Credit Rating Impact on CDO Evaluation

Daniel Rösch

University of Hannover

Harald Scheule

University of Melbourne

Key Words: Business Cycle, Collateralised Debt Obligation, Correlation, Credit Risk, Default Probability, Point-in-Time, Portfolio, Through-the-Cycle

JEL Classification: G20, G28, C51

Daniel Rösch, Institute of Banking & Finance, School of Business & Economics, Leibniz University, 30167 Hannover, Germany, Phone: +49-511-762-4668, Fax : +49-511-762-4668, Email: Daniel.Roesch@finance.uni-hannover.de

Harald Scheule, Department of Finance, Faculty of Economics and Commerce, University of Melbourne, Victoria 3010, Australia, Phone: +61-3-8344-9078, Fax : +61-3-8344-6914, Email: hscheule@unimelb.edu.au

Credit Rating Impact on CDO Evaluation

Abstract

One of the most significant developments in international credit markets in recent years has been the trade in Collateralized Debt Obligations (CDO), which has enabled financial institutions to repackage the credit risk of an asset portfolio into tranches to be transferred to investors. The present paper evaluates the credit risk of such a portfolio and the related tranches by applying two prominent prototypes for credit ratings namely the point-in-time and through-the-cycle approach. The central parameters default probability and correlation are forecast for multiple years and related forecasting errors included. The article's main findings are that banks which transfer debt tranches but retain an equity part and apply a through-the-cycle rating approach may be exposed to higher insolvency risk. Firstly, the credit risk retained may be underestimated resulting in an inadequate capital allocation. Secondly, the credit risk transferred may be overestimated resulting in additional risk-based transfer costs.

1   Introduction

One of the most significant developments in international credit markets in recent years has been the trade in Collateralized Debt Obligations (CDOs). CDOs repackage the credit risk of a portfolio into tranches and transfer it to investors. Because of the trade of portfolio characteristics, not only a single entity's risk characteristics (e.g. the probability of default), but rather correlation modelling of the underlying portfolio is of crucial importance for the accurate pricing of CDOs.

In the CDO related literature we see two major streams. One stream is concerned with modelling and estimation of the risk characteristics of a bank portfolio. Here, the focus is on individual risk parameters, such as default probabilities, loss rates given default and exposures at default, and on dependence parameters such as correlations or more general copulas. See e.g. Shumway (2001) or Crouhy/Galai/Mark (2002) for methods of bankruptcy prediction and Gordy (2000) for an alternative concept of portfolio credit risk modelling.

The second stream of the literature focuses on the pricing of CDOs where the central issue is to identify assumptions which provide accurate explanations for observed CDO spreads. These approaches use a risk-neutral pricing framework, develop pricing techniques and fit alternative models to market spreads. Recent papers of this direction include Laurent/Gregory (2005) and Hull/White (2004).

In the financial community, a lively discussion exists on how to assess the credit risk of portfolios. Two prominent prototypes for credit ratings are point-in-time (PIT) and through-the-cycle (TTC). The former approach assesses a borrower's credit risk by using all current information about the credit cycle (and is mainly employed by banks) while the latter evaluates credit risk by abstracting from the macroeconomic environment (and is mainly employed by credit rating agencies), for a discussion see Crouhy/Galai/Mark (2002) among others. Recently, primarily in the context of the new capital standards for banks (Basel II), some authors analysed effects of both rating types with respect to cyclicality of resulting risk assessments and bank capital, see e.g. Nickell/Perraudin/Varotto (2000), Rösch (2005), Rösch/Scheule (2005), or Heitfield (2005) who suggests formal definitions of both schemes. Here, we want to continue the discussion and analyse the effect of using alternative rating systems for assessment of portfolio risks and tranches thereof. We ask from the viewpoint of a bank which uses one of two rating systems, how the implied credit spreads of the CDO tranches differ by

following either rating philosophy and what the implications are. We refer to 'implied spreads' as spreads which reflect the risk characteristics of the underlying portfolio and are derived by using a simple evaluation framework. A special focus in contrasting the rating philosophies will be on the problem of forecasting future portfolio risk characteristics. Often, the underlying portfolios have maturities of multiple years and portfolio risk has to be forecast for long time horizons. We will analyse how both rating approaches perform in a forward-looking application and additionally include forecasting errors.

The rest of the paper proceeds as follows. Section 2 contains a brief summary on CDO mechanics. Section 3 presents different credit rating models. Section 4 contains the results without forecasting risk. Section 5 considers forecasting risk and Section 6 provides the discussion of the results and economic implications.

2 CDO Mechanics at a Glance

CDOs are a type of multi-name credit derivatives. A portfolio of defaultable instruments is tranched into loss bearing pieces. Via these tranches the credit risk is sold to investors, who in turn obtain an agreed periodic payment. Each tranche is defined by a lower and an upper attachment point K_l and K_u, measured in percent of portfolio loss. Buyers of a tranche with lower attachment point K_l and upper attachment point K_u are exposed to losses which exceed K_l up to K_u. The losses suffered by the holders of a tranche are therefore limited to $K_l - K_u$. The tranches might exemplarily be structured as in Table 1, which uses the lower and upper attachment points in reference to the Dow Jones CDX NA IG tranches.

Table 1: CDO Tranching Example

		Attachment points (%)	
Tranche number	Name	Lower point K_l	Upper point K_u
1	Equity	0	3
2	Mezzanine 1	3	7
3	Mezzanine 2	7	10
4	Mezzanine 3	10	15
5	Mezzanine 4	15	30
6	Senior	30	100

Consider a portfolio with value V on which a CDO is originated at the beginning of time (year) t with maturity T (years). Let L_τ denote the percentage portfolio loss at time (year) τ

and $l_\tau = \sum_{s=t}^{\tau} L_s$ the total percentage loss up to year τ. The loss suffered by the holders of tranche j of the CDO up to year τ is then

(1) $$l_{j,\tau} = \min\{l_\tau, K_{j,U}\} - \min\{l_\tau, K_{j,L}\}$$

where $K_{j,U}$ and $K_{j,L}$ are the upper and lower attachment points of tranche j. The losses are paid by the tranche holders with a predetermined frequency η in years. Then the payment of the holders of tranche j at time $\tau + \eta$ is the fraction $l_{j,\tau+\eta} - l_{j,\tau}$ of total portfolio value.

The holders of the tranche are compensated for bearing the risk of losses by receiving a periodical payment which is a fixed spread s_j on the outstanding amount $\Gamma_{j,\tau}$ of the tranche j at each time τ where

(2) $$\Gamma_{j,\tau} = (K_{j,U} - K_{j,L}) \cdot V - l_{j,\tau} \cdot V$$

Thus, each payment date during the life of the CDO the tranche holders receive an amount

(3) $$s_j \cdot \eta \cdot \Gamma_{j,\tau}$$

and pay an amount

(4) $$(l_{j,\tau} - l_{j,\tau-\eta}) \cdot V .$$

The spread $s_{j,\tau}$ usually does not vary over the life of the CDO but the outstanding amount will change over time.

The usual methodology to derive CDO spreads is to assume the absence of arbitrage and to employ risk-neutral pricing techniques. In this paper, we want to analyse implications of various parameter settings on prices which a bank calculates for CDOs. We therefore do not explicitly need the assumption of perfect markets.

From the perspective of the bank the payments are reversed. The bank pays a periodical amount (2) and receives a compensation (3) for the credit losses of the portfolio. If future losses are deterministic then the bank can simply calculate the present values of the amount to

be received and to be paid. The maximum spread would be the spread at which both present values are equal. However, future losses are random and thus, the bank has to account for the risk inherent in the cash flows. If the bank was risk neutral, then it would be enough to discount *expected* payments by the risk-free rate. Otherwise it should either account for risk by using certainty equivalent cash flows or adjust the discount factor by a risk premium. Here we propose the second way of evaluation which calculates present values of pay-offs for each tranche by using expected pay-offs which are discounted by a risk-adjusted discount factor.

Let $\beta(t_0,t_k)$ be the risk adjusted discount factor from t_0 to t_k which are ex-ante given by the bank. Then the present value of the fixed leg is given by

(5)
$$
\begin{aligned}
B_{F,j} &= \sum_{k=1}^{K} \beta(t_0,t_k)\cdot s_j \cdot \eta \cdot E\big[\Gamma_{j,t_k}\big] \\
&= \sum_{k=1}^{K} \beta(t_0,t_k)\cdot s_j \cdot \eta \cdot E\big[\big(K_{j,U} - K_{j,L} - l_{j,t_k}\big)\cdot V\big]
\end{aligned}
$$

and the present value of the floating leg is

(6)
$$
B_{V,j} = \sum_{k=1}^{K} \beta(t_0,t_k)\cdot E\big[\big(l_{j,t_k} - l_{j,t_{k-1}}\big)\cdot V\big].
$$

Thus the break-even spread can be calculated as

(7)
$$
s_j = \frac{\displaystyle\sum_{k=1}^{K} \beta(t_0,t_k)\cdot\big(E\big[l_{j,t_k}\big] - E\big[l_{j,t_{k-1}}\big]\big)}{\displaystyle\sum_{k=1}^{K} \beta(t_0,t_k)\cdot\eta\cdot\big(K_{j,U} - K_{j,L} - E\big[l_{j,t_k}\big]\big)}
$$

Note that the attachment points are fixed and that finding the expected tranche losses $E\big[l_{j,t_k}\big]$ is crucial. Therefore, forecasts for the term structure of portfolio loss distributions are required.

Generally speaking, percentage portfolio losses for year $t+1$ are modelled as

$$(8) \qquad L_{t+1} = \frac{1}{V} \sum_{i=1}^{N_{t+1}} D_{it+1} \cdot lgd_{it+1} \cdot ead_i$$

where D_{it+1} is the random default indicator variable for firm i in year $t+1$ with

$$(9) \qquad D_{it+1} = \begin{cases} 1 & \text{firm } i \text{ defaults in period } t+1 \\ 0 & \text{otherwise} \end{cases},$$

lgd_{it+1} is the loss (rate) given default, and ead_i is the exposure at default. The present paper focuses on forecasting the process for the default indicator via alternative credit rating systems. For simplicity we fix all exposures to one monetary unit and losses given default to 50%.

3 Stylized Rating Methodologies

3.1 Process for the Credit Quality

For the default process of the firm and describing rating methodologies we use a stylised model similar to Heitfield (2005). Since we are only interested in rating dynamics, the model is simplified in a way that we focus on time specific risk factors and treat borrowers homogeneous in the cross-section. Thus, only macroeconomic factors are considered and up- or downgrades are only due to the credit cycle. Assume that in any time period $t+1$ there is a credit quality index (or a return on assets) R_{it+1} associated with firm i

$$(10) \qquad R_{it+1} = \alpha + \beta \cdot Z_t + w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{it+1}$$

$\left(0 \le w < 1\right)$ where Z_t denotes a time-lagged observable systematic factor, F_{t+1} denotes a contemporaneous unobservable systematic factor, and U_{it+1} denotes an unobservable idiosyncratic factor. α is a constant, β is the exposure to the observable factor, and w is the exposure to the unobservable common factor. All random variables on the right hand side are i.i.d. and normalized, i.e. have standard normal cumulative density functions. The firm goes into default when the credit quality falls below zero, i.e.

$$(11) \qquad D_{it+1} = 1 \quad \Leftrightarrow \quad R_{it+1} \le 0.$$

A point-in-time probability of default is defined as the one-year default probability conditional on the current observable state of the economy, which at time t is given by z_t, that is

$$\begin{aligned}
PD_{it+1}^{PIT} &= P\left(D_{it+1} = 1 \,\middle|\, Z_t = z_t\right) = P\left(R_{it+1} \leq 0 \,\middle|\, Z_t = z_t\right) \\
&= P\left(w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{it+1} \leq -\left(\alpha + \beta \cdot z_t\right)\right) \\
&= \Phi\left(-\left(\alpha + \beta \cdot z_t\right)\right)
\end{aligned}$$

(12)

where $\Phi(\cdot)$ is the standard normal cumulative density function. The point-in-time PD of borrower i varies through time by the state of the macro economy.

A through-the-cycle probability of default does not condition on the current state of the economy and is given by

$$\begin{aligned}
PD_{it+1}^{TTC} &= P\left(D_{it+1} = 1\right) = P\left(R_{it+1} \leq 0\right) \\
&= \Phi\left(-\frac{\alpha}{\sqrt{1+\beta^2}}\right) = \Phi(c)
\end{aligned}$$

(13)

where $c = -\alpha\big/\sqrt{1+\beta^2}$ which is constant.

In the PIT approach the contemporaneous correlation between the credit quality indices of borrower i and j is

$$\begin{aligned}
&\text{Corr}\left(R_{it+1}, R_{jt+1} \,\middle|\, Z_t = z_t\right) \\
&= \text{Corr}\left(w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{it+1}, w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{jt+1}\right) \\
&= w^2
\end{aligned}$$

(14)

whereas in the TTC approach it is

$$\text{Corr}\left(R_{it+1}, R_{jt+1}\right)$$

(15)
$$= \text{Corr}\left(\beta \cdot Z_t + w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{it+1}, \beta \cdot Z_t + w \cdot F_{t+1} + \sqrt{1-w^2} \cdot U_{jt+1}\right)$$

$$= \frac{\beta^2 + w^2}{1 + \beta^2} = \xi^2$$

which is larger than w^2 when $\beta \neq 0$, i.e., when the exposure to the observable systematic factor is not zero. Thus, in the PIT approach the default probabilities vary over time but the correlation between the latent variables given the state of the economy is smaller whereas in the TTC approach the default probability is constant and the correlation is higher.

3.2   Simulation Evidence

Consider now a bank which applies one of both rating methodologies and estimates the respective input parameters from historical default experiences. We show in this section that the parameter estimates obtained in either approach are indeed estimates for the respective implied correlation.

We carry out a simulation study which consists of several steps:

Step 1:   We assume that the stylized model (10) is valid and specify the parameters α, β, and w.

Step 2:   Given our model we draw a random time series of length T of defaults for a portfolio of N loans.

Step 3:   For this time series we estimate parameters of a model with time varying default probabilities (true PIT model) and parameter for a model with constant default probability (misspecified TTC model). The estimation is done via Maximum-Likelihood as described in Gordy/Heitfield (2000) and Roesch/Scheule (2005).

Step 4:   We run 1,000 iterations of Step 2 and Step 3.

As a result we obtain 1,000 estimates for all parameters under both model specifications. We exemplarily choose a constellation where $c = -\alpha\big/\sqrt{1+\beta^2} = \Phi^{-1}(0.01) = -2.33$, that is, the average default probability over time is 1% for each borrower, with $\alpha = -2.54$, $\beta^2 = 0.2$ and $w^2 = 0.2$. This translates into a credit quality correlation of 0.2 (w^2) for the PIT model and of

0.333 in the TTC model according to (15). The portfolio contains 10,000 borrowers and we consider T=100, T=20, and T=10 periods.

Table 2: Parameter Estimates for PIT and TTC models

Parameter	Parameter Value	PIT		Parameter	Parameter Value	TTC	
		Mean Estimate	Standard Deviation			Mean Estimate	Standard Deviation
T=100							
α	-2.54	-2.556	0.058				
β	0.447	0.450	0.048	c	-2.33	-2.331	0.081
w^2	0.2	0.195	0.025	ξ^2	0.333	0.330	0.0350
T=20							
α	-2.54	-2.578	0.125				
β	0.447	0.458	0.110	c	-2.33	-2.354	0.177
w^2	0.2	0.183	0.053	ξ^2	0.333	0.320	0.076
T=10							
α	-2.54	-2.607	0.168				
β	0.447	0.473	0.159	c	-2.33	-2.383	0.240
w^2	0.2	0.163	0.068	ξ^2	0.333	0.304	0.103

The table shows that the parameter estimates are affected by a bias in the PIT model when the sample size is small as shown in Gordy/Heitfield (2000). The standard deviation is increasing with decreasing sample size.

4   CDO Spreads and Rating Methodology

4.1   Proceeding

We now investigate the effects of different rating methodologies on CDO prices from the perspective of a bank. The bank can choose one of both rating systems to evaluate forecasts for losses in order to determine the prices for each CDO tranche. Future Losses are considered to be generated by model (10).

In the PIT approach the bank knows all future realizations of the macroeconomic factors until maturity and calculates the term structure of default probabilities (given the $z_{t+j-1}, j=1,\ldots,M$, where M is the maturity of the loans in the portfolio). Using this term structure and the implied correlation (14) loss distributions are calculated for the pricing of the CDO tranches. In the TTC approach the bank has the same information but uses the average of the future default probabilities until maturity and the implied correlation (15). We then compare the resulting implied spreads which the banks derive from either approach.

In the base case we use the following parameters:
- The portfolio consists of 10,000 homogenous loans;
- The maturity of the portfolio is 5 years;
- The parameters of the default process are the same as in the simulation study of section 3.2, i.e., $\alpha = -2.54$, $\beta^2 = 0.2$, $w^2 = 0.2$;
- The discount rate is 5% each year;
- Tranching is done with attachment points 0%, 3%, 7%, 10%, 15%, 30%.

Given these parameters we simulate future paths for the development of the systematic risk factor until maturity $z_{t+j-1}, j=1,\ldots,M$. Each path can be seen as a given economic scenario.

For each scenario we calculate the implied tranche spread for the PIT and the TTC approach. 1,000 paths are generated in total.

4.2   Results for the Base Case

We now investigate the effects of different rating methodologies on CDO evaluation from the perspective of a bank which wants to find theoretical prices for the CDO tranches.

Exhibit 1 shows the implied spreads for the equity tranche for 1,000 economic scenarios for the base case and the TTC ratings. Each scenario consists of a particular term structure of the

default probability from which the average probability is calculated. This average default probability is used each year for calculating the tranche spreads. It is not surprising that the calculated spread is a monotonic function of the default probabilities. The higher the default probability the higher is the spread for the tranche.

In the PIT approach we use the actual term structure of default probabilities for calculating the tranche spreads. Here, the default probabilities used to calculate the tranche spreads vary each year. Exhibit 2 shows that the implied spreads for the equity tranche tend to increase with the average default probability. However, since the same average default probability may be derived from two different term structures (e.g., increasing vs. decreasing one-year probabilities) the spreads may differ for similar average probabilities.

A comparison of both methodologies of calculating equity spreads is given in Exhibit 3. Here, for each economic scenario the difference between the PIT and the TTC spreads are given. Most of the differences are positive, indicating that spreads derived from PIT ratings – accounting for the actual term structures of default probabilities – are higher than in the TTC case. This is because of the additional correlation effect in the TTC model as demonstrated in section 3. The higher correlation shifts more mass into the higher losses, thereby lowering the expected losses in the equity tranche.

Exhibit 4 shows the spread difference for the other tranches. Here, the effect is reversed and the higher correlation in the TTC model leads to overpricing of the more senior tranches. Table 3 contains summary statistics of the spread differences. Only for the equity tranche the differences are positive on average, whereas the other tranches are higher evaluated in the TTC case.

The effects are twofold. If a bank sells all but the equity tranche, the overpricing in the TTC case renders the bank with higher spreads to pay than in the PIT case. On the other hand, it retains the equity tranche and underestimates its risk by attributing too little losses to the tranche. Note that both effects are complementary: Firstly, the spreads the bank has to pay for the senior tranche are too high, and, secondly, risk assessment of the retained tranche is too low.

Exhibit 1: Equity tranche spreads for various economic scenarios, TTC rating

1,000 scenarios are generated using simulated paths of the macroeconomic factor; term structures of default probabilities are derived from the economic scenarios; in the TTC case the average default probability of the scenario is used for CDO tranche evaluation

Equity_const PD
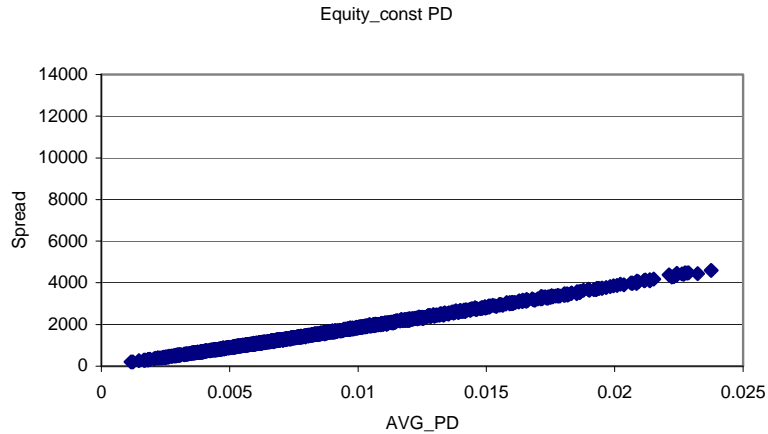
Exhibit 2: Equity tranche spreads for various economic scenarios, PIT rating

1,000 scenarios are generated using simulated paths of the macroeconomic factor; term structures of default probabilities are derived from the economic scenarios; in the PIT case the actual term structure of probabilities in each scenario is used for CDO tranche evaluation

Equity_vary PD
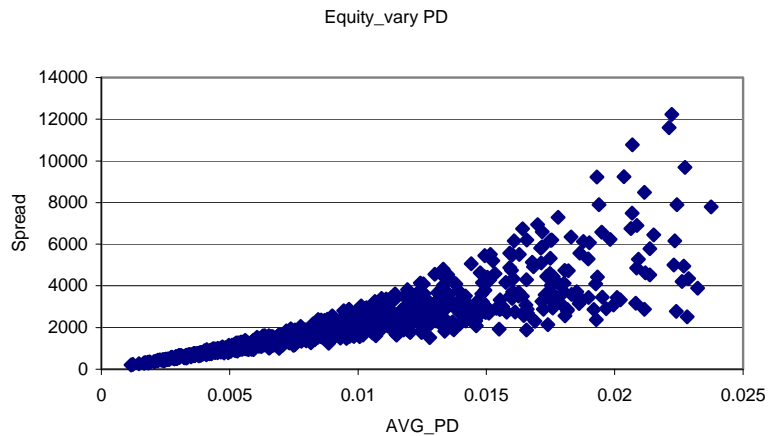
Exhibit 3: Equity spread differences for various economic scenarios

1,000 scenarios are generated using simulated paths of the macroeconomic factor; term structures of default probabilities are derived from the economic scenarios; figures shows the difference of the equity spreads from PIT and TTC rating

Equity
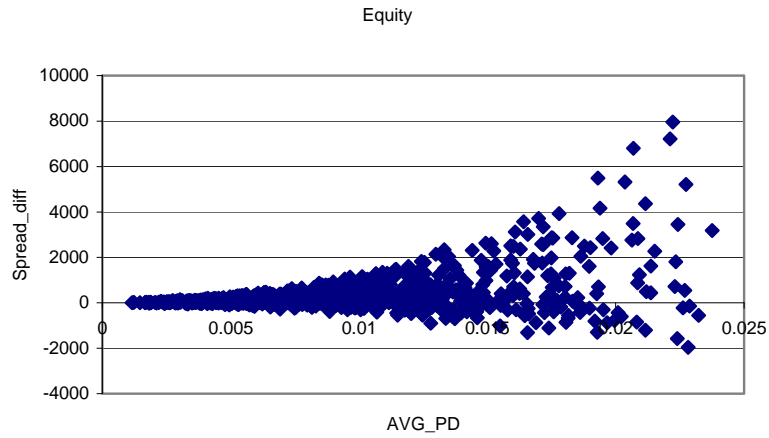
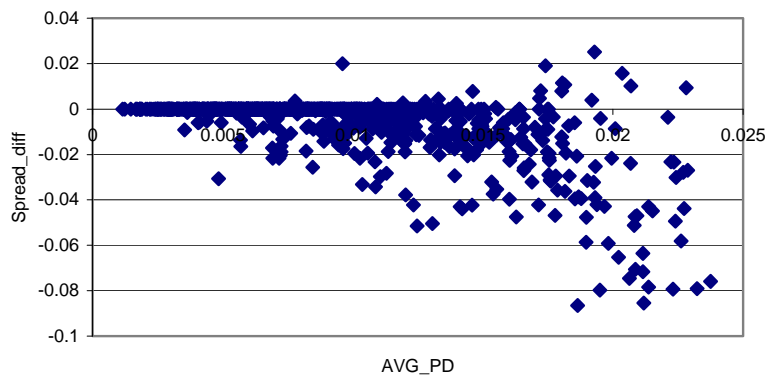**Exhibit 4: Spread differences for tranches 2 to 6 for various economic scenarios**
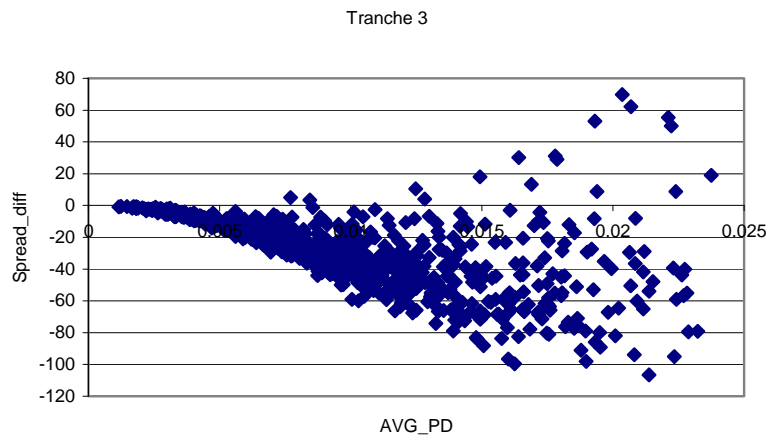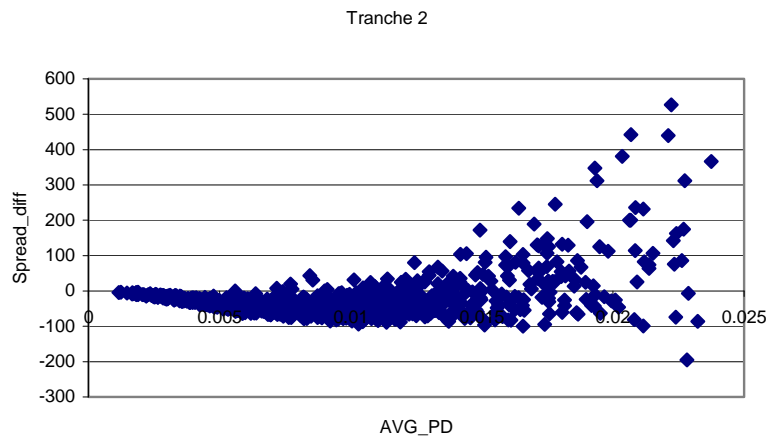
Tranche 2
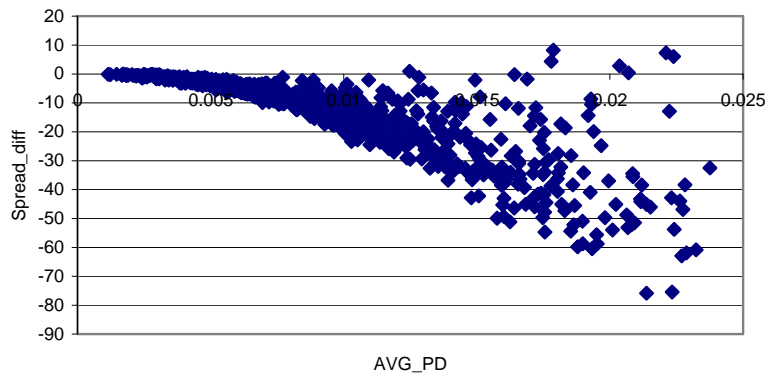
Tranche 3

Tranche 4

Tranche 5
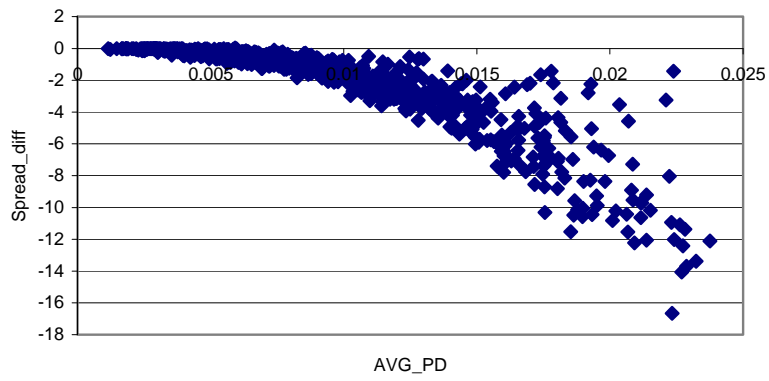
Senior

Table 3: Descriptive Statistics for Spread differences; Base Case

	Average Spread Difference	Median	Standard Deviation	Min	Max
Equity	445.15	161.40	1630.29	-7670.71	37301.81
Tranche 2	-14.54	-36.25	146.28	-940.02	3309.05
Tranche 3	-25.05	-22.19	34.80	-106.49	770.061
Tranche 4	-13.35	-8.65	16.18	-80.41	189.61
Tranche 5	-2.32	-0.93	3.86	-36.22	0.03
Senior	-0.01	0	0.09	-2.84	0.03

4.3 Sensitivity to Variations of the Base Case

An extension of the base case to different parameter settings is done in order to check the impact on implied spreads. We focus on the retained equity tranche for reasons of space restrictions. Results for other tranches are available upon request from the authors.

Exhibit 5 shows in the upper graph variations of equity tranche spread differences for various factor exposures. The solid line shows average differences for loadings to the random factor from $\omega^2 = 0$, 0.2 (base case) to 0.4. This exposure equals the base one-year asset correlation in the PIT rating. It can be seen that the differences between PIT and TTC decrease for higher correlations. In contrast, an increasing exposure to the observable factor (dashed line, 0, 0.2, 0.4) increases the differences. To the extent of a zero factor exposure PIT and TTC are congruent.

The graph in the middle shows variations of default probabilities and the discount rate. Obviously, the default probabilities change the loss distribution to a large amount and a higher PD results in a larger spread difference. Interestingly, the discount rate has only a minor impact.

Lastly, we vary the maturity of the portfolio and see that higher maturity will lead to larger differences and that the impact is already substantial for short maturity.
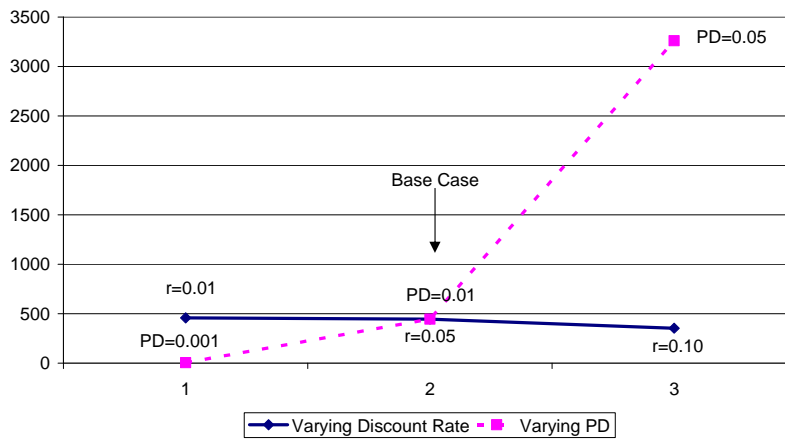
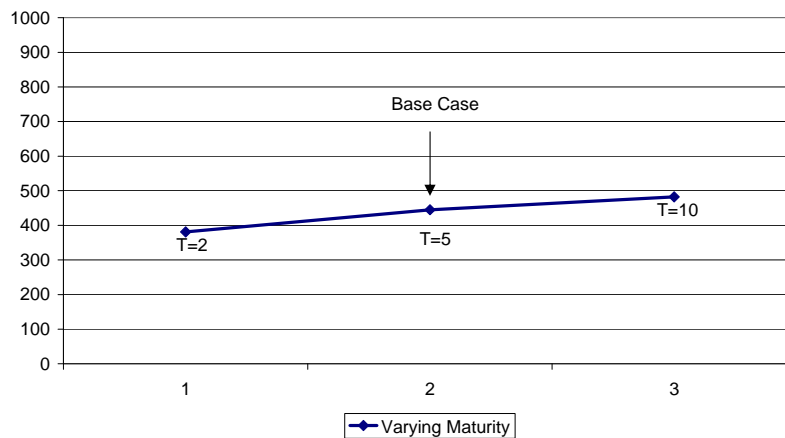**Exhibit 5: Equity tranche differences by varying parameters**

Equity Tranche Spread Differences by Varying Factor Exposure
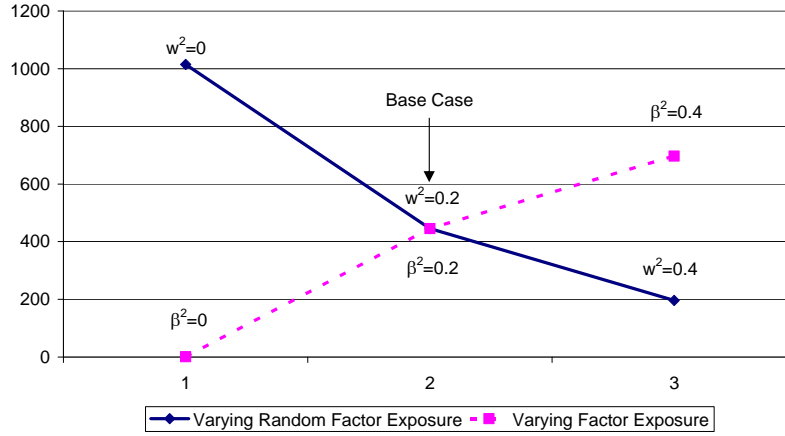
Equity Tranche Spread Differences by Varying PD and Discount Rate

Equity Tranche Spread Differences by Maturity

5 CDO and Forecasting Risk

5.1 Framework

In the preceding section we assumed for both rating systems that the future paths of the default probabilities can be accurately forecasted for both rating systems. In practice, econometric models like the one provided in section 3 can provide point forecasts only for a limited future horizon. In our example the time lag of explanatory variables is one year and for obtaining a forecast for the default probability of the next year we can plug in this year's known realization of the macroeconomic variables. Once we forecast probabilities for future periods beyond the next year, forecasts for the explanatory variables have to be established as well. This adds forecasting errors which are subject to the analysis of this section.

Consider again the process (10) for the credit quality index as a linear function of a lagged macroeconomic factor, a contemporaneous systematic factor, and an idiosyncratic factor. Let the macroeconomic variable for ease of exposition follow an AR(1) process

$$(16) \qquad Z_t = \alpha_1 \cdot Z_{t-1} + \sigma \cdot \varepsilon_t$$

where α_1 ($-1 < \alpha_1 < 1$) and $\sigma > 0$ are constants, and ε_t is an i.i.d. standard normally distributed random shock. Given the realization of year t, mean and variance of the subsequent year are given by

$$(17) \qquad E\big(Z_t\big|z_{t-1}\big) = \alpha_1 \cdot z_{t-1}$$

$$(18) \qquad \mathrm{Var}\big(Z_t\big|z_{t-1}\big) = \sigma^2 .$$

Unconditionally one obtains

$$(19) \qquad E\big(Z_t\big) = 0$$

$$(20) \qquad \mathrm{Var}\big(Z_t\big) = \frac{\sigma^2}{1 - \alpha_1^2} .$$

Consider we know the realization of the macroeconomic variable in year t and want to fore-cast the variable for the next year given the information of year t. Then we have

(21) $$Z_{t+1}|z_t = \alpha_1 \cdot z_t + \sigma \cdot \varepsilon_{t+1}$$

and

(22) $$E(Z_{t+1}|z_t) = \alpha_1 \cdot z_t$$

(23) $$\mathrm{Var}(Z_{t+1}|z_t) = \sigma^2.$$

For a two-year ahead forecast we obtain

(24) $$\begin{aligned} Z_{t+2}|z_t &= \alpha_1 \cdot Z_{t+1}|z_t + \sigma \cdot \varepsilon_{t+2} \\ &= \alpha_1 \cdot (\alpha_1 \cdot z_t + \sigma \cdot \varepsilon_{t+1}) + \sigma \cdot \varepsilon_{t+2} \\ &= \alpha_1^2 \cdot z_t + \alpha_1 \cdot \sigma \cdot \varepsilon_{t+1} + \sigma \cdot \varepsilon_{t+2} \end{aligned}$$

and

(25) $$E(Z_{t+2}|z_t) = \alpha_1^2 z_t$$

(26) $$\mathrm{Var}(Z_{t+2}|z_t) = (1 + \alpha_1^2) \cdot \sigma^2.$$

Generally one obtains for a k years ahead forecast given the value in year t

(27) $$Z_{t+k}|z_t = \alpha_1^k \cdot z_t + \sigma \cdot \sum_{j=0}^{k-1} \alpha_1^j \cdot \varepsilon_{t+k-j}$$

with expectation and variance

(28) $$E(Z_{t+k}|z_t) = \alpha_1^k \cdot z_t$$

$$(29) \qquad \mathrm{Var}\big(Z_{t+k}\big|z_t\big) = \sigma^2 \cdot \sum_{j=0}^{k-1} \alpha_1^{j^2} \ .$$

For forecasting the default probability we first specify the process for the credit quality index. Given the information about the realization of the macroeconomic variable in year t, the credit quality in year $t+1$ is

$$(30) \qquad R_{it+1}\big|z_t = \alpha + \beta \cdot z_t + w \cdot F_{t+2} + \sqrt{1-w^2} \cdot U_{it+2}$$

with mean and variance

$$(31) \qquad E\big(R_{it+1}\big|z_t\big) = \alpha + \beta \cdot z_t$$

$$(32) \qquad \mathrm{Var}\big(R_{it+1}\big|z_t\big) = 1 \ .$$

Therefore, the default probability and the correlation in year $t+1$ is

$$(33) \qquad P\big(R_{it+1} < 0\big|z_t\big) = \Phi\big(-\big(\alpha + \beta \cdot z_t\big)\big)$$

$$(34) \qquad \mathrm{Corr}\big(R_{it+1}, R_{jt+1}\big|z_t\big) = w^2 \ .$$

Note that the default probability is determined by the current state z_t of the economy. Dependence of credit qualities and default is exclusively determined by the parameter w.

Forecasting a default probability two years ahead becomes a bit more complex but is straightforward. We specify

$$(35) \qquad \begin{aligned} R_{it+2}\big|z_t &= \alpha + \beta \cdot Z_{t+1}\big|z_t + w \cdot F_{t+2} + \sqrt{1-w^2} \cdot U_{it+2} \\ &= \alpha + \beta \cdot \big(\alpha_0 + \alpha_1 \cdot z_t + \sigma \cdot \varepsilon_{t+1}\big) + w \cdot F_{t+2} + \sqrt{1-w^2} \cdot U_{it+2} \\ &= \tilde{\alpha} + \beta \cdot \sigma \cdot \varepsilon_{t+1} + w \cdot F_{t+2} + \sqrt{1-w^2} \cdot U_{it+2} \end{aligned}$$

where $\tilde{\alpha} = \alpha + \beta \cdot \big(\alpha_1 \cdot z_t\big)$. The expectation of the credit quality is

$$(36) \qquad E\!\left(R_{it+2}|z_t\right) = \tilde{\alpha}$$

and the variance is

$$(37) \qquad \mathrm{Var}\!\left(R_{it+2}|z_t\right) = \beta^2\sigma^2 + 1 .$$

The default probability then becomes

$$(38) \qquad P\!\left(R_{it+2} < 0|z_t\right) = \Phi\!\left(-\frac{\tilde{\alpha}}{\sqrt{\beta^2\sigma^2+1}}\right)$$

and the correlation between two credit qualities is

$$(39) \qquad \mathrm{Corr}\!\left(R_{it+2}, R_{jt+2}|z_t\right) = \frac{\beta^2\sigma^2 + w^2}{\beta^2\sigma^2 + 1} .$$

As can be seen from

$$(40) \qquad \frac{\partial\,\mathrm{Corr}\!\left(R_{it+2}, R_{jt+2}|z_t\right)}{\partial\,(\beta\sigma)^2} = \frac{\beta^2\sigma^2 + 1 - \left(\beta^2\sigma^2 + w^2\right)}{\left(\beta^2\sigma^2 + 1\right)^2} = \frac{1 - w^2}{\left(\beta^2\sigma^2 + 1\right)^2} \;>\; 0$$

the correlation is increasing in $(\beta\sigma)^2$. This means that the forecasting error for the process of the macroeconomic factor adds correlation, since it affects all credit qualities jointly. The higher the exposure to the macroeconomic factor and the higher the variance of its process, the more correlation is added.

For year $t+3$ we obtain

$$
\begin{aligned}
(41) \quad R_{it+3}|z_t &= \alpha + \beta\cdot Z_{t+2}|z_t + w\cdot F_{t+3} + \sqrt{1-w^2}\cdot U_{it+3} \\
&= \alpha + \beta\cdot\left(\alpha_0 + \alpha_1\cdot Z_{t+1}|z_t + \sigma\cdot\varepsilon_{t+2}\right) + w\cdot F_{t+3} + \sqrt{1-w^2}\cdot U_{it+3} \\
&= \alpha + \beta\cdot\left(\alpha_0 + \alpha_1\cdot\left(\alpha_0 + \alpha_1\cdot z_t + \sigma\cdot\varepsilon_{t+1}\right) + \sigma\cdot\varepsilon_{t+2}\right) + w\cdot F_{t+3} + \sqrt{1-w^2}\cdot U_{it+3} \\
&= \tilde{\alpha} + \beta\cdot\alpha_1\cdot\sigma\cdot\varepsilon_{t+1} + \beta\cdot\sigma\cdot\varepsilon_{t+2} + w\cdot F_{t+3} + \sqrt{1-w^2}\cdot U_{it+3}
\end{aligned}
$$

where $\tilde{\tilde{\alpha}} = \alpha + \beta \cdot \alpha_1^2 \cdot z_t$ with

$$(42) \qquad E\left(R_{it+3}\big|z_t\right) = \tilde{\tilde{\alpha}}$$

and variance

$$(43) \qquad \mathrm{Var}\left(R_{it+3}\big|z_t\right) = \beta^2 \sigma^2 \left(1 + \alpha_1^2\right) + 1 .$$

The default probability becomes

$$(44) \qquad P\left(R_{it+3} < 0\big|z_t\right) = \Phi\left(-\frac{\tilde{\tilde{\alpha}}}{\sqrt{\beta^2 \sigma^2 \left(1 + \alpha_1^2\right) + 1}}\right)$$

and the correlation now increases to

$$(45) \qquad \mathrm{Corr}\left(R_{it+3}, R_{jt+3}\big|z_t\right) = \frac{\beta^2 \sigma^2 \left(1 + \alpha_1^2\right) + w^2}{\beta^2 \sigma^2 \left(1 + \alpha_1^2\right) + 1} .$$

In general we obtain

$$(46) \qquad \begin{aligned} R_{it+k}\big|z_t &= \alpha + \beta \cdot Z_{t+k-1}\big|z_t + w \cdot F_{t+k} + \sqrt{1 - w^2} \cdot U_{it+k} \\ &= \alpha + \beta \cdot \left(\alpha_1^{k-1} \cdot z_t + \sigma \cdot \sum_{j=0}^{k-2} \alpha_1^j \cdot \varepsilon_{t+k-j}\right) + w \cdot F_{t+k} + \sqrt{1 - w^2} \cdot U_{it+k} \end{aligned}$$

with expectation and variance

$$(47) \qquad E\left(R_{it+k}\big|z_t\right) = \alpha + \beta \cdot \alpha_1^{k-1} \cdot z_t$$

$$(48) \qquad \mathrm{Var}\left(R_{it+k}\big|z_t\right) = \beta^2 \sigma^2 \cdot \sum_{j=0}^{k-2} \alpha_1^{j^2} + 1 .$$

The probability of default is

$$
(49) \qquad P\big(R_{it+k} < 0 | z_t\big) = \Phi\left(-\frac{\alpha + \beta \cdot \alpha_1^{k-1} \cdot z_t}{\sqrt{\beta^2 \sigma^2 \cdot \sum_{j=0}^{k-2} \alpha_1^{j^2} + 1}}\right)
$$

and the correlation is

$$
(50) \qquad \mathrm{Corr}\big(R_{it+k}, R_{jt+k} | z_t\big) = \frac{\beta^2 \sigma^2 \cdot \sum_{j=0}^{k-2} \alpha_1^{j^2} + w^2}{\beta^2 \sigma^2 \cdot \sum_{j=0}^{k-2} \alpha_1^{j^2} + 1}.
$$

In the TTC case the PD and the correlation in each year are given by

$$
(51) \qquad P\big(R_{it+k} < 0\big) = \Phi\left(-\frac{\alpha}{\sqrt{1 + \beta^2 \cdot \mathrm{Var}(Z_t)}}\right) = \Phi(c)
$$

$$
(52) \qquad \mathrm{Corr}\big(R_{it+k}, R_{jt+k}\big) = \frac{w^2 + \beta^2 \cdot \mathrm{Var}(Z_t)}{1 + \beta^2 \cdot \mathrm{Var}(Z_t)}.
$$

It can easily be seen, (50) converges to (52) as the time horizon grows to infinity. In other words, forecasting risk adds correlation if the maturity increases. The PIT and the TTC rating correlations match if the maturity converges to infinity.

5.2   Results

In our stylized framework the processes depend on a limited set of parameters. In this section we analyse the effects of parameter variations on CDO spreads and compare both rating approaches. It can be seen from (49) that the term structure of default probabilities in the PIT rating depends on the current state of the economy z_t while the correlation in (50) does not.

We therefore distinguish three states of the current economy, a medium condition with $z_t = 0$, low condition (recession) with $z_t = -1$ and high condition (boom) with $z_t = +1$. Note that the medium condition does not imply that the conditional default probability of PIT in (33) equals the unconditional default probability of TTC in (51). We thus interpret the terms "medium", "recession" and "boom" with respect to the macroeconomic variable instead of the probabilities. Other interpretations may exist.

Exhibit 6 and Exhibit 7 show the term structures of forecasted default probabilities and correlations for TTC default probabilities of 1% (roughly corresponding to a BB rating) and 5% (roughly corresponding to a B rating). We assume $\omega^2 = 0.2$ and a moderate size of the AR(1) process with $\alpha_1 = 0.2$. Exhibit 6 shows a medium impact of the macroeconomic variable with $\beta^2 = 0.2$. Exhibit 7 contains a high impact with $\beta^2 = 0.5$. Note that $\beta^2 = 0$ means that there is no impact at all and PIT and TTC are equivalent. We see that the value of β influences the discrepancy of PIT and TTC default probabilities in the first forecasting years. In addition, the difference between PIT and TTC is affected by the current state of the business cycle. In a recession the PIT probabilities are higher than in a boom or in the TTC case for both rating grades. The correlations are already close in the second year of the forecasting horizon.

In a next step, we analyse the effect of various parameter constellations on CDO spreads. For space reasons we focus only on the equity tranche. Details for other tranches are available upon request from the authors. Exhibit 8 shows the spreads for the PIT approaches, Exhibit 9 for the TTC approach. Exhibit 10 contains the spread differences between the PIT and the TTC rating. The underlying portfolio consists of 10,000 homogenous borrowers with a TTC default probability of 1% each and a maturity of 5 years. From Exhibit 8 and Exhibit 9 we see that the AR(1) parameter α_1, the impact β^2 of the business cycle, and the parameter ω^2 influence the equity spreads. In particular, higher β^2 and ω^2 lead to decreasing spreads due to higher implied correlations. Moreover, implied spreads are much higher in recessions than they are in booms. In booms, the current default probabilities are lower, as well as the forecasts for the next years. This leads to lower spreads than in the TTC approach. In recessions the situation is reversed. PIT default probabilities are more conservative and therefore the spreads increase.

Exhibit 6: Term Structures of Default Probabilities and Correlations

$\alpha_1 = 0.2$, $\omega^2 = 0.2$; left column: TTC default probability is 1%, right column: TTC default probability is 5%, Medium impact of macroeconomic variable with $\beta^2 = 0.2$
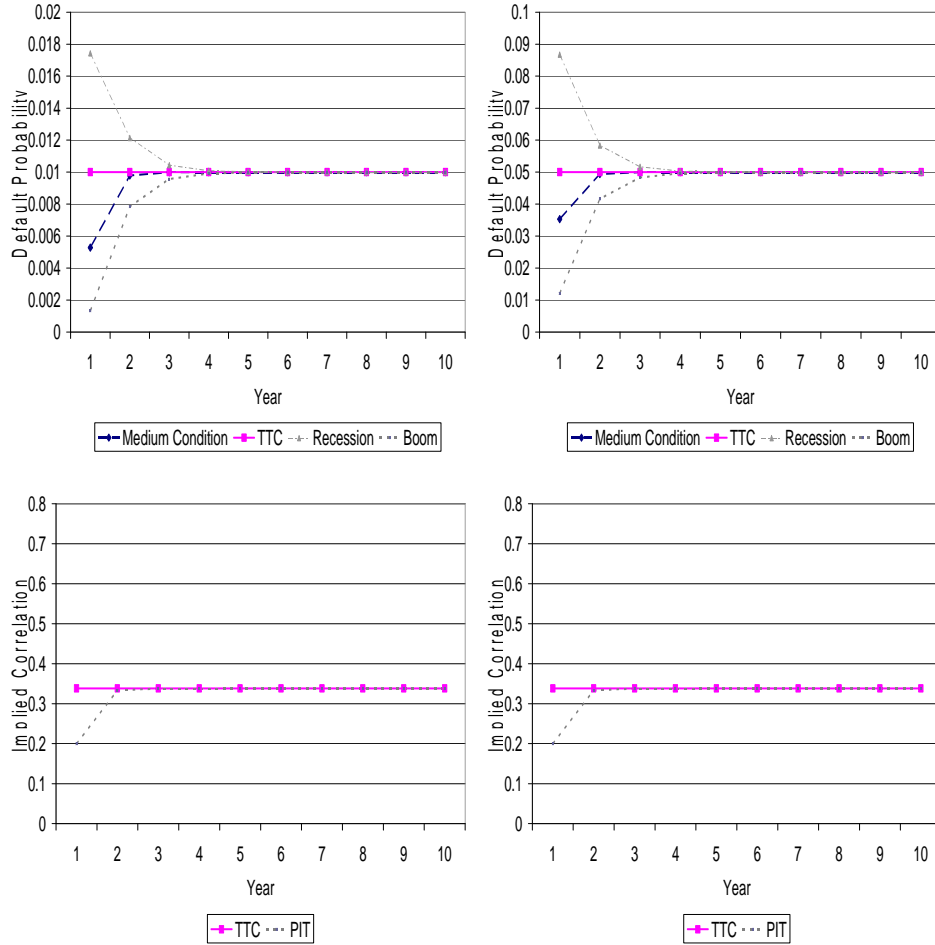
Exhibit 7: Term Structures of Default Probabilities and Correlations

$\alpha_1 = 0.2$, $\omega^2 = 0.2$; left column: TTC default probability is 1%, right column: TTC default probability is 5%, High impact of macroeconomic variable with $\beta^2 = 0.5$
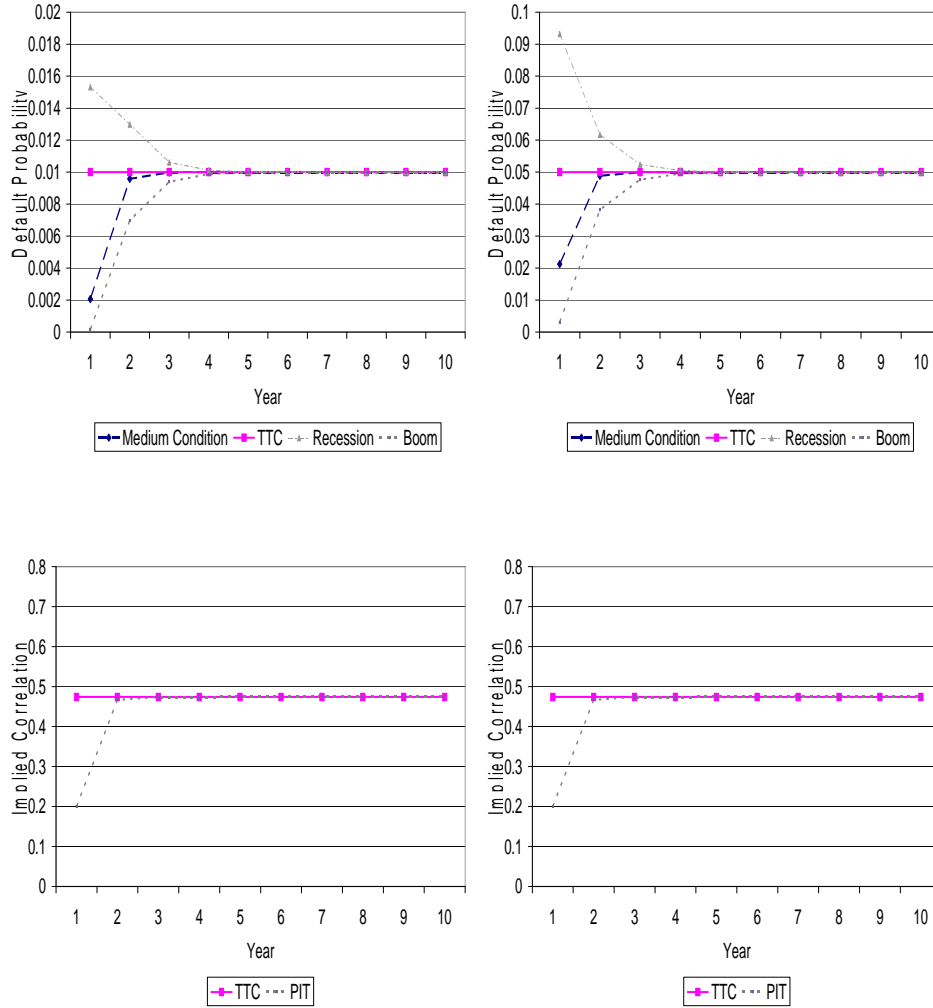
Exhibit 8: PIT CDO Equity Spreads for Various Economic Conditions

First row: boom; second row: medium condition; third row: recession; portfolio has 5 years maturity; left column: $\alpha_1 = 0$; right column: $\alpha_1 = 0.5$
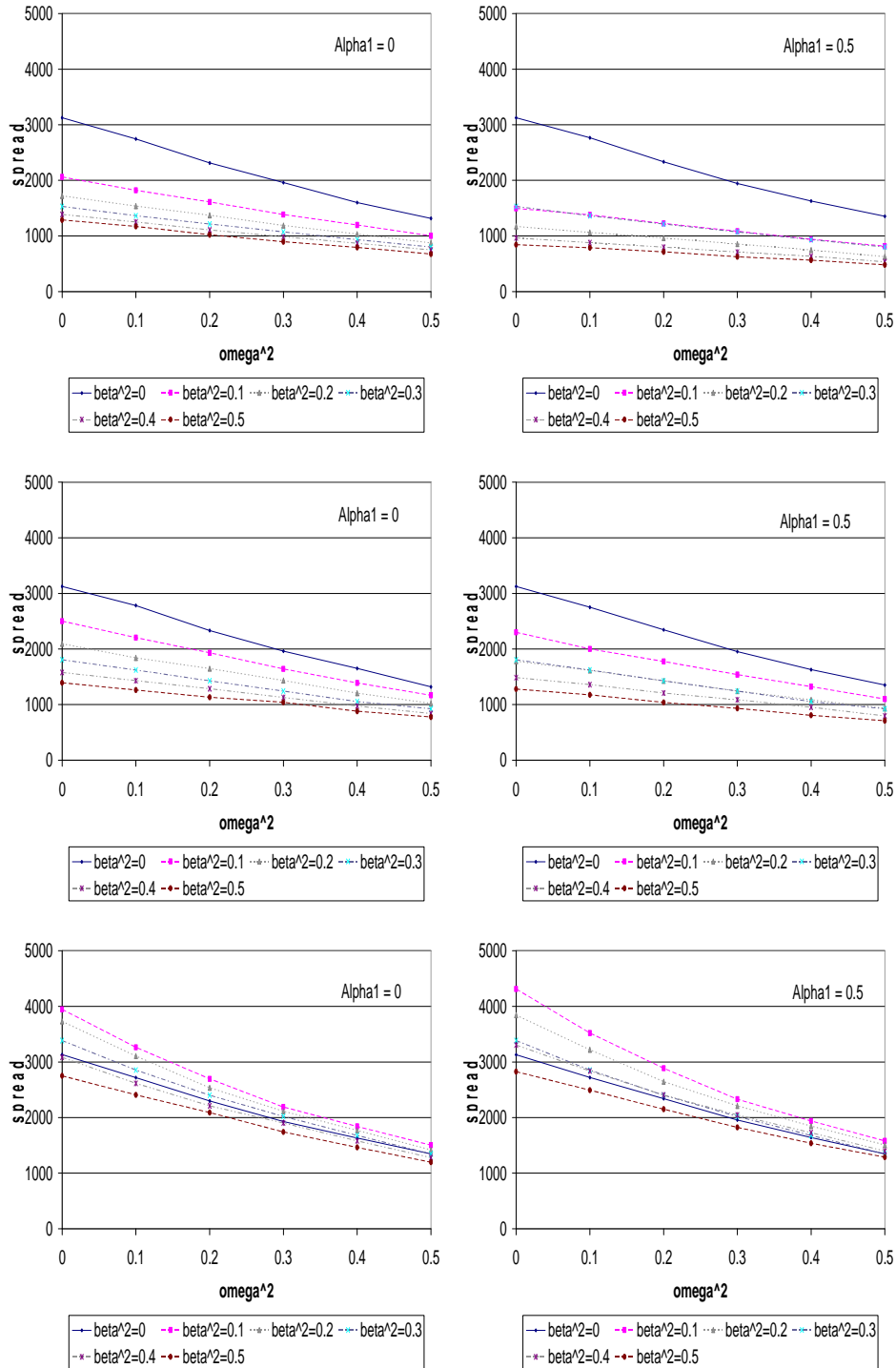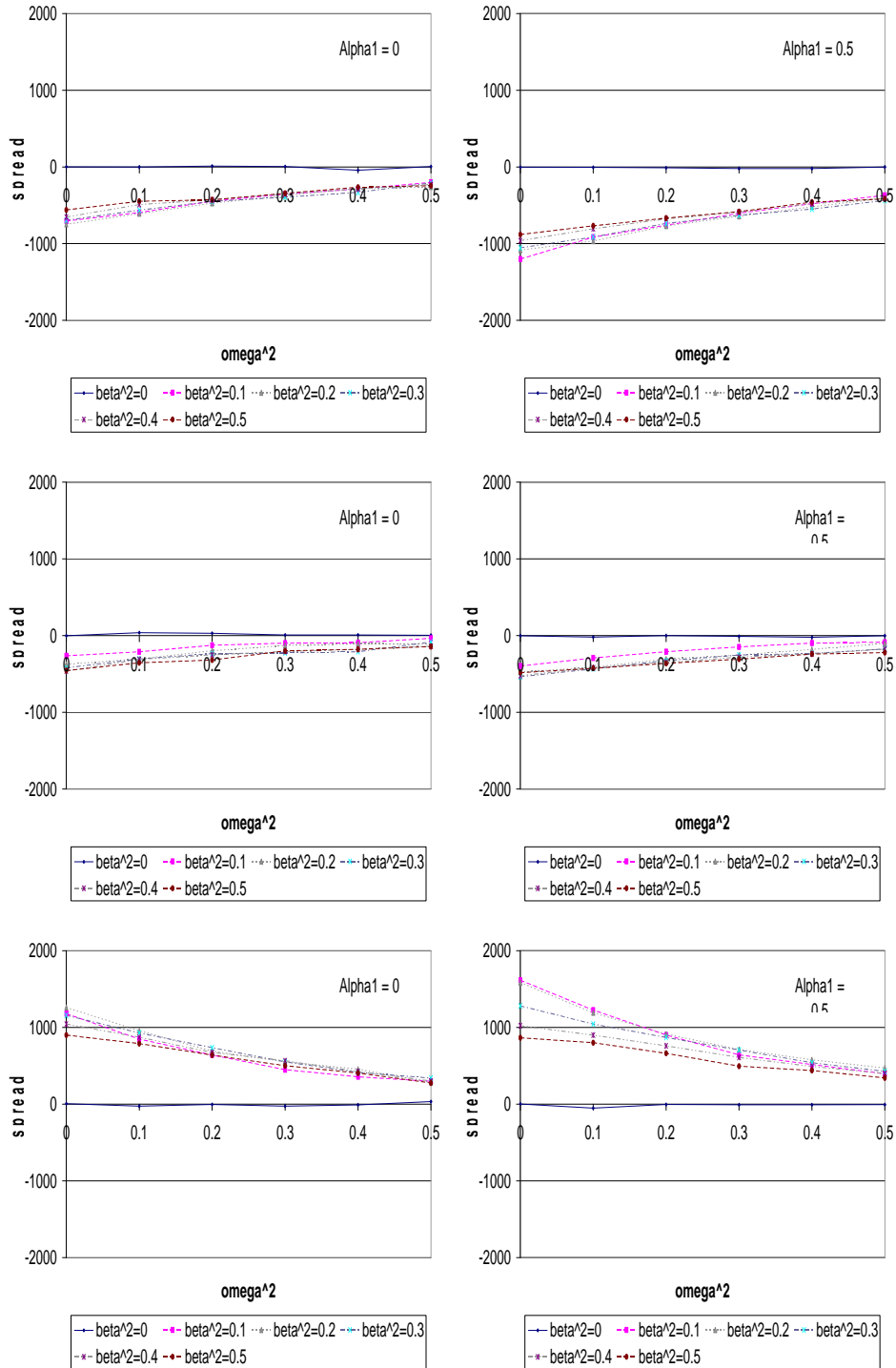
Exhibit 9: TTC CDO Equity Spreads for Various Economic Conditions

Portfolio has 5 years maturity
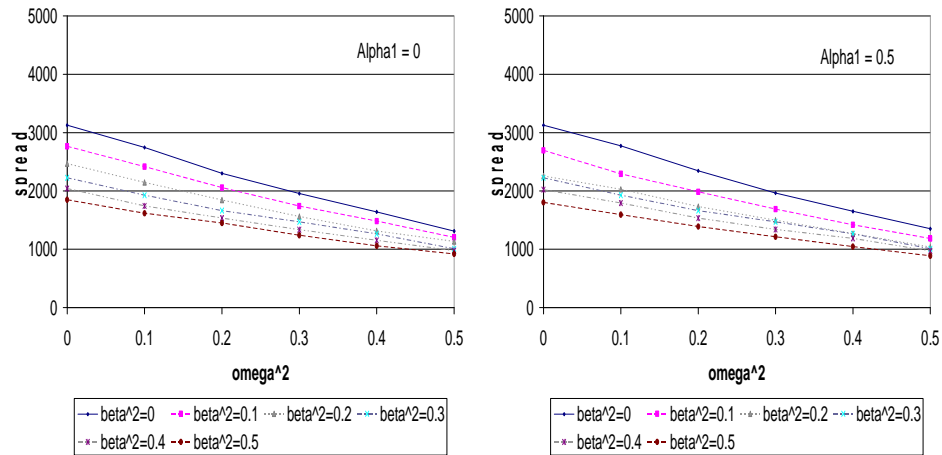
Finally, we analyse how the maturity of the portfolio, i.e. the length of the forecasting horizon, affects spread differences. The default probabilities and correlations of the PIT rating converge against the TTC parameters as the maturity increases due to forecasting risk. However, for short time series, there might be considerable differences. Exhibit 11 shows the spread differences of PIT and TTC for the three current economic conditions in dependence of ω^2 and for forecasting horizons of 2 years, 5 years (base case), and 10 years. For all maturities the differences decrease (in absolute terms) for increasing ω^2. In addition, they decrease with increasing maturity as expected. Note that even for longer maturity the difference between PIT and TTC models used for the evaluation of CDOs might still be substantial.

Exhibit 10: CDO Equity Spread Differences for Various Economic Conditions

First row: boom; second row: medium condition; third row: recession; portfolio has 5 years maturity; left column: $\alpha_1 = 0$; right column: $\alpha_1 = 0.5$

Exhibit 11: CDO Equity Spread Differences by Maturities for Various Economic Conditions

First row: boom; second row: medium condition; third row: recession; $\alpha_1 = 0.5$; $\beta^2 = 0.5$
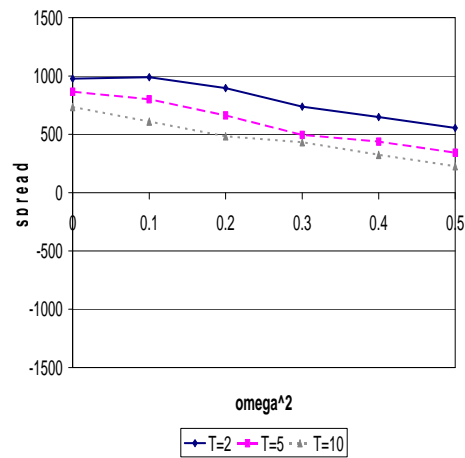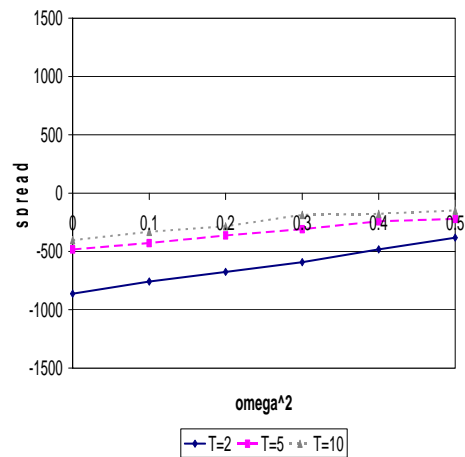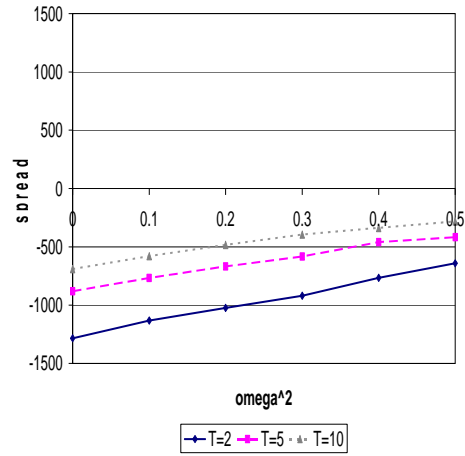
6   Conclusion

The present paper develops a framework for the evaluation of a credit portfolio and tranches thereof by comparing two competing methodologies - the point-in-time and the through-the-cycle approach. The through-the-cycle rating assumes that credit events in a given year occur jointly by random whereas the point-in-time rating tries to identify systematic risk drivers which are responsible for this co-movement and explain future credit losses by information which is known at the time the forecast is made. Applying these approaches, the central parameters default probability and correlation can be forecast for multiple years and related forecasting errors included.

Using the through-the-cycle approach as opposed to the point-in-time approach may increase the insolvency risk and therefore have severe implications for a financial institutions which transfer debt tranches but retain an equity part. Two complementary effects can be observed. Firstly, the credit risk retained may be underestimated resulting in an inadequate capital allocation. Secondly, the credit risk transferred may be overestimated resulting in additional risk-based transfer costs.

However, the maturity of a credit risk transfer acts as a mitigant to this model risk. The information which enters a point-in-time model decreases in importance over time and the forecast credit loss to a credit portfolio converges during the course of the business cycle for these two fundamental modelling methodologies.

For future work, we recommend to extend the present contribution with its evaluation framework in two directions. Firstly, the present study focused on the parameters default probability and asset correlation. Other studies (e.g., Carey, 1998, Frye, 2003, Altman et al., 2003) have shown that additional parameters such as exposures or losses given default also show cyclical behaviours and should therefore be incorporated into the evaluation framework. Secondly, the implication of the chosen CDO evaluation methodology remains to be proven empirically. However, information on the securitised loan portfolios and the internal methodologies are generally unknown. We would therefore like to encourage financial institutions and their practitioners to share their experience with the credit risk community.

References

Altman, E., Brady, B., Resti, A., Sironi, A., 2003. The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications, forthcoming Journal of Business.

Basel Committee on Banking Supervision, 2004: International Convergence of Capital Measurement and Capital Standards - A Revised Framework, Consultative Document, Bank for International Settlements, June.

Carey, M., 1998. Credit Risk in Private Debt Portfolios, Journal of Finance 53, 1363-1387.

Crouhy, M., Galai, D., Mark, R., 2001: Prototype Risk Rating System, Journal of Banking and Finance 25, 47-95.

Dietsch, M., Petey, J., 2004: Should SME Exposures be Treated as Retail or Corporate Exposures? A Comparative Analysis of Default Probabilities and Asset Correlations in French and German SMEs, Journal of Banking and Finance 28, 773-788.

Frye, J., 2003. A False Sense of security, Risk 16, 8, 63-67.

Gordy, M.B., 2003: A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules, Journal of Financial Intermediation 12, 199-232.

Gordy, M.B., 2000: A Comparative Anatomy of Credit Risk Models, Journal of Banking and Finance 24, 119-149.

Hull, J., White, A., 2004: Valuation of a CDO and nth to Default CDS Without Monte Carlo Simulation, Journal of Derivatives,12/2, 8-23.

Laurent, J.-P., Gregory, L. 2005: Basket Default Swaps, CDOs and Factor Copulas, Journal of Risk 7/4, 103-122.

Nickell, P., Perraudin, W., Varotto, S., 2000: Stability of Rating Transitions, Journal of Banking and Finance 24, 203-227.

Rösch, D., 2005. An Empirical Comparison of Default Risk Forecasts from Alternative Credit Rating Philosophies, International Journal of Forecasting 21, 37-51.

Rösch, D., Scheule, H., 2005: A Multifactor Approach for Systematic Default and Recovery Risk, Journal of Fixed Income 15/2, 63-75.

Shumway, T., 2001: Forecasting Bankruptcy More Accurately: A Simple Hazard-Rate Model, Journal of Business 74, 101-124.