ (0.015) & (0.028) & (0.011) & (0.026) & (0.007) & (0.007) \\ 0.078 & 0.469 & -0.005 & 0.249 & 0.003 & 0.030 \\ (0.166) & (0.355) & (0.072) & (0.231) & (0.093) & (0.073) \\ 0.050 & -0.041 & -0.011 & \mathbf{0.890} & 0.008 & \mathbf{0.045} \\ (0.031) & (0.076) & (0.023) & (0.048) & (0.017) & (0.013) \\ 0.076 & -0.026 & 0.062 & -0.124 & \mathbf{0.791} & \mathbf{0.146} \\ (0.175) & (0.292) & (0.116) & (0.232) & (0.063) & (0.061) \\ -0.411 & 0.560 & -0.144 & 0.523 & 0.215 & \mathbf{0.472} \\ (0.316) & (0.653) & (0.201) & (0.472) & (0.138) & (0.095) \end{pmatrix}$$

$$Q = \begin{pmatrix} \mathbf{0.431} & -0.002 & \mathbf{0.153} & 0.040 & 0.074 & -0.271 \\ (0.047) & (0.009) & (0.090) & (0.031) & (0.087) & (0.219) \\ & \mathbf{0.025} & 0.016 & -0.001 & -0.004 & 0.031 \\ & (0.002) & (0.022) & (0.007) & (0.026) & (0.054) \\ & & \mathbf{2.285} & -0.033 & -0.035 & -0.362 \\ & & (0.254) & (0.066) & (0.269) & (0.544) \\ & & & \mathbf{0.078} & 0.068 & \mathbf{-0.354} \\ & & & (0.021) & (0.074) & (0.176) \\ & & & & \mathbf{1.875} & \mathbf{-2.416} \\ & & & & (0.210) & (0.384) \\ & & & & & \mathbf{6.178} \\ & & & & & (1.176) \end{pmatrix}$$

Table 5: Yields-ER-Macro Model for the Japan

Estimated λ: 0.086 (0.003), P-value for the Null of a Diagonal Q: 0.0000

P-value of No Yields to ER: 0.2531 P-value of No Macro to ER: 0.6692

P-value of No Yields to Macro: 0.9559 P-value of No Macro to Yields: 0.7002

$$A = \begin{pmatrix} \mathbf{0.815} & -0.163 & 0.040 & -0.011 & -0.030 & 0.034 \\ (0.046) & (0.265) & (0.034) & (0.143) & (0.070) & (0.042) \\ 0.012 & \mathbf{0.912} & 0.003 & 0.003 & -0.001 & 0.005 \\ (0.008) & (0.037) & (0.005) & (0.018) & (0.011) & (0.007) \\ -0.183 & 0.358 & -0.024 & 0.133 & 0.311 & 0.030 \\ (0.142) & (0.648) & (0.009) & (0.343) & (0.172) & (0.135) \\ -0.022 & 0.091 & 0.005 & \mathbf{0.871} & -0.016 & \mathbf{0.031} \\ (0.016) & (0.081) & (0.011) & (0.048) & (0.025) & (0.014) \\ 0.017 & 0.062 & 0.004 & 0.021 & \mathbf{0.867} & \mathbf{0.085} \\ (0.030) & (0.112) & (0.017) & (0.101) & (0.038) & (0.021) \\ 0.048 & -0.385 & -0.045 & 0.115 & 0.236 & \mathbf{0.788} \\ (0.070) & (0.317) & (0.043) & (0.208) & (0.084) & (0.043) \end{pmatrix}$$

$$Q = \begin{pmatrix} \mathbf{1.652} & 0.016 & 0.113 & 0.037 & -0.049 & 0.050 \\ (0.193) & (0.023) & (0.400) & (0.056) & (0.075) & (0.181) \\ & \mathbf{0.032} & -0.031 & 0.006 & -0.005 & 0.022 \\ & (0.003) & (0.051) & (0.007) & (0.010) & (0.027) \\ & & \mathbf{11.500} & -0.106 & 0.013 & 0.138 \\ & & (1.068) & (0.138) & (0.202) & (0.592) \\ & & & \mathbf{0.200} & \mathbf{-0.081} & -0.074 \\ & & & (0.061) & (0.027) & (0.071) \\ & & & & \mathbf{0.379} & \mathbf{-0.458} \\ & & & & (0.057) & (0.081) \\ & & & & & \mathbf{1.748} \\ & & & & & (0.281) \end{pmatrix}$$

Table 6: Yields-ER-Macro Model for the UK

Estimated λ: 0.073 (0.006), P-value for the Null of a Diagonal Q: 0.0000

P-Value of No Yields to ER: 0.5718 P-Value of No Macro to ER: 0.6582

P-value of No Yields to Macro: 0.0046 P-value of No Macro to Yields: 0.8176

$$A = \begin{pmatrix} \mathbf{0.844} & -0.273 & 0.012 & \mathbf{0.350} & 0.034 & \mathbf{0.047} \\ (0.036) & (0.176) & (0.029) & (0.092) & (0.048) & (0.024) \\ -0.002 & \mathbf{0.955} & -0.002 & 0.018 & -0.004 & 0.004 \\ (0.007) & (0.032) & (0.005) & (0.021) & (0.007) & (0.004) \\ -0.004 & 0.316 & 0.075 & -0.003 & 0.024 & -0.020 \\ (0.137) & (0.463) & (0.074) & (0.366) & (0.117) & (0.067) \\ 0.015 & -0.065 & -0.017 & \mathbf{0.966} & 0.290 & 0.015 \\ (0.036) & (0.138) & (0.030) & (0.113) & (0.023) & (0.019) \\ 0.006 & 0.155 & 0.018 & 0.011 & \mathbf{0.924} & 0.031 \\ (0.085) & (0.247) & (0.046) & (0.273) & (0.046) & (0.038) \\ -0.021 & 0.523 & 0.052 & -0.314 & -0.113 & \mathbf{0.787} \\ (0.263) & (0.907) & (0.166) & (0.874) & (0.154) & (0.127) \end{pmatrix}$$

$$Q = \begin{pmatrix} \mathbf{1.012} & 0.006 & -0.110 & 0.001 & -0.009 & -0.022 \\ (0.097) & (0.018) & (0.277) & (0.103) & (0.141) & (0.567) \\ & \mathbf{0.027} & -0.015 & 0.005 & 0.018 & -0.001 \\ & (0.002) & (0.005) & (0.013) & (0.020) & (0.006) \\ & & \mathbf{8.307} & 0.027 & -0.045 & 0.220 \\ & & (0.833) & (0.229) & (0.346) & (1.133) \\ & & & \mathbf{0.309} & \mathbf{0.286} & \mathbf{-1.718} \\ & & & (0.095) & (0.092) & (0.476) \\ & & & & \mathbf{1.265} & \mathbf{-2.851} \\ & & & & (0.209) & (0.431) \\ & & & & & \mathbf{14.648} \\ & & & & & (2.070) \end{pmatrix}$$

Table 7: RMSE of the Models and Random Walk Forecasts

Horizon	*Macro + Yields*	*Macro*	*Yields*	*RandomWalk*
		Canada		
3	10.547	**10.441**	10.971	11.197
6	7.289	**6.969**	7.767	7.841
12	5.416	**4.899**	5.921	5.947
24	4.590	**3.820**	5.149	5.019
		Japan		
3	24.108	24.209	**23.719**	24.827
6	**15.791**	17.222	16.398	17.913
12	**8.331**	10.979	9.730	11.766
24	**6.574**	9.532	7.045	8.649
		UK		
3	13.339	13.370	**13.137**	13.259
6	8.977	8.968	**8.685**	8.944
12	6.524	6.431	**6.110**	6.404
24	5.157	5.109	**4.571**	4.883

Note: The sample for the UK starts after the ERM crisis (1992M10).

Table 8: Expectations or Term Premia - Exchange Rate Change

	Wald test p-value				
Horizon	No Expectations	No Long Premia	No Short Premia	R^2	R^2 with only factors
Canada					
3	0.0733	0.0347	0.1993	0.040	0.014
6	0.0297	0.0694	0.0316	0.218	-0.042
12	0.0650	0.0907	0.0104	0.373	0.011
Japan					
3	0.0385	0.2509	0.0262	0.072	0.029
6	0.0521	0.2701	0.2253	0.085	0.062
12	0.0000	0.0003	0.0003	0.661	0.205
UK					
3	0.0558	0.4860	0.1749	0.019	-0.037
6	0.0774	0.3887	0.0802	0.211	0.038
12	-	-	-	-	-

Note: The sample for UK starts after the ERM crisis (1992M10), and the sample size is too small for the 12-month horizon. All the samples are non-overlapping, using the first month of each quarter, half year and year.

Table 9: Expectations or Term Premia - Excess Return

	Wald test p-value				
Horizon	No Expectations	No Long Premia	No Short Premia	R^2	R^2 with only factors
Canada					
6	0.0095	0.0968	0.0248	0.310	0.082
12	0.0342	0.1099	0.0107	0.456	0.143
Japan					
6	0.0219	0.2778	0.2288	0.153	0.131
12	0.0000	0.0004	0.0003	0.729	0.355
UK					
6	0.0381	0.3347	0.0533	0.279	0.105
12	-	-	-	-	-

Note: The sample for UK starts after the ERM crisis (1992M10), and the sample size is too small for the 12-month horizon. All the samples are non-overlapping, using the first month of each quarter, half year and year.

Figure 1: In-Sample Fitting of the Yields-ER-Macro Model (Canada)

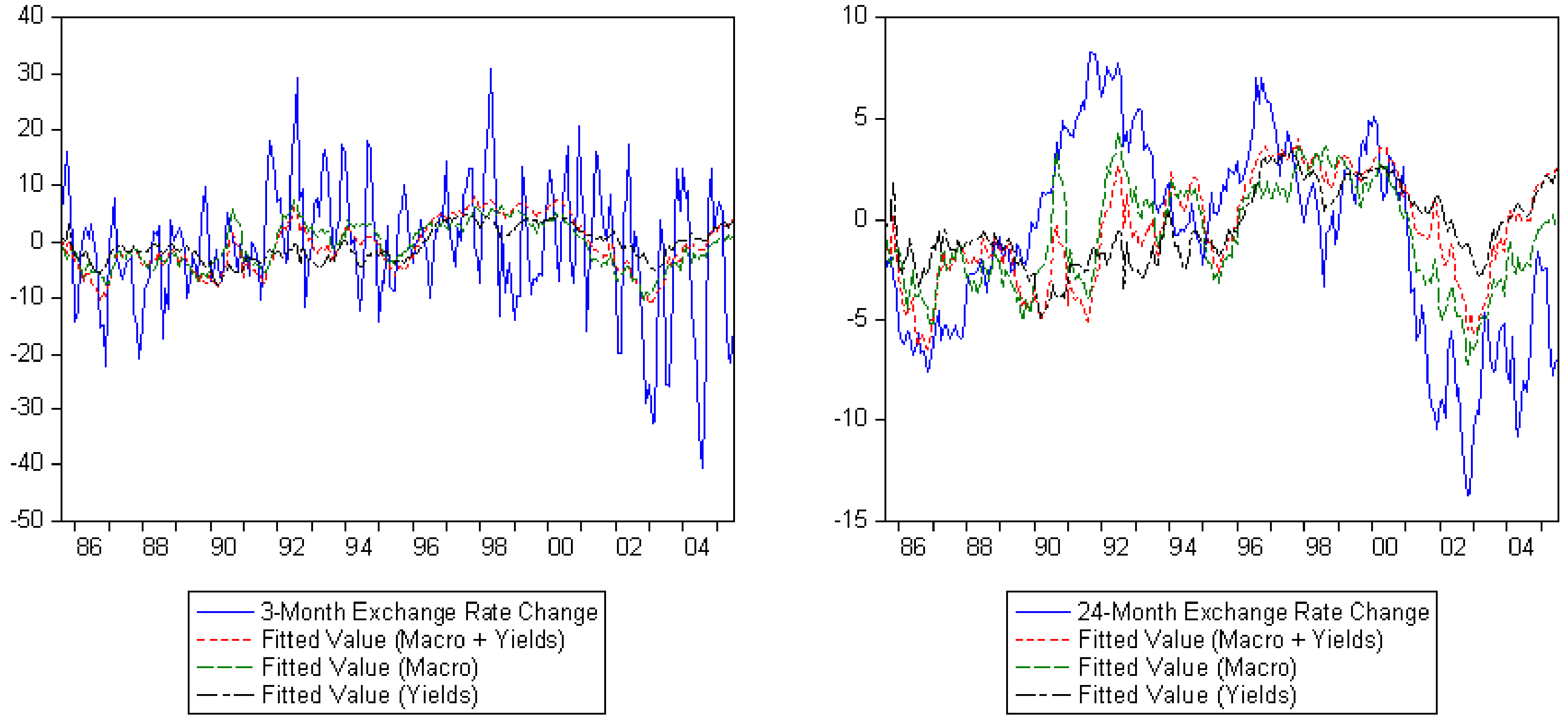


Figure 2: In-Sample Fitting of the Yields-ER-Macro Model (Japan)

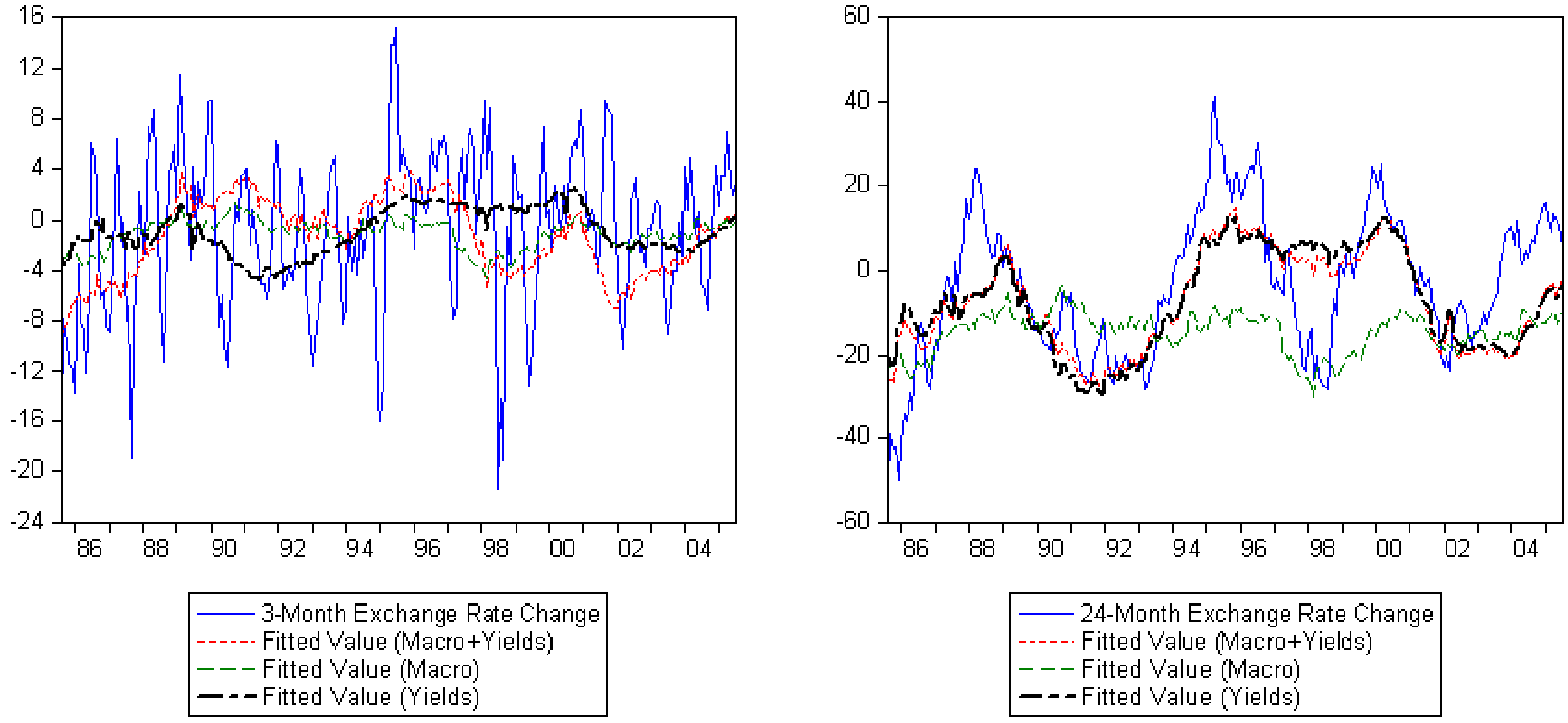
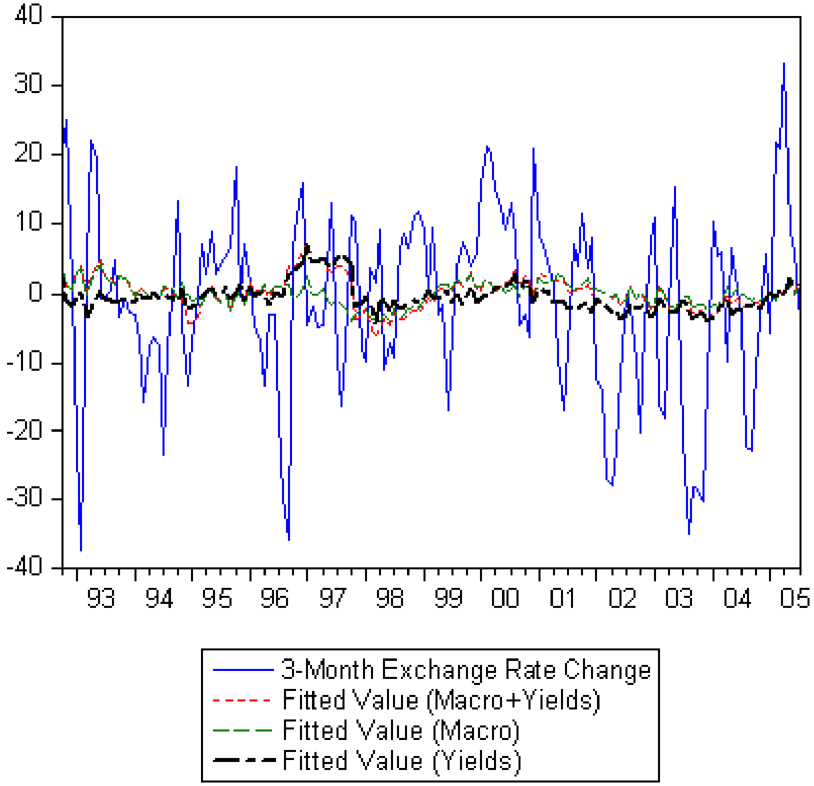
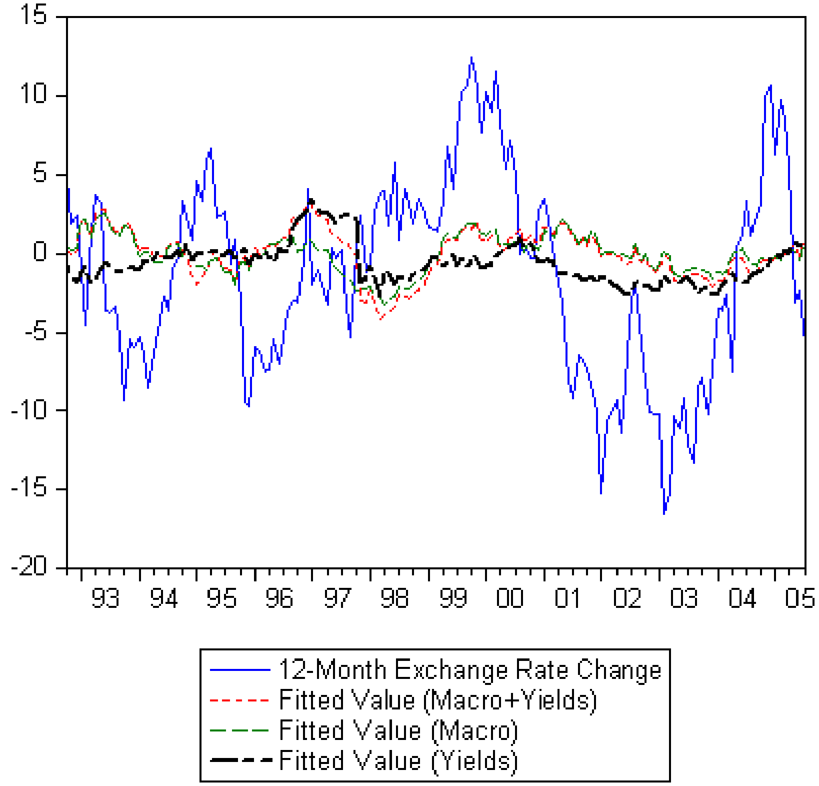


Figure 3: In-Sample Fitting of the Yields-ER-Macro Model (UK)





Note: The sample for the UK starts after the ERM crisis (1992M10).

Figure 4: Relative Term Premium (Canada)

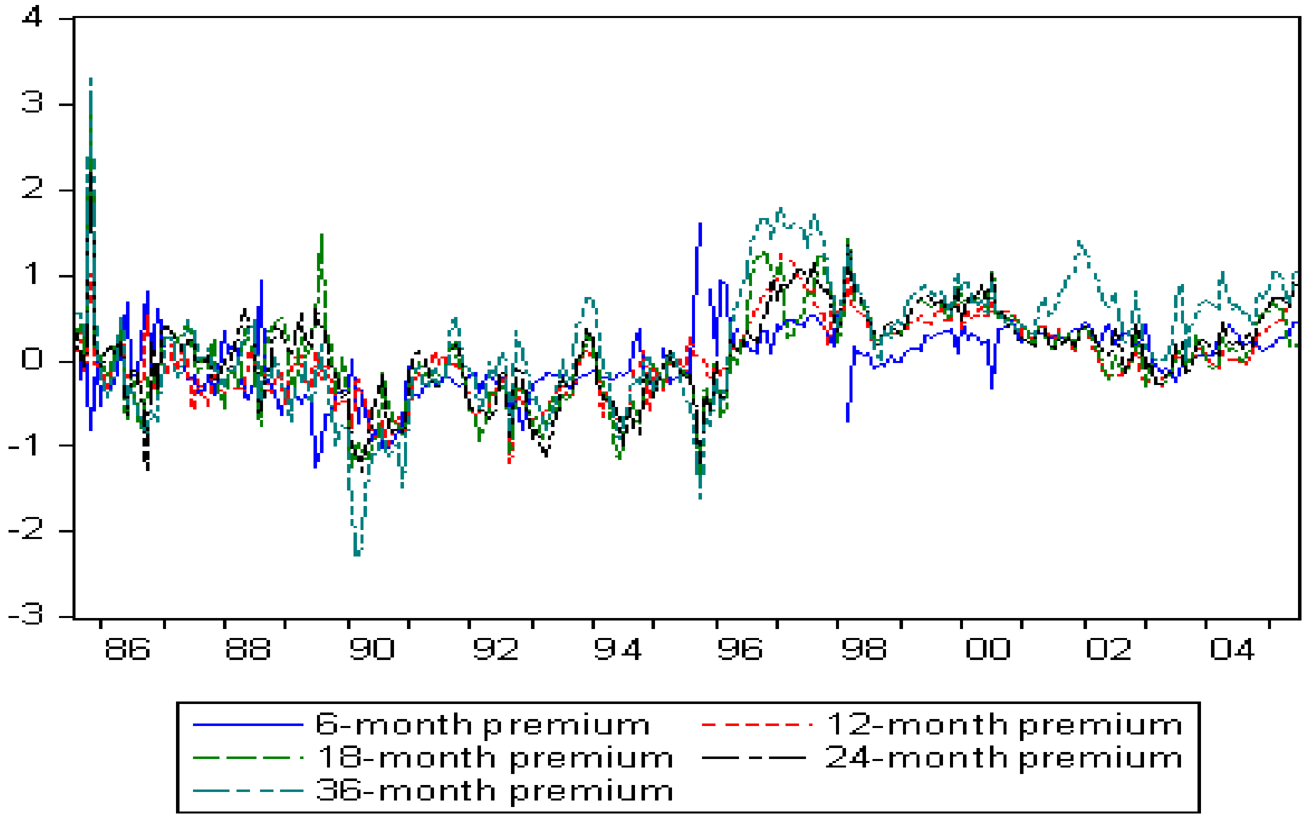


Figure 5: Relative Term Premium (Japan)

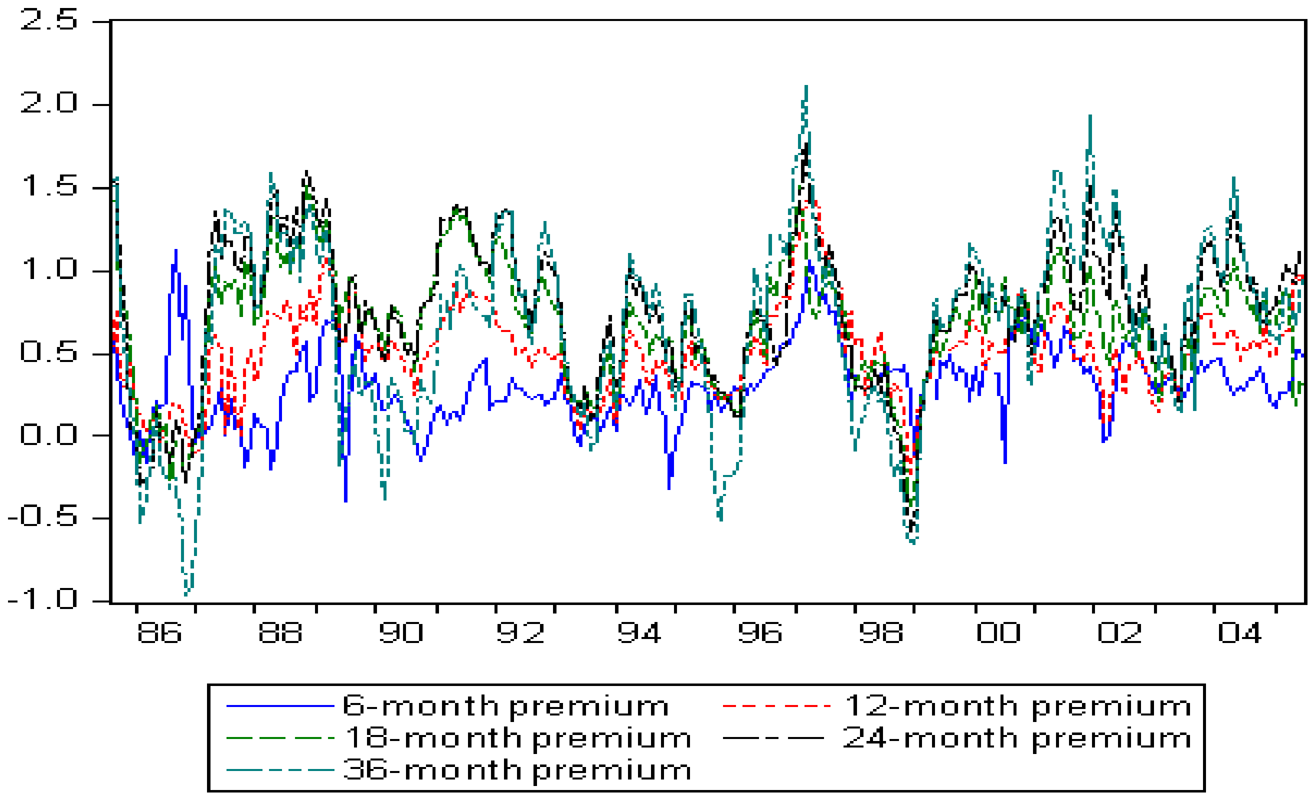


Figure 6: Relative Term Premium (UK)

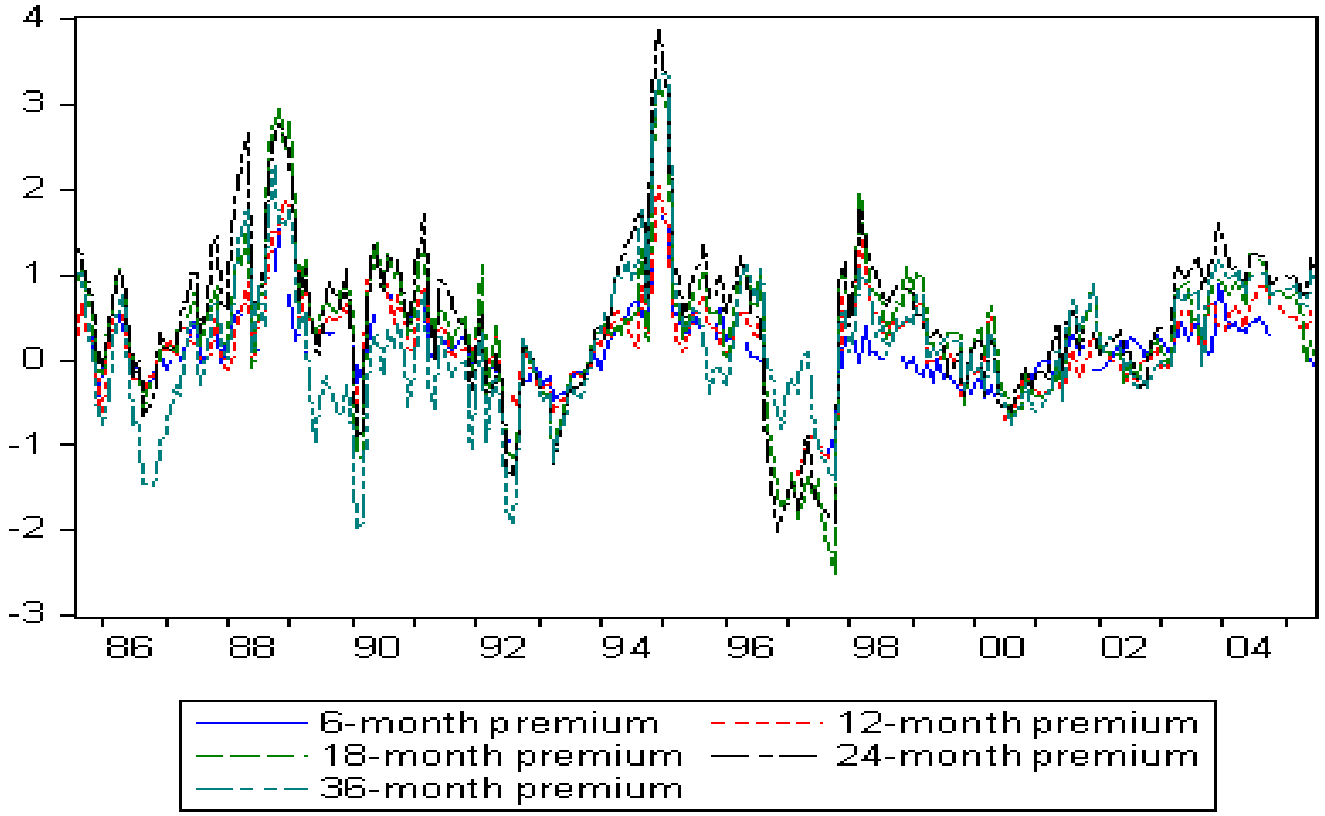


Figure 7: Excess Returns and Term Premia (Canada)

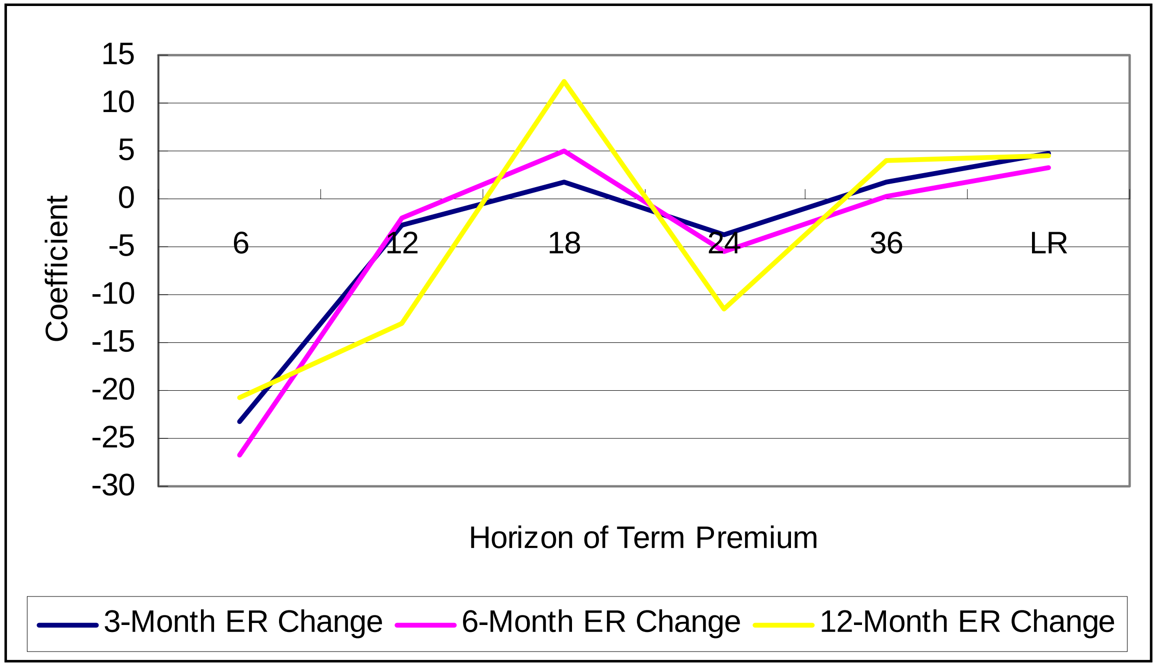


Figure 8: Excess Returns and Term Premia (Japan)

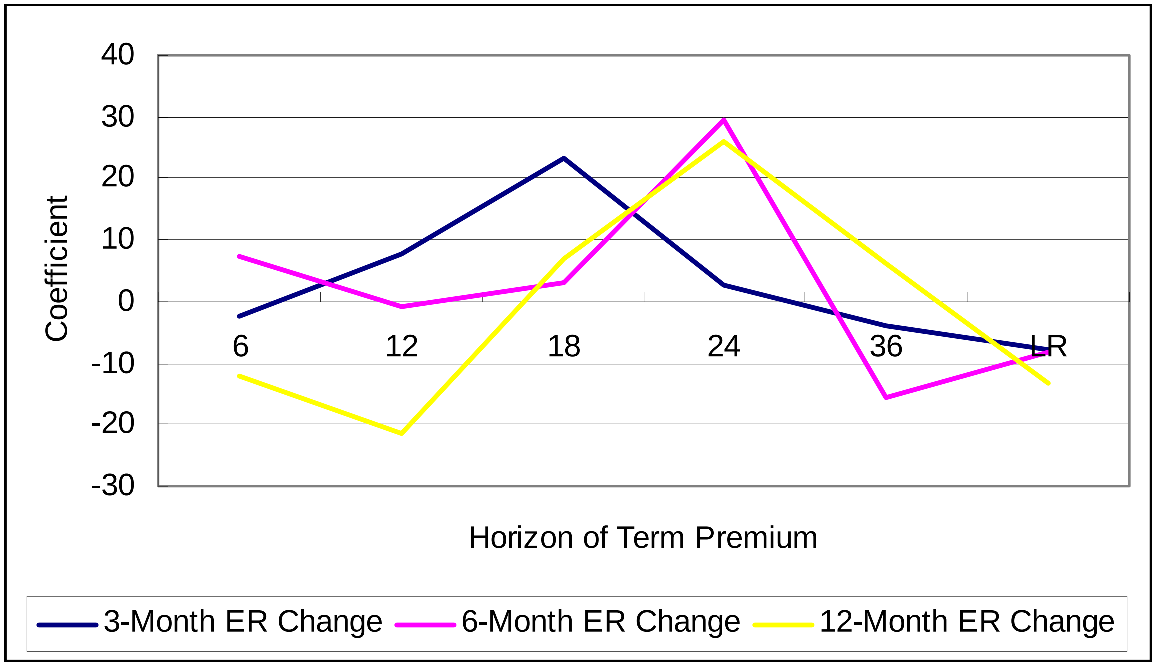


Figure 9: Excess Returns and Term Premia (UK)